Proceedings of the 8th IEEE Mediterranean Conference
on Control and Automation (MED 2000)
Rio, Patras, GREECE 17-19 July, 2000

TUNING OF PID CONTROLLER FOR INTERVAL PLANTS

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Abstract: The paper deals with the problem of controller design for guaranteed phase margin when the plant include parameter uncertainty. Some results from the area of parametric robust control related to interval systems are combined with the Astrom-Hagglund method to design robust PID controller.

Keywords: Interval systems; Robust control; Astrom-Hagglund method

1. INTRODUCTION

In modelling physical systems for control purposes, one normally needs to generate mathematical representations of the system. However, in general, it is difficult to obtain an exact mathematical description of a physical process. Therefore, it is necessary to make some simplifications in order to obtain a description of the system which is tractable. The difference between an exact model and its simplified form is called a perturbation. The requirement for a control design to be successful is that it must cope with any changes such as parametric variations which may occur in a system. This ability of a control system is known as robustness. Thus, one of the main problems in control theory is to design a fixed controller which guarantees acceptable performance in the presence of uncertainty. Uncertainties in control systems can be broadly classified under two categories. They are:

i) structured (or parametric) uncertainty, representing lack of precise knowledge of the actual parameters. For example, the uncertain parameters can be the coefficients of a transfer function of a system.

ii) unstructured (or nonparametric) uncertainty which represents unmodelled dynamics, nonlinearities and error due to linearizations etc. These types of uncertainties are usually given as norm bounded perturbations.

In recent years, a substantial amount of research [1,3,5,6] concerning robustness analysis of control systems affected by real parametric uncertainty has been done. The main methods used in the research have been the well-known Kharitonov theorem [8] on interval polynomials and the edge theorem [4] for affine polynomials. However, it is necessary to mention that a large part of the literature in the field of robust parametric control has been devoted to the robust stability analysis paradigm, rather than the robust performance paradigm. This is not because the robust performance problems of systems with parametric uncertainty have been solved, but simply because the research has had little success in this field so far.

In this paper, some results from the area of parametric robust control related to interval systems are used together with the Astrom-Hagglund controller tuning method [2] to find the parameters of a PID controller which gives a desired phase margin for an interval plant family. An example is given to illustrate the application of the method.

2. SOME RESULTS FROM INTERVAL SYSTEMS

In this section, a brief summary of the essential results on interval systems is given. Consider a unity feedback system with

$$C(s) = \frac{N_c(s)}{D_c(s)} \tag{1}$$

and

$$G(s,q,r) = \frac{N(s,r)}{D(s,q)} = \frac{r_m s^m + r_{m-1}s^{m-1} + ... + r_0}{q_n s^n + q_{n-1}s^{n-1} + ... + q_0} \tag{2}$$

where $r_i \in [\underline{r_i}, \overline{r_i}], i = 0,1,2,....,m$ and $q_i \in [\underline{q_i}, \overline{q_i}], i = 0,1,2,....,n$. The numerator, $N_c(s)$, and the denominator, $D_c(s)$, are fixed polynomials in s. Let the Kharitonov polynomials associated with $N(s,r)$ and $D(s,q)$ be respectively:

$$\begin{aligned} N_1(s) &= \underline{r_0} + \underline{r_1}s + \overline{r_2}s^2 + \overline{r_3}s^3 + ... \\ N_2(s) &= \underline{r_0} + \overline{r_1}s + \overline{r_2}s^2 + \underline{r_3}s^3 + ... \\ N_3(s) &= \overline{r_0} + \underline{r_1}s + \underline{r_2}s^2 + \overline{r_3}s^3 + ... \\ N_4(s) &= \overline{r_0} + \overline{r_1}s + \underline{r_2}s^2 + \underline{r_3}s^3 + ... \end{aligned} \tag{3}$$

and

$$\begin{aligned} D_1(s) &= \underline{q_0} + \underline{q_1}s + \overline{q_2}s^2 + \overline{q_3}s^3 + ... \\ D_2(s) &= \underline{q_0} + \overline{q_1}s + \overline{q_2}s^2 + \underline{q_3}s^3 + ... \\ D_3(s) &= \overline{q_0} + \underline{q_1}s + \underline{q_2}s^2 + \overline{q_3}s^3 + ... \\ D_4(s) &= \overline{q_0} + \overline{q_1}s + \underline{q_2}s^2 + \underline{q_3}s^3 + ... \end{aligned} \tag{4}$$

