

Integrated Design of Robust Internal-Loop Compensator and Synchronizing Motion Controller for Twin-Servo Systems

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INTEGRATED DESIGN OF ROBUST INTERNAL-LOOP COMPENSATOR AND SYNCHRONIZING MOTION CONTROLLER FOR TWIN-SERVO SYSTEMS

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Abstract. In this paper, we propose an integrated design method of robust internal-loop compensator and synchronizing motion controller to cancel out skew motion of twin-servo system caused by different dynamic characteristics of two driving systems. And also, we focus on the modeling of the twin-servo system and propose its network representation. The proposed control method consists of separate feedback motion controller of each driving system and skew motion compensating controller between two systems. Robust tracking controller based on robust internal-loop compensator is proposed as a separate motion controller and its disturbance attenuation property is shown. Skew motion compensation scheme is also designed to maintain the synchronizing motion during high speed operation, and the stability of the whole closed loop system is proved based on passivity theory. Finally, experimental results are shown to verify control performance.

Key Words. Twin-servo system, network representation, synchronizing motion control, robust internal-loop compensator.

1 INTRODUCTION

High precision control systems emphasizing high performance and high productivity have introduced twin-servo mechanism in many current application areas. Twin-servo mechanism is used to increase the payload capacity and speed of high precision system [4]. This consists of two driving motors controlled independently for one reference input. Difficulties are mainly due to the fact that the system of interest requires wide range and high speed motions under significant nonlinear characteristics. Moreover, precise description of the nonlinear effect is not available because uncertainties always exist and cannot be neglected. Consequently, the control algorithm for twin-servo with high performance must address both synchronizing motion control performance under the dynamic unbalance of twin-servo system and robustness issue under the nonlinearities and uncertainties.

In this paper, we propose an integrated design method of robust internal-loop compensator (RIC) and synchronizing motion controller, which con-

sists of separate feedback controller and skew motion compensator to meet performance specifications and cancel out the skew motion of two driving systems. We also propose the modeling of twin-servo system using network representation. Trajectory tracking controller based on RIC is proposed as the separate feedback controller. Using the equivalent structure of RIC with Q function, we show the disturbance attenuation property of the proposed separate controller in the frequency domain. To compensate the skew motion, a symmetric type skew motion compensating controller is designed. The stability of the whole closed loop system is analyzed based on passivity based approach. In the next section, we propose a separate RIC based tracking controller for each driving system. In Section 3, modeling of twin-servo system using network representation is presented. In Section 4, the skew motion compensating controller of twin-servo system is proposed and the stability analysis of the whole closed loop system is represented. Experimental results are shown in Section 5, and conclusion follows.

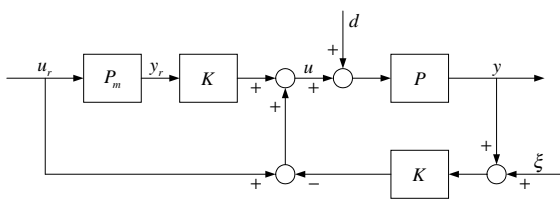


Fig. 1: Robust internal-loop compensator

2 SEPARATE FEEDBACK CONTROL

2.1 Robust Internal-Loop Compensator

Fig. 1 shows the form in which we will design RIC (robust internal-loop compensator). The plant is represented by the transfer function P and its output signal y . The function u_r represents a reference control input signal, u represents a control input signal, d represents a disturbance signal, and ξ represents measurement error. P_m and K represent dynamic systems which are to be designed. We refer to these as the reference or nominal model and the controller of RIC. Laplace variable s is dropped for clarity. Therefore, the control input for SISO (single-input, single-output) system has the form of

$$u = u_r + K(y_r - y) \quad (1)$$

where y_r is output of reference model P_m . In this paper, the structure with the control input of (1) is defined as RIC.

