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ANALYTIC SYNTHESIS, BY MEANS OF NORMALIZED DIAGRAMS, OF STANDARD REGULATORS FOR SISO CONTROL SYSTEMS

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Abstract In this paper we will present some procedures for the analytic synthesis, by means of feedforward and feedback standard regulators, of some SISO control systems when the order of the system itself doesn't go beyond the third.
These procedures utilize normalized diagrams obtained in [12] which supply the relations between the parameters of the above systems and their outputs to canonical inputs. Consequently, these procedures supply, for each of the above systems, proper normalized diagrams which, therefore, can be utilized whatever the numerical values of the parameters are.

Key Words. Analytic synthesis, Standard regulators, PD, PI, Hybrid regulators.

1. INTRODUCTION

As we all know, in cases in which, in order to obtain a required behavior of a SISO linear stationary system, it is possible to assign the system poles without having to vary or add on zeros, it is sufficient to use a suitable algebraic feedback of the states. On the contrary, when it is necessary to vary or add on zeros, or when not all states are available and for some reason it is not convenient to utilize an observer, the synthesis can be made by means of the so-called *regulators* (*standard regulators* or *compensating networks*); with these systems, placed in the feedforward or in the feedback loop, it is possible to assign not only some of the required poles, but also some of the required zeros.
As we also know, standard regulators are algebraic or dynamic systems of the first or second order and their output is proportional to the input, or to the derivative (or to the integral) of the input itself, or to a linear combination of these functions.
As to the design method of standard regulators, if the feedback system is one of the types as in [8], that is of the systems for which it is possible to determine the relations between the system parameters and the parameters of the system outputs to canonical inputs, we can determine the regulator parameters analytically by imposing that the parameters of the overall system lead to (at least approximately) the required behavior. Moreover, it is possible to supply normalized diagrams of the obtained relations so that it is very easy to utilize this design procedures.
In the following paragraphs we will examine the analytic synthesis procedures of feedforward and feedback standard regulators. As to the feedforward ones, we will examine *proportional derivative* (*PD*) regulators and *proportional integral* (*PI*) regulators while, as to the feedback regulators, we will examine PD and the so called *hybrid* regulators which are made up of a proportional regulator and of a PD. In order to simplify, we will only examine systems with algebraic sensors and, moreover, the plant transfer function $G_s(s)$ has no zeros, but only real poles.

2. FEEDFORWARD REGULATORS

2.1 Design procedures by means of PD regulators

In this paragraph we will present some analytic design procedures of PD standard regulators in the situation in which the system plant is of the second (or third) order, type 1. As to the design specifics, we will assume that the maximum overshoot, the rise time and the settling time are given, while the regulator's transfer function, as we all know, is given by:

$$G_r(s) = K_p(1 + T_d s) \tag{1}$$

SECOND ORDER, TYPE 1 PLANT
If the plant transfer function $G_s(s)$ is of the second order, type 1, considering (1), the feedforward-loop transfer function is:

$$G_r(s)G_s(s) = \frac{K_s K_p(1 + T_d s)}{s(1 + \tau s)} \tag{2}$$

where K_s and τ are, respectively, the gain constant and the time constant of $G_s(s)$. Consequently, naming K_t the sensor constant, the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_t} \frac{\frac{K}{\tau}(1 + T_d s)}{s^2 + \frac{1 + KT_d}{\tau} s + \frac{K}{\tau}} \tag{3}$$

where:

$$K = K_s K_p K_t \tag{4}$$

As we can see, leaving aside $1/K_t$, (3) is a second order polynomial function with one zero and we can write it in the form:

$$\bar{G}(s) = \frac{\omega_n^2 (1 + \frac{s}{z\delta\omega_n})}{s^2 + 2\delta\omega_n s + \omega_n^2} \tag{5}$$

assuming:

$$\frac{K}{\tau} = \omega_n^2 \qquad 2\delta\omega_n = \frac{1 + KT_d}{\tau}$$

$$\frac{1}{T_d} = z\delta\omega_n \tag{5'}$$

Since K and T_d are the only unknown variables of the three equations (5'), generally, it is not possible to solve these equations for any tern of values of the parameters z, δ and ω_n; that means it is not possible to assign arbitrarily the three system parameters (its three poles) properly assuming K and T_d. Therefore, we will name *admissible terns* the terns of z, δ and ω_n values in correspondence of which, not only ω_n is positive and $0 \le \delta \le 1$, but also it is possible to obtain values of the regulator parameters K and T_d satisfying equations (5').
In order to determine the admissible terns of δ, ω_n and z values, it is better to rewrite the equations (5') in a proper form; in particular, assuming:

$$T = \omega_n \tau \tag{6}$$

we obtain:

$$K = \omega_n T$$

$$\frac{T_d}{\tau} = \frac{2\delta T - 1}{T^2} \qquad z = \frac{T}{\delta(2\delta T - 1)} \tag{7}$$

