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A SIMPLE AUTOMATIC TUNING METHOD FOR A SMITH PREDICTOR

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Abstract: In this paper a new approach is presented based on relay autotuning of a plant to find parameters for its control using a Smith predictor. A Smith predictor configuration is represented as its equivalent internal model controller, IMC, which provides the parameters of the PI or PID controller to be defined in terms of the desired closed-loop time constant, which can be adjusted by the operator, and the parameters of the process model. This means that only one parameter, namely the desired closed-loop time constant, is left for tuning. The ISE criterion was used to find the filter parameter, and simple equations were obtained to tune the Smith predictor. The method is very simple and has given improved results compared with some previous approaches.

Keywords: Relay autotuning, PID controller, Smith predictor, IMC Control

1. INTRODUCTION

Plants with long time-delays can often not be controlled effectively using a PID controller. The main reason for this is that the additional phase lag contributed by the time delay tends to destabilize the closed loop system. The stability problem can be solved by decreasing the controller gain but this results in a very sluggish response.

The Smith predictor, shown in Fig. 1, is well known as an effective dead-time compensator for a stable process with long time-delays [1]. The performance of the Smith predictor control strategy is affected by the accuracy with which the model represents the plant. Based on the assumption that the model used matches perfectly the plant dynamics, the closed loop transfer function is given by

$$T(s) = \frac{G_c(s)G_m(s)e^{-\theta_m s}}{1 + G_c(s)G_m(s)}. \qquad (1)$$

According to eqn. (1), the parameters of the primary controller, $G_c(s)$, which is typically taken as PI or PID, may be determined using a model of the delay free part of the plant.

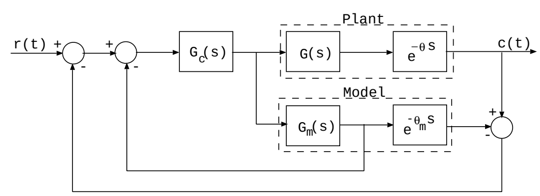

Fig. 1. The Smith predictor control scheme

Many possible approaches for determining or tuning the parameters of an appropriate controller, $G_c(s)$, have been given in the literature and some recent contributions include references [2, 3, 4, 5]. However, only a few investigations have been carried out on autotuning of the Smith predictor, which recently include [6, 7, 8, 9]. In reference [6] the relay autotuning of Åström and Hägglund [10] for simple single input single output systems was extended to Smith predictors. In references [7, 8, 9] first a FOPDT or SOPDT transfer function model is found from relay autotuning based on approximate describing function analysis, then the controller parameters, which include a user-specified constant for design methods of [8, 9], are calculated using parameters of the obtained model.

In this paper a new approach is presented based on autotuning to find the controllers parameters for a Smith predictor. A relay feedback test is performed on the plant and the frequency and amplitude of the resulting limit cycle are measured. Then the A-Locus method, an exact method for giving the parameters of a limit cycle, is used to estimate the parameters of the process model, assumed to be either a FOPDT or SOPDT transfer function [11]. However, the details of the parameter estimation is not given here and interested readers may refer to reference [11]. Once the model of the process is found, the parameters of the controller, usually a PI or PID, are found to complete the design. Tuning parameters, are found by representing the Smith predictor as its equivalent internal model controller, IMC, [12, 13], which provides the parameters of the PI or PID controller to be defined in terms of the desired closed-loop time constant, which can be adjusted by the operator, and the parameters of the process model.

The method has the advantage when compared with the methods of Hang *et al* [8], and Lee *et al* [9], of not requiring any user specified value. Secondly, the estimation method used requires less time for model parameter estimation, since only one relay feedback test is performed. Also more accurate parameter estimations can be achieved since an exact limit cycle investigation method is used. Finally, in the case of a change in the actual process parameters, the model parameters can be recalculated using a relay feedback test and thus performing the retuning to obtain a better closed loop performance.

2. INTERNAL MODEL CONTROL AND DETERMINING CONTROLLER PARAMETERS

The basics of the IMC controller design are presented in Rivera *et al* [12], and Morari and Zafiriou [13]. The block diagram of IMC control strategy is given in Fig. 2.

