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DYNAMIC MOTION PLANNING FOR MOBILE ROBOTS USING POTENTIAL FIELD METHOD

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Abstract: The potential field method is widely used for autonomous mobile robot path planning due to its elegant mathematical analysis and simplicity. However, most researches were focused on solving the motion planning problem in a stationary environment, where both targets and obstacles are stationary. This paper proposes a new potential field method for motion planning of mobile robots in a dynamic environment where the target and obstacles are moving. Firstly, the new potential function and the corresponding virtual force are defined. Then, an on-line motion planning algorithm based on the new potential field method is presented. Finally, computer simulation is used to demonstrate the effectiveness of the dynamic motion planning scheme based on the new potential field method.

Keywords: Potential Fields, Motion Planning, Moving Obstacle Avoidance

1 INTRODUCTION

The potential field method has been studied extensively for autonomous mobile robot path planning in the past decade [1]-[14]. The basic idea of the potential field method is to fill the robot's workspace with an artificial potential field in which the robot is attracted to its target position and is repulsed away from the obstacles [1]. This method is particularly attractive because of its elegant mathematical analysis and simplicity. In the previous studies, potential field methods are used to deal with mobile robot path planning in stationary environments, where targets and obstacles are all stationary. However, in many real-life implementations, the environments are dynamic. Not only the obstacles are moving, so does the target.

In this paper, we propose a new potential field method for motion planning of mobile robots in a dynamic environment where the target and obstacles are moving. The attractive potential is defined as a function of the relative position and velocity of the target with respect to the robot. The repulsive potential is also defined as the relative position and velocity of the robot with respect to the obstacles. Accordingly, the virtual force is defined as the negative gradient of the potential with respect to both position and velocity rather than position only. The new definitions of the potential functions and the virtual forces allow the robot to track the target in a desired manner.

2 PROBLEM STATEMENT

The motion planning problem for a mobile robot in a dynamic environment is to plan and control the robot motion from an initial position to track a moving target in a desired manner while avoiding moving obstacles. To simplify the analysis, we have the following assumptions:

Assumption 1 *The robot is a point mass which can move omni-directionally, whose mass m_{rob}, position $\mathbf{p}$ and velocity $\mathbf{v}$ are known and its maximum velocity v_{max} is greater than that of the target. The acceleration of the robot, $\mathbf{a}$, is omni-directional and its maximum value is a_{max}.*

Assumption 2 *The point target moves at con-*

stant velocity and its position $\mathbf{p}_{tar}$ and velocity $\mathbf{v}_{tar}$ are known.

Assumption 3 *The obstacles are assumed to be balls of radius r_i centered at $\mathbf{p}_{obsi}$, with $i = 1, 2, \ldots, n_{obs}$, where n_{obs} is the number of obstacles. The positions $\mathbf{p}_{obsi}$ and velocities $\mathbf{v}_{obsi}$ of the obstacles can be measured accurately.*

Assumption 4 *At each time instant, only one obstacle is close enough to the robot and needs to be avoided. The rest are assumed to be farther away and their influences are neglected for that instant.*

3 ATTRACTIVE POTENTIAL FUNCTION

Conventionally, the attractive potential is defined as a function of the relative distance between the robot and the target only where the target is a fixed point in space. In our research, it was found that it is beneficial to have the velocities of the robot and the target considered in the construction of the potential field. In doing so, additional degrees of freedom are introduced for different tracking performances. When the target is moving, the conventional pure position based potential function is not directly applicable and has to be modified. In this paper, the potential field functions are presented as follows

$$U_{att}(\mathbf{p},\mathbf{v}) = \alpha_p \|\mathbf{p}_{tar}(t) - \mathbf{p}(t)\|^m + \alpha_v \|\mathbf{v}_{tar}(t) - \mathbf{v}(t)\|^n \tag{1}$$

where $\mathbf{p}(t)$ and $\mathbf{p}_{tar}(t)$ denote the positions of the robot and the target, respectively; $\mathbf{p} = [x \quad y \quad z]^T$ in a 3-dimensional space or $\mathbf{p} = [x \quad y]^T$ in a 2-dimensional space; $\mathbf{v}(t)$ and $\mathbf{v}_{tar}(t)$ denote the velocities of the robot and the target, respectively; $\|\mathbf{p}_{tar}(t) - \mathbf{p}(t)\|$ is the Euclidean distance between the robot and the target at time t; $\|\mathbf{v}_{tar}(t) - \mathbf{v}(t)\|$ is the magnitude of the relative velocity between the target and the robot at time t; α_p and α_v are scalar positive parameters; and m and n are positive constants which satisfy $m, n > 1$.

