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MODELLING AND CONTROL OF A THIN PIEZO-ACTUATED STRUCTURE

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Abstract. Vibration damping for a thin piezo-actuated cantilever plate is pursued through either purely passive or purely active control. The design is carried out by using suitable reduced order models of the coupled electro-mechanical structure. Simulation results are presented in order to evaluate the proposed control laws.

Key Words Vibration damping, piezoelectric finite-elements, LQG control.

1 INTRODUCTION

In this paper, a cantilever plate actuated by means of a piezoelectric actuator is considered. An accurate model is obtained by means of the finite-element formulation (Mindlin-type, with special care in order to avoid locking phenomena) proposed in [3]: the underlying variational formulation is briefly recalled in Section 2. Such a model, after suitable order reduction, is used in Section 3 in order to optimize the parameters of a purely passive control law, obtaining results fully equivalent to the ones in [9], and in Section 4 in order to design a simple LQG control law, by using the technique proposed in [12] in order to choose the performance index. Both the passive and the active control laws are evaluated through simulations performed on a higher order model.

2 MODEL OF THE SYSTEM

The goal of this section is to describe the system under study (see Fig. 1) and the finite-element model which has been preferred in order to design and evaluate in simulation the passive and hybrid control laws described in the rest of the paper.

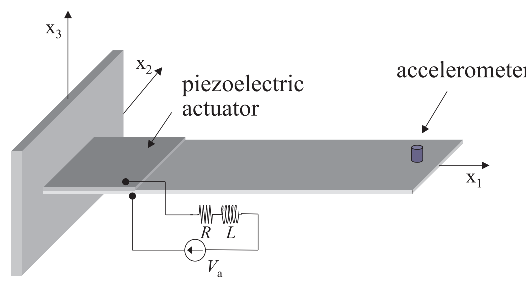


Figure 1: Structure of the experimental system.

2.1 Physical properties

The physical system to be modelled is a steel cantilever plate having sides 0.250m, 0.04m and 0.0015m, Young modulus $E = 210GPa$, Poisson ratio $\nu = 0.3$ and mass density $\rho = 7850\text{Kg/m}^3$). Let Ω be the middle cross-section of the plate. A Cartesian frame is chosen as shown in Fig. 1. A transversely-isotropic, linearly-piezoelectric, homogeneous actuator, having sides 0.046m, 0.033m and 0.000127m, mass density $\rho^p = 7700\text{Kg/m}^3$ and capacity $C^p = 126\text{nF}$, is bonded on the upper surface of the plate, as shown in Fig. 1. The relevant closed-circuit/clamped material constants [11], denoted by c^p_{11}, c^p_{33}, c^p_{44}, c^p_{12}, c^p_{13}, ε^p_{11}, ε^p_{33}, e^p_{31}, e^p_{33} and e^p_{15}, according to the classical Voigt notation, are given in Table 1. Here the superscript p is used to distinguish any quantity relevant to the piezo-actuator.

c^p_{11} [GPa]	133.	ε^p_{11} [nF/m]	9.97
c^p_{12} [GPa]	77.5	ε^p_{33} [nF/m]	8.70
c^p_{13} [GPa]	87.0	e^p_{31} [C/m^2]	-7.22
c^p_{33} [GPa]	127.	e^p_{33} [C/m^2]	15.10
c^p_{44} [GPa]	26.7	e^p_{15} [C/m^2]	13.37

Table 1: piezo-actuator (ACX). Elastic, dielectric and piezoelectric properties

In addition, the following material constants, relevant to the situation of negligible transversal stress inside the piezo-actuator, are introduced [2]:

$$\begin{array}{ll} \overline{\varepsilon}^p_{33} = \varepsilon^p_{33} + (e^p_{33})^2/c^p_{33}, & \overline{\varepsilon}^p_{11} = \varepsilon^p_{11} + (e^p_{15})^2/c^p_{44} \\ \overline{c}^p_{11} = c^p_{11} - (c^p_{13})^2/c^p_{33}, & \overline{c}^p_{12} = c^p_{12} - (c^p_{13})^2/c^p_{33} \\ \overline{e}^p_{31} = e^p_{31} - c^p_{13}e^p_{33}/c^p_{33}, & \nu^p = \overline{c}^p_{12}/\overline{c}^p_{11}. \end{array}$$

