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PATH GENERATION IN JOINT COORDINATES FOR A PRESCRIBED MOTION OF THE END-EFFECTOR OF A ROBOT

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Abstract. In this paper there is presented a method for the determination of the time histories of the internal co-ordinates of a serial type robot when the end-effector is moving along a spatial trajectory. This allow us to command the trajectory of the end-effector practically along any trajectory in the three-dimensional space. The method presented is simple and can be used for any type of robot. There is also demonstrated the existence of the solutions for the problem mentioned above.

Key words. Industrial robot , Internal coordinates , Generation of trajectories , Control of the motion.

From the kinematic point of view the end-effector of an industrial robot, has to carry out tasks inside the working space along certain trajectories and with certain velocities. In most of the situations it is important to control not only the starting point and the final point but also the path between these two points.

Path generation can be carried out in two ways:

1. In joint coordinates
2. In external coordinates

Path generation using joint coordinates generates simple algorithms and a smaller volume of computations. But this method has the disadvantage that it does not allow the control of the motion between the initial and the final point. That is why in this cases there exists the risks of collision with certain obstacles inside the working space.

Sometimes the end-effector has to move along precise trajectories in the space (welding). The control of the motion along such trajectories necessitates a large volume of computation and the conversion of the external coordinates in joint coordinates for an important number of points, which means that we will have to solve the inverse kinematic problem each time.

In order to eliminate this disadvantage we will try to find another method for path generation. The proposed method will eliminate the disadvantage of the first method and use the advantage of the other one. That means that the control of the motion will be carried out in joint coordinates for a space trajectory of the end-effector.

Most of the space trajectories can be achieved as reunions of lines and arch circles. Thus, any trajectory can be approximated satisfactory by lines and arch circles.

In what follows we will refer to the most common robot structure: the decoupled one. The decoupled structure is widely spread because for these types of structure the inverse kinematics is obtainable. This structure has the axis of the last three degrees of freedom concurrent. That is why the problem of orientation can be separated from the one of positioning. For a well known robot, Puma 600, according to [4] the position of the manipulated point can be determined by means of the relations:

$$P_x = \mathrm{Cos}[q_1]\,\mathrm{Cos}[q_2]\,(\,0{,}02\,\mathrm{Cos}[q_3] - 0{,}432\,\mathrm{Sin}[q_3]+0{,}432\,) - \mathrm{Cos}[q_1]\,\mathrm{Sin}[q_2]\,(\,0{,}02\,\mathrm{Sin}[q_3] + 0{,}432\,\mathrm{Cos}[q_3]\,) - 0{,}149\,\mathrm{Sin}[q_1]$$

P_y = Sin[q_1] Cos[q_2] (0,02 Cos[q_3]-0,432 Sin[q_3]+0,432) - - Sin[q_1] Sin[q_2] (0,02 Sin[q_3] + 0,432 Cos[q_3])- 0,149 Cos[q_1] (1)
P_z = - 0,02 Sin[q_2] Cos [q_3]+ 0,432 Sin[q_3] Sin[q_2] - 0,432 Sin[q_2] - - Cos[q_2] (0,02 Sin[q_3] + 0,432 Cos[q_3])

For this type of robot the distance from the origin of the fix reference frame to the manipulated point is:

$$D^2 = P_x^2 + P_y^2 + P_z^2 \quad (2)$$

$$P_x = C1g_1 - S1g_2$$
$$P_y = S1g_1 + C1g_2 \quad (3)$$
$$P_z = g_3 + r_1$$

For the Puma Robot (1) becomes:

$$g_1 = F_1$$
$$g_2 = F_2 \quad (4)$$
$$g_3 = F_3$$

Replacing (4) into (2) we get:

$$D^2 = P_x^2 + P_y^2 + P_z^2 = g_1^2 + g_2^2 + g_3^2 = F_1^2 + F_2^2 + F_3^2 \quad (5)$$

$$F_1 = C2\ f_1 – S2\ f_2$$
$$F_2 = S2\ f_1 + C2f_2 \quad (6)$$
$$F3= f_3+b_2$$

Thus,

$$D^2 = P_x^2 + P_y^2 + P_z^2 = g_1^2 + g_2^2 + g_3^2 = F_1^2 + F_2^2 + F_3^2 = f_1^2+f_2^2+(f_3+b_2)^2 \quad (7)$$

Replacing the D-H parameters of the the Puma robot in (1) we get:

$$f_1 =- C_3a_4 + S_3b_4 +a_3$$
$$f_2 = S3a_4 – C3b_4 \quad (8)$$
$$f_3 = 0$$

From (8) and (7) we get:

$$D^2 = P_x^2 + P_y^2 + P_z^2 = g_1^2 + g_2^2 + g_3^2 = F_1^2 + F_2^2 + F_3^2 = f_1^2+f_2^2+(f_3+b_2)^2 = 2a_3a_4\cos q_3+2a_3b_4\sin q_3+(a_4^2+a_3^2+b_4^2) \quad (9)$$

