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ROBUST POSITION/SPEED CONTROL FOR PERMANENT MAGNET DC MOTORS VIA P-D FEEDBACK

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Abstract. For a permanent magnet DC motor the design goal of perfect position/speed control with simultaneous load torque rejection is studied, using a P-D feedback law. The problem is proved to be always solvable. The P-D feedback law solving the problem is determined. Stability properties and robustness are investigated. The above results are illustrated via simulation for a 75 watt DC motor.

Key Words. DC motor, linear systems, disturbance rejection, position control, speed control, robust control.

NOMENCLATURE

B_m : Viscous friction constant
i_r : Rotor current
J : Motor moment of inertia
κ_1, κ_2 : Constants related to the characteristics of the DC motor (dimensions, shape, material ,etc.)
L_r : Rotor inductance
V_r : Rotor voltage
R_r : Rotor resistance
r : Transmission ratio
T_l, T_m : Load torque, torque generated by the motor
V_b : Induced voltage (back emf.)
θ_l, θ_m : Position of the load, motor shaft position
$\omega_l,\ \omega_m$: Angular velocity of the load, angular velocity of the motor shaft

1. INTRODUCTION

Position/speed control of DC motors appears to be a basic control application having attracted significant interest during the last decades (see f.e [1]-[7] and the reference therein).

In this paper, for a permanent magnet DC motor, perfect position/speed control with simultaneous load torque rejection is studied, using a P-D feedback law. The problem is proved to be always solvable. The P-D feedback law solving the problem is determined. Stability properties and robustness are investigated. The above results are illustrated via simulation for a 75 watt DC motor (35NT2R82- 426SP).

The contribution of the present paper, as compared to existing results in the field (see f.e. [8] where the position error is fed back via a PD controller), is the complete rejection of the load torque and perfect command following. The disadvantage (as compared to results using static controllers see f.e [1]) can be summarised to the difficulties in implementing the derivative term. With regard to this issue it has been proven that that the speed and/or position are linear with respect to the current derivative error.

2. POSITION-SPEED CONTROL OF A PERMANENT MAGNET DC MOTOR

2.1 Model description

The system of permanent magnet DC motor is characterized by a set of three linear first order differential equations. The first equation is the definition of the angular velocity of the rotor and it's position $\frac{d\theta_m}{dt} = \omega_m$. The second equation is the application of the third Newton's law for the load i.e. $J\frac{d\omega_m}{dt} + B_m\omega_m = T_m - rT_l$. The third equation describes the electrical balance in the rotor winding $L_r\frac{di_r}{dt} + R_r i_r = V_r - V_b$.

The coupling between electrical and mechanical equations, is rising from the nonlinear characteristics of the motor, is expressed by the following relations $T_m = \kappa_1 \phi i_r$ and $V_b = \kappa_2 \phi \omega_m$. Hence, the set of the equations describing the DC motor is:

$$\frac{d\theta_m}{dt} = \omega_m \quad , \quad \frac{d\omega_m}{dt} = -\frac{B_m}{J}\omega_m + \frac{k_m}{J} i_r - \frac{r}{J} T_l \quad ,$$
$$\frac{di_r}{dt} = -\frac{k_b}{L_r}\omega_m - \frac{R_r}{L_r} i_r + \frac{1}{L_r} V_r \qquad (2.1)$$

where $k_m = \kappa_1 \phi$ and $k_b = \kappa_2 \phi$. The performance output is the angular velocity of the load, let ω_l, or the position of the load, let θ_l. These variables are related with ω_m and θ_m via the transmission ratio r (between gears) as follows

$$\omega_l = \frac{1}{r}\omega_m \quad , \quad \theta_l = \frac{1}{r}\theta_m \qquad (2.2)$$

