Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

CONSTRAINED OPTIMAL CONTROL FOR MULTI-STAGE HYBRID MANUFACTURING SYSTEM MODELS[1]

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Abstract. We consider a multi-stage manufacturing process where each job has a physical state characterized by time-driven dynamics and a temporal state by event-driven dynamics, thus giving rise to a hybrid system model. A common problem is to derive an optimal control strategy to trade off the conflicting objectives of minimizing job completion times (satisfaction of customer demand) against the quality of the completed jobs. Extending our past work, in this paper we derive necessary conditions for optimality for a multi-stage system where the control inputs are bounded. The issue of nondifferentiability due to the nature of the event-driven state dynamics creates serious analytical difficulties which we can no longer effectively resolve using nonsmooth optimization methods. Instead, we establish some properties of the optimal control sequence that have interesting implications in terms of designing control policies for this class of hybrid systems.

Key Words. Hybrid Systems, Optimal Control

1 Introduction

The term "hybrid" is used to characterize systems that combine time-driven and event-driven dynamics. The former are represented by differential (or difference) equations, while the latter may be described through various frameworks used for Discrete Event Systems (DES), such as timed automata, max-plus equations, or Petri nets (see [3]). Broadly speaking, two categories of modeling frameworks have been proposed to study hybrid systems: Those that extend event-driven models to include time-driven dynamics; and those that extend the traditional time-driven models to include event-driven dynamics; for an overview, see [1][2].

The hybrid system modeling framework we will consider in this paper is largely motivated by the structure of many manufacturing systems. In these systems, discrete entities (referred to as jobs) move through a network of workcenters which process the jobs so as to change their physical characteristics according to certain specifications. Associated with each job is a temporal state and a physical state. The temporal state of a job evolves according to event-driven dynamics and includes information such as the waiting time or departure time of the job at the various workcenters. The physical state evolves according to time-driven dynamics modeled through differential (or difference) equations which, depending on the particular problem being studied, describe changes in such quantities as the temperature, size, weight, chemical composition, or some other measure of the "quality" of the job. The interaction of time-driven with event-driven dynamics leads to a natural trade-off between temporal requirements on job completion times and physical requirements on the quality of the completed jobs. For example, while the physical state of a job can be made arbitrarily close to a desired "quality target", this usually comes at the expense of long processing times resulting in excessive inventory costs

[1] This work is supported in part by NSF under grant ACI-9873339, AFOSR under grant F49620-98-1-0387, AFRL under contract F30603-99-C-0057 and EPRI/DOD under contract WO8333-03

or violation of constraints on job completion deadlines. Our objective, therefore, is to formulate and solve optimal control problems associated with such trade-offs.

In earlier work [5],[7] we used this framework to analyze a single-stage manufacturing process assuming a deterministic setting, i.e., a known job arrival schedule and controllable processing times for all jobs. In [4], we extended our previous results to a network consisting of two servers in tandem. While this may at first seem like a simple extension, it turns out to be a much more difficult problem than the single-server case as far as obtaining explicit solutions is concerned. The issue of nondifferentiability due to the nature of the event-driven state dynamics creates serious analytical difficulties which we can no longer effectively resolve using nonsmooth optimization methods.

In this paper we derive necessary conditions for optimality for a multi-stage system consisting of $M \geq 2$ servers with the additional constraint that the control inputs are bounded. We establish some properties of the optimal control sequence that have interesting implications in terms of designing control policies for this class of hybrid systems.

2 Problem Formulation

Consider an M stage manufacturing system whose objective is to process N total jobs. Each stage processes one job at a time on a first-come first-served nonpreemptive basis (i.e., once a job begins service, the server cannot be interrupted, and will continue to work on it until the operation is completed). Jobs arriving when the server at some stage is busy wait in a queue whose capacity is larger than N.

