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SELECTIVE INVARIANCE IN MULTIVARIABLE CONTROL SYSTEMS WITH INTERNAL MODELS

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Abstract. The problem of disturbance decoupling in multivariable control systems is considered. It has been shown that different two-degree-of-freedom control structures used for unmeasurable disturbance estimation and compensation may be treated as a particular case of a general Inverse Model Control approach. The decomposition of the problem into the separate disturbance state and model estimation is suggested. Moreover the connection between inverse model design problems and unknown input observer theory has been established in order to give a practical way to inverse model parameterization and design. The properties of closed-loop system with model-based controllers have been also investigated with the aim of attainable accuracy estimation.

Key Words: disturbance decoupling, inverse systems, invariance, model-based control, unknown-input observers.

1. INTRODUCTION

Recently a number of innovative model-based control methods have been developed for multivariable systems taking into account the requirements of accuracy, dynamic performance, stability and robustness [8]. The role of model-based control methods is essentially increased when the control problem under uncertainty is considered. Because uncertainties of the plant may be treated as a parametric disturbance of nominal plant model, the disturbance decoupling has become on of the most important problem in advanced process control theory.

There are two main approaches to such a problem. First, namely disturbance attenuation methods, use the available *a priory* information about disturbances in statistical or deterministic (set-membership) form. At that the design solution is obtained in a class of simple feedback control structures and is formalized as an optimization problem with the averaged or guaranteed cost function. The demands of controller internal stability are used as a supplementary restriction. In practice, the cost functions in the form of a norm of closed-loop transfer function are widely used and a corresponding solution may be obtained using H_2 or H^∞ optimal control methods [2].

It is necessary to underline that the systems, which are optimal with respect to a class of disturbances, usually doesn't ensure the high accuracy for all disturbances realizations. The most difficult case is the situation where the spectrums of reference signal and disturbances are essentially intersected. This situation is typical for many process control applications.

Another approach is based on the utilization of current information about disturbances obtained by the direct or indirect measurements. Such an approach realized in non-traditional control structures known as "two-degree-of freedom controllers" [12] is the generalization of combined feedback and feedforward control method. The corresponding design methods using the different types of plant's and disturbances models in control loop (internal model-based control) are very popular in robust process control. At that, the dynamic models are used both for disturbance estimation (indirect measurement) and for prediction and compensation in order to ensure selective invariance properties of closed-loop system [11], i.e. decoupling for a certain class of disturbance. The idea of selective invariance was initially developed for SISO systems with scalar disturbance [9,11] and it generalization for multivariable systems are of the great interest.

In this paper, we analyze from the unified point of view the different model-based disturbance decoupling methods for multivariable systems via selective invariance approach and show that they may be treated as the modifications of the general inverse model control (IMC) method [5,7]. The IMC includes model-based input disturbance estimation, output plant's reaction prediction and disturbances influence compensation. The corresponding control structure consists of the disturbance observer and feedforward controller both based on the designed inverse models of the controlled plant's channels. Such an approach ensures not only the closed-loop system stabilization, but also high accuracy arbitrary reference signal tracking and unmeasurable arbitrary disturbance decoupling.

The advantage of the proposed method became brightly apparent in the case when disturbance model is unknown and should be identified using only current measurements of output variables. In such a case for multivariable systems the problem of simultaneous disturbance state and model parameter estimation is appeared. This problem under the conventional approach is reduced for the complex nonlinear adaptive filtering problem. The IMC approach ensures the decomposition of the problem into the separate disturbance state and model linear estimation realized by the well-developed algorithms.

