Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

ROBUST GAIN AND PHASE MARGINS FOR SYSTEMS WITH PARAMETRIC UNCERTAINTY

Nusret Tan and Ibrahim Kaya

Inonu University, Engineering Faculty,
Department of Electrical and Electronics Engineering,
44100, Malatya, Turkey.
ntan@inonu.edu.tr; ikaya@inonu.edu.tr

Abstract: Gain and phase margins are two important frequency domain specifications which are widely used for controller design. In this paper, a procedure is given for computing the robust gain and phase margins of systems with an uncertain transfer function by using the $2q$-convex parpolygonal value set of polynomials with affine linear uncertainty. An example is included to illustrate the benefit of the method presented.

Keywords: Uncertain systems; Nyquist envelope; Robust gain margin; Robust phase margin

1. INTRODUCTION

The subject of robust stability of control systems with parameter variations has been a focus of attention of researchers [1-4] in recent years following the publication of Kharitonov's theorem [5] on interval polynomials and the edge theorem [6] for affine polynomials. However, beyond stability, it is of interset to guarantee some measure of robust performance for systems with parameter uncertainties. In classical control theory, gain and phase margins are two important frequency domain performance measures widely used for controller designs. For systems with a nominal transfer function these margins are computed from the Nyquist or Bode plots of the open loop transfer function. However, in the case of systems with parametric uncertainties, the computation of the gain and phase margins becomes much more complicated. To overcome this difficulty, one needs to compute the frequency response of uncertain systems, such as the Nyquist or Bode envelopes, or to convert the problem to one of robust stability of uncertain polynomials.

In this paper the computation of the robust gain and phase margins of systems with uncertain transfer functions whose numerator and denominator polynomials are a polynomial family of the form

$$P(s,q) = a_0(q) + a_1(q)s + ... + a_n(q)s^n \quad (1)$$

is considered. Here the coefficients $a_i(q)$ depend linearly on $q = [p_1, p_2, ..., p_q]^T$ and the uncertainty box is $Q = \{q : p_i \in [\underline{p_i}, \overline{p_i}], i = 1, 2, ..., q\}$ where $\underline{p_i}$ and $\overline{p_i}$ are specified lower and upper bounds of the ith perturbation p_i, respectively. In other words, the system's transfer function is assumed to be

$$G(s,q,r) = \frac{N(s,r)}{D(s,q)} = \frac{b_0(r) + b_1(r)s + ... + b_m(r)s^m}{a_0(q) + a_1(q)s + ... + a_n(q)s^n} \quad (2)$$

where $q = [p_1, p_2, ..., p_q]^T$ and $r = [d_1, d_2, ..., d_r]^T$ and $Q = \{q : p_i \in [\underline{p_i}, \overline{p_i}], i = 1, 2, ..., q\}, R = \{r : d_i \in [\underline{d_i}, \overline{d_i}], i = 1, 2, ..., r\}$. In the literature, motivated by the Kharitonov and the edge theorems, several papers [7-13] have studied the computation of the frequency response and the robust gain and phase margins of control systems under parametric uncertainty. The majority of these papers deals with interval transfer functions, where the numerator and denominator polynomial coefficients change independently. However, in practical feedback system analysis and design problems the coefficients of the plant transfer function do not necessarily vary independently. For the dependent case, the edge theorem has been used, however, the main difficulty associated with the edge theorem is computational complexity which is due to the exponential growth of exposed edges with respect to the number of uncertain parameters. On the other hand, in [14-16], by using the $2q$-convex parpolygonal value set of a polynomial of the form of Eq.(1) (for each $s = j\omega$, the $2q$-convex parpolygon is defined as the outer edges of the image of exposed edges ($q2^{q-1}$ edges)), the Nyquist, Bode and Nichols envelopes, the controller synthesis and the absolute stability problem for systems with the transfer function of Eq.(2) were studied. The distinguishing features of the results of [14-16] and of this paper is to make use of the $2q$-convex parpolygonal value set which greatly reduces the computational complexity. Here, in particular, it is dealt with the computation of the extremal gain and phase margins for the systems with uncertain transfer function of Eq.(2). Under the assumption of no *transition frequency* [14], it is shown that these margins can be computed from the extremal subset of the family of Eq.(2) which is identified by using a single value of frequency within $(0, \infty)$. And using this result, the Astrom-Hagglund method is used for controller design.

