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A MULTI-OBJECTIVE CONTROL ALGORITHM: APPLICATION TO A LAUNCHER WITH BENDING MODES

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Abstract. A multi-objective control synthesis algorithm is first presented: it allows to avoid conservatism using different Lyapunov functions by combining the Youla parameterization, an observer-based structure and congruence transformations to obtain an LMI formulation. The efficiency of this approach is then tested by considering the problem of robustly stabilizing an aerospace launcher during the atmospheric flight.

Key Words. Robust control, LMI optimization, Youla parameterization, Aerospace.

1. INTRODUCTION

Control design often involves tradeoffs among conflicting objectives. Most of the time the controller is required to satisfy simultaneously different performance and robustness objectives which are imposed on different channels of the closed loop plant. Some discussion about multi-objective control first appeared in [1, 6, 10]. There is at yet no exact solution and existing methods use various approximations to find upper and lower bounds. In particular, the mixed H_2/H_∞ problem has received many attentions. Tractable convex optimization formulations have been derived in the literature but such methods are generally conservative: they use a single common Lyapunov function for each synthesis objective and a change of variable which simultaneously affects this Lyapunov function and the controller [9, 15], or they use infinite dimensional optimization [13, 14].

More recently, Youla parameterization has been proved to be useful to reduce this conservatism [8, 4, 14]; so this paper presents an algorithm based on this parameterization [5] and an application to an aerospace problem. The method uses an independent Lyapunov function for each objective, a change of variables on these functions but not on the controller, and an observer-based structure which allows to reduce the degree of the controller. The solution is obtained using LMI optimization that is now a computationally tractable framework [2].

The efficiency of this approach is then evaluated by designing a robust autopilot for the atmospheric flight of an European space launcher type. The application and specifications are first presented and then the control synthesis is explained. This study is the result of a collaboration with the *CNES* (French Space Agency) and *Aerospatiale Matra Lanceurs*.

2. NOTATIONS AND DEFINITIONS

All plants considered in the paper are LTI finite dimensional, and described by a discrete-time state-space representation (with sample frequency $f_s = 1/T_s$). Closed-loop plants will be represented by the standard diagram of figure 1, where vector u denotes the control input, vector y the measured output, and transfers T_i from vector w_i (external input) to vector z_i (controlled output) are used to specify two different robustness or performance objectives.

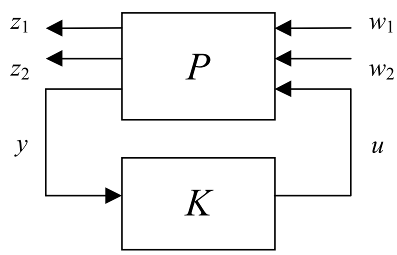

Fig. 1. Closed loop representation

State-space representations of the open-loop plant P and the controller K will be noted as:

$$P = \left(\begin{array}{c|ccc} A & B_1 & B_2 & B_u \\ \hline C_1 & D_{11} & D_{12} & D_{1u} \\ C_2 & D_{21} & D_{22} & D_{2u} \\ C_y & D_{y1} & D_{y2} & D_{yu} \end{array}\right) \quad (1)$$

$$K = \left(\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right) \quad (2)$$

Without loss of generality, it is assumed in the paper that $D_{yu} = 0$.

Interconnection of two plants will be noted by the Redheffer star product. As a particular case, the closed loop plant of Fig. 1 is noted $P * K$. The objectives under consideration in this paper are H_∞ norm constraints, which are useful to enforce robustness and to express frequency domain specifications. They are considered below with a LMI formulation. Other objectives like H_2 norm or time-domain constraints can be also translated into LMI formulations and can be used in the proposed multiobjective control approach [4].

3. MULTI-OBJECTIVE CONTROL

The goal of the multiobjective synthesis is the Pareto optimal controller which is well representative of the tradeoff among both objectives. It consists in minimizing a linear combination of different objectives.

