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CONTINUOUS – TIME SELF – TUNING CONTROLLER*

VLADIMÍR BOBÁL, PETR DOSTÁL, KAREL KOLOMAZNÍK

Brno University of Technology, Faculty of Technology Zlín, Department of Automatic Control, nám. TGM 275, 762 72 Zlín, Czech Republic, bobal@zlin.vutbr.cz

Abstract. A self – tuning controller algorithm has been derived in this paper. The process is identified by the regression (ARX) continuous – time model using the recursive least squares method (RLSM) with applied directional forgetting. The recursive parameter estimates of the continuous – time model (differential equation) are used to controller synthesis. Controller synthesis is designed on the basis of pole – placement (assignment) method. The algorithm is suitable for the automatic setting of analog controllers for deterministic processes or the adaptive control of stochastic and nonlinear processes without or with time delay. One modification of the controller has been verified by computer simulation.

Key words. Continuous – time ARX model, recursive least squares, pole assignment method, self - tuning control.

1. INTRODUCTION

Most of the self – tuning controllers designed up to now and in many cases successfully applied are digital controllers based on recursive identification of the discrete ARX or ARMAX models. These models are used mostly in the form of a *z* – transform function. The simple model structure, easy recursive identification using measurable data, suitability for the synthesis of the discrete control loop as well as for the description and expression of different types of stochastic process, including disturbance modelling, are all advantages of the z – transform function. Self – tuning controllers based on the recursive identification of the *z* – models have been derived in [1, 2].

The step *z* - transform functions have some disadvantages as the sampling period decreases:

- the *Z* - transformation parameters do not converge as the sampling period decreases to the Laplace – transformation continuous parameters from which they were derived,
- very small sampling periods yield very small numbers from the transfer function numerator,
- the poles transfer function approach the unstable domain as the sampling period decreases.

These disadvantages can be avoided by introducing a more suitable discrete model. For this purpose the δ - model is the most suitable (see Middleton and Goodwin [3] or Feuer and Goodwin [4]). Parameter δ converges with decreased sampling period T_0 to a continuous operator *s*. A self – tuning PID controller based on the recursive identification of the δ - model and of a modified Ziegler – Nichols criterion has been designed in [5] and similar PID pole placement controllers in [6]. A delta model may also be considered as a kind of half – way house between continuous – time and discrete models.

It is obvious that analog controllers are best suited to control for continuous – time processes. However, their self – tuning modifications require recursive continuous – time system identification. The theoretical background for continuous – time system identification can be found in the monograph of Unbehauen and Rao [7] or Sinha and Rao [8]. The self – tuning controllers designed in this contribution use the recursive continuous – time identification method derived by Wahlberg [9].

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2. RECURSIVE PARAMETER ESTIMATION OF CONTINUOUS – TIME MODELS

2.1. Filtering the continuous – time variables

Consider the continuous – time ARX model in the form of the differential equation

$$A(\sigma)y(t) = B(\sigma)u(t) + n(t) \qquad (1)$$

where $u(t)$ and $y(t)$ are the continuous – time input and output signals respectively. Signal $n(t)$ represents the stochastic part – a white noise source, σ denotes the differentiation operator. A and B are polynomials in the variable σ. After the Laplace – transformation of (1) expression

$$A(s)Y(s) = B(s)U(s) + N(s) + O_1(s) \qquad (2)$$

can be obtained where A, B are polynomials in the complex variable s and O_1 is the transform of the initial conditions. The Laplace - transform of the output has the form

$$Y(s) = \frac{B(s)}{A(s)}Y(s) + \frac{N(s)}{A(s)} + \frac{O_1(s)}{A(s)} \qquad (3)$$

where

$$G(s) = \frac{B(s)}{A(s)} \qquad (4)$$

is the Laplace - transfer function of the system where the condition of the properness must be fulfilled

$$\deg B \leq \deg A; \qquad \deg O_1 \langle \deg A \qquad (5)$$

Because input $u(t)$ and output $y(t)$ derivatives are not directly measurable, therefore to obtain the continuous – time variables and their derivative approximations it is necessary to introduce a so – called state – variable filter, i. e. to define

