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TUNING SMITH PREDICTORS USING SIMPLE FORMULAE

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Abstract: Good control of processes with long dead time is often achieved using a Smith predictor configuration. However, not much work has been carried out on obtaining simple tuning rules for a Smith predictor scheme. This paper develops optimal analytical tuning formulae for PID controllers in a Smith predictor configuration assuming perfect matching. These formulae have been obtained by carrying out repeated optimizations on the error transfer function of a Smith predictor, assuming perfect matching, to find optimal relations between the normalized gain and the remaining parameters of the controller. Then the least square fitting method was used to find the constants in the assumed formulae to fit the graphical data obtained. Some examples are given to show the value of the approach presented.

Keywords: Predictive control; Integral performance indices; Delay compensation; PID controllers

1. INTRODUCTION

Plants with long time-delays can sometimes not be controlled effectively using a PID controller in the conventional single feedback loop structure. The main reason for this is that the additional phase lag contributed by the time delay tends to destabilize the closed loop system. The stability problem can be solved by decreasing the controller gain. However, in this case the response obtained is very sluggish.

The Smith predictor, shown in Fig. 1, is well known as an effective dead-time compensator for a stable process with long time-delays (Smith, 1959).

Based on the assumption that the model used matches perfectly the plant dynamics, the closed loop transfer function of Fig. 1 is given by

$$T_o(s) = \frac{G_c(s)G_m(s)e^{-\theta_m s}}{1 + G_c(s)G_m(s)}. \qquad (1)$$

According to eqn. (1), the parameters of the primary controller, $G_c(s)$, which is typically taken as PI or PID, may be determined using a model of the delay free part of the plant.

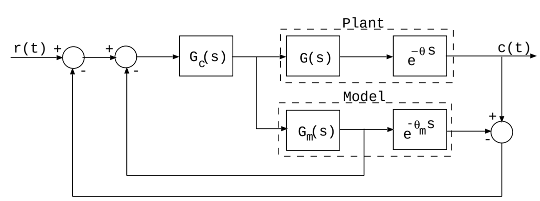

Fig. 1. The Smith predictor control scheme

Many possible approaches for determining or tuning the parameters of an appropriate controller, $G_c(s)$, have been given in the literature and some recent contributions include (Kaya and Atherton, 1999; Hang *et al*, 1995; Palmor and Blau, 1994; Hägglund, 1992). The method proposed by Kaya and Atherton (1999) replaces the conventional controller, $G_c(s)$, by a PI-PD or PI-P structure where the PD or P part is implemented in an inner feedback loop. The choice of selection of the controller structure in the inner loop, that is to choose a PD or only a P, depends on the model order. The tuning parameters of the PI-PD or PI-P controllers have been obtained using standard forms (Graham and Lathrop, 1953) and it was shown that the PI-PD or PI-P structure can give superior performance. The difficulty with the design is to involve a trade off between selected values of K_p and T_i, respectively, the gain and integral time constant of the PI controller in the forward path. Hägglund's method is based on a FOPDT model which is obtained from a step response test. The step response test is an open loop test, therefore, if there is any external disturbance during the identification procedure, it may lead to large errors.

Many studies have been devoted to the development of tuning rules based on optimization. The main disadvantage of using optimization as a design criterion is that the transfer function of the plant must be known. To eliminate this disadvantage, a possible approach is to use the relay autotuning method to estimate an FOPDT model (Kaya and Atherton, 1998) and then optimize the controller parameters based on this assumed model transfer function, (Smith and Corripio, 1985; Zhuang and Atherton, 1993). In section 3, this approach is carried out for the delay free part of the FOPDT plant transfer function which can be used to find tuning parameters of a PI or PID controller in a Smith predictor configuration if perfect matching is assumed. In references (Smith and Corripio, 1985; Zhuang and Atherton, 1993), a FOPDT model was also used to approximate higher order plant transfer functions. A better approximation may be achieved with a SOPDT plant transfer function, which is considered in this paper and it is shown how simple tuning formulae can be obtained.

