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NEW PI CONTROLLER TUNING FORMULAS FOR UNSTABLE FIRST-ORDER PLUS DEAD-TIME PROCESSES

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Abstract: New methods for tuning PI controllers for unstable first order plus dead-time (UFOPDT) processes are reported. In contrast to known methods that result on overshoot in the closed-loop response or require the modification of the feedback structure, the proposed methods ensure smooth closed-loop response to set-point changes, fast attenuation of step load disturbances and robustness against parametric uncertainty while retaining the classical PI controller structure. This enhanced performance is plausible by the use of a first order set-point filter and by the application of some new accurate approximations of the crossover frequencies of the Nyquist plot for UFOPDT models. The proposed methods require small computation effort and they are particularly useful for on-line applications. Finally, they are favorably compared to the already known PI controller tuning methods.

Keywords: PI controller tuning, process control, unstable processes, dead-time processes.

1. INTRODUCTION

Many chemical and agricultural processes, such as exothermic chemical processes and biological reactors, often operate around unstable steady states. To approximate their dynamics for the purpose of controller design, unstable first order dead-time (UFOPDT) transfer function models are used [5], [7], [8]. Control of such models, is a very active research area [2], [3], [6], [9], due to the fact that, classical PID tuning methods, like the Ziegler-Nichols or the Cohen-Coon methods, are not applicable to these models, due to its peculiarity. The common feature of the existing tuning methods for UFOPDT systems is that they give excessive overshoot. Jacob and Chidambaram were the first to point out this drawback and proposed a new tuning rule, which incorporates both a two-stage P-PI control structure and the internal model controller (IMC) tuning rule [4].

The purpose of this study is to propose several new methods for tuning PI controllers for UFOPDT processes. In contrast to known PI tuning rules that result on overshoot in the closed-loop response or require the modification of the feedback control stru-cture, the proposed methods ensure smooth response and robustness against parametric uncertainty while retaining the classical PI controller structure. This improved performance is plausible by the use of a first order set-point filter and by the application of some new accurate approximations of the crossover frequencies of the Nyquist plot for UFOPDT models. The proposed tuning rules either are expressed in terms of an adjustable parameter that can be selected to ensure a desired damping ratio of the closed-loop response or minimization of the integral of squared error plus normalized square controller output deviation criterion, or they are based on the simultaneous satisfaction of gain and phase margin specifications. Note that explicit formulas for the selection of the adjustable parameters are also proposed. The proposed methods require small computational effort and they are particularly useful for on-line applications. A variety of simulation studies have been performed and the performance of the proposed methods is compared to that of both the conventional PI controller and the two-stage IMC method. This comparison reveals that the proposed methods provide fast attenuation of step load disturbances, in addition to enhanced closed-loop response in set-point changes. As it is shown in the paper, the proposed methods are favorably compared to the already known tuning methods in terms of stability robustness. Finally, an application of the proposed methods on an open loop unstable biological reactor with hard input constraint and significant measurement delay is also presented.

2. PI CONTROL WITH SET-POINT FILTER

The transfer function of an unstable, first order plus dead-time (UFOPDT) system, is given by

$$G_P(s) = \frac{Ke^{-ds}}{Ts - 1}$$

where K, d and T are the process gain, time delay and time constant, respectively. In order to control an UFOPDT system, we next propose the PI control-ler with set-point filter based feedback scheme, depi-cted in Fig. 1, wherein, $G_C(s)$ is the transfer func-tion of the PI controller having the form

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

where K_C is the controller gain and τ_I is the controller reset time, while $G_{SPF}(s)$ is the transfer function of the so-called set-point filter, having the form

$$G_{SPF}(s) = 1/(\tau_I s + 1)$$

In what follows, our aim is the tuning of the PI controller parameters. To this end, we next analyze the feedback control structure of Fig. 1. The closed loop system response upon a set point change is

$$\frac{Y(s)}{R(s)} = G_{CL}(s) = \frac{G_{SPF}(s)G_C(s)G_P(s)}{1 + G_C(s)G_P(s)} \qquad (1)$$

With a first-order set point filter, relation (1) takes the form

$$G_{CL}(s) == \frac{KK_C \exp(-ds)}{\tau_I s(Ts - 1) + KK_C(\tau_I s + 1)\exp(-ds)} \qquad (2)$$

Using the first order approximation of the form

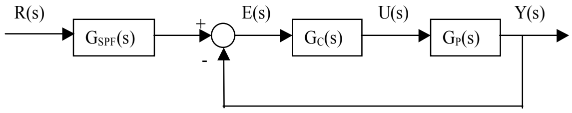

Fig. 1. Proposed feedback control structure.

