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DISTURBANCE REJECTION WITH SIMULTANEOUS DECOUPLING OF A DISTILLATION COLUMN

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Abstract. The problem of input–output Decoupling with simultaneous Disturbance Rejection (DRD) of a Distillation Column is studied. The results reported are first in the field due to the fact that the combined DRD problem is resolved for the case where time delays introduced in the column are taken into account. The controller derived is realistic to implement, while it yields a significant improvement in the system's performance over other reported distillation column studies.

Key Words. Multivariable Control, Time–Delay Systems, Decoupling and Disturbance Rejection, Linear Systems, Distillation Column Control.

1. INTRODUCTION

The problem of input–output Decoupling is a very important control design problem, since it aims in reducing a multi–input multi–output system to a set of single–input single–output systems, thus facilitating the control strategy. The problem of Disturbance Rejection is another important and applicable control design problem, since it aims in eliminating the influence of the disturbances in the system's output. The combined problem of Disturbance Rejection and Decoupling (***DRD***) is of obvious importance, since it aims in simultaneously eliminating the influence of the disturbances in the system's output while reducing a multi–input multi–output control problem to that of controlling a set of scalar systems.

In [6] the solution of the DRD problem for time–delay systems via realizable controllers has been derived for the first time. In particular, the *necessary* and *sufficient* conditions for the solvability of the problem via realizable proportional state–feedback controllers have been established. Also, the *general analytical expressions* of the realizable state–feedback *controllers* as well as of the respective input–output decoupled with rejected disturbances *closed–loop system* have been derived.

In Distillation Columns decoupling and disturbance rejection design techniques have been applied to improve the system's performance. For input–output decoupling several case studies have been reported [1], [3]–[5]. For disturbance rejection fewer studies have been reported [9], [10]. For the combined problem (DRD) in the study of Distillation Columns incorporating time delays it appears that no results have as yet been reported.

In this paper the DRD problem of Distillation Columns involving time delays is studied. The particular Distillation Column model used is the linearized model derived in [9] modified so as to take into account the time delays that occur. The design procedure applied in this paper is that reported in [6]. This procedure greatly facilitates the derivation of the general form of the controller matrices, of the general analytical expressions of the input–output decoupled with partially rejected disturbances closed–loop system as well as of the closed–loop system properties. The resulting closed–loop system yields a significant improvement in the following: It eliminates the interaction between the control loops, it completely eliminates the influence of one of the disturbances in the system's outputs while it confides the influence of the second disturbance to only one of the system's outputs and thus reduces the operating cost of the column as well as the production of off–specification products.

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2. THE MATHEMATICAL MODEL OF THE DISTILLATION COLUMN

The problem of modelling and control of distillation columns has been extensively studied in the past. The controller design techniques reported include optimal control, adaptive control, robust control, I/O decoupling, disturbance rejection, e.t.c. With regard to the decoupling techniques, several case studies have been reported. With regard to disturbance rejection techniques, fewer studies have been reported. For the combined problem (DRD) in the study of Distillation Columns incorporating time delays it appears that no case study has as yet been reported.

In this paper the DRD problem of an N–tray Distillation Column will be studied using the technique reported in [6]. The state–space mathematical model used in this paper is a modified version of the linearized model derived in [9] to include time delays. The state–space mathematical model in the frequency domain has the form

$$s\boldsymbol{X}(s)=\boldsymbol{A}(\boldsymbol{e}^{-s\boldsymbol{T}})\boldsymbol{X}(s)+\boldsymbol{BU}(s)+\boldsymbol{DU}(s) \tag{1}$$
$$\boldsymbol{Y}(s)=\boldsymbol{CX}(s) \tag{2}$$

where

$$\boldsymbol{X}(s)=\begin{bmatrix} \text{Ä}\times_D(s) \\ \text{Ä}X_1(s) \\ \cdot \\ \cdot \\ \cdot \\ \text{Ä}X_N(s) \\ \text{Ä}X_B(s) \end{bmatrix}, \quad \boldsymbol{U}(s)=\begin{bmatrix} \text{Ä}R(s) \\ \text{Ä}V(s) \end{bmatrix}$$