From (1), the attractive potential $U_{att}(\mathbf{p},\mathbf{v})$ approaches its minimum zero if and only if the relative distance and velocity between the robot and the target are zero. The attractive potential $U_{att}(\mathbf{p},\mathbf{v})$ increases as the relative distance or velocity between the robot and the target increases.

It is easy to see that if $\alpha_v = 0$ and $m = 2$, the new attractive potential function (1) degenerates to the conventional quadratic form

$$U_{att}(\mathbf{p},\mathbf{v}) = U_{att}(\mathbf{p}) = \alpha_p \|\mathbf{p}_{tar}(t) - \mathbf{p}(t)\|^2 \tag{2}$$

which does not contain the velocity information of the robot or the target. The corresponding conventional virtual attractive force is defined as the negative gradient of the attractive potential in terms of position

$$\mathbf{F}_{att}(\mathbf{p}) = -\nabla U_{att}(\mathbf{p}) = -\frac{\partial U_{att}(\mathbf{p})}{\partial \mathbf{p}} \tag{3}$$

As the new attractive potential $U_{att}(\mathbf{p},\mathbf{v})$ is a function of both the position $\mathbf{p}$ and velocity $\mathbf{v}$ of the robot, we shall define the corresponding virtual attractive force as the negative gradient of the attractive potential with respect to both position and velocity as follows:

$$\begin{aligned} \mathbf{F}_{att}(\mathbf{p},\mathbf{v}) &= -\nabla U_{att}(\mathbf{p},\mathbf{v}) \\ &= -\nabla_p U_{att}(\mathbf{p},\mathbf{v}) - \nabla_v U_{att}(\mathbf{p},\mathbf{v}) \end{aligned} \tag{4}$$

where

$$\nabla_p U_{att}(\mathbf{p},\mathbf{v}) = \frac{\partial U_{att}(\mathbf{p},\mathbf{v})}{\partial \mathbf{p}} \tag{5}$$

$$\nabla_v U_{att}(\mathbf{p},\mathbf{v}) = \frac{\partial U_{att}(\mathbf{p},\mathbf{v})}{\partial \mathbf{v}} \tag{6}$$

with the subscripts p and v denoting the gradient with respect to position and velocity, respectively.

Substituting (1) into (4), we have

$$\mathbf{F}_{att}(\mathbf{p},\mathbf{v}) = \mathbf{F}_{att1}(\mathbf{p}) + \mathbf{F}_{att2}(\mathbf{v}) \tag{7}$$

where

$$\mathbf{F}_{att1}(\mathbf{p}) = m\alpha_p \|\mathbf{p}_{tar}(t) - \mathbf{p}(t)\|^{m-1} \mathbf{n}_{RT} \tag{8}$$

$$\mathbf{F}_{att2}(\mathbf{v}) = n\alpha_v \|\mathbf{v}_{tar}(t) - \mathbf{v}(t)\|^{n-1} \mathbf{n}_{VRT} \tag{9}$$

with $\mathbf{n}_{RT}$ being the unit vector pointing from the robot to the target and $\mathbf{n}_{VRT}$ being the unit vector denoting the relative velocity direction of the target with respect to the robot. The relationship between the attractive force and the position and velocity of the robot and the target in a 2-dimensional space is illustrated in Figure 1. The attractive force $\mathbf{F}_{att}$ consists of two components: while the first component, $\mathbf{F}_{att1}(\mathbf{p})$, pulls the robot to the target and shortens the distance between them, the second component, $\mathbf{F}_{att2}(\mathbf{v})$, "tries" to drive the robot to move at the same velocity of the target.

