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ROBUST CONTROL OF THE LATERAL MOTION OF AN AIRCRAFT WITH ACTUATOR FAILURE

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Abstract. A robust controller, facilitating roll aircraft manoeuvres without affecting the respective sideslip attitude, is designed for the case of the lateral motion of an aircraft with actuator failure. To handle the issue the robust input output triangular decoupling technique is applied to the lateral motion dynamics of the aircraft. The condition under which the problem is solvable via state feedback appears to be generically true . The general form of the robust static decoupling controllers, as well as the general form of the resulting triangular decoupled closed loop system, are derived. Simulation are derived for a four engined executive jet aircraft.

Key Words. Actuator failure, Flight Control, Robust Control, I/O Triangular Decoupling.

1. INTRODUCTION

Independent control of the flight variables of an aircraft is a central problem in flight control systems [1]-[9]. With regard to the lateral motion of an aircraft it is noted that there is a coupling between the lateral velocity and the roll modes in the sense that they are all influenced via aileron and rudder commands. This is an undesirable effect in many lateral manoeuvres [8]. It is important to eliminate the coupling between the roll rate and the sideslip angle, thus, allowing the pilot to perform manoeuvres by applying simple commands (see f.e. [1]-[4]). This design goal is required to be satisfied independently of actuator failure. In particular, the error of the actuator is considered to be due to malfunctions of aileron actuator.

In this paper, a static state feedback law yielding robust input output triangular decoupling between the sideslip angle and the roll rate of an aircraft is proposed. The problem is proven to be solvable for almost all flight conditions in the sense that an inequality involving known aerodynamic stability derivatives must be satisfied. According to aerodynamic data this inequality is almost always satisfied. The explicit characterisation of all the independent of the uncertainties static state feedback controllers solving the problem, is derived in terms of the aerodynamic derivatives of the aircraft as well as free parameters that can be used to satisfy pole assignment requirements. The results are illustrated via simulations.

2. THE AIRCRAFT MODEL AND CONTROL OBJECTIVE

2.1. Lateral motion model

Consider the case of linear systems with nonlinear uncertain structure, i.e. the case of systems described by

$$\dot{x}(t) = A(q)x(t) + B(q)u(t) \quad , \quad y(t) = C(q)x(t) \qquad (2.1)$$

where $x \in \Re^n$, $u \in \Re^m$ and $y \in \Re^m$ are the state, input, and performance output vector, respectively, and where $\Re$ denotes the set of real numbers. The matrices $A(q) \in [\wp(q)]^{n\times n}$, $B(q) \in [\wp(q)]^{n\times m}$ and $C(q) \in [\wp(q)]^{m\times n}$ are function matrices depending upon the uncertainty vector $q = (q_1, \ldots, q_l) \in \mathcal{Q}$, where $\mathcal{Q}$ is the uncertainty domain and $\wp(q)$ is the set of all functions of q.

The lateral linearized motion of an aircraft with actuator failure can be expressed by system (2.1) since the model depends upon errors of the actuator. For the above mentioned uncertain case, the lateral-directional equation of motion of a fixed-wing aircraft [8] turns out to a system of equations of the form (2.1), with

$$x(t) = \begin{bmatrix} \beta(t) & p(t) & r(t) & \phi(t) \end{bmatrix}^T ,$$
$$u(t) = \begin{bmatrix} \delta_A(t) & \delta_R(t) \end{bmatrix}^T \qquad (2.2)$$

and with

$$A = \begin{bmatrix} Y_\upsilon & 0 & -1 & g\cos\gamma_0/U_0 \\ L_\upsilon & L_p & L_r & 0 \\ N_\upsilon & N_p & N_r & 0 \\ 0 & 1 & \tan\gamma_0 & 0 \end{bmatrix},$$

$$B(q) = \begin{bmatrix} 0 & Y_{\delta_R} \\ qL_{\delta_A} & L_{\delta_R} \\ qN_{\delta_A} & N_{\delta_R} \\ 0 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The variable $\beta(t)$ is the sideslip angle increment, $p(t)$ is the roll rate increment, $r(t)$ is the yaw rate increment, $\phi(t)$ is the roll (bank) angle increment, $\delta_A(t)$ and $\delta_R(t)$ are the rudder and aileron commands, respectively. The parameter γ_0 is the nominal value of the flight path angle. The parameters Y_i, L_i, N_i ($i \in \{\upsilon, p, r, \delta_R, \delta_A\}$) are lateral directional stability derivatives [8]. The parameter g is the gravitational acceleration and U_0 is the forward velocity. The variable q is an uncertain parameter which appears due to malfunctions of the aileron actuator.

