# PERFORMANCE ANALYSIS OF THE DSSS RECEIVER USING TWO-STAGE DFB FILTER UNDER THE NARROWBAND AR INTERFERENCE

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*ABSTRACT: In this paper the performance of the direct sequence spread spectrum receiver containing the two stage decision feedback filter in the presence of the narrowband autoregressive interference is analyzed. The receiver performance is expressed in regard to the Signal/(interference+noise) ratio and the bit error probability. Based on the theoretical analysis results it can be concluded that the proposed receiver performance is good, compared to the receivers containing the classical decision feedback or two sided transversal filter. The adaptive version of the proposed receiver based on the Widrow-Hoff LMS algorithm is also analyzed. The results obtained show that the overall good receiver performance is not degraded by the variation of the filter coefficients caused by the adaptation process.*

## I. INTRODUCTION

The frequency band used by the direct sequence spread spectrum (DSSS) communication system is often occupied by one or more strong narrowband signals. These narrowband interference (NBI) signals, originating from the conventional communication systems or being purposely generated as the jamming signals, are often so strong that even the inherent DSSS processing gain cannot provide the acceptable transmission quality. In that case some additional means of NBI suppression has to be implemented, [1,2].

In paper [3], the DSSS receiver containing the combination of classical decision feedback and two-sided transversal filter (DFB+2-TF) is presented and its performance in the presence of the single tone interference is analyzed. In this paper the analysis of that receiver performance is extended to the more realistic model of the narrowband autoregressive (AR) interference at the receiver input. The comparative analysis of the proposed receiver and the receivers containing the decision feedback (DFB) or two-sided transversal filter (2-TF), described in [1], is made.

The receiver performance is expressed in terms of the,

(i) signal/(NBI+noise) improvement, and
(ii) bit error probability.

The results obtained show that the receiver with DFB+2-TF is comparably better, regarding the Signal/(NBI+noise) improvement. This receiver is also comparably better regarding the bit error probability, except in the case of the extremely poor input Signal/NBI ratio.

The results considering the adaptive receiver performance in the presence of the nonstationary electromagnetic environment are also presented. The adaptation algorithm considered is the Widrow-Hoff least mean square (LMS), [5].

The paper is organized as follows. The proposed receiver is described in Sec. II. Signal analysis at the receiver input is performed in Sec. III. In Sec. IV, the description of the performance criteria is given. The comparative analysis of the receiver with DFB+2-TF, DFB and 2-TF is made in Sec. V. Performance of the proposed receiver in the presence of the nonstationary environment is presented in Sec. VI. In the last section some concluding remarks are made.

## II. PROPOSED RECEIVER DESCRIPTION

Block diagram of the proposed receiver is given at Fig.1.

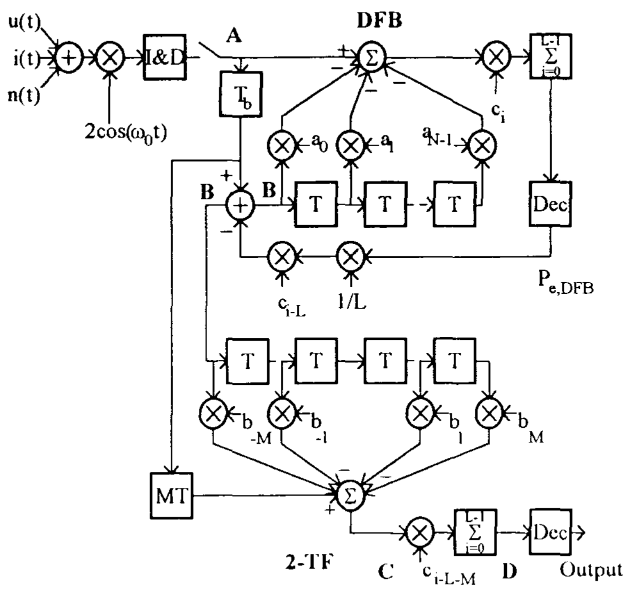


**Fig.1:** *Block diagram of the proposed DSSS receiver with DFB+2-TF. I&D is integrate and dump circuit, Dec is the decision circuit.*

The upper half of the receiver shown in Fig.1 contains the DFB filter. The main purpose of this filter is to produce the reference interference signal. This reference is fed to 2-TF, and based on the 2-TF output samples the received symbol detection is made. Using the DFB filter output as the reference signal enables avoidance of the intersymbol interference (ISI) that would be normally introduced by the 2-TF and the overall receiver performance improvement, [3].