$$\exp(-ds) = 1 - ds$$

in the denominator of (2), after some easy algebraic manipulations we obtain

$$G_{CL}(s) = \frac{1}{\rho^2 s^2 + 2\xi\rho s + 1}\exp(-ds) \qquad (3a)$$

$$\rho = \sqrt{\tau_I\left(\frac{T}{KK_C} - d\right)}\ ,\ \ \xi = \frac{\tau_I - d - \dfrac{\tau_I}{KK_C}}{2\sqrt{\tau_I\left(\dfrac{T}{KK_C} - d\right)}} \qquad (3b)$$

Moreover, the loop transfer function $G_L(s)$ has the form

$$G_L(s) = G_C(s)G_P(s) = \frac{KK_C(\tau_I s + 1)}{\tau_I s(Ts - 1)}\exp(-ds)$$

3. PROPOSED TUNING RULES

Observe that the argument and the magnitude of the loop transfer function $G_L(s)$ are

$$\phi_L(\omega) = -\frac{3\pi}{2} - d\omega + \tan^{-1}(T\omega) + \tan^{-1}(\tau_I\omega) \qquad (4a)$$

$$A_L(\omega) = |G_L(j\omega)| = KK_C\frac{\sqrt{1 + (\tau_I\omega)^2}}{\sqrt{1 + (T\omega)^2}}\frac{1}{\tau_I\omega} \qquad (4b)$$

Let now ω_{min} and ω_{max} be the smallest and the largest frequency, respectively, for which the Nyquist plot of the loop transfer function $G_L(s)$ crosses the negative real axis. Obviously, $\phi_L(\omega_{min}) = -\pi$ and $\phi_L(\omega_{max}) = -\pi$. However, the values of ω_{min} and ω_{max} cannot be obtained from (4a), since the later is obviously nonlinear and has no analytic solution. To avoid numerical solution of (4a) at $\omega = \omega_{min}$ and $\omega = \omega_{max}$ we propose the following approximation of the $\tan^{-1}$ function

$$\tan^{-1}(x) \approx \begin{cases} \dfrac{\pi}{2} - \dfrac{\pi}{3x}\ , & \text{for } x > 1 \\ x\ , & \text{for } x < 1 \end{cases} \qquad (5)$$

At frequency ω_{min}, equation (4a) yields

$$-\frac{\pi}{2} - d\omega_{min} + \tan^{-1}(T\omega_{min}) + \tan^{-1}(\tau_I\omega_{min}) = 0$$

Approximation of the $\tan^{-1}$ function at ω_{min} yields

$$-\frac{\pi}{2} - d\omega_{min} + T\omega_{min} + \left(\frac{\pi}{2} - \frac{\pi}{3\tau_I \omega_{min}}\right) = 0 \text{, or}$$

$$\omega_{min} = \sqrt{\frac{\pi}{3(T-d)\tau_I}} \qquad (6)$$

provided that T>d. Using a similar analysis, at frequency ω_{max}, we obtain

$$-\frac{\pi}{2} - d\omega_{max} + \left(\frac{\pi}{2} - \frac{\pi}{3T\omega_{max}}\right) + \left(\frac{\pi}{2} - \frac{\pi}{3\tau_I \omega_{max}}\right) = 0 \qquad (7)$$

or $6dT\tau_I\omega_{max}^2 - 3\pi T\tau_I\omega_{max} + 2\pi(T+\tau_I) = 0$, which has the solution

$$\omega_{max} = \frac{\pi}{4d}\left[1 + \sqrt{1 - \frac{16d(T+\tau_I)}{3\pi T\tau_I}}\right] \qquad (8)$$

provided that $\tau_I > \frac{16dT}{3\pi T - 16d}$ and $T > \frac{16d}{3\pi}$.

In Figs. 2 and 3, the exact solution of (4a) is compared with the approximations obtained by (6) and (8) (in the case where $\tau_I = 15T$). The accuracy of the approximation is remarkable in both cases.

Having obtained accurate analytic expression for ω_{min} and ω_{max}, we are able to calculate the PI controller parameters. More precisely, when, at frequencies ω_{min} and ω_{max}, the magnitude of the loop transfer function, equals unity, then the Nyquist plot of $G_L(s)$ crosses –1+j0 and the closed-loop system becomes unstable. In this case, from (4b), we obtain

$$K_{C,min} = \frac{1}{K}\frac{\sqrt{1+(T\omega_{min})^2}}{\sqrt{1+(\tau_I\omega_{min})^2}}\tau_I\omega_{min} \qquad (9a)$$

$$K_{C,max} = \frac{1}{K}\frac{\sqrt{1+(T\omega_{max})^2}}{\sqrt{1+(\tau_I\omega_{max})^2}}\tau_I\omega_{max} \qquad (9b)$$

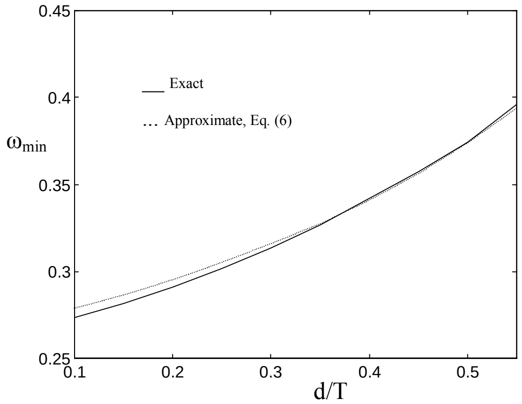

Fig. 2. Estimation of the frequency ω_{min} using (6) and exact solution.