It is also well known that this problem is hard and non convex [1]. In order to perform a synthesis via convex optimization, it is necessary to transform the initial multiobjective problem. A combination of the following tools presented in this section leads the synthesis algorithm given in section 3.3:

- *the Youla parameterization gives specific properties to the system*
- *the observer based structure allows to reduce the degree of the controller*
- *the optimization of the Youla parameter is expressed as a LMI problem*

Contrary to the usual approaches, the proposed method allows to choose different Lyapunov functions for each objective without loosing convexity; this is a crucial point to reduce the conservatism. Indeed each objective can be considered independently.

3.1. Youla parameterization

The set of all stabilizing controllers for P can be parameterized [11] as $K = J * Q$ (see figure 2) where the Youla parameter Q is any stable system, and the non-observable and non-controllable subspaces of $G = P * J$ are supplementary.

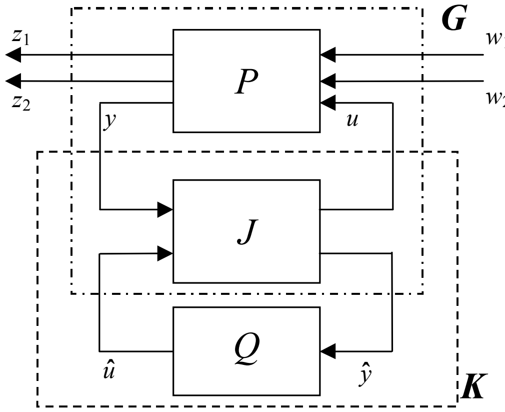

Fig. 2 - Closed-loop structure including Youla parameterization

As shown in [3], any stabilizing controller can be expressed as the interconnection of an observer based structure and a Youla parameter Q. Suppose an initial controller has been designed using any classical mono-objective method. Such a controller can then be expressed as $K = J * Q$ with J of minimum degree (i.e. the degree of P).

Finally the Youla parameterization gives a specific form to each channel state pace representation of G (from w_i to z_i) as follows:

$$G_i = \left(\begin{array}{cc|cc} A_1 & A_3 & B_{i1} & \hat{B}_u \\ 0 & A_2 & B_{i2} & 0 \\ \hline C_{i1} & C_{i2} & \hat{D}_{z_i w_i} & \hat{D}_{z_i u} \\ 0 & \hat{C}_y & \hat{D}_{y w_i} & 0 \end{array}\right) \quad (3)$$

3.2. LMI formulation of the objective

This section focuses on the LMI formulation of a H_∞ objective. The following theorem is applied for each objective (i.e. each channel). Subscript i will be now omitted for simplicity.

Theorem
Let G the i[th] channel of the system in Youla form (see eq 3) and Q a static output feedback, then

$$\|G * Q\|_\infty \le \gamma \quad (4)$$

if and only if there exist $R = R^T$, $T = T^T$ and S (with adequate dimensions) such that

$$
\left(\begin{array}{cc|cc|c|c}
-R & 0 & A_1R & A_1S - SA_2 + A_3 + \hat{B}_u Q\hat{C}_y & B_1 + \hat{B}_u Q\hat{D}_{yw} - SB_2 & 0 \\
0 & -T & 0 & TA_2 & TB_2 & 0 \\
\hline
* & * & -R & 0 & 0 & (C_1R)^T \\
* & * & 0 & -T & 0 & (C_{12} + \hat{D}_{zu} QC_y + C_1S)^T \\
\hline
* & * & * & * & -\gamma I & (\hat{D}_{zw} + \hat{D}_{zu} Q\hat{D}_{yw})^T \\
\hline
* & * & * & * & * & -\gamma I
\end{array}\right) < 0 \qquad (5)
$$

Sketch of proof

The entire proof can be found in [5]. Let consider an H_∞ objective, from w to z and the Lyapunov function X partitioned into

$$
X = \begin{pmatrix} W & Z \\ Z^T & Y \end{pmatrix} \text{ according to } A = \begin{pmatrix} A_1 & A_3 \\ 0 & A_2 \end{pmatrix} \qquad (6)
$$

