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THE IMPACT OF SCHEDULED MAINTENANCE ON THE FAILURE PROCESS OF ELECTRIC RAIL VEHICLES

CH. N. STAVROPOULOS and **S.D. FASSOIS**

Stochastic Mechanical Systems (SMS) Group, Department of Mechanical and Aeronautical Engineering, University of Patras, GR 265 00 Patras, Greece, {xstavrop, fassois}@mech.upatras.gr

Abstract. This paper addresses the stochastic modeling and impact assessment of scheduled maintenance actions on the reliability of electric rail vehicles, the latter expressed in terms of recorded Times Between Failures (TBFs). The study is based upon historical time series data from the Athens Electric Railways and intervention analysis within a novel non-stationary Functional Series modeling framework, which allows for the modeling, scheduled maintenance impact assessment, analysis, as well as failure time prediction. The results of the study indicate that intervention models incorporating scheduled maintenance effects are significantly better than their unaccounting counterparts. Furthermore, the statistical significance of the maintenance effects is demonstrated, and reliability prediction is shown to be feasible.

Key Words. Maintenance effects, stochastic reliability, failure process analysis, intervention analysis, non-stationary time series.

1 INTRODUCTION

In repairable systems the failure process, which may be expressed in terms of the series of Times Between Failures (TBFs), is expected to be affected by major repairs and maintenance actions and policies. The stochastic modeling and assessment of these relationships may be an important tool for evaluating maintenance policies and designing future actions.

In the context of stochastic point processes [12], a unit is mainly supposed to be in either a GAN (Good As New) or BAO (Bad As Old) post-maintenance state, while maintenance policies depend heavily on the underlying assumptions of the stochastic process modeling the pattern of system failures [1, 9, 5].

Time series modeling has been shown to provide a proper framework for effectively describing the *stochastic dependencies* present in reliability (such as TBF) series [13, 11], and may be thus used for reliability modeling, maintenance impact assessment through the introduction of proper explanatory variables, underlying data structure analysis, as well as prediction.

Within such a time series context, explanatory variables have been considered in studies such as Singpurwalla [10], which investigates the interrelationships between a series of operating times and a series of maintenance (down) times for a complex system using transfer function and cross spectral analysis, Khoshgoftaar *et al.* [6], which investigates the incorporation of explanatory variables (program complexity measures) within a software quality model using ARIMAX (Integrated AutoRegressive Moving Average with eXogenous inputs) models [3], and Okogbaa and Peng [8], which uses a time series intervention analysis methodology for preventive/predictive maintenance management of multiunit systems.

The *aim* of the present study is the stochastic modeling and impact assessment of the scheduled maintenance actions on system reliability. The study is based upon the paradigm of electric rail vehicle reliability using retrospective series of times (km traveled) between failures (TBFs) for vehicles of the Athens Electric Railways, along with the corresponding scheduled maintenance records. The approach used is based upon a novel non-stationary Functional Series (FS) time series framework [11] within which the scheduled maintenance actions are treated as deterministic *interventions* [4].

The study constitutes an extension of the work reported in Stavropoulos and Fassois [11] on non-stationary Functional Series type reliability modeling, focusing on the following specific objectives: (a) Stochastic modeling and impact assessment of the scheduled maintenance actions on system reliability; (b) critical comparison with stochastic reliability modeling via representations that do not account for maintenance actions; and, (c) model-based TBF series analysis and prediction.

2 THE FUNCTIONAL SERIES MODELING FRAMEWORK

The Functional Series framework postulates modeling via non-stationary Functional Series (FS) Time-dependent AutoRegressive Moving Average (TARMA) models, which are presently augmented with an exogenous excitation part (the corresponding models are refereed to as TARMAX) in order to account for the presence of the deterministic intervention representing maintenance actions. TARMAX models are of the conventional ARMAX form, but with parameters being explicit functions of time, that is,

$$A(\mathcal{B},t)\cdot Y_t = B(\mathcal{B},t)\cdot X_{t-n_d} + C(\mathcal{B},t)\cdot W_t \quad (1)$$

