Robust Predictive Regulator Design for Unstable Plants with Input Saturation

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ABSTRACT

This paper presents two techniques for designing robust predictive regulators for unstable plants subject to unstructured modeling uncertainties. One technique is based on H_∞ theory, and is applicable to systems where the inputs and outputs are unconstrained. It is possible to include an integrator in the robustified regulators, hence guaranteeing the steady-state rejection of asymptotically constant exogenous disturbances. The control design problem reduces to a Nehari extension problem, and is solved using a rigorous and systematic approach. The second technique is based on l_1/H_∞ theory, and is developed for systems where the input is subject to saturation constraints. An H_∞ constraint in the form of a Nehari extension problem is used to ensure the robust stability of the regulation loop, and an l_1 optimization objective is used to address the input constraints. The l_1/H_∞ problem is solved via an iterative algorithm with guaranteed convergence properties. A numerical example demonstrates relevant features of the robust-design procedure and illustrates the controller performance.

1. INTRODUCTION

Predictive control techniques have gained remarkable acceptance in industry. Currently there are hundreds of predictive controllers deployed in oil refineries and petrochemical plants alone (Seborg, 1994). The underlying principle of predictive control is to determine a number of future control increments that optimize an open-loop performance functional over a finite prediction horizon. Although more than one control increment is usually calculated, only the first one is implemented. At the next sampling instant, the output measurement is used to update the prediction equations and the same procedure is repeated. Since the control principle involves sequential optimizations carried out at each sampling interval, predictive controllers are particularly adept at handling input and output constraints. Currently there is an increasingly visible interest to robustify predictive controllers through design techniques that guarantee the stability and/or adequate performance of the closed-loop system when the plant model is uncertain. The robustness work reported in the literature can be conveniently classified in two groups depending on whether the predictive-control design takes into consideration process constraints.

Robinson and Clarke (1991) investigate the effect of a polynomial prefilter T on the robustness of unconstrained Generalized Predictive Control (GPC). Only two specific control designs are analyzed, namely, a dead-beat and a mean-level controller, which can be interpreted as special cases of GPC with specific choices of tuning parameters. Although the analysis is insightful, it is not strictly applicable to general choices of tuning parameters. Kouvaritakis *et al.* (1992) introduce a more rigorous approach to the robustification of unconstrained predictive controllers of the GPC type. The technique consists of reparametrizing the controller in terms of a Q-parameter (Francis, 1987), and then searching for a parameter that robustly stabilizes the system with respect to unstructured model perturbations. In order to simplify the design, the authors restrict the Q- parameter to be a polynomial or a fixed-order transfer function, and then find the undetermined coefficients using least-squares methods. Also the predictive control design is based on a pre-stabilized plant. In a very recent paper, Yoon and Clarke (1995) elaborate on the robust implementation of unconstrained GPC and give a comparison of designs obtained through the filter T and an approximated Q-parameter as in Kouvaritakis *et al.*.

The robustification of constrained predictive controllers is a yet more challenging problem, and the results are even more scarce. Although there exists a rich theory for the robust control of linear systems, little is known for the robust control of systems with constraints. Zafiriou (1990) includes process constraints in a Dynamic Matrix Control scheme, and investigate the robustness of the control system using a contraction-mapping technique. The approach leads to robust stability conditions which are rather conservative and may not be practical for controller design because of computational complexity problems. The analysis is not applicable to unstable plants because the design is based on FIR models. Genceli and Nikolaou (1993)and Zheng and Morari (1993) derive conditions for the robust stability of a predictive control system that uses a linear objective functional rather than the more conventional quadratic objective. Both of these approaches are also based on an FIR model of the plant, where the uncertainty in the impulse-response model is expressed in terms of an interval polynomial.

This paper presents procedures for robustifying constrained and unconstrained predictive regulators of the GPC type, with a focus on the case where the plant model is unstable and is affected by unstructured uncertainty. The proposed approach uses the Youla technique for parametrizing an unconstrained nominal predictive regulator which is designed using well-known strategies. In this sense, the design resembles that of Kouvaritakis *et al.* (1992), except that no pre-stabilizing compensator is used. The parametrized form of the controller is then used to synthesize two robust predictive controllers: (i) an unconstrained predictive regulator designed using H_∞ theory, and (ii) a constrained predictive regulator synthesized using emerging results from l_1/H_∞ theory. In both cases, the control-design techniques follow a systematic procedure, and do not require *ad hoc* approximations.

The manuscript is organized as follows. Section 2 presents a brief review of the method used for designing nominal predictive regulators, and Section 3 discusses the details of the technique used for parametrizing the nominal regulator. Section 4 describes the proposed algorithms for synthesizing robust unconstrained (Section 4.1) and constrained (Section 4.2) predictive regulators. An example is given in Section 5 followed by concluding remarks in Section 6.

