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ANTICIPATORY 1D AND 2D LINEAR SYSTEMS

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Abstract. Notions of anticipatory systems for discrete-time and continuous-time 1D linear systems and 2D discrete linear systems are introduced. A discrete-time system is called anticipatory if its state vector and output vector depend on the future values of inputs. A continuous-time system is called anticipatory if its state vector and output vector depend on the derivatives of inputs. Necessary and sufficient conditions for the anticipation of singular discrete-time and continuous-time 1-D linear systems are established. It is shown that the discrete-time system obtained by discretisation from continuous-time one is anticipatory for any value of the discretisation step if and only if the continuous-time system is anticipatory. Necessary and sufficient conditions for the anticipation of the singular 2D Roesser model are established.

Key Words. 1D and 2D anticipatory systems, Roesser model, dicretisation.

1. INTRODUCTION

The analysis of discrete-time and continuous-time invariant singular systems has been considered in many papers and books [1,2,7-13,17,18,24-30,34,35]. Some applications of this analysis to circuit and system design, including robotics can be found in [30,29,35].

In recent years a dynamic development of the theory of anticipatory systems specially the theory of anticipatory discrete-time linear systems has been observed [32,3,4]. The definitions of anticipatory systems are different and usually not very precise [32]. Dubois in [3,4] has introduced the concepts of incursion and hyperincursion for dynamical systems. In this paper precise definitions of anticipatory continuous-time and discrete-time linear systems will be proposed. A discrete-time system will be called anticipatory if its state vector and output vector depend on the future values of inputs. A continuous-time system will be called anticipatory if its state vector and output vector depend on the derivatives of inputs. In [9-12] it has been shown that the state vectors may depend on the future values of inputs and in singular continuous-time systems may depend on the derivatives of inputs. Let a singular continuous-time linear system be an anticipatory system. By discretisation of this singular continuous-time system we obtain a suitable singular discrete-time system. Will be the obtained discrete-time system also anticipatory? In this paper an answer to the question will be given and necessary and sufficients conditions for the anticipation of singular discrete-time and continuous-time linear systems will be established. It will be shown that the discrete-time system obtained by discretisation from continuous-time one is anticipatory for any value of the discretisation step if and only if the continuous-time system is also anticipatory.

Necessary and sufficient conditions for the anticipation of the singular 2D Roesser model will be established.

2. DISCRETE-TIME SYSTEMS

Let $R^{p\times n}$ be the set of real $p\times n$ matrices and $R^{p}:=R^{p\times 1}$.

Consider the discrete-time linear system

$$Ex_{i+1} = Fx_i + Gu_i \quad (1a)$$

$$y_i = Cx_i + Du_i \qquad i \in Z_+ := \{0,1,2,\ldots\} \quad (1b)$$

where $x_i \in R^n$, $u_i \in R^m$, $y_i \in R^p$ are the state vector, input vector and output vector at the point *i*, respectively and $E, F \in R^{n\times n}$, $G \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{p\times m}$.

If $\det E \neq 0$ then the system (1) is called standard and if $\det E = 0$ then the system is called singular.

It is assumed that the pencil (*E*,*F*) is regular, i.e.

$$\det[Ez - F] \neq 0 \quad (2)$$

for some $z \in \mathbf{C}$ (the field of complex numbers).

If the condition (2) is satisfied then

$$[Ez - F]^{-1} = \sum_{i=-\mu}^{\infty} \Phi_i z^{-(i+1)} \quad (3)$$

where μ is the nilpotence index and Φ_i is the fundamental matrix defined by

$$E\Phi_i - F\Phi_{i-1} = \Phi_i E - \Phi_{i-1} F = = \begin{cases} I_n & for \quad i = 0 \\ 0 & for \quad i \neq 0 \end{cases} \quad (4)$$

I_n - is the $n \times n$ identity matrix.

The solution of (1a) has the form [13,12,24-26,28]

$$x_i = \Phi_i E x_0 + \sum_{k=0}^{i+\mu-1} \Phi_{i-k-1} G u_k \quad (5)$$

From (5) it follows that if $\mu > 1$ then the solution x_i depends on the future values of inputs u_k for *k*>*i*.