$$C(\sigma)u_f(t) = u(t); \qquad C(\sigma)y_f(t) = y(t) \qquad (6)$$

where $C(\sigma)$ is the polynomial in σ, u_f is the filtered input and y_f is the filtered output. After the Laplace
transformation of (6) expression

$$C(s)U_f(s) = U(s) + O_2(s) \qquad (7)$$

can be obtained where O_2 is the polynomial of the initial conditions for the filtered input and O_3 is polynomial of the initial conditions for the filtered output. For polynomials O_2 and O_3

$$\deg O_2 < \deg C; \quad \deg O_3 < \deg C \qquad (8)$$

is valid. The following must be valid for polynomial $C(s)$:

1. Polynomial $C(s)$ must be stable.

2. The degree of polynomial C can be greater than of equal to the degree of polynomial A (deg $C \geq$ deg A). This fact issues from the theory of differential equations. In the interests of practicality we choose deg C = deg A.

3. The filter time constants must be less than the time constants of the identified system. If the time constants of the system are unknown it is necessary either to base the initial estimate on apriori information (e. g. from the transient characteristic) or to select a sufficiently small value.

By substituting the filtered variables into expression (2), equation

$$A\lfloor CY_f(s) - O_3 \rfloor = B\lfloor CU_f(s) - O_2 \rfloor + N(s) + O_1 \qquad (9)$$

after substitution

$$O = \frac{O_1 - BO_2 + AO_3}{C} \qquad (10)$$

can be rearranged into the form

$$Y_f(s) = \frac{B}{A}U_f(s) + \frac{1}{A}N(s) + \frac{O}{A} \qquad (11)$$

From equation (11) it follows

$$G_f(s) = G(s) = \frac{B}{A} \qquad (12)$$

which indicates that the transfer function of system for filtered variables is the same as for unfiltered variables. Only the initial conditions are different for filtered and unfiltered variables.

By transferring equation (11) into time domain equation

$$A(\sigma)y_f(t) = B(\sigma)u_f(t) + \varepsilon(t) \qquad (13)$$

can be obtained. Expression (13) is the basic equation to estimate parameters a_i, b_j. Variable $\varepsilon(t)$ expresses the difference between filtered and unfiltered variables. Equation (14) can be expressed in the form

$$\sum_{i=0}^{n} a_i y_f^{(i)}(t) = \sum_{j=0}^{m} b_j u_f^{(j)}(t) + \varepsilon(t) \qquad (14)$$

where n = deg a, m = deg b. Filtered values y_f, u_f are subtracted in discrete steps t_k

$$\sum_{i=0}^{n} a_i y_f^{(i)}(t_k) = \sum_{j=0}^{m} b_j u_f^{(j)}(t_k) + \varepsilon(t_k)\,; \quad t_k = kT_0 \tag{15}$$

for $k = 0,1,\ldots$, where T_0 is the sampling period.

Expression (15) must be rearranged into a form suitable for the recursive least squares method. There are two possible ways of doing this:

1) Polynomial A is normed in highest power s, a_n=1

$$y_f^{(n)}(t_k) = -\sum_{i=0}^{n-1} a_i y_f^{(i)}(t_k) + \sum_{j=0}^{m} b_j u_f^{(j)}(t_k) + \varepsilon(t_k) \tag{16}$$

2) Polynomial A is normed into absolute term, $a_0 = 1$

$$y_f(t_k) = -\sum_{i=1}^{n} a_i y_f^{(i)}(t_k) + \sum_{j=0}^{m} b_j u_f^{(j)}(t_k) + \varepsilon(t_k) \tag{17}$$

Experience shows that expression (16) gives better results.

