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# SELF-MODULATION IN NONLINEAR FEEDBACK SYSTEMS WITH JUMP RESONANCE

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**Abstract.** The paper proposes to present in a compact form, the phenomenon of self-modulation that is likely to appear in nonlinear feedback systems presenting jump resonance. The system contains a time slow variable parameter depending on the modification by jump of the output signal. Due to this, the output signal undergoes a process of self-modulation in amplitude and phase through resonance jumps.

**Key Words.** Nonlinear feedback system, jump resonance, self-modulation.

## 1. INTRODUCTION

In harmonically excited nonlinear control systems, the phenomenon of jump resonance may be made evident under certain circumstances,[1]. In such systems, the authors have illustrated the possibility of producing the self-modulation by resonance jumps, for signal of constant amplitude and frequency.
The paper aims to present some aspects of the mentioned phenomenon, the circumstances it takes place and the numerically simulated results.

## 2.THEORETICAL CONSIDERATIONS

A nonlinear feedback system is considered, Fig.1, where N and L stand for the system nonlinear and linear element respectively.
It is assumed that the system is stable, subharmonic oscillations do not take place and the linear element plays the role of the low-pass filter.
If at the system input the signal

$$g(t) = R \sin \omega t \qquad (1)$$

is applied, then, the input signal of the nonlinearity is:

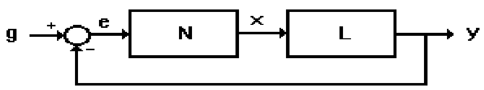



Fig.1 Nonlinear feedback system.

$$e(t) = A \sin(\omega t + \phi) \qquad (2)$$

We consider the system in Fig.1 containing a nonlinearity of saturation type. Considering the mentioned assumptions, N(A) being the nonlinearity describing function (the nonlinearity is static, non-phase-shifting) and H (jω) the frequency response of the linear element, the relations can be written:

$$\frac{E}{G}(j\omega, A) = \frac{1}{1 + H(j\omega)N(A)} \qquad (3)$$

$$\frac{Y}{G}(j\omega, A) = \frac{H(j\omega)\,N(A)}{1 + H(j\omega)\,N(A)} \qquad (4)$$

or, considering the absolute values,

$$|Y(j\omega, A)| = |E(j\omega)|\,|N(A)H(j\omega)| \qquad (5)$$

Is it noticed that | Y(jω, A) | directly depends on the solution of the equation (3). Defining:

$$H(j\omega)=\left| H(j\omega) \right| e^{j\arg H(j\omega)} = H\, e^{j\theta}$$

$$N(A)=\left|N(A)\right| e^{j\arg N(A)} = N \qquad (6)$$

The relation (3) can be written in absolute values:

$$A\,N(A)=\frac{-A\cos\theta}{H} \pm \frac{1}{H}\sqrt{R^2 - A^2\sin^2\theta} \qquad (7)$$

Noting:

$$f_n(A)=A\,N$$

$$f_l(A)=\frac{-A\cos\theta}{H} \pm \frac{1}{H}\sqrt{R^2 - A^2\sin^2\theta} \qquad (8)$$

The relation (7) becomes:

$$f_n(A)=f_l(A) \qquad (9)$$

It is well known that the relation between the input amplitude R and the amplitude A at a certain frequency under which jump resonance occurs may be of an S-shaped form as shown in Fig.2 and the jump resonance condition is given by:

$$(\delta R / \delta A)_{\omega=const} < 0 \qquad (10)$$

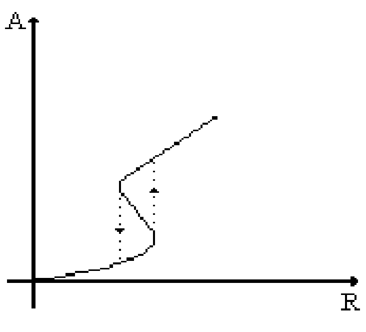



Fig.2. R-A curve.

If the linear or nonlinear element contains an parameter depending on the amplitude A,p = f(A), its monotonous variation determines the modification of the characteristic A(R), Fig.3, the frequency of the input signal staying constant. Taking into account the position of the characteristics A(R) related to the line R = $R_1$ , and the sign of the term $\delta^2 R / \delta A^2$, the sign + (-) is adopted if:

$$(\delta^2 R / \delta A^2)_{R=R_1} > 0\,(<0)$$

$$\text{at } (\delta R / \delta A)_{R=R_1} = 0 \qquad (11)$$

The modification through jump of the amplitude A determines the slow variation of the parameter p so that the condition (11) are successively fulfilled.
As a result, a process of self-modulation in amplitude and phase takes place in the system through resonance jumps of the output signal, [4].
The phenomenon of self-modulation appears in systems containing nonlinearities of both type I and type II, [3].

