Multivariable adaptive control and estimation of a nonlinear wastewater treatment process.

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Abstract. In this paper, an approach for estimating biological state and parameter variables and for controlling a nonlinear wastewater treatment process is developed. Combination of a nonlinear estimation procedure and a multivariable reference model control law provides favourable performances for tracking a given model-based reference model despite disturbances and system parameter uncertainties. Convergence of both estimation and control scheme are demonstrated via Lyapunov's method. Simulation study with additive measurements noises and parameter jumps shows the efficiency and significant robustness of the control methodology developed for this nonlinear process.

Keywords. Model reference adaptive control, nonlinear adaptive estimation, Lyapunov's method, nonlinear biological process.

1. Introduction

In recent years, the application of modern adaptive control theories to biotechnological processes received an increasing attention. A major difficulty for controlling microbial processes lies in the scarceness of on-line measurement of some key biological variables. Furthermore, control design come up against the nonlinear and nonstationary character of the process dynamic behaviour. Therefore, identification of several nonlinear parameters, often time-varying, is not straighforward. To overcome measurement difficulties, joint state and parameter estimation procedures were applied for biological processes, e.g. least-squares observer[1], observer-based estimator[2] or extended Kalman filter[3]. Controlling a partially unknown and nonlinear system most often requires adaptive methods. Control technics based on linear approximation of the nonlinear system, may provide satisfactory results[4]. But for highly nonlinear processes, conventional feedback controllers must be detuned significantly to ensure stability. Therefore, performance is often severly deteriorated. The use of control strategies based on direct exploitation of the nonlinear structure of the process model can be expected to yield significantly improved performances. Several applications in the field of bioprocesses have been reported in the literature, e.g. nonlinear control schemes based on standard pole placement[5], predictive[6] or model-reference[7] type arguments.

In this paper, a method for controlling adaptively a continuous flow wastewater treatment process is developed and simulated. The control approach consists of a model-based estimation algorithm combined with a model reference adaptive control method. This type of approach, originally introduced by Duarte and Narendra[8], results in a combined direct and indirect adaptive control scheme. The estimator provides estimates of the xenobiotic substrate concentration and of the time-varying reaction rates which serve in the control law. The estimation error model based on the Narendra's error model[9] is obtained by an extended linearization technique using Kronecker's calculation. The appropriate selection of adaptive gains in the state estimator results in the error system with prescribed stable eigenvalues. The control objective is to track both biomass and pollutant xenobiotic substrate residual concentrations at prespecified trajectories. The desired profiles are generated by a reference model of the same type as the system model. The provided convergence proofs of both estimation and control issues so as the parameter estimator equations are based on Lyapunov's method.

The paper is organized as follows. In §2, process model is briefly described. The next section reviews the joint adaptive state and parameter estimation method. The structure of the multivariable model reference adaptive control law is developed in §4. Finally, in §5, a number of features of the proposed adaptive control scheme are discussed and its efficiency demonstrated via simulation study. A general conclusion ends the paper.

2. Process model

The biological system studied is a continuous flow wastewater treatment bioreactor containing a mixed microbial population (X) growing on a blending of two types of substrates, a xenobiotic one (S) and an energetic one (E). Combination of growth with dilution governs the change in concentration of these main biological variables. Mass-balance considerations give the following nonlinear differential system

$$\begin{aligned} dX_p/dt &= (\mu_{sp}+\mu_{Ep})X_p - X_p u_1(t) - X_p u_2(t) \\ dS_p/dt &= -R_S\,\mu_{sp} X_p - S_p u_1(t) + (S^m - S_p)u_2(t) \\ dE_p/dt &= -R_E\,\mu_{Ep} X_p - E_p u_1(t) + (E^m - E_p)u_2(t) \end{aligned} \tag{1}$$

with

$$\mu_{sp} = \mu_s(S_p, E_p, \theta_p),\ \mu_{Ep} = \mu_E(S_p, E_p, \theta_p) \tag{2}$$

The energetic substrate concentration E_p and the biomass concentration X_p are the measurable output signals of the plant. The xenobiotic substrate concentration S_p is unavailable on-line. The dilution rate (u_1) of a solution without susbtrates and the dilution rate (u_2) of a solution with both substrates at a concentration $S^m = E^m$, serves as the control variables of the plant[10]. The specific reaction rates models μ_{sp} and μ_{Ep} are nonlinear functions of the states S_p, X_p and several constant kinetic parameters noted by the parameter vector $\theta_p \in R^m$. R_S and R_E are known yields constant coefficients.