For the comparison purposes, the receivers with the DFB or 2-TF only are analyzed too. Decision feedback filter is contained in the receiver on Fig.1, and its output is labeled by $P_{e,DFB}$. Receiver with the two-sided transversal filter can be obtained from the receiver in Fig.1 by feeding the 2-TF part of the receiver with the signal at point A.

## III. THEORETICAL ANALYSIS

Signal at the receiver input is demodulated and integrated at the pseudonoise sequence (PN) chip duration interval, T. The order of the DFB filter is N, and 2-TF order is 2M. Message bit duration is $T_b=LT$, where L stands for the processing gain.

Input signal consists of three components. First one is the desired DSSS signal, given by,

$$u(t) = Ud(t)PN(t)\cos(\omega_0 t), \tag{1}$$

where U and $\omega_0$ represent the DSSS carrier amplitude and angular frequency. Random message d(t) is given by,

$$d(t) = \sum_i d_i \Pi_b(t - iT_b),\ d_i \in \{-1,1\},\ \Pi_b(t) = \begin{cases} 1, & |t| \le T_b/2, \\ 0, & |t| > T_b/2. \end{cases} \tag{2}$$

At the sampler output, point A in the receiver, the desired signal is represented by its samples $u(k) \in \{-UT, +UT\}$ of power $P_u$.

Pseudonoise sequence is given by,

$$PN(t) = \sum_i c_i \Pi_c(t - iT),\ c_i \in \{-1,1\},\ \Pi_c(t) = \begin{cases} 1, & |t| \le T/2, \\ 0, & |t| > T/2. \end{cases} \tag{3}$$

Second component of the received signal is the narrowband interference, (NBI). This NBI is modeled as the narrowband autoregressive process of the fourth order [4], with two double poles $z_1=0.99\exp(2\pi f_d T)$ and $z_2=0.99\exp(-2\pi f_d T)$, where $f_d T$ stands for the normalized NBI carrier to DSSS carrier frequency offset. For $f_d T=0$, NBI reduces to the second order AR interference at the DSSS carrier with double pole $z_0=0.99$.

NBI Autocorrelation function is given by,

$$R_i(k) = P_{i,in}\left[1 - \left(1 - \frac{1.98}{(1+0.9801)z_0}\right)k\right] z_0^k \cos(k 2\pi f_d T), \tag{4}$$

where $P_{i,in}$ stands for the NBI average power at the DFB+2-TF input (Fig.1, point A), while $z_0=0.99$.

Third component of the received signal is the white Gaussian noise:

$$n(t) = n_c(t)\cos(\omega_0 t) + n_s(t)\sin(\omega_0 t). \tag{5}$$

The in-phase and quadrature components of this noise, $n_c(t)$ and $n_s(t)$ are statistically independent. One-sided power spectral density of this noise is $\eta$, and its power at the DFB+2-TF filter input (Fig.1, point A) is $P_{n,in}$.

All three components of the received signal are mutually independent and wide-sense stationary.

