Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

ON THE PERFORMANCE OF MULTIMODEL ADAPTIVE CONTROL

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Abstract. The performance of multimodel adaptive control based on switching and tuning will be studied via several simulation examples for a flexible transmission system. The effects of some design parameters like number of fixed and adaptive models and forgetting factor will be considered. The performance of a recently developed parameter adaptation algorithm based on closed loop output error will be compared with the classical least squares prediction error algorithm in the multimodel adaptive control.

Key Words. Adaptive control, multimodel, switching, closed loop identification.

1. INTRODUCTION

The plants subject to abrupt and large parameter variations are generally very difficult to control. A classical adaptive controller or a fixed robust controller encounter the difficulties to solve this problem. An adaptive controller is not fast enough to follow the parameter variations and unacceptable transients occur. Whereas a fixed robust controller normally leads to poor performances because of large uncertainties.

A solution based on switching between different controllers for this type of plants has been probably proposed for the first time in [5]. The main problem of switching is to decide when a controller should be switched to the plant. The approach based on multiple models and switching will allow the transient responses to be improved in the presence of large and fast parametric variations [6,7,9]. In this approach, we suppose that a set of models for different operating points is a priori known. Then at every instant a controller corresponding to the model yielding the minimum of a performance index is used to compute the control input. The precision of the control can be further improved using an adaptive model (i.e. a model whose parameters are updated with a parameter adaptation algorithm) in the set of models. This method together with a stability analysis was proposed by Narendra and Balakrishnan in [8].

Although it has already been shown [9,8,2] that the performance of a system can be significantly improved using the multiple model adaptive control based on switching and tuning, the design parameters selection for this approach has not been investigated in details. In this paper we try to study the effects of some design parameters of this approach on the performance of a flexible transmission system. This system is very interesting because the frequency characteristics of its model change drastically with load and makes it a suitable laboratory setup for robust and adaptive control.

The design parameters we consider for our study, are: number of fixed and adaptive models, type of parameter adaptation algorithm for adaptive models, and forgetting factor in the switching rule.

The rest of the paper is organized as follows. The flexible transmission is described in Section 2. The principles of the multiple models adaptive control based on switching and tuning will be presented briefly in Section 3. Section 4 explains the basis of the closed loop output error parameter

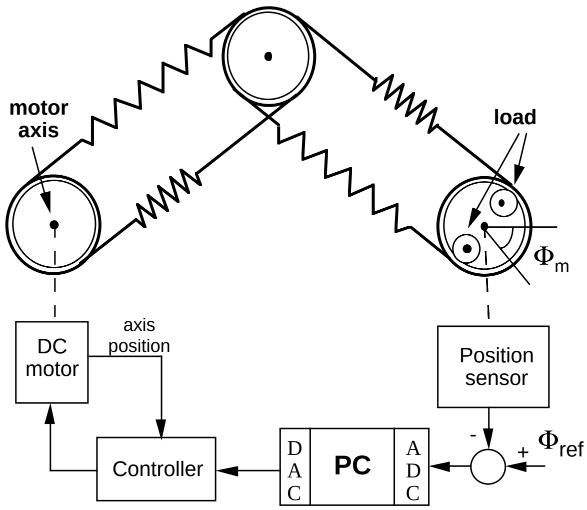

Fig. 1. Schematic diagram of the flexible transmission

estimation algorithm. The simulation results are given in Section 5 and finally, Section 6 presents the concluding remarks.

2. SYSTEM DESCRIPTION

We consider the flexible transmission system built at Laboratoire d'Automatique de Grenoble (INPG-CNRS), France, which has been used for a benchmark in robust digital control at European Control Conference in Rome, 1995 [4]. The system consists of three horizontal pulleys connected by two elastic belts (see Fig. 1). The first pulley is driven by a D.C. motor whose position is controlled by local feedback. The objective is to control the position of the third pulley which may be loaded with small disks (maximum 12 disks of 300gr). The system input is the reference for the axis position of the first pulley. A PC is used to control the system. The sampling frequency is 20Hz.

The system has a pure time delay equal to two sampling periods and an unstable zero. The system is characterized by two low damped vibration modes (with damping factors of less than 0.05), subject to a large variation in the presence of load. A variation of about 100% of the frequency of the first vibration mode occurs when passing from the full loaded case (12 disks) to the unloaded case.