2. PROBLEM STATEMENT

Consider a linear discrete-time multivariable system described by the state-space model

$$\begin{aligned} x_{k+1} &= Ax_k + B_1 u_k + B_2 w_k, \\ y_k^1 &= C_1 x_k, \quad y_k^2 = C_2 x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbf{R}^n$ - state vector in time k, $u_k \in \mathbf{R}^{m_1}$ - control, $w_k \in \mathbf{R}^{m_2}$ - disturbance vector, $y_k^1 \in \mathbf{R}^{q_1}$, $y_k^2 \in \mathbf{R}^{q_2}$ - output controlled and measured variables respectively. It will be assumed that system (1) has relative order 1 and the simplest type of invertibility condition for (1) takes place, i.e. $\operatorname{rank} C_i = q_i$, $\operatorname{rank} B_i = m_i$ and $\operatorname{rank}(C_i B_i) = m_i \le q_i$. Such an assumption is not very restrictive and used only for the simplicity of statement. The disturbance is described by the state-space model $w_{k+1} = \Phi w_k$, where matrix Φ is unknown.

The output control problem is to find the control sequence $\{u_k\}$, depending from the measured variables, which ensure the reference signal y_k^* tracking and disturbances w_k decoupling. The requirement of closed-loop system stabilization along with the disturbance decoupling leads to the *disturbance decoupling problem with stability* (DDPS). If, besides stability, arbitrary pole placement is demanded, the *disturbance decoupling problem with pole placement* (DDPPP) may be stated. Moreover, as long as the state vector of the system can't be measured directly and the formulation of the *disturbances decoupling problem by measurement feedback* (DDPM) can be defined. The conditions for solvability of the problems mentioned above are well known [1]. Nevertheless in spite of the existence of general solution of the DDP in term of invariant subspaces, the determination of analytical expressions for the controllers that solve DDP is of the great interest. Besides of a complete characterization of the solution the most important step of the design procedure is the parameterization of corresponding state feedback or dynamic compensator matrices. From practical point of view it is desirable to decompose the DDPM into the structural synthesis of the designed controller renders the fixed and free parameters and parametric synthesis based on the appropriate parameters tuning methods in order to satisfy the design goals, such as pole placement, performance optimization and so on.

3. MODEL-BASED FEEDFORWARD CONTROL

Consider at first the output control problem when the disturbance can be measured directly. Such an approach is realized in feedforward control structures and closely connected with the problem of dynamic system inversion.

3.1. Local optimal control

In accordance with the local optimal control (LOC) method [4] the control signal is found from the local criteria minimization problem

$$\begin{aligned} J_k &= \left\| y_{k+1}^* - C_1 A x_k - S_{11} u_k - S_{12} w_k \right\|^2 + \\ &+ \alpha \left\| u_k \right\|^2 \to \min \end{aligned} \tag{2}$$

where α is a weight coefficient, $S_{ij} = C_i B_j$. The corresponding control law is given by

$$\begin{aligned} u_k &= D_1(\alpha)\left(y_{k+1}^* - C_1 A x_k - S_{12} w_k\right), \\ D_1(\alpha) &= \left(\alpha I_{m_1} + S_{11}^{\mathrm{T}} S_{11}\right)^{-1} S_{11}^{\mathrm{T}}, \end{aligned} \tag{3}$$

From (1), (3) the equation of closed-loop system follows

$$\begin{aligned} x_{k+1} &= \Pi_1(\alpha) A x_k + B_1 D_1(\alpha) y_{k+1}^* + \\ &+ \Pi_1(\alpha) B_2 w_k \end{aligned} \tag{4}$$

The equation (4) coincides with the regularized inverse model of the system (1) control channel [5]. Consider the stability condition of closed-loop system (4). Without the restriction of generality it may be assumed that the (1) is stable, in over case it may be guaranteed by using the stabilizing feedback. As it has been shown in [5], the non-zero part of the spectrum of $\Pi_1 A$, where projection matrix $\Pi_1 = \Pi_1(0) = I_n - B_1 S_{11}^{-1} C_1$, coincides with the transmission zeroes of system (1). Thus for minimum-phase plants the stability of closed-loop system (8) is guaranteed for any α, in over cases the stability margin value α^* exists. At that, the limited attainable accuracy of control is determined from the equation for control error $e_k^1 = y_k^* - y_k^1$

$$\begin{aligned} e_{k+1}^1 &= E(\alpha)\left(y_{k+1}^* - C_1 A x_k - S_{12} w_k\right), \\ E(\alpha) &= \alpha\left(\alpha I_m - S_{11} S_{11}^{\mathrm{T}}\right)^{-1} \end{aligned} \quad . \tag{5}$$

It is necessary to underline that the LOC approach leads to the changing the poles of closed-loop system and for nonminimum-phase case its dynamics my be unsatisfactory.