If the DFB+2TF filter coefficients have their optimal values, given in the mean-square sense, signal at the 2-TF output, Fig.1, point C, is given as the sum of the four components:

- desired DSSS signal $s_C(k)$,

$$s_C(k) = d(k)c(k); \tag{6}$$

- intersymbol interference $e_C(k)$,

$$e_C(k) = -\sum_{i=1}^{M} b_i\left[d(k-i)c(k-i) + d(k+i)c(k+i)\right] + e_{DFB}(k); \tag{7}$$

where $e_{DFB}(k)$ stands for the ISI resulting from the error propagation in the DFB filter part:

$$e_{DFB}(k) = \begin{cases} 0, & \text{with no error in DFB} \\ -2\sum_{i=0}^{N-1} a_i d(k-i-L)c(k-i-L), & \text{with error in DFB} \end{cases}; \tag{8}$$

- colored Gaussian noise $n_C(k)$,

$$n_C(k) = n(k) - \sum_{\substack{i=-M \\ i\neq 0}}^{M} b_i n(k-i); \tag{9}$$

- residual NBI $i_C(k)$,

$$i_C(k) = i(k) - \sum_{\substack{i=-M \\ i\neq 0}}^{M} b_i i(k-i); \tag{10}$$

In eq.(7-10) $a_0 \ldots a_{N-1}$ stand for the DFB filter coefficients, and $b_1 \ldots b_M$ stand for the 2-TF coefficients at Fig.1.

## IV. NBI SUPPRESSION MEASURES

There are two measures used for the NBI suppression qualification. Namely,

(i) signal/(NBI+noise) improvement, and
(ii) bit error probability.

### *Signal/(NBI+noise) improvement*

This measure is defined as the ratio of Signal/(NBI+noise) at the DFB+2-TF output and input. On the condition of equal desired signal power, this ratio becomes,

$$G = \frac{P_{i,in} + P_{n,in}}{P_{i,out} + P_{n,out}}. \tag{11}$$

where $P_{i,in}$ and $P_{n,in}$ stand for the NBI and noise power at the DFB+2-TF input (Fig.1, point A), and $P_{i,out}$ and

$P_{n,out}$ stand for the NBI and noise power at the DFB+2-TF output (Fig.1, point C).

### *Bit error probability*

Bit error probability at the receiver output can be calculated as the function of the chosen PN sequence. The conditional error probability for the accepted PN sequence can be expressed as,

$$P_{e,PN} = \frac{1}{2} erfc\left( \frac{E\{S(T_b)\}}{\sqrt{2Var\{S(T_b)\}}} \right). \quad (12)$$

where $S(T_b)$ stands for the sample at the decision circuit input (Fig.1, point D):

$$S(T_B) = \sum_{k=1}^{L} c(k)\left[s_C(k) + e_C(k) + i_C(k) + n_C(k)\right]. \quad (13)$$

Symbols E{} and Var{} are the expectation and variance operators for the known PN sequence, and *erfc*() is the complementary error function. The probability of the error propagation in the DFB part of the receiver is also contained in the expectation operator in (7).

Total error probability is then calculated as the average of the conditional error probability for all possible PN sequences,

$$P_e = \frac{1}{K} \sum P_{e,PN}. \quad (14)$$

where K is the number of all possible combinations of PN chip contained in the memory cells of DFB and 2-TF relevant for the sample at the decision circuit input.

## V. NBI SUPPRESSION RESULTS

In the calculation of the two described NBI suppression measures for the receiver at Fig.1, all the relevant parameter values are accepted in concordance with the ones accepted in [1-3], and DFB+2-TF filter taps have their optimal values in the least square error sense. This has been done to enable the comparing of results presented in this paper with results obtained by other authors. Accepted parameter values are:

- Processing gain L=7.
- Signal/noise ratio at the DFB+2-TF input: $A_n$=12dB.
- Signal/NBI ratio at the DFB+2-TF input, Γ=-40...0dB.
- DFB filter order: N=4.
- 2-TF filter order: 2M=4.

Obtained results are presented for two cases: AR NBI at the DSSS carrier frequency and AR NBI with carrier frequency offset.

### *AR NBI at the DSSS carrier frequency*

At Fig.2 the Signal/(NBI+noise) improvement for receivers with DFB+2-TF, DFB and 2-TF is presented as the function of the Signal/NBI ratio at the filter input.

