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TRANSITION CONTROL USING A LOCALLY LINEARIZED REFERENCE SYSTEM MODEL PREDICTIVE CONTROLLER

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Abstract. This paper introduces a Reference System based Model Predictive Controller to the nonlinear transition control problem. The proposed control strategy is based on an online model reference evaluation, called reference system, using a factorization of a locally linearized version of the nonlinear model. The output feedback problem is addressed using an l_p norm as a performance control measurement. The online implementation of the algorithm requires only the solution of an LP or QP problem. By evaluating locally the achievable closed-loop performance, the RS-MPC effects the transition between steady states with enhanced performance over a wide operating region without the necessary retuning of standard MPC algorithms. A chemical reactor example is presented to illustrate the control application.

Keywords. MPC, Transition Control, Reference System.

1. INTRODUCTION

It is undoubtedly known that nonlinearities play an important role in chemical processes. The inherent nonlinearity of processes, the operational requirements to satisfy constraints, and desire to cover a broader range of operating conditions are essential requirements for most of the Model Predictive Control (MPC) online applications. Therefore, an important practical benefit expected from MPC algorithms is the ability to effectively control the process over a wide range of operating conditions with *uniform performance and without retuning*. This is a very desired controller feature and probably more significant than the desire to provide a better closed-loop performance at a single operating point.

With the above motivation in mind, the issue of achieving uniform closed-loop performance at different operating points becomes one of critical importance. Otherwise the controller will need to be retuned on-line every time the process changes its operating steady state. The need to intermittently tune a large-scale MPC, is neither an easy task for the expert nor is it a desirable chore from the plant operator's point of view. Consequently, there is a substantial desire for MPC formulations and algorithms that inherently provide such uniform closed-loop characteristics without controller retuning.

This problem has been addressed earlier in the literature by incorporating the desired closed-loop behavior and/or the controller performance requirements at the level of the controller design. This controller class is known in the literature for a long time and it is referred to as model matching control [17], model-following control, model tracking control, and direct adaptive control [7]. The same idea was also implemented by the reference system approach, which with some small differences, is called in the literature as Reference Synthesis System (RSS) [1], Globally Linearized Control (GLC) [9], or Generic Model Control (GMC) [10]. All these three approaches were conceptually developed to handle nonlinear systems outside of the predictive framework. More recently, the GLC algorithm was also extended to a MAC formulation [15]. This paper addresses the transition problem in a predictive control framework.

The idea of a designed closed-loop behavior in MPC has been implemented in a variety of ways through the use of reference trajectory control strategies: the Model Algorithm Control (MAC), which uses an adjustable and fixed first order reference model as a tracking trajectory, the Generalized Predictive

Control (GPC) based on a performance model [2], which implements a pole placement design objective rather than a performance optimization objective, the GMC-MPC [3], which introduced the GMC specification curves with a formulation of positive and negative slack variables in the deviation of the process model from a Proportional-Integral (PI) reference trajectory, and more recently, the RSS extension to the Reference System Model Predictive Control (RS-MPC) [6], which also uses a PI reference trajectory formulation.

One of the main justification in formulating the MPC control problem as an RS-MPC problem is the ability to control highly nonlinear systems in a wider range of operating conditions without the retuning for performance guarantee. The choice of formulating the control problem in the move suppression or the reference model framework has practical implications. The move suppression formulation of MPC is a natural extension of the LQR/LQG framework. However, for nonlinear systems, the degree of necessary move suppression changes from one operating point to another. A simple example of the necessity to vary the move suppression can be given in a transition control problem, when the process operating condition goes from a minimum-phase (MP) local dynamics, to a nonminimum-phase (NMP) local dynamics. The same is true in the reversed transition. Such a problem is usually addressed by using a conservative controller tuning. This implies that the controller performance will be conservative in the overall transition region.

This paper offers an extension of the ideas presented above for predictive controllers by introducing to the control design a set o requirements, which will set the desired closed-loop behavior to the achievable performance. The main characteristic of this approach is the fact that the controller design explicitly uses information about the structural dynamic limitations of the process, consequently it does not force the controller to requirements that are known to be a limiting factor of its performance. Additionally, during transitions the algorithm does not use a fixed reference trajectory. The reference trajectory is generated on-line based on the local limits of plant invertibility. By taking into consideration the closed-loop limitations, the RS-MPC algorithm directly aims for an achievable behavior and does not need an explicit move suppression in the performance index.

