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OBSERVERS FOR A CLASS OF NONLINEAR SYSTEMS WITH TIME-VARYING DELAY

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Abstract. In this paper, observers design for a class of nonlinear systems with time-varying delay is addressed. We use the Razumikhin approach to deduce general conditions for asymptotic convergence of the observer. These conditions are expressed in terms of the existence of a positive definite matrix solution of a Riccati-type equation.

Key Words. Observers, nonlinear systems, time delay, Riccati-type equation.

1.INTRODUCTION

Several control processes encountered in practice, for example in biology, mechanical and chemical engineering (see [5],[6]) involve delays. Their presence may affect the performances of control laws or even be a source of instability. Generally, the control of such systems, includes the design of an observer which must asymptotically estimate the state variables from the output and the input measurements. During the last decades, reconstruction of the state variables of linear systems with time delays has been the subject of many papers, see [1],[2],[3],[10],[11],[12] and the references therein. Very few works, however, have been performed to deal with the state estimation of nonlinear time delay systems and even less when delay is time-varying. Till now, this remains an open research problem.

In this paper, we focus our attention on observers design for a class of nonlinear systems with time-varying delay. To consider a large class of systems, the delay appears in both the linear and nonlinear parts. This work may be seen as a generalization of the results performed for a class of standard nonlinear systems [7],[8]. One of the features of the proposed approach concerns the use of a Lyapunov-Razumikhin type function that leads to general conditions for asymptotic stability. The obtained sufficient condition is expressed in terms of the existence of a positive definite matrix solution to a Riccati-type equation and therefore 'algebrize' the convergence results.

The organization of the paper is as follows. In Section 2 we present the class of systems considered and recall some basic notions. The main result is the body of Section 3. In this section, we propose sufficient conditions to guarantee the convergence of the observer Finally, some concluding remarks are given in Section 4.

2.PRELIMINARIES

The system under consideration is of the form :

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_R x(t-h(t)) + f\big(t, x(t), u(t)\big) \\ &+g\big(t, x(t-h(t)), u(t)\big) \\ &y(t) = Cx(t) \\ &x(t) = \phi(t), \quad t \in [-H, 0] \end{aligned} \tag{1}$$

$y(t) \in \mathbb{R}^q$ is the vector of the measurements and $u(t) \in \mathbb{R}^m$ is the input vector. Matrices A, A_R and C are real of appropriate dimensions. $h(.)$ is a scalar differential function and represents the delay. It is assumed to be known and satisfies $0 \leq h(t) \leq H$ for all $t > 0$. f and g are given nonlinear continuous functions, respectively k_f and k_g lipschitzian with respect to their second argument. In the following, $\mathcal{C}([-H, 0], \mathbb{R}^n)$

will denote the banach space of continuous function mapping $[-H, 0]$ into $\mathbb{R}^n$, with the norm $\|\phi\| = \sup_{t\in[-H,0]} |\phi(t)|$. The euclidean norm of $\phi(t) \in \mathbb{R}^n$ is denoted by $|\phi(t)|$. We also assume that $f(t,0,0)$=$g(t,0,0)$=0, $\forall t \in \mathbb{R}$.

Before proceeding further, we will give some preliminary results. Consider the nonlinear delay systems of the general form :

$$\dot{x}(t) = f(t, x_t) \qquad (2)$$

where $f : \mathbb{R} \times \mathcal{C}([-H, 0], \mathbb{R}^n) \mapsto \mathbb{R}^n$ is continuous with respect to the first argument, lipschitzian with respect to the second and satisfy $f(t, 0) = 0$ for all $t \in \mathbb{R}$.

For $t \geq \sigma - H$, we denote by $x(\sigma, \phi)(t)$, its solution at time t with initial data ϕ, specified at time σ, i.e., $x(\sigma, \phi)(\sigma + \theta) = \phi(\theta)$, $\forall \theta \in [-H, 0]$. For $\theta \in [-H, 0]$, $x_t(\theta) = x(t + \theta)$ and represents the state of the delay system. For all $\delta > 0$, let us denote by $\mathcal{B}(0, \delta)$, the ball $\mathcal{B}(0, \delta) = \{\phi \in \mathcal{C}([-H, 0], \mathbb{R}^n) : \|\phi\| < \delta\}$. $\mathcal{A}$ will designate in the following, the class of scalar nondecreasing functions α of $\mathcal{C}([0,\infty), \mathbb{R})$, satisfying $\alpha(s) > 0$ for $s > 0$ and $\alpha(0) = 0$.

