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CONTROL OF NESTED SYSTEMS

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Abstract. This paper presents an input-output point of view for the problem of optimal command following and disturbance rejection of systems which are comprised of subsystems that affect each other in a nested manner. In such a nested manner, a subsystem affects the subsystems that are exterior to it but not the subsystems that are interior to it. By using model matching methods the problem is shown to be convex. Techniques to solve this convex yet non-standard problem are discussed.

1. INTRODUCTION

In this paper, we take an input-output approach to consider the optimal disturbance rejection problem in a feedback structure where there are subsystems that are interconnected in a nested fashion to form an overall system. In this nested fashion the the subsystems connect to one another so that the flow of information and control action goes from inside to outside. Any control action at a particular nest (subsystem) depends only on the information of the subsystems inside the nest and not on any information outside of it. Also, the control action at a particular nest (subsystem) does not affect the subsystems (nests) inside it but only the exterior ones. Several engineering applications may posess such a nested structure where inner feedback loops as well as external (outer) are present. The motivating application for this work is the case of integrated flight propulsion control (e.g., [2]) where there is a natural interior system, the engine, and an exterior one, the airframe dynamics. These can be thought of in a nested structure since the airframe controls, such as the aerodyanamic surfaces, do not affect the engine.

The problem of interest here is minimize the norm of the overall map from all external commands and disturbances to the system variables that need to be regulated, subject to internal stability. The basic observation is that this problem can be casted as a closed loop MIMO control problem with additional constraints on the structure of the controller. Input-output methods can then be used to transform the problem to a model matching problem with convex constraints in the Youla parameter. In the paper we provide a more detailed exposition of how to solve the problem in the case of $\mathcal{H}^2$ and ℓ^1 optimal control.

2. PROBLEM DEFINITION

To illustrate the problem in simple terms we consider only two nests. The generalization to n nests is straightforward. Thus we consider the case of Figure 1 where there is a system comprised of two nests (subsystems.) The internal subsystem consists of a plant G_1 together with its controller C_1 whereas the external consists of the plant G_2 with the controller C_2. The internal and external subsystems have control inputs u_1, u_2 and measured outputs y_1 and y_2 respectively. Due to the nested structure depicted in the figure, the control input u_1 depends only on the measurement y_1 whereas u_2 can depend on both y_1 and y_2. Moreover, y_1 is affected only by u_2 while y_2 is affected by both u_1 and u_2. The overal system is subjected to exogenous inputs (e.g., commands, disturbances, sensor noise) and there are also outputs to be regulated. In particular, we allow for inputs w_1 affecting directly the internal subsystem, inputs w_2 that affect the external subsystem only, and, inputs w_{12} that affect both subsystems. Similarly, the outputs of interest z_1, z_2 and z_{12} are related respectively directly to the internal, directly to the external and to combination of both subsystems. Denote by

$$w := \begin{pmatrix} w_1 \\ w_{12} \\ w_2 \end{pmatrix},$$

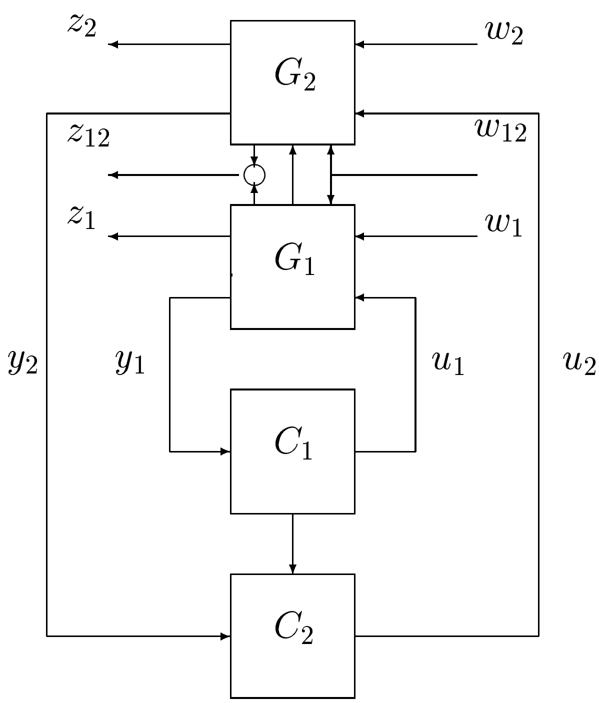

Fig. 1: Nested Structure

$$z := \begin{pmatrix} z_1 \\ z_{12} \\ z_2 \end{pmatrix},$$

and let the map from w to z be denoted as Φ.