2.2. Control objective

The objective of the present design scheme is to control independently the sideslip angle and the roll rate (or angle) in the presence of the uncertainty q. The independent control of the lateral motion variables facilitates the aircraft placement and maintenance to desired orientation, following precisely the pilot's commands.

To system (2.1) apply the static state feedback law

$$u(t) = Fx(t) + G\omega(t) \qquad (2.4)$$

where $\omega(t) = \begin{bmatrix} \beta_c(t) & p_c(t) \end{bmatrix}^T$ is the external command vector and where $\beta_c(t)$ and $p_c(t)$ denote the commands driving the performance variables $\beta(t)$ and $p(t)$, respectively. The design objective can be formulated as an input-output decoupling design scheme which must robustly be satisfied for every value of the uncertain parameter q.

3. NECESSARY AND SUFFICIENT CONDITION FOR ROBUST TRIANGULAR DECOUPLING OF THE SIDESLIP AND ROLL RATE

In this section it is investigated under which condition (over the lateral stability derivatives) robust triangular decoupling between $\beta(t)$ and $\phi(t)$ is achieved. Applying the results presented in [10] to the present case, the necessary and sufficient condition for robust input-output triangular decoupling between the sideslip and the angle, via a static state feedback law (independent of the uncertainty q), is

$$L_{\delta_A} Y_{\delta_R} \neq 0 \qquad (3.1)$$

According to aerodynamic data (see for example [8]) the condition (4.1) is almost always satisfied.

4. EXPLICIT CHARACTERISATION OF THE ROBUST TRIANGULAR DECOUPLING CONTROLLERS

It is of great importance to derive the explicit characterisation the controllers matrices yielding triangular decoupling between $\beta(t)$ and $p(t)$ i.e. independent control of $\beta(t)$ and $p(t)$ by the external commands $\beta_c(t)$, and $p_c(t)$. Based on the results of [6] and the data in (2.2) then the general explicit formulae of the robust triangular decoupling controllers matrices F and G, are

$$G = \begin{bmatrix} \dfrac{-p_{2,1}}{L_{\delta_A} p_{1,1} p_{2,2}} & \dfrac{1}{L_{\delta_A} p_{2,2}} \\ \dfrac{1}{Y_{\delta_R} p_{1,1}} & 0 \end{bmatrix} \qquad (4.1)$$

$$F = \begin{bmatrix} \dfrac{\lambda_{2,1}}{L_{\delta_A}} & \dfrac{\lambda_{2,2}}{L_{\delta_A}} & \dfrac{\lambda_{2,3}}{L_{\delta_A}} & \dfrac{\lambda_{2,4}}{L_{\delta_A}} \\ \dfrac{\lambda_{1,1}}{Y_{\delta_R}} & 0 & \dfrac{1}{Y_{\delta_R}} & \dfrac{-g\cos\gamma_0}{U_0 Y_{\delta_R}} \end{bmatrix} \qquad (4.2)$$

where $\lambda_{2,1}$, $\lambda_{2,2}$, $\lambda_{2,3}$, $\lambda_{2,4}$, $\lambda_{1,1}$ and $p_{2,1}$ are arbitrary parameters and $p_{1,1}$, $p_{2,2}$ are arbitrary different than zero parameters.

Relations (4.1) and (4.2) are explicit formulae implementable by elementary operations upon the stability derivatives and the values of g, γ_0 and U_0. The matrices F and G depend upon the parameters of the aircraft model, which is linearized around an equilibrium (operating) point. For a manoeuvre involving more than one operating point, the values of the controller have to be renewed by look up tables. This task can be carried out by an adjustment mechanism (in a real time computer) assigning also the closed loop poles. The explicitness of (4.1 and 2), allows the adjustment mechanism to be executable in very short time.

5. GENERAL FORM OF THE CLOSED LOOP SYSTEM

The general analytical expression of the triangularly decoupled closed loop transfer function is

$$H_{cl}(s,q) = \begin{bmatrix} h_{1,1}(s) & 0 \\ h_{2,1}(s,q) & h_{2,2}(s,q) \end{bmatrix}$$

where

$$h_{1,1}(s) = \frac{(p_{1,1})^{-1}}{s - \lambda_{1,1} - Y_\upsilon},$$

$$h_{2,2}(s,q) = \frac{(p_{2,2})^{-1}(a_0 s^2 + a_1 s + a_2)}{\beta_0 s^3 + \beta_1 s^2 + \beta_2 s + \beta_3},$$