$$\boldsymbol{U_D}(s)=\begin{bmatrix} \text{Ä}\times_F(s) \\ \text{Ä}F(s) \end{bmatrix}, \quad \boldsymbol{Y}(s)=\begin{bmatrix} \text{Ä}X_D(s) \\ \text{Ä}X_B(s) \end{bmatrix}$$

where $n=N+2$ and where x_D is the liquid composition in the condenser, x_B is the is the liquid composition in the reboiler, x_i (i=1,...,N) is the liquid composition on the *i-th* tray, R is the liquid flow rate in the enriching section V is the vapour flow rate in the stripping section (mol/min), x_F is the liquid composition of the feed and F is the liquid flow rate of the feed. On the above "Δ" suggests deviation from the steady state.

The elements of the matrices $\boldsymbol{A}(\boldsymbol{e}^{-s\boldsymbol{T}})$, $\boldsymbol{B}$ and $\boldsymbol{D}$ are

$a_{11}=-\underline{V}/H_D$, $a_{12}=(\underline{V}M_1/H_D)exp(-s\tau_1)$, $a_{n-1,n}=\underline{V}M_B/H_N$,

$a_{nn}=-(\underline{V}M_B+\underline{B})/H_B$, $\alpha_{\iota,\iota+1}=\underline{V}M_i/H_{i-1}$; $(i=2,...,n-2)$,

$a_{ii}=-(\underline{V}M_{i-1}+\underline{L}_{i-1})/H_{i-1}$; $(i=2,...,n-1)$, $a_{n,n-1}=\underline{L}_N/H_B$,

$a_{21}=(\underline{R}/H_1)exp(-s\tau_2)$, $\alpha_{\iota,\iota-1}=\underline{L}_{i-2}/H_{i-1}$; $(i=3,...,n-1)$,

$b_{21}=(\underline{x}_D-\underline{x}_1)/H_1$, $b_{n1}=(\underline{x}_N-\underline{x}_B)/H_B$, $b_{n-1,2}=(\underline{\psi}_B-\underline{\psi}_N)/H_N$,

$b_{i2}=(\underline{\psi}_i-\underline{\psi}_{i-1})/H_{i-1}$; $(i=2,...,n-2)$, $b_{n2}=(\underline{x}_B-\underline{\psi}_B)/H_B$,

$b_{i1}=(\underline{x}_{i-2}-\underline{x}_{i-1})/H_{i-1}$ $(i=3,...,n-1)$, $d_{f+1,1}=\underline{F}/H_f$,

$d_{n2}=(\underline{x}_N-\underline{x}_B)/H_B$, $d_{f+1,2}=(\underline{x}_F-\underline{x}_f)/H_f$, $d_{i2}=(\underline{x}_{i-2}-\underline{x}_{i-1})/H_{i-1}$; $i=f+2,...,n-1$.

$$M_i=\frac{\mathrm{a}}{[1+(\mathrm{a}-1)x_i]^2}\ ,\ \ \mathrm{i}=1,...,\mathrm{N}$$

$$M_B=\frac{\mathrm{a}}{[1+(\mathrm{a}-1)x_B]^2}$$

$$L_i=\begin{bmatrix} R(t),\ \ \mathrm{i}=1,...,f\text{-}1 \\ R(t)+F(t),\ \ \mathrm{i}=f,...,N \end{bmatrix}$$

where H_i is the liquid holdup at the *i-th* plate, H_D is the liquid holdup at the condenser (mol) and H_B is the liquid holdup at the reboiler, a is the relative volatility and where τ_1, τ_2 are the delays of the transportation line from the first tray to the condenser and from the condenser to the first tray, respectively. The undersigned quantities denote standardized steady state values. From the study of the steady state of the system the following relations are derived

$\underline{\psi}_1=\underline{x}_D$

$\underline{L}_{i-1}b_{i-1}+\underline{V}b_{i2}=0$; $i=2,...,n-1$

$\underline{L}_N b_{n1}+\underline{V}b_{n2}=0$; $i=n$

From the above relations and due to the fact that compositions x_i *and* ψ_i increase from the bottom to the top of the column we conclude the following relations

$$b_{i1}>0\ ;\ i=2,...,n \tag{3}$$
$$b_{i2}<0\ ;\ i=2,...,n \tag{4}$$

It is remarked that the above model has been obtained by taking a mass balance for a low–boiling component over each of the trays, by linearizing around the steady–state value and under several assumptions, which are:

1. The gas–liquid equilibrium relationship is given by $\psi=\mathrm{a}x/\{1+(\mathrm{a}-1)\,x\}$ where a is constant.
2. Each of the trays is an ideal tray.
3. The vapor–phase holdup on each of the trays can be ignored, while the liquid holdup is constant and independent of the time and of the tray.
4. Both the vapor and the liquid molar flow rates within the Column are constant in the enriching as well as in the stripping section.
5. The condenser is a total condenser.
6. The feed liquid is at its boiling point.

7. The number of states n include the condenser and the reboiler.

3. DECOUPLING WITH SIMULTANEOUS DISTURBANCE REJECTION OF THE DISTILLATION COLUMN

Application of the theoretical results presented in [6] to the state–space mathematical model of the distillation column presented in Section 2 yields the following:

3.1 Necessary and sufficient conditions

It holds that d_1=1 and d_2=0 and hence that

$$\boldsymbol{C}^*(z)=\begin{bmatrix} a_{11} & a_{12}z_1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{C}^*(z)\boldsymbol{B}(z)=\begin{bmatrix} a_{12}b_{21}z_1 & a_{12}b_{21}z_1 \\ b_{n1} & b_{n2} \end{bmatrix}$$

$$\boldsymbol{C}^*(z)\boldsymbol{D}(z)=\begin{bmatrix} 0 & 0 \\ 0 & d_{n2} \end{bmatrix}$$

Checking for the decoupling conditions [6] we have that $\det \boldsymbol{C}^*(z)\boldsymbol{B}(z)=a_{12}z_1(b_{21}b_{n2}-b_{n1}b_{22})$. The factor $d=(b_{21}b_{n2}-b_{n1}b_{22})$ is always different than zero since

$$d=b_{21}b_{n2}-b_{n1}b_{22}=(-F/V)\, b_{21}b_{n1}<0 \qquad (5)$$

where use was made of relation (3,4). Therefore, system (1,2) can be decoupled. Furthermore, and since relation (5) is always and unconditionally true, the system is structurally decouplable. Checking the disturbance rejection condition we observe that since the matrix $\boldsymbol{C}^*(z)\boldsymbol{D}(z)$ is not identically zero, we conclude that complete disturbance rejection can not be achieved. However, we can reject one of the disturbances, namely Δx_F.

The mathematical solution to the problem at hand is given by the following relations

$$\boldsymbol{G}(\boldsymbol{e}^{-s\boldsymbol{T}})=\gamma^{-1}\begin{bmatrix} p_{10}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}})b_{n2} & -p_{20}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}})a_{12}b_{22}e^{-s\tau_1} \\ -p_{10}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}})b_{n1} & p_{20}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}})a_{12}b_{21}e^{-s\tau_1} \end{bmatrix}$$

$p_{10}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}}),\ p_{20}^{-1}(\boldsymbol{e}^{-s\boldsymbol{T}}) \neq 0$ are arbitrary parameters.

$$\boldsymbol{F}(\boldsymbol{e}^{-s\boldsymbol{T}})=\begin{bmatrix} f_{11} & f_{12} & f_{13} & 0 & \ldots & 0 & f_{1,n-1} & f_{1n} \\ f_{21} & f_{22} & f_{23} & 0 & \ldots & 0 & f_{2,n-1} & f_{2n} \end{bmatrix}$$

where $\gamma= d\alpha_{12}exp(-s\tau_1)$.