From the block diagram in Fig. 1, the input-output relationship from reference control input u_r , external disturbance d , and measurement error ξ to the plant output can be expressed as

$$y = G_{u_r y} u_r + G_{d y} d + G_{\xi y} \xi \quad (2)$$

where

$$\begin{aligned} G_{u_r y} &= \frac{P(1 + P_m K)}{1 + PK} \\ G_{d y} &= \frac{P}{1 + PK} \\ G_{\xi y} &= -\frac{PK}{1 + PK}. \end{aligned} \quad (3)$$

The difference between plant output and reference model output is defined as model following error:

$$e_r = y_r - y. \quad (4)$$

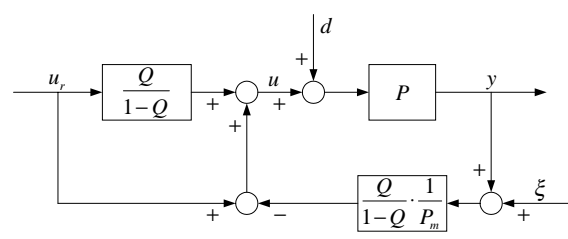
Rewriting (4) with (2), it has the form of

$$e_r = S(P_m - P)u_r - SPd + T\xi \quad (5)$$

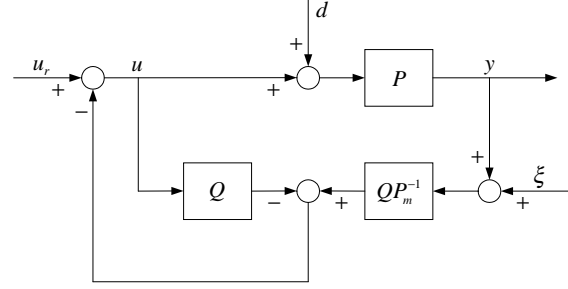
where

$$S = \frac{1}{1 + PK}, \quad T = \frac{PK}{1 + PK}. \quad (6)$$

Note that these represents the sensitivity and complementary sensitivity functions of the typical feedback system.



(a) Equivalent structure of RIC using Q



(b) Disturbance observer

Fig. 2: Equivalent structure of RIC

Consider an imaginary transfer function Q , which controls P_m using feedback controller K . Hence Q is expressed as

$$Q = \frac{P_m K}{1 + P_m K}. \quad (7)$$

Recalculating this for K , it has the form of

$$K = \frac{Q}{P_m(1 - Q)}. \quad (8)$$

If we substitute K into Fig. 1, we obtain Fig. 2(a), which is an equivalent structure of RIC. This figure can be also transformed equivalently to Fig. 2(b) which is a well known structure of DOB (disturbance observer) [3]. This means that if we select K as (8), the RIC becomes DOB and the characteristics will be the same. Let's investigate the properties in detail. It is well known that DOB makes a system robust using Q which cuts off the disturbance in low frequency region. Rewriting (3) with the controller (8), it has the form of

$$\begin{aligned} G_{u_r y} &= \frac{PP_m}{P_m + (P - P_m)Q} \\ G_{d y} &= \frac{PP_m(1 - Q)}{P_m + (P - P_m)Q} \\ G_{\xi y} &= -\frac{PQ}{P_m + (P - P_m)Q}. \end{aligned} \quad (9)$$

Below the cutoff frequency region of Q , $Q \approx 1$ is achieved. Hence $G_{d y} \approx 0$ is obtained in (9). This indicates that low frequency disturbances are attenuated and mismatch between plant and reference model is compensated in the low frequency region. This provides the robustness of RIC through this gain selection, the characteristics of RIC and

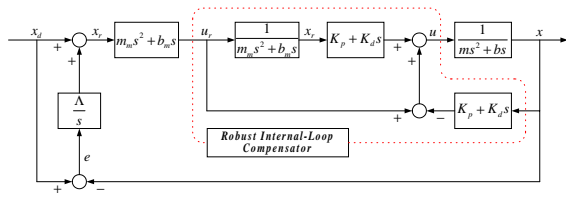


Fig. 3: RIC based tracking control structure

DOB will be the same. Consequently, unlike the empirical selection procedure for Q in DOB design, RIC has a way to design this Q .

2.2 RIC Based Tracking Control

Consider the dynamics of the primary or secondary system actuated by a separate feedback controller without skew error feedback:

$$m\ddot{x} + b\dot{x} = f \quad (10)$$

where f is a separate feedback control input to follow the desired trajectory. Tracking error is defined as

$$e = x_d - x \quad (11)$$

where x_d is a desired trajectory. We assume that x_d , its first, and second derivatives are all bounded as function of time.

From (1), the proposed RIC based feedback controller is formulated as

$$f = f_r + K(x_r - x) \quad (12)$$

where f_r is reference control input designed as

$$f_r = m_m\ddot{x}_r + b_m\dot{x}_r. \quad (13)$$

This control input is used to generate internal model state. Since m_m and b_m are designed values, x_r becomes the state of implicit internal model of (13).