In this paper, the proposed RS-MPC is presented addressing a nonlinear transition control problem using a local model representation at every single state space location. Output feedback is performed using classical filtering theory. The incorporation of the stability constraint ensures that nominal stability is always guaranteed. The analysis herein uses local arguments, the extension to a complete nonlinear description follows similar ideas and it will be reported elsewhere.

2. PRELIMINARIES

Consider a discrete-time representation of a continuous process given by the following model:

$$\begin{aligned}\hat{\mathbf{x}}_p(k+1) &= \mathbf{f}[\hat{\mathbf{x}}_p(k), \hat{\mathbf{u}}(k)] \\ \hat{\mathbf{y}}_p(k) &= \mathbf{h}[\hat{\mathbf{x}}_p(k)]\end{aligned} \tag{1}$$

where $\hat{\mathbf{x}}_p(k) \in \mathbb{R}^{n_p}$ is the state vector, $\hat{\mathbf{u}}(k) \in \mathbb{R}^{n_u}$ is the input vector, and $\hat{\mathbf{y}}(k) \in \mathbb{R}^{n_y}$ is the plant output vector. In this paper we consider square control systems, i. e., $n_y = n_u$. The origin is an equilibrium point and the system is constraint by $\hat{\mathbf{x}}_p(k) \in \mathbf{X}, \hat{\mathbf{u}}(k) \in \mathbf{U}$, and $\hat{\mathbf{y}}(k) \in \mathbf{Y}$ for all instant k. Where $\mathbf{X}, \mathbf{U}$ and $\mathbf{Y}$ contain the origin as an interior point and are also compact sets of $\mathbb{R}^{n_p}, \mathbb{R}^{n_u}$, and $\mathbb{R}^{n_y}$, respectively. Models of the form of equation (1) can be obtained from discretization of state-space continuous-time models.

Let the control law be $\hat{\mathbf{u}} = \mathbf{g}(\hat{\mathbf{x}}_p)$, with $\mathbf{g}(\mathbf{0}) = \mathbf{0}$. The closed-loop system that uses this control can be represented by:

$$\begin{aligned}\hat{\mathbf{x}}_p(k+1) &= \mathbf{f}[\hat{\mathbf{x}}_p(k), \mathbf{g}(\hat{\mathbf{x}}_p(k))] = \mathbf{T}[\hat{\mathbf{x}}_p(k)] \\ \hat{\mathbf{y}}_p(k) &= \mathbf{h}[\hat{\mathbf{x}}_p(k)]\end{aligned} \tag{2}$$

The control $\hat{\mathbf{u}} = \mathbf{g}[\hat{\mathbf{x}}_p(k)]$ is called stabilizing if the origin of (2) is an asymptotically stable equilibrium point. The most usual way to stabilize a system is by using a control law based on a local linearization of the system. In this paper, when not indicated otherwise, a stable, minimal order, discrete-time, linear local representation $\Sigma_p(\mathbf{x}_{p,j}, \mathbf{u}_j)$ will be used as follows:

$$\begin{bmatrix} \mathbf{x}_{p,j}(k+1) \\ \mathbf{y}_{p,j}(k) \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{A}_{p,j} & \mathbf{B}_{p,j} \\ \hline \mathbf{C}_{p,j} & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{x}_{p,j}(k) \\ \mathbf{u}_j(k) \end{bmatrix} \tag{3}$$

Here the index j indicates the current conditions of the local model, and $\mathbf{x}_{p,j}(k) \in \mathbb{R}^{n_p}$ is the plant model state vector at j, $\mathbf{u}_j(k) \in \mathbb{R}^{n_u}$ is the manipulated input at j, $\mathbf{y}_{p_j}(k) \in \mathbb{R}^{n_y}$ is the plant model output vector at j, which relates to the output performance of the system to be controlled. The local model will be referred as $\Sigma_{p,j}(\mathbf{x}_{p,j}, \mathbf{u}_j)$: $(\mathbf{A}_{p,j}, \mathbf{B}_{p,j}, \mathbf{C}_{p,j})$ or, more compactly, $\Sigma_{p,j}$, with a transfer function matrix representation given by $\mathbf{G}_{p,j}(z)$. It is assumed to have a relative degree (order) vector with elements given by r_i^j, where i refers to the i^{th} output. The following definitions are also necessary in the subsequent development.