3. State and parameter estimation

The model-based control law that will be presented afterwards should incorporate real-time information on the state S_p and parameters μ_{sp} and μ_{Ep}. However, measurements of neither variable is available. Consequently, an adaptive state and parameter estimator is needed. Instead of estimating the reaction rates directly as time-varying entities, they are reconstructed on the basis of their analytical expression via estimation of the parameter vector θ_p. This viewpoint forms the basic idea of the underlaying adaptive estimation procedure.

3.1 *Preliminar transformation*

For observability purposes, a preliminar useful change of coordinates is necessary. It consists of decoupling the kinetic part of the unmeasured states from the specific reaction rates[2]. This is generally possible for continuous flow processes since linear relations link the reaction rates and the different biological variable dynamics. In our case, the following auxiliary state

$$Z_p = X_p + S_p/R_S + E_p/R_E \tag{3}$$

have a dynamical behavior described by

$$dZ_p/dt = -Z_p\left(u_1(t)+u_2(t)\right) + S^m\left(1/R_S+1/R_E\right)u_2(t) \tag{4}$$

which is independent from parameter vector θ_p. The new state vector is denoted $\xi_p^T=\left[Z_p\, X_p\, E_p\right]$.

3.2 *Estimation reference model*

As a state estimator the following model is used

$$\begin{aligned}
dZ_e/dt &= -Z_e(u_1+u_2)+S^m\left(1/R_S+1/R_E\right)u_2+\alpha_1(t)\varepsilon_{X_e}+\alpha_2(t)\varepsilon_{E_e}\\
dX_e/dt &= (\mu_{se}+\mu_{Ee})X_p-X_p(u_1+u_2)+\beta_1(t)\varepsilon_{X_e}+\beta_2(t)\varepsilon_{E_e}\\
dE_e/dt &= -R_E\,\mu_{Ee}X_p-E_pu_1+\left(E^m-E_p\right)u_2+\gamma_1(t)\varepsilon_{X_e}+\gamma_2(t)\varepsilon_{E_e}
\end{aligned} \tag{5}$$

where $\varepsilon_{X_e}=X_e-X_p$ and $\varepsilon_{E_e}=E_e-E_p$ are the output estimation errors. The specific reaction rate estimates $\mu_{Se}=\mu_S\left(Z_e,E_p,\theta_e\right)$ and $\mu_{Ee}=\mu_E\left(Z_e,E_p,\theta_e\right)$ have the same structure as in the original system model, but substitute the unknown state and parameters by their respective estimate. The goal in estimation design is to choose appropriate time-varying gains $\alpha_i(t)$, $\beta_i(t)$ and $\gamma_i(t)$ $(i=1,2)$ and a parameter adjustment law of θ_e in such a way that, when $t\to\infty$, the limits $\varepsilon_{Z_e}=Z_e-Z_p\to 0$, $\varepsilon_{\mu_S}=\mu_{Se}-\mu_{Sp}\to 0$ and $\varepsilon_{\mu_E}=\mu_{Ee}-\mu_{Ep}\to 0$ are obtained.

3.3 *Estimation error equations*

The structure of the error system is demonstrated and described with more details in Zeng *et al* [11]. An extended linearization technique using Kronecker's calculation yields to the linear error system of the form

$$d\varepsilon_e/dt = A_e\varepsilon_e + B_e\varepsilon_{\theta_e} \tag{6}$$

where

$$\varepsilon_e^T=\left[\varepsilon_{Z_e}\ \varepsilon_{X_e}\ \varepsilon_{E_e}\right],\ \varepsilon_{\theta_e}=\theta_e-\theta_p \tag{7}$$

$$A_e=\begin{bmatrix} -(u_1+u_2) & \alpha_1 & \alpha_2\\ A_{X_e} & \beta_1 & \beta_2\\ A_{E_e} & \gamma_1 & \gamma_2\end{bmatrix},\ B_e=\begin{bmatrix}0\\ B_{X_e}\\ B_{E_e}\end{bmatrix} \tag{8}$$