Now we want to design a model following controller for the system given in (10). To begin, we select the reference trajectory:

$$x_r = x_d + \Lambda \int_0^t e \, dt \quad (14)$$

where Λ is an appropriate gain. The difference between the state of internal model (13) and the state of system (10) is given by

$$x_r - x = e + \Lambda \int_0^t e \, dt \triangleq e_r. \quad (15)$$

Therefore, these formulations lead to the following dynamic controller based on RIC as shown in Fig. 3,

$$f = m_m\ddot{x}_r + b_m\dot{x}_r + K_p e_r + K_d \dot{e}_r. \quad (16)$$

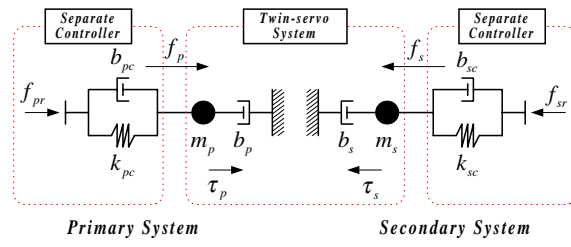


Fig. 4: Twin-servo motion control system

Remark 1 From (12) and (16), reference model and controller of RIC are given by

$$P_m = \frac{1}{m_ms^2 + b_ms}, \quad K = K_p + K_ds. \quad (17)$$

Using this equation and (7), we can get the following DOB filter Q :

$$Q = \frac{K_ds + K_p}{J_ms^2 + (B_m + K_d)s + K_p}. \quad (18)$$

Remark 2 The proposed separate dynamic controller can have the form of

$$f_f - f = b_c\dot{x} + k_c x \quad (19)$$

where f_f is feedforward command, b_c and k_c are viscous coefficient and stiffness of the separate dynamic controller, respectively, which are expressed as

$$\begin{aligned} f_f &= m_m(\ddot{x}_d + \Lambda\dot{x}_d) + (b_m + K_r)(\dot{x}_d + \Lambda x_d) \\ b_c &= m_m\Lambda + K_r \\ k_c &= (b_m + K_r)\Lambda. \end{aligned} \quad (20)$$

3 TWIN-SERVO SYSTEM

3.1 Modeling of Twin-Servo System

A twin-servo system consists of the primary and secondary servo system with control loop closed separately around them as shown in Fig. 4. The dynamic equations of two systems are expressed as

$$\begin{aligned} m_p\ddot{x}_p + b_p\dot{x}_p &= \tau_p + f_p \\ m_s\ddot{x}_s + b_s\dot{x}_s &= \tau_s + f_s \end{aligned} \quad (21)$$

where x_p and x_s denote the displacement of the primary and secondary motor, respectively. And m_p and b_p represent mass and viscous coefficient of the primary motor, respectively, whereas m_s and b_s are those of secondary motor. f_p denotes the force that separate feedback controller applies to the primary motor, and f_s denotes the force that separate feedback controller applies to the secondary motor. Driving forces for synchronizing motion of primary and secondary motor are represented by τ_p and τ_s , respectively.

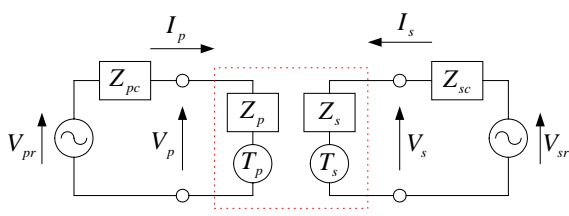


Fig. 5: Circuit representation of twin-servo system

Table 1: Correspondence between mathematical modeling and circuit representation

Mathematical modeling		Circuit representation
\dot{x}_p, \dot{x}_s	\longleftrightarrow	I_p, I_s
f_{pr}, f_{sr}	\longleftrightarrow	V_{pr}, V_{sr}
f_p, f_s	\longleftrightarrow	V_p, V_s
τ_p, τ_s	\longleftrightarrow	T_p, T_s

3.2 Skew Motion Compensating Control

Synchronizing motion controller is used to synchronize the motion of two motors by cancelling out the skew motion. Hence, this has to recognize skew motion in real time and compensate dynamic difference during high-speed motion. The separate robust feedback controllers compensate different dynamic characteristics of two motors and the skew motion compensating controller is appended to this.