Definition 1. The system (1) is called anticipatory if the state vector x_i and output vector y_i at the point *i* depends on the future values of u_k for *k*>*i*.

The standard system (1) is not anticipatory. If $\det E \neq 0$ then $\Phi_i = \begin{cases} \left(E^{-1}F\right)^i E^{-1} & for \quad i \geq 0 \\ 0 & for \quad i < 0 \end{cases}$ and μ=0.

From (5) it follows that in this case x_i (and also y_i) does not depend on the future values of inputs.

Theorem 1. The singular system (1) is anticipatory if and only if

$$\text{rank}E > \text{deg.det}[Ez\text{-}F] \quad (6)$$

where deg.det[*Ez*-*F*] denotes the degree of the polynomial det[*Ez*-*F*].

Proof. If the condition (2) is satisfied then there exists nonsingular matrix $P, Q \in R^{n\times n}$ such that [13]

$$P[Ez - F]Q = \begin{bmatrix} I_{n_1} z - A_1 & 0 \\ 0 & Nz - I_{n_2} \end{bmatrix} \quad (7)$$

where $n_1 = \text{deg.det}[Ez - F]$, $n_2 = n - n_1$, $A_1 \in R^{n_1\times n_1}$ and $N \in R^{n_2\times n_2}$ is the nilpotent matrix with index μ, $N^{\mu-1} \neq 0$, $N^{\mu} = 0$. The index μ is equal to maximal dimension of the Jordan block corresponding to the zero eigenvalue of the pair (*E*,*F*) [13]. From (7) it follows that rank $E = n_1$ if and only if *N*=0 and μ=1. The condition (7) is satisfied if and only if μ>1. From (5) it follows that in this case x_i depends on u_k for *k*>*i*. □

3. CONTINUOUS-TIME SYSTEMS

Consider the continuous-time linear system

$$E\dot{x} = Ax + Bu, \quad x(0) = x_0 \quad (8a)$$

$$y = Cx + Du \quad (8b)$$

where $\dot{x} = \frac{dx}{dt}$, $x = x(t) \in R^n$, $u = u(t) \in R^m$, $y = y(t) \in R^p$ are the state vector, input vector and output vector, respectively and $E, A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{p\times m}$.

If $\det E \neq 0$ the system (8) is called standard and if $\det E = 0$ the system is called singular.

It is assumed that the pencil (*E*,*A*) is regular, i.e.

$$\det[Es - A] \neq 0 \text{ for some } s \in \mathbf{C} \quad (9)$$

If the condition (9) is satisfied then

$$[Es - A]^{-1} = \sum_{i=-\mu}^{\infty} \Phi_i s^{-(i+1)} \quad (10)$$

where μ is the nilpotence index and Φ_i is the fundamental matrix defined by [18,24-26,28]

$$E\Phi_i - A\Phi_{i-1} = \Phi_i E - \Phi_{i-1}A = \\ = \begin{cases} I_n & for \quad i=0 \\ 0 & for \quad i \neq 0 \end{cases} \tag{11}$$

The solution x(t) of the equation (8a) has the form [18]

$$x(t) = e^{\Phi_0 At}\Phi_0 Ex_0 + \int_0^t e^{\Phi_0 A(t-\tau)}\Phi_0 Bu(\tau)d\tau + \\ + \sum_{j=1}^{\mu}\Phi_{-j}\left(Bu^{(j-1)} + Ex_0\delta^{(j-1)}\right) \tag{12}$$

where $u^{(j)} = \dfrac{d^j u}{dt^j}$, $\delta^{(j)}$ denotes the derivative of the *j*-th order of the Dirac impulse $\delta(t)$.

From (12) it follows that if μ>1 then the solution *x(t)* depends on the derivatives of *u(t).*

Definition 2. The system (8) is called anticipatory if the state vector *x* and the output vector *y* depends on the derivatives of u.

In a similar way as for (1) it can be shown that the standard system (8) is not anticipatory.