2.2. Recursive least squares method (RLSM) algorithm

The least squares method is one method of recursive analysis suitable for examining the static and dynamic relations between the variables of the plant under consideration. Consider a discrete SISO stochastic ARX process

$$y(k) = \Theta^T(k)\phi(k-1) + n(k) \tag{18}$$

where

$$\Theta^T(k) = [a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n] \tag{19}$$

and

$$\begin{aligned}\phi^T(k-1) = [&-y(k-1), -y(k-2), \ldots, -y(k-n),\\ &u(k-1), u(k-2), \ldots, u(k-n)]\end{aligned} \tag{20}$$

are the vector of the parameters and the repressor.

The algorithm of the RLSM can usefully be extended to include adaptive directional forgetting [10]. The value of the directional forgetting factor $\varphi(k)$ basically depends on the level of conformity achieved between the model and the real behaviour of the system. In this case it is minimized criterion

$$J_k(\Theta) = \sum_{i=k_0}^{k} \varphi^{2(k-i)} n^2(i)\,; \quad 0 \langle \varphi^2 \le 1 \tag{21}$$

where k_0 is the identification start and

$$n(i) = y(i) - \Theta^T \phi(i) = \begin{bmatrix} 1 & -\Theta^T \end{bmatrix} \begin{bmatrix} y(i) \\ \phi(i) \end{bmatrix} \tag{22}$$

The algorithm of the RLSM with directional forgetting then consists of the following steps in each sampling period:

Step 1. Choosing of the initial vector of parameter estimates $\hat{\Theta}(0)$, the main diagonal of the covariance matrix $C_{ii}(0)$, directional forgetting factor $\varphi(0)$, $\lambda(0)$, $\nu(0)$ and ρ.

Step 2. Calculating the prediction error from the following expression

$$\hat{e}(k) = y(k) - \hat{\Theta}^T(k)\phi(k-1) \tag{23}$$

Step 3. Calculating auxiliary variables from the following relations

$$\begin{aligned}&\xi(k-1) = \phi^T(k-1)C(k-1)\phi(k-1)\,;\\ &\upsilon(k) = \varphi(k)\left[(\upsilon(k-1)+1\right]; \qquad \eta(k) = \frac{\hat{e}^2(k)}{\lambda(k)}\,;\\ &\lambda(k) = \varphi(k)\left[\lambda(k-1) + \frac{\hat{e}^2(k-1)}{1+\xi(k-1)}\right]\end{aligned} \tag{24}$$

Step 4. Calculating the directional forgetting factor

$$\begin{aligned}\varphi(k) = \{&1 + (1+\rho)[\ln(+\xi(k-1))] + \\ &+\left[\frac{(\upsilon(k-1)+1)\eta(k-1)}{1+\xi(k-1)+\eta(k-1)} - 1\right]\frac{\xi(k-1)}{1+\xi(k-1)}\}^{-1}\end{aligned} \tag{25}$$

Step 5. Calculating the auxiliary variable

$$\varepsilon(k) = \varphi(k) - \frac{1-\varphi(k)}{\xi(k-1)} \tag{26}$$

Step 6. If $\xi(k-1) > 0$ then the covariance matrix is actualized using expression

$$C(k) = C(k-1) - \frac{C(k-1)\phi(k-1)\phi^T(k-1)C(k-1)}{\varepsilon^{-1}(k) + \xi(k-1)} \tag{27}$$

if $\xi(k-1) = 0$, then $C(k) = C(k-1)$.

Step 7. The actualization of the parameter estimates vector

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \frac{C(k-1)\phi(k-1)}{1+\xi(k-1)}\hat{e}(k-1) \tag{28}$$

The start - up conditions for the most commonly used identification methods are the initial parameter estimates and their covariance matrix. Although most users understand the importance of the initial parameter estimates and with a certain amount of effort are usually able to assign realistic values using their technical expertise, the importance of the covariance matrix is often neglected and is difficult to design. Although the issue of a priori information in the selection of start - up conditions has been discussed in [2], it is a good idea to choose the following conditions for the start of the algorithm: the elements of the main diagonal of the covariance matrix should be $C_{ii}(0) = 10^3$, start value for the directional forgetting factor $\varphi(0) = 1$, $\lambda(0) = 0.001$, $\upsilon(0) = 10^{-6}$, $\rho = 0.99$. The start estimates for the parameter estimates vector $\hat{\Theta}(0)$ is chosen according to a priori information. The relations given above can be directly programmed as an m - function in the MATLAB system

The algorithm of the RLSM with directional forgetting is in detail introduced in [1].