The bounded real lemma [2] applied to the interconnection between G and Q gives a non linear matrix inequality; using the following bijective change of variable:

$$
\begin{pmatrix} W & Z \\ Z^T & Y \end{pmatrix} \rightarrow \begin{pmatrix} R & S \\ S^T & T \end{pmatrix} = \begin{pmatrix} W^{-1} & -W^{-1}Z \\ -Z^T W^{-1} & Y - Z^T W^{-1} Z \end{pmatrix} \qquad (7)
$$

$$
\text{and defining: } M = \begin{pmatrix} R & 0 \\ S^T & I \end{pmatrix} \qquad (8)
$$

$$
\text{and } \Pi = diag\left(M, M, I, I\right) \qquad (9)
$$

then pre-multiplying and post-multiplying the non linear matrix inequality by Π^T and Π yields to the LMI (5). ■

Considering now both channels of G, one obtains two LMIs of the form (5) with the same Youla parameter Q, two H_∞ levels γ_1, γ_2 and different matrices R_1, S_1, T_1, R_2, S_2, T_2. So we have shown that the multiobjective problem depends affinely on these variables and Q.

Remark: the fundamental property on the state space structure of G allows to extend this approach to a Youla parameter Q including dynamics (see [14] and [5]).

3.3. Algorithm

The previous results lead to a multiobjective control design algorithm:

(i) *Initial synthesis*: design an initial controller using conventional techniques such as LQG methods, H_2 or H_∞ optimization, or multiobjective control with a common Lyapunov function.

(ii) *Observer structure parameterization*: obtain the Youla parameterization (figure 2)

(iii) *Convex optimization*: obtain the Youla parameter Q by solving the multiobjective control problem using the LMIs of section 3.2.

(iv) *Controller reconstruction:* the final controller is obtained by $K = J * Q$.

4. APPLICATION TO AN AEROSPACE LAUNCHER

4.1. Presentation

The application is developed for the yaw axis of an European space launcher (figure 3) whose dynamics include a rigid mode and two bending modes (sloshing modes are not considered). It concerns automatic control of the launcher, which has the function of keeping the process around its center of gravity, following the guidance reference trajectory.

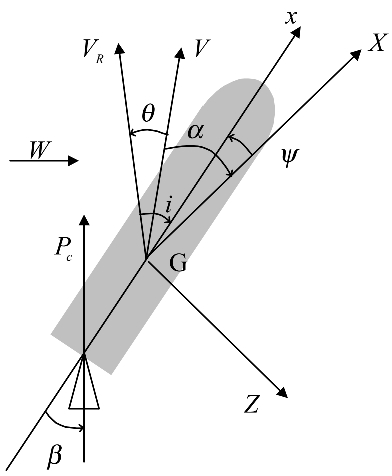

Fig. 3 – Simplified launcher representation

G is the center of gravity, V and V_r are respectively the absolute and the relative velocity, W represents the wind velocity; i represents the angle of attack of the launcher and Ψ the deviation of its yaw axis with respect to the guidance attitude reference. The control variable is the thruster angle of deflection β. This launcher is aerodynamically unstable.

As described in [12], during the atmospheric flight phase, launcher control objectives are to insure:

- closed loop stability with sufficient stability margins,

- performance with respect to disturbance due to wind and gusts,
- good tracking of guidance reference,
- robustness with respect to uncertainties on the rigid and bending modes coefficients.