Definition 2.1. The transmission zeros of the system (3), referred in this paper as zeros of the system, arise when competing internal effects are such as to make the output to be zero even when the inputs to the system are not zero. The zeros are defined as a set of complex numbers α, satisfying $rank(\mathbf{P}_j) < n_p + n_y$, where:

$$\mathbf{P}_j \equiv \begin{bmatrix} \alpha\mathbf{I} - \mathbf{A}_{p,j} & -\mathbf{B}_{p,j} \\ -\mathbf{C}_{p,j} & \mathbf{0} \end{bmatrix} \tag{4}$$

Let α_j be a transmission zero of (3), and

$$\begin{bmatrix} \nu^T & \omega^T \end{bmatrix} \begin{bmatrix} \alpha_j\mathbf{I} - \mathbf{A}_{p,j} & -\mathbf{B}_{p,j} \\ -\mathbf{C}_{p,j} & \mathbf{0} \end{bmatrix} = \mathbf{0} \tag{5}$$

where ν is the left state zero direction and ω is the output zero direction normalized so that $\omega^H\omega = 1$. The system (3) is said to be nonminimum phase (NMP) if at least one of its transmission zeros is outside the closed unit disk in the complex plane. Note that in this paper we call a minimum phase system (MP) if it has no finite zeros outside of the unit circle.

Consider N_z NMP zeros $\alpha_1, \ldots, \alpha_{N_z}$ (including multiplicities) of $\Sigma_{p,j}$. One can factorize the system model at each condition j as $G_{p,j}(z) = G_{c,j}(z)G_{M,j}(z)$, where $G_{c,j}(z)$ is inner [1], with the same NMP zeros of $G_{p,j}(z)$. The collection of all NMP zeros of a system into a stable all-pass factor using the *Blaschke* products can be performed as follows: Factor out the NMP zeros of (3) at j, one at a time [19]:

$$\begin{aligned} G_{p,j}(z) &= G_{c,j}^{(1)}(z)G_{M,j}^{(1)}(z), \\ G_{M,j}^{(1)}(z) &= G_{c,j}^{(2)}(z)G_{M,j}^{(2)}(z), \\ &\cdots\cdots\cdots \\ G_{M,j}^{(N_z-1)}(z) &= G_{c,j}^{(N_z)}(z)G_{M,j}^{(N_z)}(z) \end{aligned} \tag{6}$$

where $G_{M,j}^{(i)}(z) \equiv \mathbf{C}_{M,j}^{(i)}(z\mathbf{I} - \mathbf{A}_{p,j})^{-1}\mathbf{B}_{p,j}, \mathbf{C}_{M,j}^{(0)} \equiv \mathbf{C}_{p,j}$, $i = 1, \ldots, N_z$, and

$$\begin{aligned} G_c^{(i)} &= \mathbf{I} - \left(\frac{\alpha_i\bar{\alpha}_i - 1}{\alpha_i + 1}\right)\left(\frac{z+1}{\bar{\alpha}_i z - 1}\right)\bar{\eta}_i\eta_i^T \\ \mathbf{C}_M^{(i)} &= \mathbf{C}_M^{(i-1)} - \left(\frac{\alpha_i\bar{\alpha}_i - 1}{\bar{\alpha}_i + 1}\right)\bar{\eta}_i\theta_i^T(\mathbf{A_p} + \mathbf{I}) \end{aligned} \tag{7}$$

where θ_i and η_i are the left state zero direction and the output zero direction of zero α_i using $\mathbf{C}_M^{i-1}$ in the output direction problem, equation (5). The index j was omitted from $G_{c,j}^{(i)}$ and $\mathbf{C}_{M,j}^{(i)}$ for brevity. By using this *all-pass* factorization, one can express the system $\Sigma_{p,j} : G_{p,j}(z)$ as $G_{p,j}(z) = G_{c,j}(z)\mathbf{C}_{M,j}(z\mathbf{I} - \mathbf{A}_{p,j})^{-1}\mathbf{B}_{p,j}$, and $G_{c,j} = \prod_{i=1}^{N_z} G_{c,j}^{(i)}(z)$.