2.2 Speed Control

In this subsection, the problem of controlling the angular velocity of the load is studied. To the system (2.1) apply the P-D feedback law

$$V_r = f_1 \frac{d\omega_m}{dt} + f_2 \frac{di_r}{dt} + k_1 \omega_m + k_2 i_r + gw \qquad (2.3)$$

where f_1, f_2 and k_1, k_2 are the degrees of freedom of the derivative and proportional term, respectively. The parameter g is the precompensator gain and w is the external command. Choosing the degrees of freedom to be

$$f_1 = 0, \quad f_2 = L_r, \quad k_1 = k_b - \tfrac{1}{r}, \quad k_2 = R_r \qquad (2.4)$$

the P-D feedback controller takes on the form

$$V_r = L_r \frac{di_r}{dt} + (k_b - \frac{1}{r})\omega_m + R_r i_r + gw \qquad (2.5)$$

Substituting the controller (2.5) to the DC motor description (2.1) the resulting closed loop system becomes

$$i_r = \frac{J}{k_m} rg \frac{dw}{dt} + \frac{B_m}{k_m} rgw + \frac{r}{k_m} T_l \quad , \; \omega_m = rgw \qquad (2.6a)$$

while the output variable ω_l takes on the form

$$\omega_l = gw \qquad (2.6b)$$

From (2.6b) it is concluded that, using the feedback law (2.5), perfect output control of the angular velocity of the load, has been achieved. For the formal definition of the problem of perfect output control, see [9 and 10].

With regard to the BIBO stability of the closed loop system, it is mentioned that if w is bounded, the boundness of ω_m and ω_l is guaranteed. Furthermore if the first derivative of w is bounded then, according to (2.6a) the current i_r is bounded. With regard to asymptotic stability it is observed that for $w = 0$, it holds that

$$\omega_l(t) = 0, \quad \omega_m(t) = 0, \quad i_r(t) = \frac{r}{k_m} T_l(t) \; (\forall t > 0)$$

With regard to the robustness of the closed loop system two types of uncertainties are nominated. The first is the viscous friction B_m which is in general not known but not significantly varying upon the time. The second is a controller implementation error resulting in computing the derivative of the current. In practice the current derivative is implemented in the controller as follows: $\frac{di_r}{dt} + e_d(t)$ (instead of $\frac{di_r}{dt}$). The quantity $e_d(t)$ is the implementation error of the derivative of the current. Clearly, this is a highly time varying uncertainty. Taking into account the derivative error the controller takes on the form $V_r = L_r \frac{di_r}{dt} + L_r e_d + (k_b - \frac{1}{r})\omega_m + R_r i_r + gw$.

Substituting the controller to the open loop system (2.1), the resulting closed loop system takes on the form $\omega_m = rgw + L_r r e_d$ and the output variable ω_l is $\omega_l = gw + L_r e_d$.

The output performance error $|\omega_l(t) - gw(t)|$ is equal $|L_r e_d|$. This is linear with respect to e_d and independent upon B_m. Note that the controller is also independent of B_m. Clearly the robustly good performance of closed loop system is guaranteed for any B_m and for small current derivative implementation error.

For the implementation of the control law, it is mentioned that measurements of ω_l (eq. ω_m) and i_r are used. The first variable can be measured via a potentiometer or a high resolution (great number of holes per circle) optical encoder while the second via ammeter.

2.3 Position Control

In this subsection the problem of controlling the position of the load is studied. To the system (2.1) apply the P-D feedback law

$$V_r = f_1 \frac{d\theta_m}{dt} + f_2 \frac{d\omega_m}{dt} + f_3 \frac{di_r}{dt} + k_1 \theta_m + k_2 \omega_m + $$
$$+ k_3 i_r + gw \qquad (2.7)$$

where f_1, f_2, f_3 and k_1, k_2 , k_3 are the degrees of freedom of the derivative and proportional term of the controller, respectively. The parameter w is the external command. Choosing the degrees of freedom to be

$$f_1 = k_b - \lambda_1, \quad f_2 = 0, \quad f_3 = L_r, \quad k_1 = -\frac{1}{r}, \quad k_2 = \lambda_1,$$
$$k_3 = R_r \qquad (2.8)$$

where λ_1 is a degree of freedom, the control law takes on the form

$$V_r = (k_b - \lambda_1)\frac{d\theta_m}{dt} + L_r \frac{di_r}{dt} - \frac{1}{r}\theta_m + \lambda_1 \omega_m + R_r i_r + gw \qquad (2.9)$$