From equations (8) and (9), since $m > 1$ and $n > 1$, we can see that when the robot approaches the target, i.e. $\|\mathbf{p}_{tar}(t) - \mathbf{p}(t)\|$ approaches zero, $\mathbf{F}_{att1}$ approaches zero, and when the velocity of the robot approaches that of the target, $\mathbf{F}_{att2}$ approaches zero. Then, when both the position and velocity of the robot approach those of the target, the attractive force $\mathbf{F}_{att}$ approaches zero, i.e. when the robot catches the target and at the same

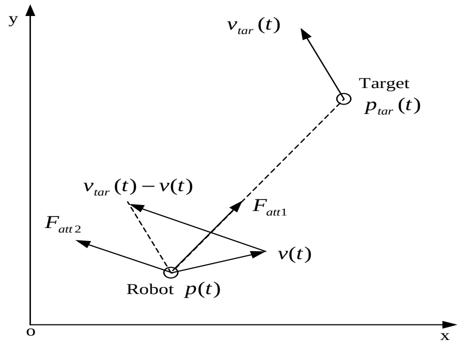

Figure 1: New attractive force in 2D space.

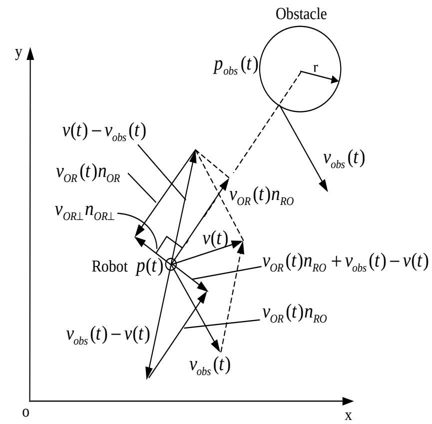

Figure 2: Vectors for defining the new repulsive potential.

time travels at the same velocity of the target, the attractive force is zero and thus the robot keeps the velocity and moves together with the target. Such choices of m and n are necessary for soft-landing problems where the velocity of the robot is required to be the same as that of the target at landing.

4 REPULSIVE POTENTIAL FUNCTION

To avoid moving obstacles, an intuitive way is to take into account the positions and velocities of the robot and the obstacles when constructing the repulsive potential function. Two pioneering repulsive potential functions were presented in [8] and [9] by taking the velocity information into consideration. In [8], though the velocity of the obstacle is considered when building the repulsive potential, the velocity of the robot is not taken into account. This is inadequate because the possibility of the collision between the robot and obstacle depends on the relative position and velocity between them. The repulsive potential function in [9] makes fully use of the velocity information of the robot and the obstacle. However, it was assumed that (i) the relative velocity of the robot with respect to the obstacle is invariant irregardless of the position of the robot and (2) its partial derivatives with respect to position is zero. These assumptions are unrealistic as the relative velocity is actually a function of the position of the robot and its derivatives with respect to position cannot be considered as zero all the time. Both methods deal with the obstacle avoidance problem with a stationary target.

To overcome these limitations, we present another repulsive potential which also takes the relative position and velocity of the robot with respect to the obstacles into consideration. In many circumstances, obstacles are not close to each other. We shall assume that at each time instant, the robot is influenced by only one obstacle which is the nearest one to the robot. Assume also that the position $\mathbf{p}_{obs}(t)$ and velocity $\mathbf{v}_{obs}(t)$ of the obstacle are accurately known. The relative velocity between the robot and the obstacle in the direction from the robot to the obstacle is given by

$$v_{RO}(t) = [\mathbf{v}(t) - \mathbf{v}_{obs}(t)]^T \mathbf{n}_{RO} \tag{10}$$

where $\mathbf{n}_{RO}$ is a unit vector pointing from the robot to the obstacle. If $v_{RO}(t) \leq 0$, i.e. the robot is moving away from the obstacle, no avoidance motion is needed. If $v_{RO}(t) > 0$, i.e. the robot is moving close to the obstacle, avoidance motion needs to be implemented.

Assume that at time t, the robot is moving toward the obstacle as shown in Figure 2. The shortest distance between the robot and the body of the obstacle is denoted by $\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t))$ which is given by

$$\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) = \|\mathbf{p}(t) - \mathbf{p}_{obs}(t)\| - r \tag{11}$$

where r is the radius of the obstacle. When $v_{RO}(t) > 0$, the robot needs to decreases its projection of relative velocity $v_{RO}(t)$ to zero before it touches the obstacle.