For a more general description, the local factorization can be performed by a generic inner-outer factorization algorithm. A recursive zeros dislocation technique, applicable to both proper and strictly proper systems [18] can be used. Another possibility for the factorization problem is the use of the generalized interactor matrix factorization [16]. If the given system $\Sigma_{p,j}$ has zeros on the unit circle, we assume that these zeros are included in the outer factor $\Sigma_{M,j}$.

With each component $\mathbf{y}_i$ one can associate a delay order r_i called *relative order of the system* with respect to the control $\mathbf{u}$. The relative order r_i [11] is the *smallest number of sampling periods* after which a manipulated input move $\mathbf{u}(k)$ affects the output y_i. If $r_i = \infty$, this implies that y_i is not controllable. Therefore, it is necessary that all outputs $\mathbf{y}$ possess finite relative order.

Remark 2.1. : The exact sampled-data representation of a dead-time-free, continuous-time system with finite relative orders always has relative order equal to 1, $r_i = 1; i = 1, ..., n_y$ [13]. In addition, each finite relative order $r_i, i = 1, ..., n_y$, satisfies the inequality $r_i \leq n_p$ [8].

It is of interest to use such information locally, during the design of an appropriate reference system. Consider a square linear system represented by $\Sigma_{p,j}$, the relative degree of the system with respect to each output is the smallest integer r_i such as $\mathbf{c}_i\mathbf{A}_{p,j}^{r_i-1}\mathbf{B}_{p,j} \neq [\mathbf{0}, \cdots, \mathbf{0}]$, where $\mathbf{c}_i$ is the ith row of matrix $\mathbf{C}_{p,j}$. This definition is in complete agreement with the notion of relative order usually used, i.e., for a given discrete SISO system, the *relative order* of the linear system is the difference between the order of the denominator and the order of the numerator.

3. MAIN RESULTS

The RS-MPC is a control methodology in which a control move is calculated such as the controlled outputs coincide with the output of a system with characteristics designed as to satisfy certain performance specifications (the reference model). A reference model in simple terms is a model that specifies the desired performance of the control system.

Consider a continuous process represented by a discrete model given by Equations 1. For simplicity of notation, in the remaining of the paper the index j, representing the local conditions, will be only used when its absence would make the formulation dubious. Consider a local representation of the process model given by $\Sigma_p = \Sigma_c\Sigma_M$. The MPC problem will be formulated using the minimum-phase stable model given by $\Sigma_M(\mathbf{x}_M, \mathbf{u}) : (\mathbf{A}_M, \mathbf{B}_M, \mathbf{C}_M)$, the invertible part of the process. Because model-based controllers may present robustness problems, the selection of a reference model will indirectly address that issue. A slower reference system will enhance the robustness of the closed-loop.

Augmenting the plant model for the achievable behavior with the reference system model $\Sigma_r(\mathbf{x}_r, \mathbf{u}_r) : (\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r)$, such as $\mathbf{x} = [\mathbf{x}_M^T \quad \mathbf{x}_r^T]^T$, gives the following system Σ_a:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{G}\mathbf{u}_r(k) \\ \mathbf{e}(k) = \mathbf{F}\mathbf{x}(k) \end{cases} \tag{8}$$

where $\mathbf{x}_r(k) \in \mathbb{R}^{n_r}$ is the state vector of the reference model, $\mathbf{u}_r(k) \in \mathbb{R}^{m_r}$ is the input of the reference model, $\mathbf{y}_r(k) \in \mathbb{R}^q$ is the reference model output, $\mathbf{x}(k) \in \mathbb{R}^n, \mathbf{u}(k) \in \mathbb{R}^m, \mathbf{e}(k) \in \mathbb{R}^q$, $n = n_M + n_r$, $\mathbf{F} = [\,\mathbf{C}_M | -\mathbf{C}_r\,]$, and

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_M & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{A}_r \end{array}\right], \mathbf{B} = \left[\begin{array}{c} \mathbf{B}_M \\ \hline \mathbf{0} \end{array}\right], \mathbf{G} = \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{B}_r \end{array}\right] \tag{9}$$