Defining the state vector $\xi_s^T=\left[Z_e\, X_p\, E_p\right]$, the elements of matrices A_e and B_e are given by:

$$A_{X_e}=X\left.\frac{\partial(\mu_S+\mu_E)}{\partial Z}\right|_{\xi_s,\theta_e},\ A_{E_e}=X\left.\frac{\partial(-R_E\mu_E)}{\partial Z}\right|_{\xi_s,\theta_e} \in R \tag{9}$$

$$B_{X_e}=X\left.\frac{\partial(\mu_S+\mu_E)}{\partial \theta}\right|_{\xi_s,\theta_e},\ B_{E_e}=X\left.\frac{\partial(-R_E\mu_E)}{\partial \theta}\right|_{\xi_s,\theta_e} \in R^{1\times m} \tag{10}$$

3.4 *Selection of estimation gains*

The first issue of estimation design is to choose the appropriate expression of the adaptive gains $\alpha_i(t)$, $\beta_i(t)$ and $\gamma_i(t)$ $(i=1,2)$ in order to stabilize the dynamical feature of the time-varying matrix A_e. This task is realized by imposing via state feedback desired stable eigenvalues of A_e. The following adaptive gains

$$\begin{aligned}
&\alpha_1(t) = -\frac{1}{A_{E_e}}\left[p_1p_2-(u_1+u_2)(p_1+p_2-u_1-u_2)\right],\ \alpha_2(t)=0\\
&\beta_1(t) = -(p_1+p_2-u_1-u_2),\ \beta_2(t)=0,\\
&\gamma_1(t)=0,\ \gamma_2(t)=-p_3
\end{aligned} \tag{11}$$

lead to a characteristic polynomial of the form

$$\det\left(pI-A_e\right)=(p+p_1)(p+p_2)(p+p_3),\ p_1,\ p_2,\ p_3>0 \tag{12}$$

which roots are located in the left-half of the complex plane.

The remaining problem is to find a suitable parameter estimator resulting, together with error system (6), in the convergent estimates of the xenobiotic substrate and the time-varying reaction rates. This is realized via Lyapunov's method. Consider the following parameter adjustment law

$$d\varepsilon_{\theta_e}/dt = -\Gamma_e B_e^T P_e \varepsilon_e \tag{13}$$

where P_e and Γ_e are symmetric positive-definite matrices. Let the quadratic function V_e be a Lyapunov function candidate of the form

$$V_e\left(\varepsilon_e,\varepsilon_{\theta_e}\right)=\varepsilon_e^TP_e\varepsilon_e+\varepsilon_{\theta_e}^T\Gamma_e^{-1}\varepsilon_{\theta_e} \tag{14}$$

Evaluating the time derivative of V_e along the trajectories of (6) and (13) we obtain

$$dV_e/dt=\varepsilon_e^T\left(A_e^TP_e+P_eA_e\right)\varepsilon_e \tag{15}$$

Since A_e is asymptotically stabilized, the solution of the matrix equation

$$A_e^TP_e+P_eA_e=-Q_e<0 \tag{16}$$

where Q_e is any symmetric positive-definite matrix, yields to a symmetric positive-definite matrix P_e. Hence we have

$$dV_e/dt = -\varepsilon_e^T Q_e \varepsilon_e \leq 0 \tag{17}$$

then, according to Narendra's theorem[12], the following conclusions can be obtained.

(i) If B_e is persistently exciting, i.e.

$$\forall\ \tau>0, \exists\ \eta, s>0 \int_{\tau}^{\tau+s} B_e^T(t)\, B_e(t)dt \geq \eta\, I \tag{18}$$

the error system (6) with (13) is uniformly asymptotically stable, i.e. we have when $t\to\infty$, $\varepsilon_e\to 0$ and $\varepsilon_{\theta e}\to 0$. It means that we can estimate the unknown constant parameters vector and hence the time-varying parameters, i.e. $\varepsilon_{\mu_e}^T = [\,\varepsilon_{\mu_{S_e}}\ \varepsilon_{\mu_{E_e}}] \to 0$.

(ii) In the case that the former condition is not satisfied the error system (6) and (13) is uniformly stable, i.e.when $t\to\infty$, $\varepsilon_e\to 0$ and $\varepsilon_{\theta e}\to$ some constant vector. However, if the state vector $\xi_p(t)$ is persistently exciting, i.e.

$$\forall\ \tau>0, \exists\ \eta, s>0 \int_{\tau}^{\tau+s} \xi_p(t)\, \xi_p^T(t)dt \geq \eta\, I \tag{19}$$

the time-varying parameters can be estimated, i.e. $\varepsilon_{\mu_e}\to 0$ albeit the constant parameters are not directly estimated. In each case (i) and (ii), estimation of the time-varying specific reaction rates, which is our design, is ensured. The estimate of variable S_p is directly reconstructed using relation (3) with the auxilliary state estimate Z_e.

