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INDEPENDENT CONTROL OF THE LATERAL MOTION OF AN AIRCRAFT

M. G. SKARPETIS , F. N. KOUMBOULIS , T. LADOPOULOS

University of Thessaly, Department of Mechanical & Industrial Engineering, 383 34 Pedion Areos, Volos, Greece.

Abstract. The design goal of independent control of sideslip angle, roll angle and yaw angle of an aircraft is considered. To handle the issue the input-output decoupling technique is applied to the lateral motion dynamics of the aircraft. The conditions under which the problem is solvable via state feedback appears to be generically true. The general form of the static controllers, as well as the general form of the resulting decoupled closed loop system, are derived. The results has been illustrated by simulation to the data of 5th-order FPCC aircraft.

Key Words. I/O Decoupling, Flight Control, Linear System, Multvariable Control

1. INTRODUCTION

Independent control of the flight variables of an aircraft is a central problem in flight control systems [1]-[7]. With regard to the lateral motion of an aircraft it is noted that there is a coupling between the yaw angle, the roll angle and the sideslip angle, in the sense that they are influenced via aileron, rudder and canard commands. This is an undesirable effect in many lateral manoeuvres [8]. It is important to eliminate the coupling between the sideslip angle, the yaw angle and the roll angle, thus, allowing the pilot to perform manoeuvres by applying simple commands (see f.e. [1]-[4]).

In this paper, a static state feedback law is proposed yielding input output decoupling between the yaw angle, roll angle and the sideslip angle of an aircraft. The problem is proven to be solvable for almost all flight conditions in the sense that an inequality involving known aerodynamic stability derivatives must be satisfied. According to aerodynamic data this inequality is almost always satisfied. The explicit characterisation of all static state feedback controllers solving the problem, is derived in terms of the aerodynamic derivatives of the aircraft as well as free parameters that can be used to satisfy pole assignment requirements.

2. MODEL AND CONTROL OBJECTIVE

The lateral linearized motion of an aircraft can be expressed by a linear time invariant system of the following form

$$\dot{x}(t) = Ax(t) + Bu(t) \quad , \quad y(t) = Cx(t) \tag{2.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ are the state, input, and performance output vector, respectively, and where $\mathbb{R}$ denotes the set of real numbers. The lateral-directional equation of motion of a fixed-wing aircraft [8] turns out to a system of equations of the form (2.1), with

$$x(t) = \begin{bmatrix} \beta(t) & p(t) & r(t) & \phi(t) & \psi(t) \end{bmatrix}^T ,$$
$$u(t) = \begin{bmatrix} \delta_A(t) & \delta_R(t) & \delta_C(t) \end{bmatrix}^T ,$$
$$y(t) = \begin{bmatrix} \beta(t) & \phi(t) & \psi(t) \end{bmatrix}^T \tag{2.2}$$

and with

$$A = \begin{bmatrix} Y_v & Y_p & -1+Y_r & g/U_0 & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} ,$$

$$B = \begin{bmatrix} 0 & Y_{\delta_R} & Y_{\delta_C} \\ L_{\delta_A} & L_{\delta_R} & L_{\delta_C} \\ N_{\delta_A} & N_{\delta_R} & N_{\delta_C} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The variable $\beta(t)$ is the sideslip angle increment, $p(t)$ is the roll rate increment, $r(t)$ is the yaw rate increment, $\phi(t)$ is the roll angle increment, $\psi(t)$ is the yaw angle increment, $\delta_A(t)$, $\delta_R(t)$ and $\delta_C(t)$ are the aileron, rudder and canard commands, respectively. The parameters Y_i L_i and $N_i(\, i \in \{\upsilon,\, p,\, r,\, \delta_R,\, \delta_A,\, \delta_C\})$ are lateral directional stability derivatives. The parameter g is the gravitational acceleration and U_0 is the forward velocity. The objective of the present design scheme is to control independently the yaw angle, roll angle and sideslip angle The independent control of the lateral motion variables facilitates the aircraft placement and maintenance to desired orientation, following precisely the pilot's commands. To system apply (2.1) the static state feedback law

$$u(t) = Fx(t) + G\omega(t) \tag{2.3}$$

where $\omega(t) = \begin{bmatrix} \beta_C(t) & \phi_C(t) & \psi_C(t) \end{bmatrix}^T$ is the external command vector and where $\beta_C(t)$, $\phi_C(t)$ and $\psi_C(t)$ denote the commands driving the performance variables $\beta(t)$ $\phi(t)$ and $\psi(t)$, respectively.