A variational formulation for the coupled electro-mechanical system is developed under the following simplifying assumptions:

i) the thickness t^p of the piezo-actuator is negligible with respect to the thickness t of the plate. Thus, only the in-plane (membranal) behavior of the piezo-actuator is considered;

ii) the in-plane displacement $\mathbf{u}^p = (u^p_1, u^p_2)$ of the piezo-actuator is constant inside the thickness of the actuator;

iii) the in-plane stiffness of the piezo-actuator is negligible with respect to the one of the plate;

iv) the displacement field (s_1, s_2, s_3) of the plate is represented according to Mindlin's hypotheses:

$$\begin{array}{rcl} s_1(x_1,x_2,x_3) & = & \varphi_1(x_1,x_2)\,x_3 \\ s_2(x_1,x_2,x_3) & = & \varphi_2(x_1,x_2)\,x_3 \\ s_3(x_1,x_2,x_3) & = & w(x_1,x_2) \end{array}$$

where w is the deflection of the middle plane of the plate and $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)$ is the rotation of the fibers parallel to x_3;

v) the electric potential v^p is linear across the thickness of the piezo-actuator and hence the electric field along the x_3 axis can be expressed as $-v^p/t^p$;

vi) the transversal stress σ_{33} in the plate and in the piezo-actuator is negligible.

Under the previous assumptions, the electro-mechanical potential energy $\mathcal{E}$ of the piezoelectric laminate [2] may be considerably simplified and turns out to be:

$$\mathcal{E} = \mathcal{E}_m + \mathcal{E}^p_m + \mathcal{E}^p_e + \mathcal{E}^p_{em} + \mathcal{U}_m + \mathcal{U}^p_e \qquad (1)$$

where different terms may be recognized:

– $\mathcal{E}_m$ is the elastic potential energy of the plate:

$$\mathcal{E}_m = \frac{tk\mu}{2}\int_\Omega ||\boldsymbol{\varphi} + \nabla w||^2\, da + \frac{Et^3}{24(1-\nu^2)}\int_\Omega [(1-\nu)||\hat{\nabla}\boldsymbol{\varphi}||^2 + \nu(\mathrm{div}\,\boldsymbol{\varphi})^2]\, da,$$

where the first integral is the shear energy and the second integral is the bending energy, $\mu = E/[2(1+\nu)]$ is the shear modulus, $k = 5/6$ is the shear factor, $||\cdot||$ denotes the norm, ∇ and div are, respectively, the gradient and the divergence operators with respect to the x_1, x_2 variables and a hat denotes the symmetric part of a tensor.

– $\mathcal{E}^p_m$ is the elastic potential energy in the piezoelectric layer:

$$\mathcal{E}^p_m = \frac{t^p\overline{c}^p_{11}}{2}\int_\Omega [(1-\nu^p)||\hat{\nabla}\mathbf{u}^p||^2 + \nu^p(\mathrm{div}\,\mathbf{u}^p)^2]\, da,$$

– $\mathcal{E}^p_e$ is the electrostatic potential energy in the piezoelectric layer:

$$\mathcal{E}^p_e = -\frac{\overline{\varepsilon}^p_{33}}{2t^p}\int_\Omega (v^p)^2\, da - \frac{\overline{\varepsilon}^p_{11}t^p}{24}\int_\Omega ||\nabla v^p||^2\, da,$$

where the first integral takes into account the energy associated with electric field along x_3 and the second integral is due to the in-plane electric field.