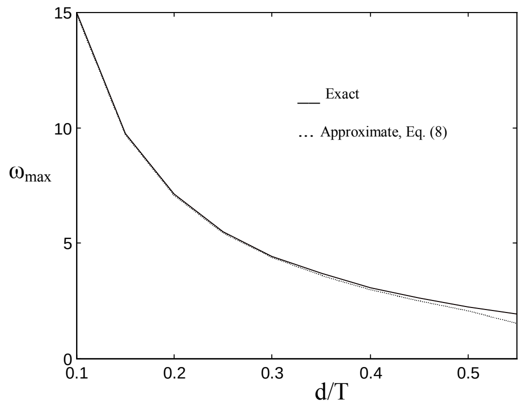

Fig. 3. Estimation of the frequency ω_{max} using (8) and exact solution.

which define the range of K_C, in which the closed-loop system remains stable. Therefore, the PI controller gain K_C can be calculated as follows

$$K_C = \sqrt{K_{C,min}K_{C,max}} \qquad (10)$$

From relations (9a) and (9b), it becomes clear that the controller gain depends of the specific value of the reset time τ_I, which can be computed by using one of the following four methods.

Method 1.

We first assume a large value for τ_I, say $\tau_I = 100T$. Then, ω_{min}, ω_{max}, $K_{C,min}$, $K_{C,max}$ can be calculated using relations (7)-(9), and K_C can be obtained using (10). Observe now that, K_C can further be written as

$$K_C = \frac{1}{K}\frac{\sqrt{1+(T\omega_F)^2}}{\sqrt{1+(\tau_I\omega_F)^2}}\tau_I\omega_F$$

where ω_F is the maximum positive real root of

$$f_1T^2\omega^4 + (f_1 - f_2\tau_I^2)\omega^2 - f_2 = 0 \qquad (11)$$

$$f_1 = \sqrt{1 + \tau_I^2(\omega_{min}^2 + \omega_{max}^2) + \tau_I^4(\omega_{min}\omega_{max})^2}$$

$$f_2 = \omega_{min}\omega_{max}\sqrt{1 + T^2(\omega_{min}^2 + \omega_{max}^2) + T^4(\omega_{min}\omega_{max})^2}$$

We next check for which value of the reset time, say $\tau_{I,max}$, equality $\omega_F = \omega_{max}$ is satisfied, or

$$\omega_F = \frac{\pi}{4d}\left[1 + \sqrt{1 - \frac{16d(T+\tau_{I,max})}{3\pi T\tau_{I,max}}}\right] \qquad (12)$$

Solving (12) with respect to $\tau_{I,max}$, we obtain

$$\tau_{I,max} = \left(\frac{3\pi}{16d}\left[1 - \left(\frac{4d\omega_F}{\pi} - 1\right)^2\right] - \frac{1}{T}\right)^{-1}$$

Set $\tau_I = \tau_{I,max}/\beta_1$, where β_1 is an adjustable parameter and repeat this procedure until a convergence.

Method 2.

We repeat all the steps of Method 1, up to the calculation of ω_F on the basis of (11). Next, we check for which value of the reset time, say $\tau_{I,min}$, equality $\omega_F = \omega_{min}$, is satisfied, or, by using (6)

$$\omega_F = \sqrt{\frac{\pi}{3(T-d)\tau_{I,min}}} \tag{13}$$

Solving (13) with respect to $\tau_{I,min}$, we obtain

$$\tau_{I,min} = \frac{\pi}{3(T-d)\omega_F^2}$$

Set $\tau_I = \tau_{I,min}/\beta_2$, where β_2 is an adjustable parameter and repeat this procedure until a convergence.

Method 3.

We repeat all the steps of Method 1, up to the calculation of K_C on the basis of (10). We next propose that the reset time τ_I can be calculated through a Ziegler-Nichols type relation of the form

$$\tau_I = \frac{P_{u,min}}{\beta_3} \tag{14}$$

where β_3 is an adjustable parameter and $P_{u,min} = 2\pi/\omega_{min}$. On the basis of (6), equation (14) yields

$$\tau_I = \frac{2\pi}{\beta_3}\sqrt{\frac{3(T-d)\tau_I}{\pi}}$$

Therefore,

$$\tau_I = \left(\frac{12\pi}{\beta_3^2}\right)(T-d)$$

We repeat the above procedure until a convergence.