2. SIMPLE TUNING FORMULAE USING OPTIMIZATION

Optimization has always been a powerful design method to determine controller parameters. The integral of squared error, ISE, criterion is one of the most well known criteria, but it generally results in a significant overshoot and a relatively long settling time. The time weighted versions of the ISE criterion give relatively smaller overshoots and comparable settling times. In the next subsections, simple tuning formulae are obtained using the ISTE and IST^{2}E criteria for a PI or PID controller in a Smith predictor configuration based on either FOPDT or SOPDT models.

2.1 *Tuning for a PI Controller*

First, a FOPDT model, $G_m(s)e^{-\theta_m s} = Ke^{-\theta_m s}/(Ts+1)$, and a PI controller with the ideal transfer function

$$G_c(s) = K_p(1 + \frac{1}{T_i s}) \qquad (2)$$

are considered.

Assuming a perfect matching between the plant and model dynamics, the error for the Smith predictor structure from Fig. 1 is

$$E(s) = \frac{R(s)}{1 + G_c(s)G_m(s)} \qquad (3)$$

Repeated optimizations were carried out on this error for $R(s)$ a step input, using eqn. (2) for $G_c(s)$ and $K/(Ts+1)$ for $G_m(s)$, for different values of normalized gain, $\kappa = KK_p$. Fig. 2 shows the relationship between the normalized gain κ and T/T_i for the ISTE and IST^{2}E criteria for a set point change over the range of $2.50 - 5.00$ for KK_p. The following formula

$$\frac{1}{T_i} = \frac{a}{T}(KK_p)^b \qquad (4)$$

was obtained to fit the graphical results, using a least square fit technique. In the graph the continuous curve shows the fitting produced by the formula and $'*'$ shows T/T_i as a function of KK_p. To obtain a good fit to the data for κ

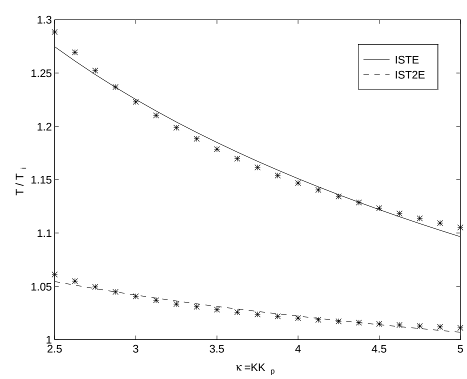

Fig. 2. PI controller parameters when a FOPDT model is used over the range $\kappa = 2.50 - 5.00$

between 1.5 and 15, the formula was used over three ranges and the coefficients are listed in Table 1 for the ISTE and IST^{2}E criteria. Once the controller gain is specified, which is chosen so that the normalized gain κ falls in one of the ranges given in Table 1, the controller integral time constant T_i can be calculated from eqn. (4).

When a SOPDT model, $G_m(s)e^{-\theta_m s} = Ke^{-\theta_m s}/[(T_1 s+1)(T_2 s+1)]$, is used, calculations can be carried out in a similar way to that used for the

FOPDT model. Using $K_p(1 + 1/T_i s + T_d s)$ for $G_c(s)$ and $K/[(T_1 s + 1)(T_2 s + 1)]$ for $G_m(s)$, the error given by eqn. (3) was minimized for the ISTE and IST^{2}E critea and some of the results for T_1/T_i versus $\kappa = KK_p$ is given in Fig. 3. The following equation

$$\frac{1}{T_i} = \frac{a}{T_1}(KK_p)^b(\frac{T_2}{T_1})^c \tag{5}$$

was used to fit the graphical data using least square curve fitting method. The coefficients in the equation are listed in Table 2 for various ranges of normalized gain, κ. A close investigation

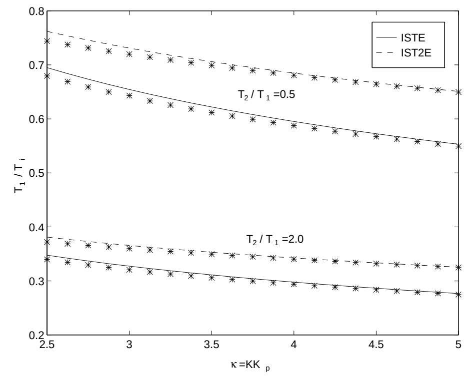

Fig. 3. PI controller parameters when a SOPDT model is used

on the table reveals that the value of c is always around -0.50 for both criteria, thus eqn. (5) can be rearranged as

$$\frac{1}{T_i} = \frac{a}{T_1}(KK_p)^b(\frac{T_1}{T_2})^{1/2} \tag{6}$$

The required a and b values are given in Table 2.