– $\mathcal{E}^p_{em}$ is the electro-mechanical coupling potential energy:

$$\mathcal{E}^p_{em} = \overline{e}^p_{31}\int_\Omega v^p \mathrm{div}\,\mathbf{u}^p\, da,$$

– $\mathcal{U}_m$ is the potential energy of the external load q, normal to the plate:

$$\mathcal{U}_m = -\int_\Omega qw\, da,$$

– $\mathcal{U}^p_e$ is the potential energy of the free electric charge ω on the piezoelectric surfaces:

$$\mathcal{U}^p_e = \int_\Omega \omega v^p\, da,$$

The in-plane displacement $\mathbf{u}^p$ of the piezoelectric layer is given by:

$$\mathbf{u}^p = \frac{t}{2}\boldsymbol{\varphi},$$

due to the requirement of continuous displacements through the thickness of the structure.

The kinetic energy of the coupled system is $\mathcal{T} = \mathcal{T}_m + \mathcal{T}^p_m$, where:

– $\mathcal{T}_m$ is the kinetic energy of the plate:

$$\mathcal{T}_m = \frac{\rho}{2}\int_\Omega [t\dot{w}^2 + \frac{t^3}{12}||\dot{\boldsymbol{\varphi}}||^2]\, da,$$

– $\mathcal{T}^p_m$ is the kinetic energy of the piezo-actuator:

$$\mathcal{T}^p_m = \frac{\rho^p}{2}\int_\Omega [t^p\dot{w}^2 + \frac{t^2t^p}{4}||\dot{\boldsymbol{\varphi}}||^2]\, da,$$

where a dot denotes differentiation with respect to the time.

2.2 Modal analysis

From the foregoing variational formulation a finite-element formulation can be obtained. It is based on a two-dimensional, quadrangular, four-node, Mindlin-type finite-element, with four degrees of freedom per node. Locking phenomena are avoided by adopting a linked interpolation method and enriching the interpolation scheme of the rotational field with some internal degrees of freedom [3].

The middle cross-section Ω of the plate is discretized by using a regular mesh. The potential energy $\mathcal{E}$ and the kinetic energy $\mathcal{T}$ are then evaluated as functions of the nodal values of the unknown fields w, $\boldsymbol{\varphi}$ and v^p. Finally, the discrete differential equations of motion are obtained by using the Hamilton principle:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ddot{x}_n \\ \ddot{v}_n \end{pmatrix} + \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}_n \\ \dot{v}_n \end{pmatrix} + \begin{pmatrix} K_{mm} & K_{me} \\ K_{me}^T & K_{ee} \end{pmatrix}\begin{pmatrix} x_n \\ v_n \end{pmatrix} = \begin{pmatrix} f_n \\ -q_n \end{pmatrix}, \qquad (2)$$

where a superscript T denotes transposition, x_n, v_n, f_n and q_n are respectively the nodal mechanical degrees of freedom, the nodal electric potentials of the actuator, the nodal external forces, and the nodal electric charges, M is the mass matrix, K_{mm} is the stiffness matrix, $-K_{ee}$ is the permittivity matrix, K_{me} is the piezoelectric coupling matrix and D takes into account the mechanical damping (a proportional damping is assumed).

By constraining all the electroded nodes to have the same electric potential (2) becomes

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ddot{x}_n \\ \ddot{v} \end{pmatrix} + \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}_n \\ \dot{v} \end{pmatrix} + \begin{pmatrix} K_{mm} & \mathcal{K}_{me} \\ \mathcal{K}_{me}^T & -C^p \end{pmatrix}\begin{pmatrix} x_n \\ v \end{pmatrix} = \begin{pmatrix} f_n \\ -q \end{pmatrix} \qquad (3)$$

where q denotes the electric charge on the actuator, v is the difference of electric potential between the electrodes, the matrix K_{me} is transformed into the column vector $\mathcal{K}_{me}$ and the scalar C^p is the electric capacity of the actuator at fixed structure.