2. NOMINAL PREDICTIVE CONTROL DESIGN

Typically, predictive controllers are deployed by executing at every sampling instant an algorithm that solves a quadratic optimization problem. It is desirable to represent the algorithmic controller in terms of transfer functions, allowing the utilization of classical z-domain tools to analyze stability and performance. Consider the nominal process model

$$y(z) = g_0(z)u(z) + d(z) \tag{2.1}$$

where

$$g_0(z) = \frac{B(z)}{A(z)} \tag{2.2}$$

and $y(z)$, $u(z)$, and $d(z)$ are the process output, input and disturbance, respectively, and $A(z)$ and $B(z)$ are coprime polynomials

$$A(z) = z^n + a_{n-1}z^{n-1} + \dots + a_0 \tag{2.3}$$

$$B(z) = b_m z^m + b_{m-1}z^{m-1} + \dots + b_0 \tag{2.4}$$

of order n and m, respectively, where $n>m$. The nominal plant $g_0(z)$ is assumed to be unstable. The nominal predictive control design is based on minimizing the quadratic predictive-control functional

$$J(t) = \sum_{i=1}^{N_y}\left[r(t+i) - y(t+i|t)\right]^2 + \lambda\sum_{i=1}^{N_u}\left[\Delta u(t+i)\right]^2 \tag{2.5}$$

where $\{r(t+i)\}$ is the sequence of future set point values, $\{y(t+i|t)\}$ is the sequence of predicted future values of the output, $\{\Delta u(t+i)\}$ is the sequence of future control increments, λ is the move-suppression parameter, and parameters N_y and N_u are the prediction and control horizons, respectively. The terms in the first summation penalize future predicted errors, and the terms in the second summation penalize excessive control energy. It can be shown that the predictive-control law that minimizes (2.5) can be expressed in terms of three polynomials $R(z)$, $S(z)$, and $T(z)$, arranged in the configuration shown in Figure 1. Following the development in (Crisalle *et al.*, 1989) it is possible to write the resulting control law in terms of transfer-function operators in the form

$$\frac{R(z)}{z^n}u(z) = T(z)r(z) - \frac{S(z)}{z^n}y(z) \tag{2.6}$$

which includes the polynomial operators

$$R(z) = z^n + r_{n-1}z^{n-1} + \dots + r_0 \tag{2.7}$$

$$S(z) = s_n z^n + s_{n-1}z^{n-1} + \dots + s_0 \tag{2.8}$$

$$T(z) = t_{N_y} z^{N_y} + t_{N_y-1} z^{N_y-1} + \dots + t_1 z \tag{2.9}$$

where

$$R(1) = 0 \tag{2.10}$$

$$T(1) = S(1) \tag{2.11}$$

and where the coefficients of polynomials $R(z)$, $S(z)$, and the set-point advancement polynomial $T(z)$ are functions of the tuning parameters N_y, N_u, and λ, and of the model polynomials $A(z)$ and $B(z)$. Note that (2.10) implies that the predictive control law (2.6) includes an integrator. A block-diagram representation of the predictive control structure is shown in Figure 1a. The predictive controller (2.6) is in regulation mode when $r(z) = 0$.

It is useful to remark that the nominal model (2.1) and the functional (2.5) are simpler versions of more elaborate formulations that improve the design performance at the expense of added complexity. Typical enhancements are the inclusion of a lower prediction-horizon parameter (Clarke *et al.*, 1987), the inclusion of a weighted end-point term in (2.5) to guarantee stability for arbitrary parameter choices (Demircioglu and Clarke, 1993), and the use of an auxiliary (filtered) set point.

When the predictive controller is used as a regulator the set-point $r(t)$ is identically zero at all times, and the closed-loop dynamics are fully characterized by the equations

$$[\mathrm{A}(z)\mathrm{R}(z)+\mathrm{B}(z)\mathrm{S}(z)]\,y(z)=\mathrm{A}(z)\mathrm{R}(z)\,d(z) \quad (2.12)$$

$$[\mathrm{A}(z)\mathrm{R}(z)+\mathrm{B}(z)\mathrm{S}(z)]\,u(z)=\mathrm{A}(z)\mathrm{S}(z)\,d(z) \quad (2.13)$$

The stability of the closed-loop for a given nominal predictive controller can thus be easily checked by calculating the roots of the characteristic polynomial $\mathrm{A}(z)\mathrm{R}(z)+\mathrm{B}(z)\mathrm{S}(z)$. Furthermore, due to the presence of integral action in the controller, perfect steady-state disturbance rejection is guaranteed for all disturbance signals that reach a steady-state.