As job i is processed at stage j, its **physical** state, denoted by $z_{ij} \in \mathrm{R}$ (chosen scalar for simplicity), is assumed to evolve according to time-driven dynamics

$$\dot{z}_{ij} = g_{ij}(z_{ij}, u_{ij}) \quad z_{ij}(\tau_{ij}) = \zeta_{ij} \tag{1}$$

where τ_{ij} is the time processing begins and ζ_{ij} is the initial state at that time. The control variable u_{ij} (assumed here to be scalar for simplicity) is used to attain a final desired physical state corresponding to a target "quality level". For simplicity, we assume that the physical state does not change while waiting in the queue.

On the other hand, the **temporal** state of the ith job at state j is denoted by x_{ij} and represents the time when the job completes processing and departs from stage j. Letting $\{\alpha_1, \ldots, \alpha_N\}$ be the given arrival time sequence for all jobs at stage 1, the event-driven dynamics describing the evolution of the temporal state are given by the following "max-plus" recursive equation:

$$x_{i,j} = \max(x_{i,j-1}, x_{i-1,j}) + s_{ij}(u_{ij}) \tag{2}$$

for $i = 1, ..., N$ and $j = 1, ..., M$ where $x_{i,0} = \alpha_i$ and $x_{0,j} = -\infty$ for $i = 1, ..., N$ and $j = 1, ..., M$.

This system is **hybrid** in the sense that it combines the time-driven dynamics (1) with the event-driven dynamics (2), the two being coupled through the choice of the control sequence. The optimal control problem we consider has the general form

$$\min_{u_{ij}, s_{ij}} \sum_{i=1}^{N} \left[\sum_{j=1}^{M} \phi(u_{ij}, s_{ij}) \right] + \psi(x_{i,M}) \tag{3}$$

subject to (1)-(2) and an additional constraint

$$u_{i,j} \leq K_j < \infty \text{ for } j = 1, ..., M \text{ and } i = 1, ..., N \tag{4}$$

Assuming that s_{ij} is monotonically decreasing in u_{ij}, the constraint on the controls in (4) can be translated to a constraint on the service times

$$s_{ij} \geq S_j > 0 \tag{5}$$

In (3), a cost $\psi(x_{i,M})$ is imposed on the departure time of the ith job from the last stage only, since this corresponds to the final output (intermediate stage departure times are not penalized).

In order to solve this optimal control problem, we proceed along the lines of a natural hierarchical decomposition of a hybrid system (see [6]) into a lower-level component representing the physical processes characterized by time-driven dynamics as in (1), and a higher-level component representing events related to these physical processes and characterized by dynamics as in (2).

Lower-level problem: We define

$$\theta(s_{ij}) = \min_{u_{ij}(t)} \phi(u_{ij}, s_{ij})$$

subject to (4).

Higher-level problem: Setting

$$\phi(u_{ij}, s_{ij}) = \theta(s_{ij})$$

in (3), we form the optimal control problem:

$$\min_{s_{i,j}} \sum_{i=1}^{N} \left[\sum_{j=1}^{M} \theta(s_{i,j}) \right] + \psi(x_{i,M})$$

subject to (2) and (5).

We illustrate this decomposition method for a class of problems with quadratic costs

$$\phi(u_{ij}, s_{ij}) = \int_0^{s_{ij}} \frac{1}{2} r_j u_{ij}^2(t) dt$$

$$\psi(x_{i,M}) = \beta x_{i,M}^2$$

and the simple physical dynamics

$$\dot{z}_{ij} = b_j u_{ij}, \quad z_{ij}(0) = z^{j-1}, \quad z_{ij}(s_{ij}) = z^j \tag{6}$$

The Hamiltonian for the lower-level problem of minimizing $\phi(u_{ij}, s_{ij})$ is

$$H_{ij}(t) = \frac{1}{2} r_j u_{ij}^2(t) + \nu_{ij}(t) b_j u_{ij}(t)$$

giving the necessary conditions for optimality:

$$\begin{aligned} \dot{\nu}_{ij}(t) &= -\frac{\partial H}{\partial z_{ij}} = 0 \\ \frac{\partial H}{\partial u_{ij}} &= r_j u_{ij}(t) + b_j \nu_{ij}(t) = 0 \end{aligned}$$