Consider the case where the robot moves in the direction toward the obstacle and the robot is approaching the obstacle, i.e. $v_{RO}(t) > 0$. If the maximum deceleration of magnitude a_{max} in the direction from the obstacle to the robot is applied to the robot to decrease its velocity toward the obstacle, the distance traveled by the robot when the velocity projection v_{OR} decreases to zero is

$$\rho_m = \frac{v_{RO}^2(t)}{2a_{max}} \tag{12}$$

The repulsive potential generated by the obstacle can then be defined as follows

$$U_{rep}(\mathbf{p}, \mathbf{v})$$

$$
= \begin{cases} 0, \\ \quad \text{if } \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m \geq \rho_0 \text{ or } v_{RO}(t) \leq 0 \\ \eta \left(\dfrac{1}{\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m} - \dfrac{1}{\rho_0} \right), \\ \quad \text{if } 0 < \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m < \rho_0 \\ \qquad \text{and } v_{RO}(t) > 0 \\ \text{not defined}, \\ \quad \text{if } v_{RO}(t) > 0 \text{ and } \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) < \rho_m \end{cases} \tag{13}
$$

where ρ_0 is a positive constant describing the influence range of the obstacle; $\mathbf{p}_{obs}(t)$ and $\mathbf{v}_{obs}(t)$ denote the position and velocity of the obstacle; $\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) = \|\mathbf{p}(t) - \mathbf{p}_{obs}(t)\| - r$ is the minimum distance between the robot and the obstacle; r is the radius of the obstacle; and constant $\eta > 0$ is a design parameter.

Note that (i) when $\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) < \rho_m$, the repulsive potential is not defined, since there is no possible solution to avoid collision with the obstacle in the aforementioned case where the robot moves toward the robot; (2) when the robot is far away from the obstacle, i.e. $\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m \geq \rho_0$, the robot is not influenced by the obstacle, and therefore no avoidance motion is implemented; and (iii) when the robot is within the influence range of the obstacle and $\rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t))$ approaches ρ_m, the repulsive potential approaches infinity and as the projection of relative velocity of the robot $v_{RO}(t)$ increases, the repulsive potential increases. In addition, even if the distance between the robot and the obstacle does not approach zero, the repulsive potential approaches infinity if the projection of relative velocity $v_{RO}(t)$ is large enough.

Similar to the definition of the new attractive force, the corresponding new repulsive force is defined as the negative gradient of the repulsive potential with respect to both position and velocity

$$
\begin{aligned} \mathbf{F}_{rep}(\mathbf{p}, \mathbf{v}) &= -\nabla U_{rep}(\mathbf{p}, \mathbf{v}) \\ &= -\nabla_p U_{rep}(\mathbf{p}, \mathbf{v}) - \nabla_v U_{rep}(\mathbf{p}, \mathbf{v}) \end{aligned} \tag{14}
$$

To derive the virtual repulsive force, we need to derive the gradient of $v_{RO}(t)$ with respect to position and velocity, respectively. The relative velocity of the robot with respect to the obstacle on the unit vector from the robot to the obstacle, $v_{RO}(t)$, can be written as

$$
\begin{aligned} v_{RO}(t) &= (\mathbf{v}(t) - \mathbf{v}_{obs}(t))^T \mathbf{n}_{RO} \\ &= (\mathbf{v}(t) - \mathbf{v}_{obs}(t))^T \frac{(\mathbf{p}_{obs}(t) - \mathbf{p}(t))}{\|\mathbf{p}_{obs}(t) - \mathbf{p}(t)\|} \end{aligned} \tag{15}
$$

The gradient of $v_{RO}(t)$ with respect to velocity is

$$
\nabla_v v_{RO}(t) = \mathbf{n}_{RO} = -\mathbf{n}_{OR} \tag{16}
$$

The gradient of $v_{RO}(t)$ with respect to position is

$$
\nabla_p v_{RO}(t) = \frac{1}{\|\mathbf{p}(t) - \mathbf{p}_{obs}(t)\|} v_{RO\perp} \mathbf{n}_{OR\perp} \tag{17}
$$

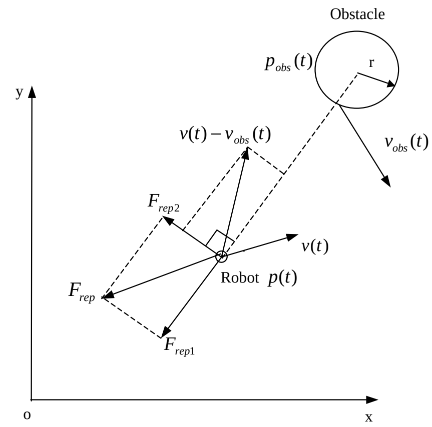

Figure 3: New repulsive force in 2D space.

where $v_{RO\perp} = \sqrt{\|\mathbf{v}(t) - \mathbf{v}_{obs}(t)\|^2 - v_{RO}^2(t)}$ is the magnitude of the component of the relative velocity of the robot with respect to the obstacle which is perpendicular to the line passing through the robot and the obstacle; and $\mathbf{n}_{OR\perp}$ is the corresponding unit vector. Figure 2 clearly shows the relationship between these vectors.