If $\mathbf{u}_r(k)$ is equaled to the desired setpoint trajectory of the closed-loop system, then $\mathbf{y}_r(k)$ describes the desired performance of the system output in the closed-loop. The goal in the RS-MPC design is to calculate the set of control moves $\mathbf{U}_k^{H_c}$ over a control horizon H_c, such that one minimizes a norm of the output deviation $\mathbf{e}(i), i = \{k, k+1, \ldots k + H_p\}$, over a prediction horizon H_p. Here $\mathbf{e}(i)$ is the difference between $\mathbf{y}_M(i)$, the estimated output of process Σ_M, and the desired one, $\mathbf{y}_r(i)$.

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[1] $G_{c,j}(z)$ is inner if it is stable and $G_{c,j}^{\sim}G_{c,j} = \mathbf{I}$, where $G_{c,j}^{\sim}(z) \equiv G_{c,j}^T(1/z)$.

Consider an objective function that minimizes the sum of distances between the predicted output error and the desired behavior given by the reference system. This distance can be represented by $d(\Sigma_r, \Sigma_r, H)$. A generic objective function defined for a prediction horizon $H_p \geq max_i(r_{M_i})$ and control horizon H_c ($H_p \geq H_c$) would be given by $\min_{\Pi_p}\{d(\Sigma_M, \Sigma_r, H_p, H_c)\}$, subject to necessary models description and appropriate input-output constraints. Consider that the set of inputs defining the controller Π_p is given by $\mathbf{U}_k^{H_c} \triangleq \left\{\mathbf{u}_{k|k}, \mathbf{u}_{k+1|k}, \ldots, \mathbf{u}_{k+H_c-1|k}\right\}$. The RS-MPC can then be defined as follows:

Definition 3.1. The Reference System Model Predictive Controller is defined to be the one in which the control move $\mathbf{u}(k)$, at sample time k, is given by the first element $\mathbf{u}(k|k)$ of $\{\mathbf{u}(k|k), \mathbf{u}(k+1|k), \ldots, \mathbf{u}(k+H_c-1|k)\}$, which is the solution of $\min_\Pi d(\Sigma_a, H_p, H_c)$, subject to $\mathbf{y}(i|k) \in \mathbb{Y}$, and the input constraints $\mathbf{u}(i|k) \in \mathbb{U}, i = k, k+1, \ldots, k+H_c-1$, where $\mathbb{U}$ defines a compact polyhedral region given by:

$$\mathbb{U} = \mathbf{u}(k) \in \mathbb{R}^{n_u} : \begin{cases} \mathbf{u}_{min} \leq \mathbf{u}(k) \leq \mathbf{u}_{max} \\ |\Delta u_i(k)| \leq \Delta u_{i,max} \\ \quad i = 1, \ldots, n_u \end{cases} \tag{10}$$

and satisfying the following assumptions for all local models Σ_p and Σ_p:
• Σ_p and Σ_r are asymptotically stable and given by minimal state space realizations. Additionally, Σ_M and Σ_r have finite and equal relative orders, with local characteristic matrices $\mathcal{C}_M$ and $\mathcal{C}_r$ nonsingular and $\infty > H_p \geq max_i(r_{M_i}) > 0$. The matrices $\mathcal{C}$ are defined using $(\mathbf{A}_M, \mathbf{B}_M, \mathbf{C}_M)$ and $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r)$ as $\mathcal{C} = \left[(\mathbf{c}_1\mathbf{A}^{r_1-1}\mathbf{B})^T \quad \cdots \quad (\mathbf{c}_{n_y}\mathbf{A}^{r_{n_y}-1}\mathbf{B})^T\right]^T$.
• Assume that there exists a triplet $(\mathbf{x}_s, \mathbf{u}_s, \mathbf{u}_{rs}) \in \mathbb{R}^{n_m+n_r+m+m_r}$ such that the set:
$\triangle_f \equiv \left\{\mathbf{F}\mathbf{x}_s = \mathbf{0}; \mathbf{x}_s = (\mathbf{I}-\mathbf{A})^{-1}\left[\mathbf{B}u_s + \mathbf{G}u_{rs}\right]\right\}$ has a nonempty interior.
• Assume that there exists $\mathbf{U}_k^{H_c} \in \mathbb{R}^{H_c n_m}$ such that: $\mathbf{F}\mathbf{x}_{k+H_p|k} = \mathbf{0}$, and $\lim_{k\to\infty}(\hat{\mathbf{y}}_p - \mathbf{y}_p) < \delta < \infty$.
Here the subscript s refers to the locally predicted steady-state value. The assumptions above guarantee that at the length of the prediction horizon H_p, the Σ_M outputs match the reference model ones.