3. MULTIMODEL ADAPTIVE CONTROL

The main idea of this method is to choose the best model for the plant from an a priori known set of models at every instant and apply the output of the corresponding controller to the plant. Since the number of available models is finite but the number of possible models is generally infinite, the identification is performed in two steps: (1) the model with smallest error with respect to

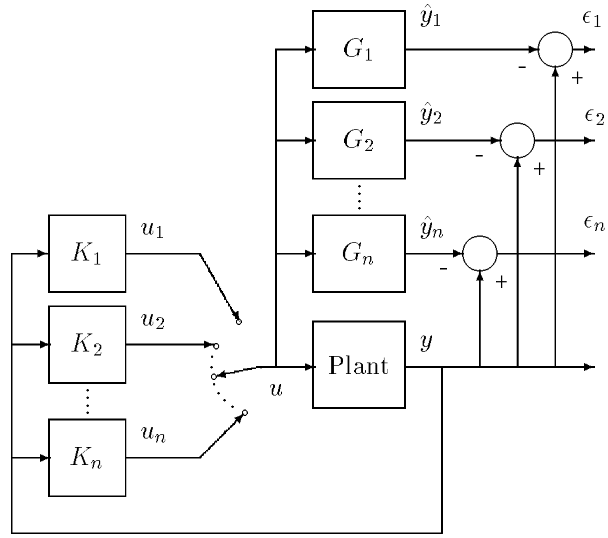

Fig. 2. Schematic diagram of the multiple models approach

a criterion is rapidly chosen (switching), (2) the parameters of the model are adjusted using a parameter adaptation algorithm (tuning).

The block diagram of this method is presented in Fig. 2. The input and output of the plant are $u(t)$ and $y(t)$, respectively. The control system contains n models $G_1, \ldots, G_n$ which are either fixed or adaptive models. The identification error is defined as the difference between the output $\hat{y}_i$ of the model G_i and the plant output ($\epsilon_i = y(t) - \hat{y}_i(t)$). For each model G_i, there is a controller K_i that satisfies the control objective for G_i (instead of n controller we may have a parameterized controller $K(G_i)$). The performance criterion $J_i(t)$ which is used as the switching rule may be defined as follows [8]:

$$J_i(t) = \sum_{j=0}^{t} e^{-\lambda(t-j)} \epsilon_i^2(j) \quad \lambda > 0 \qquad (1)$$

Where λ is a forgetting factor which also assures the boundedness of the criterion for bounded $\epsilon_i(t)$. Another design parameter T_d (dwell time), the minimum time delay between two switchings, plays an important role on the stability analysis of the system. For more details see [8,10].

4. CLOE ALGORITHM

The closed loop output error recursive adaptation algorithm (CLOE) presented in [3] is based on a reparameterized adjustable predictor for the closed loop system in terms of a known fixed controller K and an adjustable plant model $\hat{G}$. Fig. 3 shows the block diagram which is often used in closed loop identification. The main advantages of this algorithm with respect to the classical recursive least squares (RLS) algorithm are as follows:

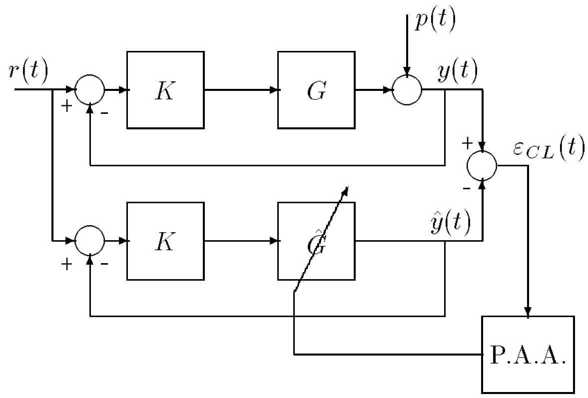

Fig. 3. Closed loop output error identification scheme

- The parameter estimates are unbiased in the presence of noise when the identified model is in the model set [3].
- The frequency distribution of the modeling error is weighted by two sensitivity functions when the identified model is not in the model set. This leads to identify a suitable model for robust control design [1].
- The use of this algorithm in indirect adaptive control removes the need for adaptation freezing [2].

A recursive algorithm minimizing ε_{CL} in Fig. 3 together with a stability and a convergence analysis have been given in [3].

5. SIMULATION RESULTS

In this section several simulation examples will be performed in order to show the effects of the design parameters on the performance of the flexible transmission system. The simulations are carried out by VisSim [11] software. For this purpose different functions have been developed in order to realize the multimodel adaptive control on this software.