By taking all combinations of the $N_i(s)$ and $D_i(s)$ for $i,j = 1,2,3,4$, one obtains the sixteen Kharitonov plants as

$$G_K(s) = \{G_{ij}(s) | G_{ij}(s) = \frac{N_i(s)}{D_j(s)}\} \tag{5}$$

The Kharitonov segments for the numerator and denominator of $G(s,q,r)$ can be written as

$$\lambda N_i(s) + (1-\lambda)N_j(s) \tag{6}$$

and

$$\lambda D_i(s) + (1-\lambda)D_j(s) \tag{7}$$

where $\lambda \in [0,1]$ and $(i,j) \in \{(1,2),(1,3),(2,4),(3,4)\}$. And the following 32 subsets of the family of interval plants $G(s,q,r)$ can be obtained by using Kharitonov segments. These subsets are

$$G_{E1}(s) = \frac{N_i(s)}{\lambda D_j(s) + (1-\lambda)D_k(s)} \tag{8}$$

and

$$G_{E2}(s) = \frac{\lambda N_j(s) + (1-\lambda)N_k(s)}{D_i(s)} \tag{9}$$

where $\lambda \in [0,1]$, $i = 1,2,3,4$ and $(j,k) \in \{(1,2),(1,3),(2,4),(3,4)\}$ and

$$G_E(s) = G_{E1}(s) \cup G_{E2}(s) \tag{10}$$

The closed loop characteristic equation of the system is denoted as

$$\delta(s) = D_c(s)D(s,q) + N_c(s)N(s,r) \tag{11}$$

Then the closed loop system is stable for all $C(s)G(s,q,r)$ if and only if it is stable for all $C(s)G_E(s)$ [5]. If the controller is a *P, PI, Lag* or *Lead* then $C(s)G(s,q,r)$ is stable if $C(s)G_K(s)$ is stable [3,5]. If the interval plant is a proper stable plant and the controller is a proportional controller then the outer boundary of the Nyquist envelope is covered by the Nyquist plots of the Kharitonov plants [7,9]. The whole boundary of the Nyquist and Nichols envelopes of $G(s,q,r)$ and $C(s)G(s,q,r)$ are generated from the boundary of $G_E(s)$ and $C(s)G_E(s)$ and the Bode envelope can be obtained from the rectangular value sets of the numerator and the denominator of the interval plant [5].

3. DESIGN OF PID CONTROLLER FOR INTERVAL PLANTS

In this section, it is assumed that the process transfer function is an interval plant of the form of Eq.(2) and the objective is to find the parameters of a PID controller which gives the specified phase margin (φ_m) for overall system using the Astrom-Hagglund method [2]. The Astrom-Hagglund controller tuning method is based on the idea that a point on the Nyquist plot of a given transfer function can be moved to a selected point in the complex plane by choosing suitable controller parameters. Such an appropriate point for tuning is the intersection of the Nyquist curve with the negative real axis which is traditionally described as the *critical point.* Generally, if the Nyquist plot of a transfer function is a '*good curve*' (see Fig.1) then it is possible to design a PID controller which moves the critical point in the third quadrant and gives a guaranteed phase margin. However, for an interval plant, there are many Nyquist curves which cross the negative real axis or for a fixed frequency there are many points in the Nyquist plane. Therefore, it is necessary to move a Nyquist template to a selected position in the complex plane instead of a point. In order to do this, one needs to find a *critical member* of the family of Eq.(2) for which the designed PID

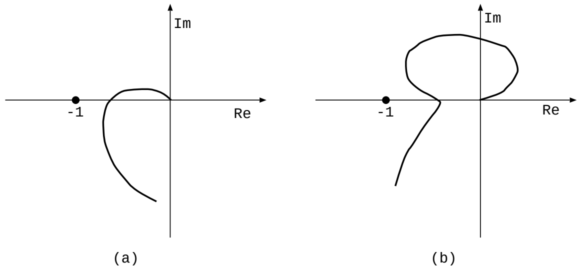

Fig. 1. a) a good Nyquist curve b) a bad Nyquist curve

controller will be robust in the sense of the phase margin. The critical member is one of the sixteen Kharitonov plants since the robust gain margin of an inerval plant family achieves at the sixteen Kharitonov plants family.