Consider the following control schemes as general expressions which determine compensating forces to synchronize motions:

$$\begin{aligned}\tau_p &= \left(K_{pp}^p + K_{pp}^v \frac{d}{dt} \right) x_p - \left(K_{ps}^p + K_{ps}^v \frac{d}{dt} \right) x_s \\ \tau_s &= \left(K_{sp}^p + K_{sp}^v \frac{d}{dt} \right) x_p - \left(K_{ss}^p + K_{ss}^v \frac{d}{dt} \right) x_s\end{aligned}\quad (22)$$

where $K_{pp}^{(p,v)}$ and $K_{ps}^{(p,v)}$ are feedback gains of the primary motor's position and velocity, whereas $K_{sp}^{(p,v)}$ and $K_{ss}^{(p,v)}$ are gains of the secondary motor, respectively.

3.3 Network Representation

Let's consider a two-terminal-pair network which is connected to a power source at each terminal pair as shown in Fig. 5. By regarding the power source as a reference command and two-terminal-pair network as a twin-servo system, the whole system can be replaced by the electrical circuit in Fig. 5. The correspondence between the mathematical modeling and the circuit representation in this figure is shown in Table 1.

(21) and (22) can be transformed from time do-

main into s domain:

$$\begin{aligned}T_p + V_p &= (m_p s + b_p) I_p \triangleq Z_p I_p \\ T_s + V_s &= (m_s s + b_s) I_s \triangleq Z_s I_s\end{aligned}\quad (23)$$

$$\begin{aligned}T_p &= \left(K_{pp}^v + K_{pp}^p \frac{1}{s} \right) I_p - \left(K_{ps}^v + K_{ps}^p \frac{1}{s} \right) I_s \\ &\triangleq P_p I_p - R_p I_s \\ T_s &= \left(K_{sp}^v + K_{sp}^p \frac{1}{s} \right) I_p - \left(K_{ss}^v + K_{ss}^p \frac{1}{s} \right) I_s \\ &\triangleq P_s I_p - R_s I_s.\end{aligned}\quad (24)$$

By eliminating T_p and T_s from (23) and (24), the impedance matrix of the twin-servo system is obtained as

$$\mathbf{Z} = \begin{bmatrix} Z_p - P_p & R_p \\ -P_s & Z_s + R_s \end{bmatrix}. \quad (25)$$

4 STABILITY ANALYSIS

4.1 Passivity Based Approach

The motion of primary motor is affected by two control inputs: f_p , the separate control command of the primary and τ_p , the skew motion compensating command of twin-servo mechanism. Control inputs of secondary motor are similar to those of the primary. Since the primary and secondary motor are interconnected in a feedback loop, the dynamics of the whole closed loop system should be considered.

From electric circuit representation of Section 3, the twin-servo system can be expressed as

$$\mathbf{b} = \mathbf{S} \mathbf{a} \quad (26)$$

where the matrix \mathbf{S} is called scattering matrix, \mathbf{a} and \mathbf{b} are input and output wave defined as

$$\begin{aligned}\mathbf{a} &= [a_1, a_2]^T \triangleq \frac{\mathbf{V} + \mathbf{I}}{2} \\ \mathbf{b} &= [b_1, b_2]^T \triangleq \frac{\mathbf{V} - \mathbf{I}}{2}\end{aligned}\quad (27)$$

where $\mathbf{V} = [V_p, V_s]^T$ and $\mathbf{I} = [I_p, I_s]^T$. The scattering matrix \mathbf{S} of the system is given by

$$\begin{aligned}\mathbf{S} &= \frac{1}{D + z_{11} + z_{22} + 1} \\ &\times \begin{bmatrix} D + z_{11} - z_{22} - 1 & 2z_{12} \\ 2z_{21} & D - z_{11} + z_{22} - 1 \end{bmatrix}\end{aligned}\quad (28)$$

where

$$D \triangleq |\mathbf{Z}| = (Z_p - P_p)(Z_s + R_s) + P_s R_p. \quad (29)$$

The system is passive if the following inequality is satisfied,

$$\begin{aligned}\|\mathbf{S}\|_\infty &= \bar{\sigma}(\mathbf{S}(j\omega)) \\ &= \sup_\omega \lambda^{1/2}(\mathbf{S}(j\omega)^* \mathbf{S}(j\omega)) \leq 1.\end{aligned}\quad (30)$$

Therefore, if the system is reciprocal, that is, \mathbf{S} is symmetric, we can analyze the stability of the system using (30) [1].