Selection of appropriate values for the design parameters depends upon some information about the plant, like: plant model in different operating points, speed of parameter variations of the plant model, existence and type of the reference signal, existence and type of the output disturbances and variance of the output noise.

For the flexible transmission system, in order to simulate the above mentioned characteristics, thirteen discrete time identified models of plant numbered from 0 to 12 (related to the number of disks on the third pully) are considered. Then we suppose that plant is initially unloaded and the small disks are placed on the third pulley one by one until the system becomes full loaded (with 12 disks). Next, the disks are taken off one by one and system again becomes unloaded. It is supposed that this load changing is repeated cyclically with period T_c and we refer to $f_c = 1/T_c$ as the parameter changing rate. Therefore a small value for f_c indicates a system with nonfrequent (spaced) parameter variations and a large value for f_c simulates a system with frequent and large parameter variations. The reference signal is either null or a filtered square wave signal (filtered by a reference model) with an amplitude of 1 and a period of 10s. The output disturbance signal is also either null or a pulse train with an amplitude of 0.5 and a period of 20s. A zero-mean normally distributed white noise is added to the plant output. The noise variance is varied in different simulations to study the noise effect.

The objective of the control system is to follow the reference input and to reject the output disturbances as fast as possible. Thus, in order to compare different design parameters a performance index is defined as follows:

$$J_c = \left(\frac{1}{T_f}\int_0^{T_f} \varepsilon_c^2(t)dt\right)^{1/2} \tag{1}$$

where $\varepsilon_c(t) = r(t) - y(t)$ and T_f is the simulation time.

Design of a multimodel adaptive control system consists of the following steps: (1) determine the number of fixed and adaptive models, (2) choose the adaptation algorithm (RLS or CLOE), (3) determine the forgetting factor λ, (4) determine the minimum time between switchings T_d, (5) choose a controller for each model or determine a control law based on the model parameters.

In this paper we are not going to discuss about the design of the controllers (step 5), although it affects significantly the overall performance of the system. In fact, we suppose that readers are able to design a controller for a fixed model which satisfies the specifications. However, in the simulations we use a two-degree of freedom digital robust controller designed by the pole placement with sensitivity function shaping method described in [2]. This controller is robust with respect to additive uncertainties and contains an integrator to reject constant disturbances and to obtain a zero steady state error. In what follows, we show how the performance of a multimodel adaptive control system is related to the design parameters.

5.1 Number of fixed and adaptive models

The first step in the multimodel control design is to determine the number of fixed and adaptive models. Principally, better performances will be achieved with more fixed models. However, the price is a more complex control system which leads to more computation time and less reliability. An adaptive model can reduce the number

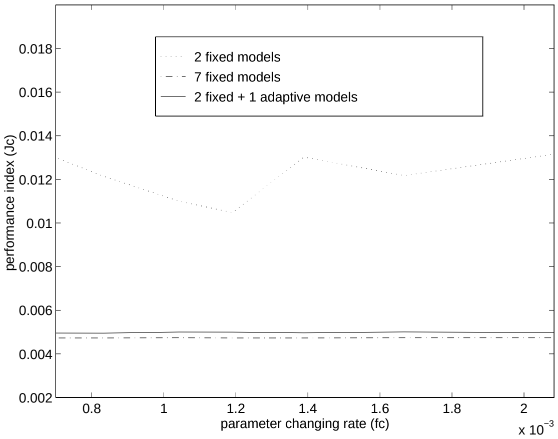

Fig. 4. Performance index versus parameter changing rate (spaced parameter variations)

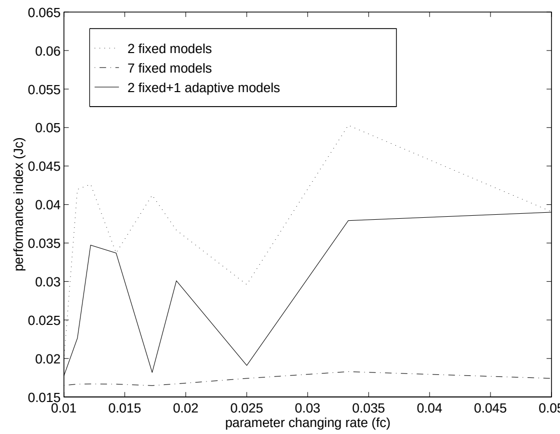

Fig. 5. Performance index versus parameter changing rate (frequent parameter variations)

of fixed models under the conditions that there exists an excitation signal on the reference input and the abrupt changes of parameters are sufficiently spaced in the time (i.e. there is enough time between two changes for parameter adaptation). Therefore for the systems in regulation (with a fixed reference signal) adaptive models should not be used in the models set. The following simulations show that when the parameter variations are sufficiently spaced, one adaptive model can reduce the number of fixed models without changing in the overall performance. But when the parameter variations are frequent, an adaptive model has less effect.

