Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

ROBUST ANALYSIS OF CONTROLLERS FOR LINEAR SYSTEMS WITH TIME DELAY *

ROMAN PROKOP†, PETR HUSTÁK†, ZDENKA PROKOPOVÁ† PETR DOSTÁL†

†Department of Automatic Control, Faculty of Technology Zlín, Technical University Brno, nám. TGM 275, 762 72 Zlín, Czech Republic, prokop@zlin.vutbr.cz

Abstract. The contribution deals with the robust control design for linear systems with time delays. The proposed control synthesis is based on general solutions of Diophantine equations in the ring of Hurwitz stable and proper rational functions R_{ps}. The methodology is suitable for stable and unstable transfer functions and covers both tracking and disturbance rejection problems. Several approximations of the time delay term is investigated and compared for stable and unstable processes. Perturbations and robustness are studied through the infinity (H_∞) norm and by open loop Nyquist plots. Resulting control laws for first order systems are of a general PID type and a scalar tuning parameter influencing robustness and properties of the closed loop system was introduced. Simulations were performed in Matlab and Simulink environment.

Key Words. Time-delay systems, linear approximations, robustness, PID controllers, Diophantine equations.

1.INTRODUCTION

Linear controllers of the PID type are still widely used in many industrial applications. However, a good part of technological plants is nonlinear and exhibits a short or long time delay as an inherent feature of the reality. The dynamics of such processes can be adequately approximated by a linear transfer function plus a time delay term. The dynamics of many technological plants can be adequately approximated by first order transfer functions plus dead-time.

There are several available design and tuning methods for stable systems without or with time delay, see e.g. [1],[3]. The dead-time can be treated in various ways. One of them is using the well-known Smith predictor improved by Watanabe and Ito [12] and Aström et al. [2]. However, Smith predictor type controllers suffer from several drawbacks, e.g. lack of robustness and difficulties in applying for unstable systems. Another way tries to approximate the time-delay term $e^{-\tau s}$ by Padé or Taylor series and to design a linear but robust controller. Unstable time delay systems are solved in [10].

In this contribution, a robust technique is proposed for stable first order time-delay systems. The control design is performed in the ring of proper and Hurwitz stable rational functions where the H_∞ norm serves as a tool for perturbation evaluation. Moreover, a scalar parameter $m > 0$ is defined for control and robust tuning. The theoretical background of algebraic notions can be found in [4], [5], [6], [11]. Some applications and utilization for PID settings are introduced in [8], [9].

2.SYSTEM DESCRIPTION AND CONTROL DESIGN

Let $R_m(s)$ denote a ring of Hurwitz stable and proper rational functions having no poles in the region **Re** $s > -m$; $m \geq 0$. A transfer function of a linear

*) This work was supported by the Europoly project INCO Copernicus No. CP 977 010 and by the Grant Agency of Czech Republic under-grant No. 102/00/0526.

continuous time traditionally modeled by a ratio of two polynomials system is then expressed a ratio of two elements of $R_m(s)$:

$$H(s)=\frac{B(s)}{A(s)};\quad A(s)=\frac{a(s)}{m(s)};\quad B(s)=\frac{b(s)}{m(s)} \tag{1}$$

where *a, b* and *m* are polynomials in *s* with the condition deg *m* = max {deg *a*; deg *b*}. As a simple example, the second order system is expressed

$$H(s)=\frac{b_1 s+b_0}{a_2 s^2+a_1 s+a_0}= \\ =\frac{\dfrac{b_1 s+b_0}{(s+m)^2}}{\dfrac{a_2 s^2+a_1 s+a_0}{(s+m)^2}}=\frac{B(s)}{A(s)} \tag{2}$$

Suppose a two degree-of-freedom control system (FBFW structure) depicted in Fig.1. Note that the traditional one degree-of-freedom system (FB structure) is obtained simply by $R=Q$.

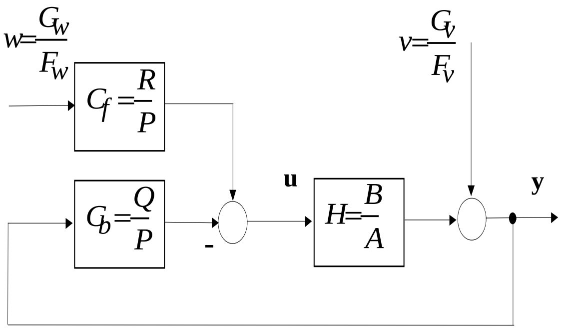

Fig.1. Feedback feedforward closed loop system.