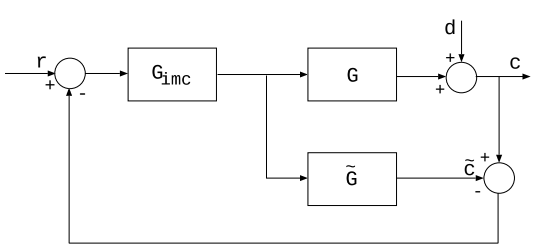

Fig. 2. IMC control strategy

Here, G and $\tilde{G}$ are the actual process and process model transfer functions respectively. When $G = \tilde{G}$, that is perfect modelling, and $d = 0$, the system is open loop. This provides the open loop advantages, such as a fast and accurate set point tracking. When $G \neq \tilde{G}$, the system is a closed loop system. Thus, the IMC control strategy has the advantages of both the open loop and closed loop structures. Another advantage of the IMC control design, and possibly the most important one, is to have only one parameter to be tuned. The first step in the IMC controller design is to factor the process model

$$\tilde{G} = \tilde{G}_+ \tilde{G}_- \tag{2}$$

where $\tilde{G}_+$ contains all the time delays and right-half plane zeros. The second step is to define the IMC controller as

$$G_{imc} = \tilde{G}_-^{-1} f \tag{3}$$

where f is a low pass filter with a steady state gain of one. The simplest filter has the following form [12, 13]

$$f = \frac{1}{(\lambda s + 1)^n} \tag{4}$$

It is straightforward to illustrate that the IMC controller, G_{imc}, is related to the classic controller, G_c, through the transformation

$$G_c = \frac{G_{imc}}{1 - G_{imc} G_m} \tag{5}$$

Fig. 3 shows the block diagram of the IMC representation of a Smith predictor.

First a FOPDT transfer function is considered. In order to obtain the IMC controller, the process model, $\tilde{G} = K_m e^{-\theta_m s}/(T_m s+1)$, must be factored as in eqn. (2). If a first order Taylor series expansion is used for the time-delay approximation, then the following equations are obtained

$$\tilde{G}_+ = (1 - \theta_m s) \tag{6}$$

$$\tilde{G}_- = \frac{K_m}{T_m s + 1} \tag{7}$$

The IMC controller can be obtained from eqn. (3), assuming a filter with $n = 1$, as

$$G_{imc} = \frac{T_m s + 1}{K_m(\lambda s + 1)} \tag{8}$$

Eqn. (8) shows that, once the parameters of the model, K and T, are known, then only the filter parameter, λ, remains to be selected. The classic controller, G_c, can then be obtained using eqn. (5) to give

$$G_c = \frac{T_m s + 1}{K_m \lambda s} \tag{9}$$

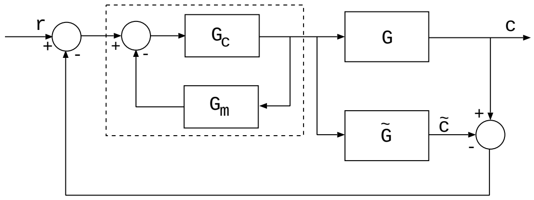

Fig. 3. IMC representation of Smith predictor

Eqn. (9) can be rearranged as a PI controller, which has the following controller parameters

$$K_p = \frac{T_m}{K_m \lambda} \tag{10}$$

and

$$T_i = T_m. \tag{11}$$

Here, the Integral Squared Error, ISE, criterion, which is given by

$$J_{ISE} = \int_0^\infty [r - c(t)]^2 dt \tag{12}$$

is used to find an optimal solution for the filter parameter, λ. The Laplace form of the output signal, $C(s)$, in the Smith predictor configuration can be obtained from Fig. 1,

$$\frac{C(s)}{R(s)} = \frac{G_c G_m}{1 + G_c G_m} e^{-\theta_m s} \tag{13}$$

assuming a perfect matching between the process and model. Substituting the proper values for $G_c(s)$ and $G_m(s)$ into eqn. (13) and assuming a unit step change into the system gives

$$C(s) = \frac{1}{s(\lambda s + 1)} e^{-\theta_m s} \tag{14}$$

The time domain solution is obtained by assuming a first order Taylor series expansion.

$$c(t) = 1 - (1 + \frac{\theta_m}{\lambda}) e^{-t/\lambda} \tag{15}$$

Putting eqn. (15) in to eqn. (12) results in

$$J_{ISE} = \frac{(\lambda + \theta_m)^2}{2\lambda} \tag{16}$$

Taking the derivative of eqn. (16) with respect to λ, produces $\lambda = \theta_m$. Finally the PI controller parameters are

$$K_p = \frac{T_m}{K_m \theta_m} \tag{17}$$

$$T_i = T_m \tag{18}$$

Processes with SOPDT transfer functions are also very common. This is why a similar result as for a FOPDT transfer function is derived for SOPDT transfer function too. Following the same procedure as for FOPDT transfer function and assuming $\tilde{G} = K_m e^{-\theta_m s}/(T_{1m}s + 1)(T_{2m}s + 1)$, it can easily be shown that the classical controller can now be implemented as a PID controller with the following parameters