$$h_{2,1}(s,q) = \frac{(p_{1,1})^{-1}(p_{2,2})^{-1}(\delta_0 s^3 + \delta_1 s^2 + \delta_2 s + \delta_3)}{(s - \lambda_{1,1} - Y_\upsilon)(\beta_0 s^3 + \beta_1 s^2 + \beta_2 s + \beta_3)}$$

and where

$$
\begin{aligned}
&a_0 = -qL_{\delta_A}U_0Y_{\delta_R}\ ,\\
&a_1 = [L_{\delta_A}(N_{\delta_R} + N_rY_{\delta_R}) - L_{\delta_R}N_{\delta_A} - L_rN_{\delta_A}Y_{\delta_R}]U_0q\ ,\\
&a_2 = q\sin(\gamma_0)g(-L_{\delta_A}N_{\delta_R} + L_{\delta_R}N_{\delta_A})\\
&\beta_0 = -L_{\delta_A}U_0Y_{\delta_R}\ ,\\
&\beta_1 = U_0\{\lambda_{2,3}N_{\delta_A}Y_{\delta_R}q + L_{\delta_A}[N_{\delta_R} + (L_p + N_r + \lambda_{2,2}q)Y_{\delta_R}]\}\\
&\beta_2 = -gL_{\delta_A}L_{\delta_R}\cos\gamma_0 - gL_{\delta_A}N_{\delta_R}\sin\gamma_0 +\\
&\qquad U_0\{-L_{\delta_A}L_pN_{\delta_R} + L_{\delta_A}L_{\delta_R}N_p\} +\\
&\qquad U_0\{\lambda_{2,2}q(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R}) + L_{\delta_A}Y_{\delta_R}(L_rN_p - L_pN_r) +\\
&\qquad +\lambda_{2,4}qL_{\delta_A}Y_{\delta_R} - \lambda_{2,3}qY_{\delta_R}(L_pN_{\delta_A} - L_{\delta_A}N_p) +\\
&\qquad \lambda_{2,2}qY_{\delta_R}(L_rN_{\delta_A} - N_rL_{\delta_A}) + \tan\gamma_0\lambda_{2,4}N_{\delta_A}qY_{\delta_R}\}\\
&\beta_3 = g\cos\gamma_0(-L_{\delta_A}N_{\delta_R}L_r + L_{\delta_A}N_rL_{\delta_R} +\\
&\qquad \lambda_{2,3}q(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R}))\ +\\
&\qquad \lambda_{2,4}qU_0(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R} + Y_{\delta_R}(L_rN_{\delta_A} - L_{\delta_A}N_r))\ +\\
&g\sin\gamma_0(L_{\delta_A}N_{\delta_R}L_p - L_{\delta_A}N_pL_{\delta_R} - \lambda_{2,2}q(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R}))\ +\\
&\qquad \lambda_{2,4}(-L_pN_{\delta_A} + L_{\delta_A}N_p)qU_0Y_{\delta_R}\tan\gamma_0\\
&\qquad \delta_0 = U_0L_{\delta_A}(-L_{\delta_R}p_{2,2} + p_{2,1}Y_{\delta_R})\\
&\delta_1 = -U_0\{N_{\delta_A}q[(\lambda_{2,3}p_{2,2} + p_{2,1})L_{\delta_R} + p_{2,1}Y_{\delta_R}L_r] +\\
&\quad L_{\delta_A}p_{2,2}(-L_{\delta_R}N_r + \lambda_{2,3}N_{\delta_R}q + L_\upsilon Y_{\delta_R} + \lambda_{2,1}qY_{\delta_R} - L_{\delta_R}Y_\upsilon) +\\
&\qquad L_{\delta_A}p_{2,2}L_rN_{\delta_R} + L_{\delta_A}p_{2,1}[qN_{\delta_R} + (\lambda_{1,1} + N_r + Y_\upsilon)qY_{\delta_R}]\}\\
&\delta_2 =\\
&-U_0p_{2,2}\{N_{\delta_A}q[\lambda_{2,1}(L_{\delta_R} - L_rY_{\delta_R}) - \lambda_{2,3}(L_\upsilon Y_{\delta_R} - L_{\delta_R}Y_\upsilon)]\ +\\
&\qquad -L_{\delta_A}\{L_\upsilon N_{\delta_R} - L_{\delta_R}N_\upsilon + q\lambda_{2,1}(N_{\delta_R} + N_rY_{\delta_R}) -\\
&\qquad \lambda_{2,3}(N_\upsilon Y_{\delta_R} - N_{\delta_R}Y_\upsilon) + N_r(L_\upsilon Y_{\delta_R} - L_{\delta_R}Y_\upsilon) -\\
&\quad L_r(N_\upsilon Y_{\delta_R} - N_{\delta_R}Y_\upsilon)\}\ - \lambda_{2,4}q\tan\gamma_0(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R})\}\ -\\
&\quad p_{2,1}q\{U_0\lambda_{2,1}(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R} + L_rY_{\delta_R}N_{\delta_A} - N_rY_{\delta_R}L_{\delta_A})\\
&\quad -Y_\upsilon U_0(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R} + L_rY_{\delta_R}N_{\delta_A} - N_rY_{\delta_R}L_{\delta_A}) +\\
&\qquad g\sin\gamma_0(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R})\}\\
&\delta_3 = \tan\gamma_0|\{\lambda_{2,4}qU_0p_{2,2}[-L_{\delta_A}N_\upsilon Y_{\delta_R} + L_\upsilon Y_{\delta_R}N_{\delta_A}\\
&\quad -N_{\delta_A}Y_\upsilon L_{\delta_R} + L_{\delta_A}N_{\delta_R}Y_\upsilon] -\ p_{2,2}g\cos\gamma_0[L_{\delta_A}N_{\delta_R}L_\upsilon -\\
&\qquad L_{\delta_A}N_\upsilon L_{\delta_R} + \lambda_{2,1}q(L_{\delta_R}N_{\delta_A} - L_{\delta_A}N_{\delta_R})]\ -\\
&\quad p_{2,1}g\cos\gamma_0[-qL_{\delta_R}N_{\delta_A}(Y_\upsilon + \lambda_{1,1}) + qN_{\delta_R}(L_{\delta_A}\lambda_{1,1} + Y_\upsilon)]\}
\end{aligned}
$$