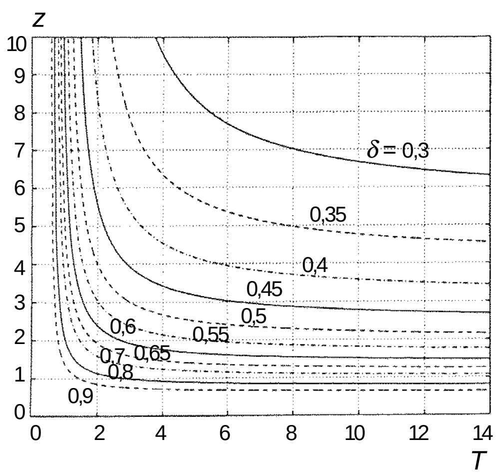

Fig.1

On the other hand, if we suppose z to be positive, it comes out:

$$T > \frac{1}{2\delta} \tag{8}$$

and, therefore, as could be easily obtained from the first two equations in (7), also K and T_d are positive. Considering (8), figure 1 represents the graphs of the last equation in (7).
We may sum up by noting that if, as already mentioned, the design specifics are the rise time t_s, the settling time t_a and the maximum overshoot S, in order to determine the admissible terns which satisfy them, we can proceed as follows:

– by utilizing the graphs of the figures represented in the Appendix of [12] as to the second order systems with one zero, we can determine the couples of δ and z values for which we can obtain the desired maximum overshoot
– in correspondence with every couple δ and z obtained, from the graphs of figure 1, we can obtain the T values (and, therefore the ω_n ones)
– by utilizing the graphs of the figures of the Appendix mentioned above, we can choose the admissible tern which approximates in the best way the desired t_s and t_a values
– in correspondence with the chosen tern, from the first and the fourth equation in (5) we can obtain the T_d and K values; lastly, from equation (4) we can obtain the K_p value.

EXAMPLE
On the basis of a unitary feedback system whose plant has a transfer function given by:

$$G_s(s) = \frac{1}{s(1+s)}$$

Table1

S = 20%						
δ	z	ω_n [s^{-1}]	t_s [s]	t_a [s]	T_d	K_p
0,9	0,57	20	0,05	0,25		
0,8	0,75	14	0,06	0,3		
0,7	1,05	12	0,07	0,4		
0,6	1,7	7	0,1	0,9		
0,56	2,3	3,5	0,23	1,7	0,23	13,5
0,5	3,5	2,3	0,43	2,6		

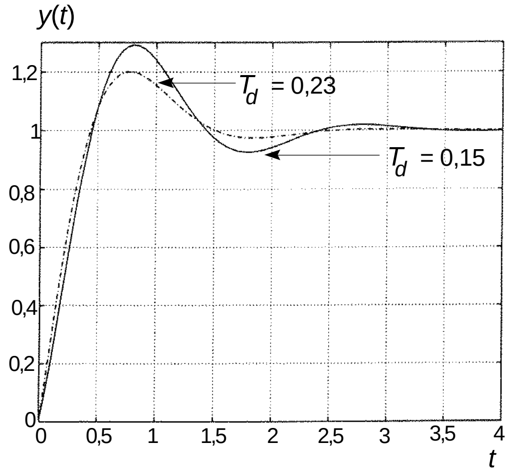

Fig.2

we want to achieve a system design, by means of a PD regulator, that will make the system output to a unit step input present a rise time t_s = 0,2 s, a settling time t_a = 2 s and a maximum overshoot S = 20%.

By applying the above mentioned design procedure, we can write table 1.
As could be easily verified, a good approximation of the desired specifics is given by K_p = 13,5 and T_d = 0,23.
Please note that the t_s, t_a and S values present in the table correspond, approximately, to the ones resulting from the system output to a unit step input shown in figure 2. In the same figure it is also shown that a decrease of the regulator time constant entails an increase of the maximum overshoot without a substantial variation of the rise time.

THIRD ORDER, TYPE 1 PLANT
If the plant transfer function $G_s(s)$ is of the third order, type 1, considering (1), the feedforward-loop transfer function is:

$$G_r(s)G_s(s) = \frac{K_s K_p(1+T_d s)}{s(1+\tau_1 s)(1+\tau_2 s)} \tag{9}$$

where K_s,τ_1 and τ_2 are, respectively, the gain constant and the time constants of $G_s(s)$.
Consequently, naming K_t the sensor constant, the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_t}\,\frac{\dfrac{KT_d}{\tau_1\tau_2}s + \dfrac{K}{\tau_1\tau_2}}{s^3 + \dfrac{\tau_1+\tau_2}{\tau_1\tau_2}s^2 + \dfrac{1+KT_d}{\tau_1\tau_2}s + \dfrac{K}{\tau_1\tau_2}} \tag{10}$$

As we can see, leaving aside $1/K_t$, (10) is a third order polynomial function with one zero and we can write it in the form:

$$\bar{G}(s) = \frac{k\omega_n^2 s + \omega_n^3 kz\delta}{s^3 + (kz+2)\delta\omega_n s^2 + (1+2\delta^2 kz)\omega_n^2 s + \omega_n^3 kz\delta} \tag{11}$$

assuming:

$$\frac{K}{\tau_1\tau_2} = \omega_n^3\, kz\delta \qquad \frac{KT_d}{\tau_1\tau_2} = k\omega_n^2$$
$$\frac{\tau_1+\tau_2}{\tau_1\tau_2} = (kz+2)\,\delta\omega_n \qquad \frac{1+KT_d}{\tau_1\tau_2} = (1+2\delta^2 kz)\omega_n^2 \tag{11'}$$

Since K and T_d, are the only unknown variables of the four equations (11'), generally, it is not possible to solve these equations for any values of the four parameters k, z, δ and ω_n; that means it is not possible to assign arbitrarily the four system parameters (its three poles and the zero) properly assuming K and T_d. Therefore, we will name *admissible four values* the values of the four parameters k, z, δ and ω_n in correspondence of which, not only ω_n is positive, $0 \le \delta \le 1$, k and z must be of the same sign to guarantee the system stability, but it is also possible to obtain values of the regulator parameters K and T_d satisfying equations (11').

