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NONINTERACTING CONTROL FOR SUBMARINE IN STRAIGHT HORIZONTAL COURSE

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Abstract. It is proposed to perform accurate manoeuvring of a submarine in straight horizontal course, with independent control of elevation angle and heave velocity. Independent control is accomplished via a noninteracting control technique using a static state feedback law. The general analytic expression of the feedback controllers satisfying the I/O decoupling requirement is derived. The necessary and sufficient conditions for noninteracting control with simultaneous stability are explicitly determined in terms of the stability derivatives of the submarine.

Key Words. Noninteracting control, Linear Systems, Submarine control, Stability

NOMENCLATURE

u, w, q, ϑ : x, z-axis velocities, elevation rate, elevation angle
m, I_{yy}, g, B : submarine mass, y-axis moment of inertia, gravitational acceleration, buoyancy
X, Z, M : x- and z-axis external hydrodynamic and propulsion forces, y-axis hydrodynamic and propulsion moment
δ_b, δ_s : bow and stern hydroplane angle
$Z_i, X_i, M_i (i = u, w, \dot{w}, q, \delta_b, \delta_s)$: stability derivatives

1. INTRODUCTION

Position/speed Successful manoeuvring of a submarine depends upon the precise control of its course variables, Fig. 1. The problem of controlling the submarine variables has attracted considerable attention during the last decade (see f.e. [1-7]). In the present paper, the problem is studied for the case of straight horizontal course. The precision of the submarine's manoeuvres can be obstructed by the inherent coupling between the course variables, particularly between elevation angle and heave velocity.

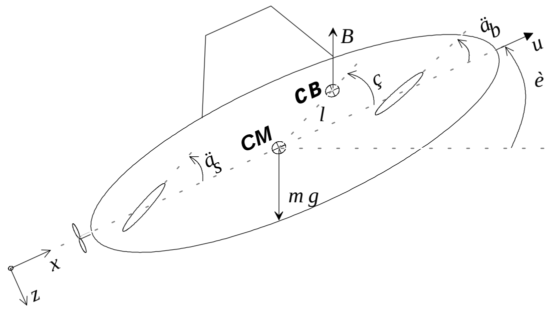

Fig. 1. Submarine Variables Configuration

In Fig. 1, CM denotes the centre of mass of the submarine while CB denotes its centre of buoyancy. The parameter η denotes the angle between the CM-CB axis and the horizontal axis of the submarine. The parameter l denotes the distance between CM and CB.

The present design goal is to control independently elevation angle and heave velocity by appropriate external commands. To accomplish this, a static state feedback law is applied using the I/O decoupling technique. The necessary and sufficient condition for the problem to be solvable is established in the form of an inequality of the stability derivatives of the submarine. The general forms of all static feedback controllers, yielding decoupling, are analytically determined in a simple rational form involving the submarine's stability derivatives and arbitrary parameters. The conditions for decoupling with simultaneous stability are also established. The results are illustrated, by simulation of the elevation pointing manoeuvre and the heave translation manoeuvre.

2. MODEL DESCRIPTION

Using the Euler equations for a submerged solid body, the non-linear equations (body axis system) describing the longitudinal motion of a submarine are :

$$X + (B - mg)\sin(\vartheta) = m(\frac{du}{dt} + qw) \quad (1a)$$

$$Z + (mg - B)\cos(\vartheta) = m(\frac{dw}{dt} - qu) \quad (1b)$$

$$M + B\cos(\vartheta + \eta)l = I_{yy}\frac{dq}{dt} \quad (1c)$$

$$\frac{d\vartheta}{dt} = q \quad (1d)$$

To linearize the above equations, small disturbance theory is used where the submarine variables are expressed in terms of nominal values plus small perturbations :

$$X = X_0 + \Delta X,\ Z = Z_0 + \Delta Z,\ M = M_0 + \Delta M,\ \vartheta = \vartheta_0 + \Delta\vartheta,$$
$$q = q_0 + \Delta q,\ u = u_0 + \Delta u,\ w = w_0 + \Delta w \quad (2)$$