with $t \geq t_o$ indicating discrete time (presently failure number), t_o the starting time, Y_t the non-stationary reliability (TBF) series modeled, X_t the exogenous excitation (intervention variable), and W_t an innovations (uncorrelated) sequence with zero mean and possibly varying variance. n_d represents the time delay between the intervention variable and Y_t, and $A(\mathcal{B},t)$, $B(\mathcal{B},t)$ and $C(\mathcal{B},t)$ the time-dependent AutoRegressive (AR), eXogenous (X), and Moving Average (MA) polynomials of orders n_a, n_b, and n_c, respectively,

$$A(\mathcal{B},t) \triangleq 1 + a_1(t)\cdot\mathcal{B} + ... + a_{n_a}(t)\cdot\mathcal{B}^{n_a} \quad (2)$$

$$B(\mathcal{B},t) \triangleq b_0(t) + b_1(t)\cdot\mathcal{B} + ... + b_{n_b}(t)\cdot\mathcal{B}^{n_b} \quad (3)$$

$$C(\mathcal{B},t) \triangleq 1 + c_1(t)\cdot\mathcal{B} + ... + c_{n_c}(t)\cdot\mathcal{B}^{n_c} \quad (4)$$

with $\mathcal{B}$ indicating the backshift operator ($\mathcal{B}\cdot Y_t \triangleq Y_{t-1}$), and $a_{n_a}(t)\neq 0$, $b_{n_b}(t)\neq 0$, $c_{n_c}(t)\neq 0$, for some $t\geq t_o$.

In Functional Series TARMAX models the model parameters belong to a p-dimensional subspace spanned by the basis functions $\{G_1(t), G_2(t), \ldots G_p(t)\}$,

$$\begin{aligned} a_i(t) &\triangleq \textstyle\sum_{j=1}^{p} a_{ij}\cdot G_j(t) && (1\leq i\leq n_a)\\ b_i(t) &\triangleq \textstyle\sum_{j=1}^{p} b_{ij}\cdot G_j(t) && (0\leq i\leq n_b)\\ c_i(t) &\triangleq \textstyle\sum_{j=1}^{p} c_{ij}\cdot G_j(t) && (1\leq i\leq n_c)\end{aligned}$$

with the corresponding model being referred to as TARMAX$(n_a,n_b,n_c)_p$.

The identification of the intervention (TARMAX) model (1) is based upon the modeling framework of Ben Mrad *et al.* [2], which accounts for parameter estimation as well as functional basis dimensionality, basis function, and model order selection. During identification each reliability and intervention series is divided into two disjoint parts: The first is used for model estimation (*estimation set*), whereas the latter is reserved for model validation (*validation set*).

Parameter estimation is achieved via the Polynomial-Algebraic (P-A) method, followed by Prediction Error (PE) refinement [2]. Model selection is based upon minimization of the Residual (prediction error) Sum of Squares (RSS) and the Akaike Information Criterion (AIC) [7, p. 419]. The final acceptance of the selected model depends upon successful validation, which

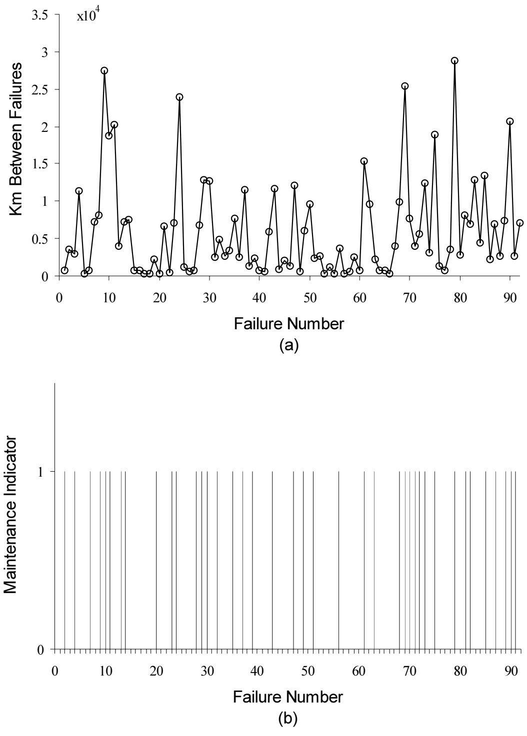

Figure 1: (a) TBF series A; (b) scheduled maintenance intervention series.

is based upon the a-posteriori examination of the zero-mean, stationarity, and uncorrelatedness hypotheses for the model residuals, as well as upon examination of the model's predictive performance within the validation set.