In this simulation example, it is supposed that the period of the cyclic changes in the model parameters is $T_c \geq 480s$ (there is at least 20s between the parameter variations). For the models set of the control system three combinations are considered as follows: (1) Two fixed models (No. 0,9), (2) Seven fixed models (No. 0,2,4,6,8,10,12), (3) Two fixed (No. 0,9) and one adaptive models.

The forgetting factor λ is chosen equal to 0.5 and $T_d = 1$ sampling period (50 ms). The simulation time T_f is 1440s. The performance index J_c versus the parameter changing rate ($f_c = 1/T_c$) is plotted for the three cases in Fig. 4. One can observe that the performance is improved when the number of fixed models is increased. It should be mentioned that the performance cannot be improved significantly using more than 7 fixed models, because the controllers are robust and give also a good performance even when the plant model is not among the fixed models of the control system. It is also observed that in the third case (one adaptive model and 2 fixed models) we have almost the same performance as the second case (7 fixed models) which shows that one adaptive model can replace five fixed models.

The second simulation example is performed under the same condition as the last simulation with the difference that the period T_c of the parameter cyclic changes is between 20s and 100s ($20 \leq T_c \leq 100$) which indicates very frequent parameter variations. A shorter simulation time $T_f = 120s$ and a greater forgetting factor $\lambda = 0.1$ are also chosen. Fig. 5 depicts that in this case an adaptive model does not change so much the performances of the system. The reason is that, the frequent parameter variations of the plant cause rapid switching between the models of the control system and the adaptive model has not enough time to adapt their parameters between two switchings. Thus in such a situation only fixed models should be employed.

5.2 Parameter adaptation algorithm

The second step of the multimodel adaptive control design is to choose the type of adaptation algorithm for the adaptive model (or models). In this section we will show that the CLOE adaptation algorithm gives better performances than the classical RLS algorithm in the presence of noise. We consider 3 fixed (0,6,12) and one adaptive model in the models set of the control system. Two distinct simulations are carried out, one using CLOE algorithm in the adaptive model and the other with RLS adaptation algorithm. The plant model is supposed to be fixed on the model No.3 which does not belong to the fixed models of the control system. Therefore the switching will be stopped after a time on the adaptive model and the parameters of the adaptive model will be tuned by the adaptation algorithm. The parameters of the switching part are chosen as follows: $\lambda = 0.05$, $T_d = 1$ sampling period and the simulation time T_f is 120s. The variance of the output noise is increased from 0 up to 0.07 and the performance index of the system is plotted versus the noise variance in Fig. 6. As depicted in this figure, increasing the noise variance will deteriorate the system performances in both cases, but CLOE algorithm gives better performances especially when the noise variance is high. Fig. 7 shows the output, the reference and

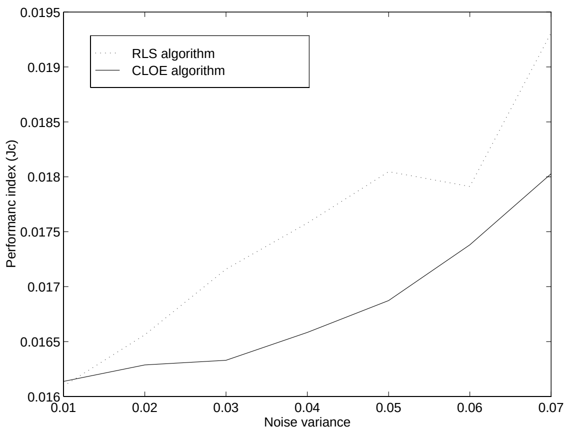

Fig. 6. Performance index versus noise variance (comparison of the adaptation algorithms)