The paper is organized as follows: In Section 2, the construction procedures of the $2q$-convex parpolygon and Nyquist envelope are given. The computation of the robust gain and phase margins for a control system with an uncertain transfer function of Eq.(2) is discussed in Section 3. In Section 4, an example is given to illustrate the benefit of the method presented. Section 5 includes concluding remarks.

2. CONSTRUCTION OF CONVEX PARPOLYGON AND NYQUIST ENVELOPE

For systems defined by a nominal transfer function, the gain and phase margins are computed from the Nyquist plot or the Bode plot of the open loop system. However, when the system parameters are subject to perturbations, one needs to construct the Nyquist or Bode envelopes in order to compute the worst case or the robust gain and phase margins. Therefore, in this section, we first investigate the construction of the $2q$-convex parpolygonal value set of the polynomial family given by Eq.(1). Then, using the $2q$-convex parpolygonal value set, the Nyquist envelope of a given transfer function of the form of Eq.(2) is discussed.

2.1 *Construction of 2q-convex parpolygon*

The corresponding polytope of a family of polynomials of Eq.(1) in the coefficient space has 2^q vertices and $q2^{q-1}$ exposed edges and it can be rewritten as

$$P(s,q) = f_0(s) + f_1(s)p_1 + ... + f_q(s)p_q, q \in Q \quad (3)$$

The 2^q vertex polynomials of the polytope of $P(s,q)$ can be written in the following pattern

$$\begin{aligned}
c_1(s,q) &= f_0(s) + f_1(s)\underline{p_1} + f_2(s)\underline{p_2} + ... + f_q(s)\underline{p_q} \\
c_2(s,q) &= f_0(s) + f_1(s)\overline{p_1} + f_2(s)\underline{p_2} + ... + f_q(s)\underline{p_q} \\
&\cdots \\
c_{2^q}(s,q) &= f_0(s) + f_1(s)\overline{p_1} + f_2(s)\overline{p_2} + ... + f_q(s)\overline{p_q}
\end{aligned} \quad (4)$$

The value set of Eq.(1) can be obtained by mapping the $q2^{q-1}$ exposed edges in the complex plane or taking the convex hull of complex plane images of the vertices of the parameter box for each $s = j\omega$ and the outer edges of the value set define a $2q$-convex parpolygon. The $q2^{q-1}$ edges in the complex plane can be divided into q groups where each group has the same number of edges, 2^{q-1} edges, [17]. All edges in group $i (i = 1, 2, ..., q)$ are parallel to each other with the same slope. Thus, knowing one edge from each group is sufficient to construct the $2q$-convex parpolygon. For example, let $e(c_i, c_j)$ denote the edge with end points c_i and c_j and for clarity of presentation consider Fig.1a which is the image of the exposed edges of a polytope with $q = 3$ parameters. It can be seen that the edges $e(c_1, c_2)$, $e(c_3, c_4)$, $e(c_5, c_6)$ and $e(c_7, c_8)$ are parallel to each other as shown in Fig.1a. Two of them which have the maximum and minimum intersections with the imaginary axis identify two edges of a $2q$-convex parpolygon as shown in Fig.1a and Fig.1b. Similarly, the other edges needed for construction of the $2q$-convex parpolygon can be identified. If there are vertical edges which have no intersection with the imaginary axis, in this case from the maximum and the minimum intersections with the real axis, the two required edges can be found. A general formula [17] for the intersection point of the edge line with the imaginary axis is

$$y^i = \frac{\omega}{E_i} \sum_{k=1}^{q} (O_k E_i - O_i E_k)(p_k - \underline{p_k}), \ k \neq i \quad (5)$$

and with the real axis is

$$x^i = \frac{1}{O_i}\sum_{k=1}^{q}(O_iE_k - O_kE_i)(p_k - \underline{p_k}),\ k{\neq}i \quad (6)$$

