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OBSERVERS INTERPOLATION FOR STATE RECONMSTRUCTION OF DISCRETE TIME LPV SYSTEMS: A LMI APPROACH

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Abstract. This paper invistigates a state observation design problem for discrete time linear parameter varying (LPV) systems. The main contribution of this paper consists in providing an interpolation scheme to build the LPV observer. We show that an appropriate choice of the interpolation functions allow to use available quadratic stability conditions to design an LPV observer.

Key Words. LPV systems, LPV Observers, Interpolation, Quadratic stability.

1.INTRODUCTION

Among recent advance in control theory, parameter varying modeling appeared to be well suited in describing the behaviour of a large class of physical systems, see [1] and references therein. Linear Parameter Varying systems theory has been motivated by the gain scheduling approach which attempts to provide a systematic methodology to design parameter-dependent control laws that guarantee stability and performances specifications [2], [3], ...

In this paper we are interested in state reconstruction of discrete time LPV systems. This problem can be investigated using different approaches. One can seek to develop a switching strategy which consists in a collection of linear time invariant (LTI) observers that are scheduled based on the measurement of the parameters. The main difficulty consists in the choice of a switching strategy that ensures at least the stability property. One can also choose to avoid switching difficulties and look for methods that guarantee smooth transition between several observers using, for example, interpolation techniques... Here, we propose a solution to state reconstruction of discrete time LPV systems. The state observer is also parameter varying and is obtained using interpolation of a collection of LTI observers.

In [4], a linear interpolation technique is used to design state feedback observer-based controllers for LPV systems. In the case of a scalar scheduling variable, a stability preserving interpolation in terms of frozen values of the scheduling variable is proposed. An application of interpolation based observer to the design of a gain scheduled robust observer-based controller is also proposed in [5]. These contributions deal with continuous time LPV systems and the used techniques are different from the one developped in this paper.

The paper is organized as follows. Section 2 gives the problem statement and develops a solution for the interpolation based LPV observation problem. Conditions allowing the design of the parameter varying observer are given in terms of LMI conditions. Different error convergence criteria are discussed. Section 3 is devoted to a numerical example.

2.PROBLEM FORMULATION

We consider LPV systems given by:

$$\begin{array}{rcl} x(k+1) & = & A(\rho(k))x(k) + Bu(k) \\ y(k) & = & Cx(k) \end{array} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control vector and $y(k) \in \mathbb{R}^p$ is the measured output vector. The dynamical matrix depends on a time varying parameter ρ and is given by:

$$A(\rho(k)) = A_0 + \rho(k)\Delta_1,\ \rho(k) \in [\rho_1, \rho_2] \tag{2}$$

We assume that ρ is bounded and real-time measurable. We look for a parameter varying observer with the following structure

$$\begin{array}{rcl} \hat{x}(k+1) & = & A(\rho(k))\hat{x} + Bu + \\ & & L(\rho(k))(y(k) - \hat{y}(k)) \\ \hat{y}(k) & = & Cx(k) \end{array} \tag{3}$$

The parameter varying gain $L(\rho(k))$ is obtained by interpolation of an off-line computed constant gains, namely those corresponding to the observers estimating the state of the original system when $\rho(k) = \rho_1$ and $\rho(k) = \rho_2$:

$$\begin{array}{rcl} x(k+1) & = & A_i x(k) + Bu(k) \\ y(k) & = & Cx(k) \end{array} \tag{4}$$

with

$$A_i = A(\rho_i) \quad i = 1, 2$$

The problem to be dealt with in this paper can be formulated as follows: *Find a collection of linear time invariant observers built for the extremal values of the time varying parameter $\rho(k)$, such that an interpolation procedure leads to a parameter varying observer for the original LPV system.*

Let L_i be the constant gains corresponding to the observers estimating the state of the original system for the extremal values (4).

$$\begin{array}{rcl} \hat{x}(k+1) & = & A_i\hat{x}(k) + Bu(k) + \\ & & L_i(y(k) - \hat{y}(k)), \\ \hat{y}(k) & = & C\hat{x}(k) \end{array} \tag{5}$$

The parameter varying gain matrix $L(\rho(k))$ is obtained by an interpolation of the gains L_i

$$L(\rho(k)) = \mu_1(\rho(k))L_1 + \mu_2(\rho(k))L_2 \tag{6}$$

where the interpolation functions $\mu_1(\rho(k))$ and $\mu_2(\rho(k))$ allow a smooth transition between the extremal observers according to actual value of $\rho(k)$. We choose linear interpolation functions as depicted in figure 1.