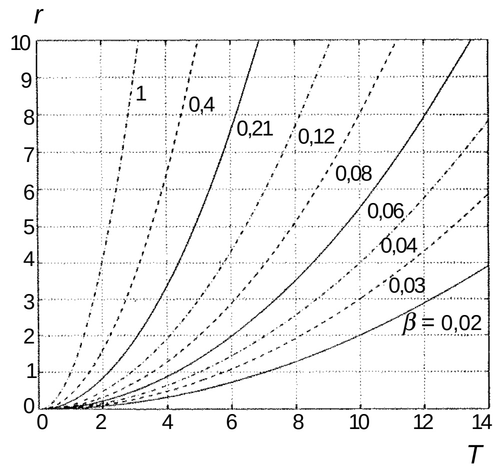

Fig.3

In order to determine the admissible four values of the parameters k, z, δ and ω_n, it is better to rewrite equations (11') in a proper form; in particular, supposing $\tau_1 \ge \tau_2$ and assuming:

$$\tau = \tau_1 \qquad r = \tau/\tau_2 \qquad T = \tau\omega_n \tag{12}$$

$$\gamma = \frac{r+1}{kz+2} \qquad \beta = 1 + k(2\delta^2 z - 1) \tag{13}$$

we obtain:

$$K = \frac{\delta kzT^3}{\tau r} \qquad T_d = \frac{\tau}{\delta zT}$$
$$\gamma = \delta T \qquad r = \beta T^2 \tag{14}$$

On the other hand, from (13) we get:

$$z = \frac{r+1-2\gamma}{\gamma(1-\beta) + 2\delta^2(r+1-2\gamma)} \tag{15}$$

$$k = \frac{\gamma(1-\beta) + 2\delta^2(r+1-2\gamma)}{\gamma}$$

If we suppose z, k to be positive, a sufficient condition to obtain this, assuming $r \geq 1$, is:

$$\beta \leq 1 \qquad\qquad 0 \leq \gamma \leq (r+1)/2 \qquad\qquad (16)$$

Considering the first equation in (16), in figures 3 and 4 are shown the graphs of the two last equations in (14).

In conclusion, if as already mentioned, the design specifics are rise time t_s, settling time t_a and maximum overshoot S, in order to determine the admissible four values which satisfy them we can proceed as follows:

- from figure 3 we can determine the couples of β and T values (and therefore the ω_n ones) corresponding to the given r value
- in correspondence with every T value obtained, from figure 4 we can determine the relative couples of γ and δ values; this way, we obtain a set of four γ, δ, β and T values split up into subsets with the same values as β and T
- in correspondence with every combination of four values obtained, we can determine, by utilizing relations (15) the relative combinations of the four parameters k, δ, z and ω_n
- having determined the admissible four values in this way, by utilizing the approximate results shown in the Appendix of [12] as to third order systems with one zero, we can obtain for each of them the relative t_s, t_a and S values and we can choose, among the admissible four values, the one which approximates the desired specifics in the best way
- in correspondence of the four values chosen, from the first two equations in (14) we can determine the K and T_d values; finally, we can obtain the K_p value from relation (4).

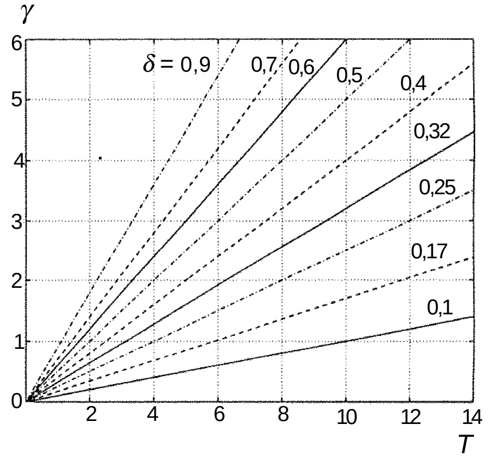

Fig.4

EXAMPLE

On the basis of a unitary feedback system whose plant has a transfer function given by:

$$G_s(s) = \frac{0{,}02}{s\,(1+0{,}2s)(1+0{,}1s)}$$

we want to achieve a system design, by means of a PD regulator, that will make the system output to a unit step input present a settling time t_a = 2 s and a maximum overshoot S = 5%.

Table 2

r = 2							
ω_n	δ	z	k	S [%]	t_a [s]	T_d	K_p
7	0,9	0,62	0,81	0	1		
7	0,7	1,02	1,05	5	1,5	0,2	258,5
7	0,6	1,4	3,36	11	0,8		
7	0,5	2	4,56	30	1,13		

By applying the above mentioned design procedure, we can write table 2 from which we evince that a good approximation of the desired specifics is given by T_d = 0,2 and K_p = 258,5.

