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GLOBAL ASYMPTOTIC STABILIZATION OF CHEN'S CHAOTIC SYSTEM VIA INVERSE OPTIMAL CONTROL

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Abstract: In this paper, a novel approach is developed for global asymptotic stablization of Chen's chaotic system, which in priciple works for other complex nonlinear systems as well. Based on a recently introduced methodology of inverse optimal control for nonlinear systems, a very simple stabilization control law is derived for the desired global asymptotic stabilization. Computer simulation is given for illustration and verification.

Keywords: Chaos, Nonlinear Systems, Inverse Optimal Control, Stability.

1. INTRODUCTION

Chaotic systems have been studied for quite a long time in the mathematical and physical communities, and controlling this kind of complex dynamical systems has recently attracted a great deal of attention within the engineering society. Different techniques have been proposed to achieve chaos control. For instance, linear state space feedback [3], Lyapunov function methods [11], adaptive control [13] and bang-bang control [12], among many others [4].

On the other hand, control methods of general nonlinear systems have been extensively developed since early 1980's, for example based on differential geometry theory.[9] Recently, the passivity approach has generated increasing interest for synthesizing control laws for nonlinear systems [2], [6], [10]. An important problem in this field is how to achieve robust nonlinear control in presence of unmodelled dynamics and external disturbances; along the same line there is the so-called H∞ nonlinear control [7], [1]. One major difficulty with this approach, alongside its possible system structural instability issue, seems to be caused by the requirement of solving the associated PDE equations. In order to alleviate this computational problem, the so-called inverse optimal control technique was recently developed based on the input-to-state stability (iss) concept [8].

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In this paper, the aforementioned inverse optimal control technique is employed to derive a very simple new control law for stabilization of the chaotic Chen's system, which in principle works for other dynamical system as well. Computer simulation is also given for the purposes of illustration and verification.

2. SYSTEM DESCRIPTION

A new chaotic system, referred to as Chen's system by other authors, has recently been discovered [5]. This system is described by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - xz + cy \\ \dot{z} &= xy - bz \end{aligned} \tag{1}$$

which has a chaotic attractor as shown in Fig. 1 when $a = 35, b = 3, c = 28$. It has been experienced that this chaotic system is relatively difficult to control as compared to the Lorenz and Chua's system due to the prominent three-dimensional and some complex features of its attractor.

We are interested in global asymptotically stabilizing this system to one of its unstable equilibrium points, $(0,0,0)$. Henceforth, we add a control input to the second state, so that the controlled system becomes

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - xz + cy + u \\ \dot{z} &= xy - bz \end{aligned} \tag{2}$$

3. INVERSE OPTIMAL CONTROL

We first find a Lyapunov function candidate that satisfies all the requirements to be an input-to-state control Lypaunov function (iss-clf). Such a function is essential for the design of a globally asymptotically stabilizing control law. We choose the following candidate:

$$V = \tfrac{1}{2}\left(x^2 + y^2 + z^2\right) \tag{3}$$

The time derivative of this function is

$$\begin{aligned} \dot{V} &= \tfrac{\partial V}{\partial X}\dot{X} \\ &= \tfrac{\partial V}{\partial X}\left(f(X) + g(X)u\right) \\ &= L_f V + (L_g V)u \end{aligned} \tag{4}$$

where $X = (x,y,z)^\top$,

$f(X) = (a(y-x), (c-a)x - xz + cy, xy - bz)^\top$,

$g(X) = (0,1,0)^\top$, $\frac{\partial V}{\partial X} f(X) = L_f V$, $\frac{\partial V}{\partial X} g(X) = L_g V$.

It follows from (2, 3, 4) that $L_f V = -a(x + \frac{cy}{2a})^2 - bz^2 + (c + \frac{c^2}{4a})y^2$, $L_g V = y$. Hence

$$\dot{V} = -a\left(x - \frac{cy}{2a}\right)^2 - bz^2 + \left(c + \frac{c^2}{4a}\right)y^2 + yu \tag{5}$$

In (5), it is easy to verify $L_g V = 0 \Longrightarrow L_f V < 0$ for all $X \neq 0$. This implies that (3) is indeed an iss-clf.

Next, we suggest the following simple linear state feedback control law:

$$\begin{aligned} u &= -\left(c + \frac{c^2}{4a} + k_0\right)y \\ &\triangleq -\beta R(X)^{-1}(L_g V)^T \end{aligned} \tag{6}$$

where k_0, β are positive constants and $R(X)^{-1}$ is a function of X in general, but here it is chosen to be

$$R(X)^{-1} = \frac{1}{\beta}\left(c + \frac{c^2}{4a} + k_0\right)$$

The motivation for this choice of the control law will be seen from the optimization discussed below.

Now, substituting (6) in (5), we obtain

$$\dot{V} = -a(x - \frac{cy}{2a})^2 - k_0 y^2 - bz^2$$

which implies $\dot{V} < 0$ for all $X \neq 0$. This means that the proposed control law (6) can globally asymptotically stabilize the system (2).

Note that (2) is input-to-state stabilizable, because the iis-clf fulfills the small control property [8]. Besides, the inverse optimal assignment problem, defined below, is solvable.

