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PID CONTROL FOR INDUCTION MOTORS IN FIELD COORDINATES

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Abstract. The problem of flux and speed control of induction motors modelled in field coordinates, is studied. First, a P-I controller is applied that satisfies the requirement of current command following. Additionally a P-D controller is applied. The P-D controller provides flux and speed command following with simultaneous rejection of the load torque. Finally, a discrete observer being suitable for on line implementation is proposed. The present results are illustrated via simulation for a M3541 Baldor industrial motor.

Key Words. AC-motors, PID Control, Nonlinear Systems, Speed/Position Control, Disturbance Rejection

NOMENCLATURE

$V_1, V_2, V_3;\ i_1,\ i_2, i_3$	: stator 3-phase voltages; stator 3-phase currents
$V_{sa}, V_{sb}, i_{sa}, i_{sb}$	: stator voltages and currents in (a,b) stator coordinate system
$V_{sd}, V_{sq}, i_{sd}, i_{sq}$	: stator voltages and currents in (d,q) field coordinate system
ψ_{rd}	: rotor flux linkage in (d,q) field coordinate system
ρ	: rotor flux angle in (d,q) field coordinate system
$R_s,\ R_r;\ L_s,\ L_r;\ M$	: stator and rotor resistance; stator and rotor self-inductance; mutual inductance
$n_p,\ J,\ D$	: number of pole pairs, rotor moment of inertia, damping coefficient
$\omega,\ T_L$	: rotor angular speed, load torque
$\sigma,\ \alpha,\ \beta$	: $1-M^2/L_sL_r$ (leakage factor), R_r/L_r (rotor time constant), $M/\sigma L_s L_r$
$\gamma,\ \mu$	: $(M^2R_r/\sigma L_s L_r^2)+(R_s/\sigma L_s)$, $3n_pM/2JL_r$

1. INTRODUCTION

The problem of controlling induction motors, via nonlinear control laws, has attracted considerable attention since the early 70’s. There is a variety of techniques as well as design goals related to the treatment of such electromechanical systems (see f.e. [1]-[8] and the references there in).

In this paper a PID control design scheme is proposed. The problem of flux and speed control of induction motors, modelled in field coordinates, is studied. First, a P-I controller is applied that satisfies the requirement of current command following. Additionally a P-D controller is applied. Using the P-D controller, flux and speed command following with simultaneous rejection of the load torque is accomplished. Finally a discrete observer, being suitable for on line implementation, is proposed. The present results are illustrated via simulation for a M3541 Baldor industrial motor.

It is important to mention that the P-I controller satisfying the requirement of current command following, appears to be a standard strategy in the field (see f.e. [1], [10]). The use of the additional P-D feedback law appears to be a beneficial result in the sense that the load torque is rejected. Comparing the present results with those in [9], where the controller is of pure P-D type, it is mentioned that the present results appear to be more robust with respect to errors in the implementation of the derivative term.

2. TRANSFORMATIONS

The three stator currents i_1, i_2, i_3, taking under consideration the restriction from the isolated neutral $(i_1 + i_2 + i_3 = 0)$, can be transformed to two phase (i_{sa}, i_{sb}) stator quantities as follows (see also [1-3] and [8])

$$\begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/\sqrt{3} & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad (1)$$

The currents, can be transformed from the stator frame (i_{sa}, i_{sb}) to the field frame (i_{sd}, i_{sq}) as follows [10]

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} \qquad (2)$$

where ρ is the angle of the rotor flux (d–axis) with respect to the a–axis of the stator. The analogous transformations (inverse) for the stator voltages are [1-3],[8],[10]

$$\begin{bmatrix} V_{sa} \\ V_{sb} \end{bmatrix} = \begin{bmatrix} \cos\rho & -\sin\rho \\ \sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} \qquad (3)$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{sa} \\ V_{sb} \end{bmatrix} \qquad (4)$$

3. INDUCTION MOTOR MODEL IN FIELD COORDINATES

The model of the induction motor in the rotor flux field oriented coordinate system is [1-3],[8],[10]

$$\frac{d}{dt}\begin{bmatrix} \omega \\ \psi_{rd} \\ \rho \\ i_{sd} \\ i_{sq} \end{bmatrix} =$$

$$= \begin{bmatrix} \mu\psi_{rd} i_{sq} - (D/J)\omega - (1/J)T_L \\ -\alpha\psi_{rd} + \alpha M i_{sd} \\ n_p\omega + \alpha M i_{sq}/\psi_{rd} \\ -\gamma i_{sd} + \alpha\beta\psi_{rd} + n_p\omega i_{sq} + \alpha M i_{sq}^2/\psi_{rd} \\ -\gamma i_{sq} - \beta n_p\omega\psi_{rd} - n_p\omega i_{sd} - \alpha M i_{sq} i_{sd}/\psi_{rd} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/\sigma L_s & 0 \\ 0 & 1/\sigma L_s \end{bmatrix} \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} \qquad (5)$$