The problem of interest is as follows:
Problem: *Find C_i, $1 \leq i \leq 2$ such that, subject to internal stability, the norm $||\Phi||$ is minimized.*

Throughout the paper we assume all systems to be linear time invariant and described in discrete-time. Also, all the signals in Figure 1 are allowed to be vector-valued. The norm $||\Phi||$ may refer to any norm, e.g., $\mathcal{H}^2$, $\mathcal{H}^\infty$ or ℓ^1. Since its particular type is not important at this stage we will use the norm symbol generically. By internal stability here we mean that all signals in the system remain (finite gain) bounded in the presence of bounded w and possible bounded additive disturbances in y_i and u_i for all $1 \leq i \leq 2$. A necessary assumption for the problem to make sense is that for all $i = 1, 2$ there exists a controller K_i operating on y_i alone to produce u_i that stabilizes G_i alone.

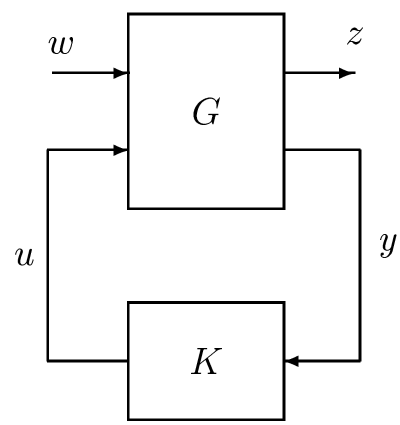

Fig. 2: Standard Framework

3. PROBLEM SOLUTION

3.1. Problem transformation

The above defined problem can be put in a standard $G - K$ control design framework of Figure 2 with the following signal identifications:

$$y := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$

$$u := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix},$$

and w and z as before.

The problem objective can be equivalently transformed to finding a stabilizing controller K that minimizes the map from w to z i.e., Φ. However, for the controller K to correspond to the nested structure of Figure 1 K should be of the form

$$K = \begin{pmatrix} K_1 & 0 \\ K_{12} & K_2 \end{pmatrix}$$

i.e., it should be an lower (block) triangular (l.b.t.) system.

Considering the equivalent problem in Figure 2 all stabilizing controllers, K, not necessarily with the lower triangular structure required, are given by the Youla parametrization [8]

$$K = (Y_l - D_l Q)(X_l - N_l Q)^{-1} = (X_r - Q N_r)^{-1}(Y_r - Q D_r)$$

where Q is a stable free parameter and $Y_l,\ D_l,\ X_l,\ N_l,\ X_r,\ N_r,\ Y_r,\ D_r$ can be obtained from a doubly coprime factorization (e.g., [4, 7]) of G_{22}, where G_{22} is the map from u to y in Figure 2. Considering the structure of G_{22} we have that it is of the form

$$G_{22} = \begin{pmatrix} * & 0 \\ * & * \end{pmatrix}$$

i.e., G_{22} has a lower triangular structure. It is precisely this structure that leads to the following key lemma in resolving the problem

Lemma 3.1 *There exist a coprime factorization of G_{22} such that all stabilizing controllers K of G with lower triangular structure are given by*

$$K = (Y_l - D_l Q)(X_l - N_l Q)^{-1} = (X_r - QN_r)^{-1}(Y_r - QD_r).$$

where Q is lower (block) triangular i.e., has the same structure as K.

