Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

SYMBOLIC IMPLEMENTATION OF LEVERRIER-FADDEEV ALGORITHM AND APPLICATIONS

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Abstract. In this paper we propose two algorithms for the computation of the Drazin inverse, based on the Leverrier-Faddeev algorithm. These algorithms represent extensions of paper [3] and a continuation of the paper [4]. A few matrix equations which include rational matrices are solved by means of the Drazin inverse and the Moore-Penrose inverse of rational matrices.
Keywords. Drazin inverse, Leverrier-Faddeev algorithm, matrix equations.

1 Introduction

Let C be the set of complex numbers, $C^{m\times n}$ be the set of $m \times n$ complex matrices, and $C_r^{m\times n} = \{X \in C^{m\times n} : \ \mathrm{rank}(X) = r\}$. As usual, $C[s]$ (resp. $C(s)$) denotes the polynomials (resp. rational functions) with complex coefficients in the indeterminate s. The $m\times n$ matrices with elements in $C[s]$ (resp. $C(s)$) are denoted by $C[s]^{m\times n}$ (resp. $C(s)^{m\times n}$). By I_r and I we denote the identity matrix of the order r, and an appropriate identity matrix, respectively. By O is denoted an appropriate null matrix. We denote by $\mathrm{Tr}(A)$ the trace of A.

For any matrix A of the order $m\times n$ consider the following equations in X, where $*$ denotes conjugate and transpose:

$$\begin{array}{llll} (a) & AXA = A & (b) & XAX = X \\ (c) & (AX)^* = AX & (d) & (XA)^* = XA, \end{array}$$

and if $m = n$, also

$$(e) \quad AX = XA \quad (f) \quad A^{k+1}X = A^k.$$

If X satis es (a) is said to be a $\{1\}$-inverse of A, whereas $X = A^\dagger$ is said to be the Moore-Penrose inverse of A if it satis es (a) (d). The group inverse $A^\#$ (known as $\{1,2,5\}$ inverse of A), is unique, satis es {a,b,e} and exists iff $\mathrm{ind}(A) = \min_k\{k : \mathrm{rank}(A^{k+1}) = \mathrm{rank}(A^k)\} = 1$. A matrix $X = A^D$ is said to be the Drazin inverse of A if (f) (for some positive integer k), (b) and (e) are satis ed. In the case $\mathrm{ind}(A) = 1$, the Drazin inverse of A is equal to the group inverse of A.

Penrose [5] and Decell [2], have adapted the Leverier-Faddeev algorithm (also called Souriau-Frame) to the calculation of the Moore-Penrose inverse of a Hermitian matrix, while Karampetakis [4] extend the results to the rational matrices. A modi cation of the Leverrier-Faddeev algorithm for obtaining the Drazin inverse of a constant square was given in [3].

It is known that the Leverrier-Faddeev algorithm is ill-conditioned. This fact is a motivation for the use of symbolic programming languages in its implementation [4] where a) variables can be stored without loss of accuracy during calculations and b) symbolic computation is free from the truncation error.

Main aim of this paper is the representation and characterization of the Drazin inverse of rational matrices. The paper is organized as follows. In the second section we restate well-known modi cation of the Leverrier-Faddeev algorithm for computation of the Drazin inverse, introduced in [3]. In the third section we investigate an extension of the Grevile s modi cation of the Leverrier-Faddeev algorithm, to the set of one-variable nonregular rational matrices. The proposed results can be considered as a continuation of the paper [4], and a generalization of the results from [3]. Finally, in the last section we investigate solutions of a few system of matrix equations, by means of the Drazin inverse and the Moore-Penrose inverse of rational matrices.

2 Preliminaries

In this section we consider constant square complex matrices of the form $A(s) \equiv A \in C^{n\times n}$. In [3] is introduced the following representation of the Drazin inverse.