The elements of the matrix $\boldsymbol{F}(\boldsymbol{e}^{-sT})$ are given by the following relations:

$$f_{11}=\frac{-b_{n2}(a_{11}^2+\lambda_{11}a_{11}+\lambda_{12})e^{s\tau_1}-b_{n2}a_{12}a_{21}e^{-s\tau_2}}{a_{12}d}$$

$$f_{21}=\frac{b_{n1}(a_{11}^2+\lambda_{11}a_{11}+\lambda_{12})e^{s\tau_1}+b_{n1}a_{12}a_{21}e^{-s\tau_2}}{a_{12}d}$$

$$f_{12}=\frac{-b_{n2}(a_{11}+a_{22}+\lambda_{11})}{d}\ ,\quad f_{13}=\frac{-b_{n2}a_{23}}{d}$$

$$f_{22}=\frac{b_{n1}(a_{11}+a_{22}+\lambda_{11})}{d}\ ,\quad f_{23}=\frac{b_{n1}a_{23}}{d}$$

$$f_{1,n-1}=\frac{b_{22}a_{n,n-1}}{d}\ ,\quad f_{2,n-1}=-\frac{b_{21}a_{n,n-1}}{d}$$

$$f_{1n}=\frac{b_{22}(a_{nn}+\lambda_{21})}{d}\ ,\quad f_{2n}=-\frac{b_{21}(a_{nn}+\lambda_{21})}{d}$$

Where λ_{11}, λ_{12}, λ_{21} are arbitrary parameters.

Due to the existence of the factor $exp(s\tau_1)$ in the general form of the matrices $\boldsymbol{F}(\boldsymbol{e}^{-s\boldsymbol{T}})$ and $\boldsymbol{G}(\boldsymbol{e}^{-s\boldsymbol{T}})$ the realizability of the matrices $\boldsymbol{F}(\boldsymbol{e}^{-s\boldsymbol{T}})$ and $\boldsymbol{G}(\boldsymbol{e}^{-s\boldsymbol{T}})$ depends upon the choice of the arbitrary parameters. To check the conditions presented in [6] regarding the realizability of the matrices $\boldsymbol{F}(\boldsymbol{e}^{-s\boldsymbol{T}})$ and $\boldsymbol{G}(\boldsymbol{e}^{-s\boldsymbol{T}})$ we start by considering only the nonzero columns of the matrix $\boldsymbol{F}(\boldsymbol{e}^{-s\boldsymbol{T}})$, denoted by $\boldsymbol{F}^*(\boldsymbol{e}^{-s\boldsymbol{T}})$ and perform the calculations necessary to achieve the ***right birealizable unitarizing transformation***. We start by rearranging the matrix $\boldsymbol{F}^*(\boldsymbol{e}^{-s\boldsymbol{T}})$ as follows ([6])

$$\boldsymbol{f}^*(\boldsymbol{e}^{-s\boldsymbol{T}})=\boldsymbol{\psi}(\boldsymbol{e}^{-s\boldsymbol{T}})\boldsymbol{N}(\boldsymbol{e}^{-s\boldsymbol{T}})-\boldsymbol{v}(\boldsymbol{e}^{-s\boldsymbol{T}})$$

where $\boldsymbol{f}^*(\boldsymbol{e}^{-s\boldsymbol{T}})$ is a 1×10 row vector, $\boldsymbol{\psi}(\boldsymbol{e}^{-s\boldsymbol{T}})$ is a 1×3 row vector with arbitrary elements, $\boldsymbol{v}(\boldsymbol{e}^{-s\boldsymbol{T}})$ is a fixed 1×10 row vector and $\boldsymbol{N}(\boldsymbol{e}^{-s\boldsymbol{T}})$ is a fixed 3×10 matrix. The form of the matrix $\boldsymbol{N}(\boldsymbol{e}^{-s\boldsymbol{T}})$ is as follows:

$$\boldsymbol{N}(\boldsymbol{e}^{-sT})=\begin{bmatrix} v_{11} & v_{12} & 0 & 0 & 0 & v_{16} & v_{17} & 0 & 0 & 0 \\ v_{21} & 0 & 0 & 0 & 0 & v_{26} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{35} & 0 & 0 & 0 & 0 & v_{3,10} \end{bmatrix}$$

the nonzero elements of the matrix $\boldsymbol{N}(\boldsymbol{e}^{-s\boldsymbol{T}})$ are given in the following relations:

$$v_{11}=\frac{-b_{n2}a_{11}e^{s\tau_1}}{a_{12}d},\ v_{12}=\frac{-b_{n2}}{d},\ v_{16}=\frac{b_{n1}a_{11}e^{s\tau_1}}{a_{12}d}$$