For the purpose of assigning the control gain, following [8], we consider the control law (6) and define a cost functional as follows:

$$\begin{aligned} J(u) &= \lim_{t\to\infty}\{2\beta V(X) + L(X)\} \\ L(X) &= \int_0^t \left(l(X) + u^\top R(X)u\right)d\tau \end{aligned} \tag{7}$$

with

$$\begin{aligned} l(X) \triangleq\ & -2\beta L_f V + 2\beta L_g V (R(X))^{-1}(L_g V) \\ & + \beta(\beta - 2)L_g V(R(X)^{-1})(L_g V) \end{aligned}$$

$$= -2\beta\left[-a\left(x-\frac{cy}{2a}\right)^2 - bz^2 + \left(c+\frac{c^2}{4a}\right)y^2\right]$$
$$+\beta\left[c+\frac{c^2}{4a}+k_0\right]y^2$$
$$= 2a\beta\left(x-\frac{cy}{2a}\right)^2 + 2b\beta z^2 - 2\beta\left(c+\frac{c^2}{4a}\right)y^2$$
$$+\beta\left[c+\frac{c^2}{4a}+k_0\right]y^2$$

According to the basic idea of the inverse optimal control theory, it is required that $l(X)$ be radially unbounded, i.e: $l(X) > 0$ for all $X \neq 0$ and $l(X) \to \infty$,as $X \to \infty$. Hence, we select

$$k_0 = \left(c+\frac{c^2}{4a}\right)+1 \tag{8}$$

which implies that

$$l(X) = 2a\beta\left(x-\frac{cy}{2a}\right)^2 + 2b\beta z^2 + \beta y^2$$

which satisfies the required condition.

Substituting (6) in (4), we obtain:

$$\begin{aligned}\dot{V} &= L_fV + L_gVu \\ &= L_fV + L_gV(-\beta R(X)^{-1})(L_gV) \\ &= L_fV - \beta(R(X)^{-1})(L_gV)^2\end{aligned}$$

Then multiplying $\dot{V}$ by -2β to obtain

$$-2\beta\,\dot{V} = -2\beta L_fV + 2\beta^2 L_gV(R(X)^{-1})(L_gV)$$

And then taking in account (6), which implies

$$u^\top R(X)\,u = \beta^2(R(X)^{-1})(L_gV)^2$$

we arrive at

$$l(X) + uR(X)u^T = -2\beta\,\dot{V} \tag{9}$$

To this end, substituting (9) in (7), we obtain:

$$J(u) = \lim_{t\to\infty}\{2\beta V(X(t)) + l_1(t)\}$$
$$l_1(t) = \int_0^t -2\beta\,\dot{V}\,d\tau$$
$$J(u) = 2\beta V(X(0))$$

Thus, the minimum of the cost functional is given by $J(u) = 2\beta V(X(0))$, for the optimal control law (6).

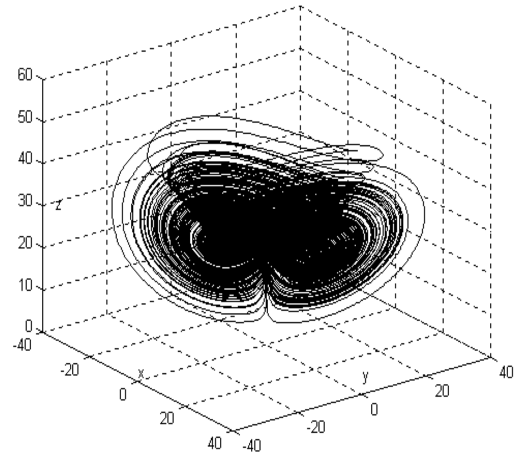

Fig. 1. Attractor of Chen's Chaotic System

In summary, taking into account (8), the optimal and stabilizing control law is finally obtained as

$$u = -\left(2c+\frac{c^2}{2a}+1\right)y \tag{10}$$

which is a very simple linear state feedback controller.

4. SIMULATIONS RESULTS

In order to verify the control applicability of the proposed control law (10), we consider system (1) currently in its chaotic state, i.e.,

$$\begin{aligned}\dot{x} &= 35(y-x) \\ \dot{y} &= -34.6y - 7x - xz \\ \dot{z} &= -3z + xy \\ & \quad x(0) = -10,\ y(0) = 0,\ z(0) = 37\end{aligned}$$

Its chaotic attractor is shown in Fig. 1.

With the control law (10) applied, the chaotic orbit of the system is quickly driven to the zero equilibrium point, as expected. In Fig. 2, we show the simulation results for different initial conditions.

As can be seen, the proposed control law global asymptotically stabilizes the originally chaotic system (1).

5. CONCLUSIONS

We have presented a novel control law for global asymptotic stabilization of the chaotic Chen's system. This control law is developed based on the inverse optimal control approach, and it is remarkably simple as compared to other existing chaos control methods. Due to its generic nature, this control law can be applied to other complex dynamical systems as well.

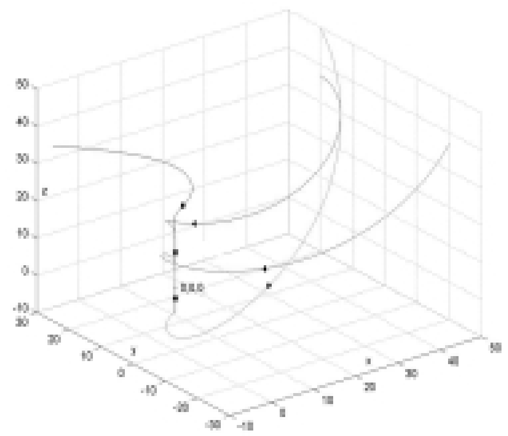

Fig. 2. Global Asymptotic Stable System

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