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AUTOMOTIVE ENGINE AND POWER-TRAIN CONTROL: A COMPREHENSIVE HYBRID MODEL*

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Abstract

The design of engine control systems has been traditionally carried out using heuristic techniques validated by simulation and prototyping using approximate average-value models. Increasing demands on controllers performance call for more robust techniques and the use of cycle–accurate models. In this paper we present a hybrid model of the engine and powertrain in which both continuous and discrete time–domain as well as event-based phenomena are modeled in a separate but integrated manner.

1 INTRODUCTION

Hybrid systems have been the subject of intensive study in the past few years by both the control and the computer–science communities. Particular emphasis has been placed on a unified representation of hybrid models ([1, 2, 3]). Some classical problems such as reachability analysis ([4]), stability and safety ([5, 6]) have been investigated and tools for their solutions, i.e. HyTech, Kronos, Checkmate, developed.

In this paper, we focus on an application domain for hybrid system theory that is of great industrial interest: automotive engine and power–train control. The engine control problem is very complex (see e.g. [7, 8]). The goals for the control strategy are, in general, given in terms of emissions and torque but it is often the case that sub–goals are given by car manufacturers on all the sub–systems composing the power–train. In order to extend the performance and the functionality of engine embedded controllers, we believe that it is important to use more accurate models than the ones proposed so far. An accurate model of a four–stroke gasoline engine has a "natural" hybrid representation because

- pistons have four modes of operation corresponding to the stroke they are in. Hence their behavior can be represented with a finite state model;
- power–train and air dynamics are continuous–time processes.

In addition, these processes interact tightly. In fact, the timing of the transitions between two phases of the pistons is determined by the continuous motion of the power–train, which, in turn, depends on the torque produced by each piston. In this paper, we present a hybrid model for the power–train.

2 BACKGROUND

Hybrid systems can be viewed as formalisms for describing a complex system using combinations of models of computation (MOCs) when a single one is not powerful, expressive or practical enough. We use the theory behind the notion of models of computation to identify the role of the interfaces among the different components of the power–train hybrid model.

2.1 The Tagged-Signal Model

The tagged-signal model (TSM) proposed by Lee and Sangiovanni-Vincentelli [9] defines a semantic framework within which models of computation can be studied and compared. The fundamental entity in the TSM is an event: a value/tag pair (v, t). Tags are often used to denote temporal behavior. Given a set of *values* V and a set of *tags* T, an *event* is an element of $V \times T$. A *signal* s is a set of events, and thus is a subset of $V \times T$. A *functional* (or deterministic) *signal* is a (possibly partial) function from T to V. The set of all signals is denoted S. A *tuple* of n signals is denoted $\mathbf{s}$, and the set of all such tuples is denoted S^n.

The issue of time representation has been central to all modeling efforts. We argue that representing specifications using physical time equivalents may result in over specifications and as a consequence, less efficient designs. For example, data manipulation operations can often be performed concurrently as long as certain precedence relations are satisfied.

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Processes. A *process* P is a subset of the set of all n-tuples of signals S^n for some n. A particular $\mathbf{s} \in S^n$ is said to *satisfy* the process P if $\mathbf{s} \in P$. An $\mathbf{s}$ that satisfies a process is called a *behavior* of the process. Thus a *process* is a set of possible *behaviors*, or a relation between signals. In a *timed process* T is totally ordered, in an *untimed process*, T is only partially ordered. Intuitively, a set of processes operate *concurrently*, and constraints imposed on their signal tags define *communication* among them. For many (but not all) applications, it is natural to partition the signals associated with a process into *inputs* and *outputs*. A process with i inputs and o outputs is a subset of $S^i \times S^o$, where (S^i, S^o) is a partition of S^n and $n = i + o$. Thus, a process defines a *relation* between input signals and output signals. An $\mathbf{s}$ can be written $\mathbf{s} = (\mathbf{s}_1, \mathbf{s}_2)$, where $\mathbf{s}_1 \in S^i$ is an i-tuple of *input signals* for process P and $\mathbf{s}_2 \in S^o$ is an o-tuple of *output signals* for process P. A process F is *functional* (or deterministic) with respect to an input/output partition if it is a single-valued, possibly partial, mapping from S^i to S^o.