Method 4.

We repeat all the steps of Method 1, up to the calculation of K_C on the basis of (10). We next propose that the reset time τ_I can be calculated through a Ziegler-Nichols type relation of the form

$$\tau_I = \frac{P_{u,max}}{\beta_4} \tag{15}$$

where β_4 is an adjustable parameter and $P_{u,max} = 2\pi/\omega_{max}$. On the basis of (8), equation (15) yields

$$\tau_I = \frac{8d}{\beta_4\left(1+\sqrt{1-\frac{16d(T+\tau_I)}{3\pi T\tau_I}}\right)}$$

or equivalently

$$\tau_I^2 + T\left(1-\frac{3\pi}{\beta_4}\right)\tau_I + \frac{12\pi dT}{\beta_4^2} = 0 \tag{16}$$

Then, τ_I can be obtained as the minimum real root of (16), which has an admissible solution if

$$T > \frac{16d}{3\pi} \text{ and } \beta_4 < 3\pi - 4\sqrt{3\pi\frac{d}{T}}$$

We repeat the above procedure until a convergence.

In the above tuning methods, it is clear that, in each iteration, the value of the reset time τ_I depends on the adjustable parameter β_i, i=1,2,3,4. In general, the value of β_i's can be selected arbitrarily, thus permitting on-line tuning. However, it would be useful for the designer to have some precise rules for the choice of β_i's. Such rules must rely of some criteria relative to the closed-loop system performance.

An obvious criterion is related to the responsiveness of the closed-loop system. From the previous analysis, it becomes clear that the closed-loop response of (3) (and therefore of the closed loop system) can be made the way we want by choosing the value of the damping ratio ξ (which depends on K_C, τ_I and hence on the adjustable parameters β_i's), to be the desired one, say ξ_{des}. For example, we can choose $\xi=1$ in order to insure overdamped nature of the closed loop response while maintaining the maximum responsiveness of the controller at the same time. When $\xi<1$ there are oscillations in the response, and the critically damped response of $\xi=1$, represents the marginal case between overdamped and underdamped responses. Larger values than unity for ξ produce more sluggish responses than necessary.

To satisfy $\xi = \xi_{des}$, the adjustable parameter β_1, in each iteration of Method 1, must be selected as the minimum real root (whenever it exists) of

$$d^2\beta_1^2 - \left[2d\left(1-\frac{1}{KK_C}\right) + 4\xi_{des}^2\left(\frac{T}{KK_C}-d\right)\right]\tau_{I,max}\beta_1 + \left(1-\frac{1}{KK_C}\right)^2\tau_{I,max}^2 = 0 \tag{17}$$

Similarly, the adjustable parameter β_2, in each iteration of Method 2, must be selected as the mini-mum real root (whenever it exists) of

$$d^2\beta_2^2 - \left[2d\left(1-\frac{1}{KK_C}\right) + 4\xi_{des}^2\left(\frac{T}{KK_C}-d\right)\right]\tau_{I,min}\beta_2 + \left(1-\frac{1}{KK_C}\right)^2\tau_{I,min}^2 = 0 \tag{18}$$

Moreover, to satisfy the same criterion, one must select the adjustable parameter β_3, in each iteration of Method 3, as the minimum real root (whenever it exists) of the biquadratic equation

$$d^2\beta_3^4 - 12\pi(T-d)\left[2d\left(1-\frac{1}{KK_C}\right)+4\xi_{des}^2\left(\frac{T}{KK_C}-d\right)\right]\beta_3^2 + 144\left(1-\frac{1}{KK_C}\right)^2\pi^2(T-d)^2 = 0$$

and the adjustable parameter β_4, in each iteration of Method 3.4, as the minimum real root (whenever it exists) of the fourth order equation

$$\left(\delta^2 + T\delta\varepsilon + T^2\delta\right)\beta_4^4 - \left(3\pi T\delta\varepsilon + 6\pi T^2\delta\right)\beta_4^3 + \left(12\pi dT^2\varepsilon - 24\pi dT\delta + 12\pi dT\varepsilon^2 + 9\pi^2T^2\delta\right)\beta_4^2 - 36\pi^2 dT^2\varepsilon\beta_4 + 144\pi^2d^2T^2 = 0$$

where

$$\delta = \frac{d^2}{\left(1-\frac{1}{KK_C}\right)^2} \quad \varepsilon = \frac{2d\left(1-\frac{1}{KK_C}\right)+4\xi_{des}^2\left(\frac{T}{KK_C}-d\right)}{\left(1-\frac{1}{KK_C}\right)^2}$$