The virtual repulsive force generated by the obstacle is then given by

$$
\begin{aligned} &\mathbf{F}_{rep}(\mathbf{p}, \mathbf{v}) \\ &= \begin{cases} 0, \\ \quad \text{if } \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m \geq \rho_0 \text{ or } v_{RO}(t) \leq 0 \\ \mathbf{F}_{rep1} + \mathbf{F}_{rep2}, \\ \quad \text{if } 0 < \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) - \rho_m < \rho_0 \\ \qquad \text{and } v_{RO}(t) > 0 \\ \text{not defined}, \\ \quad \text{if } v_{RO}(t) > 0 \text{ and } \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t)) < \rho_m \end{cases} \end{aligned} \tag{18}
$$

where

$$
\mathbf{F}_{rep1} = \eta U_{rep}^2 \left(1 + \frac{v_{RO}(t)}{a_{max}} \right) \mathbf{n}_{OR} \tag{19}
$$

and

$$
\mathbf{F}_{rep2} = \frac{\eta U_{rep}^2 v_{RO}(t) v_{RO\perp}}{a_{max} \rho_s(\mathbf{p}(t), \mathbf{p}_{obs}(t))} \mathbf{n}_{OR\perp} \tag{20}
$$

with U_{rep} being the abbreviation of $U_{rep}(\mathbf{p}, \mathbf{v})$.

The relationship of the repulsive force components in a 2D space is shown in Figure 4 where it can be seen that one repulsive force component $\mathbf{F}_{rep1}$ decreases v_{RO}, the projection of the relative velocity of the robot toward the obstacle, which will keep the robot from colliding with the obstacle, and the other component $\mathbf{F}_{rep2}$ increases $v_{RO\perp}$, the relative velocity of the robot parallel to the obstacle, which drives the robot bypassing/detouring the obstacle.

5 MOTION PLANNING BASED ON NEW POTENTIALS

At time t, the sensors on board the robot obtain the positions and velocities of the target and obstacles. The total virtual force is calculated by

$$\mathbf{F}_{total}(t) = \mathbf{F}_{att}(t) + \mathbf{F}_{rep}(t) \tag{21}$$

where $\mathbf{F}_{att}(t)$ and $\mathbf{F}_{rep}(t)$ are given by equations (7) and (18).

Usually, the robot motion is subjected to its physical limitation and the magnitude of its acceleration is upper bounded. Denote the maximum acceleration of the robot as a_{max}. According to Newton's law, the acceleration applied to the robot is given by

$$\mathbf{a}(t) = \begin{cases} \frac{\mathbf{F}_{total}}{m_{rob}}, & \text{if } \left|\frac{\mathbf{F}_{total}}{m_{rob}}\right| \leq a_{max} \\ a_{max}\frac{\mathbf{F}_{total}}{|\mathbf{F}_{total}|}, & otherwise \end{cases} \tag{22}$$

Assume that the initial velocity and position of the robot are known accurately, the velocity and position of the robot at time t are given by

$$\mathbf{v}(t) = \mathbf{v}(t_0) + \int_{t_0}^{t} \mathbf{a}(\tau)d\tau \tag{23}$$

and

$$\mathbf{p}(t) = \mathbf{p}(t_0) + \int_{t_0}^{t} \mathbf{v}(\tau)d\tau \tag{24}$$

where t_0 is the initial time; $\mathbf{v}(t_0)$ and $\mathbf{p}(t_0)$ are the the initial velocity and position of the robot, respectively.

6 SIMULATION STUDIES

In this section, we present some simulation studies on the new motion planning scheme in a 2-dimensional space. In the operational space, there is one moving target and two moving obstacles. The target is moving from point $[10, 10]^T$ at constant velocity, $\mathbf{v}_{tar} = [0.1, -0.05]^T$. The two obstacles are also moving at constant velocity. One moves from point $[5, 0]^T$ at velocity $[0, 0.1]^T$, the other moves from point $[20, 10]^T$ at velocity $[-0.05, -0.065]^T$. The robot moves from point $[1, 1]^T$ with initial velocity $[0.1, 0]^T$. The mass of the robot is $m_{rob} = 1$Kg. The parameters of the attractive potential function are chosen as $m = n = 2$, $\alpha_p = 0.0008$ and $\alpha_v = 0.04$; and the parameters of the repulsive potential function are chosen as $\eta = 0.2$ and $\rho_0 = 2m$. In the simulation, the sampling interval is $T = 0.1s$.