2.2 *Tuning for a PID Controller*

When the controller in a Smith predictor scheme is chosen as a PID controller, then the process must be modelled by a SOPDT transfer function. The reason for this is that when the delay free part of the FOPDT, $K/(Ts + 1)$, and the ideal PID form are used in optimization to minimize the error given by eqn. (3), the optimization procedure may not converge to a solution.

Some of the results for T_1/T_i and L/T_1 for set-point change, again using the ISTE and IST^{2}E criteria, are given in Fig. 4. Curve fitting in a least squares sense gives

$$\frac{1}{T_i} = \frac{a_1}{T_1}(KK_p)^{b_1}(\frac{T_2}{T_1})^{c_1} \tag{7}$$

$$T_d = T_1 a_2 (KK_p)^{b_2}(\frac{T_2}{T_1})^{c_2} \tag{8}$$

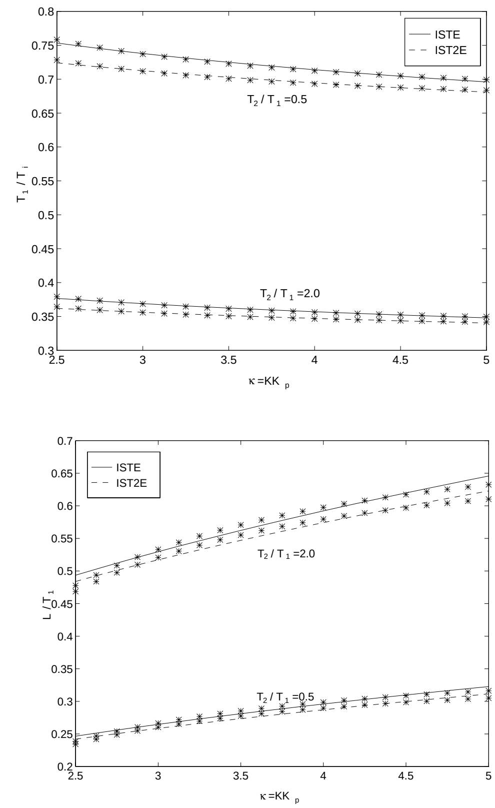

Fig. 4. PID controller parameters

The (a, b, c) coefficients are given in Table 3 for different ranges of $\kappa = KK_p$.

Again the value of c was found to be around -0.50 for T_1/T_i and 0.50 for T_d/T_1 for both criteria and ranges. Therefore eqns. (7) and (8) can be rearranged as

$$\frac{1}{T_i} = \frac{a_1}{T_1}(KK_p)^{b_1}(\frac{T_1}{T_2})^{1/2} \tag{9}$$

$$T_d = T_1 a_2 (KK_p)^{b_2}(\frac{T_2}{T_1})^{1/2} \tag{10}$$

where the a_1, a_2, b_1 and b_2 values are again given by the Table 3.

3. ILLUSTRATIVE EXAMPLES

In this section several examples are given to show the use of the method. The first example considers a FOPDT transfer function to illustrate the effect of mismatching in the time-delay. In the second example a third order system is considered to show that second order modelling gives a better approximation for processes with higher order transfer functions. In the last example a process with a second order transfer function is given to compare the performance of the proposed design method with some existing methods.

3.1 *Example 1*

This example is given to analyse the effect of mismatching in the time-delay, since this is the most detrimental to the system performance. Fig. 5 shows percentage error difference between the plant and model time delays versus ISTE criterion value. The figure shows that the ISTE criterion value increases when the mismatch between the plant and model time delays is increased. Also, it is seen from the figure that the ISTE criterion value gets larger for large time delay to time constant ratios. This means that the performance of a Smith predictor is more sensitive to a mismatch when the time delay to time constant ratio is large.