Another criterion, for the selection of the controller parameters, is based on the minimization of the integral of the squared error plus the normalized squared controller output deviation (ISENSCOD) from its final value u_∞ [10]. This integral is defined by

$$J_{ISENSCOD} = \int_0^\infty \left\{[y(t)-r(t)]^2 + K^2[u(t)-u_\infty]^2\right\}dt \quad (19)$$

and we use it next in order to tune the PI controller parameters. Since K_C depends on τ_I, which, using Methods 1-4, can be obtained as a function of β_i's, minimization of $J_{ISENSCOD}$ can be performed with respect to β_i's. Unfortunately, there is no close form of integral (19) and simulation must be used instead. Extensive simulation shows that the values of β_i's, minimizing $J_{ISENSCOD}/T$, do not depend on K or K_C. It depends only on the dimensionless parameter d/T. Based on these simulations, we can propose explicit rules for evaluating β_i's that minimize $J_{ISENSCOD}$. For example, the optimal values of β_1 as a function of the parameter d/T, can be obtained by

$$\beta_1 = \frac{0.05078 + 3.56984\left(\frac{d}{T}\right) - 12.24531\left(\frac{d}{T}\right)^2 + 9.82212\left(\frac{d}{T}\right)^3}{1 + 3.70082\left(\frac{d}{T}\right) - 24.93263\left(\frac{d}{T}\right)^2 + 27.07332\left(\frac{d}{T}\right)^3} \quad (20)$$

Remark 3.1. Simulation results show that, for a given UFOPDT process and for any given prespe-cified value of the desired damping ratio ξ_{des} of the closed loop system, all tuning methods proposed above give the same settings for the controller parameters K_C and τ_I, although they are obviously based on different algorithmic procedures. For this reason, in Sections 4 and 5 that follows, simulations in these cases will be performed on the basis of only one of the aforementioned equivalent methods.

The methods presented thus far, for tuning PI controllers for UFOPDT models rely on the satisfaction of some very important characteristics of the closed-loop system response. We next present a method for PI controller tuning based on the simultaneous satisfaction of phase and gain margin (PGM) specifications.

To present the method, let, in the sequel, G_M and P_M be the gain and phase margin, respectively, of the closed-loop system. From the basic definitions of the gain and phase margin, the following equations are obtained

$$P_M = \arg[G_c(j\omega_G)G_P(j\omega_G)] + \pi \quad (21)$$

$$G_M = \frac{1}{|G_c(j\omega_C)G_P(j\omega_C)|} \quad (22)$$

where, ω_G is given by

$$A_L(\omega_G) = |G_c(j\omega_G)G_P(j\omega_G)| = 1 \quad (23)$$

and ω_C is given by

$$\phi_L(\omega_C) = \arg[G_c(j\omega_C)G_P(j\omega_C)] = -\pi \quad (24)$$

In the case of UFOPDT models, it is obvious that $\omega_C = \omega_{max}$. So, the proposed PGM tuning method is as follows.

Method 5.

First assume a large value for τ_I, say $\tau_I = 100T$. On the basis of this value, ω_{max} and $K_{C,max}$ can be calculated using relations (8) and (9b), respectively. Then, set initially K_C=$K_{C,max}$/2. Moreover, with these initial values of τ_I and K_C, ω_G is obtained from (23), or equivalently as the square root of the maximum real root of the biquadratic equation

$$\tau_I^2T^2\omega^4 + \tau_I^2\left(1-K^2K_C^2\right)\omega^2 - K^2K_C^2 = 0 \quad (25)$$

where in producing (25) use was made of (4b). Then, in view of relations (4a), (4b), (21) and (22), re-evaluate τ_I and K_C, from relations

$$\tau_I = \frac{\tan\left[P_M + \frac{\pi}{2} + d\omega_G - \tan^{-1}(T\omega_G)\right]}{\omega_G} \quad (26)$$

$$K_C = \frac{\tau_I \omega_{max} \sqrt{1+(T\omega_{max})^2}}{KG_M \sqrt{1+(\tau_I \omega_{max})^2}} \quad (27)$$

and repeat the above procedure until a convergence. This completes the method.

In the case of P-only controllers, it has been shown [3], that the maximum of $\phi_P(\omega) = -\pi - d\omega + \tan^{-1}(T\omega)$, is achieved at frequency $\omega_P = T^{-1}\sqrt{dT^{-1}-1}$. Therefore, by choosing the P controller gain as $K_C = K^{-1}\sqrt{Td^{-1}}$, then, the phase margin P_M is maximized and is given by [3]

$$P_M = \tan^{-1}(T\omega_P) - d\omega_P \quad (28)$$

Since, for a given specification of the gain margin, a PI controller has always smaller phase margin than the P controller with the same gain margin, Method 5, is applicable only if the phase margin specifica-tion is smaller than the one calculated through (28). Moreover, in order to obtain acceptable integral ac-tion, the phase margin specification should be suffi-ciently smaller than the maximum value (28).

