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Optimal actuator guidance scheme for a 2-D thermal manufacturing processing

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Abstract
This article addresses the need for optimal actuation in distributed-parameter systems dominated by conductive heat transfer, such as in thermal processing of materials. An LQR-based approach is developed, which determines the optimal location and the power of the actuator at successive time periods, by piece-wise minimization of the quadratic performance indices corresponding to theses subintervals. This algorithm is applied to a finite dimensional representation of the temperature state and heat input distributions, yielding a suboptimal but computationally efficient heat source guidance strategy. This is shown to warrant superior temperature tracking performance over fixed —location actuation, by FEA simulations.

Thermal Control

Distributed parameter control systems have been developed in the past for thermal processing. Ummethela [1] used a linear Gaussian controller in order to develop a technique that optimally distributes heat inputs across a controlled object. The controller rejects successfully the heat loss disturbances. This technique in essence separates the spatial and temporal part of the governing partial differential equation, which results in the steady state equation for heat conduction within the controlled object. The steady state Green s function that corresponds to that equation determines the state-space model, which is used to minimize the quadratic performance index.

An LQR-based optimal actuator guidance strategy was developed in [3] and tested numerically and analytically in thermal processing of a one-dimensional material strip, dominated by conductive heat transfer. The controller provides the optimal location and power of the actuator at successive time periods for 10 1-D conductive elements corresponding to each part subregion.

In this manuscript we extend the previous result to the 2-D case. An analytical model of thermal processing has been developed, which for this case is a finite two-dimensional approximation of the associated infinite dimensional system. This model is used for the computer simulation and the off-line testing of the LQR controller. This study provides information about the design of the LQR controller and the behavior of a real-time closed loop control system.

Abstract mathematical model

The process dynamics are described by the two dimensional partial differential equation given by:

$$\rho c \frac{\partial T}{\partial t} = \nabla(k \nabla T) + hT + \delta(\xi - \xi_0) Q(t) \,, \qquad (2)$$

with Neumann boundary condition(zero flux)

$$\frac{\partial T(0,y,t)}{\partial x} = 0 = \frac{\partial T(X,y,t)}{\partial x} \quad , \quad \frac{\partial T(x,0,t)}{\partial y} = 0 = \frac{\partial T(x,Y,t)}{\partial y} \tag{3}$$

where $\delta(\xi)$ is the two dimensional delta function that denotes the location of the heat source . Q(t) is the thermal input and it is given by the difference between the actual heat input $Q_a(t)$ that corresponds to the actual temperature field in each location $T_a(\xi)$ and the desired power $Q_d(t)$, that must be generated according to
the desired temperature field $T_d(\xi)$.

The system (2), (3) can be described by an abstract evolution equation in an appropriate Hilbert [4] space H:

$$\dot{T}(t) = AT(t) + B(\xi_0)Q(t), \tag{4}$$

where T(t) is the state of the infinite dimensional system, A is the spatial operator given by the formula:

$$A\phi = \frac{\nabla(k\nabla\phi) + h\phi}{\rho c}$$

and $B(\xi_0)$ is the location-parameterized input operator given by

$$\left\langle B(\xi_0)Q,\phi \right\rangle = \int_\Omega \delta(\xi - \xi_0)Q(t)\phi(\xi)d\xi = \phi(\xi_0)Q(t)$$

for all test functions $\varphi \in L_2(\Omega)$.
The A and $B(\xi_0)$ operators can be approximately controllable, exponentially stabilizable and optimizable for certain locations ξ_0 in the domain Ω, [4]. Using an approximation scheme the preserves exponential stabilizability we arrive at the finite dimensional approximation of (4) which is given by

$$\dot{T}(t) = AT(t) + B(\xi_0)Q(t)$$

where now A is the matrix representation of the operator A and $B(\xi_0)$ is the vector representation of the operator $B(\xi_0)$, [3].