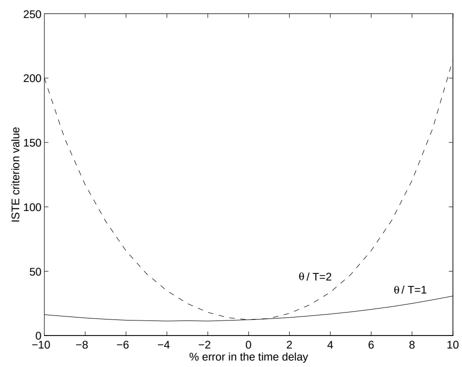

Fig. 5. ISTE criterion for different % error difference

To illustrate this, consider the following transfer function.

$$G(s) = \frac{1}{(10s+1)}e^{-\theta s}$$

First, assume that the time delay, θ, is 10.0, which gives the time delay to time constant ratio of 1.0. Limiting the controller gain, K_p, to 2.5, then integral time constant was calculated as $T_i = 7.8383$, from eqn. (4). Fig. 6 shows the step responses for the matching case, $\theta = \theta_m$, and also for $\pm10\%$ error between the plant and model time-delays. Now assume that the plant time delay is 5.0 which gives the time delay to time constant ratio of 0.5. Results for this case are given in Fig. 7, when the controller parameters are kept at the same values as before. The figure confirms the results given by Fig. 5 that for the same percentage error in the time delays, the Smith predictor performance deteriorates more for large time delay to time constant ratio.

3.2 *Example 2*

A high order transfer function is given in this example.

$$G(s) = \frac{2}{(4s+1)(3s+1)(2s+1)}e^{-5s}$$

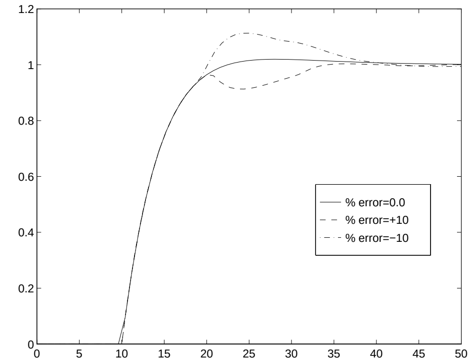

Fig. 6. Step responses when $\theta = 10$ for example 1

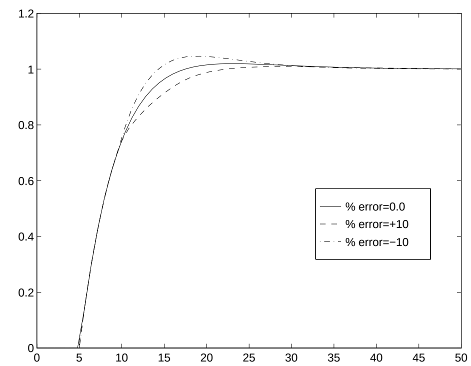

Fig. 7. Step responses when $\theta = 5$ for example 1

To identify the plant parameters, the plant was simulated in SIMULINK under relay feedback control. The relay had heights of $h_1 = 1$ and $h_2 = -0.5$ and no hysteresis. The input to system was zero. The measured limit cycle parameters were 0.249, 1.287, -0.680 and 10.583 for ω, a_{max}, a_{min} and Δ_{t1} respectively, see (Kaya and Atherton, 1998) for notations. These parameters were used to find a FOPDT and SOPDT model for the plant. The FOPDT model was obtained as $K = 2.001$, $T = 7.325$ and $\theta = 8.328$ and the SOPDT model as $K = 2.001$, $T_1 = 4.068$, $T_2 = 4.068$ and $\theta = 6.164$. The control design methods given in section 2 were implemented to compare the performance in each case. The controller gain was limited to 2.0 in each of the three cases. Then using equations given in section 2.1, the integral time constants for a PI controller with a FOPDT and SOPDT model were calculated as $T_i = 6.369$ and $T_i = 9.792$, respectively. The parameters of PID controller were obtained as $T_i = 8.062$ and $T_d = 1.707$, using the equations given in section 2.2. The response of the designed controllers in a Smith predictor structure are given in Fig. 8 for a set point change. The best result is achieved with a PID controller in the Smith predictor structure, as expected. A PI controller in the Smith predictor configuration, when the FOPDT model is used, gives the poorest response. This makes sense, since a Smith predictor is sensitive to modelling errors and a FOPDT can not model a higher order plant adequately. A PI controller in Smith predictor structure with the SOPDT model results in a fast response, but, the settling time is slightly longer when compared to the response of a PID controller in the Smith predictor structure. Fig. 9 shows control signals for the example. It is seen that the control signal for a PID control in Smith predictor settles down in a short time while for others takes longer.