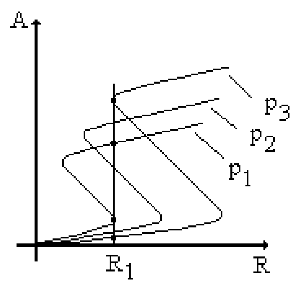
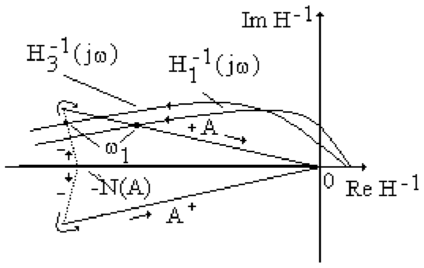
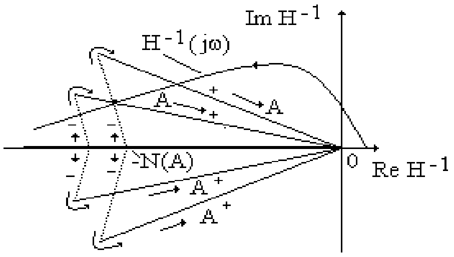



Fig.3 Effect of variable parameter p on R-A curve

The appearance of the self-modulation can be illustrated using the general criterion of jump resonance, as well, [3].
If the variable parameter p belongs to the linear element, then the self-modulation takes place if the inverse transfer locus $H^{-1}$ (jω) ($H1^{-1}$ (jω), $H_3^{-1}$ (jω) respectively) intersects the boundary of the jump resonance area having the sign + (-) at the same frequency, corresponding to the values $p_1$, $p_3$ respectively of the parameter p, Fig.4.a.
The variable parameter p is likely to belong to the system nonlinearity.
Its modification determines the modification of the jump resonance area.
The phenomenon of self-modulation takes place, only, if the inverse transfer locus $H^{-1}$(jω) goes through the intersection point of those two boundaries, having the sign +, -respectively, Fig.4.b. Using a graphical-analytical method, one can successively make evident the phenomenon of jump resonance and self modulation.




(a)




(b)

Fig.4 Illustration of the conditions of self-modulation by using the general criterion of jump resonance and self-modulation.

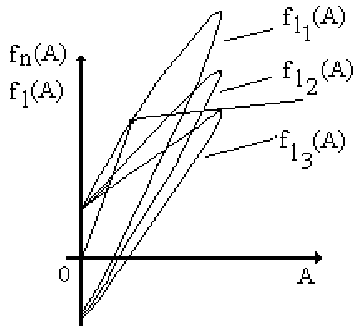
The solution of the equation (9) can be graphic analytically method obtained ($-\pi < \theta < -\pi/2$), [2], Fig.5., a, b, when the three intersection points correspond to three stable solution. If between the two characteristics there is an inner or outer tangent point, Fig.5, a, b, the solution is unstable, the resonance jump taking place.
If the linear or nonlinear element of the system contains a time slow variable parameter, aperiodic,

$$H_1\ (s) = \frac{\alpha}{T_1 T_2 s^2 + (T_1 + T_2)s + 1}$$

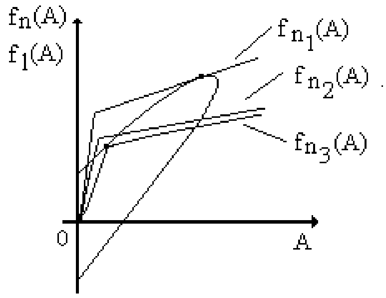
but depending on the modification of the value A through jump, p = f(A), then the inner and other tangency, Fig.5, in the tangent points, can be successively reached, thus fulfilling the condition (11),[4]. As a result, the output signal y(t) undergoes a process of self-modulation of amplitude and phase through resonance jumps. In the situations presented above, the A(R) characteristics look like in Fig.3

## 3. EXPERIMENTAL RESULTS

For the system in Fig.1, the saturation type nonlinearity, Fig.6, and the linear element of the following transfer function are considered:




(a)




(b)

Fig.5 Illustration of the conditions of self-modulation by using graphical-analytical method.

$$H(\ s\ ) = \frac{k\ e^{-\tau s}}{s\ (\ T\ s + 1\ )}$$

The input signal has the amplitude R = 1.
The slow variable parameter p, may be: k, τ, T, $m_1$, m2, δ.