The first step of the control design is to stabilize the system by a proper feedback loop. It can be formulated in an elegant way in R_{ps} by the Diophantine equations:

$$AP+BQ=1 \tag{3}$$

Details and proofs can be found e.g. in [6], [9], [11]. Let $w=G_w/F_w$ be a reference then a zero steady state of $(w\text{-}y)$ can be obtained only if F_w divides the product *AP*. Moreover, for a two degree-of-freedom controller the term *R* is given by any solutions of the second Diophantine equations:

$$F_w S+BR=1 \tag{4}$$

All feedback controllers are expressed by parameterization

$$\frac{Q}{P}=\frac{Q_0-AT}{P_0+BT} \tag{5}$$

where Q_0,P_0 are particular solutions (6) and *T* is free. Similarly, the disturbance rejection problem can be formulated in algebraic parlance. Both structures can be solved in a unified way. The problem then gives a second condition of divisibility that the product *AP* has to be divisible by F_v, where F_v is the denominator of the disturbance signal in Fig.1. The simultaneous tracking and disturbance rejection problem is solved in two steps. The first one expresses all stabilizing controllers (Youla-Kučera parameterization). The second step selects the suitable controllers according to conditions of divisibility in the given ring (see e.g [5], [6]).

3.APPROXIMATION OF THE TIME DELAY

The simplest description for process with delay can be expressed by a first order model:

$$G_1(s)=\frac{Ke^{-\tau s}}{s+\alpha} \tag{6}$$

For the linear controller design and tuning it is necessary to linearize transfer function (6). It can be done by several methods. The first one and the simplest is neglect the delay $e^{-\tau s}$. Then the time delay is considered as a perturbation of the nominal transfer function. So the nominal approximation is :

$$G_2(s)=\frac{K}{s+\alpha} \tag{7}$$

Next two approximations are based on the Taylor series approximation of $e^{-\tau s}$ in numerator or in denominator. Approximation $e^{-\tau s} \approx (1-\tau s) \approx (1+\tau s)^{-1}$ then

$$G_3(s)=\frac{K(1-\tau s)}{s+\alpha}=\frac{b_1 s+b_0}{s+\alpha} \tag{8}$$

$$G_4(s)=\frac{K}{s+\alpha}\frac{1}{1+\tau s}=\frac{K}{s^2+a_1 s+a_0} \tag{9}$$

The last model is obtained by the familiar known Padé approximation

$$G_5(s)=\frac{K}{s+\alpha}\frac{1-\dfrac{\tau}{2}s}{1+\dfrac{\tau}{2}s}=\frac{b_1 s+b_0}{s^2+a_1 s+a_0} \tag{10}$$

4.ROBUST ANALYSIS IN THE FREQUENCY DOMAIN

The fractional approach developed by Vidyasagar in [11] enables a deeper insight into control tuning and robustness. Let R_{ps} be a set of proper and Hurwitz stable rational functions. This set is a ring and the norm H_∞ can be easily defined through the frequency response.

$$\|G\| = \sup_{\mathrm{Re}s\geq 0} |G(s)| = \sup_{\omega\in E} |G(j\omega)| \tag{11}$$

$$\|G_1\, G_2\| = \sup_{\mathrm{Re}s\geq 0} \{|G_1(s)|^2 + |G_2(s)|^2\}^{\frac{1}{2}} \tag{12}$$

Almost all models differ from physical plants. Let $G(s) = B(s) / A(s)$ be a nominal plant and consider a family of perturbed systems $G'(s) = B'(s) / A'(s)$ where

$$\|A - A'\| \leq \varepsilon_1 \;\; \|B - B'\| \leq \varepsilon_2 \tag{13}$$

or $\|A - A'\; B - B'\| \leq \varepsilon$ (14)

where $\varepsilon_1, \varepsilon_2, \varepsilon$ are positive constants.

