MISMATCHED FILTERS IN SPREAD SPECTRUM COMMUNICATION SYSTEMS

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***Abstract** - In this paper the detailed survey of the methods of matched filter sidelobes suppression is presented. The methods of mismatching the receiver by means of the weighting filter which is in cascade behind the matched filter are described first. Though such a procedure is no longer applied, these methods are very significant because they set the basis for all later algorithms for designing mismatched filters. Different methods of filters mismatching are comparatively presented. They can be roughly classified into the class of filters which suppress maximal sidelobes (MINIMAX or MX filters) as well as filters which suppress RMS sidelobes (LS and similar ones). The new algorithms are offered, uniting the properties of MX and LS. The LS algorithms serve as an initial basis and converge towards the MX filters due to the iterative application of adaptive weighting coefficients. This enables considerable simplification of the designing procedure and which is more important, it can be applied to all types of sequences, which was not possible up to now. This algorithm is generalized from the suppression of sidelobes of the correlation function to the suppression of sidelobes of the ambiguity function. In such a way the algorithm for designing the Doppler optimized mismatched filter design is obtained. The paper also contains very detailed list of references in the domain mismatched filters.*

I. INTRODUCTION

Signal coding within a transmitted pulse is often used in radar, sonar and communications systems in order to spread the signal bandwith. Some code sequences can give appreciable processing gain. The major disadvantage is that the compressed pulse has range sidelobes which limit the dynamic range for closely spaced targets. The problem of sidelobe suppression occurs from the very beginning of the radar pulse compression application. In order to conceive more thoroughly the problem and its importance we are going to give a chronological and methodical survey of the previous approaches to its solving. Some of the methods are described in more details owing to the originality of the idea or because of the good results they achieve.

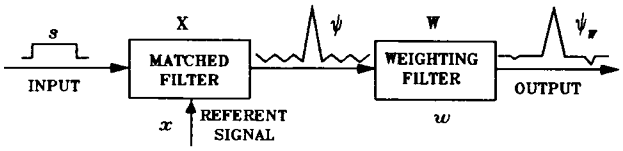

Fig.1: Sidelobe suppression with weighting filter.

In addition to this introduction this paper has eight sections. In the section 2 we analyzed the methods of sidelobe suppression by receiver mismatching, i.e. by filtering the output signal of the mismatched filter (Fig.1). Such methods are older than the others according to the time of their appearance, and they are no longer used in practice. However, they have a well - stated approach and the problem definition as well as the basically indicated methods of mismatching the matched filters itself. In the section 3 we analyzed the problem of matched filters mismatching, or in the other words, mismatched filter design. The application of the LS error criterion to the problem of mismatched filter design is analyzed separately in the section 4. The class of LS methods gives the best results so far. The Iterative Re weighted LS (IRLS) procedure, also presented in this section, yields the solution according to the minimax criterion (it minimizes maximal sidelobes). The Doppler optimized mismatched filter is described in the section 5. The design of such a type of mismatched filters which can suppress sidelobes not only for zero Doppler shift but also for the determined range of Doppler frequencies is of an exceptional importance in the domains of all military and almost all non-military applications. In the section 6 the method of minimax filter design has been presented. The filter DIRLS algorithm application in SS communication systems has been given in the section 7.

II. RECEIVER MISMATCHING OBJECTIVES

In early sidelobes suppression attempts a receiver was mismatched, and not a matched filter. It was achieved by adding the weighting filter for sidelobe suppression in cascade behind the matched filter (Fig.1). We shall refer to such a filter as a mismatching filter.

Temes in 1962 noticed [1] that the sidelobes of compressed pulse could be reduced by receiver mismatching. In such a case, obviously, we must accept certain decreasing of detection characteristics in order to reduce sidelobes i.e. to reduce the probability of false alarm. That mismatching is achieved using the weighting filter for sidelobe suppression in cascade behind the matched filter. The objective is to obtain the desired frequency receiver response.