In order to reduce complexity, it is common practice [1],[10], to use current-command strategy instead of voltage-command. This can be achieved, using fast P-I current loops of the following form

$$V_{sd} = K_{dP}(i_{sd}^* - i_{sd}) + K_{dI}\int_0^t (i_{sd}^* - i_{sd})dt \qquad (6.a)$$

$$V_{sq} = K_{qP}(i_{sq}^* - i_{sq}) + K_{qI}\int_0^t (i_{sq}^* - i_{sq})dt \qquad (6.b)$$

where i_{sd}^*, i_{sq}^* are the reference commands of the currents. K_{dP}, K_{qP} are the proportional gains and K_{dI}, K_{qI} are the integral gains of the PI current controller. Large values of these gains lead to satisfactory command following (current tracking) (i.e. $i_{sd} \simeq i_{sd}^*$ and $i_{sq} \simeq i_{sq}^*$) [10]. Considering this realistic assumption, the model of the induction motor can be approximated by the model

$$\frac{d}{dt}\begin{bmatrix} \omega \\ \psi_{rd} \\ \rho \end{bmatrix} = \begin{bmatrix} \mu\psi_{rd} i_{sq}^* - (D/J)\omega - (1/J)T_L \\ -\alpha\psi_{rd} + \alpha M i_{sd}^* \\ n_p\omega + \alpha M i_{sq}^*/\psi_{rd} \end{bmatrix} \qquad (7)$$

It can readily be observed that the last two equations of the induction motor model (5) have been eliminated. In the current-command motor model (7), it can also be observed that the first equation is activated by the disturbance (i.e. load torque, friction, inertia) while the last two equations are activated by the reference currents. In the following section a P-D controller will additionally be applied.

4. P-D CONTROLLER FOR SPEED AND FLUX COMMAND FOLLOWING

According to the literature [2], [3], [10], the most common way to control the speed and the rotor flux of the induction motor in field coordinates, is to use PI controllers. PI controllers satisfying speed and flux control (but not independently from the load torque) is of the following form

$$i_{sd}^* = K_{P1}(w_1 - \psi_{rd}) + K_{I1}\int_0^t (w_1 - \psi_{rd})dt \qquad (8.a)$$

$$i_{sq}^* = [K_{P2}(w_2 - \omega) + K_{I2}\int_0^t (w_2 - \omega)dt]/\mu\psi_{rd} \qquad (8.b)$$

where w_1, w_2 are the external commands for the rotor flux and the rotor speed, respectively.

Here, the proposed speed/rotor flux controller is not of PI type but it is of the P-D type. Particularly the proposed controller is

$$i_{sd}^* = \frac{1}{\alpha M}\Big[(\frac{d\psi_{rd}}{dt} + \alpha\psi_{rd}) + (w_1 - \psi_{rd})\Big] \qquad (9.a)$$

$$i_{sq}^* = \frac{\psi_{rd}}{\alpha M}\Big[(\frac{d\rho}{dt} - n_p\omega) + (w_2 - \omega)\Big] \qquad (9.b)$$

Using this controller, the resulting closed loop system is derived to be

$$\begin{bmatrix} \psi_{rd} \\ \omega \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{10.a}$$

$$\frac{d\rho}{dt} = n_p w_2 + \frac{\alpha M}{\mu w_1^2}\left[\frac{dw_2}{dt} + (D/J)w_2 + (1/J)T_L\right] \tag{10.b}$$

From (10a) it is observed that the flux and the speed are perfectly controlled while they are independent from the load torque.

5. FLUX AND FLUX ANGLE ESTIMATOR

For the P-D controller implementation, four quantities have to be available, namely:

a) the rotor speed ω,

b) the rotor flux ψ_{rd},

c) the derivative of the rotor flux $\frac{d\psi_{rd}}{dt}$ and

d) the derivative of the rotor flux angle $\frac{d\rho}{dt}$.

The value of rotor flux angle ρ is also necessary for the Park-transformations of currents and voltages, presented in (2) and (3), respectively. The rotor speed can be measured directly with an optical encoder, but the other quantities have to be estimated. The well known estimator [6],[8] for rotor flux ψ_{rd} and the angle ρ will be used, particularly extended to obtain also $\frac{d\psi_{rd}}{dt}$ and $\frac{d\rho}{dt}$. The proposed estimator is

$$\frac{d}{dt}\begin{bmatrix} \psi_{rd} \\ \rho \end{bmatrix} = \begin{bmatrix} -\alpha\psi_{rd} + \alpha M i_{sd} \\ n_p\omega + \alpha M i_{sq}/\psi_{rd} \end{bmatrix} \tag{11}$$

The inputs of the estimator are the d-q axis currents (i_{sd}, i_{sq}) and the rotor speed ω. The outputs of the estimator are the flux and the flux angle. Theoretically speaking, the efficiency of the estimator can be verified after subtracting the estimator from the last two equations of the system (7). Clearly the resulting error is equal to zero.