The straight lines and the arch circles, which will help us to compose almost any trajectory in the space, can be represented in different ways but the only one acceptable for our purpose is the parametric representation. The entire curve in space (lines, circles, etc) can be represented parametrical as :

$$X = x(t)$$
$$Y = y(t) \quad (10)$$
$$Z = z(t)$$

Equating (9) and (10) we will get :

$$2a_3a_4\cos q_3+2a_3b_4\sin q_3+(a_4^2+a_3^2+b_4^2) = x(t)^2 + y(t)^2 + z(t)^2 \quad (11)$$

In equation (11) we will express $\sin q_3$ and Cosq3 using tg (q_3/2) . We will denote:

$$tg(q_3/2) = v$$
$$2a_3a_4 = A$$
$$2a_3b_4 = B \quad (12)$$
$$a_4^2+a_3^2+b_4^2 = C$$
$$x(t)^2 + y(t)^2 + z(t)^2 = K$$

Equation (11) becomes:

$$A\ [(1-v^2)/(1+v^2)] + 2Bv/(1+v^2) + C = K \quad (13)$$

Where “K” depends on time and not on “v”.

Computing in (13) we get:

$$V^2(C-A-K) + 2Bv + (A+C-K) = 0 \quad (14)$$

Equation (14) is a second-degree equation in “v”

$$\Delta = B^2 + (A+C-K)^2 > 0 \quad (15)$$

So that that equation (14) will always have two real solutions for any A, B, C, K, that means for any type of curve the end-effector is constrained to move along (line or arch circle).

Also from the equation (14) we deduce that:

$$P = v_1v_2 = -1 \quad (16)$$

Where v_1 and v_2 represent the solutions of equation (14), and

$$v_1 = v_1(t) \quad (17)$$
$$v_2 = v_2(t)$$

$$tg(q_3/2) = v_1(t) \quad (18)$$

$$q_3 = 2Arctg[v_1(t)] \quad (19)$$

Practically from the relation (16) we deduce that :

$$tg(q_3)\ tg(q`_3/2) = -1 \quad (20)$$

Where q_3 and q_3` represent the time histories of the element “3” which will constrained the end-effector to move along the prescribed space trajectory.

But (20) implies :

$$(q_3/2) = (q`_3/2) + \pi/2 \quad (21)$$

We ca use any of these two solutions. As the variation of operational coordinates is continuous, the variation of the joint coordinates should also be continuous. That means that we have to choose only one of these two times histories. Booth choises bring to equivalent results because the difference between the two solutions is a constant and we can choose the zero value for the joint coordinate anywhere.

We have thus demonstrated the existence and the unicity of the solution of equation (14).

The relation (19) represents the time history of the joint coordinate of the element "3" (q_3) which forces the end-effector to move along a space trajectory.

The time histories of the other two joint coordinates, q_1 and q_2, can be easily determined. Finally we get:

$$\begin{aligned} q_1 &= q_1(t) \\ q_2 &= q_2(t) \\ q_3 &= q_3(t) \end{aligned} \tag{22}$$

That represent the time histories of the joint coordinates who can generate the prescribed spatial trajectory of the end-effector.

Now we will be no more obliged to use the inverse kinematics so many times. Thus we will eliminate the disadvantages of the control of the motion in joint coordinates (risk of collision and luck of control of the motion between the initial and the final point) as well as the ones in operational coordinates (important volume of computation due especially to the frequent use of the inverse kinematics). The new method, which generates the motion, uses the control of the joint coordinates in order to generate a prescribed trajectory of the end-effector with a minimum of computation.

The problem of the existence of the solution being solved in what follows we will apply this new method for the most used curves:

- The line.
- The circle.

The parametric equations of a line, which contains the points M_1 (x_1, y_1, z_1) and M_2 (x_2, y_2, z_2) are:

$$\begin{aligned} x(t) &= x_1 + (x_2 – x_1) u(t) \\ y(t) &= y_1 + (y_2 – y_1) u(t) \\ z(t) &= z_1 + (z_2 – z_1) u(t) \end{aligned}$$

where u (t) is determined through the interpolation with polynomials of third degree, using the following limit conditions:

$$\begin{aligned} u(0) &= 0 \\ u(t_m) &= 1 \\ \grave{u}(0) &= 0 \\ \grave{u}(t_m) &= 0 \end{aligned} \tag{23}$$

$$u(t) = 2(t/t_m)^3 – 3 (t/t_m)^2 \tag{24}$$

Finally we get for u (t):

$$u(t`) = 2t`^3-3t`^2 \tag{25}$$

and, $t= (t/t_m)$ $t \in[0,1]$

where t is the time parameter.

