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STABILITY ANALYSIS OF TAKAGI-SUGENO FUZZY SYSTEMS WITH LINEAR INPUT-OUTPUT SUBMODELS*

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Abstract. This paper presents a method analyzing stability of Takagi-Sugeno fuzzy systems with linear input-output submodels in the consequents of rules. This method can be used for stability analysis of a Takagi-Sugeno fuzzy model of a plant and for closed-loop system, where both the plant and the controller are represented by Takagi-Sugeno fuzzy systems. It will be shown that the problem of stability analysis of such a system can be transformed to robust stability analysis of a polynomial with polynomic structure of its coefficients. Stability of such polynomials is tested by the Modified Jury criterion, the Modified Routh criterion or the Hurwitz criterion together with Sign-decomposition. A necessary condition for stability of the Takagi-Sugeno closed loop systems is obtained.

Keywords. Takagi-Sugeno fuzzy systems, stability analysis, polynomials, polynomic uncertainty

1. INTRODUCTION

Stability is one of the most important issues in analysis and design of control systems. There is no particular stability theory of fuzzy systems, where the controller and the plant is considered to be a fuzzy system. There exist several approaches for testing stability of closed-loop systems, where the plant is modeled by standard mathematical tools, e.g. by the state space model (linear or nonlinear) or the matrix of transfer functions, and the controller is described by a fuzzy model. These methods are based on principles known from the theory of nonlinear systems, for example stability indices ([3],[4]), the circle criterion ([4]), hyperstability theory ([1]) etc. Chen, *et. al.* in [7] developed a method for stability analysis of systems, where the plant and the controller are described as linguistic fuzzy systems. This method is based on fuzzy relations. The next approaches ([2],[8],[9],[10]) are designed for Takagi-Sugeno fuzzy systems, where the consequents of rules of the plant and of the controller are considered to be linear state space submodels and the state feedback to each of them respectively. Stability analysis of such systems uses usually the Lyapunov theory and it is based on finding a common Lyapunov function for all subsystems. This function can be found for example by LMI.

In this paper a new method for stability analysis of Takagi-Sugeno fuzzy systems based on robust stability of polynomials will be presented. The Takagi-Sugeno fuzzy systems will be introduced in section 2. In the section 3 it will be shown, that the characteristic polynomial of Takagi-Sugeno fuzzy closed-loop system can be expressed as a polynomial with polynomic structure of its coefficients. The section 4 is devoted to stability analysis of such a polynomial.

2. TAKAGI-SUGENO FUZZY SYSTEMS

In this work only Takagi-Sugeno fuzzy systems will be analyzed, where the consequents of rules are represented by local linear input-output relations of nonlinear systems.

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Let suppose, without loss of generality, the following closed-loop system:

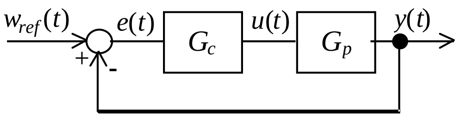

Fig. 1 Closed-loop scheme

where G_c is the controller, G_p is the plant.

The Takagi-Sugeno fuzzy model of the plant is considered, where the rules are written in the following form:

$$\begin{aligned}&R_i : \text{IF } z_1(t) \text{ is } M_1^i \text{ and} \cdots \text{and } z_g(t) \text{ is } M_g^i\\ &\text{THEN } y_i(t) = -\sum_{j=1}^{n_a} a^i_{n_a-j} y^{(j)}(t) + \sum_{j=0}^{n_b} b^i_{n_a-j} u^{(j)}(t)\end{aligned} \tag{1}$$

$$\begin{aligned}&R_i : \text{IF } z_1(t) \text{ is } M_1^i \text{ and} \cdots \text{and } z_g(t) \text{ is } M_g^i\\ &\text{THEN } y_i(t) = -\sum_{j=1}^{n_a} a^i_{n_a-j} y(t-j) + \sum_{j=0}^{n_b} b^i_{n_a-j} u(t-j)\end{aligned} \tag{2}$$

$i = 1,2,\ldots,R$, for continuous case and for discrete case respectively.