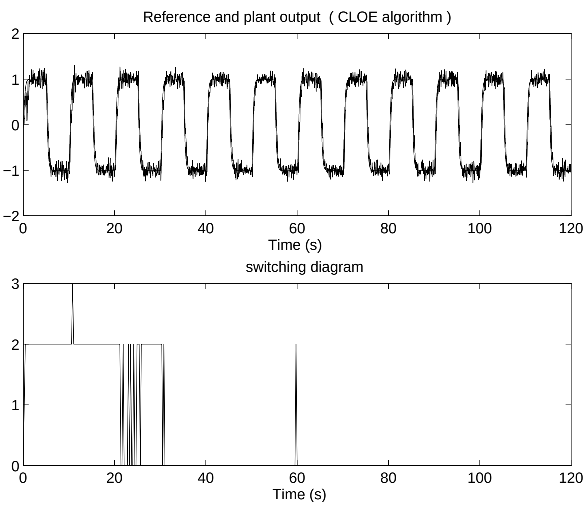

Fig. 7. Simulation results using the CLOE adaptation algorithm

the switching diagram for this simulation using the CLOE algorithm with a noise variance of 0.07. The switching diagram shows the best model chosen by the supervisor at each instant. In this diagram 0 corresponds to the adaptive model and 1, 2 and 3 correspond respectively to model no. 0,6,12. This figure can be compared with Fig. 8 corresponding to the RLS algorithm. It can be observed that the larger variations of the output (using RLS algorithm) lead to the unwanted switchings which consequently deteriorate the performances.

5.3 Forgetting factor λ

The forgetting factor λ in the switching rule plays an important role on the performance of the control system. The speed of parameter changes, the variance of output noise and the type of output disturbances affect the choice of λ. In order to study these effects four simulations are performed. Like the preceding simulation example three fixed models (No. 0,6,12) and one adaptive model (using CLOE adaptation algorithm) are considered in the

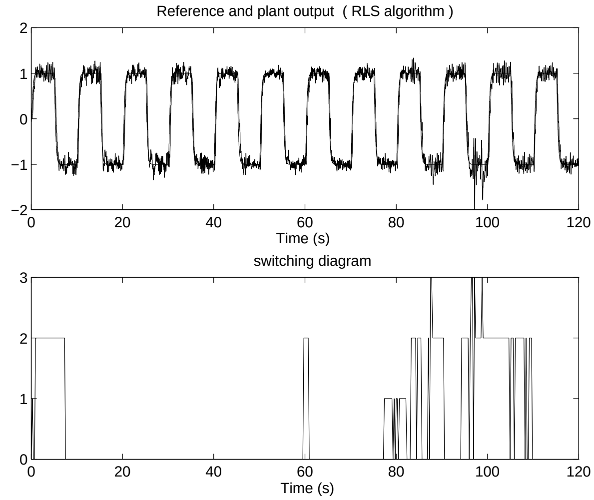

Fig. 8. Simulation results using the RLS adaptation algorithm

set of models of the control system. The dwell time is $T_d = 1$.

In the first part we choose the period of the parameter changes $T_c = 480s$ which represents spaced parameter variations. The simulation results for $T_f = 480s$ are presented in Fig. 9(a). One can observe that for this type of parameter variations we should select a small value for λ. Because for small λ the switching criterion approaches to a model identification criterion which leads to select the best model among the models set for the plant.

In the second part, in order to simulate frequent and fast parameter variations, the period of the parameter changes is chosen equal to 20s. It means the model of the plant changes from model No. 0 to 12 and return to 0 in 20s. In Fig. 9(b) the performance index J_c for $T_f = 120s$ is plotted for different values of λ. It clearly shows that the larger values for λ lead to the better performances of the system. The reason is that for a large value of λ the latest errors have more weightings in the switching criterion which cause a very quick response to the abrupt parameter changing.

In the next simulation, the output disturbance is modeled as a square wave signal with 0.5 amplitude and 20s period added to the output of the plant. The plant model is fixed (model No. 3) and the parameters of the switching part is the same as preceding simulations. The simulation results for $T_f = 100s$ (Fig. 9(c)) illustrate that smaller λ prevents the unwanted switchings and rejects better the disturbances.