Figure 5 shows the system output to a unit step input when we choose, as regulator parameters, the ones obtained above; as could easily be verified, the maximum overshoot and settling time values are quite similar to the ones in table 2.

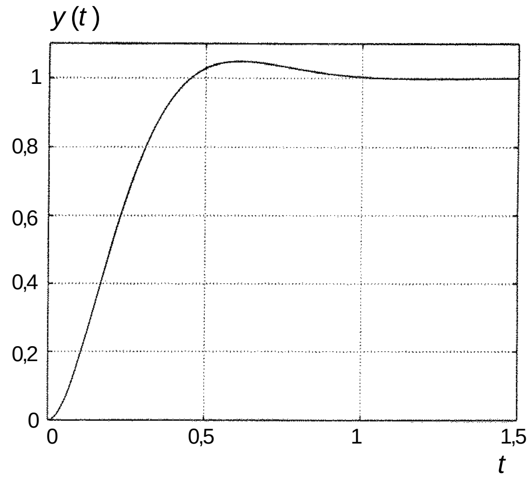

Fig.5

2.2 Design procedures by means of PI regulators

In this paragraph we will present some analytic design procedures of PI standard regulators in the case in which the system's plant $G_s(s)$ is of the second (or third) order, type 0.

As to the design specifics, we will assume that maximum overshoot, rise time and settling time are given, while the regulator transfer function, as we all know, is given by:

$$G_r(s) = K_p\left(1 + \frac{1}{T_i\,s}\right) = \frac{K_p}{T_i\,s}\,(1 + T_i\,s) \qquad (17)$$

SECOND ORDER, TYPE 0 PLANT

If the plant transfer function $G_s(s)$ is of the second order, type 0, considering (17), the feedforward-loop transfer function is:

$$G_r(s)G_s(s) = \frac{K_s K_p (1 + T_i\, s)}{T_i\, s(1+\tau_1 s)(1+\tau_2 s)} \qquad (18)$$

where K_s, τ_1 and τ_2 are, respectively, the constant gain and the time constants of $G_s(s)$. Consequently, naming K_t the sensor constant and assuming:

$$K = \frac{K_s K_p K_t}{T_i} \qquad (19)$$

the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_t}\,\frac{\dfrac{KT_i}{\tau_1\tau_2}s + \dfrac{K}{\tau_1\tau_2}}{s^3 + \dfrac{\tau_1+\tau_2}{\tau_1\tau_2}s^2 + \dfrac{1+KT_i}{\tau_1\tau_2}s + \dfrac{K}{\tau_1\tau_2}} \qquad (20)$$

On the other hand, as could be easily seen, equation (20) is quite similar to (10) and, therefore, we can determine the K_p and T_i regulator parameters by utilizing the procedure shown in Paragraph 2 and considering relation (19).

EXAMPLE

On the basis of a unitary feedback system whose plant has a transfer function given by:

$$G_s(s) = \frac{2{,}85}{s\,(1 + 0{,}022s)(1+0{,}004s)}$$

we want to achieve a system design, by means of a PI regulator, that will make the system output to a unit step input present a maximum overshoot S = 20% and a rise time $t_s = 0{,}008$ s.

As stated above, we can determine the K_p and T_i regulator parameters by applying the procedure shown in Paragraph 2 as to the systems of third order, type 1 plant. Since, from the second equation in (16) it must be $\gamma \le 3{,}25$, we can choose $\gamma = 2$ as a tentative value and, therefore, we can write table 3.

Table 3

$\gamma = 2$							
δ	ω_n $[s^{-1}]$	z	k	K_p	T_i	S [%]	t_s [s]
0,8	136	0,6	2	1,115	0,014	16	0,009
0,7	140	0,7	1,6	1,025	0,013	18	0,008
0,6	150	0,8	1,5	1,039	0,013	19	0,008
0,5	181	0,9	1,2	1,298	0,011	25	0,007

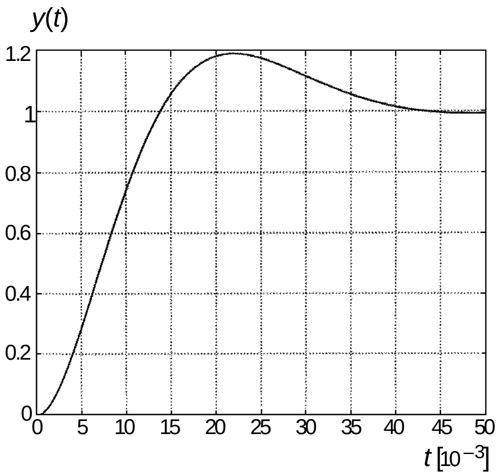

Fig.6

Table 3 shows that, choosing K_p = 1,039 and T_i = 0,0133, we can obtain the maximum overshoot S value and the rise time t_s value nearest to the ones desired.

Figure 6 shows the system output to a unit step input when we choose, as regulator parameters, the ones obtained above; as can easily be verified, the maximum overshoot and rise time values are quite similar to the ones in table 3.