4. NUMERICAL EXAMPLES

To demonstrate the effectiveness of the proposed methods and to provide a comparison with the existing tuning formulas, a numerical example is elaborated. In particular, the unstable process model studied in [5], [8], [9] is considered. Parameter va-lues for this model are K=1, d=0.5 and T=1.

First, Method 1 is applied to this specific model, in order to design a PI controller, with the adjustable parameter β_1 having the optimal value minimizing $J_{ISENSCOD}$. From (20) we obtain $\beta_{1,opt} = 1.6462$. Then, the PI controller parameters obtained are K_C=1.5091 and τ_I=6.5534.

Our method will next be compared with the methods of Rotstein and Lewin (R&L) [8] and Venkatashan-kar and Chidambaram (V&C) [9], in the case where it is assumed that there is no parametric uncertainty in the process model. Applying the R&L method (for $\lambda = 2.2$), we obtain $K_C = 1.9091$, $\tau_I = 9.2400$. The V&C method gives $K_C = 1.5066$, $\tau_I = 12.5000$. In Fig. 4, the closed-loop response, to a unit step change of the set point, is given, and it is compared to those obtained by applying the methods proposed in [8], [9], which, obviously, give unacceptable over-shoot, while the method in [8] gives the most oscilla-tory response. Our method offers the better response in terms of overshoot, oscillation and settling time. In Fig. 5, the proposed method is compared to that proposed in [9], in the extreme case where 20% error in the estimation of K and 10% error in the

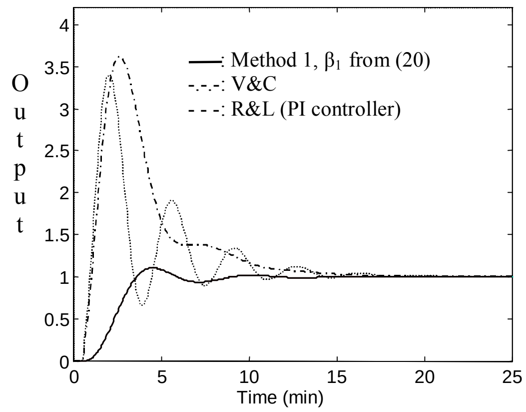

Fig. 4. Proposed Method 1 vs. the methods of Ven-katashankar/Chidambaram and Roitstein/Lewin

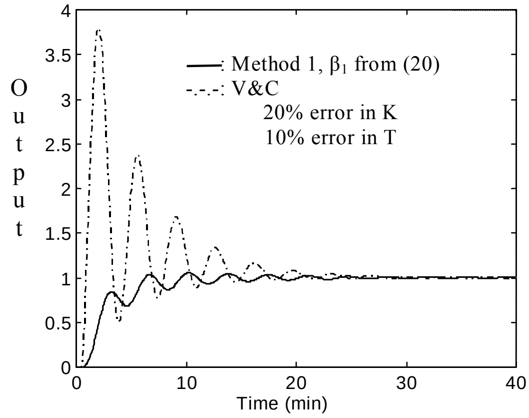

Fig. 5. Proposed Method 1 vs. the Venkatashankar/ Chidambaram method, in case of uncertainty

estimation of T are assumed. It is shown that, even in this case, our method is considerably better than the V&C method. For the assumed uncertainty, the R&L method gives unstable closed-loop response. This suggests that, although in [8] it is proposed using $\lambda = 2.2$, in order to tolerate 25% gain un-certainty, the R&L method cannot tolerate simulta-neous parameter variations.

We shall next perform a comparison of our methods with the method of Jacob and Chidambaram [4], which is based on a two stage P-PI control structure and the IMC-PID tuning rule [6]. To this end, we apply Method 2, in order to design PI controllers in the case where the closed-loop system response is desired to have the damping ratio ξ=1. The values of the PI controller parameters, obtained by applying Method 2, are $K_C = 1.5284$, $\tau_I = 7.7874$. In Fig. 6 the closed-loop system response, to unit step changes of the set point, is given in the case where no parametric uncertainty is assumed. This response is compared with those obtained by the application of the best-tuned two–stage IMC method, i.e. the tuning parameter λ taking on the values of 2.0 and 2.5. The two stage IMC P-PI controller parameters