$$v_{17}=\frac{b_{n1}}{d},\ v_{21}=\frac{-b_{n2}e^{s\tau_1}}{a_{12}d},\ v_{26}=\frac{b_{n1}e^{s\tau_1}}{a_{12}d},\ v_{35}=\frac{b_{22}}{d}$$

$$v_{3,10}=\frac{-b_{21}}{d}$$

The vector $\boldsymbol{v}(\boldsymbol{e}^{-s\boldsymbol{T}})$ has the following form

$$v(\boldsymbol{e}^{-s\boldsymbol{T}})=\begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{bmatrix}$$

The ten elements of the vector $\boldsymbol{v}(\boldsymbol{e}^{-s\boldsymbol{T}})$ are given by the following relations:

$$v_1 = \frac{b_{n2}a_{11}^2 e^{s\tau_1} + b_{n2}a_{12}a_{21}e^{-s\tau_2}}{a_{12}d}, v_2 = \frac{b_{n2}(a_{11}+a_{22})}{d},$$
$$v_4 = -\frac{b_{22}a_{n,n-1}}{d}, v_5 = -\frac{b_{22}a_{nn}}{d}, v_7 = -\frac{b_{n1}(a_{11}+a_{22})}{d},$$
$$v_6 = \frac{b_{n1}a_{11}^2 e^{s\tau_1} + b_{n1}a_{12}a_{21}e^{-s\tau_2}}{a_{12}d}, v_8 = -\frac{b_{n1}a_{23}}{d}, v_3 = \frac{b_{n2}a_{23}}{d},$$
$$v_9 = \frac{b_{21}a_{n,n-1}}{d}, v_{10} = -\frac{b_{21}a_{nn}}{d}$$

In order to apply the ***Right Birealizable Unitarizing Transformation*** we compute the following matrices

$$\boldsymbol{M}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \begin{bmatrix} e^{-s\tau_1}a_{12}d & 0 & 0 \\ -\dfrac{a_{12}d}{a_{11}} & a_{12}d & 0 \\ 0 & 0 & a_{12}d \end{bmatrix},$$

$$\Xi(\boldsymbol{e}^{-s\boldsymbol{T}}) = \xi_{ij}(\boldsymbol{e}^{-s\boldsymbol{T}}); \mathrm{i,j} = 1,...10$$

where the nonzero elements of $\Xi(\boldsymbol{e}^{-s\boldsymbol{T}})$ are

$$\xi_{11}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{n2}}{a_{11}\pi_1}, \xi_{12}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{e^{-s\tau_1}}{b_{n2}},$$
$$\xi_{22}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{a_{11}b_{n2}}{a_{12}\pi_1}, \xi_{25}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n2}^2}{\pi_1},$$
$$\xi_{38}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \xi_{47}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \xi_{89}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \xi_{9,10}(\boldsymbol{e}^{-s\boldsymbol{T}}) = 1,$$
$$\xi_{53}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{22}}{a_{12}d_2}, \xi_{61}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n2}}{a_{11}\pi_1},$$
$$\xi_{72}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{a_{11}b_{n1}}{a_{12}\pi_1}, \xi_{75}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n2}b_{n1}}{\pi_1},$$
$$\xi_{16}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{n2}^2}{\pi_1}, \xi_{54}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{21}^2}{\pi_2},$$
$$\xi_{10,3}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{-b_{21}}{a_{12}d_2}, \xi_{10,4}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{-b_{21}b_{22}}{d_2},$$
$$\xi_{66}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n1}b_{n2}}{\pi_1},$$

where $\pi_1 = b_{n1}^2 + b_{n2}^2$ and $\pi_2 = b_{21}^2 + b_{22}^2$.