The control structure is given in figure 4:

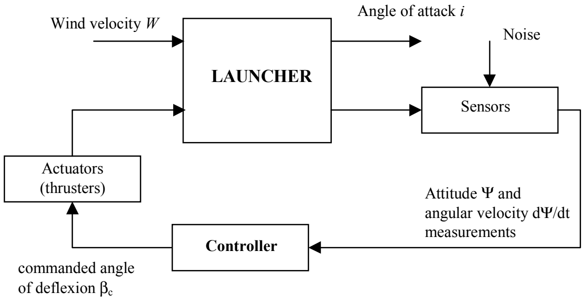

Fig. 4 – Launcher control closed loop

4.2. Multi-objective synthesis

To perform our synthesis algorithm, a representative model of the yaw axis of the launcher is chosen and the specifications are translated into automatic control objectives as follows:

1. closed-loop stability with sufficient margins: decreasing and increasing gain margins have to stay higher than given specifications.
2. control the destabilizing bending modes: the aim is to attenuate these modes under −6 dB except for the first one which can be controlled in phase with a sufficient delay margin (one sample period).
3. limit the angle of attack in case of wind (with a typical wind profile – see figure 9).
4. tracking of guidance reference
5. rejection of noise disturbances
6. all these objectives have to be robust against uncertainties (rigid and bending modes)

The objectives are translated into H_∞ criteria as follows (see fig. 5):

(i) <u>*S and T modeling*</u>
- limiting the angle of attack and reference tracking are done by penalizing the sensitivity S using a low-pass filter W_1.
- bending modes attenuation and noise rejection are obtained by penalizing the complementary sensitivity T with a roll-off filter W_2.

The first criterion is then

$$\left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_\infty < \gamma_1 \qquad (9)$$

(ii) <u>*Positivity*</u>

A positivity criterion [7] allows to control the first bending mode. Indeed, defining the transfert $H = S - T$, the following property is hold:

$$\forall \omega \in \mathbf{R}$$

$$\left| H(e^{j\omega T}) \right| < 1 \Rightarrow -\frac{\pi}{2} < \arg\left(L\left(e^{j\omega T}\right)\right) < \frac{\pi}{2} \qquad (10)$$

where L is the open loop transfer function. This property allows to control in phase a bending mode by forcing the positivity on some intervals of frequencies, using a band-pass filter W_3. The second H_∞ criterion is then:

$$\| W_3 (S - T) \|_\infty < \gamma_2 \qquad (11)$$

The chosen structure is given in figure 5 and the weightings are shown in figure 7. The first criterion corresponds to the transfer between w_1 and $(z_1 \ z_2)^t$, while the second is between w_3 and z_3.

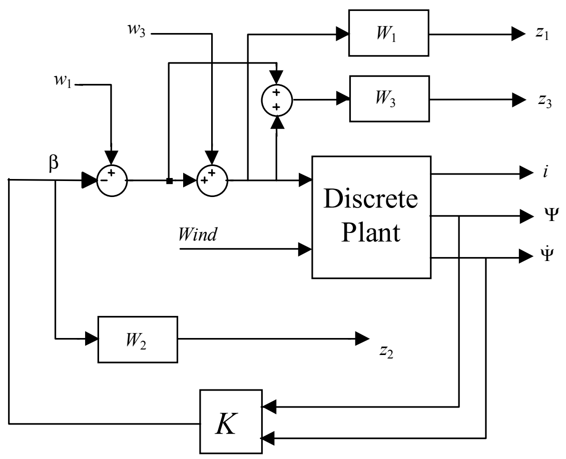

Fig. 5 – Closed loop and synthesis structure

A first synthesis is based on classical H_∞ synthesis; then the second optimization is used to separate the objectives and improve the controller with a static Youla parameter. It has been noticed that the tuning of the weightings is made easy due to the objectives decoupling.

4.3. Results and comments

The obtained controller responses are given in figure 6 and the closed loop transfer responses in figure 7 (the frequency scale has been normalized with respect to the sample frequency f_s). To test the robustness, the later are made over different models where rigid and bending modes uncertain parameters are perturbed. In all cases, the H_∞ objectives are satisfied.

The open loop frequency response is given in figure 8, with margins and roll-off specifications given with stars and a horizontal line respectively: the control law respects the second bending modes attenuation and the phase placement of the first one. Delay margin of this mode vary from 0.8 to1.52 sample period.