Theorem 2. The singular system (8) is anticipatory if and only if

$$\mathrm{rank}E > \deg.\det[Es-A] \tag{13}$$

Proof. In a similar way as for the system (1) it can be shown that the condition (13) is satisfied if and only if the nilpotence index μ>1. From (11) it follows that in this case *x* depends on the derivatives of u. □

4. INFLUENCE OF THE VALUE OF THE STEP DISCRETISATION ON THE ANTICIPATION

Substituting the derivative $\dot{x}$ in (8a) by $\dfrac{x_{i+1}-x_i}{\Delta t}$ we obtain the equation (1a) in which

$$F = E + \Delta tA,\ G = \Delta tB \tag{14}$$

Let the continuous-time system (8) be anticipatory. The following question arises. Does the discrete-time system (1) obtained by the discretisation from the continuous-time system (8) be anticipatory system for any value of the discretisation step Δt?

Theorem 3. The discrete-time system (1) obtained by discretisation from the continuous-time system (8) is anticipatory for any value of discretisation step $\Delta t > 0$ if and only if the continuous-time system is anticipatory.

Proof. By theorems 1 and 2 it is enough to shown that

$$\deg.\det[Ez-F] = \deg.\det[Es-A] \tag{15}$$

Using (14) we may write

$$\det[Ez-F] = \det[Ez-(E+\Delta tA)] = \\ = (\Delta t)^n \det\left[E\frac{z-1}{\Delta t} - A\right] = (\Delta t)^n \det[Es-A]$$

and (15) holds, where $z = 1 + \Delta ts$. □

5. SINGULAR ROESSER MODEL

Consider the 2D linear system described by the equations [27,13]

$$E\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = A\begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix} + Bu_{ij} \tag{16a}$$

$$y_{ij} = C\begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix} + Du_{ij} \tag{16b}$$

where $x_{ij}^h \in R^{n_1}$ is the horizontal state vector, $x_{ij}^v \in R^{n_2}$ is the vertical state vector, $u_{ij} \in R^m$ is the input vector, $y_{ij} \in R^p$ is the output vector and

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}, \begin{matrix} E_{11} \in R^{n_1\times n_1} \\ E_{22} \in R^{n_2\times n_2} \end{matrix},\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ \begin{matrix} A_{11} \in R^{n_1\times n_1} \\ A_{22} \in R^{n_2\times n_2} \end{matrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \begin{matrix} B_1 \in R^{n_1\times m} \\ B_2 \in R^{n_2\times m} \end{matrix}, \\ C \in R^{p\times(n_1+n_2)}, D \in R^{p\times m}$$

The model (16) is called the singular Roesser model if $\det E = 0$ and it is called the standard Roesser model if $\det E \neq 0$.

It is assumed that

$$d(z_1,z_2)=\det\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}= \\ =\sum_{i=0}^{r_1}\sum_{j=0}^{r_2} d_{ij}z_1^i z_2^j \tag{17}$$

and $d_{r_1r_2}\neq 0$ for some positive integers $r_1,r_2(r_1\le n_1, r_2\le n_2)$.

If the assumption is satisfied then [27]

$$\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}^{-1}= \\ =\sum_{i=-\mu_1}^{\infty}\sum_{j=-\mu_2}^{\infty} T_{ij}z_1^{-(i+1)}z_2^{-(j+1)} \tag{18}$$

where the pair (μ_1,μ_2) is the nilpotence index of (16) and the transition matrices T_{ij} are defined by [27,13]

$$[E_1,0]T_{i,j-1}+[0,E_2]T_{i-1,j}-AT_{i-1,j-1}= \\ =\begin{cases} I_n & for\ \ i=j=0 \\ 0 & for\ \ i\neq 0\ \ or/and\ \ j\neq 0\end{cases} \tag{19}$$

and $T_{ij}=0$ for $i<-\mu_1$ or/and $j<-\mu_2$

$$E_1=\begin{bmatrix} E_{11} \\ E_{21}\end{bmatrix},\ E_2=\begin{bmatrix} E_{12} \\ E_{22}\end{bmatrix}$$

The solution x_{ij} of (16a) with the boundary conditions

$$x_{0j}^h,\ j\in Z_+,\ x_{i0}^v, i\in Z_+ \tag{20}$$

is given by [27,13]

$$x_{ij}=\sum_{k=0}^{i+\mu_1-1}\sum_{l=0}^{j+\mu_2-1} T_{i-k-1,j-l-1}Bu_{kl}+\sum_{l=0}^{i+\mu_1-1} T_{i,j-l-1}E_1x_{0l}^h \\ +\sum_{k=0}^{i+\mu_1-1} T_{i-k-1,j}E_2x_{k0}^v \tag{21}$$

From (21) it follows that if $\mu_1>1$ or/and $\mu_2>1$ then the solution x_{ij} depends on the future values of inputs u_{kl} for $k>i, l>j$.