3 RAIL VEHICLE RELIABILITY MODELING

Two retrospective series (series A and B) of Times (km traveled) Between Failures (TBFs) for vehicles of the Athens Electric Railways, along with their corresponding intervention series formed from scheduled maintenance actions, are examined. Each intervention series (X_t) is formed using an indicator variable which takes the value of 0 or 1 depending upon whether a scheduled maintenance action did not or did, respectively, precede the $t-th$ failure.

3.1 TBF Series A Modeling

The electrical failures of a rail vehicle, incurred between January 1995 and December 1999, are included in the 92-sample-long series A (Figure 1a). The corresponding intervention

Table 1: AIC and RSS/SSS in the estimation and validation sets for selected models (series A; minimal values in boldface).

Model	AIC $\times 10^{-3}$	RSS/SSS Estim. (%)	RSS/SSS Valid. (%)
TARX(1,0)$_3$ cheb	**1.365**	48.79	**64.63**
TARX(0,1)$_3$ cos	**1.365**	**48.54**	67.80
TARX(1,0)$_3$ sin	1.367	49.93	75.66
TAR(2)$_3$ cheb	1.410	84.40	91.75
TARMA(1,1)$_3$ cos	1.411	85.29	90.76
TARMA(1,1)$_3$ sin	1.407	81.29	85.10

(maintenance) series is shown in Figure 1b. The first 80 samples constitute the estimation set, while the latter 12 samples the validation set.

A preliminary analysis analogous to that in Stavropoulos and Fassois [11] confirms the series non-stationarity. Subsequently, TAR/TARMA models of various orders, functional subspaces consisting of Chebyshev II polynomials or sine/cosine functions, and basis dimensionalities of $p = 2$ or 3, are fitted to the constant-mean-corrected TBF series A. The number of estimated parameters is maintained less than 8, in order to ensure a Samples Per Parameter (SPP) value greater than 10.

Candidate models are examined in terms of their achieved RSS/SSS (Residual Sum of Squares normalized by the mean-corrected Series Sum of Squares) and AIC values within the estimation set. The best candidate TAR/TARMA models are, for different fumctional bases, shown in Table 1 (lower part).

In order to assess the effects of scheduled maintenance on the failure process, TARX$(n_a, n_b)_p$ intervention models with various functional subspaces are fitted to the TBF – scheduled maintenance data. The time delay n_d is set to zero to allow for possibly immediate effects.

The functional subspaces of the AR part of the examined intervention models are identical to those used in their TARMA counterparts, while different subspaces are used in the intervention (exogenous) part.

Candidate intervention models are examined in terms of their achieved RSS/SSS and AIC values

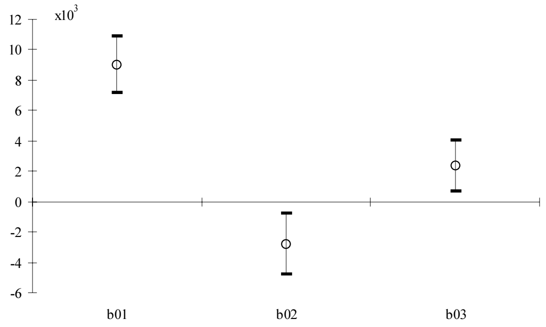

Figure 2: TBF series A: Exogenous (intervention) coefficients of projection for the TARX$(1,0)_3$ model (95% confidence intervals).

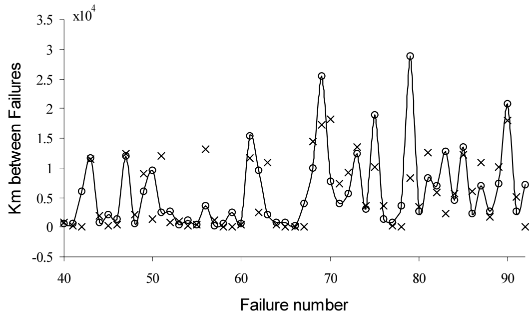

Figure 3: Actual TBF series A (—o—) and the Chebyshev TARX$(1,0)_3$ model-based 1-step-ahead predictions (—×—).

within the estimation set. Certain candidate TARX models are shown in Table 1 (upper part), from which a TARX$(1,0)_3$ model characterized by Chebyshev II functional subspaces is selected as best. The AR functional basis of this model consists of the 0th, 17th, and 36th Chebyshev II polynomials, whereas the X functional basis consists of the 0th, 45th, and 69th polynomials. The model is successfully validated by confirming residual stationarity and uncorrelatedness.