Definition(see [4])
The equilibrium solution, $x \equiv 0$ of the delay differential equation (2) is said to be :
1.*stable*, if for any $\sigma \in \mathbb{R}$, $\varepsilon > 0$, there is a $\delta = \delta(\varepsilon, \sigma)$ such that $\phi \in \mathcal{B}(0, \delta)$ implies $x_t(\sigma, \phi) \in \mathcal{B}(0, \varepsilon)$ for $t \geq \sigma$.
2.*asymptotically stable*, if it is stable and there exists $b_0 = b_0(\sigma) > 0$ such that $\phi \in \mathcal{B}(0, b_0)$ implies $x(\sigma, \phi)(t) \to 0$ as $t \to \infty$. 3.*uniformly stable*, if the number δ is independant of σ.
4.*uniformly asymptotically stable*, if it is uniformly stable and there is $b_0 > 0$ such that for every $\eta > 0$ there is a $t_0(\eta)$ such that $\phi \in \mathcal{B}(0, b_0)$ implies $x_t(\sigma, \phi) \in \mathcal{B}(0, \eta)$ for $t \geq \sigma + t_0(\eta)$ for every $\sigma \in \mathbb{R}$.

Theorem 1 (see [9])
Suppose $u, v, w : \mathbb{R}^+ \to \mathbb{R}^+$ are continuous, nondecreasing functions, $u(s)$, $v(s)$ positive for $s > 0$, $u(0) = v(0) = 0$, v stricly increasing.
If there exists a continuous function $V : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}$ such that

$$u(|x|) \leq V(t, x) \leq v(|x|), \quad \forall t \in \mathbb{R}, x \in \mathbb{R}^n \qquad (3)$$

and

$$\dot{V}(t, \phi(0)) \leq -w(|\phi(0)|), \qquad (4)$$
$$\textit{if } V(t + \theta, \phi(\theta)) < (V(t, \phi(0))) \textit{ for } \theta \in [-H, 0],$$

then the trivial solution of (2) is uniformly stable. If in addition $w(s) > 0$ for $s > 0$ and if there exists a continuous nondecreasing function $p(s) > s$ for $s > 0$ such that

$$\dot{V}(t, \phi(0)) \leq -w(|\phi(0)|), \qquad (5)$$
$$\textit{if } V(t + \theta, \phi(\theta)) < p(V(t, \phi(0))) \textit{ for } \theta \in [-H, 0],$$

then the solution is uniformly asymptotically stable.

Remark 1
In [13] the authors established an improved Razumikhin-type theorem. The class of functions for which $\dot{V}$ must satisfy the inequality (4) is defined differently than by equation (5). Indeed the authors proposed a set of functions ϕ for which there exists a real $q > 1$ such that

$$|\phi(s)| < q|\phi(0)|, \qquad \forall \theta \in [-H, 0]. \qquad (6)$$

With this new formulation of Theorem 1, they show that the improved theorem is better than the original one in conservatism reduction. This theorem will be used in the following.

The following notations will be used throughout the paper. For $v \in \mathbb{R}^n$, v^T denote the transpose of v. If M is a positive definite matrix, then $M^{\frac{1}{2}}$ denote a square root of M. For any matrix M, M^T designate its transpose. For a matrix $M \in \mathbb{R}^{n\times n}$, $\lambda_{max}(M)$ and $\lambda_{min}(M)$ denote the maximum and minimum eigenvalue of M. The $\mathbb{R}^n$-valued identity matrix will be denoted by I_n.

3.OBSERVER DESIGN

Consider the system given by (1). The state observer that we propose is of the form :

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + A_R\hat{x}(t - h(t)) + f(t, \hat{x}(t), u) \\ &\quad + g(t, \hat{x}(t - h(t)), u) + L(y(t) - C\hat{x}(t)). \qquad (7)\end{aligned}$$

where the observed state is denoted by $\hat{x}$, and $L \in \mathbb{R}^{n\times q}$ is the observer gain matrix.
Then the state estimation error, $e = x - \hat{x}$, has the following dynamics :

$$\begin{aligned}\dot{e}(t) &= (A - LC)\, e(t) + A_R e(t - h(t)) \\ &\quad + F(t, e(t), u) + G(t, e(t - h(t)), u) \qquad (8)\end{aligned}$$

with

$$F(t, e(t), u) = f(t, x(t), u) - f(t, x(t) - e(t), u)$$

and

$$\begin{aligned}&G(t, e(t - h(t)), u) = g(t, x(t - h(t)), u) \\ &- g(t, x(t - h(t)) - e(t - h(t)), u).\end{aligned}$$

We can remark that F and G are respectively k_f and k_g lipschitzian with respect to their second componant and that $F(t, 0, u) \equiv G(t, 0, u) \equiv$

$0, \forall t \in \mathbb{R}^n, u \in \mathbb{R}^m$. A sufficient condition for the asymptotic convergence of the observer is presented in the following Theorem.