THIRD ORDER, TYPE 0 PLANT

If the plant transfer function $G_s(s)$ is of the third order, type 0, considering (17), the feedforward-loop transfer function is:

$$G_r(s)G_s(s) = \frac{K_s K_p (1 + T_i\, s)}{T_i\, s(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)} \qquad (21)$$

where K_s, τ_1, τ_2 and τ_3 are, respectively, the gain constant and the time constants of $G_s(s)$. Since (21) is a fourth order system, in order to design it analytically, it is necessary to reduce its dynamic order. Therefore, assuming:

$$T_i = \tau_3 \qquad K'_p = \frac{K_p}{T_i} \qquad (22)$$

we can cancel the zero of the regulator and one pole (generally the fastest) of the plant and, therefore, we can write (21) in the form:

$$G_r(s)G_s(s) = \frac{K_s K'_p}{s(1+\tau_1 s)(1+\tau_2 s)} \qquad (23)$$

Consequently, naming K_t the sensor constant and assuming:

$$K = K'_p\, K_s K_t \qquad (24)$$

the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_t}\,\frac{K/\tau_1\tau_2}{s^3 + \dfrac{\tau_1+\tau_2}{\tau_1\tau_2}s^2 + \dfrac{s}{\tau_1\tau_2} + \dfrac{K}{\tau_1\tau_2}} \qquad (25)$$

Leaving aside $1/K_t$, (25) is a third order polynomial function and we can write it in the form:

$$\bar{G}(s) = \frac{\omega_n^3 p\delta}{s^3 + (p+2)\,\delta\omega_n s^2 + (1+2\delta^2 p)\omega_n^2 s + \omega_n^3 p\delta} \qquad (26)$$

assuming:

$$\frac{K}{\tau_1\tau_2} = \omega_n^3\, p\delta \qquad \frac{\tau_1+\tau_2}{\tau_1\tau_2} = (p+2)\,\delta\omega_n \qquad (26')$$

$$\frac{1}{\tau_1\tau_2} = (1+2\delta^2 p)\omega_n^2$$

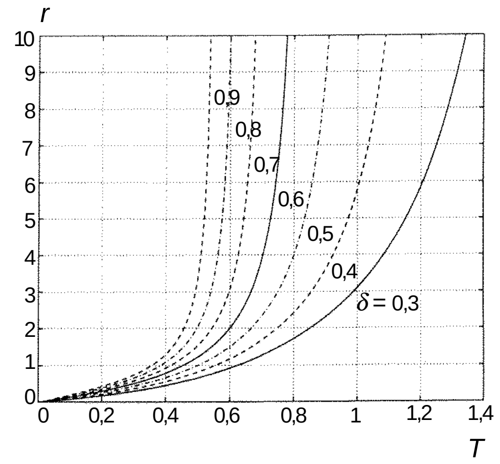

Fig.7.a

In order to determine the admissible terns of δ, ω_n and p values, it is better to rewrite equations (26') in a proper form; in particular, supposing $\tau_1 \geq \tau_2$ and assuming:

$$\tau = \tau_1 \qquad r = \tau/\tau_2 \qquad T = \tau\omega_n \qquad (27)$$

we obtain the equations:

$$K\tau = T\,\frac{1+T(T-2\delta)}{2\,\delta + T(1-4\delta^2)} \qquad p = \frac{1+T(T-2\delta)}{T\,\delta(1-2T\delta)} \qquad (28)$$

$$r = \frac{2T\delta + T^2(1-4\delta^2)}{1-2T\delta}$$

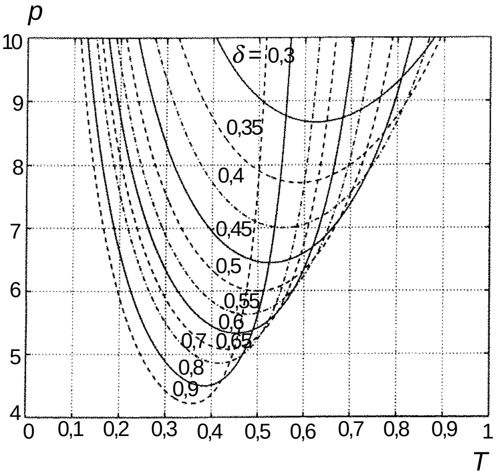

Fig.7.b

Supposing p to be positive, from the second equation in (28) we obtain:

$$0 < T < \frac{1}{2\delta} \qquad (29)$$

Consequently, from the first equation in (28), we can easily see that, in correspondence with the admissible terns, it always comes out $K > 0$.

Considering (29), figures 7.a and 7.b show the graphs of the last two equations in (28); these graphs, thanks to the use of the normalized variable T, can be utilized whatever the τ_1 and τ_2 plant parameters values are.

In conclusion, in order to determine the admissible terns, we can proceed as follows:

– in correspondence with the r given value, from figure 7.a we can determine the couples of δ and T values (and therefore the ω_n ones) and from the latter, utilizing the graphs in figure 7.b, we can determine the corresponding p values.