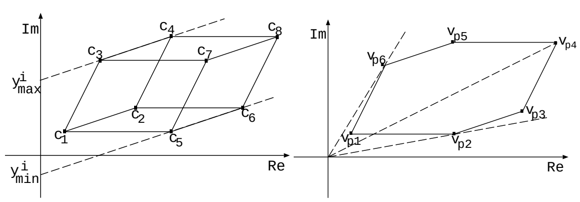
where $i = 1, 2, ..., q$, p_k takes either $\underline{p_k}$ or $\overline{p_k}$ depending on which edge it is associated with and E_i and O_i are the even and odd parts of $f_i(s)$. Further information about the value set of the uncertain polynomials and the construction of the $2q$-convex parpolygon can be found in [1] and in [17].

For different values of frequency, the edges of a $2q$-convex parpolygon may be different. The following theorem is given in order to divide the frequency axis, $\omega{\in}[0, \infty)$, into a finite number of intervals where in each interval the edges of the $2q$-convex parpolygon are the same. The proof of the following theorem can be found in [14, 17].

Theorem 1: For $i, j = 1, 2, ..., q$ and $i{\neq}j$, the positive real roots of

$$Re[f_i]Im[f_j] - Re[f_j]Im[f_i] = 0 \quad (7)$$

divide the frequency axis into finite intervals where in each interval the $2q$ edges of the $q2^{q-1}$ exposed edges which constitute the boundary of the convex parpolygon remain unchanged. The frequencies where the edges which constitute the boundary of the convex parpolygon may change will be referred to as transition frequencies.



(a) (b)

Fig. 1. a) Image of exposed edges b) $2q$-convex parpolygon

2.2 *Nyquist envelope*

Consider the transfer function given in Eq.(2) and let v_{n1}, v_{n2}, v_{n3},..., v_{n2r} and v_{d1}, v_{d2}, v_{d3},..., v_{d2q} be the vertices of the $2r$ and $2q$-convex parpolygons of $N(s, r)$ and $D(s, q)$ at $s = j\omega^*$ (see Fig.1.b), respectively. Then define the sets S_{N_V} and S_{N_E} which contain the vertices and the edges of the $2r$-convex parpolygon of $N(s, r)$ at $s = j\omega^*$ as

$$S_{N_V} = \{v_{n1}, ..., v_{n2r}\},\ S_{N_E} = \{e_{n1}, ..., e_{n2r}\} \quad (8)$$

similarly define S_{D_V} and S_{D_E} for the denominator as

$$S_{D_V} = \{v_{d1}, ..., v_{d2q}\},\ S_{D_E} = \{e_{d1}, ..., e_{d2q}\} \quad (9)$$

where $e_{n1}, ..., e_{n2r}$ and $e_{d1}, ..., e_{d2q}$ are the edges of $2r$ and $2q$-convex parpolygons, respectively. Then, it is shown in [15] that

Theorem 2: At $s = j\omega^*$,

$$\partial G(j\omega^*, q, r) {\subset} (\frac{S_{N_V}}{S_{D_E}} {\cup} \frac{S_{N_E}}{S_{D_V}}) \quad (10)$$

where ∂ denotes the boundary and S_{N_V}, S_{N_E}, S_{D_V} and S_{D_E} are defined in Eqs.(8-9). This means that the number of vertices of the corresponding polytopes of the numerator(denominator) polynomials, which need to be considered is reduced from $2^r(2^q)$ to $2r(2q)$ and the number of edges which need to be considered is reduced from $r2^{r-1}(q2^{q-1})$ to $2r(2q)$.

The results of Section 2.1 enable one to identify the edges which constitute a $2q$-convex parpolygon of a polytopic polynomial family of Eq.(1). Therefore, the advantage of theorem 2 is that we do not need to consider all the exposed edges and vertices of the corresponding polytopes of the numerator and denominator polynomials. For example, in order to find a Nyquist template of a transfer function of Eq.(2) with $r = 3$ and $q = 4$ uncertain parameters then using known results one needs to find the image of $7(2^6) = 448$ edges, however, from theorem 2, one needs to find the image of only 96 edges.