Therefore,

$$u_{ij}^*(t) = -\frac{b_j}{r_j} \nu_{ij} \text{ , } u_{ij} \text{ (constant)}$$

We can evaluate the optimal control by solving (6) to get $z^j = s_{ij} b_j u_{ij} + z^{j-1}$and, therefore,

$$u_{ij} = \frac{z^j - z^{j-1}}{s_{ij} b_j} = \frac{q_j}{s_{ij}}$$

where we set

$$q_j = \frac{z^j - z^{j-1}}{b_j}$$

Thus, the optimal control will generate a cost

$$\theta(s_{ij}) = \int_0^{s_{ij}} \frac{1}{2} r_j u_{ij}^2 dt = \frac{1}{2} r_j u_{ij}^2 s_{ij} = \frac{1}{2} r_j \frac{q_j^2}{s_{ij}}$$

If we define

$$\gamma_j = \frac{1}{2} r_j q_j^2$$

then

$$\theta(s_{ij}) = \frac{\gamma_j}{s_{ij}} \tag{7}$$

Then, the higher-level problem can be formulated as

$$\min_{s_{i,j}} \sum_{i=1}^{N} \left[\sum_{j=1}^{M} \frac{\gamma_j}{s_{ij}} \right] + \beta x_{i,M}^2$$

subject to (2) and (5) where $S_j = \frac{q_j}{K_j}$.

3 Necessary Conditions For Optimality

Let us form the augmented cost

$$\begin{aligned} \bar{J}(s, x, \lambda, \mu) &= \sum_{i=1}^{N} [\psi(x_{i,M}) \\ &\quad + \sum_{j=1}^{M} [\theta(s_{i,j}) \\ &\quad + \lambda_{i,j} (\max(x_{i,j-1}, x_{i-1,j}) \\ &\quad + s_{i,j} - x_{i,j}) \\ &\quad + \mu_{i,j} (S_j - s_{i,j})]] \end{aligned}$$

The optimality equations are

$$\begin{aligned} \mu_{i,j} &\geq 0, \quad \mu_{i,j}(S_j - s_{i,j}) = 0, \\ \frac{\partial \bar{J}}{\partial s_{i,j}} &= 0, \quad \frac{\partial \bar{J}}{\partial x_{i,j}} = 0 \end{aligned}$$

for $i = 1, ..., N$, $j = 1, ..., M$ which yield

$$\frac{\partial \bar{J}}{\partial s_{i,j}} = \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} + \lambda_{i,,j} - \mu_{i,j} = 0 \tag{8}$$

$$\lambda_{i,j} = \lambda_{i,j+1} \frac{\partial \max(x_{i,j}, x_{i-1,j+1})}{\partial x_{i,j}} \tag{9}$$

$$+ \lambda_{i+1,j} \frac{\partial \max(x_{i+1,j-1}, x_{i,j})}{\partial x_{i,j}} \tag{10}$$

for $i < N$ and $j < M$, and

$$\lambda_{i,M} = \frac{\partial \psi(x_{i,M})}{\partial x_{i,M}} + \lambda_{i+1,M} \frac{\partial \max(x_{i+1,M-1}, x_{i,M})}{\partial x_{i,M}} \tag{11}$$

for $i < N$,

$$\lambda_{N,j} = \lambda_{N,j+1} \frac{\partial \max(x_{N,j}, x_{N-1,j+1})}{\partial x_{N,j}} \tag{12}$$

for $j < M$, and

$$\lambda_{N,M} = \frac{\partial \psi(x_{N,M})}{\partial x_{N,M}} \tag{13}$$

We define the ith job to be critical at the jth stage if the optimal solution is such that $x_{i,j} = x_{i+1,j-1}$. It turns out [7],[5] that critical jobs, which cause the nondifferentiability in the costate equations due to the $\max(x_{i,j}, x_{i+1,j-1})$ term, are a common occurrence in an optimal trajectory, in which case a standard gradient-based procedure for solving the Two-Point Boundary-Value Problem (TPBVP) defined by the equations above will not work. For this reason, identification of the conditions leading to critical jobs is an essential component of the analysis and solution of such problems (see [5]).