The time. In classical transformational systems, such as personal computers, the correct result is the primary concern—when it arrives is less important (although *whether* it arrives *is* important). By contrast, embedded systems are usually real-time systems, where the time at which a computation takes place is very important. As mentioned previously, different models of time become different order relations on the set of tags T in the tagged-signal model. Implicit communication generally requires totally ordered tags (*timed processes*), usually identified with physical time. The tags in a *metric timed process* have the notion of a "distance" between them. Two events are *synchronous* if they have the same tag. Two signals are synchronous if each event in one signal is synchronous with an event in the other signal and vice versa. A *synchronous process* is one in which every signal in the process is synchronous with every other signal in the process. An *asynchronous process* is a process in which no two events can have the same tag. Note that asynchronous processes in our framework are a subset of non synchronous processes.

2.2 Models of computation used in our model

Timed MOCs will be used in the description of our system[2]. In particular, we use:

Finite State Machines. An FSM is a synchronous TSM process in which the tags take values in the set of integers and the sets of inputs, outputs and states are finite. The tags represent the ordering of the sequence of events in the signals, not physical time, and are globally ordered.

Sequential Systems. An SS is a synchronous TSM process in which the tags take values in the set of integers and the inputs, outputs and states assume values on infinite sets;

Discrete-Event Systems. In a DES, tags are order-isomorphic with the natural numbers and assume values on the set of real numbers. The tags represent the values of the time at which the events occur.

Continuous–Time Systems. A CTS is a metric timed TSM process, where T is a connected set.

3 POWERTRAIN HYBRID MODEL

In this section, we propose a new model of a power–train with a N–cylinder 4–stroke engine. The overall system is composed of four main interacting blocks, namely the *intake manifold*, the *cylinders*, the *power-train* and the *actuators* (Figure 1). In this paper systems are represented as interacting processes of heterogeneous MOCs. The general properties of the overall system depend critically on such interaction.

The manifold pressure p is controlled by the throttle valve, which, in the electronic–throttle case, is powered by an electrical motor. We denote by V_α and α the motor input voltage and the throttle-valve position, respectively. The mass of air loaded in the cylinders depends on the pressure p and on the crankshaft revolution speed n. The torque T produced by the engine is given by $\Sigma_{i=1}^{N} T^i$, where T^i is the torque generated by the i–th cylinder, which is determined by the mass of loaded air m^i, the mass of fuel q^i injected in the cylinder, and the $spark^i$ ignition command[3]. The timing sequence of the four strokes of each cylinder is determined by the continuous motion of the crankshaft. We denote by θ the crankshaft angular position, which is obtained by the integration of the crankshaft velocity n. The ignition subsystem generates, for each cylinder, the spark signal $spark^i$ at the instant of time specified by the desired spark advance $\tilde{\varphi}^i$. The injection subsystem injects an amount of fuel q^i according to the desired value $\tilde{q}^i$. By measuring the crankshaft angle θ, both actuation systems synchronize with the evolution of the cylinders. Finally, the power-train dynamics and the crankshaft revolution speed n, controlled by the generated torque T, are subject to the sum of a number of load torques, T_l, and depend on the position of the *clutch* and the selection of the *gear*.

A detailed description of the four blocks that compose the engine and power-train system and of their interactions follows.

[2] Note that the specification for the controller may use untimed models. In this paper we focus on physical models and this is the reason for the use of timed models.

[3] From this point on, we use the superscript i to indicate variables related to the i–th cylinder.