3.2. Inverse model control

The inverse model control (IMC) method is the generalization of combined control with inverse model [7]. The control law is accepted in the form

$$u_k = -K_1 e_k + u_k^*, \tag{6}$$

where the first component is realized the output feedback with ensures the desired dynamic properties of closed-loop system, and the second component is used for reference signal tracking and disturbance compensation. Such a control signal is formed by the feedforward controller, based on the inverse model of control channel of system (1):

$$\begin{aligned} \tilde{x}_{k+1} &= \Pi_1 A \tilde{x}_k + B_1 S_{11}^{-1} y_{k+1}^* + \Pi_1 B_2 w_k, \\ u_k^* &= S_{11}^{-1}\left(y_{k+1}^* - C_1 A \tilde{x}_k - S_{12} w_k\right) \end{aligned} \tag{7}$$

From (1) and (7) follows the equation for closed-loop system

$$\begin{aligned} \theta_{k+1} &= A\theta_k + B_1 K_1 e_k^1, \\ e_{k+1} &= -C_1 A \theta_k - S_{11} K_1 e_k^1, \end{aligned} \tag{8}$$

where $\theta_k = x_k - \tilde{x}_k$.

Taking into account the evident balance property $C_1\theta_{k+1} + e_{k+1} = 0$, the equation (8) may be represented in the equivalent form

$$\theta_{k+1} = (A - B_1 K_1 C_1)\theta_k, \quad e_{k+1} = -C_1\theta_k. \tag{9}$$

Therefore, IMC method ensures the arbitrary reference signal tracking and disturbances decoupling if the invertibility conditions of system (1) take place. However, the control law (6) may be realized in the only case when the feedforward compensator (7) is stable. Thus the direct IMC can be used only for minimum-phase plants. In general case a regularized feedforward compensator may be designed using the similar technique as for LOC:

$$\begin{aligned} \tilde{x}_{k+1} &= \Pi_1(\alpha) A \tilde{x}_k + B_1 D(\alpha) y_{k+1}^* + \\ &\quad + \Pi_1(\alpha) B_2 w_k, \\ u_k^* &= D(\alpha)\left(y_{k+1}^* - C_1 A x_k - S_{12} w_k\right), \end{aligned} \tag{10}$$

with parameters matrices

$$\begin{aligned} D(\alpha) &= \left(\alpha I_m + S_{11}^{\mathrm{T}} S_{11}\right)^{-1} S_{11}^{\mathrm{T}}, \\ \Pi_1(\alpha) &= I_n - C_1 D(\alpha) B_1. \end{aligned}$$

In such a case the equations for control error dynamics may be obtained as:

$$\begin{aligned} \theta_{k+1} &= (A - B_1 K_1 C_1)\theta_k + B_1 K_1 f_k, \\ e_k &= C_1\theta_k + f_k, \end{aligned} \tag{11}$$

where $f_k = E_1(\alpha)\left(y_k^* - C_1 A \tilde{x}_{k-1} - S_{12} w_{k-1}\right)$ is the equivalent disturbance. Using (1) it is easy to estimate the attainable accuracy of combined IMC in the dependence of the desired stability margin of feedforward compensator.

4. DISTURBANCE IDENTIFICATION

Consider the internal model-based control system design when the disturbances w_k can't be measured directly. The corresponding modifications of control law have to use the estimations of disturbances $\hat{w}_k$, obtained from the measured data $\{y_k^2\}$ (the method of indirect disturbance measurement). In accordance with the concept of internal model the indirect disturbances measurement may be realized using either internal dynamic plant model [8], or static two-input matching model [11].