Figures 4 - 7 record the paths of the robot, obstacles and target at time shots t =26s, 34s, 84s, and 100s, respectively. In Figures 4 and 5, it is shown that when the robot approaches close to Obstacle 1, it speeds up and detours the obstacle in front of it. In Figures 6 and 7, it is shown that the robot slows down near Obstacle 2 and let the obstacle passes first. Figure 8 shows the overall simulation result, where the robot tracks the target successfully.

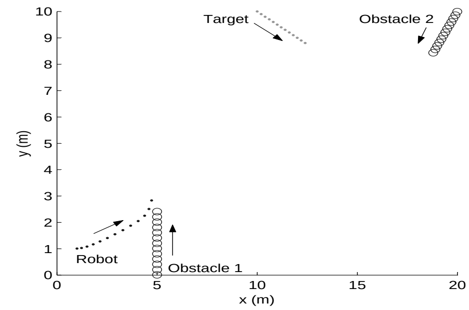

Figure 4: Paths of robot, obstacles and target at time $t = 26s$.

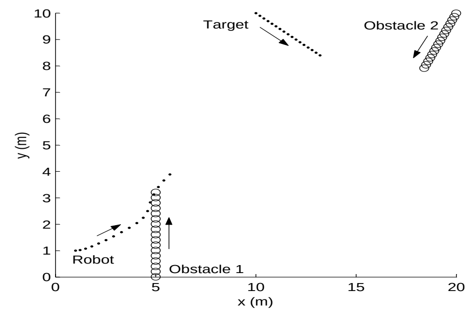

Figure 5: Paths of robot, obstacles and target at time $t = 34s$.

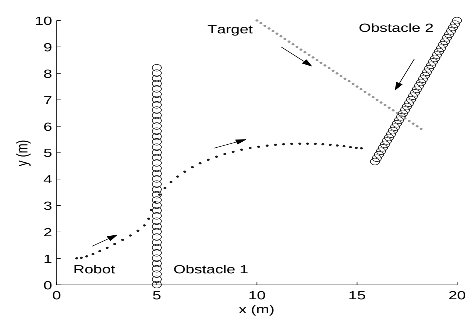

Figure 6: Paths of robot, obstacles and target at time $t = 84s$.

7 CONCLUSION

In this paper, a new potential field method has been proposed for mobile robot motion planning in a dynamic environment where the target and the obstacles are moving. The new potential functions take into account the relative velocities of the target with respect to the robot and the relative velocities of the robot with respect to the obstacles. The virtual forces are defined by the negative

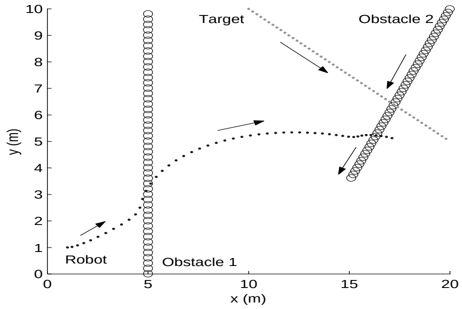

Figure 7: Paths of robot, obstacles and target at time $t = 100s$.

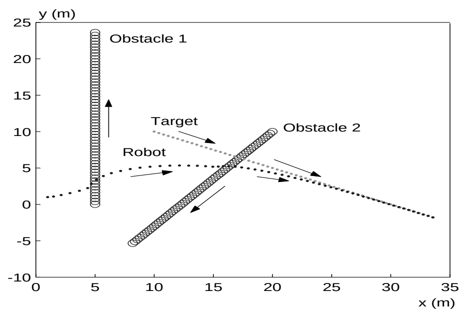

Figure 8: Mobile robot moves in a dynamic environment.

gradients of the potential functions with respect to both position and velocity. The combination of the new potential function and the new definition of virtual force enables target tracking and moving obstacle avoidance possible. Computer simulation results demonstrate the effectiveness of the mobile robot motion planning scheme based on the new potential field method.

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