In this case, the functions $\mu_1(\rho(k))$ and $\mu_2(\rho(k))$ are given by:

$$\mu_1(\rho(k)) = \frac{\rho(k) - \rho_1}{\rho_1 - \rho_2}, \quad \mu_2 = 1 - \mu_1 \tag{7}$$

Using the LPV observer (3), the dynamic of the observation error $e(k) = x(k) - \hat{x}(k)$ is given by:

$$e(k+1) = \big(A(\rho(k)) - L(\rho(k))C\big)e(k)$$

which can be rewritten, using (7), as :

$$e(k+1) = \big(\mu_1(\rho(k))\tilde{A}_1 + \mu_2(\rho(k))\tilde{A}_2\big)e(k) \tag{8}$$

with

$$\tilde{A}_1 = A_1 - L_1C, \text{ and } \tilde{A}_2 = A_2 - L_2C$$

and

$$\mu_i(\rho(k)) > 0, \quad \sum_i \mu_i(\rho(k)) = 1, \quad i = 1, 2 \tag{9}$$

The interpolated parameter varying observer determination reduces to the computation of the constant gains L_i, such that the polytopic uncertain system (8) is asymptotically stable. As the parameter $\rho(k)$ is time varying, and no limitation has been considered on the rate of variation, we look for a single Lyapunov function to prove stability. A new discrete time Lyapunov condition developped in [6] is used. The following Theorem recalls such a condition.

Theorem 1 *[6] The following conditions are equivalent*

i) There exists a symmetric matrix $P > \mathbf{0}$ such that

$$A'PA - P < \mathbf{0} \tag{10}$$

ii) There exist a symmetric matrix $P >$ and a matrix G such that

$$\begin{bmatrix} -P & A'G' \\ GA & P - (G + G') \end{bmatrix} < \mathbf{0} \tag{11}$$

As stated in [6], due to the presence of an extra degree of freedom, namely the matrix G, results based on the evaluation of condition ii) includes as a particular case those based on the classical quadratic stability coniditon i). The following proposition makes use of this new condition to provide the gains matrices L_i.

Proposition 1 *Suppose that there exist matrices $P = P'$, G and R_i, $i = 1, 2$, such that*

$$\begin{bmatrix} -P & A_i'G' - C'R_i' \\ GA_i - R_iC & P - (G + G') \end{bmatrix} < \mathbf{0}, \tag{12}$$

then the extremal LTI observers are given by (6) with $L_i = G^{-1}R_i$. Moreover, the global error dynamic (8) is asymptotically stable and the LPV observer is given by (3) with (6) and (7).

Proof: If condition (12) is satisfied then it is quite easy to check that

$$(A_i - L_iC)'P(A_i - L_iC) - P < \mathbf{0}, \ i = 1, 2$$

which means that the error dynamic is asymptotically stable for the extremal values of $\rho(k)$. In this case, the corresponding observers are given by (6). Since the lyapunov matrix P is common and according to (9), the global error dynamic (8) is asymptotically stable. Hence, the LPV observer is given by (3)with (6) and (7). ■

The error dynamics obtained using a parameter varying observer built as indicated previously converges to the origin asymptotically. One can also look for a faster convergence rate. Depending on the nature of variation of the parameter $\rho(k)$, one can look for different kind of performance criteria.

2.1.Slow parameter variations

In the case of slow parameter variations such that the system can be considered as time invariant over time intervals, a pole placement constraint can be imposed to improve the convergence of the observation error. One can look for a pole placement of the dynamical error observation matrix in a disk $\mathcal{D}(\alpha, r)$ centered at $(\alpha, 0)$ and with radius r such that $|\alpha| + r < 1$. Such a region allows to specify good transient behaviour by mean of managing speed and damping indexes [7], [8]...

Introduce the following change of variables:

$$A_{ir} = \frac{A_i - \alpha\mathbf{I}}{r}, \qquad C_r = \frac{C}{r} \tag{13}$$

The problem of interpolated parameter varying state observer synthesis reduces to find the gains L_i such that the poles of the dynamical error matrix $(A(\rho) - L(\rho)C)$ belong to a specified disk $D(\alpha, r)$. This problem is equivalent to ensure that the poles of the modified slowly varying parameter matrix

$$\frac{(A(\rho) - L(\rho)C - \alpha\mathbf{I})}{r}$$

belong to the unit disk $\mathcal{C}(0, 1)$. Using the change of variables (13), we have the following proposition.