Now, assume that neither $N(s, r)$ nor $D(s, q)$ has any *transition frequency*. This assumption guarantees that the edges for a single frequency which constitute the $2r$ and $2q$-convex parpolygons remain unchanged for all $\omega{\in}(0, \infty)$. Under the assumption of no *transition frequency*, the following theorem is given in [16] for characterizing the boundary of the Nyquist envelope.

Theorem 3: Assume that neither $N(s, r)$ nor $D(s, q)$ has any *transition frequency*. Then,

$$\partial G(j\omega, q, r) {\subset} G_E(j\omega) = (\frac{S_{N_V}}{S_{D_E}} {\cup} \frac{S_{N_E}}{S_{D_V}}) \quad (11)$$

where ∂ denotes the boundary, G_E means extremal system and S_{N_V}, S_{N_E}, S_{D_V} and S_{D_E} are defined in Eqs.(8-9).

3. ROBUST GAIN AND PHASE MARGINS

As stated before, the gain and phase margins are two important frequency domain specifications. This section deals with the calculation of the robust gain and phase margins for systems with an uncertain transfer function of the form of Eq.(2) using the theory presented in the previous sections.

Suppose that a closed loop system with an uncertain plant of the form of Eq.(2) is stable then the robust gain margin is the largest value of the gain K greater than 1 for which the stability of $KG(s,q,r)$ is preserved and the robust phase margin is the largest value of phase θ for which the uncertain system with $e^{-j\theta}G(s,q,r)$ is robustly stable. Thus, the worst case gain margin K^* and phase margin θ^* can be stated as

$$K^* = inf_{G(s)\in G(s,q,r)} K_G$$
$$\theta^* = inf_{G(s)\in G(s,q,r)} \theta_G \qquad (12)$$

where K_G stands for gain margin of $G(s)$ and θ_G stands for phase margin of $G(s)$.

Using the following theorem, the values of K^* and θ^* can be computed from the extremal system, $G_E(s)$ of Eq.(11).

Theorem 4: Suppose a unity feedback system with $G(s,q,r)$ is stable and assume that neither $N(s,r)$ nor $D(s,q)$ has any *transition frequency* (if there is transition frequency, see Remark 1). Then, the robust gain and phase margins are

$$K^* = inf_{G(s)\in G_E(s)} K_G$$
$$\theta^* = inf_{G(s)\in G_E(s)} \theta_G \qquad (13)$$

where $G_E(s) = (S_{N_V}/S_{D_E})\cup(S_{N_E}/S_{D_V})$ and S_{N_V}, S_{N_E}, S_{D_V} and S_{D_E} are defined in Eqs.(8-9).

Proof: Let A and B be the two complex plane polygons with vertex sets S_{A_V} and S_{B_V}, and edge sets S_{A_E} and S_{B_E}, respectively. Then, from the complex plane geometry, the following is known

$$\partial(A+B)\subset(S_{A_E}+S_{B_V})\cup(S_{A_V}+S_{B_E}) \qquad (14)$$

Now, for the calculation of the gain margin, one needs to find the maximum value of K greater than 1 for which

$$\Lambda(s) = KN(s,r) + D(s,q) \qquad (15)$$

is Hurwitz stable. From theorem 1, it is clear that if there is no *transition frequency* then the identified edges which constitute the $2r$ and $2q$-convex parpolygons for a single frequency remain unchanged for all $\omega\in(0,\infty)$. The multiplication of a $2r$-convex parpolygon with a fixed K is still a $2r$-convex parpolygon. Thus, for a fixed value of K, one can write

$$S_{A_V} = KS_{N_V},\ S_{B_V} = S_{D_V}$$

and

$$S_{A_E} = KS_{N_E},\ S_{B_E} = S_{D_E}$$

and from Eq.(14), the following equation can be written

$$\Lambda(j\omega)\subset\Lambda_E(j\omega) = (KS_{N_E}+S_{D_V})\cup(KS_{N_V}+S_{D_E}) \qquad (16)$$

Therefore, the stability of $\Lambda_E(s)$ implies the stability of $\Lambda(s)$. For the phase margin calculation, the gain K of Eq.(15) will be a complex gain such as $K = e^{-j\theta} = cos(\theta) - jsin(\theta)$ and the same proof will be valid. □

The following procedure is given for computing the robust gain and phase margins:

1) Rewrite $N(s,r)$ and $D(s,q)$ in the form of Eq.(3).