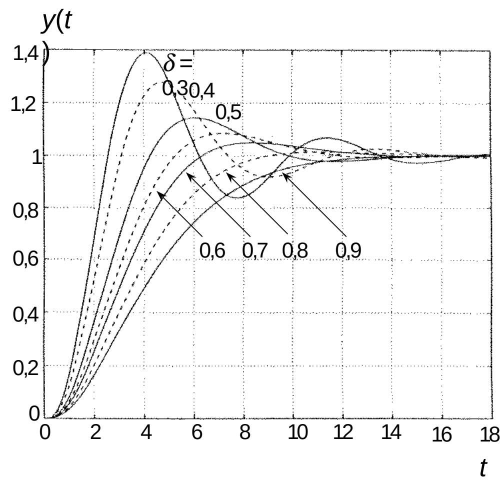

Fig.7.c

Having determined the admissible terns of δ, ω_n and p values in this way, if, as already mentioned, the design specifics are rise time t_s, settling time t_a and maximum overshoot S, in order to satisfy the same specifics, we can proceed as follows:

– in correspondence with each admissible tern, from the figures represented in the Appendix of [12] as to the third order systems, we can obtain the relative t_s, t_a and S values

– having chosen the tern corresponding to the t_s, t_a and S values which satisfy the desired specifics in the best way, from the first equation in (28) we can determine the K value and, lastly, from (22) and (24) the K_p value.

EXAMPLE

On the basis of a unitary feedback system whose plant has a transfer function given by:

$$G_s(s) = \frac{2{,}85}{(1+0{,}004s)(1+0{,}010s)(1+0{,}022s)} \qquad (30)$$

we want to achieve a system design, by means of a PI regulator, that will make the system output to a unit step input present a maximum overshoot S = 25% and a rise time t_s = 0,02 s.

By applying the above mentioned design procedure as to the systems with third order, type 0 plant, considering the first equation in (22) we obtain:

$T_i = 0{,}004$

Thus, (23) comes out:

$$G_s(s)G_r(s) = \frac{2{,}85K_p'}{s(1+0{,}01s)(1+0{,}022s)} \tag{31}$$

We can then write table 4 from which we evince that a good approximation of the desired specifics is given by $K_p = 0{,}53$.

Figure 8 shows the system output to a unit step input when we choose the regulator parameters as the ones obtained above; as could easily be verified, the maximum overshoot and rise time values are quite similar to the ones in table 4.

Table 4

δ	ω_n [s^{-1}]	p	S [%]	t_s [s]	K_n
0,5	60	6,2	15		
0,4	90	8,5	25	0,021	0,53
0,3	75	9	35		

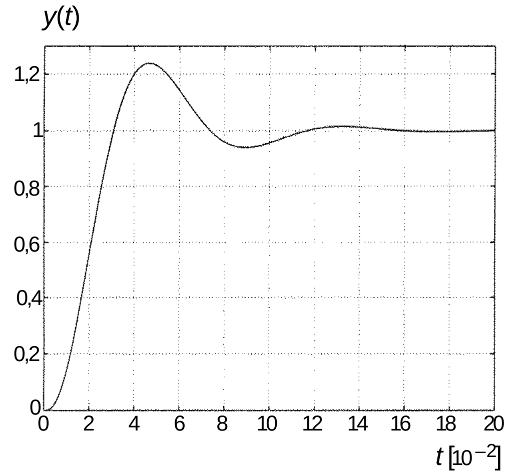

Fig.8

3 FEEDBACK REGULATORS

3.1 Design procedures by means of PD regulators

In this paragraph we will present an analytic design procedure of PD standard regulators whose transfer function, as already mentioned, is given by relation (1).

As to the design specifics, we will assume that the maximum overshoot, the rise time and the settling time are given and, moreover, we will assume the plant transfer function $G_s(s)$ is of the third order, type 1. Consequently we can write the plant transfer function in the form:

$$G_s(s) = \frac{K_s}{s(1+\tau_1 s)(1+\tau_2 s)} \tag{32}$$

where K_s, τ_1 and τ_2 are, respectively, the gain constant and the time constants of $G_s(s)$. Moreover, naming K_t the sensor constant and taking into account relation (1), the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_pK_t}\,\frac{\dfrac{K}{\tau_1\tau_2}}{s^3+\dfrac{\tau_1+\tau_2}{\tau_1\tau_2}s^2+\dfrac{1+KT_d}{\tau_1\tau_2}s+\dfrac{K}{\tau_1\tau_2}} \tag{33}$$

where K is given by relation (4).

As we can see, leaving aside $1/K_p K_t$, (33) is a third order polynomial function and we can write it in the (26) form assuming:

$$\frac{K}{\tau_1\tau_2} = \omega_n^3\,p\delta \qquad \frac{\tau_1+\tau_2}{\tau_1\tau_2} = (p+2)\,\delta\omega_n$$
$$\frac{1+KT_d}{\tau_1\tau_2} = (1+2\delta^2 p)\omega_n^2 \tag{34}$$

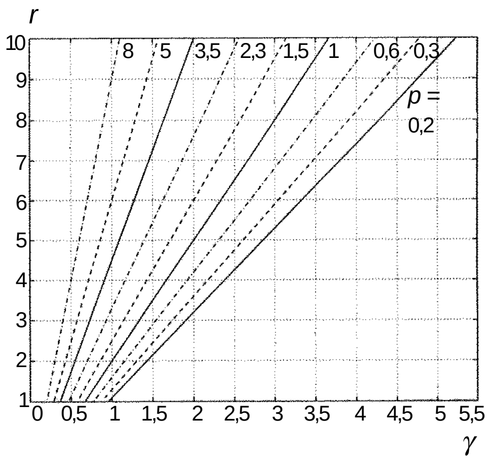

Fig.9

In order to determine the admissible terns of δ, ω_n, p values (see Paragraph 2.1 above), it is better to rewrite equations (34) in a proper form; in particular, assuming $\tau_1 \geq \tau_2$ and moreover:

$$\tau = \tau_1 \qquad r = \tau/\tau_2$$
$$T = \tau\omega_n \qquad \gamma = \delta T \tag{35}$$

we get:

$$K = \frac{\delta p T^3}{\tau r} \qquad T_d = \frac{T^2(1+2\delta p)-r}{T\delta p\omega_n}$$
$$r = \gamma(p+2)-1 \tag{36}$$

On the other hand, assuming p positive, from the third equation in (36) we get:

$$0 \leq \gamma \leq (r+1)/2 \tag{37}$$

Considering (37), figures 9 and 10 represent the graphs of the third equation in (36) and of the fourth equation in (35).

