Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

Distributed Optimization and Control Using Only a Germ of Intelligence

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Abstract

Foraging can be modeled as an optimization process where an animal seeks to maximize energy obtained per unit time spent foraging. Search strategies form the basic foundation for foraging decisions. Here, the chemotactic behavior of *E. coli*, i.e., how it forages, is explained and a computer program that emulates its foraging optimization process is presented and applied to solve a function minimization problem. Then, it is explained how biomimicry of bacterial foraging can be used to provide adaptive control strategies, and methods for distributed coordination and control of autonomous vehicles. Next, we endow our forager with higher cognitive functions (e.g., learning and planning) and discuss how this impacts coordination, control, and swarming behavior for autonomous vehicles. Foundations in optimization theory are discussed. Finally, we explain how to perform stability analysis of swarms, thereby providing mathematical foundations for the study of social foraging.

1 Foraging Theory

Animals search for and obtain nutrients in a way that maximizes

$$\frac{E}{T}$$

where E is energy obtained, and T is time spent foraging (or, they maximize long-term average rate of energy intake). Evolution optimizes foraging strategies since animals that have poor foraging performance do not survive.

Generally, a foraging strategy involves finding a "patch" of food (e.g., group of bushes with berries), deciding whether to enter it and search for food (do you expect a better one?), and when to leave the patch. There are predators and risks, energy required for travel, and physiological constraints (sensing, memory, cognitive capabilities). Foraging scenarios can be modeled and optimal policies can be found using, for instance, dyanamic programming. Search and optimal foraging decision-making of animals can be broken into three basic types: cruise (e.g., tunafish, hawks), saltatory (e.g., birds, fish, lizards, and insects), and ambush (e.g., snakes, lions). In cruise search the animal searches the perimeter of a region, an in ambush it sits and waits. In saltatory search an animal typically moves in some direction, stops (or slows down), looks around, and then changes direction. It searches throughout a whole region.

Some animals forage as individuals and others forage as groups. While to perform social foraging an animal needs communication capabilities, it can gain advantages in that it can essentially exploit the sensing capabilities of the group, the group can "gang-up" on large prey, individuals can obtain protection from predators while in a group, and in a certain sense the group can forage with a type of collective intelligence. Social foragers include birds, bees, fish, ants, wildebeasts, and primates. Note that there is a type of "cognitive spectrum" where some foragers have little cognitive capability, and other higher life forms have significant capabilities (e.g., compare the capabilities of a single ant with those of a human). Generally,

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endowing each forager with more capabilities can help them succeed in foraging, both as an individual and as a group. From an engineering perspective both ends of such a spectrum are interesting.

2 Chemotactic Behavior of E. coli

Here, we consider the foraging behavior of *E. coli*, which is a common type of bacteria (it lives in your gut) with a diameter of $1\mu m$ and a length of about $2\mu m$, and which under appropriate conditions can reproduce (split) in 20 min. Its ability to move comes from a set of up to six rigid $100-200$ rps spinning falgella, each driven by a biological "motor." An *E. coli* bacterium alternates between running (at $10-20$ μmeters/sec, but they cannot swim straight) and tumbling (changing direction). When the flagella rotate clockwise (counterclockwise) they unbundle (bundle into a "propeller") and hence tumble (run).

Chemotactic actions:

1. If in neutral medium alternate tumbles and runs $\Rightarrow$ Search
2. If swimming up nutrient gradient (or out of noxious substances) swim longer (climb up nutrient gradient or down noxious gradient) $\Rightarrow$ Seek increasingly favorable environments
3. If swimming down nutrient gradient (or up noxious substance gradient), then search $\Rightarrow$ Avoid unfavorable environments

In this way it can climb up nutrient "hills" and at the same time avoid noxious substances. The sensors it uses are receptor protiens where are very sensitive, and overall there is a "high gain" (i.e., a small change in concentration of nutrients can cause a significant change in behavior). The sensor averages sensed concentrations and computes a *time* derivative. This is probably the best understood sensory and decision-making system in biology (it is understood and simulated at molecular level).