4.1. Internal model method

Taking the internal model in the following form

$$\hat{\tilde{x}}_{k+1} = A\hat{\tilde{x}}_k + B_1 u_k, \quad \hat{\tilde{y}}_k = C_2 x_k \tag{12}$$

one can obtained the disturbances estimate as

$$\hat{w}_k = S_{22}^{+}\left(y_{k+1}^2 - \hat{y}_{k+1}\right). \tag{13}$$

At that, the estimation error $e_k^2 = w_k - \hat{w}_k$ will include the bias proportional to w_k. In order to avoid it the corrected internal model may be used

$$\hat{x}_k = A\hat{x}_k + B_1 u_k + B_2 \hat{w}_k, \tag{14}$$

or taking (13) into account

$$\hat{x}_{k+1} = \Pi_2 A\hat{x}_k + \Pi_2 B_1 u_1 + B_2 S_{22}^{+} y_{k+1}^2, \tag{15}$$

where $\Pi_2 = I_n - B_2 S_{22}^{+} C_2$, "+" denotes the Moore-Penrouze generalized inversion.

Moreover the estimation error is given by

$$e_{k+1}^2 = \Pi_2 A e_k^2, \tag{16}$$

and will be invariant with respect to the unmeasured disturbance.

The equations(15), (16) exactly coincides with the equation of inverse model of system's (1) disturbance channel [5] so the internal model method generalization for multivariable system leads to the IMC.

4.2. Two-input static model method

In such a case the disturbance estimate is formed in accordance with the equation

$$\hat{w}_k = S_{22}^{+}\left(y_{k+1}^2 - CA\hat{x}_k - S_{21}u_k\right), \tag{17}$$

where the state vector estimate $\hat{x}_k$ is obtained by the dynamic state observer with the additional internal feedback intended for bias elimination

$$\hat{x}_{k+1} = Ax_k + B_1 u_k + L\left(y_k - C_2\hat{x}_k\right) + B\hat{w}_k \tag{18}$$

or in equivalent form

$$\hat{x}_{k+1} = F\hat{x}_k + B_2 S_{22}^{+} y_{k+1}^2 + Ly_k^2 + \Pi_2 B_1 u_k, \tag{19}$$

where $F = \Pi_2 A - LC_2$, L - is an arbitrary tuning matrix with appropriate dimension.

The equation (19) coincides with the equation of unknown-input observer (UIO) [3,6], so the disturbance observer in the form of two input static model [11] for multivariable systems converts with the combination of (17) into the UIO based tuning inverse model [5] of system's (1) disturbances channel. It is evident that if the observability conditions of matrix pair $\left(\Pi_2 A, C_2\right)$ take place, the inverse model may be designed in accordance with the pre-established dynamic properties.

If the proper inverse model is used, the corresponding disturbance estimates are formed with one step delay with respect to the current measurement. Such a delay may be compensated in the control loop in accordance with the general selective invariance idea [11] using the disturbance model $w_{k+1} = \Phi w_k$. In such a way the equations of feedforward compensator with indirect disturbances measurement may be obtained in the form

$$\begin{aligned}\tilde{x}_{k+1} &= \Pi_1(\alpha)A\tilde{x}_k + B_1 D(\alpha) y_{k+1}^{*} + \\ &+ \Pi_1(\alpha) B_2 \Phi \hat{w}_{k-1}, \\ u_k^{*} &= D(\alpha)\left(y_{k+1}^{*} - CA\tilde{x}_k - S_{12}\Phi\hat{w}_{k-1}\right),\end{aligned} \tag{20}$$

the closed-loop system equations are

$$\begin{aligned}\theta_{k+1} &= \left(A - B_1 K_1 C_1\right)\theta_k + B_1 K_1 f_k + B_2 \Phi e_{k-1}^2, \\ e_k^1 &= -C_1\theta_k + f_k - S_{12}\Phi e_{k-1}^2, \quad e_{k+1}^2 = Fe_k^2.\end{aligned} \tag{21}$$

Moreover the generalized separation principle takes place, i.e. the dynamic properties of control loop and disturbances observer may be established independently.