From the results of Table 1 it is obvious that the model prediction performance is substantially improved when the scheduled maintenance is taken into account. This is confirmed by all criteria, since the Chebyshev TARX$(1,0)_3$ model attains minimum AIC and RSS/SSS values in both the estimation and validation sets (it should be noticed that the achieved RSS/SSS values within the estimation set are almost half of those attained by models not accounting for scheduled maintenance).

The impact of maintenance actions on the failure process is formally confirmed by examining the statistical significance of the coefficients of projection of the exogenous polynomial of the model, which are all different from zero at the $\alpha = 0.05$ level (Figure 2). It is also worth mentioning that attempts to increase the exogenous order did not improve the modeling effectiveness when judged from the AIC point of view.

The Chebyshev TARX$(1,0)_3$ model based 1-step-ahead predictions are, along with the actual TBF values, presented in Figure 3 for the latter half of the data record (estimation and validation sets). The predictions tend to follow the actual TBF series, although the larger deviations are generally difficult to predict.

3.2 TBF Series B Modeling

The electrical failures of a different rail vehicle, also incurred between January 1995 and December 1999, are included in the 97-sample-long series B (Figure 4a). The corresponding intervention (maintenance) series is shown in Figure 4b. The first 80 samples constitute the estimation set, while the latter 17 samples the validation set.

A preliminary analysis, analogous to that in Stavropoulos and Fassois [11], confirms the series non-stationarity, and the TAR/TARMA modeling procedure leads to the best candidate models indicated in Table 2.

In a manner analogous to that of case A, TARX intervention models are also fitted to the TBF – scheduled maintenance data. The AR functional subspaces are identical to those used in their TARMA counterparts, while different subspaces are used in the intervention (exogenous) part.

Candidate TARX models are shown in Table 2, from which a sine TARX$(2,1)_3$ model [basis functions of the AR part are $G_1(t) = 1$, $G_2(t) = sin(\frac{22\pi t}{80})$ and $G_3(t) = sin(\frac{65\pi t}{80})$; basis functions of the exogenous part are $G_1(t) = 1$, $G_2(t) = sin(\frac{2\pi t}{80})$ and $G_3(t) = sin(\frac{29\pi t}{80})$] is selected as best. The model is succesfully validated by confirming residual stationarity and uncorrelatedness.

As in case A, the inclusion of the scheduled maintenance leads to substantially better predictive performance from both the AIC and RSS/SSS

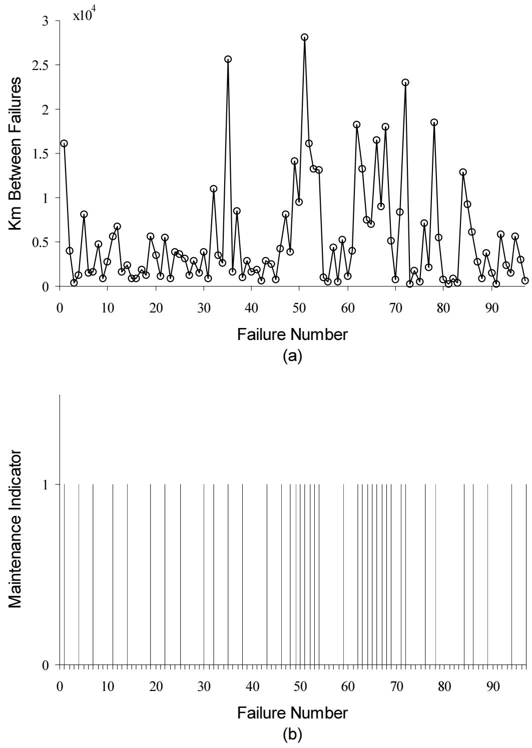

Figure 4: (a) TBF series B; (b) scheduled maintenance intervention series.

viewpoints (again, the achieved RSS/SSS values within the estimation set are almost half of those attained by models not accounting for scheduled maintenance).