In order to study the noise effect on the performance of the control system, the plant model is fixed to the model No. 3 and the noise variance is 0.1. It is shown in Fig. 9(d) that for a noisy system λ should be kept small in order to avoid the unwanted switchings. A large value for λ makes

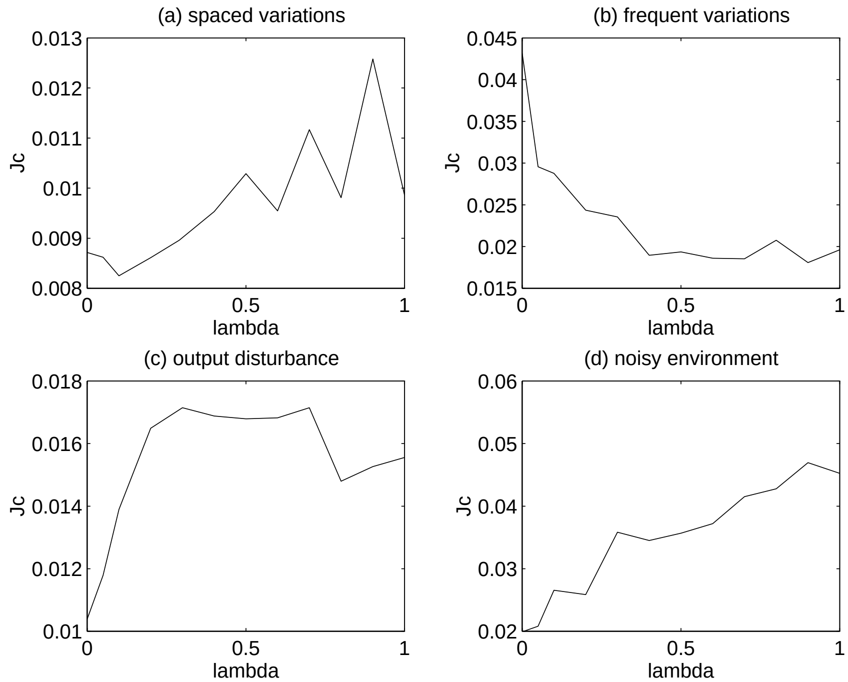

Fig. 9. Performance index versus λ in different situations

the switching criterion very sensitive to noise and leads to poor performances. It should be noticed that the choice of λ for a noisy system with spaced parameter variations subject to output disturbance is nonconflicting, but for a system with frequent and large parameter variations is conflicting. In such situations the choice of λ should be performed with precautions. However, experiences shows that a large value (greater than 0.5) for λ should be avoided.

6. CONCLUSIONS

The design parameters selection of multimodel adaptive control has been investigated via several simulations for a flexible transmission system. The effects on the overall system performance of number of fixed and adaptive models, type of adaptation algorithm and forgetting factor have been studied.

7. REFERENCES

[1] A. Karimi and I. D. Landau. Comparison of the closed loop identification methods in terms of the bias distribution. *Systems and Control Letters*, (34):159–167, 1998.

[2] A. Karimi and I. D. Landau. Robust adaptive control of a flexible transmission system using multiple models. In *37-th CDC*, Tampa, Florida USA, December 1998.

[3] I. D. Landau and A. Karimi. An output error recursive algorithm for unbiased identification in closed loop. *Automatica*, 33(5):933–938, May 1997.

[4] I. D. Landau, D. Rey, A. Karimi, A. Voda, and A. Franco. A flexible transmission system as a benchmark for robust digital control. *European Journal of Control*, 1(2), 1995.

[5] B. Martensson. *Adaptive Stabilization.* PhD thesis, Lund Institute of Technology, Lund, Sweden, 1986.

[6] A. S. Morse. Control using logic-based switching. In A. Isidori, editor, *Trends in Control.* Springer Verlag, Heidelberg, 1995.

[7] A. S. Morse, D. Q. Mayne, and G. C. Goodwin. Application of hysteresis switching in parameter adaptive control. *IEEE Transactions on Automatic Control*, 37:1343–1354, September 1992.

[8] K. Narendra and Balakrishnan. Adaptive control using multiple models. *IEEE Transactions on Automatic Control*, 42(2):171–187, February 1997.

[9] K. S. Narendra and J. Balakrishnan. Improving transient response of adaptive control systems using multiple models and switching. *IEEE Transactions on Automatic Control*, 39:1861–1866, September 1994.

[10] K. S. Narendra and G. Xiang. Adaptive control of discrete-time systems using multiple models. In *37-th CDC*, Tampa, Florida USA, December 1998.

[11] Visual Solutions, Inc., 487 Groton Rd. Westford, MA USA. *VisSim users's guide*, 1996.