$\mathbf{z}(t)=[\, z_1(t), \ldots, z_q(t)]$	variables of the premise (some measurable plant variables)
M_j^i	fuzzy sets
$y(t) \in \Re$	the output of the plant
$u(t) \in \Re$	the input to the plant

The following procedure will be derived for discrete case, but it can be followed for continuous system analogically.

The total output of the fuzzy system is:

$$\begin{aligned}y(t) &= \frac{\sum_{i=1}^{R} w_i(\mathbf{z}(t)) y_i(t)}{\sum_{i=1}^{R} w_i(\mathbf{z}(t))} = \sum_{i=1}^{R} h_i(\mathbf{z}(t)) y_i(t) =\\ &= \sum_{i=1}^{R} h_i(\mathbf{z}(t)) \left(-\sum_{j=1}^{n_a} a^i_{n_a-j} y(t-j) + \sum_{j=0}^{n_b} b^i_{n_a-j} u(t-j) \right)\end{aligned} \tag{3}$$

where

$$w_i(\mathbf{z}(t)) = \prod_{j=1}^{g} M_j^i(z_j(t)). \tag{4}$$

$M_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in M_j^i. It is assumed that

$w_i(z(t)) \geq 0$, for $i = 1,2,\cdots,R$ and $\sum_{i=1}^{R} w_i(z(t)) > 0$ for all t. Therefore

$$\sum_{i=1}^{R} h_i(z(t)) = 1. \tag{5}$$

By applying Z-transform to (3) the following transfer function is obtained:

$$\begin{aligned}G_p(z) &= \frac{Y(z)}{U(z)} = \frac{\sum_{i=1}^{R} w_i(\mathbf{z}(t)) G_p^i(z)}{\sum_{i=1}^{R} w_i(\mathbf{z}(t))} =\\ &= \sum_{i=1}^{R} h_i(\mathbf{z}(t)) G_p^i(z) =\\ &= \frac{\sum_{i=1}^{R} h_i(\mathbf{z}(t)) \sum_{j=0}^{n_b} b_j^i z^j}{\sum_{i=1}^{R} h_i(\mathbf{z}(t)) \sum_{j=0}^{n_a} a_j^i z^j} =\\ &= \frac{\sum_{i=1}^{R} h_i(\mathbf{z}(t)) b^i(z)}{\sum_{i=1}^{R} h_i(\mathbf{z}(t)) a^i(z)}\end{aligned} \tag{6}$$

where $G_p^i(z)$ denotes the transfer function of the consequent of the i-th rule.

The controller is described by the following rules:

$$\begin{aligned}&R_i : \text{IF } z_1(t) \text{ is } M_1^i \text{ and} \cdots \text{and } z_g(t) \text{ is } M_g^i\\ &\text{THEN } u_i(t) = -\sum_{j=1}^{n_q} q^i_{n_q-j} u^{(j)}(t) + \sum_{j=0}^{n_r} r^i_{n_q-j} e^{(j)}(t)\end{aligned} \tag{7}$$

or

$$\begin{aligned}&R_i : \text{IF } z_1(t) \text{ is } M_1^i \text{ and} \cdots \text{and } z_g(t) \text{ is } M_g^i\\ &\text{THEN } u_i(t) = -\sum_{j=1}^{n_q} q^i_{n_q-j} u(t-j) + \sum_{j=0}^{n_r} r^i_{n_q-j} e(t-j)\end{aligned} \tag{8}$$

The total output of the controller is

$$\begin{aligned}u(t) &= \frac{\sum_{i=1}^{R} w_i(\mathbf{z}(t)) u_i(t)}{\sum_{i=1}^{R} w_i(\mathbf{z}(t))} =\\ &= \sum_{i=1}^{R} h_i(\mathbf{z}(t)) \left(-\sum_{j=1}^{n_q} q^i_{n_q-j} u(t-j) + \sum_{j=0}^{n_r} r^i_{n_q-j} e(t-j) \right)\end{aligned} \tag{9}$$