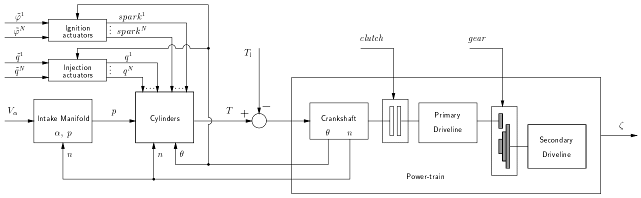

Figure 1: The power–train blocks.

3.1 The intake manifold

Manifold pressure dynamics is a continuous-time process controlled by the throttle-valve position α that changes the effective section of the intake rail of the manifold. Denoting by p the mean-value pressure, manifold dynamics is modeled as (see [10]):

$$\dot{p}(t) = a_p[n(t), p(t)]\, n(t)p(t) + b_p[p(t)]\, s[\alpha(t)] \tag{1}$$

where $s(\alpha)$ is the *equivalent throttle area*, given in terms of throttle angle α. Parameters a_p and b_p depend in a strongly nonlinear fashion on the geometric characteristics of the manifold, on the physical characteristics of the gas and atmosphere, and on the current value of the pressure p and engine speed n. The dynamics of actuation of the throttle valve is modeled by a linear first–order dynamical system.

3.2 The cylinder

The cylinder model is the most complex. The profile of the torque T^i generated by each piston depends on the phases of the cylinder, the piston position ϕ^i, the mass m^i of air, the mass q^i of fuel both loaded in the cylinder during the intake phase, and on the spark ignition timing. In a 4–stroke combustion engine, a piston reaches the *Top Dead Center* (TDC) (*Bottom Dead Center* (BDC)) when it is at its uppermost (lowermost) position. Each cylinder cycles through the following four phases:

intake (I). The piston goes down from the TDC to the BDC loading the air–fuel mix;

compression (C). The trapped mix is compressed by the piston during its upward movement;

expansion (E). The combustion takes place pushing down the piston from the TDC to the BDC;

exhaust (H). During its upward movement the piston expels combustion exhaust gases.

Let ϕ^i be the position of the i–th piston, expressed in terms of the corresponding crank angle, with respect to the last *Dead Center* (DC), that is

$$\phi^i(t) = [\theta(t) - \phi^i_0] \bmod 180^o, \tag{2}$$

where ϕ^i_0 is the value of θ for which the i-th cylinder is at a DC. This corresponds to reset ϕ^i at the beginning of each phase. Note that since the pistons are connected to the crankshaft their positions ϕ^i are related to each other.

The quantity m^i of air loaded into each cylinder at the end of the intake run depends, in a nonlinear fashion, on the evolution of the intake manifold pressure and the crankshaft speed. The amount of air loaded up to time t, denoted by $m^i(t)$, is sampled at the intake BDC time t_ℓ to obtain the loaded air for the current engine cycle. The amount of fuel q^i that can be injected is subject to constraints to limit emissions and to increase efficiency. These constraints are usually expressed in terms of the air-to-fuel ratio $A/F = \frac{m^i}{q^i}$ of the mixture. When $A/F = 14.64$, the mix is said to be at stoichiometry, which is a desirable operating point for emissions[4].

Spark ignition must occur at every cycle. It is convenient to produce a spark before the piston completes the compression stroke (*positive spark advance*), to achieve maximum fuel efficiency. Producing a spark after the piston has completed the compression phase and is in the expansion stroke (*negative spark advance*) may be used to reduce drastically the value of the torque generated during the expansion run. The spark ignition time[5]is commonly defined in terms of the spark advance φ^i, which denotes the difference between the angle of the crank at the TDC between compression and expansion and the one at the time of ignition t^i_j:

$$\varphi^i = \begin{cases} 180^o - \phi^i(t^i_j) & \text{for positive spark advance} \\ -\phi^i(t^i_j) & \text{for negative spark advance} \end{cases} \tag{3}$$

The spark advance and the amount of injected fuel is set at each cycle to control the generated torque.