In the case when disturbance model matrix Φ is unknown the suitable identification algorithms may be applied using the disturbance estimates (13) or (17). For example, if the recurrent least square method is used [4] the disturbance model identification algorithms are in the form

$$\hat{\Phi}_{k+1} = \hat{\Phi}_k + \left(\hat{w}_{k+1} - \hat{\Phi}_k \hat{w}_k\right)\hat{w}_k^{\mathrm{T}}\Gamma_k, \tag{22}$$

where

$$\Gamma_k = \Gamma_{k-1} - \left(1 + \hat{w}_k^{\mathrm{T}}\Gamma_k \hat{w}_k\right)^{-1}\Gamma_{k-1}\hat{w}_k\hat{w}_k^{\mathrm{T}}\Gamma_{k-1} \tag{23}$$

The corresponding disturbance compensation algorithm includes the disturbance prediction based on the model estimates may be treated as the adaptive selective invariance approach.

5. INVERSE MODEL DESIGN

The basic of IMC approach is the state space representation of the inverse models. If the invertibility conditions take place [10], the structure inversion algorithm may be applied, in this case the structure and parameters of inverse models are strictly determined by the parameters of the corresponding channels. So for nonminimum-phase system the inverse models will be unstable. The inverse model design method must include the suitable parameterization of its equations and free parameters are selected from the simultaneous conditions of stability and desired dynamic properties. The most general way for such parameterization is the UIO theory [3,6], then the observer equation combined with the unknown input signal estimate may be treated as the designed inverse model.

5.1. Full-order inverse model

Consider the problem of dynamic system inversion, for this purpose supposes that $w_k \equiv 0$, u_k and y_k^1 will be treated as the unknown input and measured output respectively. In the case under consideration using the UIO observer

$$\tilde{x}_{k+1} = F_1\tilde{x}_k + G_{11}y_k^1,\ \hat{x}_k = \tilde{x}_k + H_{11}y_k^1, \tag{24}$$

one can obtained the inverse models equation in the form

$$\begin{aligned} x_{k+1}^{I_1} &= F_1 x_k^{I_1}(t) + (G_{11} - F_1 H_{11})u_k^{I_1}(t) + H_{11}u_{k+1}^{I_1}, \\ y_k^{I_1} &= B_1^+\left[x_{k+1}^{I_1} - Ax_k^{I_1}\right] \end{aligned} \tag{25}$$

where $x_k^{I_1} = \hat{x}_k \in \mathbf{R}^n$, $u_k^{I_1} \in \mathbf{R}^{q_1}$, $y_k^{I_1} \in \mathbf{R}^{m_1}$ - inverse model state vector, input and output signals respectively, $u_k^{I_1} = y_k^1$.

If the parameters of the observer (24) satisfy the so-called “invariance conditions” [5,7]

$$\begin{aligned} &(I_n - H_{11}C_1)F_1 - F_1(I_n - H_{11}C_1) = G_{11}C_1, \\ &B_1 - H_{11}C_1B_1 = 0, \end{aligned} \tag{26}$$

the unknown input u_k will be eliminate from the deviation vectors $e_k^x = x_k - x_k^{I_1}$, $e_k^u = u_k - y_k^{I_1}$ which will be given by following equations:

$$e_{k+1}^x = F_1 e_k^x, \quad e_k^u = -B_1^+(F_1 - A)e_k^x. \tag{27}$$

As it has been shown in [5] in general case $m_1 \le q_1$ the system of linear algebraic equation (26) has a solution

$$\begin{aligned} &F_1(L_1) = \Pi_1 A - L_1 C_1, \quad H_1 = B_1 S_{11}^+, \\ &G_{11}(L_1) = \Pi_1 A H_1 + L_1 \Omega_1, \end{aligned} \tag{28}$$

where $\Pi_1 = I_n - B_1 S_{11}^+ C_1$, $\Omega_1 = I_n - S_{11}S_{11}^+$, $S_{11}^+ = \left(S_{11}^T S_{11}\right)^{-1} S_{11}^T$, and L_1 is the arbitrary $(n \times q_1)$ matrix of free tuning parameters. Therefore if the pair $(\Pi_1 A, C_1)$ is observable (input observability conditions), the eigenvalues of $F_1(L_1)$ may be assigned by means of tuning matrix L_1 selection via pole placement method.