4. Adaptive multivariable control

The control objective is to regulate and track the biomass and pollutant substrate concentration of the plant (X_p, S_p respectively) at desired values (X_c, S_c respectively) by acting on the control variables $u_1(t)$ and $u_2(t)$. In deriving the control law, it is first assumed that all the necessary process variables can be measured. Then the inaccessible variables are substituted by their estimates provided by the previous estimation algorithm.

4.1 *Model reference control*

The reference model that is used to generate the reference inputs $r_1(t)$ and $r_2(t)$ for $u_1(t)$ and $u_2(t)$ is based on the plant model and described by the following nonlinear equations

$$\begin{aligned} dX_c/dt &= (\mu_{sc}+\mu_{Ec})X_c - X_c r_1(t) - X_c r_2(t) \\ dS_c/dt &= -R_S\mu_{sc}X_c - S_c r_1(t) + (S^m - S_c)r_2(t) \\ dE_c/dt &= -R_E\mu_{Ec}X_c - E_c r_1(t) + (E^m - E_c)r_2(t) \end{aligned} \tag{20}$$

The reference reaction rates are determinated on the basis of the biological knowledge about the process using the same structure as the plant, i.e. $\mu_{sc} = \mu_s(S_c, E_c, \theta_c)$ and $\mu_{Ec} = \mu_E(S_c, E_c, \theta_c)$. In order to eliminate the control errors despite of the variations of process parameter, the following adaptive control law is used

$$\begin{aligned} u_1(t)+u_2(t) &= \varphi_1(t)\varepsilon_{Xc}+\psi_1(t)+r_1(t)+r_2(t)+\delta_1(t) \\ u_2(t) &= \varphi_2(t)\varepsilon_{Sc}+\psi_2(t)+r_2(t)+\delta_2(t) \end{aligned} \tag{21}$$

where $\varepsilon_{Sc}=S_c-S_p$ and $\varepsilon_{Xc}=X_c-X_p$ are the output control errors. $\varphi_i(t)$, $\psi_i(t)$ and $\delta_i(t)$ ($i=1,2$) are adaptive gains to be suitably choosen.

4.2 *Control error equations*

The adaptive feedback gains given by the expressions

$$\begin{aligned} \delta_1(t) &= -\left(\varepsilon_{\mu_{S_c}}+\varepsilon_{\mu_{E_c}}\right) \\ \delta_2(t) &= -\left[\varepsilon_{\mu_{S_c}}\left(S_p+R_S X_p\right)+\varepsilon_{\mu_{E_c}} S_p\right]/S^m \end{aligned} \tag{22}$$

with equations (20), (21) and (22), lead to the linear control error system of the form

$$d\varepsilon_c/dt = A_c\varepsilon_c + B_c\Psi \tag{23}$$

where

$$\varepsilon_c^T = [\,\varepsilon_{Xc}\ \varepsilon_{Sc}\,],\ \varepsilon_{\mu_{S_c}} = \mu_{sc}-\mu_{sp},\ \varepsilon_{\mu_{E_c}} = \mu_{Ec}-\mu_{Ep} \tag{24}$$

$$A_c = \begin{bmatrix} \mu_{sc}+\mu_{Ec}+X_p\varphi_1-(r_1+r_2) & 0 \\ -R_S\mu_{sc}+S_p\varphi_1-S^m\varphi_2 & S^m\varphi_2-(r_1+r_2) \end{bmatrix} \tag{25}$$

$$B_c = \begin{bmatrix} X_p & 0 \\ S_p & -S^m \end{bmatrix},\ \Psi^T = [\,\psi_1\ \psi_2\,] \tag{26}$$

A necessary stability condition of error system (23) is that the dynamical matrix A_c is stable. The following adaptive gains

$$\varphi_1(t) = -(\mu_{sc}+\mu_{Ec})/X_p,\ \varphi_2(t) = \varphi_2 > 0 \tag{27}$$

lead to a time-varying characteristic polynomial given by

$$det(pI - A_c) = (p+r_1+r_2)(p+r_1+r_2+S^m\varphi_2) \tag{28}$$

Since r_1, r_2, S^m and φ_2 are positive, the roots of (28) are stable eingenvalues of matrix A_c.