We are using an LQR performance index for control policy. This quadratic performance index is given by

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left[\left\langle T(\tau), ST(\tau)\right\rangle_H + RQ^2(\tau)\right] d\tau + \left\langle T(t_f), MT(t_f)\right\rangle_H$$

The optimal feedback law for the interval (t_0,t_f) is given by

$$Q(t) = -R^{-1}B^*(\xi_0)P(t)T(t)$$

where *P(t)* is the solution to the operator differential Riccati equation (DRE)

$$\frac{d}{dt}\left\langle \varphi_2, P(t)\phi_1\right\rangle = -\left\langle \varphi_2, P(t)A\varphi_1\right\rangle - \left\langle A\phi_2, P(t)\phi_1\right\rangle - \left\langle S\phi_2,\phi_1\right\rangle + \left\langle P(t)B(\ {}_0)R^{-1}B(\ {}_0)P(t)\phi_2,\phi_1\right\rangle$$

The minimum of the location-parametrized cost function for the interval (t_0,t_f) is

$$J^*(\xi_0) = \left\langle T(t_0), P(t_0,\xi)T(t_0)\right\rangle_H .$$

To find the optimal actuator location for the time interval, the above optimal cost is minimized with respect to the possible actuator locations.

Now, to find what the guidance policy should be, the above minimization procedure is performed in a smaller time subinterval of fixed length.
Therefore the optimal location is found by minimizing the above J* with respect to all possible actuator locations. This requires the solution of many differential Riccati equations in each time subinterval $[t_s, t_s+\Delta t]$.

This optimal location control signal is extremely computationally intensive. In order to reduce the computational load one can follow the procedure proposed in [5] by solving instead the algebraic Riccati equation

$$A^T P(\xi) + P(\xi)A + S - P(\xi)B(\xi)R^{-1}B^T(\xi)P(\xi) = 0$$

for all the possible locations. The optimal actuator location in each time subinterval is then found as the one that minimizes $T'(t)P(\xi)T(t)$.

Simulation of the real-time closed loop system

A plain square steel plate (1040) measuring 0.1 x 0.1 x 0.003 m has been used as the example specimen for simulation of the closed loop control system. The metallic plate has uniform geometry and conductive thermal properties. The material properties include the density ρ=8000 kg/m^3, specific heat c=500 J/kgK, thermal conductivity k=70 W/mK and heat transfer coefficient h=-10W/m^2K.

The purpose of the controller is to eliminate the difference between the desired temperature T_d and the actual temperature T_a (ξ_i,t) uniformly over the space for each subinterval t_s. Thus the actuator is driven to the respective optimal location $\xi_i(t_s)$ while the actual heat input is provided according to the expression $Q_a(t_s)=Q_d$ — $Q(t_s\ \xi_i)$. Q_d is the input, which corresponds to the desired temperature T_d at the steady state of the thermal system. For a desired temperature Td=800 K the Qd is estimated taken into account the heat transfer coefficient h=-10W/ m^2K , Q_d=-h[T_d-T(0)](0.1x 0.1)=-200 W.

The thermal control scheme is applied on a numerical model of the thermal processed specimen. The numerical model employs a 2-D finite element analysis of n_x x n_y = 4 x 4 =16 conductive elements. The ambient temperature T(0)=300 K for all elements.

Figures 1 illustrates the resulting trajectory of the heat source according to the optimal location $\xi^*(t_s)$ at times t=0,4,8,12,16,20 sec.

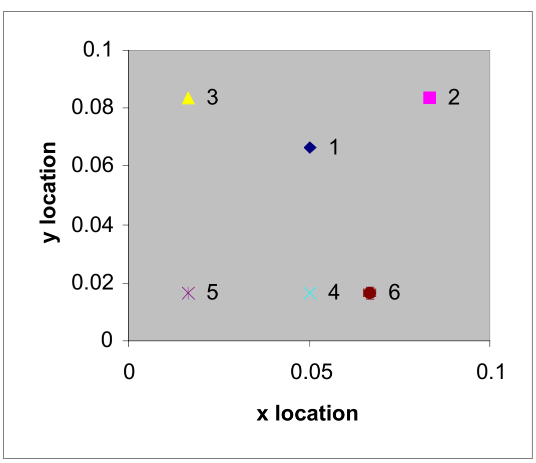

Figure 1: Simulated heat trajectory.