Now M and K_{mm} are simultaneously diagonalized:

$$V^T M V = I, \qquad V^T K_{mm} V = \Lambda \qquad (4)$$

where V is a square matrix whose columns are the eigenmodes of the actuated cantilever plate when the electrodes on the actuator are shorted, Λ is a diagonal matrix containing the squared natural frequencies and I is the identity matrix. By introducing the modal coordinates $y = V^{-1}x_n$, and adding to (3) the relevant Kirchhoff equation, we have

$$\begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L \end{pmatrix}\begin{pmatrix} \ddot{y} \\ \ddot{v} \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} \Delta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & R \end{pmatrix}\begin{pmatrix} \dot{y} \\ \dot{v} \\ \dot{q} \end{pmatrix} + \begin{pmatrix} \Lambda & \mathcal{K} & 0 \\ \mathcal{K}^T & -C^p & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} y \\ v \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ v_a \end{pmatrix} \qquad (5)$$

where the column vector $\mathcal{K}$ contains the modal coupling stiffnesses relevant to all the eigenmodes of the structure and Δ is a diagonal matrix whose elements are the modal mechanical damping coefficients. As in Fig. 1, R and L are the values of the passive components of the circuit, and v_a is the applied control voltage. Reduced discrete models are obtained from (5) by considering only some components of the vector y. In particular, taking into account only the first eigenmode of the structure, the reduced model reads explicitly as:

$$m\ddot{y}_1 + c\dot{y}_1 + \lambda y_1 + kv = 0 \qquad (6)$$

$$ky_1 - C^p v = -q \qquad (7)$$

$$L\ddot{q} + R\dot{q} + v = v_a \qquad (8)$$

where $m = 1$, c, λ, k and y_1 are, respectively, the mass, the mechanical damping coefficient, the stiffness, the coupling stiffness and the modal coordinate relevant to the first eigenmode of the structure. Solving (7) with respect to v and substituting into (6) and (8), the following two equations are obtained:

$$m\ddot{y}_1 + c\dot{y}_1 + \overline{\lambda} y_1 + \frac{k}{C^p}q = 0 \qquad (9)$$

$$L\ddot{q} + R\dot{q} + \frac{1}{C^p}q + \frac{k}{C^p}y_1 = v_a \qquad (10)$$

where $\overline{\lambda} = \lambda + k^2/C^p$.

3 PASSIVE CONTROL

The vibration passive damping of a structure can be achieved by using an external shunt circuit, whose passive electric components are a resistor R and an inductor L and the active control voltage v_a is set to zero. In practical applications, the problem arises to choose the values of R and L in such a way as to obtain the most effective vibration damping. In order to perform such an optimization, the following dimensionless version of equations (9)-(10) is adopted [4]:

$$\ddot{Y} + 2\nu\dot{Y} + Y + \kappa\omega Q = 0 \qquad (11)$$

$$\ddot{Q} + 2\zeta\omega\dot{Q} + \omega^2 Q + \kappa\omega Y = 0 \qquad (12)$$

where $\omega = \omega_e/\omega_m$, $\omega_e = \sqrt{1/(LC^p)}$, $\omega_m = \sqrt{\overline{\lambda}/m}$, $\nu = c/(2\sqrt{m\overline{\lambda}})$, $\zeta = R/(2\omega_e L)$ and $\kappa = k/\sqrt{C^p\overline{\lambda}}$. The modal coupling coefficient κ depends only on the material and geometrical characteristics of the coupled vibrating structure. On the other hand, ω and ζ depend on the parameters R and L of the external electric circuit.