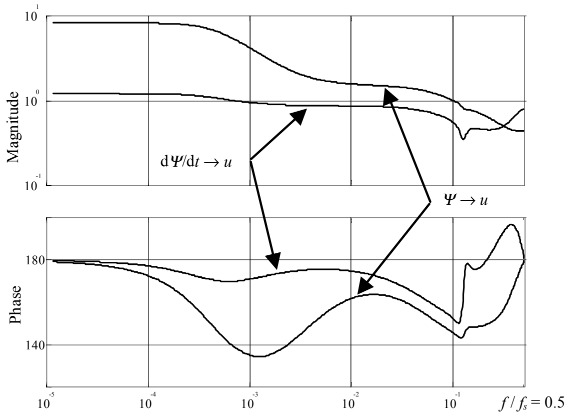

Fig 6 – Bode plot of the controller

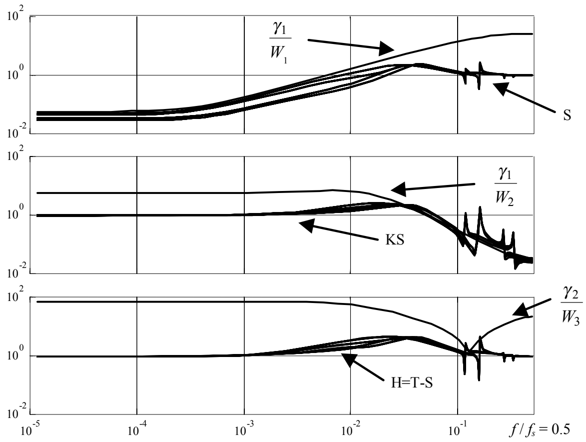

Fig 7 – Weightings and closed loop transfers

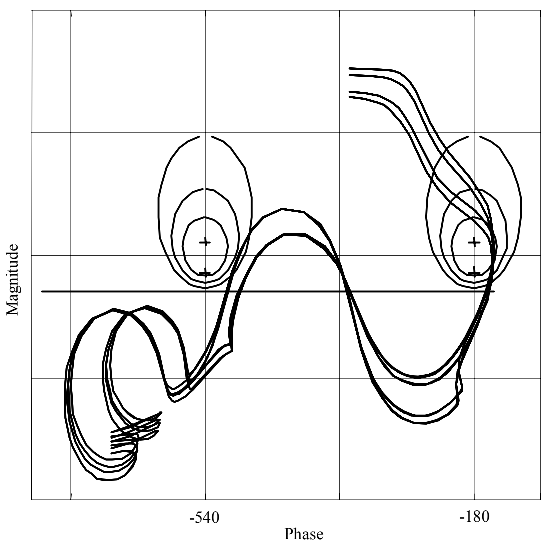

Fig 8 – Nichols open loop chart

Then, temporal aspects are analyzed by simulations including the complete launcher model with the typical wind profile given in figure 9 (the temporal scale has been normalized with respect to the sample period T_s):

- Angle of attack performance (fig. 10) is almost satisfying for the whole family of models, since the variation of the peak is from 0.9 to 1.10 (the normalized specification is 1).

- Attitude tracking (fig. 11) is well verified and the behavior of the control variables is very satisfying. Indeed, the angle of deflection and its velocity are rather far from the maximal allowables values (fig. 12 and 13).