where $X_0, Z_0, M_0, \vartheta_0, q_0, u_0, w_0$ are the nominal values and $\Delta X, \Delta Z, \Delta M, \Delta\vartheta, \Delta q, \Delta w$ are the respective perturbations. For straight horizontal course, the nominal values of ϑ, q and w are considered to be zero $(\vartheta_0 = 0, q_0 = 0, w_0 = 0)$. Since the perturbations are small, the products of perturbations can be neglected while the trigonometric approximations $\sin(\Delta\vartheta) = \Delta\vartheta,\ \cos(\Delta\vartheta) = 1$ are used. Thus, the kinematic equations become:

$$\Delta X + (B - mg)\Delta\vartheta = m\Delta\dot{u} \quad , \quad \Delta Z = m\Delta\dot{w} \ ,$$
$$\Delta M - B\sin(\eta)\Delta\vartheta l = I_{yy}\Delta\dot{q} \quad , \quad \Delta\dot{\vartheta} = \Delta q \quad (3)$$

Normalizing the forces and moment by mass and moment of inertia, respectively, the equations (3) can be rewritten as follows:

$$\Delta\tilde{X} + (\frac{B}{m} - g)\Delta\vartheta = \Delta\dot{u} \quad , \ \Delta\tilde{Z} = \Delta\dot{w} - u_0\Delta q \quad ,$$
$$\Delta\tilde{M} - \frac{Bl\sin(\eta)\Delta\vartheta}{I_{yy}} = \Delta\dot{q} \quad , \Delta\dot{\vartheta} = \Delta q \quad (4)$$

The contribution of small accelerations to the submarine dynamics can be neglected. Furthermore, assume that the angular velocity of the propeller is constant. Hence, the increments $\Delta\tilde{X}$, $\Delta\tilde{Z}$ and $\Delta\tilde{M}$ of the hydrodynamic forces and moment are functions of the bow and stern angles δ_b, δ_s and the velocities u, w and the elevation rate q. Expand the force increments and moment increment into multivariant Laurent series, with respect to $\Delta u, \Delta w, \Delta q, \Delta\delta_s$ and $\Delta\delta_b$, to yield:

$$\Delta\tilde{X} = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial q}\Delta q + \frac{\partial X}{\partial \delta_s}\Delta\delta_s + \frac{\partial X}{\partial \delta_b}\Delta\delta_b \quad (5a)$$

$$\Delta\tilde{Z} = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_s}\Delta\delta_s + \frac{\partial Z}{\partial \delta_b}\Delta\delta_b \quad (5b)$$

$$\Delta\tilde{M} = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_s}\Delta\delta_s + \frac{\partial M}{\partial \delta_b}\Delta\delta \quad (5c)$$

Substituting (5a-c) into (4), the linearized equations of the submarine motion take on the form:

$$\frac{\partial\Delta u}{\partial t} = X_u\Delta u + X_w\Delta w + X_q\Delta q + (\frac{B}{m} - g)\Delta\vartheta +$$
$$+ X_{\delta_b}\Delta\delta_b + X_{\delta_s}\Delta\delta_s$$

$$\frac{\partial\Delta w}{\partial t} = Z_u\Delta u + Z_w\Delta w + (Z_q + u_0)\Delta q + Z_{\delta_b}\Delta\delta_b + Z_{\delta_s}\Delta\delta_s$$

$$\frac{\partial\Delta q}{\partial t} = M_u\Delta u + M_w\Delta w + M_q\Delta q - \frac{B\sin(\eta)\Delta\vartheta l}{I_{yy}} +$$
$$M_{\delta_b}\Delta\delta_b + M_{\delta_s}\Delta\delta_s$$

$$\Delta\dot{\vartheta} = \Delta q$$

where $X_j = \frac{\partial X}{\partial j}$, $Z_j = \frac{\partial Z}{\partial j}$, $M_j = \frac{\partial M}{\partial j}$ $(j = u, w, q, \delta_s, \delta_b)$.