For controller enhancement, instead of the last assumption above, several other types of stability constraint can be used, refer to [4,5,14], and references therein for alternative stability constraint formulation and stability proofs.

A p-norm, indicated by $\|\cdot\|_p$, is a convenient measure for RS-MPC cost formulation. In this paper, the l_1 and l_2 norms (p=1 and p=2) will be used to formulate the correspondent RS-MPC, called as RS_1-MPC and RS_2-MPC, respectively.

The key issue of output model matching is whether a perfect match of two given systems is possible or not. Therefore, it is of fundamental importance that the formulation of the model predictive following problem assures that the controller is feasible at all times. Therefore, given the fact that it may not be feasible to follow exactly the reference system trajectory, one has to formulate the optimization problem as to minimize the mismatch between the desired behavior and the possible behavior, due to disturbances or input and output constraints.

3.1 RS_1-MPC Algorithm

The local RS_1-MPC problem can be written as

$$J(H_p, H_c, k) = \min_{\Pi_1} \sum_{i=0}^{H_p} \Upsilon_1(k,i) \tag{11}$$

$$\Upsilon_1(k,i) = \mathbf{w}_+^T(i)\mathbf{\Lambda}^+(k+i|k) + \mathbf{w}_-^T(i)\mathbf{\Lambda}^-(k+i|k)$$
$$\Lambda_+(k) = [\lambda_1^+(k), \ldots, \lambda_q^+(k)]^T, \quad \lambda_j^+(k) \geq \mathbf{0}$$
$$\Lambda_-(k) = [\lambda_1^-(k), \ldots, \lambda_q^-(k)]^T, \quad \lambda_j^-(k) \geq \mathbf{0}$$

subject to

$$\mathbf{y}_{M_i}(k + r_{M_i}) = \mathbf{y}_{r_i}(k + r_{M_i}) + \lambda_i^+(k) - \lambda_i^-(k),$$
$$i = 1, \ldots, n_y, \qquad \mathbf{u}(k) \in \mathbb{U}, \qquad \mathbf{y}(k) \in \mathbb{Y},$$
$$\mathbf{F}\mathbf{x}_{k+H_p|k} = \mathbf{0}$$

The weighting vectors $\mathbf{w}_+^T(k)$ and $\mathbf{w}_-^T(k)$ consist of nonnegative elements and $\mathbf{y}(k)$ refers to the non-factorized local model output prediction used for output constraint enforcement.

The transformation of the nonsmooth performance index into a smooth control problem adds to the RS-MPC slack variables which enhances the tracking feasibility of the optimization problem.

3.2 RS_2-MPC Algorithm

A quadratic cost function is the most popular cost function used in MPC formulations. It amplifies the penalty for large deviations, with an opposite behavior for small deviations. One could express the RS-MPC controller, as it is usual in the literature [12]. Here, to keep a consistent notation within the RSS literature, a representation similar to the one for RS_1-MPC will be maintained. The RS_2-MPC algorithm representation can be written as

$$J(H_p, H_c, k) = \min_{\Pi_2} \sum_{i=0}^{H_p} \Upsilon_2(k,i) \tag{12}$$

$$\Upsilon_2(k,i) = \left\{\mathbf{\Lambda}_\pm{}^T(k+i|k)\mathbf{W}\mathbf{\Lambda}_\pm(k+i|k)\right\}$$
$$\mathbf{\Lambda}_\pm(k) = [\lambda_1^\pm(k), \ldots, \lambda_{n_y}^\pm(k)]^T$$

subject to

$$\mathbf{y}_{M_i}(k + r_{M_i}) = \mathbf{y}_{r_i}(k + r_{M_i}) + \lambda_i^\pm(k),$$
$$i = 1, \ldots, n_y, \qquad \mathbf{u}(k) \in \mathbb{U}, \qquad \mathbf{y}(k) \in \mathbb{Y},$$
$$\mathbf{F}\mathbf{x}_{k+H_p|k} = \mathbf{0}$$

here $\mathbf{W}$ is a weighting matrix.