Bacteria are often killed and dispersed and this can be viewed as part of their motility. Mutations in *E. coli* affect, e.g., reproductive efficiency at different temperatures, and occur at a rate of about 10^{-7} per gene, per generation. *E. coli* occasionally engage in a type of "sex" called "conjugation" that affects characteristics of a population of bacteria. There are many other types of taxes that are used by other bacteria. For instance, some bacteria are attracted to oxygen (aerotaxis), light (phototaxis), temperature (thermotaxis), or magnetic lines of flux (magnetotaxis). Some bacteria can change their shape and number of flagella based on medium to reconfigure to ensure efficent foraging in a variety of media.

E. coli and *S. typhimurium* can form intricate stable spatio-temporal patterns in certain semi-solid nutrient media. They can radially eat their way through a medium if placed together initially at its center. Moreover, under certain conditions they will secrete cell-to-cell attractant signals so that they will group and protect each other. These bacteria can "swarm."

3 Bacterial Swarm Foraging for Optimization

Here, the basic goal is to find the minimum of

$$J(\theta),\ \theta \in \Re^p$$

when we do not have the gradient $\nabla J(\theta)$. Suppose θ is the position of a bacterium, and $J(\theta)$ represents an attractant-repellant profile (i.e., it represents where nutrients and noxious substances are located so $J<0$, $J=0$, and $J>0$ represent the presence of nutrients, a neutral medium, and the presence of noxious substances, respectively).

Let

$$P(j,k,\ell) = \left\{\theta^i(j,k,\ell) | i = 1, 2, \ldots, S\right\}$$

represent the positions of each member in the population of the S bacteria at the j^{th} chemotactic step, k^{th} reproduction step, and ℓ^{th} elimination-dispersal event. Let $J(i,j,k,\ell)$ denote the cost at the location of the i^{th} bacterium $\theta^i(j,k,\ell) \in \Re^p$.

Let N_c be the length of the lifetime of the bacteria as measured by the number of chemotactic steps. To represent a tumble, a unit length random direction, say $\phi(j)$, is generated; then we let

$$\theta^i(j+1,k,\ell) = \theta^i(j,k,\ell) + C(i)\phi(j)$$

so that $C(i) > 0$ is the size of the step taken in the random direction specified by the tumble. If at $\theta^i(j+1,k,\ell)$ the cost $J(i,j+1,k,\ell)$ is better (lower) than at $\theta^i(j,k,\ell)$, then another chemotactic step of size $C(i)$ in this same direction will be taken, and repeat that up to a maximum number of steps, N_s.

To model the cell-to-cell signaling via an attractant we use functions $J^i_{cc}(\theta)$, $i = 1,2,\ldots,S$. Let

$$d_{attract} = 0.1$$

be the depth of the attractant released by the cell and

$$w_{attract} = 0.2$$

be a measure of the width of the attractant signal. How does a cell repel another one? Via local consumption, and cells are not food for each other. Let

$$h_{repellant} = d_{attract}$$

be the height of the repellant effect (magnitude) and

$$w_{repellant} = 10$$

be a measure of the width of the repellant. Let

$$\begin{aligned} J_{cc}(\theta) &= \sum_{i=1}^{S} J^i_{cc} \\ &= \sum_{i=1}^{S}\left[-d_{attract}\exp\left(-w_{attract}\sum_{j=1}^{p}(\theta_j-\theta^i_j)^2\right)\right] \\ &+ \sum_{i=1}^{S}\left[h_{repellant}\exp\left(-w_{repellant}\sum_{j=1}^{p}(\theta_j-\theta^i_j)^2\right)\right] \end{aligned}$$

where $\theta = [\theta_1,\ldots,\theta_p]^\top$ is a point on the optimization domain.

For swarming we will consider minimization of

$$J(i,j,k,\ell) + J_{cc}(\theta^i(j,k,\ell))$$

so that the cells will try to find nutrients, avoid noxious substances, and at the same time try to move towards other cells, but not too close to them. The $J_{cc}(\theta^i(j,k,\ell))$ function dynamically deforms the search landscape to represent the desire to swarm.