For robust control it is necessary to choose a part of stabilizing controllers (6), (8) which stabilize perturbed plants. For perturbed plants choose such P, Q in (6) which fulfil the conditions

$$\varepsilon_1 \|P_0 + BT\| + \varepsilon_2 \|Q_0 - AT\| < 1 \tag{15}$$

or $\varepsilon \left\| \begin{matrix} P_0 + BT \\ Q_0 - AT \end{matrix} \right\| < 1$ (16)

For a deeper insight into robustness the notion of the sensitivity function:

$$\in = \frac{y}{v} = \frac{1}{1 + HC_b} = A(P_0 + BT) \tag{17}$$

can be used in the sense as in [4]. For above mentioned SISO systems sensitivity function $\in$ is a non-linear function of $m>0$ and it can be minimized by a simple scalar optimization method. In this way the “most robust“ controller of given structure can be obtained.

5.ROBUST PID DESIGN AND TUNNING

All approximated transfer function (7)-(10) can be considered as a special case of (2) – second order system with relative degree one. Let the reference be a stepwise signal with the denominator

$$F_w = s / (s+m) \tag{18}$$

Equation (3) takes the form:

$$\begin{aligned}&(s^2 + a_1 s + a_0)(p_1 s + p_0) + \\ &+ (b_1 s + b_0)(q_1 s + q_0) = (s+m)^3\end{aligned} \tag{19}$$

All stabilizing controllers could be written:

$$P = P_0 + BT = \frac{s + p_0}{s+m} + \frac{b_1 s + b_0}{(s+m)^2} T \tag{20}$$

$$Q = Q_0 - AT = \frac{q_1 s + q_0}{s+m} - \frac{s^2 + a_1 s + a_0}{(s+m)^2} T \tag{21}$$

where T is arbitrary element of R_{ps}

The divisibility condition $F_w \setminus P$ is achieved for :

$$T = t_0 = -\frac{p_0 m}{b_0} \tag{22}$$

The final solution is then:

$$\begin{aligned}
\tilde{P} &= \frac{s^2 + s(p_0 + m - p_0 m \frac{b_1}{b_2})}{(s+m)^2} = \frac{s^2 + \tilde{p}_0 s}{(s+m)^2} \\
\tilde{Q} &= \frac{s^2 (q_1 + \frac{p_0 m}{b_0})}{(s+m)^2} + \frac{s(q_0 + q_1 m + a_1 \frac{p_0 m}{b_0})}{(s+m)^2} + \\
&+ \frac{a_0 \frac{p_0 m}{b_0} + q_0 m}{(s+m)^2} = \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{(s+m)^2}
\end{aligned} \tag{23}$$

and the transfer function for controller is given:

$$\frac{Q}{P} = \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s(s + \tilde{p}_0)} \tag{24}$$

This controller corresponds with the realistic PID controller (see [1], [3])

$$C_{PID} = K(1 + \frac{1}{T_I s} + \frac{T_D}{T_f s + 1}) \tag{25}$$

The disturbance rejection problem can be explained by the following example. Consider an integrator without dead-time expressed by the transfer function (7) for $\alpha = 0$, the reference (18) and a disturbance expressed by

$$v = \frac{\omega^2}{s^2 + \omega^2} = \frac{\frac{\omega^2}{(s+m)^2}}{\frac{s^2 + \omega^2}{(s+m)^2}} = \frac{G_v(s)}{F_v(s)} \tag{26}$$

It means the problem of simultaneous regulation and disturbance rejection of a harmonic signal. All stabilizing controllers are given by (3) in the form

$$\frac{s}{s+m} p_0 + \frac{b_0}{s+m} q_0 = 1 \tag{27}$$

with the solution:

$$P = 1 + \frac{b_0}{s+m}T; \quad Q = \frac{m}{b_0} - \frac{s}{s+m}T \tag{28}$$

The condition of divisibility of AP by F_w is fulfilled generically for the integrator since functions F_w and A are the same. For the disturbance rejection it is necessary to ensure the divisibility of P by F_v. It is possible to achieve by a suitable choice of $T = \frac{t_1 s + t_0}{s+m}$ in (28), so that:

$$P = 1 + \frac{b_0}{s+m}\frac{t_1 s + t_0}{s+m} \approx \frac{s^2 + \omega^2}{(s+m)^2} \tag{29}$$