3. REGULATOR PARAMETRIZATION

In this section the nominal predictive controller (2.6) is parametrized in terms of a transfer function $Q(z)$ according to Wiener-Hopf design (Youla *et al.*, 1976). Consider a nominal predictive controller (2.6) that stabilizes the closed loop system (2.12)-(2.13). Hence, the nominal closed-loop characteristic polynomial

$$\mathrm{A}^*(z)=\mathrm{A}(z)\mathrm{R}(z)+\mathrm{B}(z)\mathrm{S}(z) \quad (3.1)$$

of degree $2n$ is Schur. In order to parametrize the controller, consider a coprime fractional representation of the nominal plant model (2.2) of the form

$$g_0(z)=\frac{N(z)}{M(z)} \quad (3.2)$$

where $N(z)$ and $M(z)$ are proper and stable transfer functions that satisfy the Diophantine equation

$$N(z)\,X(z)+M(z)\,Y(z)=1 \quad (3.3)$$

for some pair of stable and proper transfer functions $X(z)$ and $Y(z)$. (Note the use of italicized capital letters for transfer functions, and non-italicized capitals for polynomials.). A suitable $(M(z), N(z))$ pair can be readily derived from the nominal characteristic polynomial (3.1). Factoring the closed-loop characteristic polynomial in the form $\mathrm{A}^*(z)=\mathrm{A}_1(z)\mathrm{A}_2(z)$, where both $\mathrm{A}_1(z)$ and $\mathrm{A}_2(z)$ are of degree n, and then dividing both sides of (3.1) by the product $\mathrm{A}_1(z)\mathrm{A}_2(z)$ yields

$$\frac{\mathrm{A}(z)\,\mathrm{R}(z)}{\mathrm{A}_1(z)\mathrm{A}_2(z)}+\frac{\mathrm{B}(z)\mathrm{S}(z)}{\mathrm{A}_1(z)\mathrm{A}_2(z)}=1 \quad (3.4)$$

Stable and proper factorizations that satisfy (3.3) are finally obtained by defining

$$M(z):=\frac{\mathrm{A}(z)}{\mathrm{A}_1(z)},\quad N(z):=\frac{\mathrm{B}(z)}{\mathrm{A}_1(z)} \quad (3.5)$$

$$X(z):=\frac{\mathrm{S}(z)}{\mathrm{A}_2(z)},\quad Y(z):=\frac{\mathrm{R}(z)}{\mathrm{A}_2(z)} \quad (3.6)$$

where $X(z)$ and $Y(z)$ are clearly stable and proper rational transfer functions. Setting $r(z)$=0 in (2.6)to obtain the regulator form, and using (3.5)-(3.6) leads to the control law

$$Y(z)\;u(z)=X(z)\;y(z) \quad (3.7)$$

The set of all solutions to (3.3) can be written in terms of the transfer functions (3.5)-(3.6) and a proper and stable transfer-function $Q(z)$ through the well-known relations (Youla *et al.*, 1976)

$$X'(z)=X(z)+M(z)Q(z) \quad (3.8)$$

$$Y'(z)=Y(z)-N(z)Q(z) \quad (3.9)$$

Therefore, the set of all stabilizing predictive regulators with the structure (3.7) is obtained in the parametrized form

$$[Y(z)-N(z)Q(z)]\;u(z)=[X(z)+M(z)Q(z)]\;y(z) \quad (3.10)$$

to yield the control scheme shown in Figure 1b. Clearly, setting $Q(z)$=0 reduces the parametrized predictive regulator (3.10) to the nominal predictive regulator (3.7).

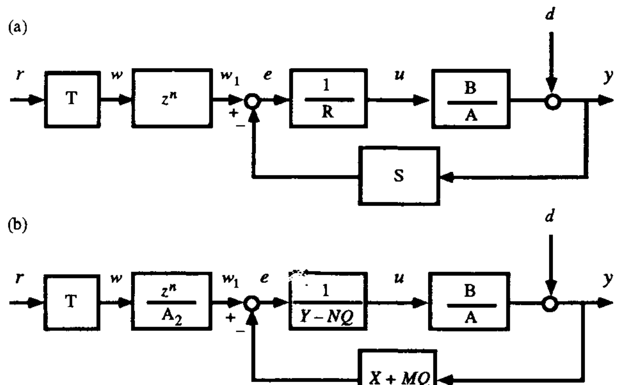

Figure 1. (a) Structure of a nominal predictive controller. (b) Structure of the parametrized predictive controller featuring the Youla parameter $Q(z)$.