After N_c chemotactic steps, a reproduction step is taken. Suppse there are N_{re} reproduction steps. For reproduction, the healthiest bacteria (ones that have lowest accumulated cost over their lifetime) split, and then we kill the same number of unhealthy ones (hence, we get a constant population size). Let N_{ed} be the number of elimination-dispersal events and for each elimination-dispersal event each bacterium in the population is subjected to elimination-dispersal (death, then random placement of a new bacterium at a random location on the optimization domain) with probability p_{ed}. Is this a biologically valid model? No, not completely. The objective is simply to capture the gross characteristics of chemotactic hill-climbing and swarming.

As an example, we try to find the minimum of the function in Figure 1 (note that the point $[15,5]^\top$ is the global minimum point and $[20,15]^\top$ is a local minimum). Standard ideas from optimization theory can be used to set the algorithm parameters. If no swarming is used, and $S = 50$, $N_c = 100$, $N_s = 4$ (a biologically-motivated choice), $N_{re} = 4$, $N_{ed} = 2$, $p_{ed} = 0.25$, $C(i) = 0.1$, $i = 1,2,\ldots,S$, with a random

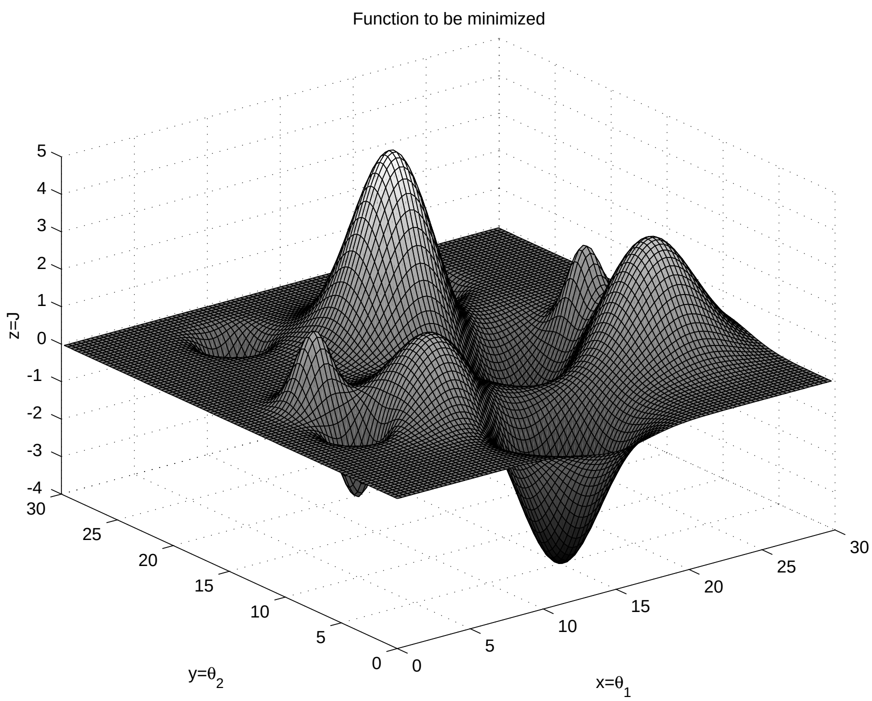

Figure 1: Nonlinear function with multiple extremum points.