Proof(Sketch) The proof in the case where G_{22} is stable follows by using the parametrization $K = -Q(I - G_{22}Q)^{-1}$. In the more general case one can construct an observer-based controller with the lower triangular structure which in turn provides the coprime factors with the lower triangular structure (the details are ommitted here.) An alternate proof can be provided along the lines of [3]. ■

With this key lemma at hand, the problem of minimizing $\|\Phi\|$ can be casted as

$$\mu := \inf_{K \text{ stabilizing, l.b.t.}} \|\Phi\| = \inf_{Q \text{ stable l.b.t.}} \|H - UQV\|$$

where H, U, V are stable systems. Therefore, the resulting problem is convex, yet infinite dimensional (the pulse response coefficients of Q.)

3.2. Approaches for solving the equivalent problem

In principle, one can solve the problem by considering truncations of the Q parameter [1] and thus approximating the problem with a finite dimensional (the pulse response coefficients of the truncated Q) convex programming problem

$$\mu_N := \inf_{Q} \|H - UQ_N V\|$$

where Q_N is a Finite Impulse Response (FIR) of length N, lower block triangular system. It can be easily checked that $\mu_N \to \mu$ monotonically from above as $N \to \infty$. Then main shortcoming of this method is that it cannot indicate how close to the optimal solution is the converging lower bound μ_N. To do so, one needs converging lower bounds as well. In the sequel we specialize the discussion to the optimal $\mathcal{H}^2$ and ℓ^1 problem to provide alternative methods.

3.2.1. $\mathcal{H}^2$-norm minimization

In this case we can invoke the projection theorem along the lines in [6] to obtain the solution directly. To this end let $U = U_i U_o$, $V = V_o V_i$ be an inner-outer factorization of U and V respectively. Define the subspace $M = \{Z : Z = U_o Q V_o\}$ with $Q \in \mathcal{H}^2$ and lower triangular. Then the following can be shown:

Theorem 3.1 *The optimal solution Z_o for the problem*

$$\mu = \inf_{Z \in M} \|H - U_i Z V_i\|$$

is given by the projection onto M

$$Z_o = \Pi_M U_i^* H V_i^*.$$

Once Z_o is found an optimal Q can be found as $Q = U_o^{-r} Z_o V_o^{-l}$ where U_o^{-r} is a right inverse of U_o and V_o^{-l} is a left inverse of U_o

3.2.2. ℓ^1-norm minimization

In this case one can use an extention of the scaled-Q method in [5] to provide converging lower and lower bounds to μ. In particular, for the problem at hand let P_N denote the Nth truncation operator and define the two finite dimensional linear programs:

$$\nu_N(\alpha) := \min \max\{\|H - R\|, \alpha \|Q\|\}$$

subject to

$$P_N(R) = P_N(HQV)$$

and

$$\mu_N(\alpha) := \min \max\{\|H - R\|, \alpha \|Q\|\}$$

subject to

$$R = UP_N(Q)V$$

where α is a scalar positive parameter. Then, using elements of duality theory the following can be shown

Theorem 3.2 *There exists an apriori computable α_0 such that for all α with $0 < \alpha \leq \alpha_0$, $\mu_N(\alpha) \to \mu$ monotonically from above and $\nu_N(\alpha) \to \mu$ monotonically from below as $N \to \infty$.*

Hence, with the above theorem one obtains close to optimal solutions to any any prespecified accuracy.

4. CONCLUSIONS AND DISCUSSION

In this paper we investigated the problem of optimal disturbance rejection when a nested subsystem structure exists. Such a structure allows the problem to be transformed equivalently to a controller design problem with the constraint that the controller be lower triangular. The equivalent problem was shown to be a convex model matching problem in the Youla parameter. Approaches via projection and duality theoremsto solve respectively the optimal $\mathcal{H}^2$ and ℓ^1 problems were given. Several remarks are in order.

For the $\mathcal{H}^\infty$ problem a Nehari-based approach [6] to get a sequence of converging lower bounds is possible. However, more efficient computaions are needed. The author is currently pursuing an extension of the Q truncation approach along the lines of [5] in order to obtain converging lower bounds.

Finally, the case of multirate operation can also be resolved along the same lines. In particular, one can consider the same problem in the case where the various nests recievesend signals with different sampling rates e.g., the exterior nests may send information at a slower rate than their interior. The details of this more compex situation are currently studied and will be the subject of future publications.

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