The values of the rotor flux, the rotor flux angle and their derivatives can be directly obtained by a discrete approximation of system (11). In particular the discretization of the estimator can be accomplished after taking into account the following approximations (forward discretization of the derivative)

$$\psi_{rd}\big|_{t=(k+1)T} = \psi_{rd}\big|_{t=kT} + \frac{d\psi_{rd}}{dt}\bigg|_{t=kT} T \tag{12.a}$$

$$\rho\big|_{t=(k+1)T} = \rho\big|_{t=kT} + \frac{d\rho}{dt}\bigg|_{t=kT} T \tag{12.b}$$

and the equations of the estimator at particular discrete time points

$$\frac{d\psi_{rd}}{dt}\bigg|_{t=(k+1)T} = -\alpha\ \psi_{rd}\big|_{t=kT} + \alpha M\ i_{sd}\big|_{t=kT} \tag{12.c}$$

$$\frac{d\rho}{dt}\bigg|_{t=(k+1)T} = n_p\ \omega\big|_{t=kT} + \alpha M \frac{i_{sq}\big|_{t=kT}}{\psi_{rd}\big|_{t=kT}} \tag{12.d}$$

where T is the sampling period and k is a greater than or equal to zero integer. Using the discretized system described in (12) the flux and the flux angle can iteratively be computed. To initiate the iterations the following initial conditions, namely the values of the variables at $k = 0$, are used:

$$\psi_{rd}\big|_{t=0} = \varepsilon\ (\varepsilon \to 0^+),\ \ \rho\big|_{t=0} = 0,\ \ \frac{d\psi_{rd}}{dt}\bigg|_{t=0} = 0,$$

$$\frac{d\rho}{dt}\bigg|_{t=0} = 0$$

It can readily be observed that the above algorithm for the implementation of the estimator appears to be simple and elegant. Higher order approximations can be used for special circumstances. The initial condition for the rotor flux ψ_{rd} can be considered to be, for the model (12), equal to ε. The number ε should be chosen to be small enough but avoid overflow in (12a).

The overall control scheme (controller + estimator) is illustrated in Fig. 1.

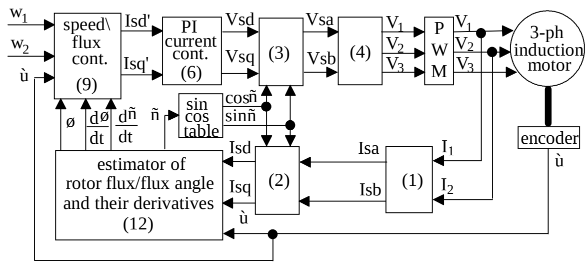

Fig. 1. P-D Field Oriented Control of Induction Motor

In Fig.1, the numbers in the parenthesis (inside the blocks) are the numbers of the respective relations in the paper. A PWM (Pulse Width Modulator) has been used, as well as a sine/cosine lookup table for the calculation of $\sin\rho$ and $\cos\rho$. A microcontroller is a suitable platform for the implementation of all parts in Fig.1

6. SIMULATIONS

The present results are applied to the case of the Baldor M3541 induction motor. This is a two-pole motor with a rated speed of 3450 rpm, rated voltage of 230 V and rated current of 2.7 A. A step load torque with a magnitude of $T_L = 0.3$ Nm can be considered to be attached to the rotor axis via suitable mechanical arrangement. The parameters of the induction motor model are [7]: $n_p = 1$, $R_s = 3.05\ \Omega$, $R_r = 2.12\ \Omega$, $L_s = 0.243$ H, $L_r = 0.306$ H, $M = 0.225$ H. The moment of inertia is considered to be $J = 2 \times 10^{-4}$ kg m^2 and the respective damping factor $D = 0.002$ N m sec rad^{-1}. The external commands are $w_1 = 0.8$ and $w_2 = 100$. Here, the desired performance is $\omega = 100$ rad/sec, $\psi_{rd} = 0.8$ Wb.

The output of the controller is a voltage source inverter driven by a PWM. A very realistic and practical case, even for large induction motors (up to 500 KW) [8], is an IGBT power unit working with a PWM frequency of 10 kHz. This corresponds to a time period of 100 ìs. This is also the period of the execution of the control algorithm, which is assumed to be implemented with a fast DSP (Digital Signal Processor). The currents are measured by an A/D converter with a quantization error of 20 mA. This is the case of a standard 10-bit A/D converter that measures currents in the range of ± 10 A It should be noted that the imposed stator voltages have limits at ± 300 V. The proportional gains of the current controllers (K_{qP}, K_{dP}) are chosen to be equal to 100 while the integral gains (K_{qI}, K_{dI}) are chosen to be equal to 1000.