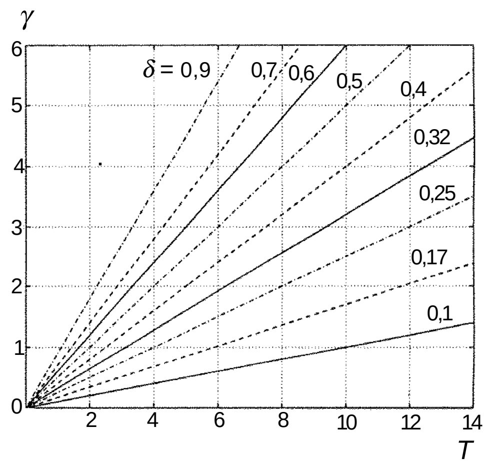

Fig.10

In conclusion, in order to determine the admissible terns, we can proceed as follows:

- in correspondence with the r given value, from figure 9 we can determine the couples of γ and p values
- in correspondence with every γ value obtained, from the figure 10 we can determine the corresponding couples of δ and T values (and therefore the ω_n ones).

Having determined the admissible terns of δ, ω_n and p values in this way, if, as already mentioned, the design specifics are rise time t_s, settling time t_a and maximum overshoot S, in order to satisfy the same specifics, we can proceed as follows:

- in correspondence with each admissible tern, from the graphs of the figures represented in the Appendix of [12] as to the third order systems, we can obtain the relative t_s, t_a and S values
- having chosen the tern corresponding to the t_s, t_a and S values which satisfy the desired specifics in the best way, from the first and the second equation in (36) we can determine the K and T_d values; lastly, from (4) the K_p value.

EXAMPLE

On the basis of a unitary feedback system whose plant has a transfer function given by:

$$G_s(s) = \frac{48{,}21}{s(1 + 2{,}7s)^2} \tag{38}$$

we want to achieve a system design, by means of a PD regulator, that will make the system output to a unit step input present a maximum overshoot $S \leq 5\%$ and a rise time $t_a \leq 12$ ms.

By applying the above mentioned design procedure we can obtain that assuming:

$$K_p = 0{,}0077 \qquad T_d = 2{,}7 \tag{39}$$

the output, to a unit step input, of the feedback system presents a maximum overshoot $S = 8\%$ and a settling time $t_a = 18$ ms (fig. 11). Since these values are a little higher than the ones desired, we can vary the K_p and T_d values; anyway, we can obtain smaller values than the ones found above only of the maximum overshoot and not of the rise time.

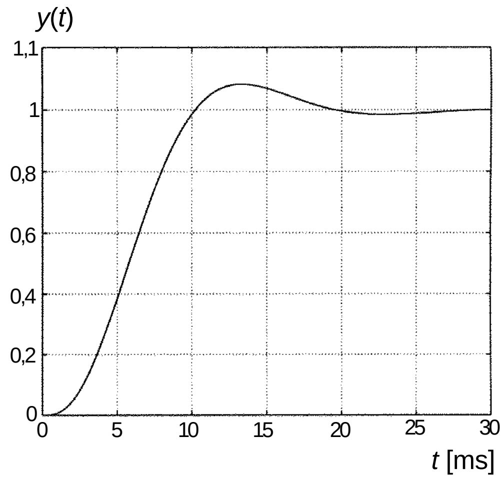

Fig. 11

In fact, as we can see from the figures of the Appendix mentioned above, all the p values which supply the maximum overshoot desired are higher than 1 and, therefore, we cannot obtain settling time values much lower than the one found above. Consequently, we can conclude that, in this situation it is impossible to satisfy completely the desired specifics by utilizing a PD regulator.

To conclude the paragraph, we can note that, sometimes, in addition to the plant output, we can measure some more variables which represent the integral and the derivative of the output itself and, therefore, we can implement the regulator by utilizing directly the measures of these variables. As we all know, the so called *tachometer-feedback control* in a position control system is an interesting example of these systems.

3.2 Design procedures by means of hybrid regulators

If, as it may happen, by utilizing a PD feedback regulator we cannot satisfy the desired specifics, we can employ the so called *hybrid* regulators. As we can see in figure 12, they are feedback regulators made up by two parts in parallel; the first is made of a proportional regulator and the second of a PD regulator whose input is the measure of the output derivative (namely, in the case of a position regulator, the velocity measure).