The RLS continuous – time identification version differs from the estimation of the discrete version. In case continuous – time version are estimated the parameters of the differential equation and also in filling of the regressor (number 1 is included to the regressor and parameter d is estimated). This parameter comprises differences between the filtered and unfiltered variable. Then the regressor has the form

$$\begin{aligned}\phi^T(t_k) = [&-y_f(t_k), -y_f'(t_k), \ldots, -y_f^{(n-1)}(t_k),\\ &u_f(t_k), u_f'(t_k), \ldots, u_f^{(m)}(t_k), 1]\end{aligned} \tag{29}$$

and the vector of parameters is

$$\Theta^T(t_k) = [a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_m, d] \tag{30}$$

The choice of initial parameters follows the same rules as for discrete model parameter identification. Problem includes in choice of the start parameter estimates (suitable choosing of the start parameter estimates influences the course of the control process). When the control law requires identification to provide the values of the time constants and the gain of the system these can be simply calculated from the parameter estimates of the process model using estimates obtained from the actual identified system.

3. ANALOG PID CONTROLLERS SYNTHESIS

Analog self – tuning controller synthesis can be designed using several methods (e. g. Ziegler – Nichols method, pole placement method, LQ criterion and dynamics inversion method). In this article the polynomial pole placement method will be used [11].

A simple control – loop with transfer function of the process

$$G_P(s) = \frac{B(s)}{A(s)} \tag{31}$$

and with transfer function of the controller

$$G_R(s) = \frac{Q(s)}{P(s)} \tag{32}$$

as shown in Fig. 1 is considered. $Y(s)$, $U(s)$, $E(s)$ and $W(s)$ are the Laplace – transforms of the process output, controller output, error and reference signal, $F(s) = s$ is compensator. Then the transfer function of the control closed – loop is given by relation

$$G_w(s) = \frac{B(s)Q(s)}{A(s)F(s)P(s) + B(s)Q(s)} \tag{33}$$

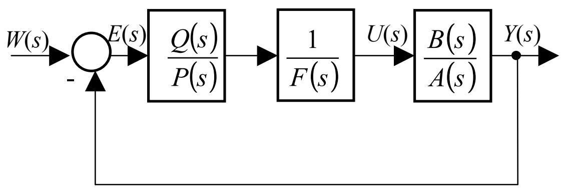

Fig. 1 Block diagram of single control closed – loop system

The denominator of the of transfer function (33) is the characteristic polynomial

$$A(s)F(s)P(s) + B(s)Q(s) = D(s) \tag{34}$$

on the whose poles determine the behaviour of the closed control loop. The polynomial $D(s)$ must be stable and can be specified by different methods. The control system satisfies the condition of internal properness only when transfer functions of all its components are proper. The degrees of polynomials of the controller function (inclusive of the compensator) must then fulfill the inequality

$$\deg Q(s) \leq \deg P(s) + \deg F(s) \tag{35}$$

From analysis of the solvability of equation (34) and taking into account condition (35), the degree of the polynomial $Q(s)$ is given as

$$\deg Q(s) = \deg A(s) + \deg F(s) - 1 \tag{36}$$

and for degree of the polynomials $P(s)$ and $D(s)$ be valid

$$\begin{aligned}&\deg P(s) \geq \deg A(s) - 1\,;\\ &\deg D(s) = 2\deg A(s) + \deg F(s) - 1\end{aligned} \tag{37}$$