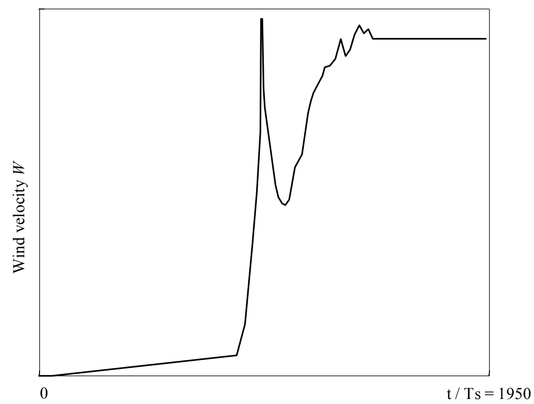

Fig 9 – Wind profile

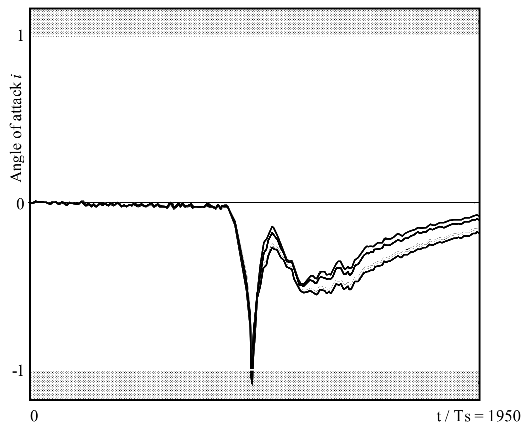

Fig 10 – Angle of attack i

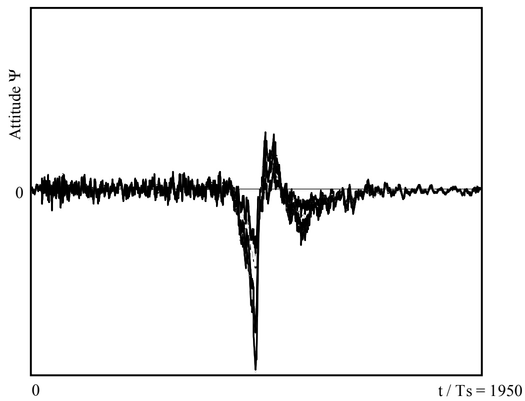

Fig 11 – Attitude Ψ

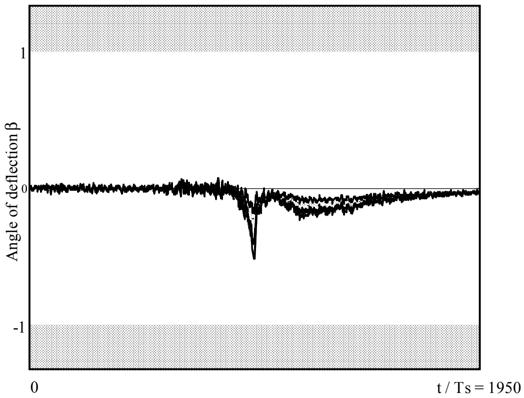

Fig 12 – Control: angle of deflection β

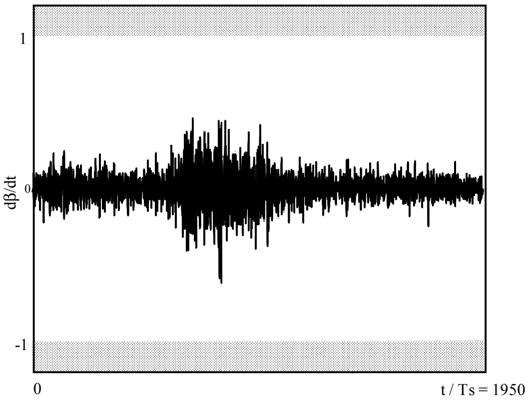

Fig 13 – Control velocity: $\dot{\beta}$

5. CONCLUSION

A solution to a general multichannel H_∞ control problem has been presented. Previous results are used to initialize the algorithm whereas the Youla parameterization allows to improve the results. Each step is based on matrix manipulations and convex optimization. Besides matrix manipulations are numerically well conditioned and convex optimization can be solved in polynomial time. So it leads to computationally tractable problems.

This approach offers advantages over existing methods. It allows to reduce the conservatism by using a particular Lyapunov function for each objective; it is not necessary to inverse the changes of variables. Note however that increasing the number of decision variables can turn to numerical problems when the plant order or the number of objectives is large.

The method gives efficient results when applied to an industrial application such as a space launcher when two H_∞ constraints are considered independently. The results are satisfying in view of the specifications.

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