Equating of coefficients in (29), the following linear equations for t_0, t_1 are:

$$\begin{aligned} 2m + b_0 t_1 &= 0 \\ m^2 + b_0 t_0 &= \omega^2 \end{aligned} \tag{30}$$

which gives $t_1 = -\frac{2m}{b_0}$ and $t_0 = \frac{\omega^2 - m^2}{b_0}$. The resulting feedback controller Q/P is then obtained:

$$P = \frac{s^2 + \omega^2}{(s+m)^2}; \; Q = \frac{q_2 s^2 + q_1 s + q0}{(s+m)^2} \tag{31}$$

where $q_2 = \frac{m}{b_0} - t_1; \; q_1 = \frac{2m^2}{b_0} - t_0; \; q_0 = \frac{m^3}{b_0}$.

The control law $Pu=Qe$ then gives the control law:

$$\begin{aligned} u(t) = -\omega^2 \quad & u(\tau)d\tau + q_2 e(t) + \\ + q_1 \quad & e(\tau)d\tau + q_0 \quad e(\tau)d\tau \end{aligned} \tag{32}$$

6.EXAMPLES AND COMPARISON

Example 1: Consider a stable transfer function with time delay

$$G_N(s) = \frac{1}{3s+1}e^{-5s} \tag{33}$$

and its two perturbations

$$G_{P1}(s) = \frac{1.5}{s^2 + 5s + 2}e^{-6s} \tag{34}$$

$$G_{P2}(s) = \frac{1.2}{1.5s^2 + 5s + 2}e^{-5s} \tag{35}$$

Using the above described method for nominal plant G_N and Padé approximation, a robust controller is obtained for m=0.32 with the transfer function:

$$C_3(s) = \frac{Q}{P} = \frac{0.522s^2 + 0.405s + 0.07864}{s^2 + 0.3873s} \tag{36}$$

Control responses for transfer function (33)-(35) and robust controller C_3 for m=0.32 are shown in Fig. 2 while Fig. 3. shows open loop Nyquist plots. A unit step setpoint is changed at time t=100 and a load disturbance v= -1 is introduced at time t=200. The distance of the open loop Nyquist plot G_a from the critical point (-1, 0) was obtained for the value m=0.32 which represents the "most robust" controller.

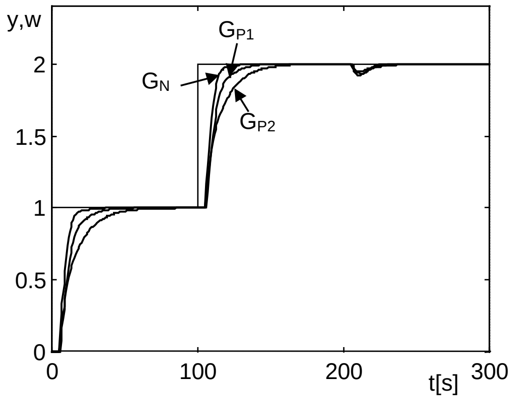

Fig. 2. Control response for the nominal plant, perturbed plants and robust controller C_3 (m=0.32).

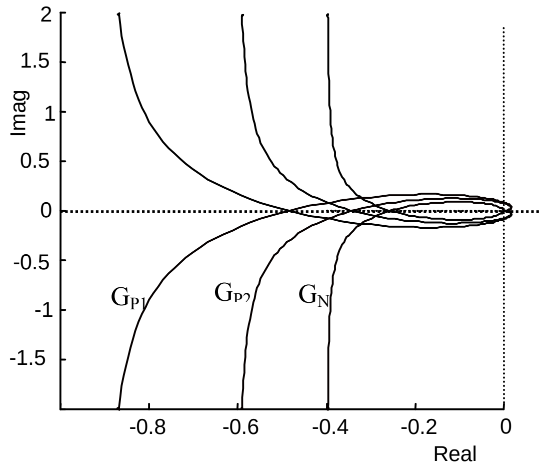

Fig. 3. Open loop Nyquist plots for Example 1.

Example 2: Let the integrating process have the parameters in (7) K=1, α=0 and time delay τ = 5. Approximation (7) gives controller (24) in a PI structure. The disturbance rejection controller was derived according to (27) - (31). In all cases, the scalar parameter m > 0 was used for tuning and finding satisfactory control responses. Figs. 4,5 represent the responses for FB and FBFW structures and m=0.1. The magnitude of the load disturbance was assumed to be -0.1 in t=200. The improvement of the response caused by the structure FBFW is evident.