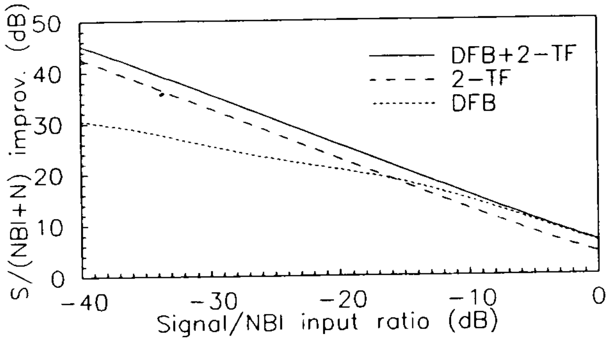


**Fig.2:** *Signal/(NBI+noise) improvement for receivers with DFB+2-TF, DFB or 2-TF as a function of input Signal/NBI ratio. N=4, M=2, L=7, $A_n$=12dB.*

It can be seen that the highest improvement is obtained in the receiver with DFB+2-TF.

At Fig.3. the bit error probability at the output of receivers with DFB+2-TF, DFB or 2-TF is presented as a function of the Signal/NBI ratio at the filter input.

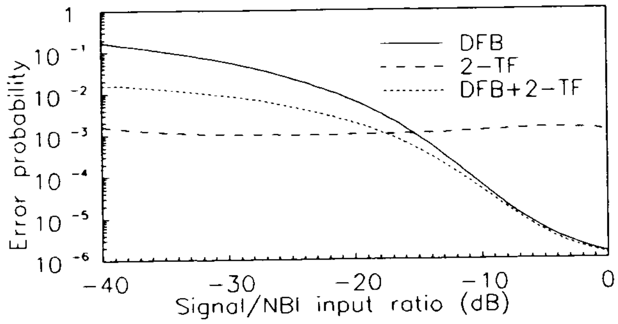


**Fig.3:** *Bit error probability as a function of the Signal/NBI ratio at the filter input N=4, M=2, L=7, $A_n$=12dB.*

Based on the Fig.3, the conclusion can be made that the choice of the most appropriate DSSS receiver in the presence of strong AR NBI depends on the input Signal/NBI ratio. For input Signal/NBI ratios greater than -18dB (for the accepted set of parameters) the best results are obtained with the DFB+2-TF in the receiver. For lesser Signal/NBI ratios, receiver with 2-TF shows the better performance. Receiver with DFB has the moderate performance, due to the error propagation phenomena.

Comparing the DFB+2-TF and 2-TF receiver performance in the presence of the AR NBI, it can be concluded that for the Signal/NBI input ratios better than Γ=-20dB the main source of error in the 2-TF receiver is the intersymbol interference. For worst Signal/NBI input ratios the main source of decision errors in the DFB+2-TF receiver is the decision error propagation. The value of Γ=-20dB can be considered as the limit of the ISI dominance as the cause of the decision errors, under the accepted conditions.

***AR NBI with the carrier frequency offset***

In the following analysis the Signal/NBI ratio at the filter input is held fixed at Γ=-20dB, and NBI frequency offset is changed. Interference is modeled as the fourth-order AR NBI, its autocorrelation function given by (4).

At Fig.4 the Signal/(NBI+noise) improvement as the function of the NBI carrier frequency offset is presented for receivers with DFB+2-TF, DFB and 2-TF, respectively.

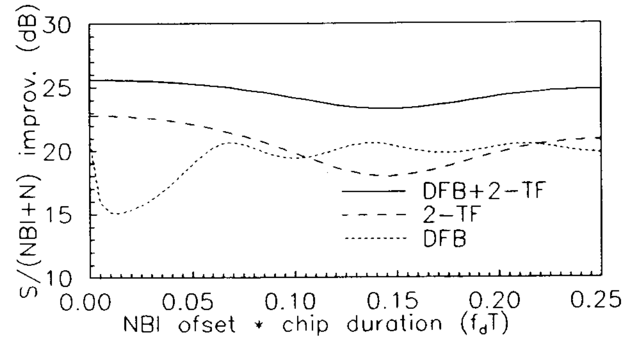


**Fig.4**: *Signal/(NBI+noise) improvement in the receiver with DFB+2-TF, DFB, and 2-TF, respectively, as a function of the normalized NBI carrier offset. $N=4$, $M=2$, $L=7$, $\Gamma=-20dB$, $A_n=12dB$.*

One can see from Fig.4 that the comparatively greatest improvement can be obtained in the receiver with DFB+2-TF.