Consequently, considering (1), naming K_t the constant of the sensor of the output derivative, K_c the constant of the output sensor, K_{ac} the gain of the preamplifier of the latter sensor, the regulator transfer function is:

$$G_r(s) = K_p K_t (1 + T_d\, s)s + K_{ac} K_c \tag{40}$$

If the plant transfer function $G_s(s)$ is of the third order, type 1, we can write it in the form (32). Consequently, considering (40), the feedback system transfer function is given by:

$$\bar{G}(s) = \frac{1}{K_{ac}K_c} \frac{\dfrac{K_s K_{ac} K_c}{\tau_1 \tau_2}}{s^3 + As^2 + Bs + \dfrac{K_s K_{ac} K_c}{\tau_1 \tau_2}} \qquad (41)$$

where:

$$A = \frac{\tau_1 + \tau_2 + K_s K_t K_p T_d}{\tau_1 \tau_2} \qquad B = \frac{1 + K_s K_t K_p}{\tau_1 \tau_2}$$

As we can see, leaving aside $1/K_{ac}K_c$, (41) is a third order polynomial function and we can write it in the (26) form assuming:

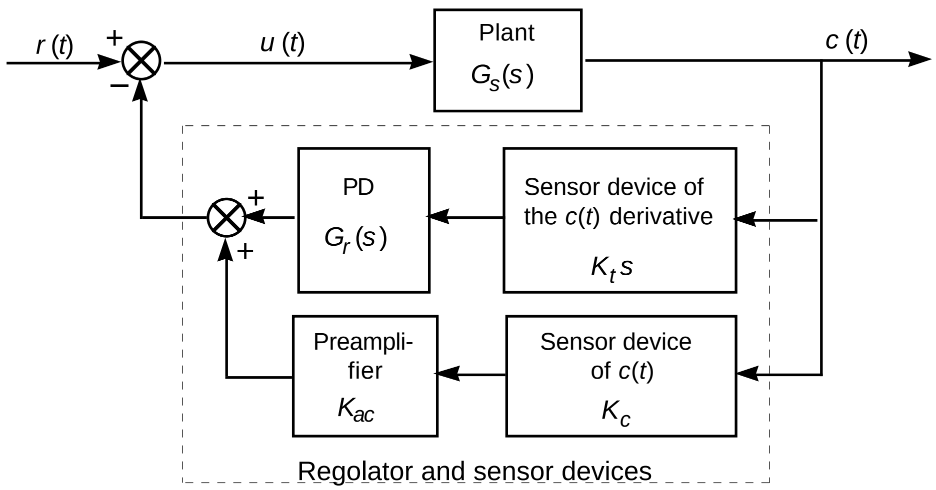

Fig. 12

$$\frac{K_s K_{ac} K_c}{\tau_1 \tau_2} = \omega_n^3\, p\delta \frac{1 + K_s K_t K_p}{\tau_1 \tau_2} = (1 + 2\delta^2 p)\omega_n^2 \qquad (42)$$

$$\frac{\tau_1 + \tau_2 + K_s K_t K_p T_d}{\tau_1 \tau_2} = (p + 2)\,\delta\omega_n$$

Consequently, we get:

$$K_{ac} = \frac{\omega_n^3 p \delta \tau_1 \tau_2}{K_s K_c} \qquad K_p = \frac{\tau_1 \tau_2 (1 + 2\delta^2 p)\omega_n^2 - 1}{K_s K_t} \qquad (43)$$

$$T_d = \frac{(p+2)\delta\omega_n\, \tau_1 \tau_2 - (\tau_1 + \tau_2)}{K_s K_t K_p}$$

and, therefore, once we have determined, by utilizing the figures of the Appendix mentioned above, any tern of δ, p e ω_n values which satisfy the desired specifics, from (43) we can calculate the K_{ac}, K_p and T_d values.

EXAMPLE

On the basis of a unitary feedback system whose plant has a transfer function given by (38) we want to achieve a system design, by means of a hybrid regulator, that will make the system output to a unit step input present a maximum overshoot $S \leq 5\%$ and a rise time $t_a \leq 12$ ms.

By applying the above mentioned design procedure we can obtain that, if the system plant is (38) and assuming:

$$K_p = 0{,}379 \quad T_d = 0{,}207 \quad K_{ac} = 0{,}224 \qquad (44)$$

the feedback system output, to a unit step input, presents a maximum overshoot S = 5% and a settling time t_a = 12 ms (fig. 13).

To conclude, we can note that, if the system plant is of the form (38), in order to satisfy the desired specifics, it is necessary to employ a derivative hybrid regulator, since, as we have seen above, it is impossible to obtain a satisfying system behavior by making use of a PD regulator.

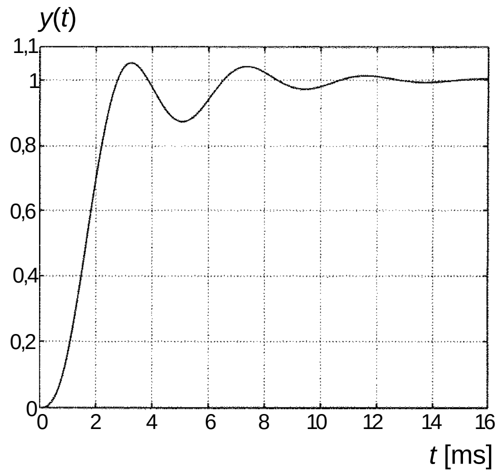

Fig.13

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