Definition 3. The model (16) is called anticipatory if the state vector $x_{ij}=\begin{bmatrix} x_{ij}^h \\ x_{ij}^v\end{bmatrix}$ and output vector y_{ij} depend on the future values of inputs, u_{kl} for $k>i, l>j$.

Theorem 4. The singular system (16) is anticipatory if and only if

$$rank\,E_i>\deg_{z_i} d(z_1,z_2)\ \text{ for }\ i=1\ \ or/and\ \ i=2 \tag{22}$$

where $d(z_1,z_2)$ is defined by (17).

Proof. It is easy to show that

$$\deg_{z_i} adj\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}=rank\,E_i$$

for $i=1,2$.

Using the well-known procedure of the division of polynomials from the formula

$$\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}^{-1}= \\ =\frac{1}{d(z_1,z_2)}adj\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix} \tag{23}$$

we obtain $\mu_i=rank\,E_i-r_i+1$ for $i=1,2$. Therefore, if (22) holds then $\mu_i>1\ or/and\ \mu_2>1$ and from (21) it follows that the solution x_{ij} depends on the future values of inputs.

Example. Consider the model (16a) with

$$E=\left[\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0\end{array}\right], \\ A=\left[\begin{array}{cc:cc} 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ \hdashline -1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 0\end{array}\right],\ B=\left[\begin{array}{c} 1 \\ 0 \\ \hdashline -1 \\ 1\end{array}\right] \tag{24}$$

In this case $n_1=n_2=2,\ m=1$

$$d(z_1,z_2)=\det\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}=$$

$$=\begin{vmatrix} z_1 & 1 & 0 & 1 \\ 0 & z_1-1 & 0 & -1 \\ 1 & 2 & z_2 & -1 \\ 0 & -1 & 0 & 0 \end{vmatrix}=z_1z_2$$

and $r_1=r_2=1$.

Using (23) we obtain

$$\begin{bmatrix} E_{11}z_1-A_{11} & E_{12}z_2-A_{12} \\ E_{21}z_1-A_{21} & E_{22}z_2-A_{22}\end{bmatrix}^{-1}=$$

$$=\begin{bmatrix} z_1 & 1 & 0 & 1 \\ 0 & z_1-1 & 0 & -1 \\ 1 & 2 & z_2 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix}^{-1}=$$

$$=\frac{1}{z_1z_2}\begin{bmatrix} z_2 & z_2 & 0 & z_1z_2 \\ 0 & 0 & 0 & -z_1z_2 \\ -1 & -z_1-1 & z_1 & -z_1^2+2z_1 \\ 0 & -z_1z_2 & 0 & -z_1^2z_2+z_1z_2 \end{bmatrix}=$$

$$=\begin{bmatrix} z_1^{-1} & z_1^{-1} & 0 & 1 \\ 0 & 0 & 0 & -1 \\ -z_1^{-1}z_2^{-1} & -z_2^{-1}-z_1^{-1}z_2^{-1} & z_2^{-1} & -z_1z_2^{-1}+2z_2^{-1} \\ 0 & -1 & 0 & -z_1+1 \end{bmatrix}=$$

$$=T_{-2-1}z_1+T_{-20}z_1z_2^{-1}+T_{-1-1}+T_{-10}z_2^{-1}+T_{-01}z_1^{-1}+T_{00}z_1^{-1}z_2^{-1}$$

where

$$T_{-2-1}=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-1\end{bmatrix}, T_{-20}=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&-1\\0&0&0&0\end{bmatrix},$$

$$T_{-1-1}=\begin{bmatrix} 0&0&0&1\\0&0&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix}, T_{-1,0}=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&-1&1&2\\0&0&0&0\end{bmatrix},$$

$$T_{0-1}=\begin{bmatrix} 1&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, T_{00}=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\-1&-1&0&0\\0&0&0&0\end{bmatrix}$$

Hence $\mu_1=rankE_1-r_1+1=2$ and $\mu_2=rank\,E_2-r_2+1=1$.