At Fig.5 the bit error probability is presented for all three receiver types as a function of the normalized NBI carrier frequency offset.

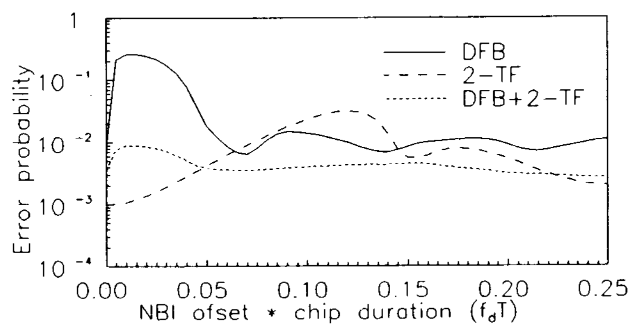


**Fig.5:** *Bit error probability at the output of the receiver with DFB+2-TF, DFB, and 2-TF as a function of the normalized NBI carrier frequency offset. $\Gamma=-20dB$, $A_n=12dB$, $L=7$, $N=4$, $M=2$.*

It can be seen from Fig.5 that the receiver with 2-TF shows better performance in the case of small NBI frequency offsets. With the increase of this offset, the receiver with DFB+2-TF shows comparatively better performance, and receivers with DFB and with 2-TF have similar performance.

## VI. ADAPTIVE DFB+2-TF RECEIVER PERFORMANCE

In this section, the performance of the adaptive DFB+2-TF receiver, Fig.1, in the presence of the nonstationary AR NBI is analyzed. Simple and robust Widrow-Hoff least mean square (LMS) adaptation algorithm is considered, described by, [5],

$$w_i(k+1) = w_i(k) + 2\mu y(k) x_i^*(k). \qquad (15)$$

In this expression $w_i(k)$ represents the i-th filter coefficient at t=kT, $x_i^*(k)$ stands for the complex conjugate sample at the i-th filter delay cell, y(k) is the filter output signal, and μ is the algorithm convergence parameter

Adaptive DFB+2-TF filter performance is tested in the presence of nonstationary AR NBI by the computer simulation. Block diagram of the modeled receiver is presented in Fig.1, input signals are described in Sec.III, and all parameter values are the same as described in Sec.V.

In Fig. 6 the process of DFB+2-TF filter adaptation is presented. This figure gives the Signal/(NBI+noise) ratio improvement as the function of the number of LMS algorithm iterations, with the μ as a parameter. Algorithm performs one iteration per input signal sample, and sampling period is equal to the PN chip duration, T. Signal, NBI and noise power are calculated as an average of ten consecutive samples. Starting values of DFB+2-TF coefficients are all set to zero.

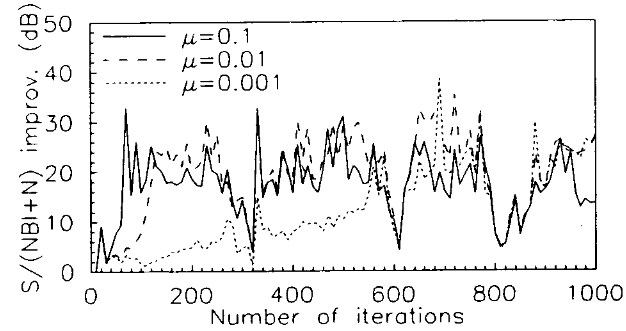


**Fig.6.** *Signal/(NBI+noise) improvement as a function of the number of LMS algorithm iterations, with μ as parameter. Interference is second-order AR on the DSSS carrier frequency. $N=2M=4$, $\Gamma=-20dB$, $A_n=12dB$, $L=7$.*