$$K_p = \frac{T_{1m} + T_{2m}}{K_m \theta_m} \tag{19}$$

$$T_i = T_{1m} + T_{2m} \tag{20}$$

$$T_d = \frac{T_{1m} T_{2m}}{T_{1m} + T_{2m}} \tag{21}$$

Some Remarks: It is known that the ISE criterion usually gives an oscillatory closed loop response with long settling time. Thus the time weighted version of the ISE criterion, that is the ISTE criterion, can be used to find a new value for the filter time constant. However, it can easily be shown from eqn. (12) that this does not give a solution. Also, the approximation used for the time delay is poor. Therefore, a higher order Taylor series expansion can be used but it again can be shown from eqn. (12) that a solution cannot be obtained. Alternatively, a 0/1, 1/1 or 1/2 Padé approximation can be used but the expressions obtained are more complicated and an analytical solution is not possible.

2.1 *Autotuning Procedure*

The block diagram for autotuning of the Smith predictor configuration is shown in Fig. 4. The autotuning procedure to find controller parameters can be carried out as follows:

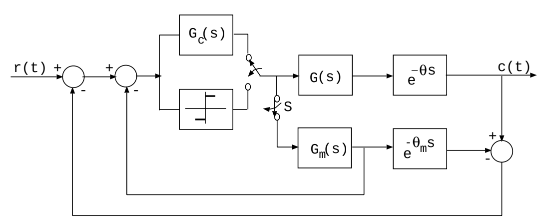

Fig. 4. Block diagram for autotuning of the Smith predictor

- When the controller needs to be tuned, switch from the controller mode to relay mode. At the same time, open the switch "S" so that the original relay feedback configuration is obtained.
- Measure the limit cycle parameters and estimate parameters for the FOPDT or SOPDT model plant transfer function using the relay feedback method proposed by Kaya and Atherton [11].

- Find tuning parameters using either eqns. (17)-(18), if the FOPDT model is used, or eqns. (19)-(21), if the SOPDT model is used.
- Switch from the relay mode to the controller mode with calculated tuning parameters for the control of the process. At the same time, close the switch "S" so that the Smith predictor configuration is reobtained.

3. ROBUSTNESS ANALYSIS OF THE PERFORMANCE

The robustness analysis of the proposed controller design is done using the block diagram shown in Fig. 1. The characteristic equation of the system given in Fig. 1 is

$$1 + G_c(s)G_m(s) + G_c(s)[P(s) - P_m(s)] = 0 \quad (22)$$

where $P(s) = G(s)e^{-thetas}$ is the actual plant transfer function and $P_m(s) = G_m(s)e^{-\theta_m s}$ is the model of the plant. If the uncertainties are given by $P(s) = P_m(s) + \delta P(s)$, where the $\delta P(s)$ is the uncertainty in $P(s)$, then eqn. (22) can be rearranged as

$$1 + G_c(s)G_m(s) + G_c(s)\delta P(s) = 0 \quad (23)$$

which then gives

$$|\delta P(s)| = \frac{|1 + G_c(s)G_m(s)|}{|G_c(s)|} \quad (24)$$

the norm bound uncertainty region [13] in order to maintain the closed loop stability.

Substituting for $G_m(s)$, $K_m/(T_m s+1)$, and $G_c(s)$, from eqn. (9), gives

$$|\delta P(s)|_{FOPDT} = \frac{K_m\sqrt{\lambda^2\omega^2+1}}{\sqrt{T_m^2\omega^2+1}} \quad (25)$$

the norm bound uncertainty region. For low frequencies the norm bound uncertainty region for $|\Delta P(s)|_{FOPDT}$ is given by the steady state gain of the model K_m. The magnitude of the modelling errors, $|P(j\omega) - P_m(j\omega)|$, at low frequencies is given by $(K - K_m)$. This illustrates that at low frequencies, the closed loop stability is only affected by the uncertainties in the steady state gains of the plant and model. Also, it is seen that very high modelling errors, that is 100%, in the plant and model steady state gains is allowed for maintaining the closed loop stability. For high frequencies the norm bound is given by $K_m\lambda/T_m$. Thus the larger the value of the filter time constant λ the larger norm bound uncertainty region, that is, the permission for larger modelling errors.