4. ROBUST DESIGN

When the nominal model (3.2) is not exact due to the presence of modeling errors, the plant transfer function $g(z)$ may be written in the form

$$g(z)=g_0(z)+\Delta(z) \quad (4.1)$$

where $g_0(z)$ is the nominal plant model, and $\Delta(z)$ is an unstructured perturbation. Without loss of generality, we treat the case of additive perturbations, described by the magnitude bound

$$|\Delta(e^{i\omega})|\le|\mathrm{W}(e^{i\omega})|\quad\forall\,\omega \quad (4.2)$$

where the uncertainty weight W(z) is a stable and proper transfer function. Following the standard approach the perturbation $\Delta(z)$ is also assumed to be stable (Francis, 1987). The case of multiplicative perturbations, as well as other typical unstructured uncertainty representations can be treated in an analogous fashion.

The objective is to design a robust predictive controller that stabilizes the closed loop for all the members of the uncertain family of plants (4.1)-(4.2). The stability robustness of the closed loop shown in Figure 2, which includes the parametrized controller (3.10) and the uncertain family of plants (4.1)-(4.2), can be analyzed using the $\mathcal{H}_\infty$-theory results summarized in Theorem 4.1 below.

Theorem 4.1. A necessary and sufficient condition for the robust stability of the closed-loop system of Figure 2 is the inequality condition (Francis, 1987)

$$\| W(z)C(z)\mathcal{S}(z)\|_\infty < 1 \tag{4.3}$$

where

$$C(z) := \frac{X(z)+M(z)Q(z)}{Y(z)-N(z)Q(z)} \tag{4.4}$$

and

$$\mathcal{S}(z) := M(z)\,[Y(z)-N(z)Q(z)] \tag{4.5}$$

This paper deals with two variants of the robust predictive regulator design. First is treated the case where the manipulated variable is unconstrained. The second problem treats the case where the robust regulator is designed taking into account input constraints.

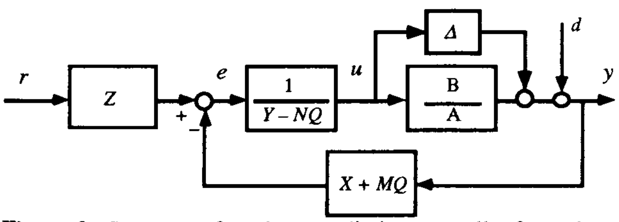

Figure 2. Structure of a robust predictive controller for a plant with an additive uncertainty. An exogenous disturbance $d(t)$ affects the plant output.

4.1 Unconstrained Regulator Design

We propose a systematic procedure for solving the robust synthesis problem without resorting to approximations for the Q-parameter. A particular challenge to the design problem posed is the objective of including an integrator in the robustified controller in order to guarantee effective disturbance rejection. In the following subsection we develop a design technique for the case of an unstable plant with no poles on the unit circle. The next subsection treats the case where the plant is unstable but has poles on the unit circle, as in the case of an integrator.

4.1.1 *Unstable Plant with no Poles on the Unit Circle.*

Consider the robust predictive controller design problem for the case where the nominal plant model $g_o(z)$ is unstable but has no poles on the unit circle. The synthesis problem is attacked by recasting the robust stability condition (4.3) in the equivalent model-matching form

$$\| T_1(z)+T_2(z)Q(z)\|_\infty < 1 \tag{4.6}$$

where

$$T_1(z) = W(z)X(z)M(z) \tag{4.7}$$

$$T_2(z) = W(z)M^2(z) \tag{4.8}$$

Inequality (4.6), which is affine in the unknown parameter $Q(z)$, is obtained by substituting equations (4.4) and (4.5) into inequality (4.3). The model-matching problem is commonly approached in the context of $\mathcal{H}_\infty$ control theory using the γ-iteration process, where (4.6) is substituted by the alternative inequality

$$\| T_1(z)+T_2(z)Q(z)\|_\infty \le \gamma \tag{4.9}$$

where γ is a positive scalar parameter selected by the designer. A robust design is obtained if a Youla parameter $Q(z)$ is found for a specified $\gamma < 1$. The key is then to be able to synthesize a Youla parameter with a reliable algorithm.

We make use of a z-domain technique proposed by Rotstein and Sideris for solving the model-matching problem (Rotstein and Sideris 1992; Rotstein, 1993). The algorithm solves the problem of approximating a stable transfer function $R(z)$ with an antistable (all poles outside the unit circle) transfer function $Q_R^{\sim}(z)$, where the tilde superscript denotes the conjugate operation $Q_R^{\sim}(z) := Q_R(1/z)$. This problem, also known as the Nehari extension problem (Maciejowski, 1989), calls for finding an antistable function $Q_R^{\sim}(z)$ such that

$$\| R(z)+Q_R^{\sim}(z)\|_\infty \le \gamma \tag{4.10}$$

The original condition (4.9) can be cast to the form (4.10) through a series of norm-preserving operations, as follows. Factor $T_2(z)$ as $T_2(z) = T_{ap}(z)T_{mp}(z)$ where $T_{ap}(z)$ is an all-pass function and $T_{mp}(z)$ is a stable minimum-phase function, then find $T_{ap}^{\sim}(z) = T_{ap}(1/z)$ and carry out the decomposition