It can be seen from Fig.6 that smaller values of parameter μ result in slower algorithm convergence, but also perform better regarding the obtainable Signal/(NBI+noise) improvement. Value of μ=0.01 can be regarded as optimal, considering the adaptation speed and achievable Signal/(NBI+noise) improvement. This value of μ enables the DFB+2-TF coefficients to reach the steady state in 100 to 150 algorithm iterations.

Once in the steady state, the average Signal/(NBI+noise) improvement obtained in the

DFB+2TF as the function of Signal/NBI ratio at the filter input and μ as parameter is presented in Fig.7.

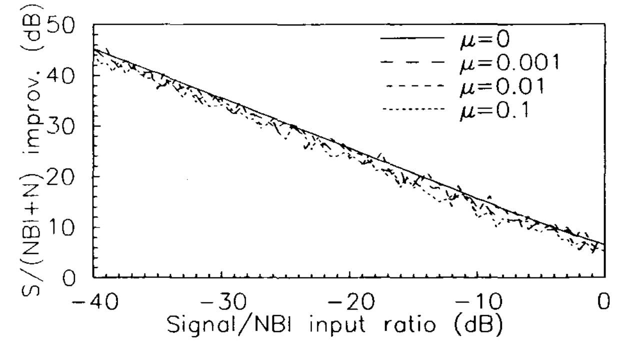


**Fig.7.** *Average Signal/(NBI+noise) improvement in the steady state as a function of the Signal/NBI ratio at the DFB+2-TF input, with $\mu$ as parameter. Interference is second-order AR on the DSSS carrier frequency. $N=2M=4$, $A_n=12dB$, $L=7$.*

It can be seen that the DFB+2-TF adaptation process results in the small Signal/(NBI+noise) improvement degradation, comparing to the case of filter coefficients having the optimal values (μ=0 in Fig.7). The performance degradation is of the order of 1dB, with smaller values of parameter μ resulting in slightly better performance.

In Fig.8 the Signal/(NBI+noise) improvement during the adaptation process is presented with the Signal/NBI ratio at DFB+2-TF input as parameter.

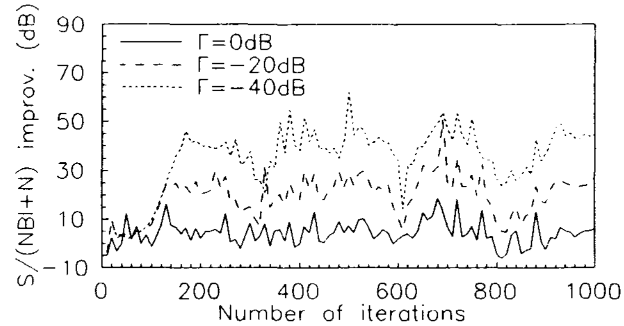


**Fig.8.** *Signal/(NBI+noise) improvement as a function of the number of LMS algorithm iterations, with the Signal/NBI ratio at the DFB+2-TF input as parameter. Interference is second-order AR on the DSSS carrier frequency. $N=2M=4$, $A_n=12dB$, $\mu=0.01$, $L=7$.*

It can be seen from Fig.8 that the Signal/(NBI+noise) improvement is at first rising almost to the obtainable limit for the given input Signal/NBI ratio, and than is oscillating bellow that maximal value. The slope at the beginning of the adaptation process is independent of the Signal/NBI ratio, so more time is needed by DFB+2-TF to obtain the steady state in the case of worst input Signal/NBI ratios.

In Fig.9 the influence of the fourth order AR NBI frequency ofset on the DFB+2-TF adaptation process is presented. Normalized NBI frequency offset is parameter in this figure.