Proposition 2 *Suppose that there exist matrices $P = P'$, G and R_i, $i = 1, 2$, such that*

$$\begin{bmatrix} -P & A_{ir}'G' - C_r'R_i' \\ GA_{ir} - R_iC_r & P - (G + G') \end{bmatrix} < \mathbf{0}, \tag{14}$$

then the extremal LTI observers are given by (6) with $L_i = G^{-1}R_i$. Moreover, the parameter varying observer is given by (3) with (6) and (7) and the poles of the error dynamical matrix belong to the specified disk $D(\alpha, r)$.

2.2.Fast parameter variations

If the parameter variations are such that one can not aproximate the system behaviour by a piece-wise LTI system behaviour, one can also look for a faster convergence rate using the decay rate criteria [9]. In the discrete time case, the decay rate is defined as the larget scalar $\gamma \geq 1$ such that

$$\lim_{k\to\infty} \gamma^k \|e_k\| = 0 \tag{15}$$

holds for all trajectories $e(k)$. The asymptotic convergence corresponds to $\gamma = 1$. We can use the quadratic Lyapunov function

$$V(e(k)) = e'(k)Pe(k)$$

to establish a lower bound on the decay rate γ. The LTI observer gains L_i are then designed using the following proposition.

Proposition 3 *Assume that the following convex optimisation problem*

$$\begin{array}{ll} Min & \beta \\ s.t & \begin{bmatrix} -\beta P & A_i'G' - C'R_i' \\ GA_i - R_iC & P - (G+G') \end{bmatrix} < \mathbf{0}, \\ & 0 < \beta < 1 \end{array} \tag{16}$$

admits a solution $\beta = \beta_*$ with the correponding matrices $P_* = P_*'$, $G = G_*$ and R_{*i}, then the interpolated parameter varying observer obtained with $L_{i*} = G_*^{-1}R_{*i}$ leads to an error convergence to the origin with a decay rate at least equal to $\beta_*^{-\frac{1}{2}}$.

Proof: If (16) admits a solution then

$$(A(\rho(k)) - L(\rho(k))C)'P(\bullet) - \beta P < 0$$

which means that

$$V(e(k+1)) < \beta V(e(k)), \quad 0 < \beta < 1 \tag{17}$$

holds for all trajectories $e(k)$. Hence,

$$V(e(k)) < \beta^k V(e(0))$$

or equivalently

$$\|e(k)\| < \beta^{\frac{k}{2}}.M$$

with $M > 0$ a scalar given by

$$M = \|P^{\frac{1}{2}}\|\|P^{-\frac{1}{2}}\|\|e(0)\| \qquad \blacksquare$$

3.ILLUSTRATIVE EXAMPLE

We consider a LPV system given by (1)-(2) where

$$A_0 = \begin{bmatrix} -0.51 & -1 & 0 \\ 0.28 & -0.1 & -1.12 \\ -0.07 & 0.005 & 0.03 \end{bmatrix},$$

$$\Delta_1 = \begin{bmatrix} -0.9 & 0 & 0.42 \\ 0.08 & 0 & -1.32 \\ -0.74 & -0.550 & 1.7 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.5 & 0 & 0.3 \end{bmatrix}, \quad \rho \in [-0.05, 0.23]$$

Solving the condition of Proposition 1, one gets the following gains

$$L_1 = \begin{bmatrix} 1.0454 \\ -0.8783 \\ 0.0276 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1.5389 \\ -1.0666 \\ 0.6230 \end{bmatrix}$$

The corresponding LPV observer has been simulated with an initial error $e(0) = \begin{bmatrix} 10 & -5 & 1 \end{bmatrix}'$, a control $u = 0$ and a fast parameter variation as shown in figure 2. The obtained behaviour is reported in figure 4. Figure 3 shows the interpolation functions $\mu_1(\rho(k))$ and $\mu_2(\rho(k))$.