From (21) we obtain

$$x_{ij}=\sum_{k=0}^{i+1}\sum_{l=0}^{j}T_{i-k-1,j-l-1}Bu_{kl}+\sum_{l=0}^{j}T_{i,j-l-1}E_1x_{0l}^h+$$

$$+\sum_{k=0}^{i+1}T_{i-k-1,j}E_2x_{k0}^v=\begin{bmatrix} u_{ij}+u_{i-1,j} \\ -u_{ij} \\ -u_{i+1,j-1}+u_{i,j-1}-u_{i-1,j-1} \\ -u_{i+1,j}+u_{ij}\end{bmatrix}$$

for $i,j>0$

It is easy to show that the singular 2D Fornasini-Marchesini type models can be recasted in singular 2D Roesser type model [21]. Therefore, the considerations can be easily extended for the singular 2D Fornasini-Marchesini type models and the singular general 2D model [16,13].

6. CONCLUDING REMARKS

The standard linear continuous-time and discrete-time systems are not anticipatory systems. The singular linear continuous-time systems are anticipatory systems if and only if the condition (13) is satisfied and the singular linear discrete-time systems are anticipatory systems if and only if the condition (6) is satisfied. It has been shown that the discrete-time system obtained by discretisation from continuous-time one is anticipatory for any value of the discretisation step $\Delta t>0$ if and only if the continuous-time is also anticipatory. Necessary and sufficient conditions for the anticipation of singular 2D Roesser model have been established.

An open problem is the extension of the considerations for singular 2D continuous-discrete linear systems [20].

7. REFERENCES

[1] Campbel S. L.: Singular systems of differential equation. Pitman Advanced Publishing Program. pp.138-143

[2] Dai L.: Singular Control Systems. Lectures Note in Control and Information Sciences. 1989 Springer-Verlag

[3] Dubois D. M.: Computing Anticipatory Systems with Incursion and Hyperincursion. CP437, Computing Anticipatory Systems: CAYS-First mt. Corif. edited by Daniel M. Dubois, The American Institute of Physics, 1998, pp.3-28

[4] Dubois D. M.: Incursive Anticipatory Control of a Chaotic Robot Arm., CP437, Computing Anticipatory Systems: CAYS-First mt. Cotif edited by Daniel M. Dubois, The American Institute of Physics, 1998, pp. 406-417

[5] Fornasini E. and Marchesini G.: State-space realization theory of two-dimensional filters,