Figure 2 to 6 illustrate the change of the temperature profile of the specimen s area at the respective time t subintervals. The initial temperature is effectively brought to the desired one T_d within the process time interval of 20 sec as it is also shown in figure 7. In order to compare the efficiency of the actuator guidance technique the LQR controller was applied to a fixed actuator positioned in the middle of the specimen. The simulations results from this application are shown in figure 8. The resulting temperature distribution profile exceeds the desired temperature level within the process time.

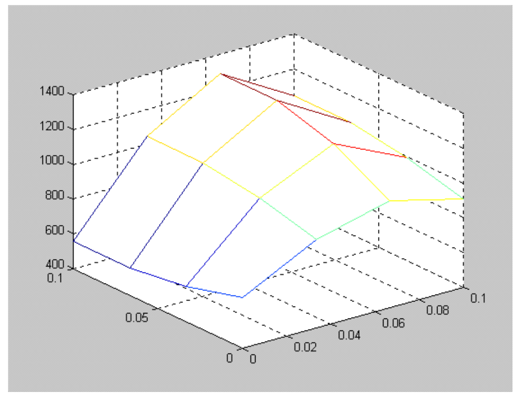

Figure 1: Temperature profile at t=0sec

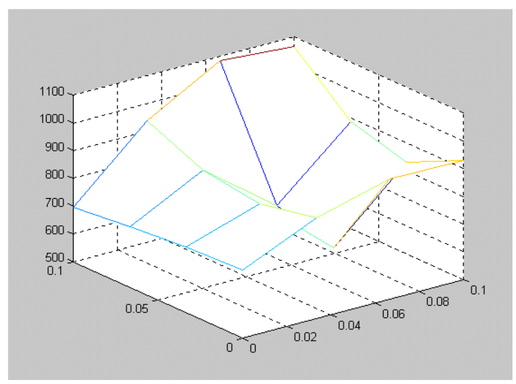

Figure 2: Temperature profile at t=4sec

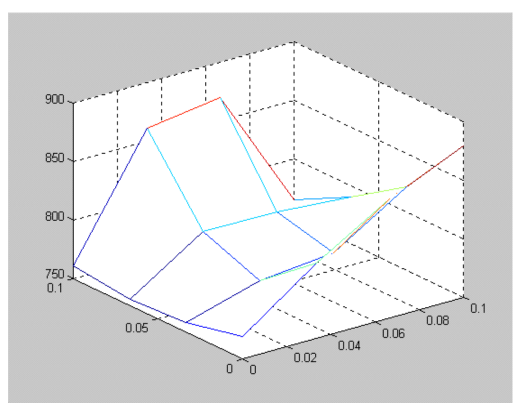

Figure 3: Temperature profile at t=8sec

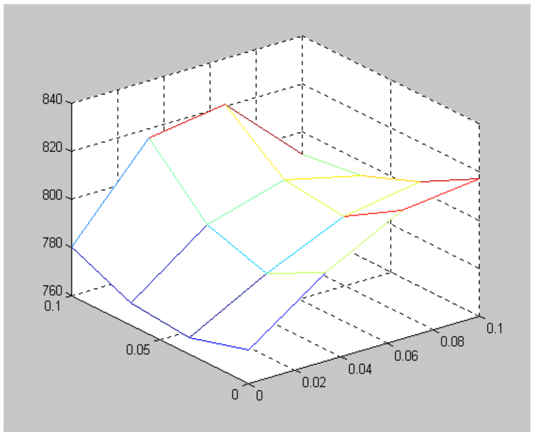

Figure 4: Temperature profile at t=12sec

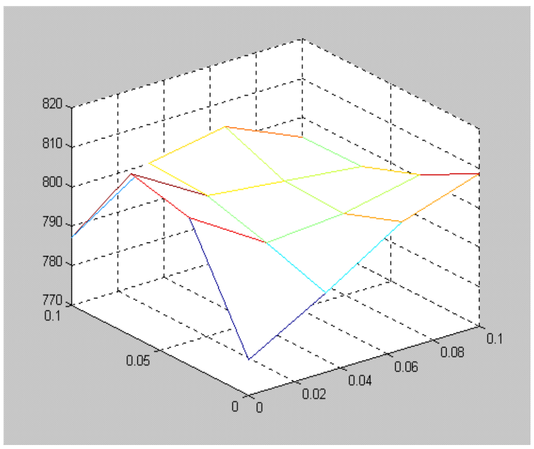

Figure 5: Temperature profile at t=16sec