Hence, the linear kinematic equations (short period approximations) describing the motion of the submarine in straight horizontal course may be written in state space form as follows:

$$\dot{x}(t) = Ax(t) + Bu(t),\ \ y(t) = Cx(t),\ \ x(0-) = x_0 \quad (6)$$

where

$$x(t) = \begin{bmatrix} \Delta u & \Delta w & \Delta q & \Delta\vartheta \end{bmatrix}^T,\ y(t) = \begin{bmatrix} \Delta w & \Delta\vartheta \end{bmatrix}^T,$$
$$u(t) = \begin{bmatrix} \Delta\delta_b & \Delta\delta_s \end{bmatrix}^T$$
$$A = \begin{bmatrix} X_u & X_w & 0 & (\frac{B}{m} - g) \\ Z_u & Z_w & Z_q + u_0 & 0 \\ M_u & M_w & M_q & -\frac{B\sin(\eta)\Delta\vartheta l}{I_{yy}} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} X_{\delta_b} & X_{\delta_s} \\ Z_{\delta_b} & Z_{\delta_s} \\ M_{\delta_b} & M_{\delta_s} \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. SOLVABILITY CONDITIONS

Consider the static state feedback law

$$u(t) = Fx(t) + G\omega(t),\ \ \omega(t) = \begin{bmatrix} w_c(t) \\ \vartheta_c(t) \end{bmatrix} \quad (7)$$

where $w_c(t)$ and $\vartheta_c(t)$ are the external inputs considered as the elevation angle and heave velocity commands, respectively. Apply the controller (7) to the system (6) to achieve independent elevation and heave control. In this section, it will be investigated under which conditions the controller (7) results into a diagonally decoupled closed loop system. The important benefit of the diagonal form is that the increment of the heave velocity is controlled only by $w_c(t)$ while elevation angle is controlled only by $\vartheta_c(t)$. The necessary and sufficient conditions for independent control for the heave velocity and elevation angle are established in the theorem stated below.

Theorem 1: Elevation angle and heave velocity can be controlled independently if and only if the following condition is satisfied:

$$M_{\delta_s} Z_{\delta_b} - M_{\delta_b} Z_{\delta_s} \neq 0 \tag{8}$$

Proof : As proven in [8] input-output decoupling is solvable if and only if $\det[C^* B] \neq 0$, where

$$C^* = \begin{bmatrix} c_1 A^{d_1} \\ c_2 A^{d_2} \end{bmatrix}, \; c_i \text{ : the i-th row of } C \text{ ,}$$
$$d_i = \left\{ \begin{array}{c} \min\{j : \; c_i A^j B \neq 0, \; j = 0, 1, ..., n-1\} \\ n-1 \text{ if } c_i A^j B = 0 \;\; \forall j \end{array} \right\} \tag{9}$$

To satisfy the condition $\det[C^* B] \neq 0$, start by observing that $c_1 B = \begin{bmatrix} Z_{\delta_b} & Z_{\delta_s} \end{bmatrix}$, $c_2 B = \begin{bmatrix} 0 & 0 \end{bmatrix}$ and $c_2 AB = \begin{bmatrix} M_{\delta_b} & M_{\delta_s} \end{bmatrix}$. Hence it holds that $d_1 = 0$, $d_2 = 1$ and $C^* B = \begin{bmatrix} Z_{\delta_b} & Z_{\delta_s} \\ M_{\delta_b} & M_{\delta_s} \end{bmatrix}$ and consequently that $\det[C^* B] = M_{\delta_s} Z_{\delta_b} - M_{\delta_b} Z_{\delta_s}$. Thus, the necessary and sufficient condition for decoupling is condition (8). ■

4. EXPLICIT CHARACTERIZATION OF THE CONTROLLER MATRICES

Assuming that the solvability condition is satisfied, i.e. $M_{\delta_s} Z_{\delta_b} - M_{\delta_b} Z_{\delta_s} \neq 0$, and using the design procedure in [9], the general solution of the feedback matrices of the form (7) yielding decoupling, is derived to be:

$$F = \begin{bmatrix} \frac{M_{\delta s} Z_u - M_u Z_{\delta s}}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} & \frac{M_{\delta s}(\lambda_{1,2} - Z_w) + Z_{\delta s} M_w}{M_{\delta s} Z_{\delta_b} - M_{\delta_b} Z_{\delta s}} & \frac{M_{\delta s}(u_0 + Z_q) + Z_{\delta s}(\lambda_{2,3} - M_q)}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} & \frac{Z_{\delta s}(Bl \sin(\eta) + I_{yy} \lambda_{2,4})}{I_{yy}(M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b})} \\ \frac{M_u Z_{\delta_b} - M_{\delta_b} Z_u}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} & \frac{M_{\delta_b}(\lambda_{1,2} - Z_w) + Z_{\delta_b} M_w}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} & \frac{M_{\delta_b}(u_0 + Z_q) + Z_{\delta_b}(\lambda_{2,3} - M_q)}{M_{\delta s} Z_{\delta_b} - M_{\delta_b} Z_{\delta s}} & \frac{Z_{\delta_b}(-Bl \sin(\eta) + I_{yy} \lambda_{2,4})}{I_{yy}(M_{\delta s} Z_{\delta_b} - M_{\delta_b} Z_{\delta s})} \end{bmatrix} \tag{10}$$

$$G = \begin{bmatrix} \frac{M_{\delta s}(p_{1,0})^{-1}}{M_{\delta s} Z_{\delta_b} - M_{\delta_b} Z_{\delta s}} & \frac{Z_{\delta s}(p_{2,0})^{-1}}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} \\ \frac{M_{\delta_b}(p_{1,0})^{-1}}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} & \frac{Z_{\delta_b}(p_{2,0})^{-1}}{M_{\delta s} Z_{\delta_b} - M_{\delta_b} Z_{\delta s}} \end{bmatrix} \tag{11}$$

where $p_{i,0}$ and $\lambda_{i,j}$ are arbitrary parameters.

Relations (10) and (11) are explicit formulae that can be easily implemented via elementary operations upon the stability derivatives, the nominal value of the forward velocity u_0, and the free parameters.

5. DECOUPLED CLOSED LOOP SYSTEM

Assuming that the decoupling condition (8) is satisfied and substituting the controller (7) with F and G given in (10) and (11), to the system (6), the decoupled closed loop system transfer function takes on the form:

$$H(s) = C(sI - A - BF)^{-1} BG = \begin{bmatrix} \frac{(p_{1,0})^{-1}}{s - \lambda_{1,2}} & 0 \\ 0 & \frac{(p_{2,0})^{-1}}{s^2 - \lambda_{2,3} s - \lambda_{2,4}} \end{bmatrix} \tag{12}$$

where $p_{i,0}$ and $\lambda_{i,j}$ are arbitrary parameters.

Clearly, these arbitrary parameters can be used to satisfy stability and amplitude requirements. The general form of $C(sI - A - BF)^{-1} BG$ has three arbitrary poles. Since the system is of fourth order, there exists a pole that is cancelled out in the general from of the closed loop transfer function. The polynomial of the cancelled out pole expressed in terms of stability derivatives of the submarine is:

$$\begin{aligned} p(s) = {} & s(M_{\delta_b} Z_{\delta_s} - M_{\delta_s} Z_{\delta_b}) - M_{\delta_s} X_{\delta_b} Z_u + M_{\delta_b} X_{\delta_s} Z_u \\ & + M_{\delta_s} X_u Z_{\delta_b} - M_u X_{\delta_s} Z_{\delta_b} - M_{\delta_b} X_u Z_{\delta_s} + M_u X_{\delta_b} Z_{\delta_s} \end{aligned} \tag{13}$$

Hence, the cancelled out pole of the closed loop system is:

$$s_0 = \frac{M_{\delta s}(X_{\delta_b} Z_u - X_u Z_{\delta_b}) + M_{\delta_b}(X_u Z_{\delta s} - X_{\delta s} Z_u) + M_u(X_{\delta s} Z_{\delta_b} - X_{\delta_b} Z_{\delta s})}{M_{\delta_b} Z_{\delta s} - M_{\delta s} Z_{\delta_b}} \tag{14}$$