2) From Eq.(7), find that there is no any transition frequency for both $N(s,r)$ and $D(s,q)$ (if there is any see Remark 1).

3) Choose an arbitrary value of frequency within $(0,\infty)$ and by using Eqs.(5) and (6), identify the $2r$ and $2q$-convex parpolygons edges.

4) From Eqs.(8-9), find vertex and edge sets ($S_{N_V}, S_{N_E}, S_{D_V}$ and S_{D_E}) and thus obtain the extremal system $G_E(s)$ of Eq.(11).

5) Use theorem 4 compute the robust gain and phase margins.

Remark 1:

a) For clarity of presentation, theorem 4 is given for the no *transition frequency* case. If there is a *transition frequency* then the Nyquist envelope can be obtained by using theorem 2. Thus, the result of theorem 4 can be reformulated for this case. However, the $G_E(s)$ of Eq.(11) may be different for different frequency intervals.

b) The result of theorem 4 can be extended to the feedback system with a fixed controller, $C(s)$, and an uncertain plant $G(s,q,r)$. In this case, the extremal system will be $C(s)G_E(s)$ [15-16].

4. EXAMPLE

Consider

$$G(s,q,r) = \frac{N(s,r)}{D(s,q)} = \frac{d_1}{p_5s^4+p_4s^3+(p_2+p_3)s^2+(p_1+0.5p_2+p_3)s+p_1}$$

where $d_1\in[0.9, 1.1]$, $p_1\in[0.965, 1.035]$, $p_2\in[0.59, 0.73]$, $p_3\in[0.5, 0.65]$, $p_4\in[0.33, 0.41]$ and $p_5\in[0.02, 0.072]$. The aim is to find the parameters of the PID controller of the form

$$C(s) = K_p(1 + sT_d + \frac{1}{sT_i}) \qquad (17)$$

for which the phase margin of the system is about $\varphi_m = 45°$ using the Astrom-Hagglund method [18]. The Astrom-Hagglund controller tuning method is based on the idea that a point on the Nyquist plot of a given transfer function can be moved to a selected point in the complex plane by choosing suitable controller parameters. Such an appropriate point for tuning is the intersection of the Nyquist curve with the negative real axis which is traditionally described as the *critical point.* However, for an uncertain plant, there are many Nyquist curves which cross the negative real axis or for a fixed frequency there are many points in the complex plane. Therefore, it is necessary to move the worst case point of the Nyquist envelope to a selected position in the complex plane. In order to do this, one needs to use the worst case specifications.

Now, since $N(s,r) = d_1$, for all $\omega \in (0,\infty)$ the sets S_{N_V} and S_{N_E} are

$$S_{N_V} = \{0.9, 1.1\} \ and \ S_{N_E} = \{0.9 + 0.2\lambda\} \quad (18)$$

where $\lambda \in [0,1]$. Using theorem 1 it was computed that there is no any *transition frequency* for $D(s,q)$. Thus, one single value of frequency within $\omega \in (0,\infty)$ is sufficient to identify the edges of the $2q$-convex parpolygons of $D(s,q)$. We found that the edges, $e(c_7,c_8)$, $e(c_{25},c_{26})$, $e(c_{22},c_{24})$, $e(c_9,c_{11})$, $e(c_{18},c_{22})$, $e(c_{11},c_{15})$, $e(c_{18},c_{26})$, $e(c_7, c_{15})$, $e(c_8,c_{24})$ and $e(c_9,c_{25})$ (here, c_7, c_8, c_9, ... are the vertex polynomials of the polytope of $D(s,q)$ which constitute the edges of a $2q$-convex parpolygon and they can be obtained by using Eqs.(5-6)), constitute the boundary of the $2q$-convex parpolygons of $D(s,q)$. Thus for all $\omega \in (0,\infty)$, the sets S_{D_V} and S_{D_E} can be obtained from Eq.(9).