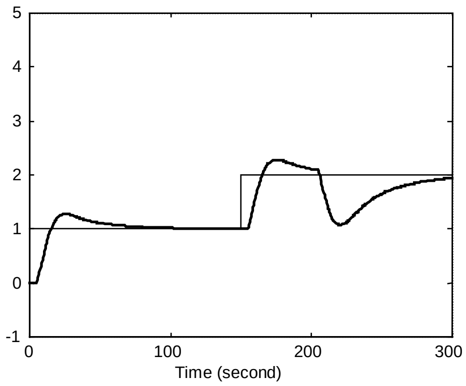

Fig. 4. Control response of the time delay integrator with the neglecting of the time delay term (FB structure and m=0.1).

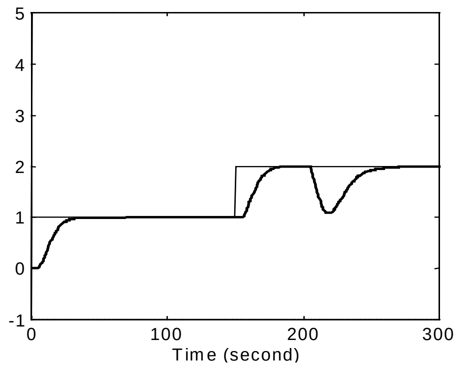

Fig. 5. Control response of the time delay integrator with neglecting of the time delay term (FBFW structure and m=0.1).

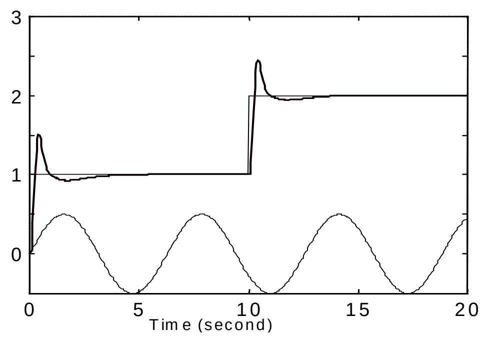

Fig. 6. Disturbance and regulation control of the integrator with FB structure and neglecting of the time delay term.

The simultaneous disturbance rejection and tracking response for the FB structure and the integrator with τ = 0.1 is shown in Fig. 6. Also the time delay term was neglected according to (7). It is clear that a scalar parameter $m>0$ incorporated in relations (31) is indeed a tuning knob for influencing controller parameters and control behavior.

Example 3: Consider an unstable system with time delay (6) with one unstable pole and K = 1, α = -1; τ = 0.8. Nyquist plots of all approximated transfer functions (7)-(10) are depicted in Fig. 7.The best control response was reached for the FBFW structure and m=0.3. Again, the step change of the setpoint was performed in t=150 and the magnitude of the load disturbance was assumed to be -0.1 in t=200.

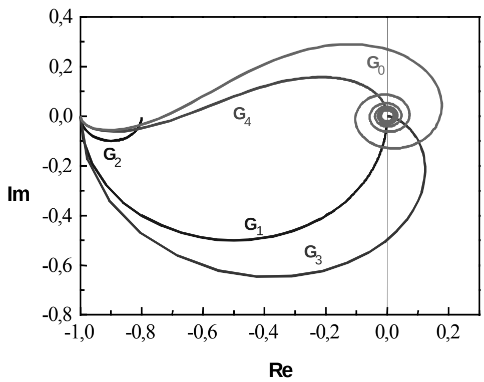

Fig. 7. Nyquist plots for transfer functions (7)-(10) K=1,α= -1, τ=0.8.

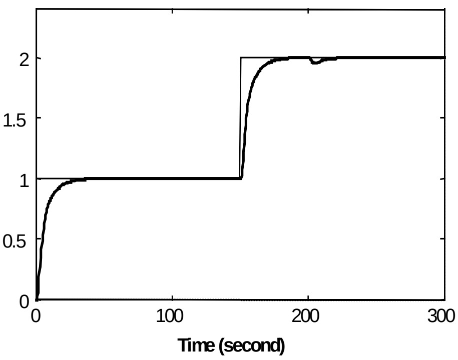

Fig. 8. Taylor denominator expansion for Example 3 with m=0.3 and FBFW structure.

