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COMPLEXITY MADE SIMPLE *

*AT A SMALL PRICE

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Abstract. This talk is motivated by the need to analyze and synthesize complex systems that require computationally intensive modeling, control, and optimization. Examples of such systems abound in communication networks, manufacturing, computer, C^4I, transportation, and logistics environments. In these environments, one encounters design, planning, resource allocation, and decision making problems whose complexity is such that computational power alone will never be able to overcome the combinatorial explosion and lack of structure in the search spaces associated with these systems. We therefore seek drastically different approaches for both *static* (parametric) and *dynamic* optimization.

Two key themes in the approaches to be overviewed are: (a) Replacing a hard optimization problem by a simpler "surrogate problem" whose solution may in fact be the same as that of the original or, sometimes, sufficiently near-optimal, (b) Exploiting the problem structure whenever this is possible to obtain computationally efficient solution procedures. In the first case, the "small price" to pay is the possible loss of optimality, whereas in the second case it is the loss of generality in the methodologies developed which are tailored to specific structural properties.

While traditional optimization techniques rely on the evaluation of the *cardinal* values of an objective function over some parameter set in order to drive a parameter-adjustment process, *ordinal optimization* is driven by the *relative order* of such estimates -- not their absolute values, allowing us to exploit robustness properties of these order statistics with respect to substantial estimation noise. Using this approach, and trading off "global optimality" for "near optimality with high probability", some highly complex problems have recently been tackled with considerable success. For a class of discrete stochastic optimization problems where structural properties can also be exploited, ordinal optimization methods can actually yield global optimality in probability or even with probability 1 under certain conditions.

Along the lines of seeking simpler "surrogate problems" when one is faced with combinatorially hard discrete optimization problems, it is reasonable to pose the following question: Is it possible to transform a *discrete* optimization problem into a "surrogate" *continuous* optimization problem, proceed to solve the latter using standard gradient-based approaches, and finally transform its solution into a solution of the original problem? Moreover, is it possible to design this process for *on-line* operation?

That is, at every iteration step in the solution of the surrogate continuous optimization problem, is it possible to immediately transform the surrogate continuous state into a feasible discrete state? This is crucial, since whatever information is used to drive the process (e.g., sensitivity estimates) can only be obtained from a sample path of the *actual* system. A recently developed general framework that accomplishes this task will be presented, illustrated by several examples and applications.

Another instance where structural properties can be exploited to solve hard dynamic optimization problems arises in a class of hybrid systems where a lower-level component with time-driven dynamics interacts with a higher-level component with event-driven dynamics. These typically arise in manufacturing environments where the lower-level component represents physical processes and the higher-level component represents events related to these physical processes. Solving a problem which aims at jointly optimizing the performance of both hierarchical components is a difficult task, complicated by nondifferentiable event-driven dynamics. We will describe how structural properties of the optimal state trajectories in these problems can be exploited to decompose them into segments and how specific algorithms can reduce the associated problem of identifying these segments from a combinatorially explosive one to a linear one.