As in earlier work [5], let us use the terms busy period and idle period to describe periods of time during which a server is busy serving jobs and not serving jobs respectively. Busy periods can further be partitioned into blocks which start with the first job after either a critical job or an idle period and end with either a critical job or the last job of the busy period. Using this definition of a block and the following assumptions we can establish some properties of the optimal control sequence.

A1. $\frac{\partial \psi(x_{i,M})}{\partial x_{i,M}} \geq 0$ for all $i = 1, ..., N$..

A2. $\frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} < 0$ and monotonically increasing with $s_{i,j}$ for all $i = 1, ..., N$ and $j = 1, ..., M$.

Lemma 1 Assume that jobs i and $i+1$ are in the same block of the jth stage for $i = 1, ..., N-1$ and $j = 1, ..., M$.. Then, the optimal service times satisfy

$$s_{i,j} \leq s_{i+1,j}$$

Proof. Note that from (8)

$$\frac{\partial \theta(s_{i+1,j})}{\partial s_{i+1,j}} - \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} = \mu_{i+1,j} - \lambda_{i+1,j} - \mu_{i,j} + \lambda_{i,j}$$

Since jobs i and $i+1$ are in the same block of the jth stage, $x_{i+1,j-1} < x_{i,j}$. Therefore, from (9) for $j < M, i < N$,

$$\lambda_{i,j} = \lambda_{i,j+1} \frac{\partial \max(x_{i,j}, x_{i-1,j+1})}{\partial x_{i,j}} + \lambda_{i+1,j}$$

So for $j < M$

$$\frac{\partial \theta(s_{i+1,j})}{\partial s_{i+1,j}} - \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} = \lambda_{i,j+1} \frac{\partial \max(x_{i,j}, x_{i-1,j+1})}{\partial x_{i,j}} + \mu_{i+1,j} - \mu_{i,j}$$

From (11),

$$\lambda_{i,M} = \lambda_{i+1,M} + \frac{\partial \psi(x_{i,M})}{\partial x_{i,M}}$$

so for $j = M$

$$\frac{\partial \theta(s_{i+1,M})}{\partial s_{i+1,M}} - \frac{\partial \theta(s_{i,M})}{\partial s_{i,M}} = \mu_{i+1,M} - \mu_{i,M} + \frac{\partial \psi(x_{i,M})}{\partial x_{i,M}}$$

Since $\frac{\partial \psi(x_{i,M})}{\partial x_{i,M}} \geq 0$, so is $\lambda_{i,j+1} \frac{\partial \max(x_{i,j}, x_{i-1,j+1})}{\partial x_{i,j}}$, therefore for both cases

$$\frac{\partial \theta(s_{i+1,j})}{\partial s_{i+1,j}} - \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} - \mu_{i+1,j} + \mu_{i,j} \geq 0$$

Now there are four cases to analyze:

1. If $s_{i,j} > S_j$ and $s_{i+1,j} > S_j$ then $\mu_{i+1,j} = 0$ and $\mu_{i,j} = 0$. In this case,

$$\frac{\partial \theta(s_{i+1,j})}{\partial s_{i+1,j}} - \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} \geq 0 \Rightarrow s_{i+1,j} \geq s_{i,j}$$

2. and 3. If $s_{i,j} = S_j$ then $s_{i+1,j} \geq s_{i,j}$.