Hence it holds that [6]

$$\boldsymbol{v}^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \begin{bmatrix} v_1^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) & v_2^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) & v_3^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) \end{bmatrix}$$

where

$$v_1^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{a_{11}^2 e^{s\tau_1} + a_{12}a_{21}e^{-s\tau_2}}{a_{12}a_{11}d}, v_3^{**}(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{a_{nn}}{a_{12}d}$$
$$v_2^{**} = \frac{a_{11}}{a_{12}d} + \frac{a_{21}e^{-s\tau_2}e^{-s\tau_1}}{d} - \frac{a_{11}(a_{11}+a_{22})}{a_{12}^2 d}$$

It also holds that [6]

$$v^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\left(v_i^*(\boldsymbol{e}^{-s\boldsymbol{T}})\right) i = 1,...,7$$

where

$$v_1^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = v_2^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = v_3^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = 0,$$
$$v_4^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{22}a_{n,n-1}}{d}, v_6^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n1}a_{23}}{d}$$
$$v_5^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{n2}a_{23}}{d}, v_7^*(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{21}a_{n,n-1}}{d}$$

According to [6] and since the vector $\boldsymbol{v}^*(\boldsymbol{e}^{-s\boldsymbol{T}})$ is realizable the third necessary and sufficient condition is satisfied and therefore a realizable solution to the DDR problem exists.

3.2 Realizable Solution

The general form of the realizable matrices achieving I/O Decoupling and partial Disturbance Rejection are given by the following relations

$$\boldsymbol{G_r}(\boldsymbol{e}^{-s\boldsymbol{T}}) = \begin{bmatrix} b_{n2}p_{r,1}^*(\boldsymbol{e}^{-s\boldsymbol{T}}) & -b_{22}a_{12}p_{r,2}^*(\boldsymbol{e}^{-s\boldsymbol{T}}) \\ -b_{n1}p_{r,1}^*(\boldsymbol{e}^{-s\boldsymbol{T}}) & b_{21}a_{12}p_{r,2}^*(\boldsymbol{e}^{-s\boldsymbol{T}}) \end{bmatrix}$$

where $p_{r,i}^*(\boldsymbol{e}^{-s\boldsymbol{T}})$ are abitrary nonzero realizable para - meters

$$\boldsymbol{F_r}(\boldsymbol{e}^{-s\boldsymbol{T}}) = f_{i,j}^r(\boldsymbol{e}^{-s\boldsymbol{T}}), \ i = 1,2 \text{ and } j = 1,...,n$$

where the nonzero elements of $\boldsymbol{F_r}$ are

$$f_{11}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = -b_{n2}a_{11}\lambda_{r,1}(\boldsymbol{e}^{-s\boldsymbol{T}})e^{-s\tau_1} - \frac{b_{n2}a_{21}e^{-s\tau_2}}{d}$$
$$f_{12}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = -b_{n2}a_{12}\lambda_{r,1}(\boldsymbol{e}^{-s\boldsymbol{T}})e^{-s\tau_1} - \frac{b_{n2}a_{12}}{a_{11}}\lambda_{r,2}(\boldsymbol{e}^{-s\boldsymbol{T}}) - \frac{b_{n2}a_{22}}{d} - \frac{b_{n2}}{d} + \frac{b_{n2}(a_{11}+a_{22})}{a_{12}d}$$
$$f_{13}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{n2} + b_{n2}a_{23}}{d}$$
$$f_{1,n-1}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{22}a_{n,n-1}}{d}$$
$$f_{1,n}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = b_{22}a_{12}\lambda_{r,3}(\boldsymbol{e}^{-s\boldsymbol{T}})$$
$$f_{21}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = b_{n1}a_{11}\lambda_{r,1}(\boldsymbol{e}^{-s\boldsymbol{T}})e^{-s\tau_1} + \frac{b_{n1}a_{21}e^{-s\tau_2}}{d}$$
$$f_{22}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = b_{n1}a_{12}\lambda_{r,1}(\boldsymbol{e}^{-s\boldsymbol{T}})e^{-s\tau_1} - \frac{b_{n1}a_{12}}{a_{11}}\lambda_{r,2}(\boldsymbol{e}^{-s\boldsymbol{T}}) + \frac{b_{n1}a_{22}}{d} + \frac{b_{n1}}{d} + \frac{b_{n1}(a_{11}+a_{22})}{a_{12}d}$$
$$f_{23}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = \frac{b_{n1}a_{23}}{d}$$
$$f_{2,n-1}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = -\frac{b_{21}a_{n,n-1}}{d}$$
$$f_{2,n}^r(\boldsymbol{e}^{-s\boldsymbol{T}}) = -b_{21}a_{12}\lambda_{r,3}(\boldsymbol{e}^{-s\boldsymbol{T}})$$

where $\lambda_{r,i}(\boldsymbol{e}^{-s\boldsymbol{T}})$ are arbitrary realizable parameters.