Finally the parameterized state-space representation of the inverse model are obtained in the form

$$\begin{aligned} x_{k+1}^{I_1} &= F_1(L_1)x_k^{I_1}(t) + L_1 u_k^{I_1} + H_1 u_{k+1}^{I_1}, \\ y_k^{I_1} &= -C_1(L_1)x_k^{I_1} + B_1^+ L_1 u_k^{I_1} + S_{11}^+ u_{k+1}^{I_1}, \end{aligned} \tag{29}$$

where $C_1(L_1) = S_{11}^+ C_1 A + B_1^+ L_1 C_1$.

For example, using the special form of system (1), which may be obtained by nonsingular state-space transformation

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ B_1 = \begin{pmatrix} B_{11} \\ B_{12} \end{pmatrix},\ C_1 = \left(I_{q_1} \mid 0_{q_1, n-q_1}\right), \tag{30}$$

the inverse models matrices may be represented as

$$\begin{aligned} \Pi_1 &= \left(\begin{array}{c|c} \Omega_{B_{11}} & 0_{q_1, n-q_1} \\ \hline -B_{12}B_{11}^+ & I_{n-q} \end{array}\right), \\ F_1(L_1) &= \left(\begin{array}{c|c} \Omega_{B_{11}} A_{11} - L_{11} & \Omega_{B_{11}} A_{12} \\ \hline \tilde{A}_{21} - L_{12} & \tilde{A}_{22} \end{array}\right), \end{aligned} \tag{31}$$

where $\Omega_{B_{11}} = I_{q_1} - B_{11}B_{11}^+$, $\tilde{A}_{1i} = A_{2i} - B_{12}B_{11}^+ A_{1i}$, $L_1^T = \left(L_{11}^T \mid L_{12}^T\right)$, $L_{11} \in \mathbf{R}^{q_1 \times q_1}$, $L_{12} \in \mathbf{R}^{n-q_1 \times q_1}$.

Thus the suitable modal control method may be used for inverse model design.

5.2. Reduced-order inverse model

The minimal state-space realization of the inverse model may be obtained by means of reduced order UIO. Let $z_k = R_1 x_k \in \mathbf{R}^{n-q_1}$ be an aggregated auxiliary variables, where R_1 is the appropriate aggregate matrix such as $\operatorname{rank}\left(C_1^T \mid R_1^T\right) = n$. Then the state vector estimate may be obtained as follows

$$\hat{x}_k = P_1 y_k^1 + Q_1 \hat{z}_k, \tag{32}$$

where $\hat{z}_k$ is given by minimal-order UIO

$$\bar{x}_{k+1} = \bar{F}_1 \bar{x}_k + \bar{G}_{11} y_k^1,\ \hat{z}_k = \bar{x}_k + \bar{H}_{11} y_k^1 \tag{33}$$

and matrices $P_1 \in \mathbf{R}^{n \times q_1}$, $Q_1 \in \mathbf{R}^{n \times n-q_1}$ are defined as

$$\begin{aligned} &(P_1 \mid Q_1) = \begin{pmatrix} C_1 \\ R_1 \end{pmatrix}^{-1}, \quad C_1 P_1 = I_{q_1}, \\ &R_1 Q_1 = I_{q_1}, \quad P_1 C_1 + Q_1 R_1 = I_n \\ &C_1 Q_1 = 0_{q_1, n-q_1}, \quad R_1 P_1 = 0_{n-q_1, q_1} \end{aligned} \tag{34}$$

The “invariance conditions” in such a case take on the form

$$\begin{aligned} &(R_1 - \bar{H}_1 C_1)A - \bar{F}_1(R_1 - \bar{H}_1 C_1) = \bar{G}_{11} C_1, \\ &R_1 B_1 - \bar{H}_{11} C_1 B_1 = 0, \end{aligned} \tag{35}$$

and a corresponding solution of (35) may be obtained as

$$\begin{aligned} &\bar{F}_1(R_1) = R_1 \Pi_1 A Q_1, \\ &\bar{H}_{11} = R_1 B_1 S_{11}^+ = R_1 H_1, \\ &\bar{G}_{11}(R_1) = R_1 \Pi_1 A\left(\bar{H}_{11} + P_1 \Omega_1\right), \end{aligned} \tag{36}$$

where matrices P_1, Q_1 are uniquely determined by R_1 selection.