By adopting a pole-placement technique, the optimal values ω_{opt} and ζ_{opt} are implicitly defined by the following equations [4]:

$$\omega_{opt}^2\,\kappa^2 = (1-\nu^2)(-1+\omega_{opt}^2+2\nu^2 - 2\nu\sqrt{\nu^2-1+\omega_{opt}^2}) \tag{13}$$

$$\zeta_{opt} = \sqrt{\nu^2-1+\omega_{opt}^2}/\omega_{opt} \tag{14}$$

It is easy to verify that in the special case $\nu = 0$, relevant to the situation of vanishing mechanical damping, the above equations reduce to

$$\omega_{opt} = \sqrt{1/(1-\kappa^2)} \tag{15}$$

$$\zeta_{opt} = \kappa \tag{16}$$

which exactly coincide with Hagood and von Flotow's formulas [9], up to a transformation between the present dimensionless parameters κ, ω and ζ, and Hagood's K_{ij}, δ and r, given by:

$$\kappa^2 = \frac{K_{ij}^2}{1+K_{ij}^2}, \qquad \omega = \frac{\delta}{\sqrt{1+K_{ij}^2}}, \qquad \zeta = \frac{r\delta}{2} \tag{17}$$

4 ACTIVE CONTROL

Here, vibration damping for the cantilever plate described in Section 2 is achieved by means of a linear dynamic feedback compensator which generates the control input v_a based on the measured output $z = \ddot{w}_P$, being w_P the vertical displacement of the accelerometer.

For the sake of simplicity, the design is based on a reduced-order model of the system, which takes into account only the first eigenmode and neglects the mechanical damping. The design is carried out by means of standard LQG techniques (see, e.g., the books [7, 13]), considering, therefore, a suitable stochastic model of the system. Letting $\eta(t) = [\,y_1(t) \quad \dot{y}_1(t)\,]^T$, be the state of the system at time t, the state space equations of our design model are as follows:

$$\dot{\eta}(t) = A\,\eta(t) + b\,v_a(t) + \xi(t), \tag{18}$$

$$z(t) = C\,\eta(t) + d\,v_a(t) + \theta(t), \tag{19}$$

where $\xi(t)$, $\theta(t)$ are stationary Gaussian random processes described by $\mathbf{E}\{\xi(t)\} = 0$, $\mathbf{E}\{\theta(t)\} = 0$, $\mathbf{E}\{\xi(t)\xi^T(\tau)\} = \Xi\,\delta(t-\tau)$ and $\mathbf{E}\{\theta(t)\theta^T(\tau)\} = \Theta\,\delta(t-\tau)$, being $\Xi \in \mathbf{R}^{2\times 2}$, $\Xi = \Xi^T$, $\Xi \geq 0$, $\Theta \in \mathbf{R}$, $\Theta > 0$, and with $\mathbf{E}\{\cdot\}$ denoting the expected value of the argument. From equations (9) and (10), setting $R = 0$, $L = 0$, and $c = 0$, we have $A = \begin{bmatrix} 0 & 1 \\ -\lambda & 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ -k \end{bmatrix}$, whereas $C = [-\lambda\tilde{V} \quad 0]$, $d = -k\tilde{V}$, being $\tilde{V}$ the vertical displacement of the first eigenmode of the plate in correspondence to the position of the accelerometer. The LQG controller proposed here is designed in order to minimize the following performance index:

$$\mathcal{J} = \mathbf{E}\left\{\frac{1}{2}\int_0^{+\infty} \left(\eta^T(\tau)Q\,\eta(\tau) + r\,v_a^2(\tau)\right)\mathrm{d}\tau\right\},$$

where $r \in \mathbf{R}$, $r > 0$ and $Q \in \mathbf{R}^{2\times 2}$, $Q = Q^T$, $Q \geq 0$, are to be seen, together with Θ and Ξ, as the design parameters, to be chosen in order to obtain a satisfactory behaviour for the closed loop system.