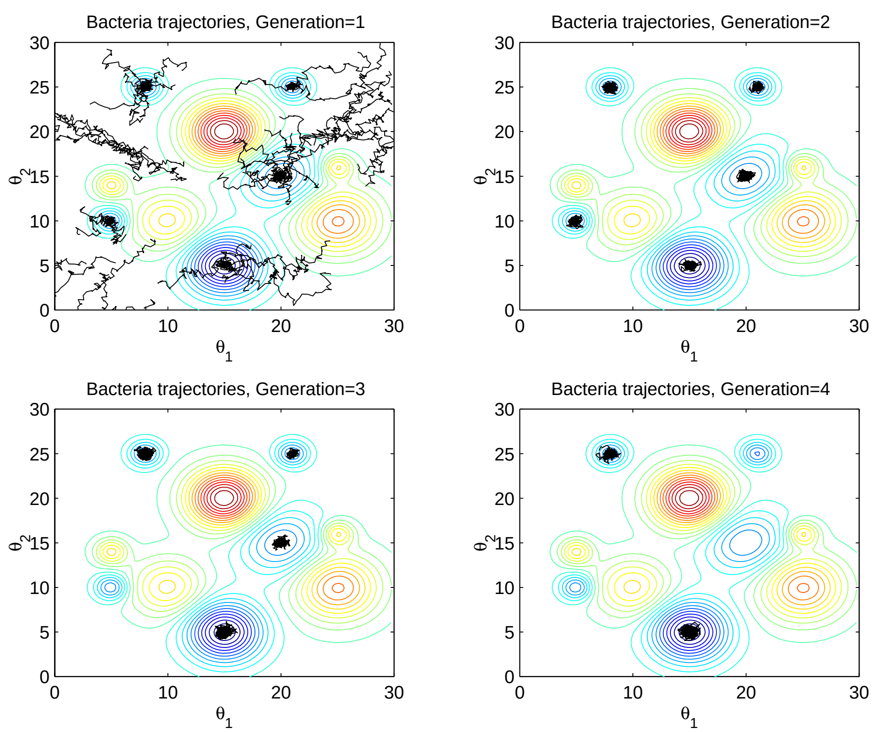

Figure 2: Bacterial motion trajectories, generations 1-4, on contour plot.

initial bacteria distribution, the results are shown in Figure 2. Note that in generation 1 the bacteria search a wide area of the optimization domain and by subsequent generations local minima are found.

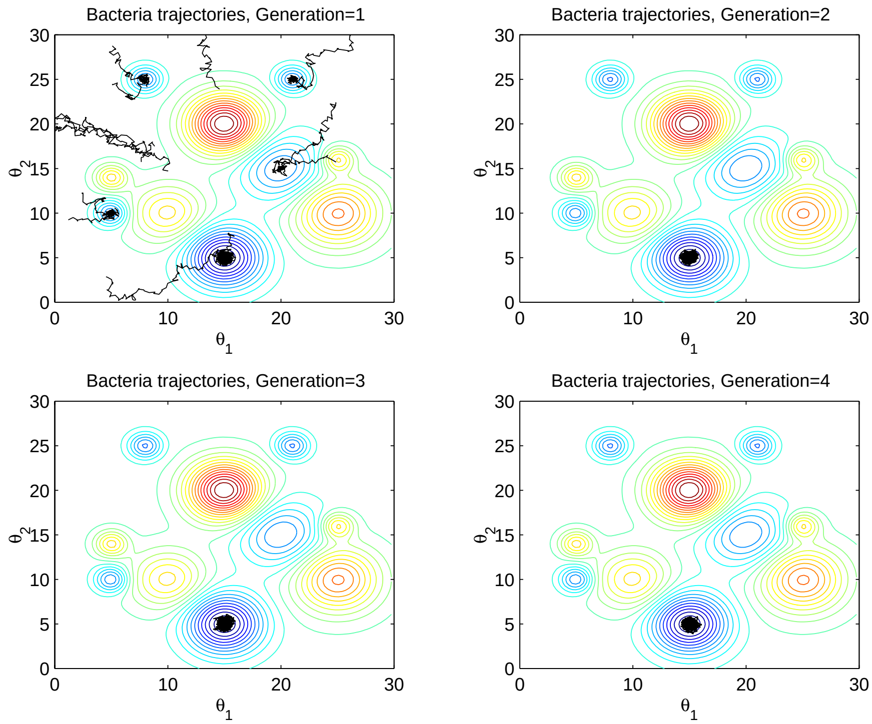

Figure 3: Bacterial motion trajectories, generations 1-4, on contour plot, after an elimination-dispersal event.

Next, an elimination-dispersal event and we get Figure 3 and after several generations all the baceria are near the global minimum so the algorithm succeeds (but at quite a computational cost).