$$T_{ap}^{\sim}(z)T_1(z) = R_a(z)+R_s(z) \tag{4.11}$$

where $R_a(z)$ and $R_s(z)$ are an antistable and a stable transfer function, respectively. Using the property that $\| T_{ap}^{\sim}(z)G(z)\|_\infty = \| G(z)\|_\infty$ it is possible to write the equalities

$$\begin{aligned}\| T_1(z)+T_2(z)Q(z)\|_\infty &= \| T_{ap}^{\sim}(z)T_1(z)+T_{mp}(z)Q(z)\|_\infty \\ &= \| R_a(z)+R_s(z)+T_{mp}(z)Q(z)\|_\infty\end{aligned} \tag{4.12}$$

Defining $R^{\sim}(z):=R_a(z)$ and $Q_R(z):=R_s(z)+T_{mp}(z)Q(z)$, and since $\|G^{\sim}(z)\|_\infty = \|G(z)\|_\infty$, (4.12) reduces to the desired form

$$\|T_1(z)+T_2(z)Q(z)\|_\infty = \|R(z)+Q_R^{\sim}(z)\|_\infty \tag{4.13}$$

After $Q_R(z)$ is found, the solution to the original model-matching problem (4.9) can simply be recovered as

$$Q(z) = T_{mp}^{-1}(z)\left(R_s(z)+Q_R(z)\right) \tag{4.14}$$

Finally the robust predictive regulator design is obtained by substituting the Youla parameter (4.14) and the factorizations (3.5)-(3.6)in the scheme (3.10).

4.1.2 *Unstable Plant with Poles on the Unit Circle*

When the nominal plant model $g_0(z)$ has poles on the unit circle, the standard $\mathcal{H}_\infty$ control theory is no longer applicable. In addition, the factorization (4.11) is no longer possible because no minimum-phase stable transfer function can possibly satisfy the equality. This difficulty is circumvented by introducing a change of variables that maps unit-circle poles to a circle of larger radius. Let $z=\delta\hat{z}$ where $\delta<1$ is a scalar, and define the operators

$$T_1'(\hat{z}) = T_1(\delta\hat{z}) \tag{4.15}$$

$$T_2'(\hat{z}) = T_2(\delta\hat{z}) \tag{4.16}$$

Then the design problem (4.9) can be posed in terms of the transformed variable $\hat{z}$ in the form

$$\|T_1'(\hat{z})+T_2'(\hat{z})Q'(\hat{z})\|_\infty < \gamma \tag{4.17}$$

and can be solved for $Q'(\hat{z})$using the base-case algorithm as described in Section 4.1.1. The z-domain Youla parameter is simply recovered by transforming the result back to the original space, *i.e.*,

$$Q(z) = Q'(\hat{z}/\delta) \tag{4.18}$$

The final robust predictive controller design for this case is obtained by substituting the Youla parameter (4.18) and the factorizations (3.5)-(3.6) in the structure (3.10).

Using the Maximum Modulus Theorem, it follows that the transformed design problem (4.17) is related to the original problem (4.6) through the inequality

$$\|T_1'(\hat{z})+T_2'(\hat{z})Q'(\hat{z})\|_\infty \geq \|T_1(z)+T_2(z)Q(z)\|_\infty \tag{4.19}$$

If no $Q'(\hat{z})$can be found that satisfies (4.17), then a larger value for δ should be adopted and the design is repeated.

As previously discussed, the closed-loop dynamics of the nominal system described by (2.12) guarantees that asymptotically constant disturbances will be rejected because R(1)=0. However, this is not necessarily the case for the robust design. From Figure 2 it follows that the transfer function for the robustified predictive controller is

$$\frac{y(z)}{d(z)} = \frac{A(z)R(z)}{A(z)R(z)+B(z)S(z)} - \frac{A(z)B(z)}{A_1^2(z)}Q(z) \tag{4.20}$$

The synthesis procedure may not necessarily yield a Youla parameter satisfying $Q(1)$=0. Then the robustified predictive regulator may display unacceptable performance at the steady state unless the nominal plant has an integrator (*i.e.*, A(1)=0) or is a self-regulating process (B(1)=0). Clearly, the robust predictive controller will attain perfect steady-state disturbance rejection for all the plants belonging to the uncertain family (4.1) only if the Youla parameter has a zero gain, *i.e.*, $Q(1)$=0. This gain constraint can be introduced in the robust predictive control design through a simple modification of the factorizations (3.5)-(3.6). First the integrator is extracted from the nominal predictive controller by writing $R(z)=(z-1)R'(z)$, and then (3.4) is rewritten in the form

$$\frac{A(z)(z-1)R'(z)}{A_1(z)A_2(z)} + \frac{B(z)S(z)}{A_1(z)A_2(z)} = 1 \tag{4.21}$$