4. SIMULATION EXAMPLE

As an example of verification by computer simulation the control of a second – order model with time delay with transfer function

$$G_P(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-T_d s} \tag{38}$$

has been used. Using approximation

$$e^{-T_d s} \approx (1 - T_d s) \tag{39}$$

transfer function (38) is possible arranged in the form

$$G_P(s) = \frac{B(s)}{A(s)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{40}$$

where

$$a_0 = \frac{1}{T_1 T_2}; \qquad a_1 = \frac{1}{T_1} + \frac{1}{T_2};$$
$$b_0 = \frac{K}{T_1 T_2}; \qquad b_1 = -\frac{K T_d}{T_1 T_2} \tag{41}$$

It is obvious that degrees of individual polynomials are:

$$\begin{aligned} &\deg A(s) = 2; \quad \deg B(s) = 1; \quad \deg F(s) = 1; \\ &\deg P(s) = 1; \quad \deg Q(s) = 2; \quad \deg D(s) = 4 \end{aligned} \tag{42}$$

Then the transfer function of the controller is in the form

$$G_R(s) = \frac{Q(s)}{P(s)} = \frac{q_2 s^2 + q_1 s + q_0}{s + p_0} \tag{43}$$

We choose characteristic polynomial in the form

$$D(s) = (s + \alpha_1)^2 (s + \alpha_2)^2 \tag{44}$$

it is we choose two real double poles in the left – side of the s – plane. Substituting individual polynomials into equation (34) we obtain equation

$$\begin{aligned} &(s^2 + a_1 s + a_0) s (s + p_0) + (b_1 s + b_0)(q_2 s^2 + q_1 s + q_0) = \\ &\quad = (s + \alpha_1)^2 (s + \alpha_2)^2 \end{aligned} \tag{45}$$

By comparing of the same powers of s then we obtain system of equations in the matrix form

$$\begin{bmatrix} 1 & 0 & 0 & -b_1 \\ a_1 & 0 & -b_1 & b_0 \\ a_0 & -b_1 & b_0 & 0 \\ 0 & b_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ q_0 \\ q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 2\alpha_1 + 2\alpha_2 - a_1 \\ \alpha_1^2 + \alpha_2^2 + 4\alpha_1\alpha_2 - a_0 \\ 2\alpha_1\alpha_2^2 + 2\alpha_1^2\alpha_2 \\ \alpha_1^2\alpha_2^2 \end{bmatrix} \tag{46}$$

The system equations (46) gives the relations for computing controller parameters p_0, q_0, q_1 and q_2.

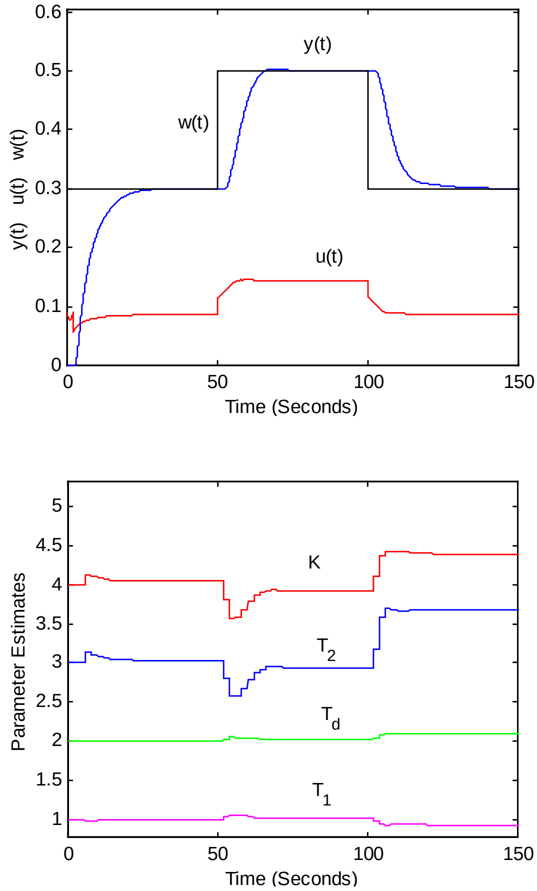

Fig. 2 Simulation results: control and parameter estimates of model (38) – initial parameter estimates: $\hat{K} = 4; \hat{T}_1 = 1; \hat{T}_2 = 3; \hat{T}_d = 2$