To study swarming effects consider optimization over the function in Figure 4. Initially, place all the cells at the peak $[15, 15]^\top$. We get Figure 5 where in the first generation the bacteria move radially and then later swarm when there is little food. Next, for Figure 1 and random initialization get Figure 6. Notice the swarming behavior in generations 1 and 2 in regions where there is little food.

Finally, note that it is important to study relationships to other optimization methods such as stochastic approximation methods, genetic algorithms, etc. Evolution made bacterial foraging search strategies "optimal" for their environment (a class of cost functions, perhaps ones that are quite different from those found in engineering applications). What is the value of such an approach? To be determined, but for now: Fun, science, conceptual framework/metaphor, biomimicry for engineering and control.

4 Bacterial Foraging for Control

You can use biomimicry of bacterial foraging strategies to provide ideas for how to solve control problems. For instance, the hill-climbing process of a single bacterium provides a stochastic optimization strategy that could be used in indirect adaptive control for identifier model parameter tuning; then coupled with a certainty equivalence controller it could provide for an adaptive control strategy. If you have a whole population of bacteria then you can use an (indirect) multiple model strategy where evolutionary characteristics (reproduction and elimination/dispersal) can be incorporated. It is also possible to use set-based optimization strategies (i.e., ones where multiple parameter vectors are updated simultaneously) for direct adaptive control. To do this you simply employ a type of model predictive control (MPC) strategy where the optimization method chooses among a finite number of controls that generate simulated responses for the system (of course, this requires a model of the plant). If in the MPC strategy you adapt the model you can obtain a combination of the direct and indirect strategies. Such methods are closely related to genetic adaptive control strategies.

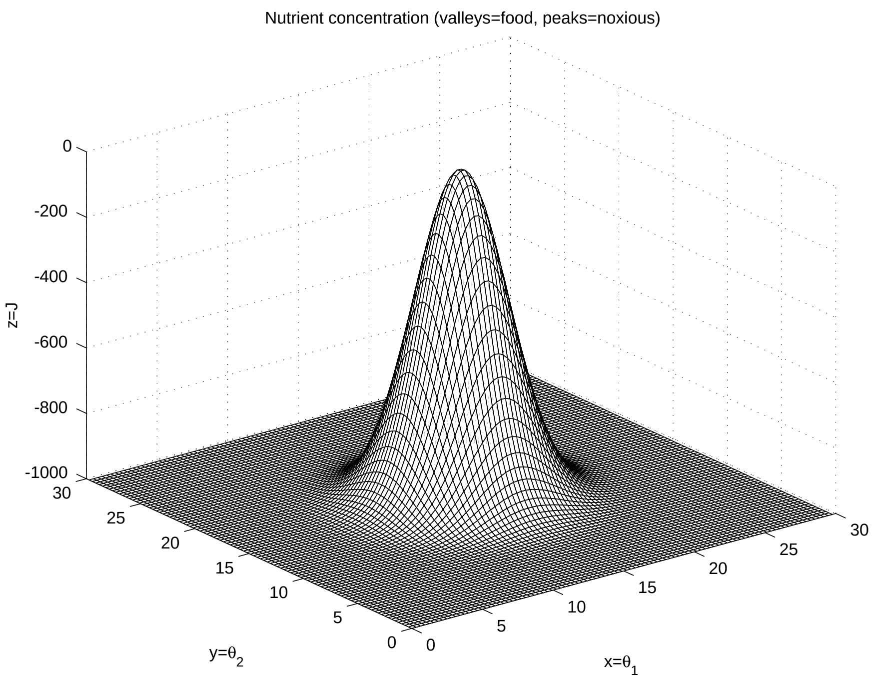

Figure 4: Function for testing swarm behavior of *E. coli*.