4.2 Stability Analysis

In this paper, skew motion compensating scheme is chosen as a symmetric type PD (proportional and derivative) control by which one motor follows the position of the other. Therefore the control algorithm is expressed as

$$\begin{aligned}\tau_p &= K_p(x_s - x_p) + K_d(\dot{x}_s - \dot{x}_p) \\ \tau_s &= K_p(x_p - x_s) + K_d(\dot{x}_p - \dot{x}_s)\end{aligned}\quad (31)$$

where K_p and K_d are PD gains, respectively. From (18), the dynamic characteristic can be assigned so that the primary and the secondary system have equivalent parameters in the low frequency range. Hence, we assume that the dynamic equations of the two systems are represented as (17). The scattering matrix is symmetric when the system is reciprocal, so that it is easier to analyze stability. Substituting the parameter of (31) into (28), we get

$$\begin{aligned}\mathbf{S} &= \frac{1}{(1 + \alpha)(1 + \alpha + 2\beta)} \\ &\times \begin{bmatrix} \alpha(\alpha + 2\beta) - 1 & -2\beta \\ -2\beta & \alpha(\alpha + 2\beta) - 1 \end{bmatrix}\end{aligned}\quad (32)$$

where

$$\alpha = m_m s + b_m, \quad \beta = \frac{k_p}{s} + k_d. \quad (33)$$

Therefore, the singular values of \mathbf{S} are given by

$$\sigma_1 = \frac{|\alpha - 1|}{|\alpha + 1|} \leq 1, \quad \sigma_2 = \frac{|\alpha + 2\beta - 1|}{|\alpha + 2\beta + 1|} \leq 1. \quad (34)$$

Both of them never violate the inequality (30). Therefore, the stability of twin-servo system when the proposed control algorithm is applied has been guaranteed.

5 EXPERIMENTAL RESULTS

The system we are dealing with in this paper is the twin-servo precision linear motor system shown in Fig. 6, which has no physical connection between the primary and the secondary motor. Fig. 7 shows the hardware configuration of the twin-servo control test bed. The host computer computes the control input every 1 msec. Then, the control input is converted by 12-bit D/A converter and applied to linear servo motor (Anorad, LEB-S-2-S-NC) through DC servo amplifier (Anorad, AM-4-BL-MS-S). The position measured by linear encoders which have $2\mu\text{m}$ resolution for primary motor and $5\mu\text{m}$ resolution for secondary motor, is fed back through counter board to main computer.

Dynamic equations of the twin-servo system can be simply expressed as second order differential equations:

$$\begin{aligned}0.55\ddot{x}_p + 0.4\dot{x}_p &= \tau_p + f_p \\ 0.45\ddot{x}_s + 0.3\dot{x}_s &= \tau_s + f_s.\end{aligned}\quad (35)$$



Fig. 6: Twin-servo precision linear motor system

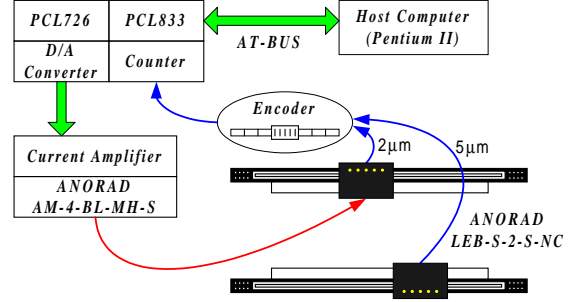


Fig. 7: Hardware structure of twin-servo system

The proposed separate dynamic controller in (16) is used to stabilize the whole system and track the desired position accurately,

$$f_{(p,s)} = m_m(\ddot{x}_d + \Lambda_d \dot{e} + \Lambda_p e) + K e_r \quad (36)$$

where m_m is 0.5, Λ_d is 100, Λ_p is 3000, and K is designed based on \mathcal{H}_∞ mixed sensitivity method [2], which is expressed as

$$K = \frac{(9.4550 \times 10^9)s + (3.0231 \times 10^{11})}{s^2 + (2.7659 \times 10^5)s + (6.0826 \times 10^7)}. \quad (37)$$

The skew motion compensator is selected as a symmetric type PD controller in (31):

$$\begin{aligned}\tau_p &= K_p e_{skew} + K_d \dot{e}_{skew} \\ \tau_s &= -K_p e_{skew} - K_d \dot{e}_{skew}\end{aligned}\quad (38)$$

where the skew error $e_{skew} \triangleq x_s - x_p$ and the gains are selected as $K_p = 50$ and $K_d = 15$.