The air-fuel mixture is loaded in the cylinder during the intake stroke while the torque generation starts after the spark is ignited. Hence, to complete the description of the torque generation process, we need to model the delay between the time at which the mixture is loaded and the time at which the corresponding active torque is generated. The overall model of the torque generation process for a single cylinder consists of four communicating processes of different MOCs: (1) an FSM, modeling the 4-stroke engine cycle, (2) a DES, modeling the discrete delay on the active torque generation, and (3) two memory-less CTSs, modeling

[4]Rich mixtures $A/F < 14.64$ produce excess of CO and HC, while lean mixtures $A/F > 14.64$ have excess of NO_x. The efficiency of three–way catalytic converters, commonly used to reduce emissions, is satisfactory only in a narrow range around the stoichiometry ($\pm 5\%$). This results in an interval $[q_{\min}, q_{\max}] = [15.4^{-1}, 13.9^{-1}]m^i$ of admissible values for the injected fuel.

[5]Note that the spark advance has to be bounded both from above and from below to prevent the mix from not burning uniformly thus causing undesired knocking (upper bound) and from misfiring (lower bound), which causes undesired pollutants.

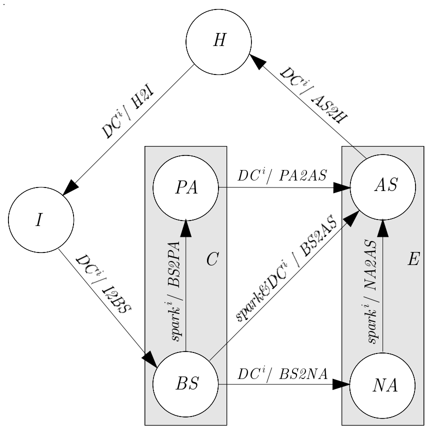

Figure 2: FSM of the i–th cylinder.

the air intake process and the profile of the generated torque.

Engine cycle FSM. This part of the cylinder model is used to capture the sequential nature of the behavior of the cylinders. Based on the events generated by the spark ignition signal and by the reaching of dead centers, the FSM takes a transition and outputs the appropriate information to coordinate the other parts. The four phases of the piston are associated to the states of an FSM that represents the behavior of the cylinder. A state transition would then occur when the piston reaches a dead center. However, the torque generated by the piston is related not only to the four phases of the piston but also to the spark generation process. Since spark ignition may occur either during the compression stroke or during the expansion stroke, a six state FSM is needed to model the possible behaviors of the cylinder. The cylinder FSM is shown in Figure 2. The FSM state s_k^i takes one of the following values: I (*Intake*), BS (*Before Spark*), PA (*Positive Advance*), NA (*Negative Advance*), AS (*After Spark*) H (*Exhaust*). The cylinder changes phase either when a spark is given (FSM input event $u_k^i = spark^i$ or $u_k^i = spark\&DC^i$ if the spark is given exactly at the dead center), or when a dead center is reached (FSM input event $u_k^i = DC^i$). The evolution of the torque produced by the cylinder depends on the transitions of the FSM, provided by the output o_k^i of the FSM that takes the following values: *BS2AS*, *BS2PA*, *BS2NA*, *PA2AS*, *NA2AS*, *AS2H*, *H2I* and *I2BS*. The next–state and output functions of the cylinder FSM

$$s_{k+1}^i = \delta(s_k^i, u_k^i), \qquad o_k^i = \lambda(s_k^i, u_k^i) \tag{4}$$

are shown in Figure 2. Note that, for the sake of notational simplicity, we dropped the superscript i, indicating the correspondence of the variable with cylinder i, from the index k.

Air-intake CTS. Assuming small variations of the crankshaft speed n during intake and recalling that p represents the pressure mean–value over the engine cycle, air intake can be described by the following memory–less CTS

$$m^i(t) = w[p(t), n(t)] \ . \tag{5}$$

This model can be further refined by describing the opening and closing of intake valves, which are synchronized with the piston position ϕ^i. In this paper, we will use $w[p(t), n(t)]$ proportional to the product $a[p(t), n(t)]\, p(t)$ with $a(\cdot)$ as in (1).