The remaining problem is to determine a suitable adjustment law of $\Psi(t)$ using the observed signal so that the control error tends to zero in time. This task is realized using the Lyapunov's method. Consider the adjustment law of $\Psi(t)$ of the form

$$d\Psi/dt = -\Gamma_c B_c^T P_c \varepsilon_c \tag{29}$$

where P_c and Γ_c are a symmetric positive-definite matrices. Evaluating the time derivative of the following Lyapunov function candidate

$$V_c(\varepsilon_c, \Psi) = \varepsilon_c^T P_c \varepsilon_c + \Psi^T \Gamma_c^{-1}\Psi \tag{30}$$

along the trajectories of (23) and (29) we obtain

$$dV_c/dt = \varepsilon_c^T\left(A_c^T P_c + P_c A_c\right)\varepsilon_c \tag{31}$$

Since A_c is an asymptotically stable matrix, the solution of the matrix equation

$$A_c^T P_c + P_c A_c = -Q_c < 0 \tag{32}$$

where Q_c is any symmetric positive-definite matrix, yields to a symmetric positive-definite matrix P_c. Hence we have

$$dV_c/dt = -\varepsilon_c^T Q_c \varepsilon_c < 0 \qquad (33)$$

we therefore conclude that the state error converge uniformely asymptotically to zero, i.e. when $t \to \infty$, $\varepsilon_c \to 0$.

5. Simulation results

5.1 *Numerical values*

Simulation results are obtained by using a 4th-order Runge-Kutta algorithm to integrate the process equations (1). The specific reaction rates are modelled as[13]

$$\mu_{Sp}(S_p, E_p, \theta_p) = \mu_{S_m} S_p / (K_S + S_p + a_E E_p) \qquad (34)$$
$$\mu_{Ep}(S_p, E_p, \theta_p) = \mu_{E_m} E_p / (K_E + E_p + a_S S_p)$$

where $\theta_p^T = [\mu_{S_m} \; K_S \; a_E \; \mu_{E_m} \; K_E \; a_S]$. The values of the parameter vector components are $\mu_{S_m}=0.1$, $K_S=2$, $a_E=10$, $\mu_{E_m}=0.3$, $K_E=1.5$ and $a_S=0.1$. The values of the yield coefficients are $R_S=1.11$ and $R_E=2.5$.The feeding substrate concentration is $S^m=E^m=6$g/l. The plant kinetic parameters are submitted to 30% jumps to test the estimator robustness. The outputs variables are polluted with a 2% signal noise for more realistic measurements. The roots of the characteristic polynomial of matrix A_e are $p_1=0.05$, $p_2=0.05$ and $p_3=0.5$. The tuning matrices of the estimator are $\Gamma_e = diag\{1.2, 400, 1e^5, 7, 100, 20\}$ and $P_e=I$. The parameters of the control law are $\varphi_2=0.03$, $P_c=I$ and $\Gamma_c = diag\{1.5e^{-3}, 1e^{-3}\}$.The values of the reference parameter vector θ_c components are $\mu_{S_mc}=0.12$, $K_{Sc}=1.6$, $a_{Ec}=12$, $\mu_{E_mc}=0.36$, $K_{Ec}=1.2$ and $a_{Sc}=0.12$.

5.2 *Simulation results*

Figures 1 and 2 illustrate the behavior of the controlled outputs X_p and S_p and their corresponding reference trajectories, X_c and S_c. The reference inputs r_1 and r_2 are depicted on figures 3 and 4 together with the actual control u_1 and u_2 applied in the system. In spite of the variation of the desired tracking trajectories, the setpoints were reached after short controller transient responses. Although the high level of coupling of the process dynamics, the effect of the coupling between the two controlled variables was attenuated. The perturbations of biomass regulation due to the jumps of μ_{S_m} at t=120h and of K_S at t=240h are favourably rejected by the adaptive controller. Figures 5 and 6 show the actual, estimate and reference reaction rates behavior. The parameter estimator is able to track suitably the reaction rates to their true simulated values with relatively short transient responses. The time-varying reaction rates identification, which is our estimation design, is not influenced by convergence of the constant parameters to biaised values (figures 7 and 8) even in presence of abrupt parameter jumps. This occurence is due to the insufficient persistently excitation of $B_e(t)$ (case (i)) However, if only one of the parameters vector component is unknown, it can be shown that the estimated parameter converges to its correct value. Simultaneously, the estimator provides good estimate of unavailable xenobiotic substrate concentration (Fig 1) via auxiliary state estimation (Fig 9). Moreover, significant robustness of the estimator is demonstrated by efficiency of tracking abrupt variations of reaction rates and of unmeasured state values when the setpoints varies.