Substituting the controller (2.9) to the DC motor description (2.1) the resulting closed loop system becomes

$$\omega_m = rg\frac{dw}{dt} \ , \ i_r = \frac{J}{k_m}rg\frac{d^2w}{dt^2} + \frac{B_m}{k_m}rg\frac{dw}{dt} + \frac{r}{k_m}T_l \ , \ \theta_m = rgw \qquad (2.10a)$$

while the output variable θ_l takes on the form and the output

$$\theta_l = gw \qquad (2.10b)$$

From (2.10b) it is concluded that using the feedback law (2.9), perfect output has been achieved for position control.

With regard to the closed loop system BIBO stability, it is mentioned that if $w(t)$ is bounded the boundness of θ_m and θ_l is guaranteed. Furthermore if the first and the second derivative of $w(t)$ is bounded then, according to (2.6a) the current i_r is bounded. With regard to asymptotic stability it is observed that for $w(t) = 0$ it holds that

$$\theta_l(t) = 0, \ \ \theta_m(t) = 0, \ \ \ i_r(t) = \frac{r}{k_m}T_l(t) \ (\forall t > 0)$$

With regard to the robustness of the closed loop system, the uncertainties B_m, $e_d(t)$ (current derivative implementation error) and $e_\omega(t)$ (position derivative implementation error) must be considered. If there exist an optical encoder for speed measurement there is no implementation error $e_\omega(t)$ in computing $\frac{d\theta_m}{dt}$. For this case the controller takes on the form: $V_r = (k_b - \lambda_1)\frac{d\theta_m}{dt} + L_r\frac{di_r}{dt} + e_dL_r - \frac{1}{r}\theta_m + \lambda_1\omega_m + R_ri_r$ $+gw$. Substituting the controller to the open loop system (2.1), the resulting closed loop system takes on the form: $\theta_m = rgw + L_rre_d$ and the output variable θ_l is $\theta_l = gw + L_re_d$.

The output performance error $|\theta_l - gw|$ is $|L_re_d|$. It is linear with respect to e_d and independent upon B_m. Note that the controller is independent of B_m, the good performance of closed loop system is guaranteed for small derivative implementation error. If only the position θ_l (eq. θ_m) is measured (f.e. via a potentiometer or an optical encoder) there is one additional uncertainty in computing ω_m, let e_ω. In this case the term $\frac{d\theta_m}{dt}$ in the controller substituted by $\frac{d\theta_m}{dt} + e_\omega$.

3. SIMULATION FOR THE 75 WATT DC MOTOR (35NT2R82-426SP)

The previous results will be applied to control a 75 Watt DC permanent magnet motor. The parameters of the motor are: $B_m = 0.002\ [kgm^2/s]$, $J = 71.4 \cdot 10^{-7}[kgm^2]$, $k_m = 0.0497[Nm^2/A]$, $r = 2$, $k_b = 0.312[Vs]$, $R_r = 2.2[\Omega]$ and $L_r = 0.0004[H]$. The unknown load torque is shown in Fig. 1.

3.1 Speed control

Choosing $g = 2$, the P-D feedback law is (given in (2.5)):

$$V_r = 0.000714\frac{di_r}{dt} + (0.312 - \frac{1}{2})\omega_m + 2.2i_r + 2w$$

The desired angular velocity of the permanent magnet DC motor is $20\ [1/s]$. The derivative term in the controller is implemented via a backward discretization (sample and hold) with sampling period equal to 0.001 [sec]. The performance of the DC motor variables, for external input $w = 10\ [V]$ and zero initial conditions, is illustrated in Fig. 2a-d, where all motor's variables are presented.