The 30 $2q$-convex parpolygons of $D(s,q)$ for $\omega \in [0,3]$ are shown in Fig. 2 and Fig. 3 shows the Nyquist template of extremal system, $G_E(s)$, at $\omega = 1.5 rad/sec$. It is clear that the extremal system has 30 systems with one unknown parameter $\lambda \in [0,1]$. On the other hand, using edge theorem, it can be seen that the extremal system has 192 system each of which has one unknown parameters namely $\lambda \in [0,1]$. So, the computational gain for this example is about 85%.

The worst case gain margin of $G(s,q,r)$ is $2.17(6.73db)$ and achieved at

$$G(s) = \frac{1.1}{0.072s^4 + 0.41s^3 + 1.09s^2 + 1.76s + 0.965}$$

and the pahse margin is $124.5°$ and achieved at

$$G(s) = \frac{1.1}{0.02s^4 + 0.41s^3 + 1.23s^2 + 1.83s + 0.965}$$

Using the Astrom-Hagglund method, we found the values of the parameters of a PID controller to be

$$K_p = 1.53, \ T_d = 0.58 \ and \ T_i = 2.33$$

Thus , the designed PID controller is

$$C(s) = \frac{2.07s^2 + 3.56s + 1.53}{2.33s} \quad (19)$$

The Nyquist envelopes of $G(s,q,r)$ and $C(s)G(s,q,r)$ are shown in Fig.4. From $C(s)G_E(s)$, it was found that the robust gain margin of $C(s)G(s,q,r)$ is $1.6(4.1db)$ and the phase margin is $45°$.

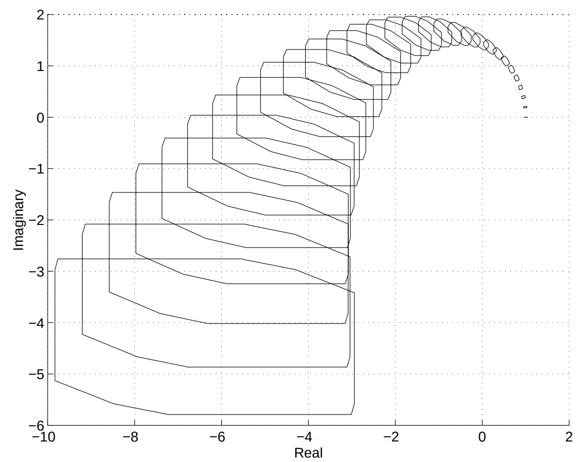


Fig. 2. $2q$-convex parpolygons of $D(s,q)$

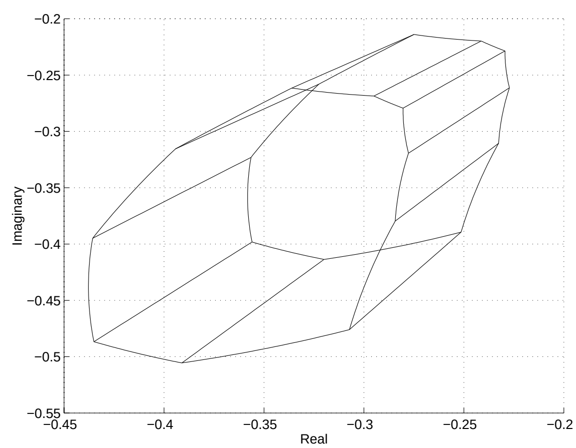


Fig. 3. The Nyquist template of $G(s,q,r)$ at $\omega = 1.5rad/sec$

5. CONCLUSION

In this paper, the problem of robust gain and phase margins for systems with parametric uncertainties defined by Eq.(2) has been studied. A novel feature of the present approach is the use of the $2q$-convex parpolygonal value set and theorem 1, to obtain an extremal subset of the uncertain family which characterize the boundary