4. If $s_{i,j} > S_j$ and $s_{i+1,j} = S_j$ then $\mu_{i,j} = 0$ and $\mu_{i+1,j} \geq 0$, therefore we have

$$\frac{\partial \theta(s_{i+1,j})}{\partial s_{i+1,j}} - \frac{\partial \theta(s_{i,j})}{\partial s_{i,j}} \geq \mu_{i+1,j} \geq 0$$

which, by A2, implies $s_{i+1,j} \geq s_{i,j}$ but $s_{i,j} > s_{i+1,j}$ thus a contradiction is observed. Hence, the result follows. ■

Lemma 2 Let jobs i and $i+1$ be in the same block of the jth stage and jobs $i-1$ and i be in the same block of the $(j+1)$th stage for $i = 2, ..., N-1$ and $j = 1, ..., M-1$. Then the optimal service times satisfy

$$s_{i+1,j} = s_{i,j}$$

Proof. By the assumption, $x_{i,j} < x_{i-1,j+1}$ and $x_{i,j} > x_{i+1,j-1}$. Therefore, from the optimality equations,

$$\lambda_{i,j} = \lambda_{i+1,j} \tag{14}$$

If $s_{i,j} > S_j$, by the previous lemma $s_{i+1,j} > S_j$, therefore $\mu_{i+1,j} = \mu_{i,j} = 0$. Then from the optimality equation (8) and (14),

$$\frac{\partial\theta(s_{i+1,j})}{\partial s_{i+1,j}} = \frac{\partial\theta(s_{i,j})}{\partial s_{i,j}} \Rightarrow s_{i,j} = s_{i+1,j}$$

If $s_{i,j} = S_j$ then $\mu_{i,j} \geq 0$ and

$$\frac{\partial\theta(s_{i,j})}{\partial s_{i,j}} = \mu_{i,j} - \lambda_{i,j}$$

Assume that $s_{i+1,j} > S_j$. Then $\mu_{i+1,j} = 0$ and

$$\begin{aligned}\frac{\partial\theta(s_{i+1,j})}{\partial s_{i+1,j}} &= -\lambda_{i+1,j} = -\lambda_{i,j} \\ &> \frac{\partial\theta(s_{i,j})}{\partial s_{i,j}} = \mu_{i,j} - \lambda_{i,j} \\ &\Rightarrow 0 > \mu_{i,j}\end{aligned}$$

which is a contradiction, and the result follows. ■

Lemma 3 In the optimal sample path, there does not exist any job that leaves a stage idle and arrives at a busy stage.

Proof. Assume that in the optimal path the ith job leaves the jth stage idle , i.e. $x_{i,j} < x_{i+1,j-1}$, and arrives at stage $j+1$ and finds it busy, i.e., $x_{i,j} < x_{i-1,j+1}$. Let us perturb the service time $s_{i,j}$ to become $s_{i,j} + \delta$ where $\delta > 0$ and observe the effect on the cost function. Note that $x_{i,j}$ will become $\bar{x}_{i,j} = x_{i,j} + \delta$. If we select

$$\delta < \min(x_{i-1,j+1} - x_{i,j}, x_{i+1,j-1} - x_{i,j})$$

then other departure times $x_{k,l}$ where $k \neq i$ and $l \neq j$ will not change. Therefore, the change in the cost will be

$$\theta(s_{i,j} + \delta) - \theta(s_{i,j}) < 0$$

which contradicts the optimality assumption. Hence, the result follows. ■

These results allow us to restrict the search for optimal policies to those satisfying the properties above. When the costs ϕ and ψ are specified, we can further restrict the search space. The following property can be established for the linear-quadratic class of systems considered in Section 2.

Theorem Let T_j be defined as

$$T_j = \max_{k=j,\ldots,M}\left(\frac{\gamma_k}{2\beta S_k^2}\right) - \sum_{k=j}^{M} S_k$$