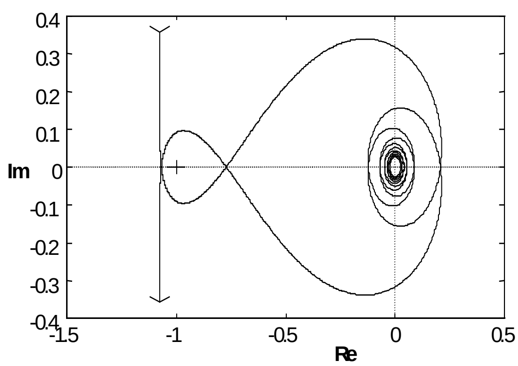

Fig. 9. Open loop Nyquist plot of Example 3.

Evidently, the resulting response is very satisfactory, without overshooting and with a good reaction to the load disturbance of the magnitude -0.1. The open loop Nyquist plot of this controller nominal setting is depicted in Fig.9. According to Nyquist criterion The critical point (-1, 0) has to be encircled once time since the controlled plant has one unstable pole. The minimal distance of the plot to the critical point is again a measure of the robustness of the proposed controller.

7.CONCLUSION

In this contribution, a non-traditional method of control design and tuning for SISO, continuous - time transfer functions having time delay has been introduced. The proposed control synthesis is based on the fractional representation of system and regulator transfer functions. Both, single feedback (FB) controller and feedback-feedforward (FBFW) controller structures were investigated. Controller transfer functions are obtained as a special solution of Diophantine equations in the ring of proper and stable rational functions. The asymptotic tracking and disturbance rejection problems are then expressed through the conditions of divisibility laid on controller denominators. For first order systems, the resulting control laws are of general PI or PID types. The time delay terms are approximated by several ways. The fractional approach enables to define a scalar positive parameter which can be seen as a ”tuning knob” which generates all controller parameters as well as control responses and robustness of the overall system.

Simulation experiments confirmed a highly robust performance of the proposed control systems in both, setpoint tracking and disturbance rejection problems. Robust analysis based on Nyquist open loop plots is added. All simulations were performed in the Matlab and Simulink environment. The designed strategy can be preferably recommended for industrial applications when parameter uncertain plants with dead - time are to be controlled.

8.REFERENCES

[1] Aström K.J., Hang C.C., Persson P., Ho W.K.. Towards intelligent PID control. Automatica, Vol. 28, No.1, 1992, pp. 1-9.

[2] Aström K.J., Hang C.C., Lim B.C. A new Smith predictor for controlling a process with an integrator and long dead-time. IEEE Trans. on AC, Vol. 39, No. 2, 1994, pp. 343-345.

[3] Aström K.J., Hägglund T. PID Controllers: Theory, Design and Tuning. Instrument Society of America, USA, 1995

[4] Doyle C.D., Francis B.A., Tannenbaum A.R. Feedback Control Theory. Macmillan, New York, 1992.

[5] Grimble M.J., Kučera V. Polynomial Methods for Control Systems Design. Springer, Berlin, 1996.

[6] Kučera V. Diophantine equations in control - A survey, Automatica, Vol. 29, No.6, 1993, pp. 1361-75.

[7] Persson P., Aström K.J. PID control revisited. In: Prepr. 12th IFAC World Congress, Sydney, Vol. 8, 1993, pp. 241-244.

[8] Prokop R., Mészáros A. Design of robust controllers for SISO time delay systems. Journal of Electrical Engineering, Vol. 47, No. 11-12, 1996, pp. 287-294.

[9] Prokop R., Corriou J.P. Design and analysis of simple robust controllers, Int. J. Control , Vol. 66, No. 6, 1997, pp. 905-921.

[10] Venkatashankar V., Chidambaram M. Design of P and PI controllers for unstable firs-order plus time delay systems, Int. J. Control, Vol. 60, No. 1, 1994, pp. 137-144.

[11] Vidyasagar M. Control system synthesis: a factorization approach. MIT Press, Cambridge, M.A, 1985.

[12] Watanabe K., Ito M. A process model control for linear systems with delay, IEEE Trans. On AC, Vol. 26, No. 6, 1981, pp. 1261-66.