The same example has been considered for slow variation of the parameter ρ. To guarantee a good convergence rate of the observation error, a disk centered at 0 with radius 0.5 has been specified. Solving the LMIs given in Proposition 2, we get the following result

$$L_1 = \begin{bmatrix} 0.7566 \\ -0.4867 \\ 0.0802 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1.2481 \\ -0.4891 \\ 0.4252 \end{bmatrix}$$

Figure 5 shows the pole location of the error dynamic as well as the behaviour obtained with the proposed LPV observer. The simulation has been performed for 60 values of ρ with the same initial error and control value as previously.

4.CONCLUSION

An interpolation based method has been proposed to design a parameter varying observer. It is based on a new LMI condition with an extra degree of freedom. The choice of linear interpolation functions has been made for simplicity reasons. Polynomial inteprolation functions has also been investigated but the improvement of the observer behaviour is not satisfactory regarding to the complexity of the mathematical developpments. Only the scalar parameter case has been considered in this paper. Extending the proposed results to the vector case as well as reducing the conservatism of the our approach will be addressed in a near future.

References

[1] D. J. Leith and W. E. Leithead, "Survey of gain scheduling analysis and design", *To appear in International Journal of Control.*

[2] G. Becker and A. K. Packard, "Robust performance of linear parametrically varying systems using parametrically dependent linear feedback", *Systems and Control Letters*, vol. 23, pp. 205–215, 1994.

[3] P. Apkarian and P. Gahinet, "A convex characterization of gain-scheduled h_infty controllers", *IEEE Trans. on Automatic Control*, vol. 40, pp. 853–864, 1995.

[4] D. J. Stiwell and W. J. Rugh, "Interpolation of observer state feedback controllers for gain scheduling", *IEEE Trans. on Automatic control*, vol. 44, pp. 1225–1229, 1999.

[5] A. R. Hyde and K. Glover, "The application of scheduled h_∞ controllers to a vstol aircraft", *IEEE Trans. on Automatic Control*, vol. 38, pp. 1021–1039, 1993.

[6] J. Bernussou M. C. de Oliveira and J. C. Geromel, "A new discrete-time robust stability condition", *Systems and Control Letters*, vol. 36, pp. 135–141, 1999.

[7] K. Furuta and S. B. Kim, "Pole assignment in a specified disk", *IEEE Tran. on Auto. Cont.*, vol. AC-32, pp. 423–427, 1987.

[8] G. Garcia, J. Daafouz, and J. Bernussou, "Output feedback disk pole assignment for systems with positive real uncertainty", *IEEE Tran. on Auto. Cont.*, vol. AC-41, pp. 1385–1391, 1996.

[9] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequality in Systems and Control Theory*, Studies in Applied Mathematics, 1994.