Theorem 1 *[3] Consider the matrix $A \in C^{n\times n}$. Assume that*

$$a(z) = \det[zI_n - A] = \\ = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \quad a_0 = 1, \quad z \in \mathbb{C}$$

is the characteristic polynomial of A. Also, consider the following sequence of $n \times n$ matrices

$$B_j = a_0 A^j + a_1 A^{j-1} + \cdots + a_{j-1} A + a_j I_n, \\ a_0 = 1, \quad j = 0, \dots, n$$

Let r denote the smallest integer such that $B_r = O$, let t denote the largest integer satisfying $a_t \neq 0$, and let $k = r - t$. Then, the Drazin pseudoinverse of A is given by

$$A^D = a_t^{-k-1} A^k B_{t-1}^{k+1} = \tag{1}$$

$$= a_t^{-k-1} A^k \left(A^{t-1} + a_1 A^{t-2} + \cdots + a_{t-1} I_n\right)^{k+1}$$

Also, in [3] is proposed the following algorithm for computation of the Drazin inverse of A.

Algorithm 2 *Consider $A \in C^{n\times n}$.*

Step 1. *Construct the sequence of complex numbers $\{a_0, a_1, \dots, a_n\}$ and the sequence of $n \times n$ matrices $\{B_0, B_1, \dots, B_n\}$ in the following way:*

$$\begin{array}{lll} A_0 = 0 & a_0 = 1 & B_0 = I_n \\ A_1 = AB_0 & a_1 = -\frac{Tr(A_1)}{1} & B_1 = A_1 + a_1 I_n \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ A_n = AB_{n-1} & a_n = -\frac{tr(A_n)}{n} & B_n = A_n + a_n I_n \end{array} \tag{2}$$

Step 2. *Let*

$$t = \max\{l : a_l \neq 0\}, \quad r = \min\{l : B_l = \mathbb{O}\} \\ k = r - t$$

Then the Drazin inverse A^D is given by

$$A^D = a_t^{-k-1} A^k B_{t-1}^{k+1}.$$

3 Drazin inverse of rational matrices

In the beginning we introduce an extension of the Theorem 1, assuming that $A(s) \in C(s)^{n\times n}$ is a rational matrix, where the variable s is an indeterminate. Also, the algorithm can be used when an arbitrary value is assigned to the variable s, in which case we obtain the Grevile s algorithm. The proof of the following theorem is similar with the corresponding one from [3].

Theorem 3 *Consider a nonregular one-variable rational matrix $A(s)$. Assume that*

$$a(z, s) = \det[zI_n - A(s)] = \\ = a_0(s) z^n + a_1(s) z^{n-1} + \cdots a_{n-1}(s) z + a_n(s), \\ a_0(s) \equiv 1, \quad z \in \mathbb{C}$$

is the characteristic polynomial of $A(s)$. Also, consider the following sequence :

$$B_j(s) = a_0(s) A(s)^j + a_1(s) A(s)^{j-1} + \cdots + a_j(s) I_n,$$

$$a_0(s) = 1, \quad j = 0, \dots, n \tag{3}$$

Let

$$a_n(s) \equiv 0, \dots, a_{t+1}(s) \equiv 0, \quad a_t(s) \neq 0.$$

De ne the following set:

$$\Lambda = \{s_i \in \mathbb{C}\text{: } a_i(s_i) = 0\} \tag{4}$$

Also, assume

$$B_n(s) \equiv \mathbb{O},\ldots,B_r(s) = 0, B_{r-1}(s) \neq 0$$

and $k = r - t$. *In the case* $s \in C\backslash\Lambda$ *and* $k > 0$, *the Drazin inverse of* $A(s)$ *is given by*

$$A(s)^D = a_t(s)^{-k-1}A(s)^k B_{t-1}(s)^{k+1} =$$
$$= a_t(s)^{-k-1}A(s)^k \left[a_0(s)A(s)^{t-1} +.+ a_{t-1}(s)I_n\right]^{k+1}$$

In the case $s \in C\backslash\Lambda$ *&* $k = 0$, *we get* $A(s)^D = O$.

For $s_i \in \Lambda$ *denote by* t_i *the largest integer satisfying* $a_{t_i}(s_i) \neq 0$, *and by* r_i *the smallest integer satisfying* $B_{r_i}(s_i) \equiv O$. *Then the Drazin inverse of* $A(s_i)$ *is equal to*

$$A(s_i)^D = a_{t_i}(s_i)^{-k_i-1}A(s_i)^{k_i}B_{t_i-1}(s_i)^{k_i+1} =$$
$$= \frac{A(s_i)^{k_i}\left[a_0(s_i)A(s_i)^{t_i-1} + .. + a_{t_i-1}(s_i)I_n\right]^{k_i+1}}{a_{t_i}(s_i)^{k_i+1}}$$

where $k_i = r_i - t_i$. ∎

In view of the results of Theorem 3 we present the following algorithm for computation of the Drazin inverse. This algorithm is a generalization of the Algorithm 2.