3.3 Closed–loop system

The resulting closed–loop system has the form

$$\begin{aligned}&\boldsymbol{C}(\boldsymbol{e}^{-sT})(s\boldsymbol{I}-\boldsymbol{A}(\boldsymbol{e}^{-sT})-\boldsymbol{B}(\boldsymbol{e}^{-sT})\boldsymbol{F}_r(\boldsymbol{e}^{-sT}))^{-1} *\\ &\left[\boldsymbol{B}(\boldsymbol{e}^{-sT})\boldsymbol{G}_r(\boldsymbol{e}^{-sT}) \;\vdots\; \boldsymbol{D}(\boldsymbol{e}^{-sT})\right]=\\ &\left[\boldsymbol{H}_\omega^*(\boldsymbol{e}^{-sT}) \;\vdots\; \boldsymbol{H}_d^*(\boldsymbol{e}^{-sT})\right]\end{aligned}$$

where

$$\boldsymbol{H}_\omega^*(\boldsymbol{e}^{-sT})=\begin{bmatrix} h_1(\boldsymbol{e}^{-sT}) & 0\\ 0 & h_2(\boldsymbol{e}^{-sT})\end{bmatrix}$$

$$\boldsymbol{H}_d^*(\boldsymbol{e}^{-sT})=\begin{bmatrix} 0 & 0\\ 0 & h_3(\boldsymbol{e}^{-sT})\end{bmatrix}$$

where

$$h_1(\boldsymbol{e}^{-sT})=\frac{p_{r,1}^*(\boldsymbol{e}^{-sT})e^{-s\tau_1}}{s^2+\lambda_1(\boldsymbol{e}^{-sT})s+\lambda_2(\boldsymbol{e}^{-sT})}$$

$$h_2(\boldsymbol{e}^{-sT})=\frac{p_{r,2}^*(\boldsymbol{e}^{-sT})e^{-s\tau_1}}{s+a_{12}d\lambda_{r,3}(\boldsymbol{e}^{-sT})-a_{nn}}$$

$$h_3(\boldsymbol{e}^{-sT})=\frac{d_{n2}}{s+a_{12}d\lambda_{r,3}(\boldsymbol{e}^{-sT})-a_{nn}}$$

and where

$$\begin{aligned}\lambda_1(\boldsymbol{e}^{-sT})&=a_{12}d\lambda_{r,1}(\boldsymbol{e}^{-sT})-a_{11}-\frac{a_{12}d}{a_{11}}\lambda_{r,2}(\boldsymbol{e}^{-sT})\\ &\quad+1-\frac{a_{11}+a_{22}}{a_{12}}\\ \lambda_2(\boldsymbol{e}^{-sT})&=a_{12}d\lambda_{r,2}(\boldsymbol{e}^{-sT})-a_{11}-a_{12}a_{21}e^{-s\tau_1}e^{-s\tau_2}\\ &\quad-\frac{a_{11}+a_{22}}{a_{12}}a_{11}\end{aligned}$$

Remark 3.1 Observing the form of the closed–loop system one may readily conclude the following: The first output y_1(t) namely the liquid composition of the first tray (top product) can be controlled by manipulating only ω_1(t). In that case the disturbances have no effect on the top product. The second output y_2(t) namely the liquid composition of the *n–th* tray (bottom product), is influenced by the second disturbance (the liquid flow rate of the feed tray). Since the system relating the bottom product with external input and disturbance is a single–input single–output (siso) system that can be made delay–free any of the well–known techniques for siso systems can be applied to suppress the influence of this disturbance on the system output. Since in practice, one usually must control the concentration and the quality of the top product, the present technique has successfully eliminated the influence of the disturbances. At the same time, quality control of the top product is easily achieved by manipulating ω_1(t).