4. IMPLEMENTATION

In the Receding Horizon formulation, H_p is the prediction horizon and H_c is the control horizon ($H_p \geq H_c$). The problem formulation can be indicated as follows. Let $u^*_{(H_p,H_c)}(i)$, for $i < H_c$, be the control sequence for the problem formulated above. In a receding horizon framework only the first control move ($i = 0$) will be implemented. In this case, the updated local augmented system will be given by:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}^*_{(H_p,H_c)}(0) + \mathbf{G}\mathbf{u}_r(k) \\ \mathbf{e}(k) &= \mathbf{F}\mathbf{x}(k) \end{aligned} \tag{13}$$

With this, the rest of the control moves, $u^*(i), i = 1, ..., H_c$, are discarded and the optimization problem is solved using the new local model and initial conditions.

The RS-MPC, can be defined in several ways. In this implementation, it is designed such as the plant model output matches the reference model output in the beginning of the prediction horizon, for each instant k. A new reference model is calculated at each sampling time. Its speed is directly related to the speed of $\Sigma_{c,j}$ and possibly the distance of the operating point. Another important aspect in this implementation is the fact that the states are not measured, and an Extended Kalman Filter (EKF) was used to estimate them.

Due to the formulation of the RS-MPC, in cases of very slow $\Sigma_{c,j}$, a too slow closed-loop behavior may take place, in that case, certain speed can be designed such as only an acceptable overshoot/undershoot behavior takes place. In general, the reference system is selected at each sampling time based on the information from $\Sigma_{c,j}$ and the distance between the current and the desired operating condition.

In the transition control implemented herein, despite the fact the process is nonlinear, a time varying linear model is used. This model that is given by the linearization of the nonlinear process model at every sampling time. In this case, the reference point for linearization is the current state and input move, i.e., an unsteady-state. Because of the inherent mismatch between the plant and its representation by local linear model used for prediction, a performance degradation is expected. This means that the reference system will be only a local representation of the process behavior and some performance degradation is expected independently of the control structure used. However, that degradation was acceptable in all cases studied, given the benefit of solving an LP/QP problem instead of a nonconvex nonlinear optimization problem, which does not have any guarantee of global optimality.

The algorithm can be represented by the following steps:
(1) Evaluation of the nonlinear feasibility of the desired operating points. In this case a feasibility analysis is performed to identify whether or not the desired transition is feasible in the available input space.
(2) Measurement of current output and estimation of plant states via an EKF using the nonlinear plant model.
(3) Prediction of future outputs via the nonlinear model and control moves calculated in the prior sample time. Comparison to the desired outputs of the reference system.
(4) Linearization of the model at current conditions.

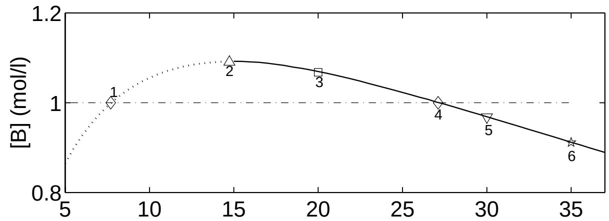

Fig. 1. Operating conditions for van de Vusse CSTR

(5) Factorization of the model and design of the reference system at current conditions.
(6) Calculation of the desired changes in the control moves to minimize the performance index. Implementation of the first move.
(7) Change in sample index. Go to item 4 or end.