Choosing $\lambda_{1,1} < -Y_\upsilon$, the stability of $h_{1,1}(s)$ is guaranteed. Using the results in [11], the parameters $\lambda_{2,1}$, $\lambda_{2,2}$, $\lambda_{2,3}$, $\lambda_{2,4}$ can be chosen such that the polynomial $\beta_0s^3 + \beta_1s^2 + \beta_2s + \beta_3$ is Hurwitz invariant (robustly stable). Since the number of transition poles is equal to the system dimension then the closed loop system can become robustly stable.

From the triangular form of the closed loop system it is concluded that

$$
\begin{aligned}
\mathcal{L}\{\beta(t)\}\ &= h_{1,1}(s,q)\mathcal{L}\{\beta_c(t)\}\ ,\\
\mathcal{L}\{p(t)\} &= h_{2,1}(s,q)\mathcal{L}\{\beta_c(t)\}\ + h_{2,2}(s,q)\mathcal{L}\{p_c(t)\}
\end{aligned}
$$

Choosing $\mathcal{L}\{\beta_c(t)\}\ = 0$ then it holds that $\mathcal{L}\{\beta(t)\}\ = 0$, $\mathcal{L}\{p(t)\} = h_{2,2}(s,q)\mathcal{L}\{p_c(t)\}$ and hence

$$
\mathcal{L}\{\phi(t)\} = \frac{1}{s}[h_{2,2}(s,q)\mathcal{L}\{p_c(t)\} + \tan\gamma_0\mathcal{L}\{r(t)\}]
$$

It is noted that due to structure of $h_{2,2}(s,q)$ and of $\mathcal{L}\{r(t)\}$ the pole at zero is simplified.

6. SIMULATION FOR A FOUR ENGINED EXECUTIVE JET AIRCRAFT (ALPHA)

Consider the four engined executive jet aircraft presented in [8]. The decoupling results will be applied to yield independent control of the lateral motion variables. This aircraft is consider to fly at see level with velocity $U_0 = 67.7$ m/sec. The parameters of the aircraft are $Y_\upsilon = -0.014$, $L_\upsilon = -4.05$, $L_p = -1.85$, $L_r = 0.52$, $N_\upsilon = 1.34$, $N_p = -0.25$, $N_r = -0.19$, $N_{\delta_R} = -0.64$, $Y_{\delta_r} = 0.034$, $L_{\delta_A} = 2.21$, $L_{\delta_R} = 1.11$, $N_{\delta_r} = -0.64$. The gravitational acceleration is $g = 9.81$ m/sec. One may easily verify that the decoupling condition (3.1) is satisfied. Choosing $\lambda_{1,1} = -9.986$, $\lambda_{2,2} = -8.93282$, $\lambda_{2,1} = 0$, $\lambda_{2,3} = 75.011$, $\lambda_{2,4} = -63.6329$, $p_{1,1} = 1$, $p_{2,1} = 0$ and $p_{2,2} = 0.019$ the root locus of the closed loop system poles for $q \in [0.8, 1.2]$ is shown in Fig. 1.