3. NECESSARY AND SUFFICIENT CONDITIONS

According to [9], for the present system models, input output decoupling is solvable if and only if $\det[C^*B] \neq 0$ where $C^* = \begin{bmatrix} c_1 \\ c_2A \\ c_3A \end{bmatrix}$ where c_i is the i-th row of the matrix C. Thus, independent control of the side slip angle, roll angle and the yaw angle of the aircraft (2.2), via static state feedback (2.3), is satisfied if and only if $-L_{\delta_R}N_{\delta_A}Y_{\delta_C} + N_{\delta_R}L_{\delta_A}Y_{\delta_C} + Y_{\delta_R}N_{\delta_A}L_{\delta_C} - L_{\delta_A}N_{\delta_C}Y_{\delta_R} \neq 0$. The condition is true for almost all stability derivatives.

4. GENERAL FORM OF THE CONTROLLER MATRICES

Assume that system (2.2) satisfies the condition $-L_{\delta_R}N_{\delta_A}Y_{\delta_C} + N_{\delta_R}L_{\delta_A}Y_{\delta_C} + Y_{\delta_R}N_{\delta_A}L_{\delta_C} - L_{\delta_A}N_{\delta_C}Y_{\delta_R} \neq 0$. Then following the results in [10] the general explicit expression of the controller matrices G and F are

$$G = \begin{bmatrix} 0 & p_1^*Y_{\delta_R} & p_1^*Y_{\delta_C} \\ p_2^*L_{\delta_A} & p_2^*L_{\delta_R} & p_2^*L_{\delta_C} \\ p_3^*N_{\delta_A} & p_3^*N_{\delta_R} & p_3^*N_{\delta_C} \end{bmatrix}^{-1} \tag{4.1}$$

$$F = \begin{bmatrix} 0 & Y_{\delta_R} & Y_{\delta_C} \\ L_{\delta_A} & L_{\delta_R} & L_{\delta_C} \\ N_{\delta_A} & N_{\delta_R} & N_{\delta_C} \end{bmatrix}^{-1} \times \begin{bmatrix} \lambda_{1,1} & -Y_p & 1-Y_r & -g/U_0 & 0 \\ -L_\upsilon & \lambda_{2,1} & -L_r & \lambda_{2,2} & 0 \\ -N_\upsilon & -N_p & \lambda_{3,1} & 0 & \lambda_{3,2} \end{bmatrix} \tag{4.2}$$

where p_1^*, p_2^*, p_3^* $\lambda_{1,1}$, $\lambda_{3,1}$, $\lambda_{3,2}$, $\lambda_{2,1}$ and $\lambda_{2,2}$ are the free parameters. Relations (4.1) and (4.2) are explicit formulae implementable by elementary operations upon the stability derivatives and the values of g and U_0. The matrices F and G depend upon the parameters of the aircraft model, which is linearized around an equilibrium (operating) point. For a manoeuvre involving more than one operating point, the values of the controller have to be renewed by look up tables. This task can be carried out by an adjustment mechanism (in a real time computer) assigning also the closed loop poles. The explicitness of (4.1, 2), allows the adjustment mechanism to be executable in very short time.

5. CLOSED LOOP PERFORMANCE

The general analytical expression of the transfer function matrix of the decoupled closed-loop is

$$H(s) = \begin{bmatrix} \frac{(p_1^*)^{-1}}{s-\lambda_{1,1}-Y_\upsilon} & 0 & 0 \\ 0 & \frac{(p_2^*)^{-1}}{s^2-s(L_p+\lambda_{2,1})-\lambda_{2,2}} & 0 \\ 0 & 0 & \frac{(p_3^*)^{-1}}{s^2-s(N_r+\lambda_{3,1})-\lambda_{3,2}} \end{bmatrix} \tag{5.1}$$

According to the form of the closed loop transfer function (5.1) it is concluded that the number of transmission poles is equal to the dimension of the model. Thus, there are no cancelled out poles and decoupling with simultaneous stability can always be achieved for the aircraft model (2.1).