The corresponding transfer function is:

$$G_c(z)=\frac{U(z)}{E(z)}=\frac{\sum_{i=1}^{R} h_i(\mathbf{z}(t))r^i(z)}{\sum_{i=1}^{R} h_i(\mathbf{z}(t))q^i(z)}. \qquad (10)$$

Now it is possible to write the transfer function of the closed-loop system:

$$\frac{Y(z)}{W_{ref}(z)}=\frac{G_c\cdot G_p}{1+G_c\cdot G_p}=\cdots= \\ =\frac{\sum_{i=1}^{R}\sum_{i=1}^{R}h_ih_jb^i(z)r^j(z)}{\sum_{i=1}^{R}\sum_{j=1}^{R}h_ih_j\left(b^i(z)r^j(z)+a^i(z)q^j(z)\right)} \qquad (11)$$

$$\frac{Y(s)}{W_{ref}(s)}=\frac{G_c\cdot G_p}{1+G_c\cdot G_p}=\cdots= \\ =\frac{\sum_{i=1}^{R}\sum_{i=1}^{R}h_ih_jb^i(s)r^j(s)}{\sum_{i=1}^{R}\sum_{j=1}^{R}h_ih_j\left(b^i(s)r^j(s)+a^i(s)q^j(s)\right)} \qquad (12)$$

for discrete system and for continuous system respectively.

3. TAKAGI-SUGENO FUZZY SYSTEMS AS UNCERTAIN POLYNOMIALS

In this section it will be shown, that the characteristic polynomial of the closed-loop Takagi-Sugeno fuzzy system introduced in last section can be expressed as a polynomial with polynomic structure of its coefficients with an uncertain interval parameter **h**. The equation (13) will be used to decrease dimension and computational complexity of the problem.

$$h_R=1-\sum_{i=1}^{R-1}h_i \qquad (13)$$

Substituing (13) to (11) the characteristic polynomial of the closed-loop system is obtained:

$$\begin{aligned}p(\mathbf{h})&=\sum_{i=1}^{R}\sum_{j=1}^{R}h_ih_j\left(b^ir^j+a^iq^j\right)=\cdots=\\ &=\sum_{i=1}^{R-1}\sum_{j=1}^{R-1}h_ih_j\begin{pmatrix}b^ir^j+a^iq^j+b^Rr^R+a^Rq^R\\-\left(b^jr^R+a^jq^R+b^Rr^j+a^Rq^j\right)\end{pmatrix}+\\ &+\sum_{j=1}^{R-1}h_j\begin{pmatrix}\left(b^jr^R+a^jq^R+b^Rr^j+a^Rq^j\right)\\-2\left(b^Rr^R+a^Rq^R\right)\end{pmatrix}\\ &+b^Rr^R+a^Rq^R\end{aligned} \qquad (14)$$

where b^i, a^i, r^j, q^j denote $b^i(z)$, $a^i(z)$, $r^j(z)$, $q^j(z)$ respectively.

The polynomial (14) can be transformed into the following form:

$$p(s,\mathbf{h})=c_n(\mathbf{h})s^n+c_{n-1}(\mathbf{h})s^{n-1}+\cdots+c_0(\mathbf{h}) \qquad (15)$$

or

$$p(z,\mathbf{h})=c_n(\mathbf{h})z^n+c_{n-1}(\mathbf{h})z^{n-1}+\cdots+c_0(\mathbf{h}) \qquad (16)$$

for continuous system and for discrete systems respectively, where n is max (n_b+n_r, n_a+n_q),

$$\mathbf{h}\in H\subset\Re^{R-1} \\ H=\left\{\begin{matrix}\mathbf{h}\in\Re^{R-1}: h_i\in[0,1], \sum_{i=1}^{R-1}h_i\le 1,\\ i=1,\ldots,R-1\end{matrix}\right\} \qquad (17)$$

and coefficients $c_k(\mathbf{h})$ $k=0,\ldots,n$ are polynomic functions of **h**.