Fig. 8 shows the robust synchronizing motion control structure. The 5th order polynomial function is used to specify the position, velocity, and acceleration at the beginning and end of path. The target position is 30 mm. Control sampling frequency is 1000 Hz and all controllers are discretized by using the bilinear transformation.

Fig. 9 shows the experimental results. Fig. 9 (a), (b), (c), and (d), respectively show the position responses, the control inputs, the tracking errors, and the skew error between the primary and the secondary systems. As can be seen here, the tracking errors show good performance within about $\pm 40\mu\text{m}$ and the maximum skew error is within about $\pm 20\mu\text{m}$.

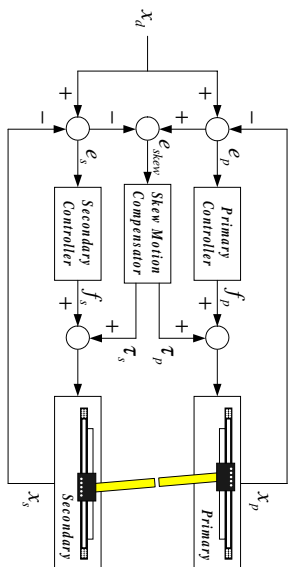


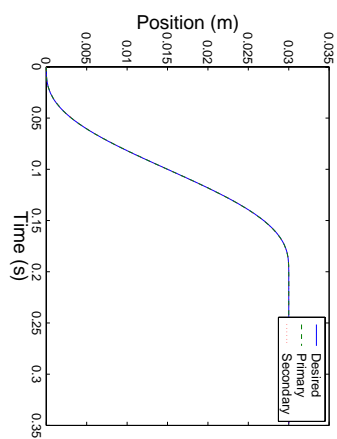
Fig. 8: Synchronizing motion control structure

6 CONCLUSIONS

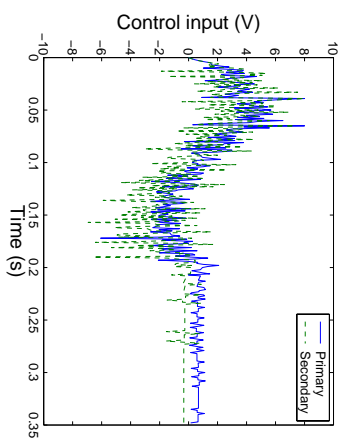
We proposed a modeling and network representation of twin-servo system including robust synchronizing motion control algorithm which consists of separate feedback controller and skew motion compensator. Trajectory tracking controller based on robust internal-loop compensator is proposed as the separate feedback controller and symmetric type PD controller which makes the system reciprocal is designed as the skew motion compensator. The stability analysis of the proposed synchronizing motion control algorithm is shown based on passivity based approach. The effectiveness of the proposed algorithm is verified through trajectory tracking experiment and the results show excellent performance under various nonlinear friction characteristics for twin-servo brushless DC linear motor system used in assembly automation line of semiconductor devices.

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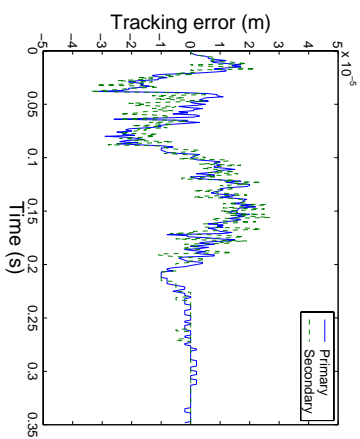
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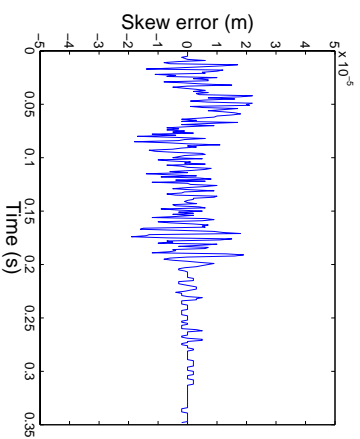
(a) Position



(b) Control input



(c) Position error



(d) Skew error

Fig. 9: Experimental results of twin-servo system