Algorithm 4 *It is assumed that* $A(s) \in C(s)^{n\times n}$ *is a given rational matrix.*

Step 1. *Construct the sequence of rationals* $\{a_0(s), a_1(s), \ldots, a_n(s)\}$ *and the sequence of* $n \times n$ *rational matrices* $\{B_0(s), B_1(s), \ldots, B_n(s)\}$ *in the following way:*

$$\begin{array}{lll} A_0 = 0 & a_0(s) = 1 & B_0 = I_n \\ A_1 = AB_0 & a_1(s) = -\frac{Tr(A_1)}{1} & B_1 = A_1 + a_1 I_n \\ \ldots.. & \ldots.. & \ldots.. \\ A_n = AB_{n-1} & a_n(s) = -\frac{tr(A_n)}{n} & B_n = A_n + a_n I_n \end{array} \tag{5}$$

Step 2. *Let*

$$t = \max\{l : a_l(s) \neq 0\},$$
$$r = \min\{l : B_l(s) = \mathbb{O}\} \quad k = r - t$$

For $s \in C\backslash\Lambda$ *the Drazin inverse* $A(s)^D$ *is given by*

$$A(s)^D = a_t(s)^{-k-1}A(s)^k B_{t-1}(s)^{k+1} \tag{6}$$

For those $s_i \in \Lambda$, *denote by* t_i *the largest integer satisfying* $a_{t_i}(s_i) \neq 0$, *and by* r_i *the smallest integer satisfying* $B_{r_i}(s_i) \equiv O$. *For the integer* $k_i = r_i - t_i$, *the Drazin inverse of* $A(s_i)$ *is equal to*

$$A(s_i)^D = a_{t_i}(s_i)^{-k_i-1}A(s_i)^{k_i}B_{t_i-1}(s_i)^{k_i+1} \tag{7}$$

Example 5 *Consider the rational matrix*

$$A = \begin{bmatrix} 1+w & \frac{1}{w} & 1+w \\ \frac{1}{w} & -1+w & \frac{1}{w} \\ 1+w & \frac{1}{w} & 1+w \end{bmatrix}$$

Applying the algorithm 4, we obtain the following intermediate results for the variables a_j *and matrices* B_j*:*

$a_1 = 1 + 3w$
$B_1 = \{\{-2w, \frac{1}{w}, 1+w\}, \{\frac{1}{w}, -2(1+w), \frac{1}{w}\}, \{1+w, \frac{1}{w}, -2w\}\}$
$a_2 = 2 + \frac{2}{w^2} - 2\ w^2$
$B_2 = \{\{-1-\frac{1}{w^2}+w^2,\ 0,\ 1+\frac{1}{w^2}-w^2\},\ \{0,\ 0,\ 0\},\ \{1 + \frac{1}{w^2} - w^2,\ 0,\ -1 - \frac{1}{w^2} + w^2\}\}$
$a_3 = 0$
$B_3 = \{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

Then $t = 2, r = 3, k = r - t = 1$ and the Drazin inverse of A is equal to

$$A^D = a_2^{-1}A^1B_1^2 =$$
$$= \begin{bmatrix} \frac{(-1+w)w^2}{4(-1-w^2+w^4)} & \frac{w}{2+2w^2-2w^4} & \frac{(-1+w)w^2}{4(-1-w^2+w^4)} \\ \frac{w}{2+2w^2-2w^4} & \frac{(1+w)w^2}{-1-w^2+w^4} & \frac{w}{2+2w^2-2w^4} \\ \frac{(-1+w)w^2}{4(-1-w^2+w^4)} & \frac{w}{2+2w^2-2w^4} & \frac{(-1+w)w^2}{4(-1-w^2+w^4)} \end{bmatrix}$$

4 Drazin inverse and some matrix equations

In this section we solve a few systems of matrix equations, using the Moore-Penrose and the Drazin inverse of a given rational matrix.

In [4] is investigated the rational matrix equation $A(s)X(s)B(s) = C(s)$, as a generalization of the matrix equation $AXB = C$, investigated in [5]. We restate here this result for the sake of completeness.