4. STABILITY ANALYSIS

A method for analysis of robust stability of polynomic interval polynomials was described in [6] for continuous case and in [5] for discrete case. The methods use the Modified Routh criterion or the Modified Jury criterion respectively. Positivity of elements of both tables is tested by Sign-decomposition.

Theorem 1 (Modified Jury criterion): The polynomial $R(z)=c_nz^n+\cdots+c_1z+c_0$ (discrete-time polynomial with constant coefficients) is stable if and only if $b_{k,0}>0$ (elements of Modified Jury table) $\forall k=1,\ldots,n$.

The corresponding Modified Jury table is defined as follows:

$$\begin{matrix}b_{0,0} & b_{0,1} & b_{0,2} & \cdots & b_{0,n-2} & b_{0,n-1} & b_{0,n}\\ b_{1,0} & b_{1,1} & b_{1,2} & & b_{1,n-2} & b_{1,n-1} & 0\\ b_{2,0} & b_{2,1} & b_{2,2} & & b_{2,n-2} & 0 & 0\\ \vdots & & & & & & \vdots\\ b_{n-1,0} & b_{n-1,1} & 0 & & 0 & 0 & 0\\ b_{n,0} & 0 & 0 & \cdots & 0 & 0 & 0\end{matrix} \qquad (18)$$

where

$$b_{0,j}=c_{n-j} \qquad j=0,\ldots,n \qquad (19)$$

and

$$b_{i,j}=b_{i-1,j}\cdot b_{i-1,0}-b_{i-1,n-i+1}\cdot b_{i-1,n-i-j+1} \\ i=1,\ldots,n \quad j=0,\ldots,n-i \qquad (20)$$

Theorem 1 can be enlarged to the polynomial (16).

Theorem 2: The Takagi-Sugeno closed-loop system (11) with the characteristic polynomial (16) is stable for an arbitrary choice of fuzzy sets in rules R^i only if the corresponding Modified Jury table is stable $\forall \mathbf{h} \in H$.

For the polynomial (16) the elements of the Modified Jury table are multivariate polynomic functions. The positivity of elements $b_{k,0}$, $k=1,\dots,n$ of Modified Jury table can be tested by Sign-decomposition.

In the similar way it is possible to define the Modified Routh table:

$$\begin{array}{c|cccc} s^n & c_n & c_{n-2} & c_{n-4} & \cdots \\ s^{n-1} & c_{n-1} & c_{n-3} & c_{n-5} & \cdots \\ s^{n-2} & b_{3,1} & b_{3,2} & \cdots & \\ s^{n-3} & b_{4,1} & b_{4,2} & \cdots & \\ \vdots & \vdots & & & \\ s^0 & b_{n+1,1} & & & \end{array} \tag{21}$$

where

$$b_{i,j} = b_{i-1,1} b_{i-2,j+1} - b_{i-2,1} b_{i-1,j+1} \qquad \forall i \geq 3 \tag{22}$$

Theorem 3: The Takagi-Sugeno closed-loop system (12) with the characteristic polynomial (15) is stable for an arbitrary choice of fuzzy sets in rules R^i only if
a) $c_i(\mathbf{h}) > 0$ for $i=0,\dots,n$ and
b) $b_{k,1}(\mathbf{h}) > 0$ for $k=3,\dots,n+1$.
$\forall \mathbf{h} \in H$.

The positivity of elements c_i , $i = 0,\dots,n$ and $b_{k,1}$, $k=1,\dots,n$ can be tested by Sign-decomposition.
Theorem 2 and Theorem 3 state only a necessary condition for stability of Takagi-Sugeno fuzzy systems.

4.1. Sign-decomposition

Sign-decomposition is a method derived by Elizondo and described in detail in [6] which makes it possible to check positivity or negativity of a class of multivariate functions on a convex set. Polynomic functions belong to this class.
Let $Q \subset \Re^l$ be a box of uncertainties. This box can be always translated to some subset of positive orthant $P \subset \Re^l$ by a linear transformation, so it is possible to consider $Q \subset P \subset \Re^l$.