As is well known, if P and Σ denote the unique positive definite solutions of the algebraic Riccati equations:

$$Q + A^T P + P\,A - \frac{1}{r}\,P\,b\,b^T P = 0,$$

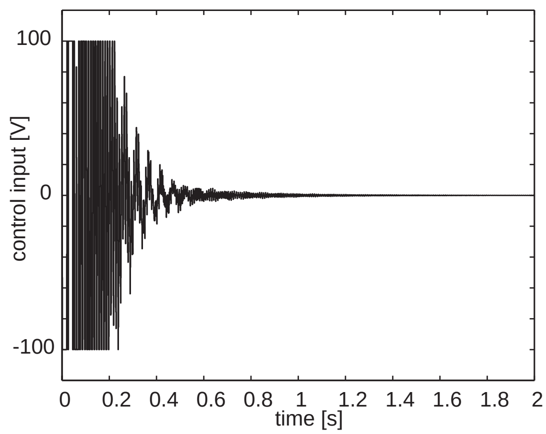
$$\Xi + A\,\Sigma + \Sigma\,A^T - \frac{1}{\Theta}\,\Sigma\,C^T C\,\Sigma = 0,$$

respectively, the LQG compensator is described, in state-space form, by the equations:

$$\dot{x}_K(t) = A_K\,x_K(t) + b_K\,z(t), \tag{20}$$

$$u_a(t) = C_K\,x_K(t), \tag{21}$$

where $x_K \in \mathbf{R}^2$, $A_K = A + b\,K_c - K_o\,C - K_o\,d\,K_c$, $b_K = K_o$, $C_K = K_c$, $K_c = -\frac{1}{r}\,b^T P$, $K_o = \frac{1}{\Theta}\,\Sigma\,C^T$ and u_a would coincide with v_a if it was possible to directly connect (i.e. with no saturation) the proposed compensator with the given system. The choice of r, Q, Θ and Ξ has to be made by considering the well known trade-off between the objective of increasing the convergence speed, which requires that the elements of Q and Ξ are taken "large", compared to r and Θ, respectively, and the objectives of keeping the absolute value of v suitable small (i.e., admissible for the piezoelectric actuator) and of reducing the "spillover" effects due to the unmodelled dynamics (which will arise when the compensator designed on the basis of the reduced order model (18)-(19) will be applied to the actual infinite dimensional system). This two last objectives, in turn, require that the elements of Q and Ξ are "small", compared to r and Θ, respectively. Without loss of generality, we have set $r = \Theta = 1$, whereas, in order to select matrices Q and Ξ, we have found very convenient to use the technique reported in [12], which allows to select such matrices in order to obtain desired closed-loop eigenvalues for the ideal overall system. Satisfactory results have been obtained by choosing as closed-loop eigenvalues $\lambda_{1,2} = -10 \pm 122\,j$ and $\lambda_{3,4} = -15 \pm 122\,j$. In order to evaluate the performance of the proposed LQG compensator when applied to the real system, simulations have been performed by using



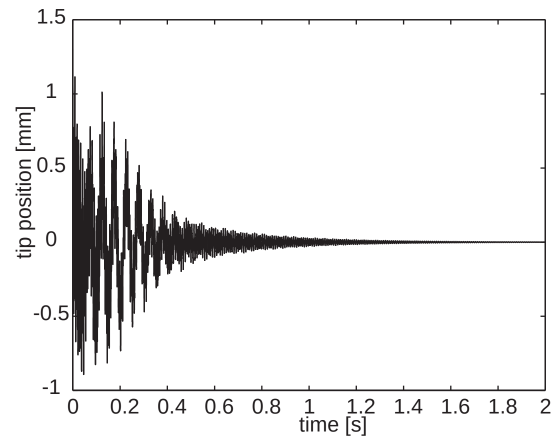


Figure 2: Simulation results for the active control law: (a) input voltage v_a, (b) vertical displacement of the accelerometer w_P.

a reduced order model which takes into account the first five bending vibration modes of the plate, and includes some mechanical damping (properly identified on the available experimental system).