3.3 Position control

According to relation (2.9) and choosing $g = 2$, $\lambda_1 = 0$ (there is no measurement of the angular velocity), the P-D feedback law is:

$$V_r = 0.312\frac{d\theta_m}{dt} + 0.0004\frac{di_r}{dt} - \frac{1}{2}\theta_m + 2.2i_r + 2w$$

The desired position of the permanent magnet DC motor is chosen to be $4\ [rad]$. The derivative terms in the controller are implemented via a backward discretization (sample and hold) with sampling period equal to 0.001 [sec]. The control input (voltage) is considered to be saturated at $18\ [V]$. Saturation is a standard characteristic of any DC motor driver. The performance of the DC motor variables (external input $w = 2\ [V]$ and zero initial conditions) are shown in Fig. 3a-3d.

4. CONCLUSIONS

The problem of perfect position or speed control with simultaneous load torque rejection (with respect to the load torque) for a permanent magnet DC motor, has been studied using P-D feedback law. The problem has been proven to be always solvable. The responses of all state variables have been determined. Stability properties of the closed loop system have been investigated. The performance of the closed loop system, in cases where the friction coefficient is unknown and the derivative term is implemented with a relative error, has been studied. It has been proven that the performance of the closed loop system is independent upon the friction coefficient and is linear with respect to the derivative implementation error. The results of the paper have been illustrated via simulations for an Escap 35NT2R82-426SP permanent magnet DC motor.

5. REFERENCES

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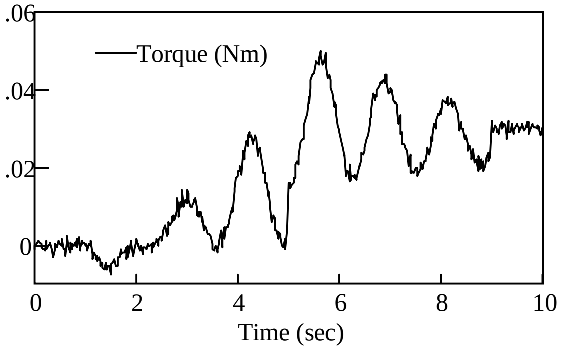

Fig. 1. Load Torque

Speed Control

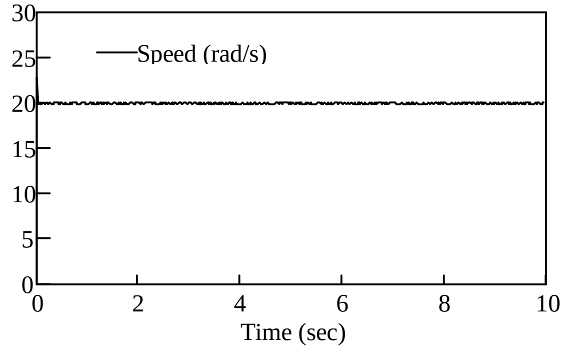

Fig. 2a. Speed

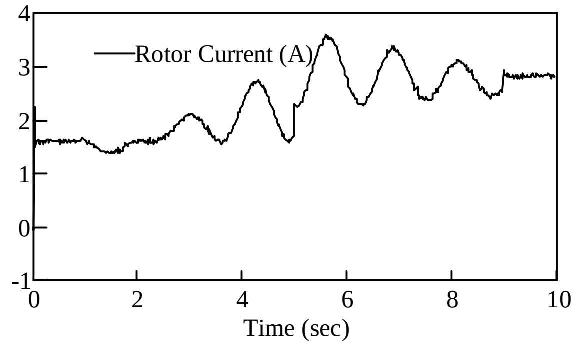

Fig. 2b. Rotor Current

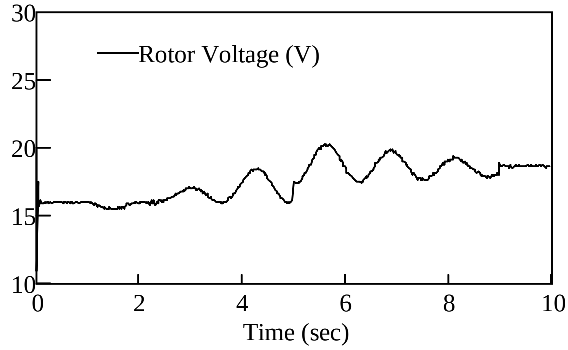

Fig. 2c. Rotor Voltage (Controller Output)