Similarly the norm bound uncertainty region for the case when the plant is modelled by the SOPDT is obtained as

$$|\delta P(s)|_{SOPDT} = \frac{K_m\sqrt{\lambda^2\omega^2+1}}{\sqrt{[1-(T_{1m}T_{2m}\omega)^2]^2 + (T_{1m}+T_{2m})^2\omega^2}} \quad (26)$$

For low frequencies the norm bound uncertainty region for $|\delta P(s)|_{SOPDT}$ is again given by the steady state gain of the model K_m. Since the modelling errors are again given by $(K - K_m)$, a very high value for modelling errors, namely 100%, is allowed at low frequencies. For $\omega \to \infty$, $|\delta P(s)|_{SOPDT} \to 0$. Thus this implies that the choice of λ has little affect on the stability of the closed loop system at high frequencies. However, the mid-frequencies are more affective on the stability of the system, therefore it can still be expected that the larger values of λ gives larger margins to maintain the closed loop system stability, as a large value of λ gives larger norm bound uncertainty region at mid-frequencies.

4. AN EXAMPLE

One example is given to illustrate the use of the method. The example is given to both illustrate the performance robustness of the presented design method in the case of a mismatch in the time delay, the most detrimental case to the system performance and compare the performance of the proposed design method with some other existing ones.

Consider the SOPDT transfer function given by

$$G = \frac{e^{-5s}}{(6s+1)(2s+1)}$$

which was simulated in SIMULINK. The constant input to the relay feedback system was 0.1 and the relay had unity heights and no hysteresis. The resulting asymmetric limit cycle parameters were 0.603, −0.508, 10.609 and 0.309 for a_{max}, a_{min}, Δ_{t1} and ω, (see reference [11] for notations). The steady-state gain, K_m, the time constants, T_{1m} and T_{2m}, and the dead-time, θ_m, were calculated as 1.000, 6.001, 1.999 and 5.001, respectively. These process parameters gave the PID controller parameters of $K_p = 1.600$, $T_i = 8.001$ and $T_d = 1.500$, using eqns. (19)-(21). Step response of the Smith predictor with these controller parameters for matching and ±50% mismatch in the time delays are shown in Fig. 5. Also, a disturbance at $t = 75$ with magnitude of $d = -0.1$ is injected into the system is shown. Obviously, the response of the system is quite satisfactory in all three cases, although, a relatively large mismatch in the time delay is assumed. Alternatively, if a better performance is required, as mentioned before in the introduction, retuning can be done to find new process model parameters and hence new controller parameters.

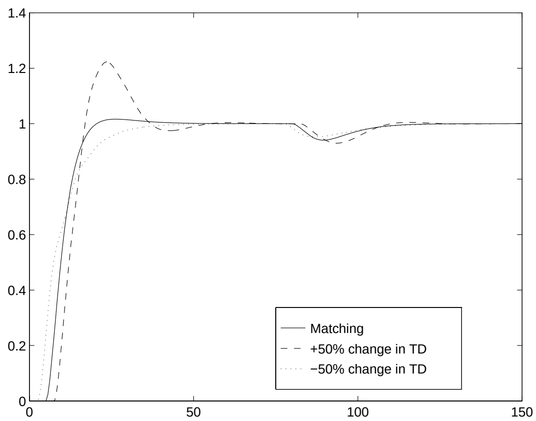

Fig. 5. Step and disturbance rejection responses for the example

To compare the performance of the proposed design method with some other existing methods, Benouarets and Atherton [6] and Lee *et al* [9], responses to a step change and disturbance rejection are given in Fig. 6. The proposed method gives faster respose to step change and disturbance rejection than the others for this example.

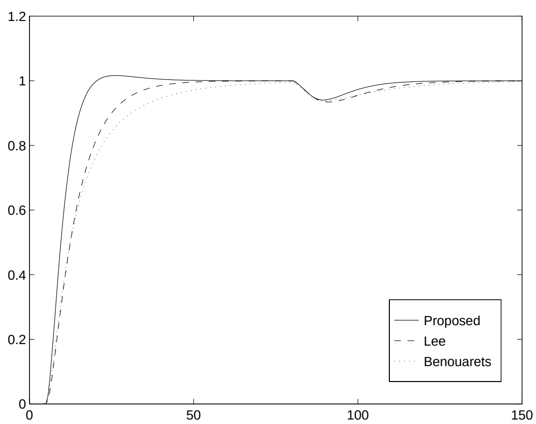

Fig. 6. Step and disturbance rejection responses for the example

5. CONCLUSIONS

An autotuning method for Smith predictor controllers has been given based on exact limit cycle analysis for FOPDT and SOPDT plants. The Smith predictor was represented as an equivalent IMC controller and this enabled to define the PI or PID controller parameters to be defined in terms of the model parameters and the filter parameter, λ. It was ssumed that a model of the plant could be found using relay autotuning method (Kaya and Atherton, 1998), this meant that only one parameter, namely the filter parameter λ, was left for tuning. The ISE criterion was used to find the filter parameter, and simple equations were obtained to tune the Smith predictor. The method is very simple and has given improved results compared with some previous approaches.

6. REFERENCES

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