Modeling the torque profile with a CTS. The profile of the torque T^i produced by the i–th piston as ϕ^i evolves, is modeled by a memory–less CTS

$$T^i(t) = g_{o_k^i}[y_k^i, \phi^i(t)] \tag{6}$$

where o_k^i is the current FSM output and y_k^i collects the values (m^i, q^i, φ^i) and changes only at the FSM transitions.

This representation is general enough to allow the accurate description of complex torque profiles. However, in this paper, we restrict ourselves to a simpler model obtained by abstracting away the details of the combustion process as well as those related to gas pumping in and out of the cylinder. We set to zero the torque T^i during the passive phases of the cylinder, but we take into account the loss of energy due to these phases by reducing the amount of torque generated during the active phase. As a consequence of this simplification, the profile T^i is described by a piece-wise constant function that is assumed to be zero everywhere except in the expansion phase when the spark ignition command has already been given, i.e., $g_{BS2PA} = g_{BS2NA} = g_{AS2H} = g_{H2I} = g_{I2BS} \equiv 0$; $g_{PA2AS} = g_{BS2AS} = g_{NA2AS} = G_f \ q^i \ \eta(\varphi^i)$ where: the gain G_f represents the potential value of the torque that can be achieved by the given mix.

Torque generation DES. The delay on active torque generation, which is characteristic of 4–stroke engine cycles, is modeled by means of a DES synchronized with the FSM transitions and whose dynamics depends on the FSM transitions:

$$\begin{array}{rcl} z_{k+1}^i & = & f_{o_k^i}(z_k^i, v_k^i) \\ y_k^i & = & h_{o_k^i}(z_k^i, v_k^i) \end{array} \tag{7}$$

where o_k^i denotes the k–th FSM transition. The components of the DES input vector v_k^i are the mass of loaded air m^i, the mass of injected fuel q^i, and the piston position ϕ^i (used to compute the spark advance φ^i according to (3). The DES state z_k^i is used to model the delay between the mixture intake and the active torque generation, while the DES output y_k^i provides the values (m^i, q^i, φ^i) to the CTS describing the profile of the engine torque (6).

The functions $f_{o_k^i}$ and $h_{o_k^i}$ describing the dynamics

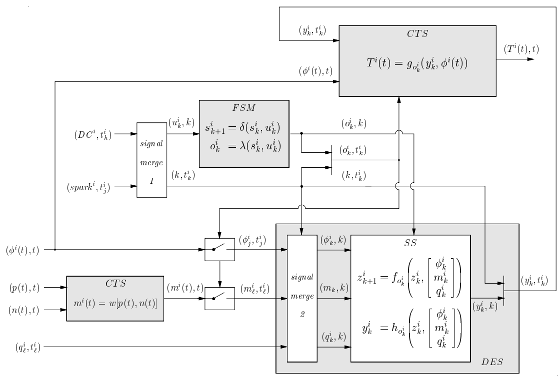

Figure 3: TSM hybrid model of the i–th cylinder.

and the output of the DES are the following:

$$\begin{array}{lllll} f_{I2BS} & = & h_{I2BS} & = & (m^i_k,\, q^i_k,\, 0) \\ f_{BS2PA} & = & h_{BS2PA} & = & z^i_k + (0,\, 0,\, 180^o - \phi^i_k) \\ f_{NA2AS} & = & h_{NA2AS} & = & z^i_k + (0,\, 0,\, -\phi^i_k) \end{array}$$

and $f_{BS2NA} = h_{BS2NA} = z^i_k$ otherwise. Consider for example the torque produced in the state AS when a positive spark advance has been applied, i.e. $T^i(t) = g_{PA2AS}[y^i_k, \phi^i(t)]$. According to the DES dynamics this torque depends on the value of the DES output y^i_k at the transition $PA \to AS$, which in turn depends on values m^i_{k-2}, q^i_{k-2} at the transition $I \to BS$, i.e.: $y^i_k = z^i_k = z^i_{k-1} + (0, 0, 180^o - \phi^i_{k-1}) = (m^i_{k-2}, q^i_{k-2}, 180^o - \phi^i_{k-1})$. This shows how the DES model captures the delays in the torque generation process: a one–step delay associated to the spark ignition and a two–step delay associated to the mix mass.