Definition 1: Let $f: \Re^l \to \Re$ be a continuous function and let Q be convex and $Q \subset P \subset \Re^l$. We say that $f(\cdot)$ is a non-decreasing function on Q if $\mathbf{x} \geq \mathbf{y}$ ($x_i \geq y_i$, $i=1,\dots,l$) implies $f(\mathbf{x}) \geq f(\mathbf{y})$.

Definition 2: Let $f: \Re^l \to \Re$ be a continuous function and let Q be convex and $Q \subset P \subset \Re^l$. Then it is said that $f(\cdot)$ has sign-decomposition in Q if $f(\cdot) = f^p(\cdot) - f^n(\cdot)$ $\forall$ $\mathbf{q} \in Q$, where $f^n(\cdot) \geq 0$ and $f^p(\cdot) \geq 0$ are non-decreasing function in Q.

$$\begin{aligned} & f(\mathbf{q}) = f^p(\mathbf{q}) - f^n(\mathbf{q}), \forall \mathbf{q} \in Q \\ & f^p(\cdot) \cong \text{positive part of } f(\cdot) \\ & f^n(\cdot) \cong \text{negative part of } f(\cdot) \end{aligned} \tag{23}$$

Denote by $\mathbf{v}^{\min}, \mathbf{v}^{\max} \in Q \subset P$ the minimum and maximum Euclidean norm vertices of Q. Then it is easy to prove that for a continuous non-decreasing function f $\min_{\mathbf{q} \in Q} f(\mathbf{q}) = f(\mathbf{v}^{\min})$ and $\max_{\mathbf{q} \in Q} f(\mathbf{q}) = f(\mathbf{v}^{\max})$.

Lemma 1: Let $f(\mathbf{q})$ be a polynomic function with $\mathbf{q} \in Q \subset \Re^l$. Then $f(\mathbf{q})$ has Sign-decomposition in Q.

Proposition 1: Let $f_1(\cdot)$ and $f_2(\cdot)$ have a Sign-decomposition then the sum $f_1(\cdot)+f_2(\cdot)$ and the pointwise product $f_1(\cdot) \cdot f_2(\cdot)$ has a Sign-decomposition.

Rectangle Theorem: Let f be a continuous function $f: \Re^l \to \Re$ with Sign-decomposition in a box $Q \subset P \subset \Re^l$ with $\mathbf{q}^{min}, \mathbf{q}^{max} \in Q$, where $\mathbf{q}^{min}, \mathbf{q}^{max}$ are the minimum and maximum Euclidean norm vertices of Q. Then:
a) $f^p(\mathbf{q}^{min}) - f^n(\mathbf{q}^{max}) < f(\mathbf{q}) < f^p(\mathbf{q}^{max}) - f^n(\mathbf{q}^{min})$, $\forall \mathbf{q} \in Q$
in the plane (f^n, f^p):
b) $f(\mathbf{q}) \subset$ rectangle with vertices $(f^n(\mathbf{q}^{min}), f^p(\mathbf{q}^{min}))$, $(f^n(\mathbf{q}^{max}), f^p(\mathbf{q}^{max}))$, $(f^n(\mathbf{q}^{min}), f^p(\mathbf{q}^{max}))$, $(f^n(\mathbf{q}^{max}), f^p(\mathbf{q}^{min}))$, $\forall$ $\mathbf{q} \in Q$
c) if vertex $(f^n(\mathbf{q}^{max}), f^p(\mathbf{q}^{min}))$ is above the 45° line then $f(\mathbf{q}) > 0$, $\forall \mathbf{q} \in Q$
d) if vertex $(f^n(\mathbf{q}^{min}), f^p(\mathbf{q}^{max}))$ is below the 45° line then $f(\mathbf{q}) < 0$, $\forall \mathbf{q} \in Q$
e) if vertex $(f^n(\mathbf{q}^{min}), f^p(\mathbf{q}^{min}))$ is below (above) the 45° line and vertex $(f^n(\mathbf{q}^{max}), f^p(\mathbf{q}^{max}))$ is below (above) the 45° line then $f(\mathbf{q})$ is neither positive nor negative $\forall \mathbf{q} \in Q$