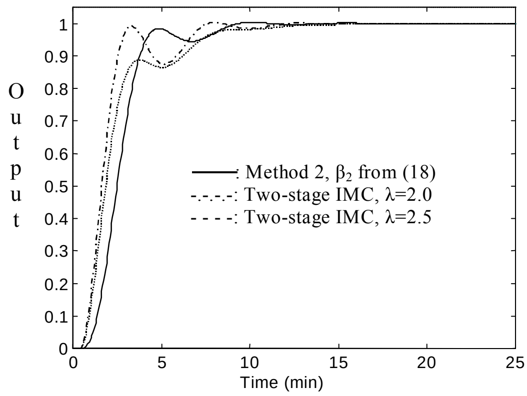

Fig. 6. Proposed Method 2 vs. the two-stage IMC method (λ=2 and λ=2.5)

are $K_C = 0.1402$, $\tau_I = 0.9571$, $K_{C,i} = 1.4142$ and $K_C = 0.1121$, $\tau_I = 0.9571$, $K_{C,i} = 1.4142$, respectively. It turns out that, in this case, the performance of our method is as good as that of the best-tuned two-stage IMC method. Note that, when no parame-tric uncertainty is assumed, the case of $\lambda = 2.0$ for the two-stage IMC method performs better than the case λ=2.5. Fig. 7 shows the regulatory control results for unit step load changes, by Method 2, along with those obtained by the two-stage IMC method. It turns out that Method 2 is comparable with the two-stage IMC method. We finally perform a comparison of the performance of the proposed method with the best-tuned two-stage IMC method of [4], in the case where the process parameters deviate from their estimated nominal values. Simulation show that, in the robustness tests the case of $\lambda = 2.5$ performs well than λ of 2.0. Fig. 8 shows the case of 20% uncertainty increase of process gain. Fig. 9 shows a comparison of Method 2 with the best-tuned two-stage IMC method, in the case of 20% decrease of time constant. Similar results can be obtained in the case of 20% increase of time delay. It turns out that the performance of our method is quite satisfactory in all cases.

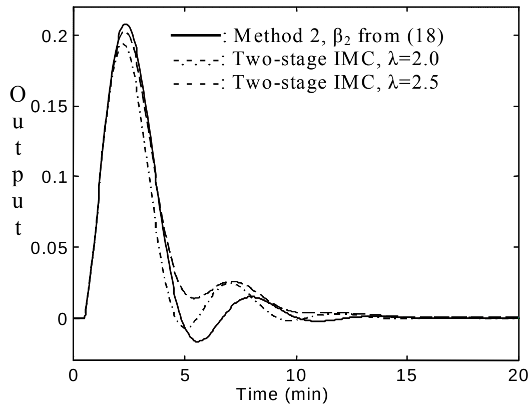

Fig. 7. Proposed Method 2 vs. the two-stage IMC method in the case of regulatory control.

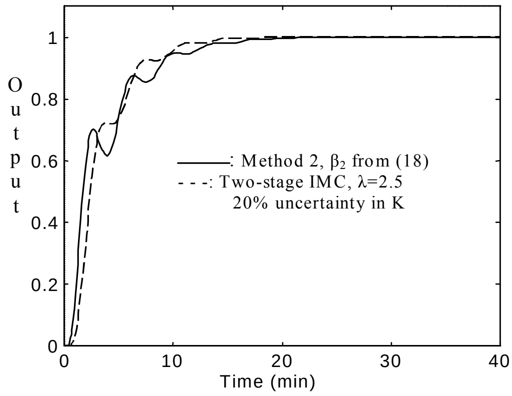

Fig. 8. Proposed Metrhod 2 vs. the two-stage IMC method in case of gain uncertainty.

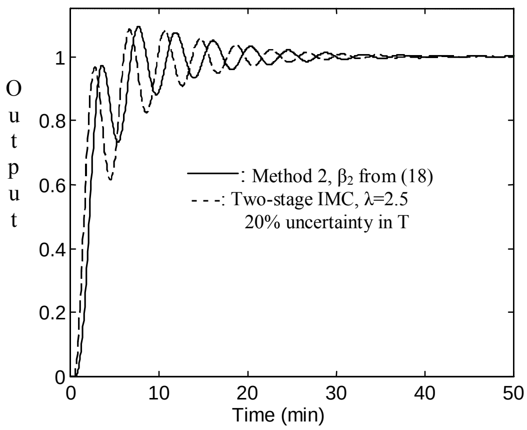

Fig. 9. Proposed Method 2 vs. the two-stage IMC method in case of time constant uncertainty.