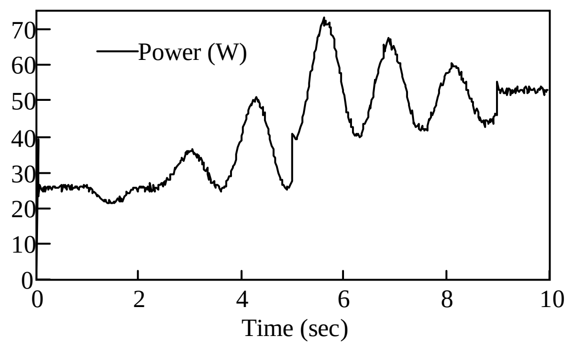

Fig. 2d. Motor Power

Position Control

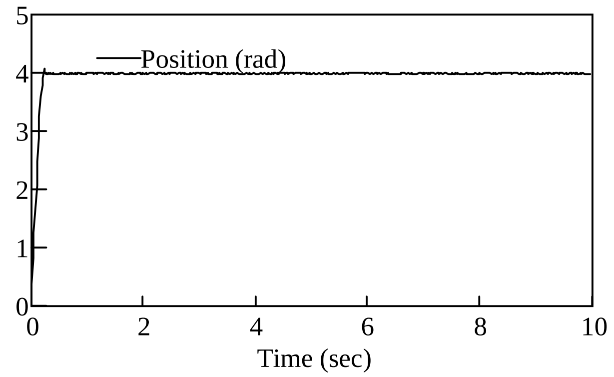

Fig. 3a. Position

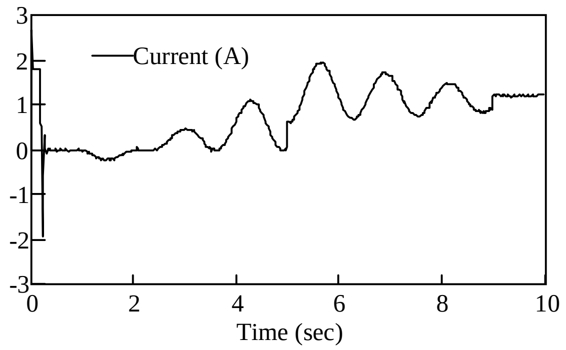

Fig. 3b. Rotor Current

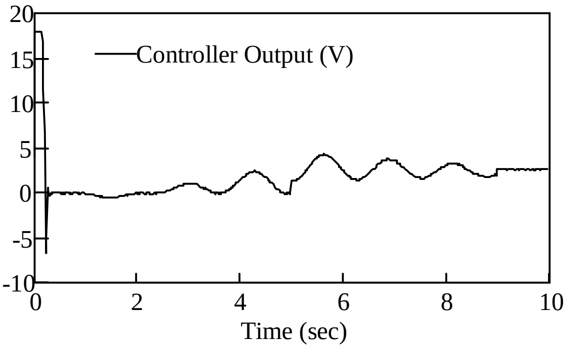

Fig. 3c. Rotor Voltage (Controller Output)

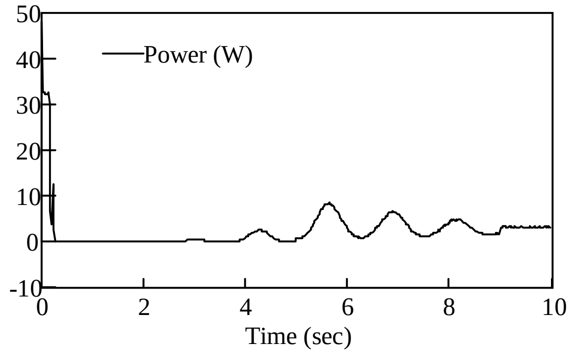

Fig. 3d. Motor Power