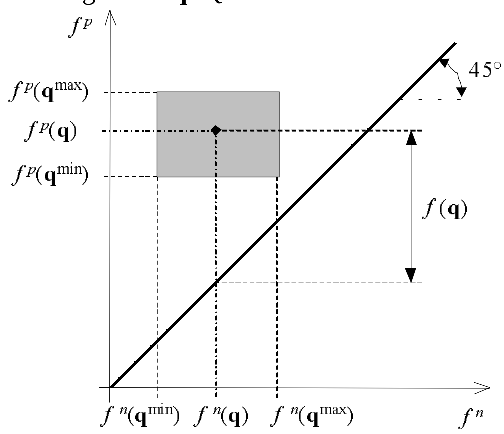

Fig. 2 Sign-decomposition

If none of conditions c), d) or e) is satisfied (vertices $(f^n(\mathbf{q}^{min}), f^p(\mathbf{q}^{min}))$ and $(f^n(\mathbf{q}^{max}), f^p(\mathbf{q}^{max}))$ are both above or both below the 45° line and the 45° line crosses the corresponding rectangle), then it is not possible to decide about positivity or negativity of function $f(\mathbf{q})$ and one can continue as follows:

Let $Q \subset P \subset \Re^l: Q = [q_1^-, q_1^+] \times [q_2^-, q_2^+] \times \cdots \times [q_l^-, q_l^+]$. Each uncertainty interval $[q_i^-, q_i^+]$ can be divided in k parts, generating k uncertainty intervals: $[q_i^-, q_i^1]$, $[q_i^1, q_i^2], \cdots, [q_i^j, q_i^{j+1}], \cdots, [q_i^{k-1}, q_i^+]$. Let $[\gamma_i^-, \gamma_i^+]$ be one of these uncertainty intervals, then one can get k^l boxes Γ: Let $\Gamma^i = [\gamma_1^-, \gamma_1^+] \times [\gamma_2^-, \gamma_2^+] \times \cdots \times [\gamma_l^-, \gamma_l^+]$ be one of the k^l boxes: $Q = \bigcup_i \Gamma^i$ and let $\boldsymbol{\mu}^{\min}$, $\boldsymbol{\mu}^{\max} \in \Gamma^i$ be the minimum and maximum Euclidean norm vertices of Γ^i.

Theorem 4: A function is positive (negative) on an uncertain box Q if and only if there exists a set of boxes Γ as described above, such that the vertex $\left(f^n(\boldsymbol{\mu}_{\max}), f^p(\boldsymbol{\mu}_{\min})\right)$ of each Γ^i is above the 45° line (all the vertices $\left(f^n(\boldsymbol{\mu}_{\min}), f^p(\boldsymbol{\mu}_{\max})\right)$ of each Γ^i are below the 45° line). A function is neither positive nor negative on an uncertainty box Q if and only if there exists at least one of boxes Γ^i, where the function is positive and one of boxes Γ^i, where the function is negative.

4.2. Computational procedure

The computational procedure for testing stability of Takagi-Sugeno fuzzy systems runs as follows:

1) Compute the characteristic polynomial of the Takagi-Sugeno fuzzy system
2) Generate the sets $I_1, I_2, \ldots, I_u$ of indices of all combination of rules which can be active simultaneously
3) Set j=1
4) For the set I_j do following steps
   a) Set $\mathbf{h}_z = 0$, where $z \in Z = \{1,2,\ldots,m\text{-}1\} - I_j$
   b) Re-compute the characteristic polynomial of the Takagi-Sugeno fuzzy system
   c) Set k=1
   d) Compute k-th row of the Modified Jury table
   e) Test positivity of element $b_{k,0}$ by Sign-decomposition. If $b_{k,0}$ is negative for some $\mathbf{h} \in H$, the procedure is terminated – the Takagi-Sugeno fuzzy system is unstable, else continue on step f)
   f) If $k = n$, then $j = j$+1 and continue on step 5), else $k = k$+1 and continue on step d)
5) If j=u, then the procedure is terminated – it is not possible to decide about stability of the closed-loop; else continue on step 4).