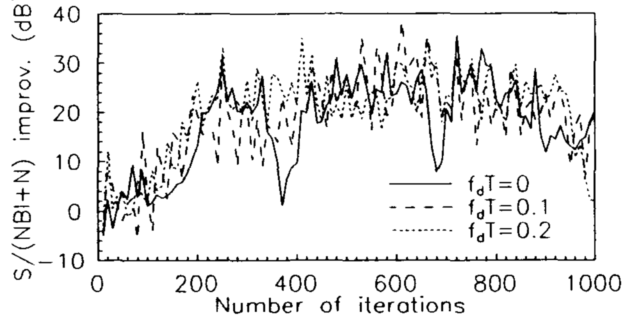


**Fig.9.** *Signal/(NBI+noise) improvement as a function of the number of LMS algorithm iterations, with the fourth order AR NBI carrier frequency to DSSS carrier frequency offset as parameter. $N=2M=4$, $A_n=12dB$, $\mu=0.01$, $\Gamma=-20dB$, $L=7$.*

It can be seen from Fig.9 that the NBI frequency offset does not have a noticeable influence on the DFB+2-TF adaptation speed.

Last group of results present the DFB+2-TF order influence on the bit error probability. The DFB filter order N and 2-TF order M are given as parameters, and the error probability is expressed through the relative error frequency obtained as the average of 100 simulation runs. In Tab.1 the results are given for the second order AR NBI, and in Tab.2 for the fourth order AR NBI with the frequency offset $f_dT=1/L$.

***Tab.1.*** *Relative error frequency for the second order AR NBI as a function of DFB+2-TF filter order. $A_n=12dB$, $\mu=0.01$, $\Gamma=-20dB$, $L=7$.*

| | 2-TF order | |
|---|---|---|
| DFB order | M=1 | M=2 |
| N=1 | $1.754*10^{-3}$ | |
| N=2 | $2.145*10^{-3}$ | |
| N=3 | $2.526*10^{-3}$ | $1.754*10^{-3}$ |
| N=4 | | $2.096*10^{-3}$ |
| N=5 | | $2.096*10^{-3}$ |

***Tab.2.*** *Relative error frequency for the fourth order AR NBI as a function of DFB+2-TF filter order. $A_n=12dB$, $\mu=0.01$, $\Gamma=-20dB$, $L=7$, $f_dT=1/7$.*

| | 2-TF order | |
|---|---|---|
| DFB order | M=1 | M=2 |
| N=1 | $3.953*10^{-3}$ | $5.683*10^{-3}$ |
| N=2 | $3.676*10^{-3}$ | $5.401*10^{-3}$ |
| N=3 | $2.188*10^{-3}$ | $4.580*10^{-3}$ |
| N=4 | $2.549*10^{-3}$ | $5.335*10^{-3}$ |

One can see from Tab.1. and Tab.2. that the DFB+2-TF filter order has the small influence on the overall bit error probability. This is the result of the very good NBI suppression performed by the DFB+2-TF, so the dominant source of error at the filter output is the intersymbol interference caused by the error propagation in the DFB part of DFB+2-TF.

## VII. CONCLUSION

In this paper the preliminary results of the DSSS receiver with the DFB+2-TF interference suppression filter performance in the presence of the narrowband AR interference are presented. The proposed receiver is compared in performance with the DSSS receivers that contain decision feedback or two sided transversal filter only. Receivers are compared regarding the Signal/(NBI+noise) improvement ratio and the bit error probability.

Based on the results obtained, it can be concluded that the proposed receiver obtains the better Signal/(NBI+noise) improvement than the receivers with DFB or 2-TF only. Regarding the bit error probability, the proposed receiver is better, except in the case of the very low input Signal/NBI ratio (less than -18dB), when the receiver with 2-TF has the better performance.

In this paper, the performance of the adaptive DFB+2-TF receiver is also analyzed. On the basis of the results, it can be concluded that the overall good receiver performance is not degraded by the variation of the filter coefficients caused by the adaptation process.

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