Lemma 6 *[4] The matrix equation* $A(s)X(s)B(s) = C(s)$, $A(s) \in C(s)^{m\times n}$, $B(s) \in C(s)^{p\times q}$, $C(s) \in C(s)^{m\times q}$ *has a solution if and only if*

$$A(s)A(s)^{\dagger}C(s)B(s)^{\dagger}B(s) = C(s) \tag{8}$$

In this case, all the solutions are given by the formula

$$X(s) = A(s)^{\dagger}C(s)B(s)^{\dagger} + \tag{9}$$

$$+Y(s) - A(s)^{\dagger}A(s)Y(s)B(s)B(s)^{\dagger}$$

where $Y(s)$ *is arbitrary and has the dimensions of* $X(s)$. *As it is in [4] mentioned, (8) and (9) hold when* $A(s)^{\dagger}$ *and* $B(s)^{\dagger}$ *are replaced by speci c* $\{1\}$-*inverses* $A(s)^{(1)}$ *and* $B(s)^{(1)}$, *respectively.*

We also use the following result.

Lemma 7 *System of rational matrix equations*

$$A(s)X(s) = B(s), \qquad X(s)D(s) = E(s) \tag{10}$$

has a common solution if and only if each of these equations has a solution and

$$A(s)E(s) = B(s)D(s) \tag{11}$$

In the following de nition we introduce a notion of the index of rational square matrices.

De nition 8 *The index of a given square rational matrix* $A(s) \in C(s)^{n\times n}$, *denoted by* $ind(A(s))$, *is equal to*

$$ind(A(s)) = \min_{k} \left(\begin{array}{c} k : \forall s \in \mathbb{C}\,, \\ rank(A(s)^k) = rank(A(s)^{k+1}) \end{array} \right)$$

In Lemma 9 and Lemma 10 we generalize known representations of the Drazin inverse and weak Drazin inverse of a given constant matrix from [1].

Lemma 9 *Consider a rational matrix* $A(s) \in C(s)^{n\times n}$ *satisfying* $ind(A(s)) = k$. *Let the rational matrix, denoted by* $A_c(s)$, *be any solution of the system of rational matrix equations*

$$A(s)^l = X(s)A(s)^{l+1} \tag{12}$$

$$A(s)X(s) = X(s)A(s) \tag{13}$$

where $l \geq k$ *is an arbitrary integer. Then* $A_c(s)$ *satis es the following equalities:*

$$A(s)^l(A_c(s))^l = (A_c(s))^l A(s)^l, \tag{14}$$

$$A(s)^l(A_c(s))^l A(s)^l = A(s)^l \tag{15}$$

P roof. (14) follows from $A(s)A_c(s) = A_c(s)A(s)$. We verify (15). Because of $\text{rank}(A(s)^{k+1}) = \text{rank}(A(s)^k)$, it is easy to conclude the existence of the matrix $A_c(s)$. Using (12) and (14), we get

$$\begin{aligned} A(s)^l(A_c(s))^l A(s)^l &= A(s)^{2l}(A_c(s))^l = \\ &= A(s)^{l-1}A(s)^{l+1}(A_c(s))^l = \\ &= A(s)^{l-1}(A_c(s))^{l-1}A_c(s)A(s)^{l+1} = \\ &= A(s)^{l-1}A(s)^l(A_c(s))^{l-1} = \\ &= A(s)^{2l-1}(A_c(s))^{l-1} = ... = A(s)^{l+1}A_c(s) = A(s)^l \end{aligned}$$

Hence (15) is veri ed. ∎

Lemma 10 *Let* $A(s) \in C(s)^{n\times n}$, $ind(A(s)) = k$ *and* $l \geq k$. *Let* $A_c(s)$ *be any solution of the system of matrix equations (12) and (13). Then*

$$A(s)^D = A(s)^l(A_c(s))^{l+1} = (A_c(s))^{l+1}A(s)^l \tag{16}$$

P roof. Using the known representation of the Drazin inverse from [1], it is not difficult to verify the following representation of the Drazin inverse $A(s)^D$:

$$A(s)^D = A(s)^l(A(s)^{(2l+1)})^{(1)}A(s)^l.$$

Also, in view of (15), we get $(A_c(s))^l \in A(s)^l\{1\}$ and

$$A(s)^D = A(s)^l(A_c(s))^{2l+1}A(s)^l$$

Now, using (14) and (15) it follows that

$$\begin{aligned} A(s)^D &= A(s)^l(A_c(s))^l A(s)^l(A_c(s))^{l+1} = \\ &= A(s)^l(A_c(s))^{l+1} \end{aligned}$$