6. Conclusion

A complete estimation and control structure for a nonlinear wastewater treatment process is developed. A model-based multivariable model reference control law is used in order to maintain both biomass and pollutant substrate concentrations at prespecified levels. This control law is combined with an adaptive estimator for on-line tracking of unavailable state and time-varying reaction rates. The persistently exciting condition of the signals determines the observability of the system. The main result is that the estimation of the time-varying parameters, under state vector persistently exciting condition, can be realized albeit the constant parameters are not directly estimated. The efficiency of the adaptive control scheme is demontrated via computer simulations. The controller is able to reject disturbances and ensure convergence with relatively short transient responses. The estimator demonstrates significant robustness for tracking abrupt jumps of the plant kinetic parameters. Moreover, on-line estimation of main state and parameter variables provides real-time informations on the culture physiology appreciated by process users.

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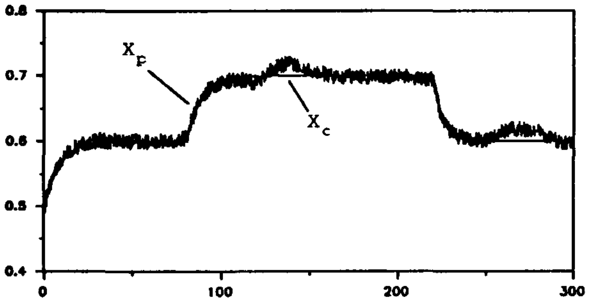

Fig.1: evolution of biomass concentration (X_p, plant; X_c, setpoint (reference model)).

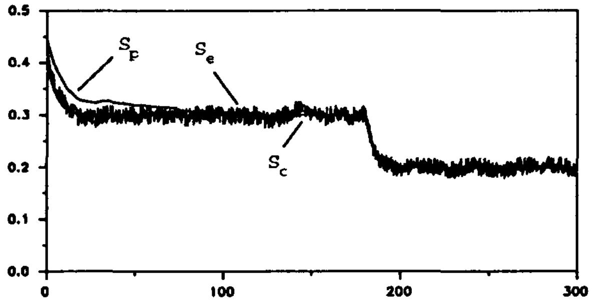

Fig. 2: evolution of xenobiotic substrate concentration (S_p, plant; S_c setpoint (model reference); S_e, estimate).

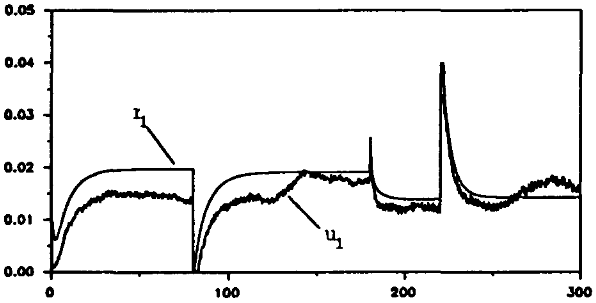

Fig. 3: evolution of the reference input $r_1(t)$ and the plant input $u_1(t)$.

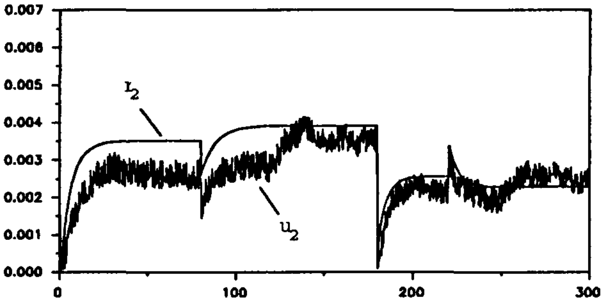

Fig. 4: evolution of the reference input $r_2(t)$ and the plant input $u_2(t)$.