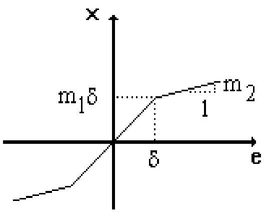
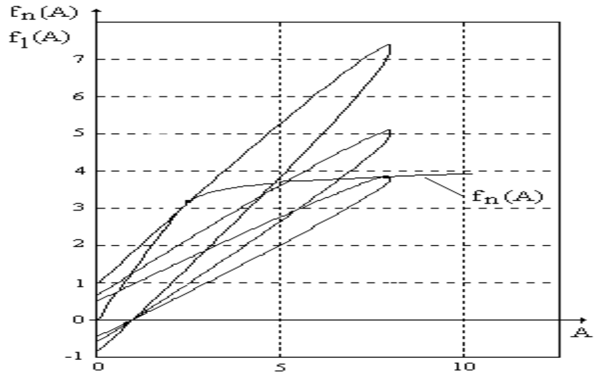



Fig.6 Static characteristic of the nonlinearity

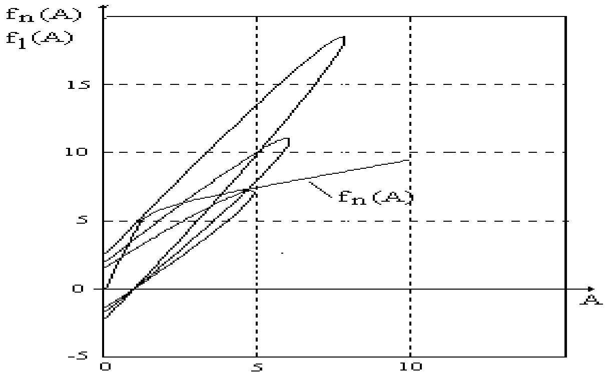
The experimental results obtained by numerical simulation are presented in Fig.7, Fig.8, Fig.9, Fig.10 and Fig.11.

## 4. CONCLUSIONS

The paper focuses on a new phenomenon that is likely to appear in nonlinear feedback systems presenting jump resonance.




(a)

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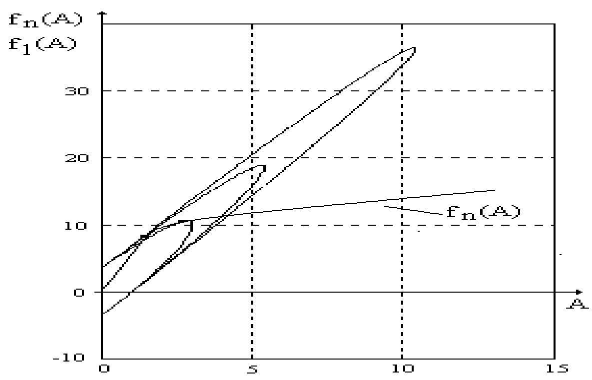
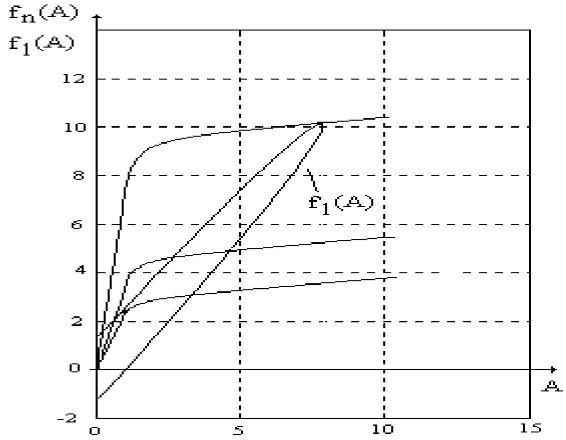
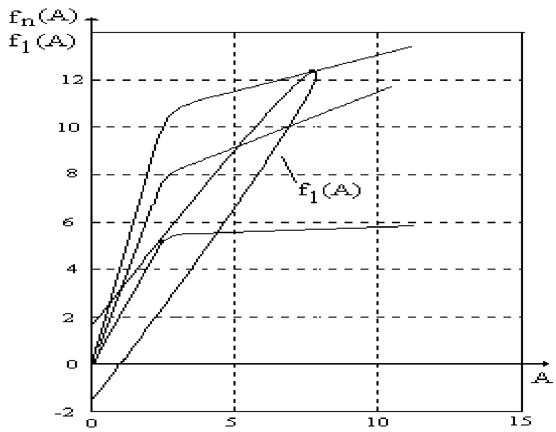



(c)

Fig.7 Numerical simulation ($\omega$=3 rad/sec)
(a) $m_1 = 1.3$, $m_2 = 0.38$, $\delta = 2.5$, T=1 sec., $\tau = 0$, k = 13; 17; 21.
(b) $m_1 = 3.3$, $m_2 = 0.56$, $\delta = 1$, k = 10, $\tau = 0$, T = 1.6; 2; 2.6 sec.
(c) $m_1 = 4.4$, $m_2 = 0.4$, $\delta = 1.5$, T = 0.2 sec., $\tau = 0.7$; 0.9; 1.6 sec.