Now, from Fig. 2, let ω_{cp} be the critical frequency of the plant within the interval plant family whose Nyquist curve passes through point A. Suppose the design goal is to design a PID controller of the form

$$C(s) = K_p(1 + sT_d + \frac{1}{sT_i}) \tag{12}$$

for which the overall system gives a phase margin of φ_m. In order to move the point A to the point B (see Fig.2) and makes its magnitude equal to one at $s = j\omega_{cp}$, the controller must satisfy

$$\begin{aligned} &C(j\omega_{cp}) = \\ &K_p + j\frac{K_p(\omega_{cp}^2 T_i T_d - 1)}{\omega_{cp} T_i} = \frac{1}{mag[A]} e^{j\varphi_m} \\ &= \frac{1}{mag[A]} cos\varphi_m + j\frac{1}{mag[A]} sin\varphi_m \end{aligned} \tag{13}$$

Thus, it can be seen that

$$K_p = \frac{1}{mag[A]} cos\varphi_m \tag{14}$$

and defining a constant ratio, α, between T_i and T_d as [2]

$$T_i = \alpha T_d \tag{15}$$

and using

$$\frac{K_p(\omega_{cp}^2 T_i T_d - 1)}{\omega_{cp} T_i} = \frac{1}{mag[A]} sin\varphi_m \tag{16}$$

the values of T_i and T_d can be found.

4. EXAMPLE

Consider an interval transfer function as

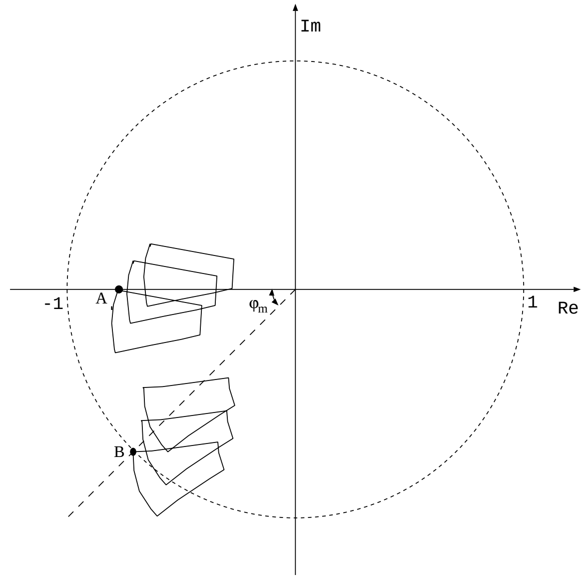

Fig. 2. Movement of the Nyquist templates

$$\begin{aligned} G(s,q,r) &= \frac{N(s,r)}{D(s,q)} = \\ &\frac{r_0}{q_4 s^4 + q_3 s^3 + q_2 s^2 + q_1 s + q_0} \end{aligned} \tag{17}$$

where $r_0 = 1$, $q_0 \in [0.965, 1.035]$, $q_1 \in [1.965, 2.035]$, $q_2 \in [1.295, 1.365]$, $q_3 \in [0.335, 0.405]$ and $q_4 \in [0.002, 0.072]$. The aim is to find the parameters of the PID controller of the form of Eq.(12) for which the phase margin of the system is at least $\varphi_m = 45°$. The four Kharitonov plats of the family are

$$\begin{aligned} G_1(s) &= \frac{1}{\underline{q_4} s^4 + \overline{q_3} s^3 + \overline{q_2} s^2 + \underline{q_1} s + \underline{q_0}} \\ G_2(s) &= \frac{1}{\underline{q_4} s^4 + \underline{q_3} s^3 + \overline{q_2} s^2 + \overline{q_1} s + \underline{q_0}} \\ G_3(s) &= \frac{1}{\overline{q_4} s^4 + \overline{q_3} s^3 + \underline{q_2} s^2 + \underline{q_1} s + \overline{q_0}} \\ G_4(s) &= \frac{1}{\overline{q_4} s^4 + \underline{q_3} s^3 + \underline{q_2} s^2 + \overline{q_1} s + \overline{q_0}} \end{aligned} \tag{18}$$