$$D^2 = x^2(t) + y^2(t) + z^2(t) = u^2(t) [(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2] + 2u(t) [x_1(x_2 – x_1) + y_1(y_2 – y_1) + z_1(z_2 – z_1)] + (x_1^2+y_1^2+z_1^2) \tag{26}$$

$$u(t) = 2t^3 - 3t^2$$

Equations (10) becomes for a line:

$$\begin{aligned} X &= x(t) \\ Y &= y(t) \\ Z &= z(t) \end{aligned} \tag{27}$$

Replacing (25) in (26) we will determine $K = D^2$, where K = K(t\`).

The parametric equations of a circle having O_1 (x_0, y_0, z_0) as center, the radius R and the normal to its plan "n " will be determined having as starting point the parametric equations of a circle, which belong to the plane XOY and have "O" as center.

The parametric equations of a circle situated in the plane XOY or in a plane parallel to that are :

$$\begin{aligned} x(t) &= x_c + R \cos(u) \\ y(t) &= y_c + R \sin(u) \\ z &= z_c \end{aligned} \tag{28}$$

where u=u(t).

In order to determine the parametric equations of a circle situated in the three dimensional space we will have as starting point the equations of a circle situated in the plane XOY, having "O" as centre, which we will denote "C_1". The equations of the circle "C_1" are :

$$\begin{aligned} x &= R \cos(u) \\ y &= R \sin(u) \end{aligned} \tag{29}$$

The circle who\`s parametric equations we want to find is $O_1$ ( $x_0$ , $y_0$ , $z_0$ ) situated in a plane who\`s normal is O_1Z_2 , which makes an "á" angle with OZ. We will denote this circle "C_3". In order to make the "C_1" circle identical with "C_3" we must apply a roto-translation to the first circle. We will attach to the circle "C_3" a $O_1X_2Y_2Z_2$ coordinate frame with the OZ_2 axis perpendicular to the plane of the circle.

Let M_1 be a generic point belonging to C_1. It will have the following coordinates :

$$\begin{aligned} x_1 &= R \cos(u) \\ y_1 &= R \sin(u) \end{aligned} \tag{30}$$

where “ü” depends on time.

In the first stage we will rotate OXYZ until it becomes identical to $OX_1Y_1Z_1$ which has OX_1Y_1 parallel to $O_1X_2Y_2$. The transformed point through the rotation. M_2 has the coordinates

$$M_2 = [x_2 , y_2 , z_2]^T \, [x_2 , y_2 , z_2]^T = R\, [x_1 , y_1 , z_1]^T \qquad (31)$$

where “R” represents the rotation matrix.

Matrix “R” can have different forms. We can use one of the following ways to define “R”:

1. Denavit- Hartemberg parameters;
2. Olinde-Rodrigues parameters;
3. Euler angles

All the methods mentioned bellow are equivalent. Using the Denavit-Hartemberg parameters the orientation of $OX_1Y_1Z_1$ against OXYZ is defined by means of two angles : α and θ. “α” is defined as the angle between the axis Z and Z_1 in the positive sense of the axis X. “θ” is defined as the angle between X and X_1 in the positive sense of the axis Z_1.

Thus the rotation matrix will become :

$$R = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \cos\alpha_i \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \\ \sin\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & \cos\alpha_i \end{bmatrix}$$

replacing (32) in the above relation we will get to :

$$\begin{bmatrix} X2 \\ Y2 \\ Z2 \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \cos\alpha_i \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \\ \sin\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & \cos\alpha_i \end{bmatrix}$$

$$\begin{bmatrix} R\cos(u) \\ R\sin(u) \\ 0 \end{bmatrix}_i \qquad (32)$$

From (32) we will deduce :

$$x_2 = R\cos(\theta)\cos(u) - R\sin(\theta)\sin(u)$$
$$y_2 = R\sin(\theta)\cos(u)\,\mathrm{Cos}(\alpha) + R\cos(\theta)\sin(u)\cos(\alpha) \qquad (33)$$
$$z_2 = R\cos(u)\sin(\alpha)\sin(\theta) + R\cos(\theta)\sin(u)\sin(\alpha)$$

After completing the rotation we will have to carry out a translation of $OX_1Y_1Z_1$ from O to O_1. The transformed M_2 (x_2 , y_2 , z_2) through the translation is M_3 (x_3 , y_3 , z_3).