(a)




(b)

Fig.8 Numerical simulation ($\omega$=3 rad/sec)
(a) k = 10, T = 1, $\tau = 0$, $\delta = 1.3$, $m_2 = 0.35$, $m_1 = 1.9$; 3.2; 6.9
(b) k = 10, T = 0.8, $\tau = 0$, $\delta = 2.5$, $m_1 = 2$; 3.1; 4.2, $m_2 = 0.2$; 0.78; 0.6

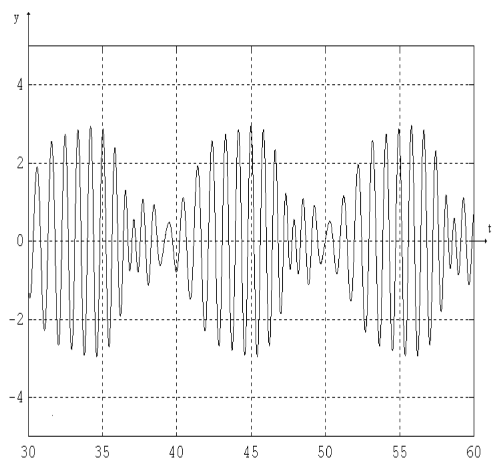



Fig.9. Numerical simulation ($\omega$=6.98 rad/sec)
T=1 sec, $\delta$=1, $\tau$=0, $m_1$=1, $m_2$=0.2, $T_1$=3 sec, $T_2$=0, $\alpha$=50, k=1, $\omega$=6.98 rad/sec;

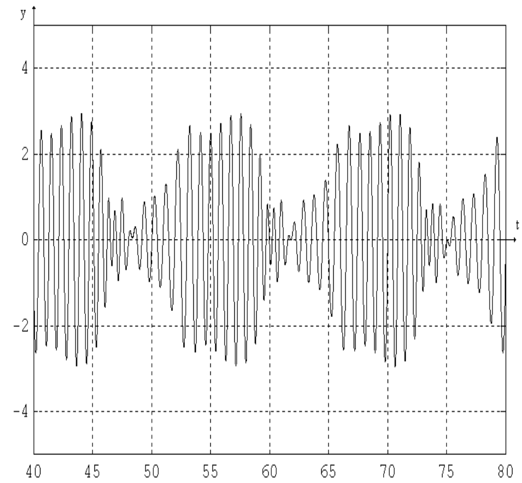



Fig10. Numerical simulation ($\omega$=6.98 rad/sec)
T=1sec, $\delta$=1, $\tau$=0, $m_1$=1, $m_2$=0.2, $T_1$=1sec,
$T_2$=3 sec, $\alpha$=50, k=1.

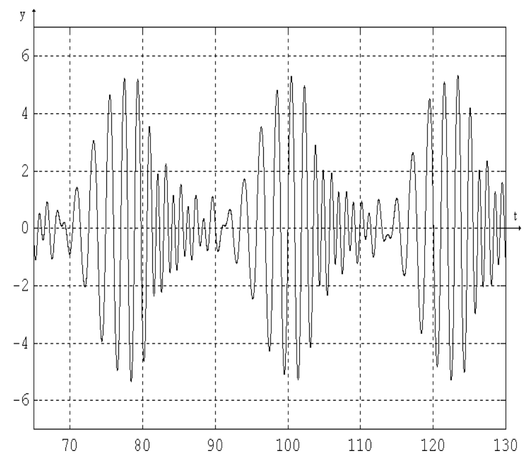



Fig.11. Numerical simulation ($\omega$=3 rad/sec)
k=20,T=2.6 sec., $\delta$=1, $\tau$=0,
for $m_1$: $\alpha$=2, $T_2$=0, $T_1$=5 sec.
for $m_2$: $\alpha$=0.05, $T_2$=0, $T_1$=5 sec.

The theoretical considerations establish the conditions of producing and existence of this phenomenon.
The experimental results obtained by numerical simulation confirm the theoretical consideration done.
The results obtained allow the understanding of the particular behavior of some processes in physics, astronomy, technique and medicine.

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