Commentary: If the positivity of all elements $b_{k,0}$, $\forall k$=1,…,n is proved, it means, that all "switched" linear systems are stable. If some rectangle is negative, the Takagi-Sugeno fuzzy system is unstable. In this case the value of **h** can be obtained, where the rectangle is negative. This value represents the combination of rules for which the system is unstable.

Presented procedure can be used analogically for stability analysis of continuous systems. In this case the Modified Jury table is replaced by the Modified Routh table and it is necessary to check positivity of $c_i(\mathbf{h})$ for i = 0,…,n and $b_{k,1}(\mathbf{h})$ for k = 3,…,n+1. In order to decrease the computional complexity it is convenient to use the Routh criterion for stability analysis of discrete systems using a linear fractional transformation z =(s+1)/(s-1).

5. EXAMPLE

To illustrate the presented method following example is considered. Define the Takagi-Sugeno fuzzy system described by two rules:

$$
\begin{aligned}
R^1:&\ \text{IF } y(t-1) \text{ is } M_1^1 \text{ THEN} \\
&\ y(t) = 2u(t-2) - y(t-1) - 3.25y(t-2) \\
R^2:&\ \text{IF } y(t-1) \text{ is } M_1^2 \text{ THEN} \\
&\ y(t) = 1.1u(t-2) - 0.8y(t-1) - 2.59y(t-2)
\end{aligned}
$$

The controller stabilizing both the consequents of the system was chosen:

$$
\begin{aligned}
R^1:&\ \text{IF } y(t-1) \text{ is } M_1^1 \text{ THEN} \\
&\ u(t) = 2.5u(t-1) + 4e(t-1) \\
R^2:&\ \text{IF } y(t-1) \text{ is } M_1^2 \text{ THEN} \\
&\ u(t) = 2.3u(t-1) + 5.3018e(t-1)
\end{aligned}
$$

The sets of indices of simultaneously active rules are I_1={1}, I_2={2} and I_3={1,2}.

Due to (14) the characteristic polynomial of the closed-loop is

$$p(z,\mathbf{h}) = z^3 + c_2(\mathbf{h})z^2 + c_1(\mathbf{h})z + c_0(\mathbf{h})$$

where

$$
\begin{aligned}
&c_2(\mathbf{h}) = -1.5 \\
&c_1(\mathbf{h}) = 0.75 + 0.04h_1 - 0.04h_1^2 \\
&c_0(\mathbf{h}) = 0.125 + 1.304h_1 - 1.304h_1^2 \\
&h_1 \in [0,1]
\end{aligned}
$$

The stability of the closed-loop Takagi-Sugeno fuzzy system for sets I_1 and I_2 is obvious from design. To decide about stability of the Takagi-Sugeno fuzzy system for the set I_3 the method described above is applied. The method indicates that the polynomial $p(z,\mathbf{h})$ is not Schur stable for $h_1 \in [0.3,0.7]$. It means that the fuzzy system is not stabilized by the controller.

6. CONCLUSION

An algorithm for stability testing of the Takagi-Sugeno fuzzy systems with linear input-output submodels was presented. It was shown that the problem of stability analysis of this system can be transformed to robust stability analysis of a polynomial with polynomic structure of its coefficients. A necessary condition of stability was derived. Presented method is based on well-known criteria used for stability analysis of linear systems with constant coefficients – the Jury and the Routh criterion.

The method can be modified for stability analysis of the Takagi-Sugeno fuzzy systems with the state space submodels.

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