∎

Theorem 11 *Let* $A(s) \in C(s)^{n\times n}$ *and* $ind(A(s)) = k$. *For arbitrary integers* $l \geq k$ *and* $m \geq k$ *each of the following matrix equations*

$$A(s)^l = X(s)A(s)^{l+1} \tag{17}$$

$$A(s)^m = A(s)^{m+1} X(s) \tag{18}$$

has the general solution, which are represented by the following expressions, respectively:

$$X(s) = A(s)^D + Y(s)(I - A(s)A(s)^D) \tag{19}$$

$$X(s) = A(s)^D + (I - A(s)A(s)^D)W(s) \tag{20}$$

where $Y(s)$ and $W(s)$ are appropriate rational matrices of the order $n \times n$. Also, the system of matrix equations (12) and (18) has the following general solution:

$$X(s) = A(s)^D + (I - A(s)A(s)^D) \times$$

$$\times Z(s)(I - A(s)A(s)^D) \tag{21}$$

where $Z(s)$ is an arbitrary $n \times n$ rational matrix.

P roof. According to Lemma 6, it is not difficult to verify consistency of the matrix equation (12). Indeed, using Lemmata 9 and 10, we get

$$A(s)^l (A(s)^{l+1})^{(1)} A(s)^{l+1} = A(s)^l (A_c(s))^{l+1} A(s)^{l+1} =$$
$$= A(s)^D A(s)^{l+1} = A(s)^l$$

which is a veri cation of Lemma 6 for this case. Also, according to Lemmata 6, 9 and 10, the general solution of (12) is equal to

$$X(s) = A(s)^l (A(s)^{l+1})^{(1)} +$$
$$+ Y(s) - Y(s) A(s)^{l+1} (A(s)^{l+1})^{(1)} =$$
$$= A(s)^l (A_c(s))^{l+1} + Y(s) \left(I - A(s)A(s)^l (A_c(s))^{l+1}\right) =$$
$$= A(s)^D + Y(s)(I - A(s)A(s)^D)$$

In a similar way one can verify that (20) is a general solution of (18). Now we derive a general solution of the system of matrix equations (12) and (18). Since $A(s)^{m+1} A(s)^l = A(s)^m A(s)^{l+1}$, according to Lemma 7 we conclude consistency of the system of matrix equations (12) and (18). After a substitution of (19) in (18) we get

$$A(s)^m = A(s)^{m+1} \left[A(s)^D + Y(s)(I - A(s)A(s)^D)\right] \Rightarrow$$
$$A(s)^{m+1} \left[Y(s)(I - A(s)A(s)^D)\right] = \mathbb{O}.$$

Applying again the result of Lemma 6, we verify consistency of this equation and get the following general solution:

$$Y(s) = Z(s) - (A(s)^{m+1})^{(1)} A(s)^{m+1} Z(s) \times$$
$$\times (I - A(s)A(s)^D)(I - A(s)A(s)^D)^{(1)}$$

where $Z(s)$ is an arbitrary rational matrix of appropriate dimensions. Since the matrix $I – A(s)A(s)^D$ is idempotent, it follows that $I \in (I - A(s)A(s)^D)\{1\}$. Also, according to Lemma 9 :

$$Y(s) = Z(s) - (A_c(s))^{m+1} A(s)^m A(s) \times$$
$$\times Z(s)(I - A(s)A(s)^D)$$

Using (16) we obtain

$$Y(s) = Z(s) - A(s)^D A(s) Z(s)(I - A(s)A(s)^D) =$$

$$= (I - A(s)A(s)^D) Z(s)(I - A(s)A(s)^D) \tag{22}$$

A substitution of (22) in (19) leads to (21). ■

Theorem 12 *Let $A(s) \in C(s)^{n \times n}$ satis es $ind(A(s)) = k$. For arbitrary integers $l \geq k$ the following system of matrix equations*

$$A(s)X(s)A(s) = A(s) \tag{23}$$

$$A(s)^l = X(s)A(s)^{l+1}$$

has the general solution

$$X(s) = A(s)^D + Z(s)(I - A(s)A(s)^D) + \tag{24}$$

$$+ A(s)^\dagger (I - A(s)Z(s))(I - A(s)A(s)^D) \times$$
$$\times A(s)A(s)^\dagger (I - A(s)A(s)^D)$$

where $Z(s)$ is an appropriate $n \times n$ rational matrix.