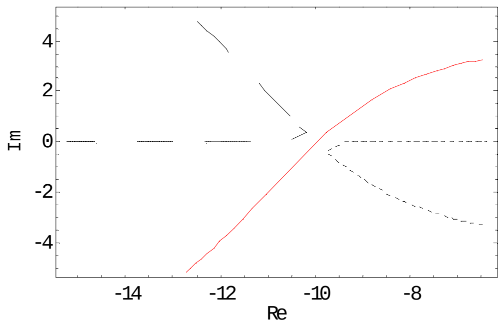

Fig. 1. Root locus of the closed loop system poles with respect to actuator uncertainty $q \in [0.8, 1.2]$

Consider the external commands $p_c(t) = 0.0157$ [rad] and $\beta_c(t) = 0$. The performance of the closed loop system for the case of a roll manoeuvre ($\phi(t) = 0.0157$ [rad], $\beta(t) = 0$) is illustrated in Fig. 2 for $q = 0.8$(dotted lines), $q = 1$(continuous lines) and $q = 1.2$ (dashed lines). According to Fig. 2 the performance of the closed loop system appears to be satisfactory.

8. CONCLUSIONS

For an aircraft in lateral motion with actuator failure, independent control between the roll rate and the sideslip angle has been achieved. The results are based on I/O robust triangular decoupling via static state feedback. The set of all robust controllers solving the problem and the respective general form of the triangularly decoupled closed-loop transfer function, have been derived. Finally all above results has been illustrated by application to the data of a four engined executive jet aircraft.

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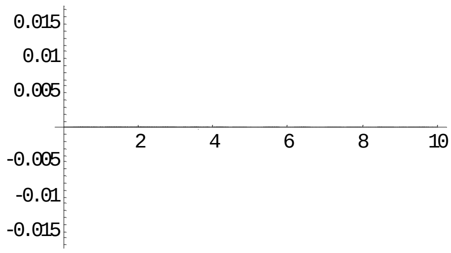
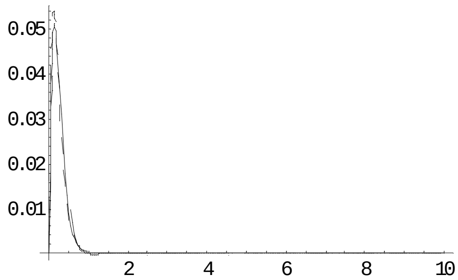
sideslip angle (rad)



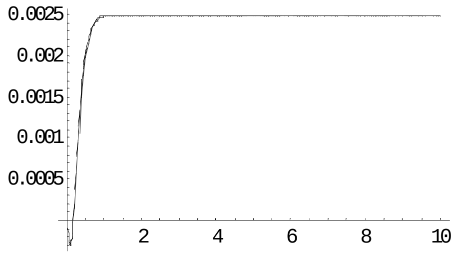
time (sec)

roll rate



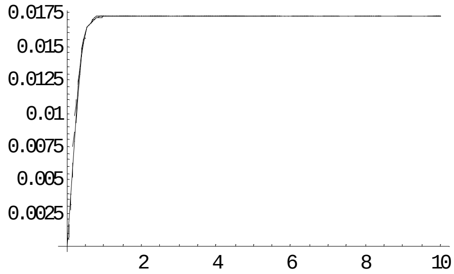
time (sec)

yaw rate (rad/sec)



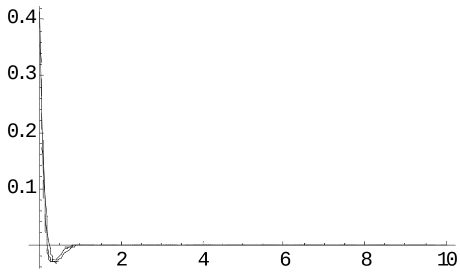
time (sec)

bank angle (rad)



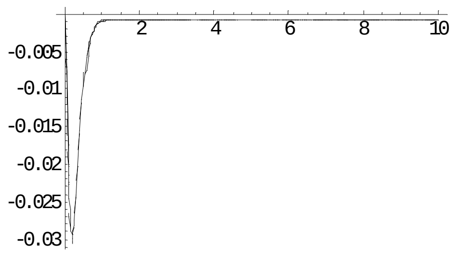
time (sec)

aileron deflection (rad)



time (sec)

rudder deflection (rad)



time (sec)

Fig. 2: Roll manoeuvre