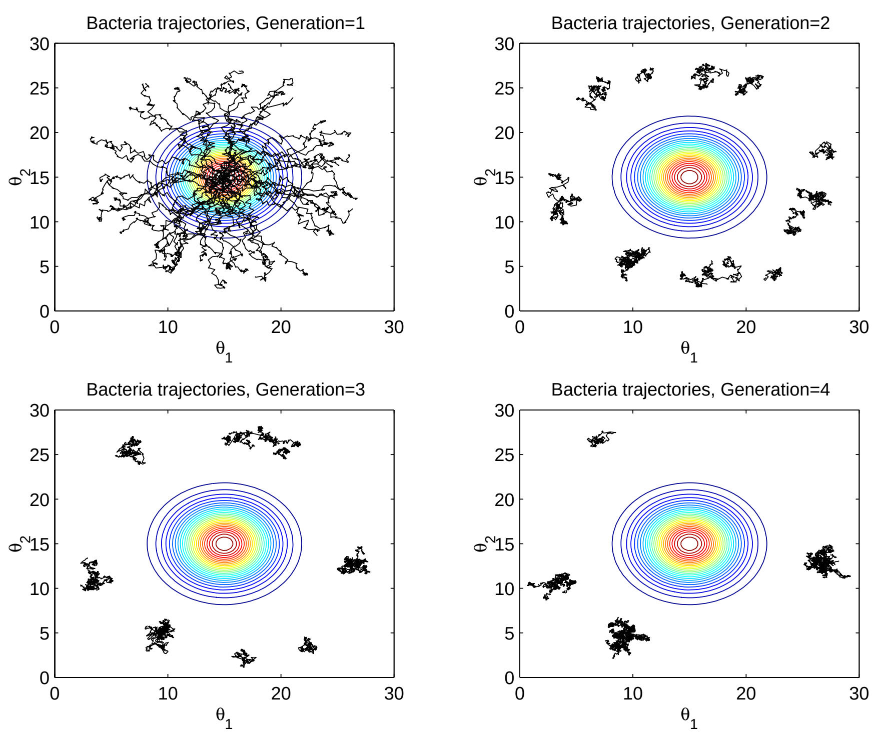

Figure 5: Swarm behavior of *E. coli* on a test function.

Next, note that if you view the bacterium as a small autonomous vehicle (a roboticist's/nanotechnologist's dream!) then its search/avoidance strategy provides a metaphor for the design of strategies for guidance and swarming of autonomous vehicles. Clearly, however the environment that the bacteria commonly forages in

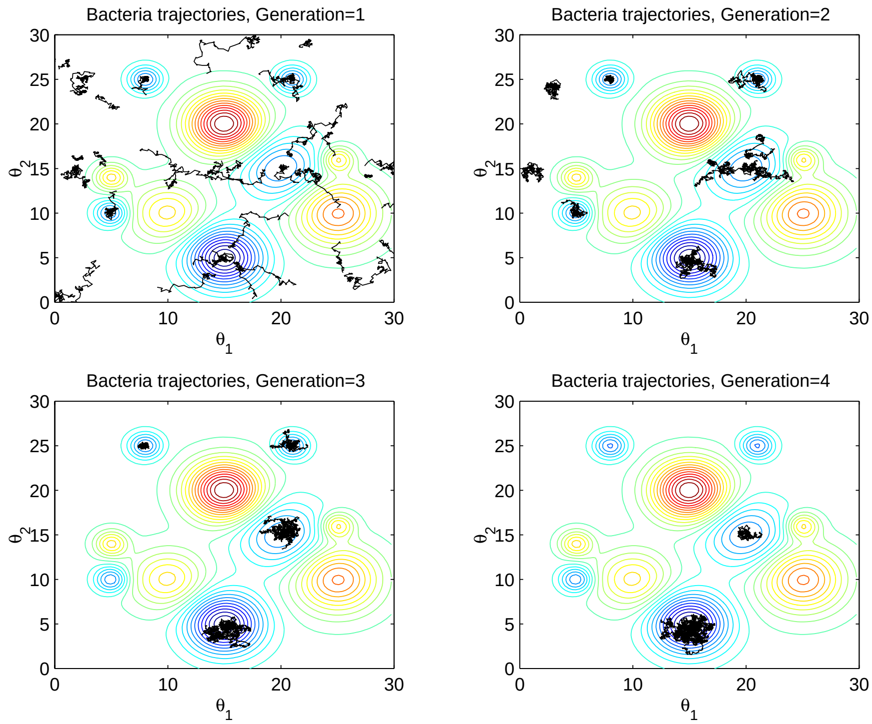

Figure 6: Swarm behavior of *E. coli* on a test function.

may be quite different from that of your autonomous vehicles operating domain; hence, other strategies are likely to be more successful. Moreover, current technology may allow more on-board functionality and hence more sophisticated foraging decision-making.