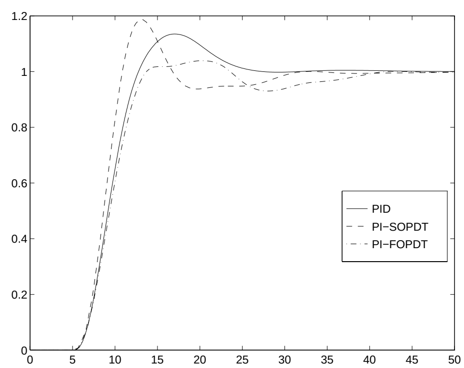

Fig. 8. Step responses for example 2

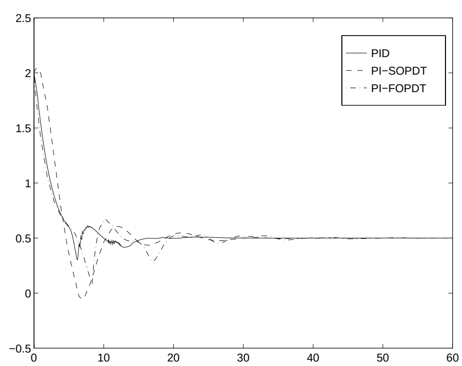

Fig. 9. Control signals for example 2

3.3 *Example 3*

A process with SOPDT transfer function given by

$$G = \frac{e^{-5s}}{(10+1)(5s+1)}$$

is considered. The model of the plant is obtained accurately using asymmetric limit cycle data from the relay feedback method (Kaya and Atherton, 1998). Constraining the controller gain K_p to 2.5 results in the remaining tuning parameters of $T_i = 13.273$ and $T_d = 2.467$, using the ISTE criterion and eqns. (9)-(10). Response of the Smith predictor with these controller parameters to a unit step and a disturbance of $d = -0.1$ at $t = 80$ are shown in Fig. 10. Similar results for the design method of Palmor (1994), Hang *et al* (1995) and Hägglund (1992) are also given in the same figure for comparison. The design methods proposed by Hägglund and Hang give slow closed loop responses. Palmor's method gives good responses but results in a slightly longer settling time than the proposed method. The control signals for the design methods are shown in Fig. 11 which shows that Palmor's method gives a larger initial control signal.