Since the poles of the closed loop transfer function can be arbitrarily assigned, in order to guarantee stability for the transfer function of the closed loop system (transmission poles), the arbitrary parameters $\lambda_{1,2}$, $\lambda_{2,3}$ and $\lambda_{2,4}$ must satisfy the conditions: $\lambda_{1,2} < 0$, $\lambda_{2,3} < 0$ and $\lambda_{2,4} < 0$. In addition, to guarantee stability of the closed loop system, the cancelled out pole must lie in the left complex half-plane. Combining the above remark with the observation that the model (6) is of fourth order, the following theorem may be stated:

Theorem 2: Independent control of the elevation angle and heave velocity with simultaneous stabilization can be achieved if and only if conditions of Theorem 1 are satisfied and $s_0 < 0$. ■

From equation (12), it is clear that by choosing $(p_{1,0})^{-1} = \lambda_{1,2}$ and $(p_{2,0})^{-1} = \lambda_{2,4}$ while restricting the poles of the polynomials $(s - \lambda_{1,2})$ and $(s^2 - \lambda_{2,3} s - \lambda_{2,4})$ to be sufficiently stable, the variables ϑ and w follow accurately the desired trajectories ϑ_c and w_c respectively.

6. SIMULATION RESULTS

In the present simulation a submarine with model description that follows equation (6) is considered and the static state feedback law (7) with controller matrices as in (10) and (11) with $\lambda_{1,2} = -10$, $\lambda_{2,3} = -30$, $\lambda_{2,4} = -20$, $p_{1,0} = 0.1$ and $p_{2,0} = 0.05$ is applied. The responses of the performance variables for heave velocity pointing and elevation pointing are presented in Fig. 2 and 3, respectively. The commands are $w_c(t) = 1$[m/s], $\vartheta_c = 0$[rad] for heave velocity translation and $w_c(t) = 0$[m/s], $\vartheta_c = 0.01$[rad] for elevation pointing. The response of the elevation angle increment in the case of heave velocity pointing is identical to zero. Also, the response of the heave velocity increment in the case of elevation pointing is identical to zero.

It is noted that the performance of the closed loop system, for the case where the angles ϑ and η occur in trigonometric (nonlinear) form, resulting after the application of the controller (7) appears to be visually identical to that presented in Fig. 2 and 3 (linear case).

7. CONCLUSIONS

To perform accurate manoeuvring of a submarine in straight horizontal course, independent control of elevation angle and heave velocity has been proposed. The desired goal has been accomplished using a static state feedback law and applying the I/O decoupling technique. The set of the stability derivatives of the submarine for which I/O decoupling is satisfied, has been explicitly determined. The general analytic expression of the feedback controllers, satisfying the decoupling requirement, has been analytically determined. The necessary and sufficient conditions for I/O decoupling and simultaneous stability have been explicitly determined in terms of the stability derivatives of the submarine. The results have been illustrated via simulations for elevation pointing and heave velocity translation.

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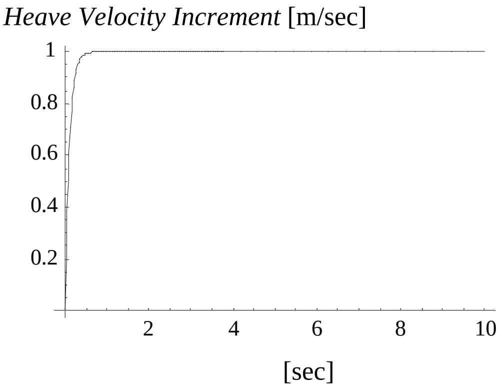

Fig. 2. Heave velocity response for step input command

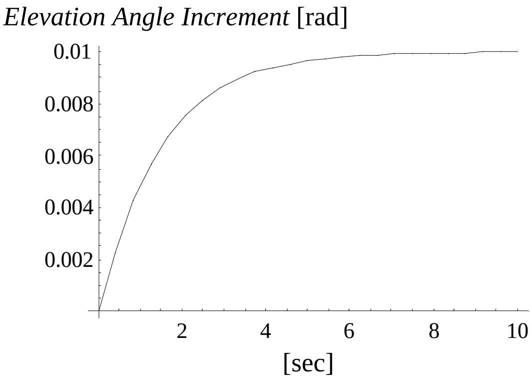

Fig. 3. Elevation response for step input command