Composing the MOCs describing the cylinder. Figure 3 shows the (non trivial) interactions among the MOCs modeling the different phenomena in the cylinder behavior. Using the TSM framework, we can underline the importance of sequencing and timing of events in the torque generation process and, hence, be more effective in the synthesis of control algorithms.

A transition in the FSM is caused by two possible events, *spark*^{i} and DC^i, which may occur at the same time. The first task of the composition of the MOCs is to collect all the events that cause a transition in the FSM (producing a single input to the FSM) and the times at which these events occur. Note that the FSM uses only the information about the sequencing of events, not their exact timing. However, torque generation does need time information since it feeds a continuous time system (the model of the power–train) and the inputs to that CTS have to be correctly placed in time. Hence, the block that takes care of collecting the events is also responsible for generating the right coupling of sequence indexes and actual times.

The input signals $(spark^i, t^i_j)$, (DC^i, t^i_h), are received by the *signal_merge_1* process at times t^i_j (spark ignition times), t^i_h (DC times), respectively. This process merges the input signals to yield the FSM input signal (u^i_k, k), where k is the index of the totally ordered sequence $\{t^i_k\} = \{t^i_j\} \cup \{t^i_h\}$, so that $\{t^i_k\}$ is the sequence of times t^i_k at which the FSM associated to the i–th cylinder takes the k–th transition. Note that u^i_k takes the value $(spark\&DC^i, t^i_j)$ when $t^i_j = t^i_h$. The signals (k, t^i_k) and (o^i_k, t^i_k) are also produced as outputs to coordinate the other parts of the model. Signal (o^i_k, k) is received by the SS to select the current dynamics $f_{o^i_k}$ and output $h_{o^i_k}$. Signal (o^i_k, t^i_k) is used by the torque profile CTS and by the two samplers. The sequence of sampling times $\{t^i_j\}$ (corresponding to the spark ignition times) is extracted from the signal (o^i_k, t^i_k), by taking the tags t^i_k for which o^i_k assumes value in $\{BS2PA, BS2AS, NA2AS\}$. Similarly, sampling times $\{t^i_\ell\}$ (corresponding to intake BDC times) are extracted from the signal (o^i_k, t^i_k), by taking the tags t^i_k for which o^i_k assumes the value $I2BS$.

The CTS that represents torque generation needs the piston position, a continuous time variable, the phase of the cylinder (provided directly by the FSM output), the amount of air and fuel that was loaded at the end of the intake phase as well as the spark advance. The DES delivers the information about the amount of air and fuel at the end of the intake phase as well as the spark advance, all appropriately placed in physical time by virtue of the signal merge taking place at its output. The inputs to the SS are generated by merging the appropriate signals so that the sequencing is synchronized with the input of the FSM. In doing so, we are "synchronizing" non synchronous signals by assigning the special value $\perp$ (absence of value) to signals that do not have a value for a particular tag that is of interest for another signal. In particular, at intake BDC times t^i_ℓ, the *signal_merge_2* process receives the signals (q^i_ℓ, t^i_ℓ) and (m^i_ℓ, t^i_ℓ) - where the value m^i_ℓ is equal to $m^i(t^i_\ell)$. At spark ignition times t^i_j, it further receives the signal (ϕ^i_j, t^i_j) (piston position at the time of the spark), to compute the spark advance according to (3). For each of the previous signals - say (a_m, t^i_m) - on the basis of the signal (k, t^i_k), the *signal_merge_2* process yields the SS input signals (ϕ^i_k, k), (q^i_k, k), (m^i_k, k) applying the following rule:

$$(a_{\overline{k}}, \overline{k}) = \begin{cases} (a_{\overline{m}}, \overline{k}) & \text{if } (a_{\overline{m}}, t^i_{\overline{k}}) = (a_{\overline{m}}, t^i_{\overline{m}}) \in (a_m, t^i_m) \\ (\perp, \overline{k}) & \text{otherwise} \end{cases}$$