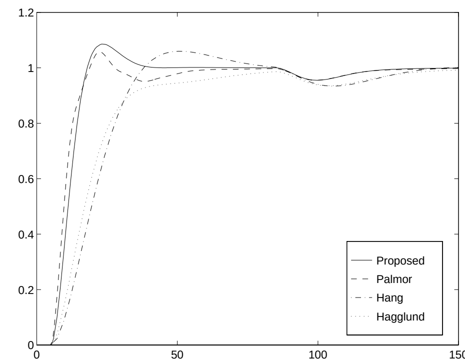

Fig. 10. Step responses for example 3

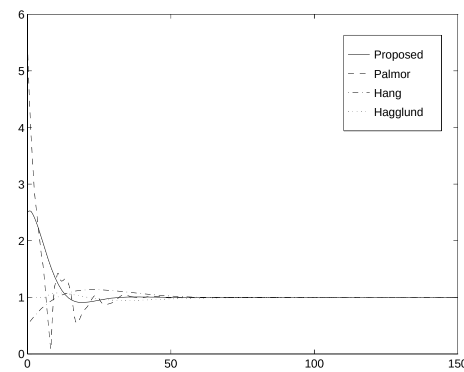

Fig. 11. Control signals for example 3

4. CONCLUSION

The paper investigated obtaining simple tuning formulae for PID controllers in a Smith predictor configuration. The ISTE and IST^{2}E criteria, which gives a small overshoot and short settling time, has been used to obtain optimal relations between the normalized dead time and the remaining controller parameters. The least square curve fitting method was used find the constants in the assumed formulae to fit the graphical data obtained. The method requires the controller gain to be specified first and then the formulae given in section 2 can be used to determine the remaining controller parameters. It is shown by examples that the method results in a fast response with a small overshoot and a short settling time provided that there is no mismatch between the plant and model dynamics.

5. REFERENCES

[1] Smith, O.J., *A controller to overcome dead time*, ISA J., vol. 6, no. 2, p. 28-33, 1959.

[2] Kaya, I., Atherton, D.P., *A new PI-PD Smith predictor for control of processes with long dead time*, 13.th IFAC World Congress, 1999.

[3] Hang, C.C., Wang, Q.G., Cao, L.S., *Self-tuning Smith predictors for processes with long dead time*, Int. J. of Adaptive and Signal Processing, vol. 9, p. 255-270, 1995.

[4] Palmor, Z.J. and Blau, M., *An Auto-tuner for Smith Dead Time Compensator*, Int. J. Control, vol. 60, no.1, pp. 117-135, 1994.

[5] Hägglund, T., *A predictive PI controller for processes with long dead-time delay*, IEEE Control Syst. Mag., vol. 12, no. 1, p. 57-60, 1992.

[6] Kaya, I., Atherton D.P., *An improved parameter estimation method using limit cycle data*, UKACC International Conf. on Control'98, Swansea, U.K., vol. 1, 682-687, 1998.

[7] Smith, A.C., Corripio, B.A., *Principles and practice of automatic process control*, Jhon Wiley & Sons, 1985.

[8] Zhuang, M., Atherton, D.P., *Automatic tuning of optimum PID controllers*, IEE Proceedings-D, vol. 140, no. 3, p. 216-224, 1993.

Table 1. PI Tuning Formulae based on a FOPDT model

$\kappa = KK_p$ range	1.50-2.50		2.60-5.00		5.10-15.00	
Criterion	ISTE	IST^2E	ISTE	IST^2E	ISTE	IST^2E
a	1.9443	1.3037	1.5556	1.1209	1.2149	1.0196
b	-0.4722	-0.2426	-0.2173	-0.0666	-0.0679	-0.0075

Table 2. PI Tuning Formulae based on a SOPDT model

$\kappa = KK_p$ range	1.50-2.50		2.60-5.00		5.10-15.00	
Criterion	ISTE	IST^2E	ISTE	IST^2E	ISTE	IST^2E
a	0.6873	0.6341	0.6345	0.6315	0.6647	0.6638
b	-0.4022	-0.2112	-0.3051	-0.1965	-0.3295	-0.2274
c	-0.5000	-0.4999	-0.5008	-0.5000	-0.4998	-0.4998

Table 3. PID Tuning Formulae based on a SOPDT model

$\kappa = KK_p$ range	1.50-2.50		2.60-5.00		5.10-15.00	
Criterion	ISTE	IST^2E	ISTE	IST^2E	ISTE	IST^2E
a_1	0.6781	0.6338	0.5916	0.5554	0.5169	0.4950
b_1	-0.2709	-0.2410	-0.1144	-0.0888	-0.0323	-0.0184
c_1	-0.5000	-0.5000	-0.5000	-0.5002	-0.5001	-0.5001
a_2	0.1058	0.1057	0.2446	0.2453	0.4049	0.3978
b_2	1.3371	1.3041	0.3877	0.3637	0.0767	0.0640
c_2	0.5003	0.4998	0.5002	0.4999	0.5000	0.4999