The recursive parameter estimates of continuous – time transfer function (40) are then inserted into relations for computing controller parameters. Here, the state – variable filter is of the second - order

$$y_f''(t) + c_1 y_f'(t) + c_0 y_f(t) = y(t) \tag{47}$$

and it can be expressed by polynomial

$$C(s) = s^2 + c_1 s + c_0 = (s + \delta)^2 = s^2 + 2\delta s + \delta^2;$$
$$c_0 = \delta^2; \quad c_1 = 2\delta \quad; \quad \delta = \frac{1}{T_f} \tag{48}$$

where T_f is the time constant of the filter. The vector of the parameters and regressor have the following form

$$\begin{aligned} &\Theta^T = [a_0, a_1, b_0, b_1, d]; \\ &\phi^T = [-y_f(t_k), -y_f'(t_k), u(t_k), u_f'(t_k), 1] \end{aligned} \tag{49}$$

The control law subsequently is given by

$$u''(t) + p_0 u'(t) = q_2 e''(t) + q_1 e'(t) + q_0 e(t) \qquad (50)$$

In our simulation verification of control model (38) using controller (43) with compensator F we chose the following parameters: $K = 3.5$; $T_1 = 1.5$; $T_2 = 2.5$; $\alpha_1 = 0.3$; $\alpha_2 = 0.8$; $T_f = 0.6$. Fig. 2 shows the control process and the parameter estimates $\hat{\Theta}(t)$ where the initial parameter estimates have been chosen on the basic of a priori information. From Fig. 2 we can clearly see excellent process control. Fig. 3 shows the control process and the parameter estimates $\hat{\Theta}(t)$ when the initial parameter estimates have been chosen without a priori information - randomly.

Comparing Fig. 2 and 3 is obvious that the choice of the initial parameter estimates influences the control process.

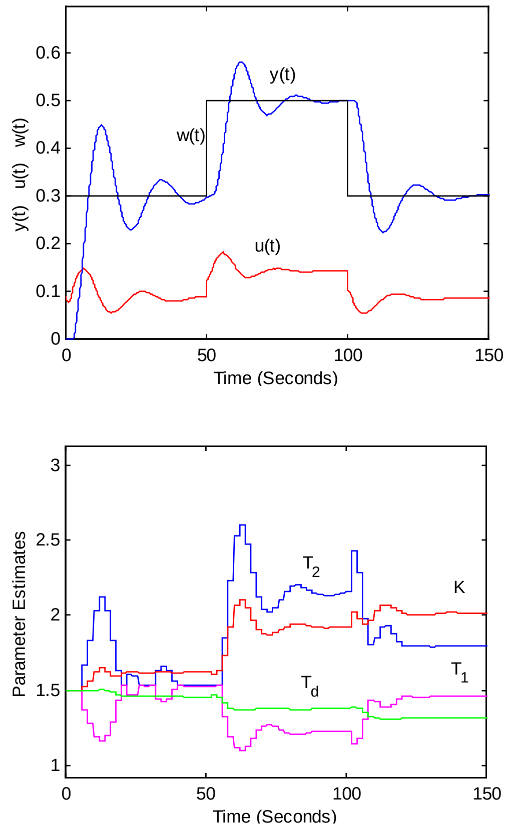

Fig. 3 Simulation results: control and parameter estimates of model (38) – initial parameter estimates: $\hat{K} = \hat{T}_1 = \hat{T}_2 = \hat{T}_d = 1.5$

5. CONCLUSIONS

The proposed self - tuning controller based on continuous – time identification and pole placement method demonstrated very good dynamic behaviour. The advantages of this controller synthesis include simplicity, accuracy, universality and the facilitation of control systems with time delay. This controller has been successfully verified for the control of the laboratory through – flow heater. The simulation results and verification under laboratory conditions operating in real time demonstrated that the initial parameter estimates have influence on the control process quality. The filter time constant T_f influences identification and control process quality, too. The proposed self – tuning continuous - time controller will be implemented for control of enzymatic digestion of chrometanned waste [12].

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