5. APPLICATION

Consider the benchmark for nonlinear control systems known as van de Vusse reactor problem. The main interesting features of this reactor is the fact that the zero dynamics change in the range of desired operation. All details on modeling and system parameters can be found in [5] and in references therein. The reactor of interest is a CSTR with the following reaction $A \overset{k_1}{\to} B \overset{k_2}{\to} C, \quad 2A \overset{k_3}{\to} D$. The CSTR dynamics is given by the following model:

$$\begin{aligned} \frac{dC_A}{dt} &= u(C_{Ao} - C_A) + R_A \\ \frac{dC_B}{dt} &= u(C_{Bo} - C_B) + R_B \\ \rho C_p \frac{dT}{dt} &= u\rho C_p(T_o - T) + \frac{Q}{V} - \sum_{i=1}^{3} \Delta H_i r_i \\ y &= C_B \end{aligned} \tag{14}$$

Where $k_i = k_{io}exp(-E_i/RT), i = 1, 2, 3,\ u = F/V,\ r_1 = k_1 C_A,\ r_2 = k_2 C_B,\ r_3 = k_3 C_A^2,\ R_A = -r_1 - r_3,\ R_B = r_1 - r_2$, and $Q = k_w A_R(T_k - T)$.

The problem is to control the concentration of B in several different scenarios, manipulating the values of u, which can assume any value in the interval $u = [5h^{-1}, 40h^{-1}]$. In addition, the only measured variable is the concentration of B. The difficulty of this problem is given not only by its strong nonlinearity, but also by the existing input multiplicity in the operating region. Figure 1 shows the steady state locus plot for the concentration of B, indicated by [B], in the range of production rate of interest. It also indicates several operating conditions of interest. The dotted region indicates that the local linear model has unstable inverse dynamics (NMP). It is desired to make a few transitions as indicated below. Figure 2 presents the closed-loop behavior using the RS_1-MPC with the following parameters ($T_s = 0.02h, H_p = H_c = 20, \mathbf{w}_+ = \mathbf{w}_- = 1, u_{min} = 1, u_{max} = 40, \Delta u_{max} = 6, y_{min} = 0.9, y_{max} = 1.09$). The solid curve indicates the system's transition towards the goal represented by the dashed line. The reference model used was a fixed system given by $G_r = \frac{0.5}{z-0.5}$ in the MP region. For the NMP region,

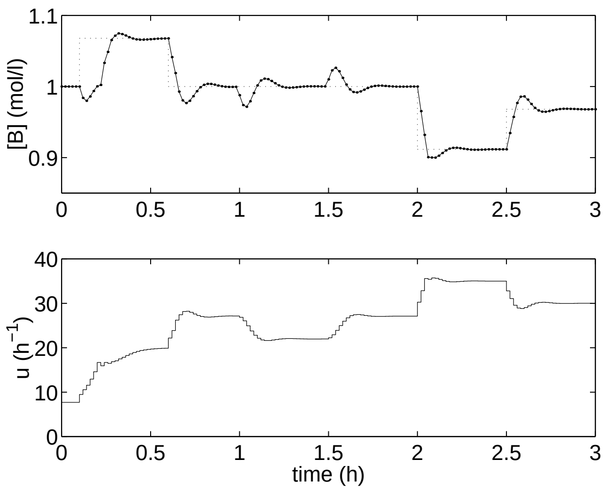

Fig. 2. Operating conditions (transitions)

the reference model used was a function of the NMP zero of the local process model, $\Sigma_{r,j} = \frac{1-(\sigma_j)^{-1}}{z-(\sigma_j)^{-1}}$, where σ_j represents the NMP dominant dynamics of the process model $\Sigma_{c,j}$.

The set of transitions are as follows: (i) transition at time $0.1h$ from operating condition 1 to operating condition 3, indicated as transition 1→3, (ii) transition 3→4, (iii) at $2h$ the operation performs a system transition 4→5, and (iv) at $2.5h$ the transition 5 →6. Furthermore, during the operation at condition 3, the feed concentration of A, C_{Ao}, suffers an unmeasured -5% disturbance at $1.0h$ (with $0.5h$ duration).

As it can be seen in Figure 2, the RS-MPC is able to make the desired transitions. If the input multiplicity is of concern in certain transitions, that issue has to be taken care directly in the performance/reference system implementation. The small oscillation shown in the response is mainly due to the used local linear approximation for the plant model (model/plant mismatch) and state estimation.

6. CONCLUSIONS

This paper introduces a RS-MPC algorithm for transition control. The proposed RS-MPC incorporates local closed-loop limits of performance to the optimization problem. It has been shown that the RS$_p$-MPC deals satisfactorily with transitions and NMP behavior can be dealt with using a simple local factorization approach.

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