Theorem 2
Consider the system (1) with $0 \leq h(t) \leq H$, $\dot{h}(t) < 1$ and its observer (7). If there exists a scalar $\varepsilon > 0$ and symmetric positive definite matrices P and Q such that

$$(A-LC)^T P + P(A-LC) + I_n + P\Big(\frac{1}{\varepsilon}A_R A_R^T + \gamma_f I_n\Big)P + Q = 0 \quad (9)$$

where $\gamma_f = 1 + k_f^2$, and if

$$\varepsilon + k_g^2 < \lambda_{min}(Q) \quad (10)$$

then the observer (7) is uniformly asymptotically convergent.

Proof of Theorem 2
Consider the Lyapunov function $V : \mathbb{R}^n \mapsto \mathbb{R}$:

$$V(x) = x^T P x. \quad (11)$$

The derivative of V along the trajectories of (8) is

$$\dot{V}(e(t)) = e(t)^T \Big((A-LC)^T P + P(A-LC)\Big)e(t) + 2e(t)^T P A_R e(t-h(t)) + 2e(t)^T P F(t, e(t), u) + 2e(t)^T P G(t, e(t-h(t)), u). \quad (12)$$

By applying the Young's inequality

$$2u^T v \leq \varepsilon u^T u + \frac{1}{\varepsilon} v^T v \qquad \forall u, v \in \mathbb{R}^n, \forall \varepsilon > 0,$$

we obtain :

$$2e(t)^T P A_R e(t-h(t)) \leq \frac{1}{\varepsilon} e(t)^T P A_R A_R^T P e(t) + \varepsilon |e(t-h(t))|^2.$$

Proceeding in the same manner and using the assumption on F and G, we get :

$$2e(t)^T P F(t, e(t), u) \leq k_f^2 e(t)^T P P e(t) + |e(t)|^2.$$

and

$$2e(t)^T P G(t, e(t-h(t)), u) \leq e(t)^T P P e(t) + k_g^2 |e(t-h(t))|^2.$$

From the above majorations we get :

$$\dot{V}(e(t)) \leq e(t)^T \Big((A-LC)^T P + P(A-LC) + I_n + P\Big(\frac{1}{\varepsilon}A_R A_R^T + (1+k_f^2)I_n\Big)P\Big)e(t) + (\varepsilon + k_g^2)|e(t-h(t))|^2. \quad (13)$$

Substituting $|e(t+\theta)| < q|e(t)|$, $\theta \in [-H, 0]$, into (13) yields :

$$\dot{V}(e(t)) \leq e(t)^T \Big((A-LC)^T P + P(A-LC) + I_n + P\Big(\frac{1}{\varepsilon}A_R A_R^T + (1+k_f^2)I_n\Big)P\Big)e(t) + q^2(\varepsilon + k_g^2)|e(t)|^2.$$

If there exists a matrix Q such that (9) is satisfied, then :

$$\dot{V}(e(t)) \leq -e(t)^T \left(Q - q^2(\varepsilon + k_g^2)I_n\right) e(t).$$

If (10) holds, then there exists a real q, such that

$$1 < q^2 < \frac{\lambda_{min}(Q)}{\varepsilon + k_g^2}.$$

Thus

$$\dot{V}(e(t)) < -\lambda_{min}(Q)\alpha |e(t)|^2 \quad (14)$$

where $\alpha = 1 - q^2 \dfrac{\varepsilon + k_g^2}{\lambda_{min}(Q)}$. This concludes the proof of Theorem 2.

Remark 2
From (14), we have :

$$\dot{V}(e(t)) < -\beta V(e(t)) \qquad \text{where} \quad \beta = \frac{\lambda_{min}(Q)\alpha}{\lambda_{max}(P)}.$$

Dividing this inequality by $V(e(t))$ and integrating from t_0 to t yields

$$V(e(t)) < V(e(t_0))e^{-\beta(t-t_0)}.$$

Hence if $e^T P e$ is thought to be a measure of the distance of the error from the origin, then the error will approach zero in magnitude exponentially with a rate that is at least as fast as $e^{-\beta t}$.

4.CONCLUSIONS

In this paper we have presented a simple observer design for a class of nonlinear systems with time-varying delay. The analysis of its convergence was obtained from Lyapunov-Razumikhin theory. Sufficient conditions, independant of the delay, expressed in terms of the existence of a solution of a certain Riccati equation have been obtained.

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