5 Intelligent Foraging for Distributed Coordination and Control

What if each agent has more capability in decision-making and communication than a simple bacterium? Even a slight improvement can significantly change the behavior of a forager or group of foragers. For instance, the bacterium *M. xanthus* is capable of a variety of cell-to-cell communications, complex vegatative swarming, and sophisticated cell colony protection methods (the "fruiting body"). Higher animals (e.g., primates) have an ability to pay attention to the most important parts of their environment, and to learn and plan. How does this affect their foraging strategies? What if we add learning and planning capabilities to an autonmous vehicle? At the basis of cognitively-sophisticated foraging strategies is optimization and search theory. For instance, simplex and pattern search methods can be thought of as search strategies for foraging. Even more relevant are the "surrogate model" methods (response surface methods) where the search algorithm builds a model of the cost function using interpolation methods and strategies for deciding where to move to take measurements. Such a method is analogous to the building of a "cognitive map" (something that behavioral ecologists and psychologists have studied for some time). For autonomous vehicles we think of having on-board computers that can build maps of their environment, and use these maps to plan and act. For social foraging we are concerned with how vehicles can share learned information and work together as a group (e.g., coordinate their planned actions over their learned maps).

6 Stable Foraging Swarms

One component of the development of the mathematical foundations for the study of social foraging is to study what is necessary in terms of communications and on-board capabilities to achieve swarming behavior (e.g., group cohesion, coordinated movements, effective group search). Swarm cohesion can be formulated as a stability problem. First, you model a single swarm member as having certain sensing and locomotion capabilties. For instance, you might define a swarm member as having an ability to sense the location of its neighbors, with a possible random delay in obtaining the neighbor's position. It may be able to move according to the sensed inter-swarm member distances to try to achieve a "comfortable" distance between itself and its neighbors (it wants to be close, but does not want to collide). Under these conditions, if a "proximity sensor" is used to avoid collisions, if the swarm member movement strategy is defined properly, and if one swarm edge member stays stationary, it is possible to show that a group of swarm members will converge to be near each other. Moreover, if you assume that the each swarm member's sensed values of its neighbor's positions is delayed by no more than a fixed value, then you can get convergence within a finite time. If you assume that swarm edge members are "leaders" in the sense that they make movements to search for nutrients, then you can study swarm stability for the case where the swarm moves to maintain cohesion, and moves to forage. Vehicle dyanamics and learning and planning capabilities can also be included; however, this significantly complicates the stability analysis.

7 Concluding Remarks

- You can do a lot with a germ of intelligence, and some communications!
- Biomimicry of optimal foraging for distributed optimization and control is useful from an engineering perspective.
- Theoretical foundations (swarm stability, optimization) are very important.
- Relevant engineering applications... autonomous vehicles, adaptive control applications, etc.

8 Simulation Code Available

The code for the bacterial swarm foraging algorithm was written in Matlab 5.2. If you would like to obtain this code (and other code for intelligent systems and control) see:

`http://eewww.eng.ohio-state.edu/~passino`

9 Relevant Literature

Foraging theory is described in [1] and the part on search strategies of foraging animals is based on [2]. "Ant colony optimization" is an optimization method based on foraging in ant colonies and it is discussed in [3]. Group behavior of organisms is discussd in the areas of swarm intelligence and artificial life [3, 4, 5, 6].

The description of the biological details of the *E. coli* bacteria and their motile behavior were taken from [7, 8, 9, 10, 11, 12, 13]. Pattern formation in *E. coli* and *S. typhimurium* is discussed in [14, 15, 16, 17, 18, 19] (and a mathematical swarm model and simulations are provided in [17]). The phrase "germ of intelligence" was borrowed from [20].

Genetic adaptive estimation and control strategies [21, 22, 23, 24] are similar to ones based on biomimicry of social foraging.

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