In order to be implemented through a digital computer, the proposed continuous-time LQG compensator needs to be discretized. This has been done by means of standard routines, choosing the sampling time as $\delta_T = 1/1024s$, which is an admissible value for the experimental equipment. Notice that the discrete-time input $z_D(k)$ of the discretized LQG compensator is the ideal sampling of the continuous-time output of the plant $z(t)$, i.e., $z_D(k) = z(k\,\delta_T)$, whereas a zero order holding mechanism converts the discrete-time output $u_{a,D}(k)$ of the discretized compensator into the continuous-time control input, i.e., $u_a(t) = u_{a,D}(k)$, for all $t \in (k\,\delta_T,\ (k+1)\,\delta_T]$. Furthermore, to be more realistic, it has been taken into account that, in order to preserve the piezo-actuator, it must be ensured that $|v| \leq 100\,V$. In an experimental setup such a saturation is guaranteed by the power amplifier which generates the input voltage, but in the simulations an ideal saturation has been considered. The results of the first two seconds of simulation, starting from null initial conditions for the compensator, and initial deformations of the plate corresponding to a suitable impulsive force applied at time $t = 0$ to the free end of the plate in central position, are reported in Fig. 2.

In order to compare such simulation results with the ones obtained by means of passive control, we have simulated, on the same five-degrees of freedom reduced order model, the behaviour of the passive control law proposed in Section 3, from the same initial conditions at time 0^+. The comparison between the behaviour of the discretized LQG compensator and the passive control law discussed in Section 3 is reported in Fig. 3.

5 CONCLUSIONS

Accurate models of a steel cantilever plate, actuated by means of a piezoelectric element, have been obtained, by taking explicitly into account in the finite-element modelling phase the electromechanical coupling of the structure.

Based on such models, purely passive and purely active control laws have been designed, and their performances have been evaluated in simulation.

Ongoing research focuses both on the experimental validation of such results, and on the development of hybrid control laws, combining the advantages of the passive and the active control.

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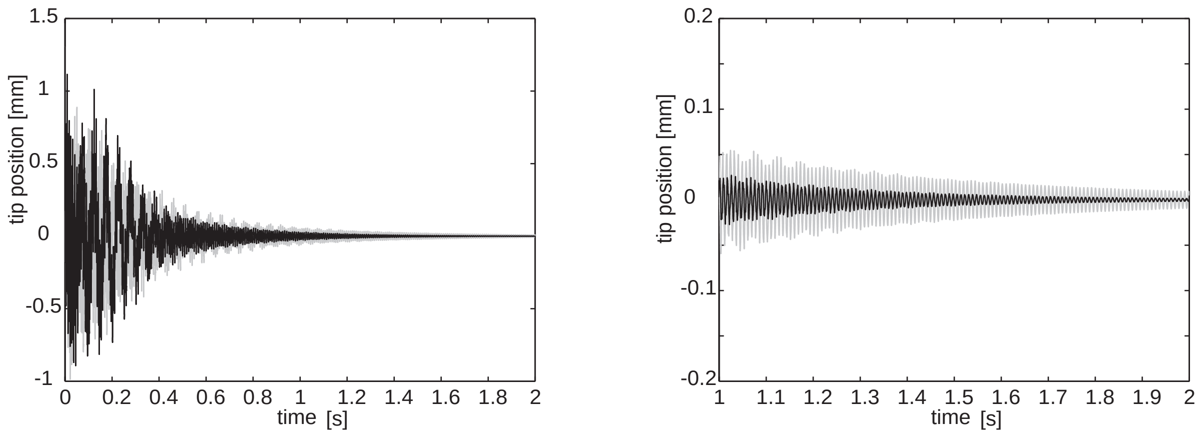

Figure 3: Comparison between the results of passive control (grey line) and the ones of active control (black line): in (a) the first two seconds of simulation are reported, for the sake of completeness, whereas in the detail (b) it can be appreciated that, once the effects of the saturation disappear, active control provides better vibration damping.

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