5. APPLICATION TO A BIO-REACTOR

The open loop behavior of a variety of constant volume continuous stirred tank fermenters (CSTF) with sterile feed can be described by the following unstructured model [1]

$$\frac{dX}{dt} = (\mu(S) - D)X \quad , \quad \frac{dS}{dt} = D(S_f - S) - \frac{\mu(S)X}{Y_{X/S}} \quad (29)$$

where, $\mu(S) = \mu_m S(K_m + S + S^2 / K_I)^{-1}$ is the specific growth rate, $Y_{X/S}$ is the cell-mass yield coefficient, μ_m is the maximum specific growth rate, K_m is the growth rate constant and K_I is the substrate inhibition constant. Typical values for the model parameters are [1]

$$Y_{X/S} = 0.4\%g/g,\ S_f = 4\%g/g,\ D=0.36h^{-1}$$
$$\mu_m = 0.53h^{-1},\ K_m = 0.12\%g/g,\ K_I = 0.4545\%g/g$$

The solution of (29) exhibits an unstable steady sta-te at $[X,S]_2$=[0.9951, 1.5122]. In the present simula-tion study it is desired to operate the CSTF at this unstable steady state. The cell mass concentration X is the controlled variable. The upper and lower con-straints to the manipulated variable D, are D_L= 0.25

h^{-1} and D^U=0.40 h^{-1}. A measurement delay of one hour is also considered in the measurement of X. The system behavior at this unstable point is fitted by local linearization to an UFOPDT model. To estimate the parameters of the model, the method of [4] is used. Then, we obtain the UFOPTD model

$$G_P(s) = \frac{-5.89}{5.86s - 1}\exp(-s)$$

To the above UFOPDT model of the CSTF we next apply Method 5 in order to design a PI controller that provides a gain margin of 6.0206 dB and a pha-se margin of 20° in the closed-loop system. Appli-cation of the proposed method yields the PI control-ler settings $K_C = -0.6558$, $\tau_I = 6.6766$. The closed-loop response of the CSTF obtained in this case, for a step change in the set point from 0.9951 to 1.1941, is given in Fig. 10. Next, we apply Method 3, in order to design a PI controller in the case where the closed-loop system is desired to have the damping ratio ξ=0.9. The values of the PI controller parame-ters, obtained by applying Method 3, are $K_C = -0.5022$ and $\tau_I =$ 10.0470. The reader could easily check that, for the same design criterion (i.e. for $\xi_{des} = 0.9$), Methods 1, 2 and 4 give the same controller parameters. The closed-loop response of the CSTF obtained in this case, for a step change in the set point from 0.9951 to 1.1941, is given in Fig. 11. Finally, it can be easily checked that Method 5, provides the above controller settings, when a gain margin of 8.6733 dB and a phase margin of 31.2332° are prespecified for the closed-loop system. Taking into consideration the difficulty of the prob-lem, the responses obtained are rather acceptable.

6. CONCLUSIONS

In this paper, the problem of tuning PI controllers for UFOPDT processes has been investigated. For addressing the problem, several new methods based on a first-order set-point filter and on some new accurate approximations of the crossover frequencies have been presented. The proposed methods require small computation rates and they are particularly useful for on-line tuning. They also ensure smooth response and fast regulatory control and they are quite robust against parametric uncertainty. Simulation results show that the proposed methods produce as good performance as the best-tuned two-stage IMC method, while retaining the classical PI con-troller structure.

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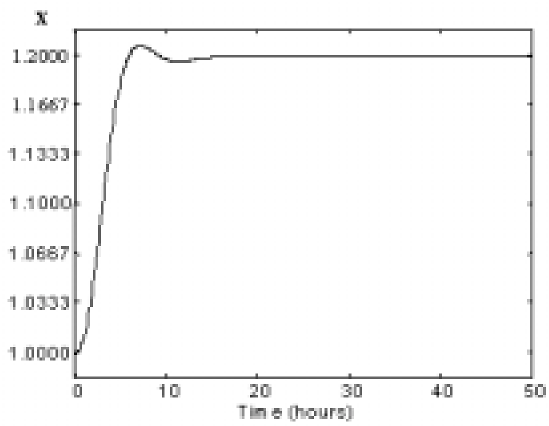

Fig. 10. Closed-loop response of the bioreactor under PI control based on stability margins specifications.

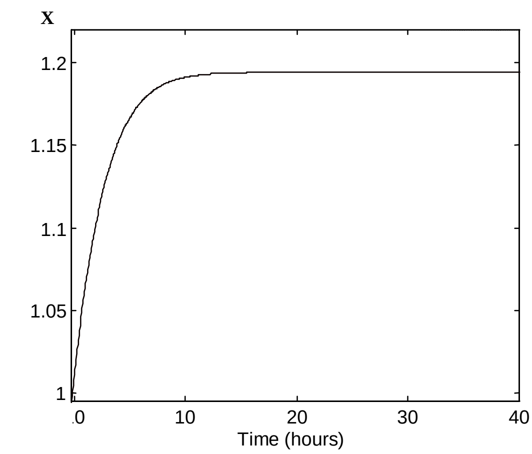

Fig. 11. Closed-loop response of the bioreactor under PI control based on Methods 1-4 with $\xi_{des} = 0.9$.