IEEE Trans. Autom. Contr. Vol. AC-21, 1976, pp. 484-491.
[6] Fornasini E. and Marchesini G.: Doubly-indexed dynamical systens: State-space models and structural properties, Math. Syst. Theory, vol. 12, 1978, pp. 59-72.
[7] Fragulis G.F: A closed formula for the determination of the impulsive solutions of linear homogeneous matrix differential equations, IEEE Trans. Automat. Contr., vol. AC-38, No 11, 1993, pp.1688-1695.
[8] Fragulis G.F. and Mertzios B.G.: Computation of the impulse behavior of multivariable linear systems using a division algorithm, Systems and Control Letters, vol. 38, 1999
[9] Kaczorek T.: Computation of fundamental matrices and rechability of positive singular discrete linear systems, Bull. Pol. Acad. TecIm. Sci. vol.46, No 4, 1998, pp. 501-511.
[10] Kaczorek T.: Electrical circuits as example of positive singular continuous-time systems. SPETO '98, Ustroń 20-22.05.98. pp.37-43
[11] Kaczorek T.: Positive linear systems and their relationship witch electrical circuits. SPETO '97, Ustroń 21-24.05.1997, pp.33-41
[12] Kaczorek T.: Positive singular discrete linear systems, Bull. Pol. Acad. Techn. Sci. vol.45; 1997 No 4, pp. 619-631
[13] Kaczorek T.: Linear Control Systems, vol. 1, 2, New York, Wiley, 1992.
[14] Kaczorek T.: Reachability and controllability of non-negative 2-D Roesser type models, Bull. Acad. Pol. Sci. Ser. Sci. Techn., vol. 44, No 4, 1996, pp. 405-410.
[15] Kaczorek T.: Two-Dimensional Linear Systems, Springer-Verlag, Berlin 1985.
[16] Kaczorek T.: Singular general model of 2-D systems and its solution, IEEE Trans. on Autom. Contr., vol. AC-33, No 11, 1988, pp. 1060-1061
[17] Kaczorek T.: Positive descriptor discrete-time linear systems. International Jouraal: Problems of Nonlinear Analysis in Engineering Systems; 1998 No 1(7), pp.38-54
[18] Kaczorek T.: Weakly positive continuous-time linear systems, Bull. Pol. Acad. Sci. Vol 46; 1998 No.2, pp.233-245
[19] Kaczorek T.: Influence of value of the discretisation sten on positivity and stability of linear dynamic systems Pomiary, Automatyka, Kontrola, PAK 12, 1999, pp. 3-7.
[20] Kaczorek T.: Singular 2-D continuous-discrete linear systems, Dynamics of continuous, discrete and impulse systems, Advances in Systems Science and Applications, 1995, pp. 103-108.
[21] Kaczorek T.: Equivalence of nD singular Roesser and Fornasini-Marchesini models, Bull. Pol. Acad. Sci. vol. 47; No 3, 1999, pp. 235-246.
[22] Klamka J.: Controllability of dynamical systems, Kluwer Academic Publ., Dordrecht, 1991.
[23] Kurek J.: The general state-space model for a two-dimensional linear digital system, IEEE Trans. Autom. Contr. AC-30, June 1985, pp. 600-602.
[24] Lewis F. L.: A survey of linear singular systems. Circuits Systems Signal Process, Vol.5; 1986 No.1, pp.1-36
[25] Lewis F. L.: Descriptor systems: Decomposition into forward and backward subsystems, IEEE Trans. Automat. Contr. Vol. AC-29; 1984, pp.167-170
[26] Lewis F.L. and Mertzios B.G.: On the analysis of discrete linear-time invariant singular systems, IEEE Trans. on Automat. Control. Vol. AC-35, 1990, pp. 506-511.
[27] Lewis F.L. and Mertzios B.G.: On the analysis of two-dimensional discrete singular systems, Circuits Systems and Signal Processing, vol.11, No.3, 1992, pp.399-419.
[28] Mertzios B. G. and Lewis F. L.: Fundamental matrix of discrete singular systems. Circuits, Syst., Signal Processing Vol.8; 1989 No.3, pp.341-355
[29] Newcomb R.W.: The semistate description of nonlinear time-variable circuits, IEEE Trans. on Circuits and Systems, vol. CAS-28, No 1, 1981, pp. 62-71.
[30] Newcomb R.W. and Dziurla B.: Some circuits and system applications of semistate theory, Circuits, Systems and Signal Proc., Special Issue: Recent Advances in Singular Systems (Editors F.L. Lewis and B.G. Mertzios), vol. 8, No 3, 1989, pp. 235-260.
[31] Roesser R.B.: A discrete state space model for linear image processing, IEEE Trans. Autom. Contr. AC-20, 1975, pp. 1-10.
[32] Rosen R.: Anticipatory Systems, Pergamon Press 1985
[33] Valcher M.E.: On the internal stability and asymptotic behavior of 2-D positive systems, IEEE Trans. on Circuits and Systems – I, vol. 44, No 7, 1997, pp. 602-613
[34] Verghese G.C, Levy B.C. and Kailath T.: A generalized state-space for singular systems, IEEE Trans. Autom. Control, vol. AC-26, No 4, 1981, pp. 811-831.
[35] Zaghloul M.E. and Newcomb R.W.: Semistate implementation: Differentiator example, Circuits, Systems and Signal Proc., Special Issue: Recent Advances in Singular Systems (Editors F.L. Lewis and R.W. Newcomb), vol. 5, No 1, 1986, pp. 171-183.