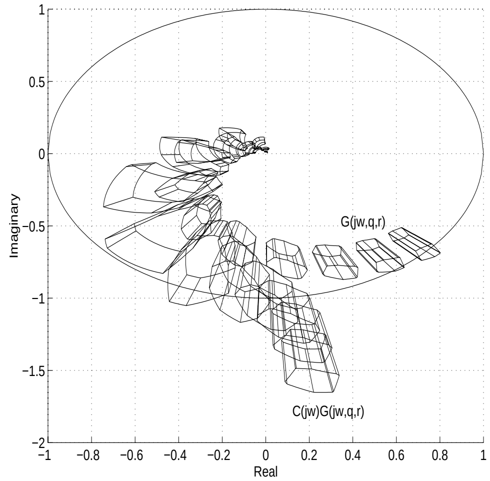


Fig. 4. Nyquist envelopes of $G(s,q,r)$ and $C(s)G(s,q,r)$

of the Nyquist envelope. Thus, the result given in the paper reduces the computational burden which occurs using the edge theorem greatly. The example given clearly shows the benefit of the method presented from a computational point of view.

6. REFERENCES

[1] Barmish, B. R. *New Tools for Robustness of Linear Systems.* MacMillan, NY, 1994.

[2] Bhattacharyya, S. P., Chapellat, H. and Keel, L. H., *Robust Control: The Parametric Approach.* Prentice Hall, 1995.

[3] Ackermann, J. *Robust Control: Systems with Uncertain Physical Parameters.* Springer-Verlag, 1993.

[4] Djaferis, T. E., *Robust Control Design: A Polynomial Approach.* Kluwer Academic Publishers, Boston, 1995.

[5] Kharitonov, V. L., "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differential Equations*, Vol. 14, pp. 1483-1485, 1979.

[6] Bartlett, A. C., Hollot, C. V. and Lin H., "Root location of an entire polytope of polynomials: it suffices to check the edges," *Mathematics of controls, Signals and Systems*, Vol. 1. pp. 61-71, 1988.

[7] Hollot, C. V. and Tempo, R., "On the Nyquist envelope of an interval plant family," *IEEE Trans. Automat. Contr.*, Vol. 39, No. 2, pp. 391-396, 1994.

[8] Keel, L. H. and Bhattacharyya, S. P., "Control system design for parametric uncertainty," *Int. J. Robust and Nonlinear Contr.*, Vol. 4, pp. 87-100, 1994.

[9] Argoun, M. B. and Bayoumi, M. M., "Robust gain and phase margins for interval uncertain systems," *1993 Canadian Conf. on Electrical and Computers Engineering*, Vol. 1, pp. 349-352, 1993.

[10] Tan, N. and Atherton, D. P., "AISTK-A software package for the analysis of interval systems," *IEE Colloquim: Robust Control-Theory, Software and Application*, London UK, Digest No:97/380, pp.4/1-4/7, 1997.

[11] Fu, M., "Computing the frequency response of linear systems with parametric perturbations," *Syst. Contr. Lett.*, Vol. 15, pp. 45-52, 1990.

[12] Bartlett, A. C., "Nyquist, Bode and Nichols plots of uncertain systems," *Proc. Amer. Contr. Conf.*, pp. 2033-2036, San Diego, CA, 1990.

[13] Bartlett, A. C., "Computation of frequency response of systems with uncertain parameters: a simplification," *Int. J. Contr.*, Vol. 57, No. 6, pp. 1293-1309, 1993.

[14] Tan, N. and Atherton, D. P., "Magnitude and phase envelopes of systems with affine linear uncertainty," *International Conf. on Control'98, UKACC*, Swansea UK, pp.1039-1044, 1-4 Sept. 1998.

[15] Tan, N. and Atherton, D. P., "Controller synthesis technique for systems with affine linear uncertainty," *14th World Congress of IFAC*, Vol. G, pp. 133-138, July 5-9, 1999.

[16] Tan, N. and Atherton, D. P., "Absolute stability problem of systems with parametric uncertainties," *European Control Conference*, Karlsruhe, Germany, 31. August-3.September, 1999.

[17] Shaw, J. and Jayasuriya, S., "A new algorithm for testing the stability of a polytope: a geometric approach for simplification," *Journal of Dynamic, Sys., Meas., and Contr.*, Vol. 118, pp. 611-614, 1996.

[18] Astrom, K. J. and Hagglund, T., "Automatic tuning of simple regulators with specification on phase and gain margins," *Automatica*, vol. 20, No. 5, pp. 645-651, 1984.