$$[x_3 , y_3 , z_3]^T = T + [x_2 , y_2 , z_2]^T \qquad (34)$$

where “T” represents the transformation matrix and is defined as: :

$$T = [x_0 , y_0 , z_0]^T$$

Thus (34) becomes :

$$x_3 = x_0 + x_2$$
$$y_3 = y_0 + y_2 \qquad (35)$$
$$z_3 = z_0 + z_2$$

Now we have got the parametrical equations of the circle C_3 expressed in the coordinates of OXYZ :

$$x_2 = x_0 + R\cos(\theta)\cos(u) - R\sin(\theta)\sin(u)$$
$$y_2 = y_0 + R\sin(\theta)\cos(u)\,\mathrm{Cos}(\alpha) + R\cos(\theta)\sin(u)\cos(\alpha) \qquad (36)$$
$$z_2 = z_0 + R\cos(u)\sin(\alpha)\sin(\theta) + R\cos(\theta)\sin(u)\sin(\alpha)$$

The geometrical elements which define the C_3 circle are :

1. The centre of the circle O_1 (x_0, y_0 , z_0);
2. The radius “R”;
3. The normal to its plane n [n_x , n_y , n_z].

Identifying (1) and (37) we will get a system who`s solution is the solution to our problem, that is the law of variation of the internal coordinates of the joints which will constrain the effector to move along a circle in the three dimensional space.

$x_0 + R\cos(\theta)\cos(u) - R\sin(\theta)\sin(u) = \mathrm{Cos}[q_1]\,\mathrm{Cos}[q_2]\,(0{,}02\,\mathrm{Cos}[q_3] - 0{,}432\,\mathrm{Sin}[q_3]+0{,}432)- \mathrm{Cos}[q_1]\,\mathrm{Sin}[q_2]\,(0{,}02\,\mathrm{Sin}[q_3] + 0{,}432\,\mathrm{Cos}[q_3])- 0{,}149\,\mathrm{Sin}[q_1]$

$y_0 + R\sin(\theta)\cos(u)\,\mathrm{Cos}(\alpha) + R\cos(\theta)\sin(u)\cos(\alpha) = \mathrm{Sin}[q_1]\,\mathrm{Cos}[q_2]\,(0{,}02\,\mathrm{Cos}[q_3]-0{,}432\,\mathrm{Sin}[q_3]+0{,}432) - \mathrm{Sin}[q_1]\,\mathrm{Sin}[q_2]\,(0{,}02\,\mathrm{Sin}[q_3] + 0{,}432\,\mathrm{Cos}[q_3])- 0{,}149\,\mathrm{Cos}[q_1]$ (37)

$z_0 + R\cos(u)\sin(\alpha)\sin(\theta) + R\cos(\theta)\sin(u)\sin(\alpha) = - 0{,}02\,\mathrm{Sin}[q_2]\,\mathrm{Cos}\,[q_3]+ 0{,}432\,\mathrm{Sin}[q_3]\,\mathrm{Sin}[q_2] - 0{,}432\,\mathrm{Sin}[q_2] -- \mathrm{Cos}[q_2]\,(0{,}02\,\mathrm{Sin}[q_3] + 0{,}432\,\mathrm{Cos}[q_3])$

Solving (37) we will get the time histories for each joint coordinate in joint coordinates so that the end-effector of the robot moves along the prescribed spatial trajectory.

If t` ∈ [0,1] , than the effector moves along the whole circle. If we constrain the effector to move along an arch M_1M_2 , than t ∈ [t_1 , t_2].

As we have determined the parametrical equations of the circle in the space, than it will be very easy to determine the speed and the acceleration by deriving the parametrical equations.

Similarly we can determine the laws of variation of the joint coordinates for the degrees of freedom “2” and “3” , $q_2(t)$ and $q_3(t)$.

Most of the calculations used in the proposed method are off-line comparing with the others methods of path generation along spatial trajectories which use on-line computations. This is another major advantage of the proposed method.

CONCLUSIONS

Path generation using joint coordinates generates simple algorithms and a smaller volume of computations. But this method has the disadvantage that it does not allow the control of the motion between the initial and the final point. That is why in this cases there exists the risks of collision with certain obstacles inside the working space.

Sometimes the end-effector has to move along precise trajectories in the space (welding). The control of the motion along such trajectories necessitates a large volume of computation and the conversion of the external coordinates in joint coordinates for an important number of points, which means that we will have to solve the inverse kinematic problem each time on-line.

In order to eliminate this disadvantage we have found another method for path generation. The proposed method eliminates the disadvantage of the first method and use the advantage of the other one. That means that the control of the motion is carried out in joint coordinates for a space trajectory of the end-effector and the computations will be off-line.

Most of the space trajectories can be achieved as reunions of lines and arch circles. Thus, any trajectory can be approximated satisfactory by lines and arch circles. That is why we have developed the algorithms for straight line and the circle.

This new method can be the support for new control methods, simpler and with computational low costs.

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