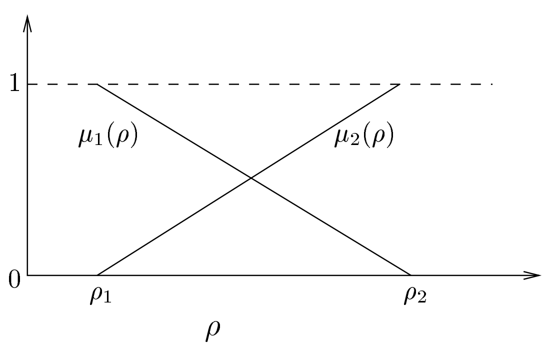

Figure 1: Interpolation functions

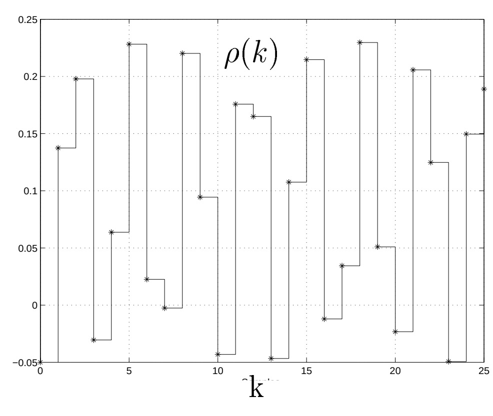

Figure 2: Parameter variation

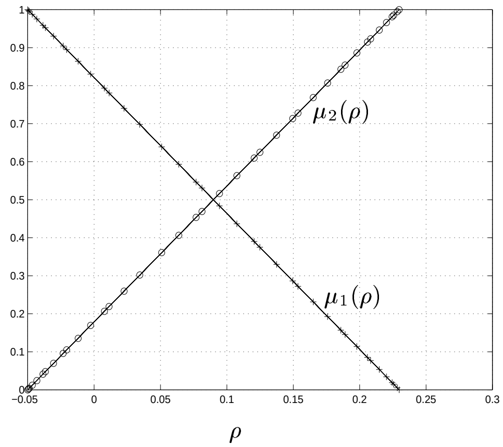

Figure 3: Interpolation function $\mu_1(\rho(k))$ and $\mu_2(\rho(k))$

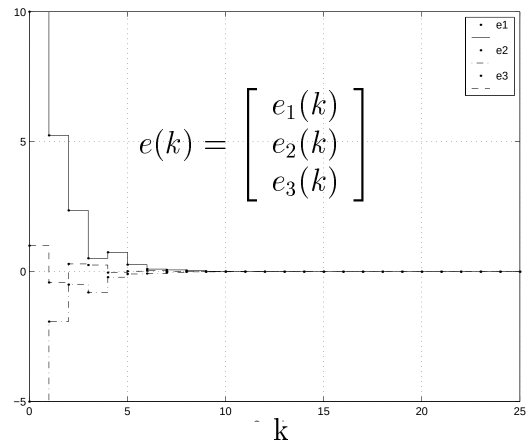

Figure 4: The LPV observer behaviour

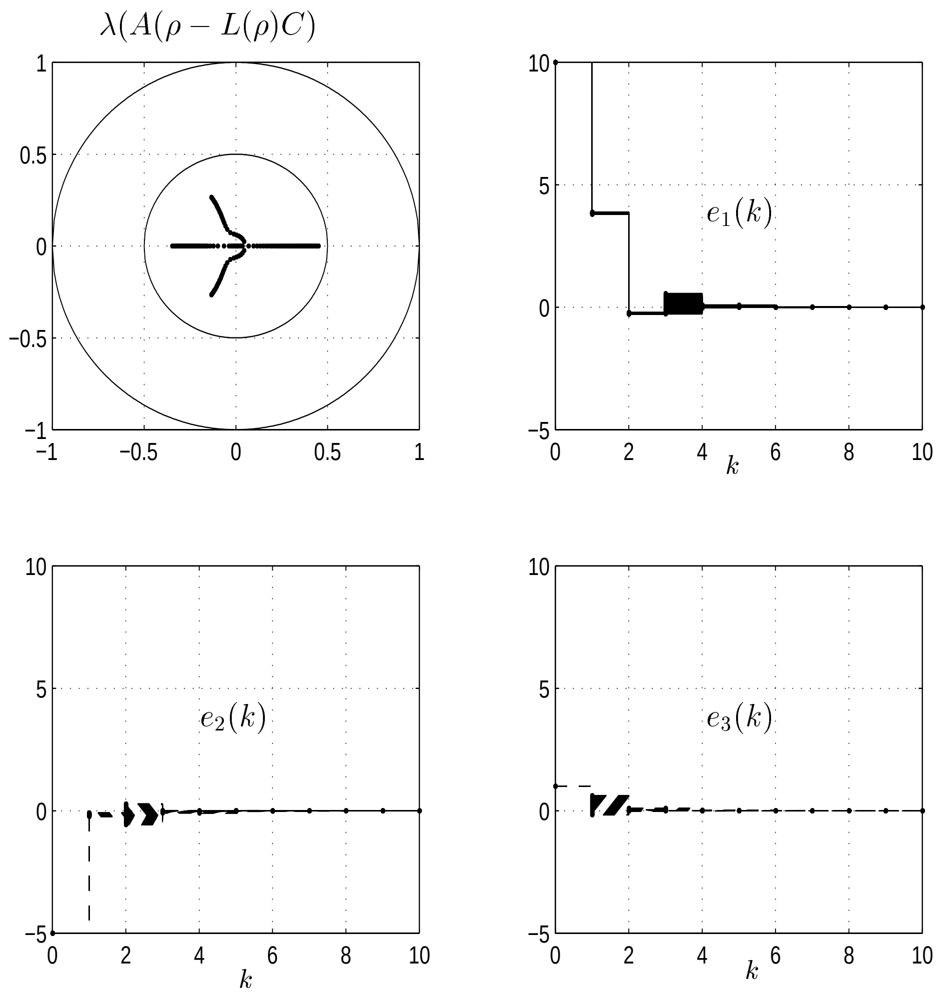

Figure 5: Slow parameter variation case