This makes the output signals of the *signal_merge_2* process and of the FSM synchronous. Note that, setting $a_{\overline{k}} = \perp$ when $t^i_{\overline{m}} \neq t^i_{\overline{k}}$ does not change the behavior of the system since the SS does not read this input when the tag k is equal to $\overline{k}$. Since $\{t^i_m\} \subset \{t^i_k\}$, then, by the previous rule, the sequence $\{a_m\}$ is appropriately augmented and defined over all times t^i_k.

From the SS output (y^i_k, k) the DES output signal (y^i_k, t_k) is generated by replacing the signal tag k with

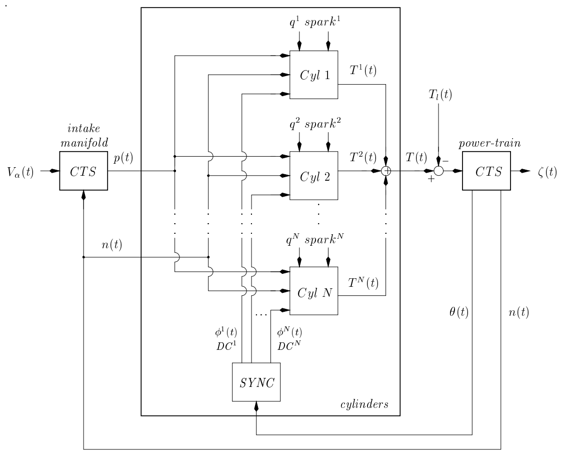

Figure 4: Composition of cylinders models. For the sake of simplicity, in this figure tags are not reported.

the corresponding time t_k. Then, this value is given as a parameter to the CTS to generate the output torque $T^i(t)$.

3.3 The power–train

For a given *gear* selection and *clutch* position, the power-train is described by the CTS

$$\begin{aligned} \dot{\zeta}(t) &= A\,\zeta(t) + b\,(T(t) - T_l(t)) - b_0 && (8) \\ \dot{\theta}(t) &= (0,\ 6,\ 0)\,\zeta(t) && (9) \end{aligned}$$

where $\zeta = (\alpha_e, n, \omega_p)^T$ includes the drive-line torsion angle, the crankshaft revolution speed, and the wheel revolution speed and θ is the crankshaft angle position. Input T is the torque produced by the engine, while T_l represents the load torque acting on the crankshaft. Vector b_0 models the resistant actions on the power-train, due to internal friction and external forces at the equilibrium point. Dynamics (8) is exponentially stable and is characterized by a real dominant pole λ_1, and a pair of conjugate complex poles $\lambda \pm j\mu$.

3.4 The engine and power–train model

Figure 4 shows the hybrid model for vehicles with 4–stroke N–cylinder gasoline engine. Such model is obtained by combining N cylinder hybrid models and it is composed of the following parts:

- N subsystems as in Figure 3 describing the behavior of the N cylinders of the engine.
- two CTS's modeling, respectively, the power-train and intake manifold dynamics.
- the block $SYNC$, that synchronizes the cylinders models by generating the piston position angles ϕ^i from the crankshaft angle θ, according to (2), and the events DC^is.

4 CONCLUSIONS

We presented the application of hybrid system techniques to an important industrial domain: automotive engine and power-train control. We argued that while in the past average models were successfully used to describe the behavior of the engine, the ever increasing demands on drive comfort, safety, emissions and fuel consumption imposed by car manufacturers require cycle–accurate models, which can only be described using a hybrid formalism.

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