The impact of maintenance actions on the failure process is formally confirmed by examining the statistical significance of the coefficients of projection of the exogenous polynomial of the model, several of which are different from zero at the $\alpha = 0.05$ level (Figure 5). In contrast to case A, the first order exogenous polynomial indicates a prolonged effect of scheduled maintenance on the failure process.

The sine TARX$(2,1)_3$ model based 1-step-ahead predictions are, along with the actual TBF values, presented in Figure 6 for the latter half of the data record (estimation and validation sets).

4 CONCLUDING REMARKS

(a) The non-stationarity and serial correlation structure present in the TBF series of two Athens Electric Railways vehicles has been effectively modeled via non-stationary Functional Series TARMA modeling.

Table 2: AIC and RSS/SSS in the estimation and validation sets for selected models (series B; minimal values in boldface).

Model	*AIC* $\times 10^{-3}$	RSS/SSS Estim. (%)	RSS/SSS Valid. (%)
TARX$(2,1)_3$ sin	**1.344**	**37.55**	64.71
TARX$(3,0)_3$ cheb	1.355	43.77	65.21
TAR$(2)_3$ sin	1.390	77.14	66.86
TARMA$(1,1)_3$ sin	1.390	76.65	70.92
TAR$(2)_3$ cheb	1.397	84.72	**63.13**
TAR$(2)_3$ cos	1.393	79.88	72.44

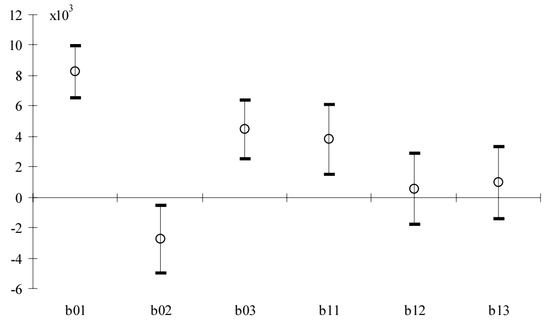

Figure 5: TBF series B: Exogenous (intervention) coefficients of projection for the TARX$(2,1)_3$ model (95% confidence intervals).

(b) The Functional Series intervention (TARX) models proved to be effective in modeling the TBF series and their corresponding scheduled maintenance. The intervention (exogenous) polynomial was of order $n_b = 0$ or 1, and of a functional subspace different from that of the AR part. The latter characteristic may indicate the decomposition of the failure process into two "parallel" subprocesses, each one with each own "internal structure": a scheduled maintenance subprocess and a degradation subprocess. In addition, the confirmation of a zero time delay ($n_d = 0$) between the intervention and the TBF series indicates the "immediate" impact of the former on the latter.

(c) The Functional Series intervention (TARX) models accounting for scheduled maintenance were shown to be superior to their unaccounting (TAR/TARMA) counterparts

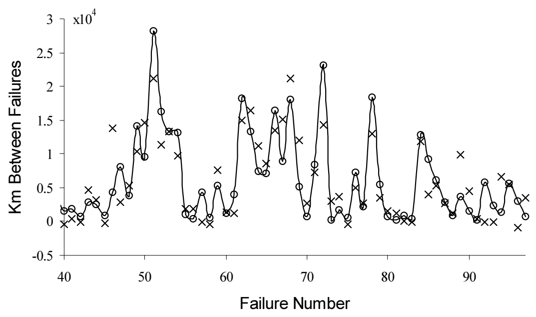

Figure 6: Actual TBF series A (—o—) and the sine TARX(2,1)$_3$ model-based 1-step-ahead predictions (—×—).

from both the RSS/SSS and AIC viewpoints; an indirect, though clear, indication of the scheduled maintenance significance. Especially worthwhile is the fact that their achieved RSS are (within the estimation set) of values about half of those attained by the latter models.

(d) The impact of the scheduled maintenance was formally confirmed by assessing the statistical significance of the intervention (exogenous) coefficients of projection at the $\alpha = 0.05$ level.

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