Remark 3.2. The characteristic polynomial of the closed–loop system is

$$P(s,\boldsymbol{e}^{-sT})=P_1(s,\boldsymbol{e}^{-sT})P_2(s,\boldsymbol{e}^{-sT})P_3(s,\boldsymbol{e}^{-sT})$$

where the three terms $P_i(s,\boldsymbol{e}^{-sT})\; i=1,2,3$ are given by

$$\begin{aligned}P_1(s,\boldsymbol{e}^{-sT})&=s^2+s\lambda_1(\boldsymbol{e}^{-sT})+\lambda_2(\boldsymbol{e}^{-sT})\\ P_2(s,\boldsymbol{e}^{-sT})&=s+a_{12}d\lambda_{r,3}(\boldsymbol{e}^{-sT})-a_{nn}\\ P_3(s,\boldsymbol{e}^{-sT})&=\det(s\boldsymbol{I}-\boldsymbol{A}_{C2})\end{aligned}$$

The cancelled out polynomial is det($s\boldsymbol{I}$–$\boldsymbol{A}_{C2}$). The form of $\boldsymbol{A}_{C2}$ is given in [6]. In general it is very tedious to check the stability of tis polynomial. It is relatively easy to study the case where the column has the minimum number of trays i.e for N=3. For this case we have:

$$P_3(s,\boldsymbol{e}^{-sT})=s^2-(a_{c33}+a_{c44})s+(a_{c33}a_{c44}-a_{c34}a_{c43})$$

Since this polynomial does not include delays its stability can be determined with the well–known criteria of regular systems. Hence the stability of the closed–loop system may be determined. In order for $P_3(s,\boldsymbol{e}^{-sT})$ to be stable the following relations must hold

$$\begin{aligned}&a_{c33}a_{c44}-a_{c34}a_{c43}>0\\ &a_{c33}+a_{c44}<0\end{aligned}$$

where

$$\begin{aligned}a_{c33}&=a_{33}-\frac{b_{31}}{b_{21}}a_{23}=-\frac{(1+K_1)VM_2+R+F}{H_2}\\ a_{c44}&=a_{44}-\frac{b_{41}}{b_{51}}a_{54}=-\frac{(1+K_2)(R+F)+VM_3}{H_3}\\ a_{c34}&=a_{34}=\frac{VM_3}{H_2},\; a_{c43}=a_{43}=\frac{(R+F)}{H_3}\end{aligned}$$

and where

$$K_1=\frac{\chi_1-\chi_2}{\chi_D-\chi_1},\; K_2=\frac{\chi_2-\chi_3}{\chi_3-\chi_B}$$

Since the liquid composition χ_i increases from the bottom to the top of the column it holds that $K_1>0$ and $K_2>0$.On the basis of these inequalities we conclude that

$$a_{c33}<0,\; a_{c44}<0$$

It also holds that

$$a_{c33}a_{c44}-a_{c43}a_{c34}=-\frac{(1+K_1)(1+K_2)VM_2(R+F)}{H_2H_3}$$
$$-\frac{(1+K_1)V^2M_2M_3+(1+K_2(R+F)^2}{H_2H_3}$$

From these last two relations we readily conclude that the cancelled out polynomial is stable.

4. CONCLUSIONS

In this work the problem of controlling a distillation column has been investigated. To control the distillation column, I/O decoupling and disturbance rejection techniques have been applied. The model used for the study has been obtained by linearizing around the steady state of a nonlinear model taking into account the existence of delays in the transportation lines. The results derived are the following:

a) Decoupling techniques can be applied independently of the number of the trays that the column consists of.
b) Decoupling can be achieved by measuring and feeding back only five concentrations, no matter what the number of the trays is. This is particularly important since the cost and the implementation of the controller does not vary with the number of trays.
c) The decoupled closed–loop system eliminates the interaction between the control loops, thus facilitating independent regulation of the system outputs. Furthermore, it improves the concentration of both the distillate and the bottom product.
d) The influence of one of the disturbances is completely eliminated while the other is restricted to affect only the bottom product. Furthermore, by using well–known siso techniques disturbance attenuation can be accomplished.
e) The aforementioned four contributions result in the reduction of both the operating cost and the production of off–specification products.

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