Introducing the modified coprime factorization

$$\hat{M}(z) := \frac{(z-1)A(z)}{zA_1(z)}, \quad \hat{N}(z) := N(z) \tag{4.22}$$

and

$$\hat{X}(z) := X(z), \quad \hat{Y}(z) := \frac{zR'(z)}{A_2(z)} \tag{4.23}$$

leads to operators that satisfy the Diophantine equation (3.3), and the design can proceed as before, solving the problem (4.10) using the modified factorizations (4.22)-(4.23). Note that the definition of $\hat{M}(z)$ in (4.22) is equivalent to designing a controller for a nominal plant which has been augmented by an integrator. Hence this plant is treated using the formulas (4.15)-(4.18). After a solution to (4.10) is found, the Youla parameter $Q(z)$ used in the parametrized predictive control structure of Figure 2 is constructed by re-associating the augmented-plant integrator with the controller to obtain

$$Q(z) = \frac{(z-1)}{z}\hat{Q}(z) \tag{4.24}$$

The resulting controller includes an integrator since (4.24) satisfies the zero-gain condition $Q(1)$=0.

4.2 Constrained Regulator Design

The objective in the present section is to design a predictive regulator with disturbance rejection properties, and such that it recognizes that the input must satisfy a constraint of the form $\|u\|_\infty \le \beta$. The exogenous signal d represents a persistent but bounded disturbance such that $\|d\|_\infty \le 1$, where without loss of generality the unity bound represents the result of a signal normalization. Let $T_{ud}(z)$ represent the closed-loop transfer function between the manipulated variable and the disturbance, *i.e.* $u(z) = T_{ud}(z)d(z)$. Carrying out elementary block diagram algebra in Figure 2, $T_{ud}(z)$ is shown to be affine with respect to the transfer function $Q(z)$ and is given by

$$T_{ud}(z) = T_{ud,1}(z) + T_{ud,2}(z)Q(z) \tag{4.25}$$

where

$$T_{ud,1}(z) = -M(z)X(z)$$
$$T_{ud,2}(z) = -M(z)X(z)M(z)$$

Notice that both $u(t)$ and $d(t)$ are l_∞-signals because they are bounded. Using the fact that the operator 1-norm $\|T_{ud}\|_1 = \sum_{i=0}^{\infty}|t_i|$ (where t_i denotes the i-th impulse response coefficient) is related to the infinity norms of the signals $u(t)$ and $d(t)$ through the relationship (*c.f.*, Dahleh and Diaz-Bobillo, 1995)

$$\|T_{ud}\|_1 := \sup_{\|d\|_\infty \le 1} \|u\|_\infty \tag{4.26}$$

it follows that constraints on the input can be effectively incorporated into the regulator design strategy by minimizing (4.26). Hence, the problem of satisfying saturation constraints on the input, guaranteeing simultaneously robust stability of the closed-loop can be precisely stated as follows:

$$\mu^0 = \inf_{Q \in \mathcal{RH}_\infty} \|T_{ud}\|_1 \tag{4.27}$$

subject to

$$\left\| R(z) + Q_R^-(z) \right\|_\infty < 1 \tag{4.28}$$

The l_1/H_∞ problem (4.27)-(4.28) calls for finding an optimal Q-parameter that minimizes the 1-norm of T_{ud} and simultaneously satisfies the robust-stability condition (4.28). If the optimal solution satisfies $\mu^0 \le \beta$, then the predictive regulator satisfies the constraint specifications on the input. On the other hand, if $\mu^0 > l$ there is no robustly stabilizing regulator that can satisfy the input constraints.

Remark 1. Notice that (4.27) is in a semi-infinite optimization form. It is possible to exploit the special structure of the problem to find a global suboptimal solution by considering a modified truncated problem, as shown in (Sznaier, 1994). First (4.27) is transformed to a finite-dimensional optimization problem by keeping only the first N impulse-response coefficients of $T_{ud}(z)$. The resulting problem is convex and simple to solve. The solution $Q_F(z) = \sum_{i=0}^{N-1} q_i z^{-i}$ to the truncated problem is then used to solve a Nehari extension problem of the form $\|R^-(z) + Q_F(z) + z^{-N}Q_s(z)\|_\infty < 1$, where $Q_S(z)$ is now the unknown parameter. The l_1/H_∞ problem (4.22)-(4.23) has thus been decoupled into a finite optimization and an unconstrained Nehari problem, with a suboptimal solution $Q_R(z)$ given by $Q_R(z) = Q_F(z) + z^{-N}Q_S(z)$.