For any job i starting process at time $t \geq T_j$ at the jth stage, if

$$t + \sum_{k=j}^{m-1} S_k \geq x_{i-1,m} \tag{15}$$

for all $m = j+1, \ldots, M$ (i.e., the ith job is guaranteed not to wait in any downstream queues), then the optimal service times are

$$s^*_{i,k} = S_k$$

for $k = j, \ldots, M$

Proof. (By Induction) Consider job i at the last stage starting process at time t. The overall cost can be written as

$$J_b(t) + \beta(t + s_{i,M})^2 + \frac{\gamma_M}{s_{i,M}} + J_a(t + s_{i,M})$$

where $J_b(t)$ is the cost incurred in the system up to time t and $J_a(t + s_{i,M})$ is the cost that the system will incur due to other jobs given that job i departs at $t + s_{i,M}$. Assume that $t \geq T_M$ and the optimal service time $s^*_{i,M} > S_M$. If we decrease the service time by $0 < \delta \leq s^*_{i,M} - S_M$ the cost will become

$$\begin{aligned}&J_b(t) + \beta(t + s^*_{i,M} - \delta)^2 \\ &+ \frac{\gamma_M}{s^*_{i,M} - \delta} + J_a(t + s^*_{i,M} - \delta)\end{aligned}$$

If the ith job departs earlier, it will not increase the cost for the other jobs, therefore

$$J_a(t + s^*_{i,M} - \delta) \leq J_a(t + s^*_{i,M})$$

so for a contradiction it suffices to show

$$\beta(t + s^*_{i,M} - \delta)^2 + \frac{\gamma_M}{s^*_{i,M} - \delta} < \beta(t + s^*_{i,M})^2 + \frac{\gamma_M}{s^*_{i,M}}$$

which after some algebra reduces to

$$t > \frac{\gamma_M}{2\beta}\frac{1}{s^*_{i,M}(s^*_{i,M} - \delta)} - (s^*_{i,M} - \frac{\delta}{2})$$

Since

$$\begin{aligned}t &\geq \frac{\gamma_M}{2\beta S_M^2} - S_M \\ &> \frac{\gamma_M}{2\beta}\frac{1}{s^*_{i,M}(s^*_{i,M} - \delta)} - (s^*_{i,M} - \frac{\delta}{2})\end{aligned}$$

for jobs that start the process after T_M, it is optimal to finish processing in minimal time.

Next, let us consider the jth stage and assume that the theorem holds for stages $j+1, \ldots, M$. We will assume that equation (15) is satisfied, i.e., the ith job will not wait in any downstream queues, $t \geq T_j$ and the optimal service time is $s^*_{i,j} > S_j$. Note that by the induction argument (it is trivial to show that the conditions are met), the optimal service times for the downstream stages are S_k for $k = j+1, ..., M$. Then, using a similar argument, for contradiction it suffices to show that for $0 < \delta \leq s^*_{i,j} - S_j$

$$\begin{aligned} & \beta(t + s^*_{i,j} - \delta + \sum_{k=j+1}^{M} S_k)^2 + \frac{\gamma_j}{s^*_{i,j} - \delta} \\ < \quad & \beta(t + s^*_{i,j} + \sum_{k=j+1}^{M} S_k)^2 + \frac{\gamma}{s^*_{i,j}} \end{aligned}$$

which reduces to

$$t > \frac{\gamma_j}{2\beta} \frac{1}{s^*_{i,j}(s^*_{i,j} - \delta)} - (s^*_{i,j} - \frac{\delta}{2}) - \sum_{k=j+1}^{M} S_k$$

Since $t \geq T_j$, the result follows. ■

Corollary After time $T_M = \frac{\gamma_k}{2\beta S_k^2} - S_M$, it is always optimal for the last machine to work at full speed K_M.

Let us separate the system into two parts (1) the bottleneck stage and its upstream stages, and (2) the downstream stages from the bottleneck. The implication of the theorem is that after some 'critical' time is passed, the departure rate from the second part will be higher than the arrival rate to the second part, therefore, the second part of the system will be drained (until the departure rate equals the arrival rate). This will ensure that equation (15) is satisfied, hence, it will be optimal to run downstream machines at full speed.

Experimental results suggest that the optimal policy for the upstream stages is not to starve the bottleneck after the critical time is passed. A formal proof of this fact is the subject of ongoing research.

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