Therefore the minimal-order inverse model is given by equations:

$$\begin{aligned} \bar{x}_{k+1}^{I_1} &= \overline{F}_1(R_1)\bar{x}_k^{I_1} + R_1\Pi_1 AP_1 u_k^{I_1} + R_1 H_{11} u_{k+1}^{I_1}, \\ y_k^{I_1} &= -\overline{C}_1(P_1)\left[C_1 AQ_1 x_k^{I_1} + C_1 AP_1 u_k^{I_1} - u_{k+1}^{I_1}\right] \end{aligned} \qquad (37)$$

where $x_k^{I_1} = z_k \in \mathbf{R}^{n-q_1}$ - state vector of the inverse model, $\overline{C}_1(P_1) = S_{11}^{+} + B_1^{+} P_1 \Omega_1$.

The deviation vectors $\bar{e}_k^x = R_1 x_k - x_k^{I_1}$, e_k^u also are invariant with respect to u_k:

$$\bar{e}_{k+1}^x = \overline{F}_1(R_1)e_k^x,\ \ e_k^u = -C_1(P_1)C_1 AQ_1 \bar{e}_k^x, \qquad (38)$$

and its dynamic properties is determined by tuning matrix R_1 selection.

Concretely define the matrices P_1, Q_1 choice, one can admit

$$\begin{aligned} &\left(P_1 \mid Q_1\right) = \begin{pmatrix} P_{11} & Q_{11} \\ P_{12} & Q_{12} \end{pmatrix}, \\ &P_{11} = I_q, \quad Q_{11} = 0_{q,n-q}, \end{aligned} \qquad (39)$$

in such a case $R_1 = Q_{12}^{-1}\left(-P_{12} \mid I_{n-q}\right)$ and P_{12}, Q_{12} are arbitrary matrices with $\det Q_{12} \neq 0$. For system representation (15) from (16), (21) follows that

$$\begin{aligned} \overline{F}_1(R_1) &= Q_{12}^{-1}\left(\overline{A}_{22} - P_{12}\overline{A}_{12}\right)Q_{12}, \\ \overline{A}_{12} &= \Omega_{B_{11}} A_{12}, \quad \overline{A}_{22} = A_{22} - B_{12} B_{11}^{+} A_{12}. \end{aligned} \qquad (40)$$

Thus the matrix Q_{12} defines the similarity transformation and doesn't change the spectrum of $\overline{F}_1(R_1)$ which completely determined by arbitrary matrix $P_2 \in \mathbf{R}^{n-q_1 \times q_1}$. The last may be chooses by pole placement method if pair $\left(\overline{A}_{22}, \overline{A}_{12}\right)$ is observable. The aggregate matrix R_1 is determined up to an arbitrary nonsingular matrix Q_{12}.

6. CONCLUSION

The proposed UIO-based approach to selective invariance properties ensuring in multivariable systems leads to the decomposition of the problem on the disturbance state estimation and model identification. As it has been shown the inverse models may be used for both disturbance estimation and compensation. Therefore the inverse model-based control method is seemed to be the most general approach to the disturbance decoupling problem in multivariable systems and may be consider as a basis for high accuracy control system design. The UIO theory may be used as a basis for inverse systems design, moreover the nonminimum-phase case may be treated in the same way. The reduced-order and regularized inverse models and multivariable model-based disturbance compensator has been developed and design method proposed using pole-placement method. It is essentially that such an approach gives a simple criterion of inverse model design problem solvability.

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