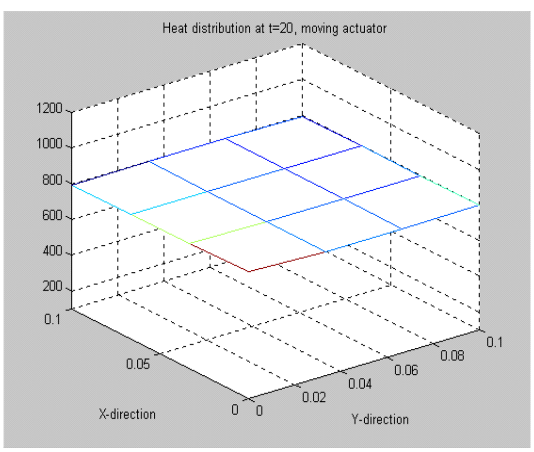

Figure 6: Temperature profile at t=20sec

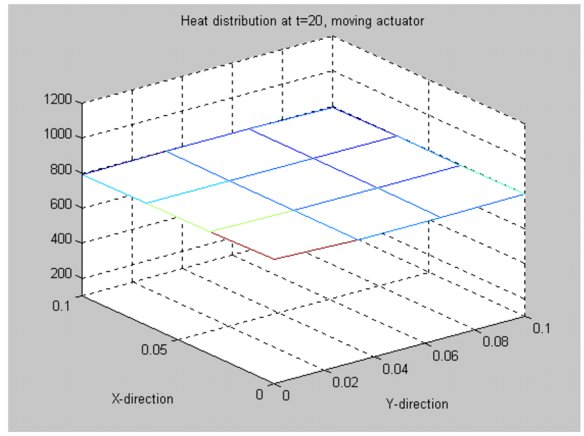

Figure 7: Simulated temperature profiles
T(ξ) of a moving actuator at time t_s=20 sec.

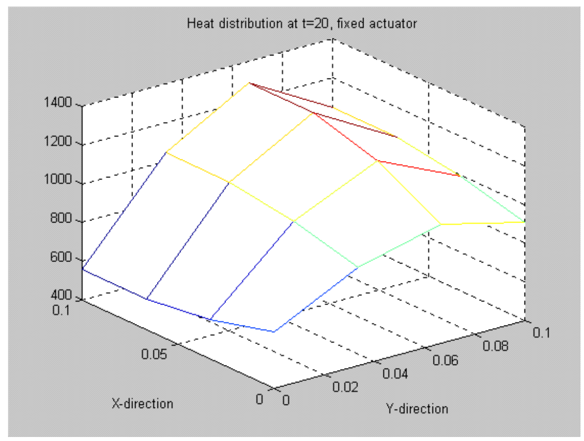

Figure 8: : Simulated temperature profiles
T(ξ)of a fixed actuator at time t_s=20 sec.

Conclusion

An LQR —optimal based actuator guidance strategy was developed in distributed parameter systems, such as in thermal processing of materials, dominated by conductive heat transfer. The optimal location and power of the actuator at successive time periods was determined by minimization of the quadratic performance indices at these subintervals. A finite —dimensional representation of the heat input distribution and the temperature state provided a computationally efficient heat source guidance strategy. This was verified by FEA simulations results that also indicate the superior temperature tracking performance when fixed location actuator is applied.

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