Remark 2. Solving the finite-dimensional problem may not guarantee satisfaction of the input constraints at times greater than the horizon N. The following simple modification yields a good overall behavior. Introduce a change of variables that places the poles of the closed-loop system inside a disk of radius smaller than one, thus forcing a decay ratio on the time response. For a given $\delta < 1$, define $H_{\infty,\delta} := \{ G(z) \in \mathcal{H}_\infty$: $G(z)$ analytic in $|z| \ge \delta$ $\}$. Then a Youla parameter $Q(z)$ is sought such that $Q(z) \in \mathcal{H}_{\infty,\delta}$. This can be interpreted as adding the requirement that the closed-loop system poles must lie inside a disc of radius $\delta < 1$. A parametrization of all possible closed-loop transfer functions that satisfy this additional requirement can be obtained by simply changing the stability region using the transformation $z = \delta\hat{z}$, where $0<\delta<1$ is a real scalar. Details can be found in (Sznaier, 1994).

The modified l_1/H_∞ problem can then be decoupled into a finite-dimensional convex optimization and an unconstrained Nehari extension problem, as shown in Remark 1, providing a suboptimal solution to (4.27)-(4.28). Once (4.27)-(4.28) is solved for a suboptimal parameter $Q(z)$ the final regulator design is obtained by substituting the Q-parameter and the factorizations (3.5)-(3.6) in the predictive regulator scheme (3.10).

5. EXAMPLE

Consider the unstable second-order nominal plant model

$$g_0(z) = \frac{z + 0.2}{z^2 - 0.6z + 1.12}$$

and the uncertainty weight

$$W(z) = \frac{0.63z + 0.6174}{z + 0.5}$$

Three regulators are designed: (i) a nominal predictive controller (NPC), (ii) a robust predictive controller (RPC), and (iii) a robust predictive controller with integral action (RPCI). The nominal predictive regulator is of form (2.6), and is realized using the design parameters N_y=4, N_u=2, and λ=0, to arrive at the polynomials

$$R(z) = z^2 - 0.8039z - 0.1961$$

$$S(z) = 0.8639z^2 - 1.579z + 1.0984$$

$$T(z) = 0.2914z^4 - 0.0156z^3 + 0.366z^2 + 0.3243z$$

which leads to a nominal predictive controller that stabilizes the closed loop when the uncertainty is neglected. However, the NPC regulator is not robustly stabilizing because it violates the robust stability condition (4.3), *i.e.* $\| W(z)C(z)S_I(z) \|_\infty = 2.9 > 1$ where $C(z)$ and $S_I(z)$ are calculated using $Q(z)=0$ in (4.4) and (4.5).

The RPC design is of the form (3.10). Since the unstable nominal plant has no poles on the unit circle, the design proceeds as discussed in Section 4.1.1. The transfer functions $T_1(z)$ and $T_2(z)$ are formed as prescribed in (4.7) and (4.8). To solve the Nehari extension problem we use $\gamma = 0.99$, The algorithm described in the main section leads to a Youla parameter $Q(z)=N_Q(z)/D_Q(z)$ of order 8. The RPC transfer functions $Y(z)-N(z)Q(z)$, and $X(z)+M(z)Q(z)$, are of order 9 in their minimal forms. The regulator is robustly stabilizing because $\| W(z)C(z)S_I(z) \|_\infty = 0.35 < 1$.

Finally, the design of the RPCI is carried out as indicated in Section 4.1.1, using again the specification $\gamma = 0.99$ and $\delta = 0.9$. The procedure leads to a Youla parameter $Q(z)=(z-1)N_Q(z)/(zD_Q(z))$ of order 9. The resulting RPCI transfer functions $Y(z)-N(z)Q(z)$, and $X(z)+M(z)Q(z)$ are of order 10 in their minimal forms, and $Q(1)=0$, as desired. The RPCI controller is robustly stabilizing because $\| W(z)C(z)S_I(z) \|_\infty = 0.49 < 1$.

Figure 3 shows the results of a closed-loop simulation test carried out to evaluate the nominal regulation performances of the three control designs. When a unit-step disturbance $d(t)$ is introduced at $t=12$, the NPC rejects the disturbance effectively, as shown in Figure 3a. In contrast, the RPC fails to reject the effect of the disturbance, and displays steady-state offset. The RPCI, on the other hand, succeeds in rejecting the disturbance, with slower dynamics than the nominal controller. Figure 3b shows that the NPC achieves the disturbance rejection at the expense of fairly energetic control actions that follow the onset of the disturbance. On the other hand, the RPCI prescribes more conservative input adjustments, typical of robust controllers. In many practical situations, the smoother dynamics of the RPCI design may be highly preferable over the more aggressive behavior of the NPC.