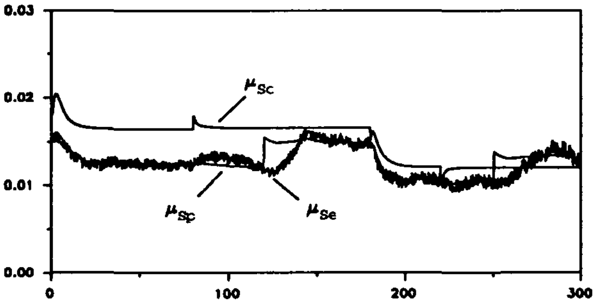

Fig. 5: evolution of the specific reaction rate (μ_{Sp}, plant; μ_{Sc}, reference model; μ_{Se}, estimate).

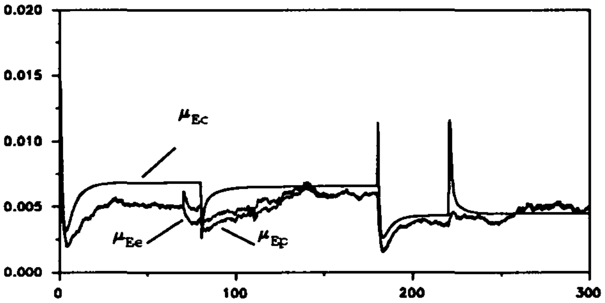

Fig. 6: evolution of the specific reaction rate (μ_{Ep}, plant; μ_{Ec}, reference model; μ_{Ee}, estimate).

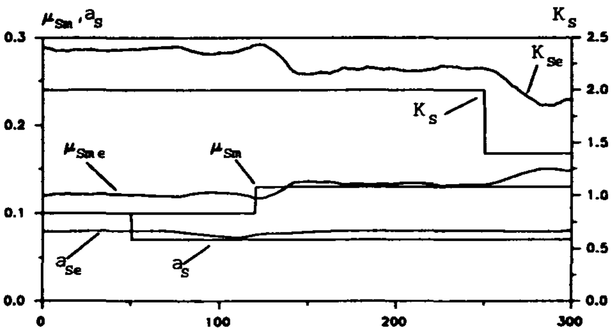

Fig. 7: evolution of the biological parameters and their estimate values

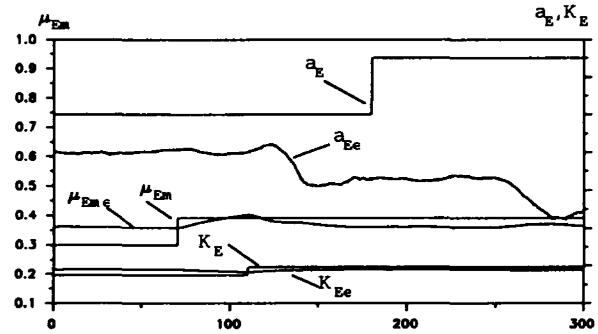

Fig. 8: evolution of the biological parameters and their estimate values

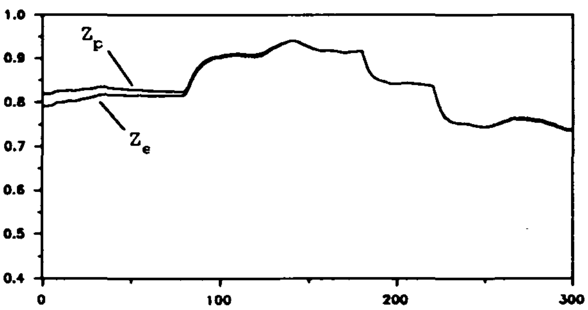

Fig. 9: evolution of auxiliary state concentration (Z_p, plant; Z_e, estimate).

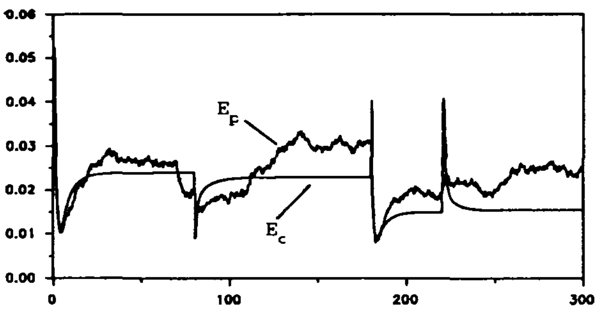

Fig. 10: evolution of energetic substrate concentration (E_p, plant; E_c, reference model)