P roof. A substitution of the general solution (19) of the equation (12) in (23) produces the following rational matrix equation

$$A(s) \left[A(s)^D + Y(s)(I - A(s)A(s)^D\right] A(s) = A(s)$$

which is equivalent to

$$A(s)Y(s)(I - A(s)A(s)^D)A(s) =$$
$$= (I - A(s)A(s)^D)A(s)$$

From Lemma 6, it is easy to verify the consistency of this equation, and derive its general solution :

$$Y(s) = A(s)^\dagger (I - A(s)A(s)^D) A(s) \times$$
$$\times \left[(I - A(s)A(s)^D)A(s)\right]^{(1)} +$$
$$+ Z(s) - A(s)^\dagger A(s) Z(s)(I - A(s)A(s)^D) \times$$
$$\times A(s) \left[(I - A(s)A(s)^D)A(s)\right]^{(1)}$$

where $Z(s)$ is an appropriate rational matrix. It is not difficult to verify

$$A(s)^{\dagger} \in \left((I - A(s)A(s)^{D})A(s)\right)\{1\}.$$
$$I \in (I - A(s)A(s)^{D})\{1\}$$

Using the above equations, we get

$$\begin{aligned} Y(s) &= A(s)^{\dagger}(I - A(s)A(s)^{D})A(s)A(s)^{\dagger} + Z(s) - \\ &- A(s)^{\dagger}A(s)Z(s)(I - A(s)A(s)^{D})A(s)A(s)^{\dagger} \Longrightarrow \\ Y(s) &= Z(s) + A(s)^{\dagger}(I - A(s)Z(s)) \times \\ &\times (I - A(s)A(s)^{D})A(s)A(s)^{\dagger} \end{aligned} \tag{25}$$

A substitution of (25) in (19) produces (24). ■

Similar with the proof of the previous theorems is the proof of the following two theorems :

Theorem 13 *Let $A(s) \in C(s)^{n\times n}$, $ind(A(s)) = k$. For arbitrary integers $l \geq k$ the following system of rational matrix equations*

$$A(s)X(s)A(s) = A(s)$$
$$A(s)^{m} = A(s)^{m+1}X(s)$$

has the following general solution

$$\begin{aligned} X(s) &= A(s)^{D} + (I - A(s)A(s)^{D})Z(s) + \\ &+ (I - A(s)A(s)^{D})A(s)^{\dagger}A(s) \times \\ &\times (I - A(s)A(s)^{D})(I - Z(s)A(s))A(s)^{\dagger} \end{aligned}$$

where $Z(s)$ is an arbitrary $n \times n$ rational matrix.

Theorem 14 *Let $A(s) \in C(s)^{n\times n}$, $ind(A(s)) = k$. For arbitrary integers $l \geq k$ the system of rational matrix equations (23), (12) and (18) has the following general solution*

$$\begin{aligned} X(s) &= A(s)^{D} + (I - A(s)A(s)^{D})Y(s)(I - A(s)A(s)^{D}) + \\ &+ (I - A(s)A(s)^{D})A(s)^{\dagger}A(s)(I - A(s)A(s)^{D}) \times \\ &\times \left[I - Y(s)(I - A(s)A(s)^{D})A(s)\right] A(s)^{\dagger}(I - A(s)A(s)^{D}) \end{aligned}$$

where $Y(s)$ is an arbitrary rational matrix.

Remark 15 *Theorems 12-14 are valid when $A(s)^{\dagger}$ is replaced by a speci c $\{1\}$-inverse $A(s)^{(1)}$ of $A(s)$.*

5 Conclusion

We introduce a Leverrier-Faddeev algorithm for the computation of the Drazin inverse of singular square rational matrices. This algorithm is an extension of the modi cation of the Leverrier-Faddeev algorithm, introduced in [3]. Several systems of matrix equations are solved by means of the Drazin inverse and the Moore-Penrose inverse for rational matrices. The above algorithms and solutions have been implemented by the authors in the package MATHEMATICA and can be distributed to anyone who might be interested.

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