Figure 4 shows the results of a closed-loop simulation test for a perturbed plant ($\Delta(z) \neq 0$), belonging to the family (4.1). As in the previous example, an external unit-step disturbance $d(t)$ is introduced at $t=12$. The figure shows that the NPC is unable to control the plant, causing unstable closed-loop dynamics. In marked contrast the RPCI regulator is stabilizing, has offset-free behavior and manages to reject the disturbance without excessive control action.

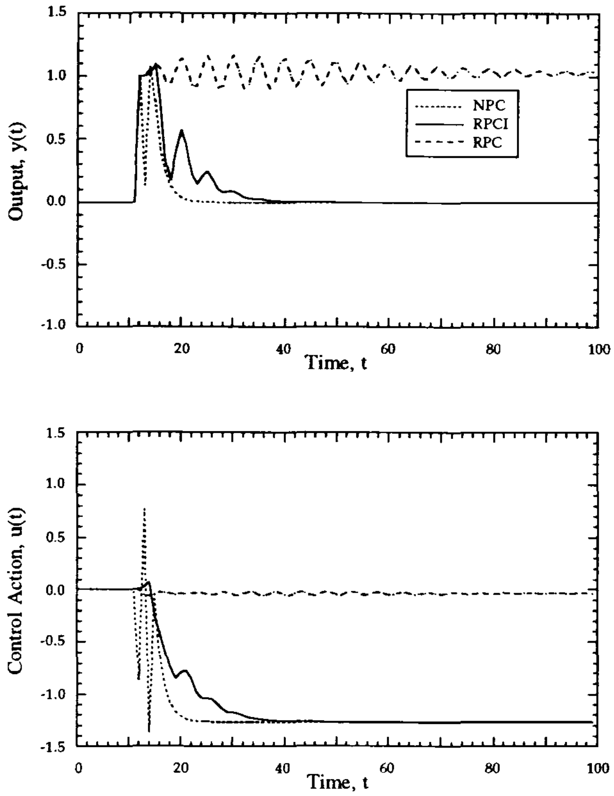

Figure 3. Comparison of the performance of the nominal predictive regulator (NPC), the robust predictive regulator (RPC), and the robust predictive regulator with integral action (RPCI) designed in the example, acting on a plant with no uncertainty. A unit-step disturbance $d(t)$ is introduced at t=12.

6. CONCLUSIONS

Systematic methods for robustifying predictive regulators for unstable plants have been proposed in this paper. Design procedures are given for both the unconstrained case and the case of constraints on the manipulated variable. Experience with these methods shows that the unconstrained design technique leads to controllers of reasonable order, whereas the constrained design often results in very high-order controllers. Although model reduction techniques may be used to approximate the l_1/H_∞ regulator, the large controller order seems to be less of an issue in predictive control applications, where high-order convolution models have been used extensively.

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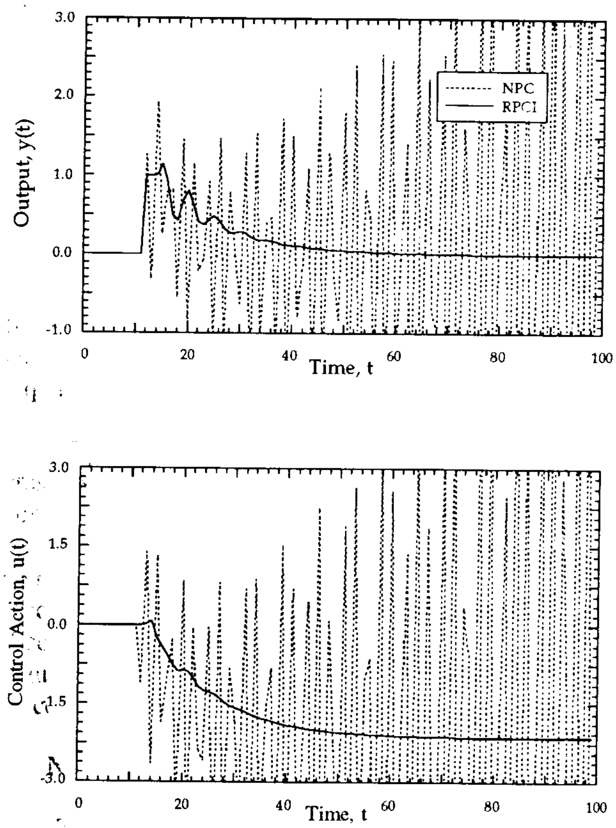

Figure 4. Comparison of the performance of the nominal predictive controller (NPC), and the robust predictive controller with integral action (RPCI), designed in the example, acting on a perturbed plant belonging to the uncertainty description. A unit-step disturbance *d(t)* (not shown) is introduced at t=12.

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