The responses of the speed and the flux are shown in Fig. 3 and 4 respectively. In Fig. 4 and 5 the speed response and the flux responses are presented, for the case where the rotor resistance increases by 100%. This case corresponds to extreme thermal conditions in the motor (maximum temperature of rotor). Finally in Fig. 6 and 7 the speed and the flux responses are shown for the same rotor resistance increase (100%) while speed measurement error is considered. The speed error is considered to be about $\pm\pi$ rad/sec. This case is met with a 20000 ppr incremental encoder at the sampling rate of 10 kHz. The results of the simulation appear to be satisfactory. In case of rotor resistance variations and speed measurement error, the good performance of the controller is preserved.

7. CONCLUSIONS

The problem of flux and speed control of induction motors modelled in field coordinates, has been studied via a PID controller. First a current command following P-I controller has been applied. Next a P-D feedback law has been applied to yield perfect output control for the speed and the flux, independently from the load torque. According to the simulation the present results appear to be robust with respect to rotor resistance variations and controller implementation errors.

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Figure 2: rotor flux

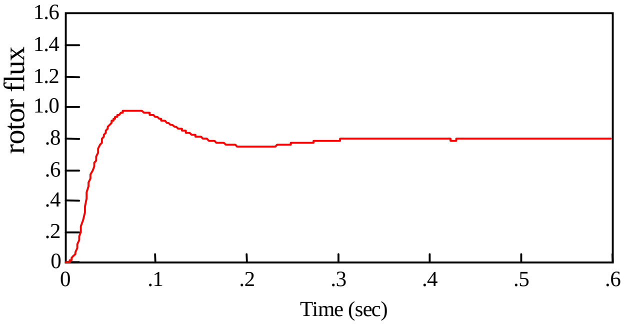

Figure 3: rotor speed

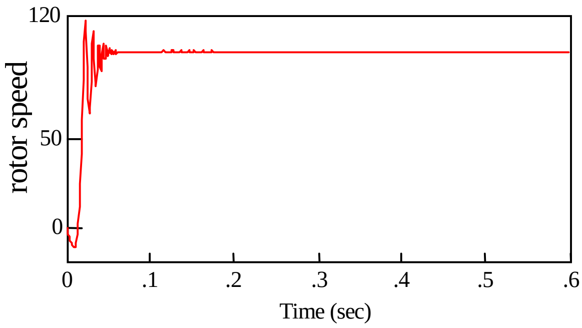

Figure 4: rotor flux (resistance variation)

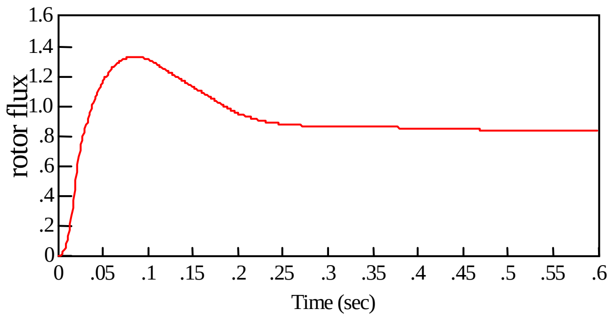

Figure 5: rotor speed (resistance variation)

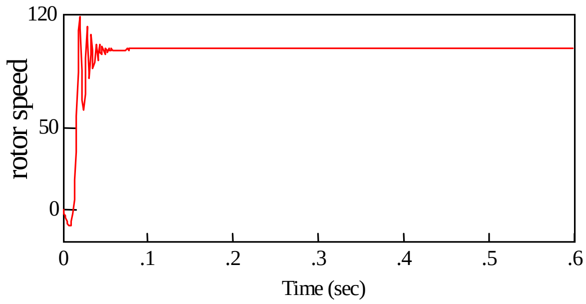

Figure 6: rotor flux (resistance variation and speed measurement error)

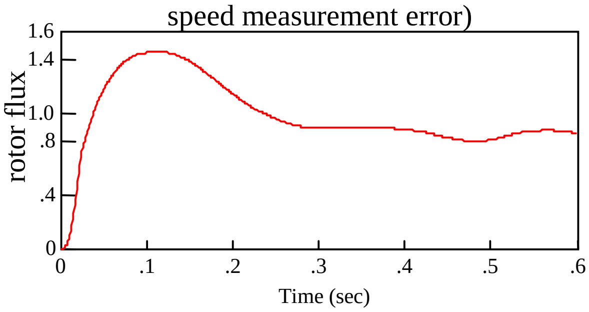

Figure 7: rotor speed (resistance variation and speed measurement error)