6. SIMULATION FOR A FIFTH-ORDER FPCC AIRCRAFT

Consider the fifth-order FPCC aircraft presented in [8]. The decoupling results will be applied to yield independent control of the lateral motion variables. The parameters of the model are $Y_\upsilon = -0.340$, $Y_p = 0.001$, $Y_r = 0.0031$, $g/U_0 = 0.0157$, $Y_{\delta_R} = 0.0755$, $Y_{\delta_C} = 0.0246$, $L_\upsilon = -2.69$, $L_p = -1.15$, $L_r = 0.738$, $N_\upsilon = 5.91$, $N_p = 0.138$, $N_r = -0.506$, $N_{\delta_R} = -5.03$, $N_{\delta_A} = 0.034$, $L_{\delta_A} = 5.22$, $L_{\delta_R} = 4.48$, $L_{\delta_C} = -0.742$, $N_{\delta_C} = 0.0984$. One may easily verify that the decoupling condition is satisfied. Choosing $\lambda_{1,1} = -0.16$, $\lambda_{2,2} = -6$, $\lambda_{2,1} = -3.85$, $\lambda_{3,2} = -6$, $\lambda_{3,1} = -4.494$, $p_1^* = 1/0.5$, $p_3^* = 1$ and $p_2^* = 1$ the closed loop transfer function poles are assigned all at -2, -3 (double poles) and -0.5. The performance of the closed loop system for the case of a heading manoeuvre is illustrated in Fig. 1. Note that the design objective of a heading manoeuvre is to command the heading angle $\gamma = \psi + \beta$, while keeping the yaw and the roll angle zero. The external commands are chosen to be $(\beta_C(t) = 0.0157[\mathrm{rad}],\, \phi_C(t) = \psi_C(t) = 0)$. As is shown in Fig. 1 the performance of the state vector is quite satisfactory since the rising times of the sideslip angle or heading angle are very short while the yaw angle, roll angle, yaw rate and roll rate are identically zero.

8. CONCLUSIONS

The yaw angle, the roll angle and the side slip angle of an aircraft have been independently controlled, via static state feedback yielding input output decoupling with simultaneous stability. The necessary and sufficient condition for the decoupling problem to be solvable are derived in a form being generically true. The set of all controllers solving the problem and the respective general form of the decoupled closed-loop transfer function, have been derived. Stability of the closed loop system is always guaranteed. Finally all above results has been illustrated by application to the data of fifth-order FPCC aircraft.

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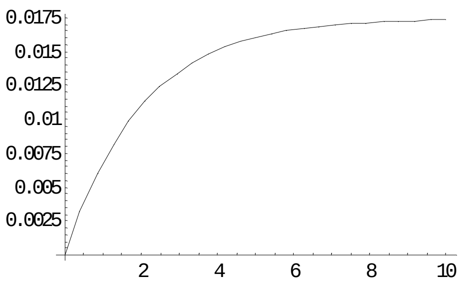
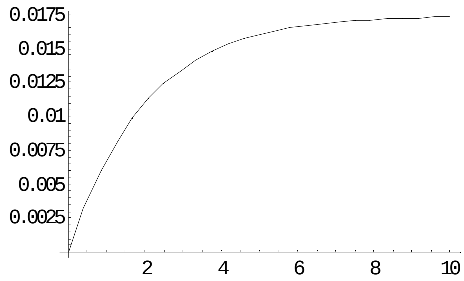
sideslip angle (rad)



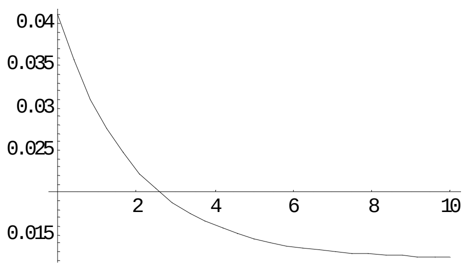
time (sec)

heading angle (rad)



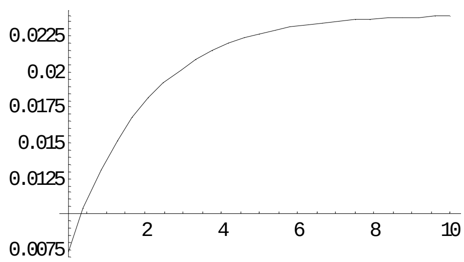
time (sec)

aileron deflection (rad)



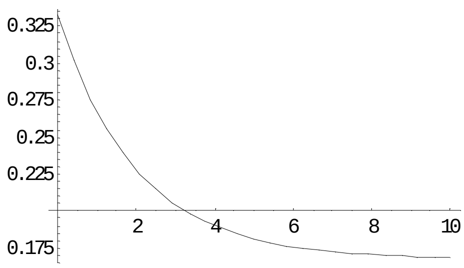
time (sec)

rudder deflection (rad)



time (sec)

vertical canard deflection (rad)



time (sec)

Fig 1. Heading manoeuvre