It was found that $G_3(s)$ gives the minimum gain margin of the system which is equal to $3.5532(11db)$ at the frequency $\omega_{cp} = 2.2027$. Using this frequency and the gain at this frequency, from Eq.(14)

$$K_p = 2.51 \tag{19}$$

For $\alpha = 4$ and using Eq.(16), it was found

$$T_d = 0.548 \;\; and \;\; T_i = 2.193 \tag{20}$$

Thus, the designed PID controller is

$$C(s) = \frac{3.012s^2 + 5.5s + 2.51}{2.193s} \tag{21}$$

The minimum phase margin of the overall system is 45.22°. Fig. 3 shows the closed loop step responses of $G_K(s)$ and Fig. 4 shows the closed loop step responses of $C(s)G_K(s)$. The Nyquist envelopes of $G(s,q,r)$ and $C(s)G(s,q,r)$ are shown in Fig. 5.

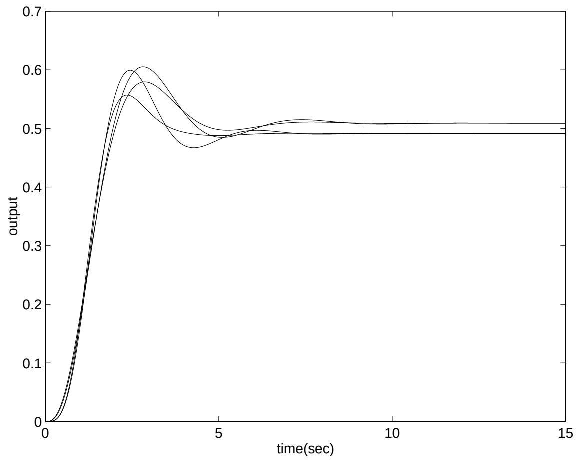

Fig. 3. Step responses of $G_K(s)$

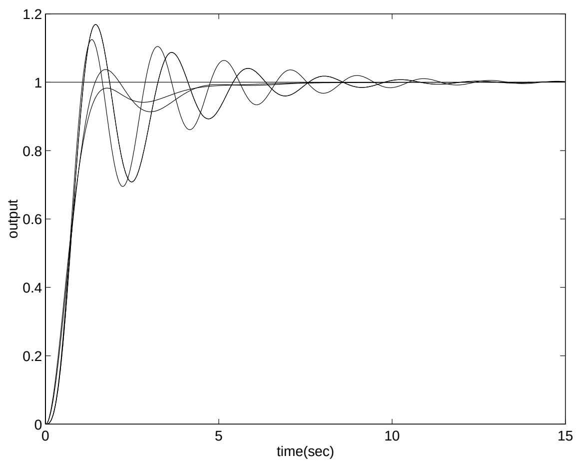

Fig. 4. Step responses of $C(s)G_K(s)$

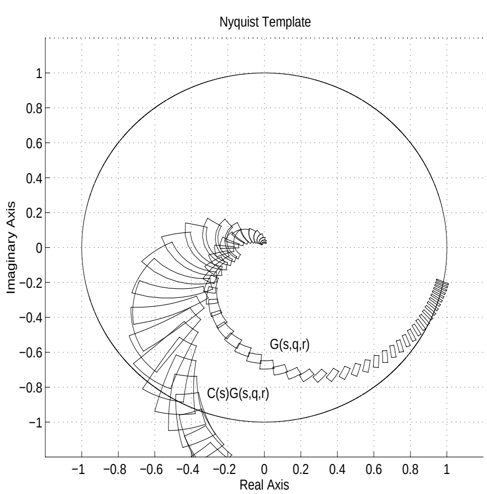

Fig. 5. Nyquist envelopes of $G(s,q,r)$ and $C(s)G(s,q,r)$

5. CONCLUSION

In this paper, a method has been presented in order to find the parameter of a PID controller for which the minimum phase margin of an interval system is equal or greater than the desired phase margin. The method is based on the Astrom-Hagglund method. This method can be extended to the control systems with more complicated uncertainty structures.

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