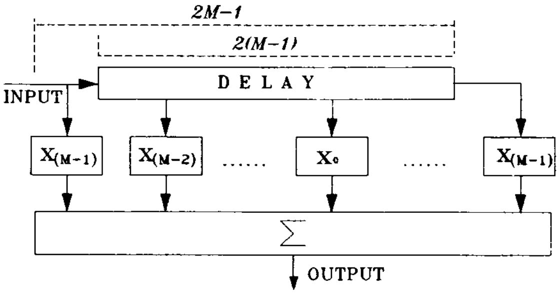

Fig.2: Filter with delay line.

Mosca in 1967 in [2] considered the problem of sidelobe suppression in radars with binary phase coded signals. The filter transfer function could be synthetisized using a filter with delay line (Fig.2). Transfer function of such filter is:

$$W(f) = e^{-j2\pi(M-1)f} \sum_{m=-(M-1)}^{M-1} x_m e^{-j2\pi m f}, \quad (1)$$

where *2M* is the total number of weighting coefficients x_m and *f* is the input signal frequency.

McAulay and Johanson [3] obtained a similar solution, four years latter although they did not refer to the earlier Moskin's work. Their optimal filter contains the matched filter followed by the transversal filter, the so-called transversal equalizator. The calculation of filter coefficients for the particular objective function is based on linear programming techniques.

Rihaczek and Golden in 1971 using the analysis procedure in frequency domain like Mosca founda solution for the filter which suppresses sidelobes on the matched filter output [4]. Good results are obtained for acceptable filter lengths. The method is applicable only to Barker's sequences which have the positive sidelobes of the autocorrelation function (ACF). It is applicable to the combinations of such Barker's sequences as well.

They obtained the coefficients of the filter with the following transfer function in Z domain:

$$H(z) = A_0 + \sum_{m=1}^{M} \frac{A_m}{2}(z^m + z^{-m}) \quad (2)$$

The realization of this function is given in Fig.3. The filter consists of M adding elements, M multiplier, and 2 times 2M dely elements.

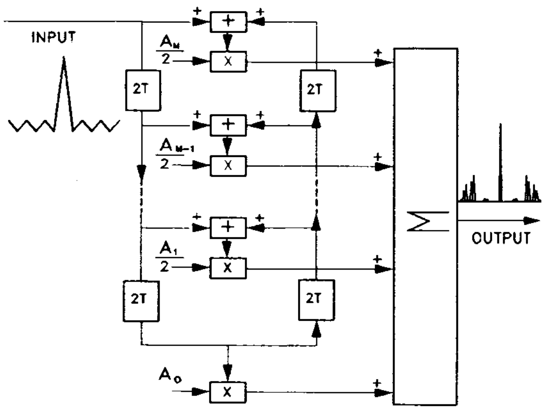

Fig.3: Schema of R-G filter.

In order to enable modeling, we have transformed Rihaczek - Golden (R-G) filter from Fig.3 into the equivalent canon form of the filter with the finite impulse response (FIR), see Fig.4. This equivalent filter consists of equal numbers of delay elements and twice more multipliers.

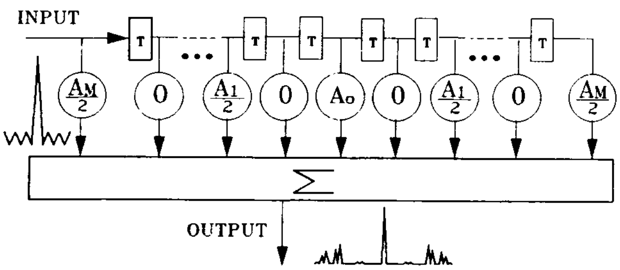

Fig.4: Schema of FIR filter equivalent to The R-G filter.

Efficiency comparison of different methods for sidelobe suppression is based on the relative mismatched filter length, or the matched filter length + the length of filter with weight function in the case of mismatched receiver. There is a dilemma what the length of *Rihaczek - Golden* filters: the number of delay elements or number of coefficients? From the point of realization, the most acceptable solution is obtained by the equivalent schema. According to this, the exact length is equivalent to the number of delay elements. From here follows that in canonical form every second element would be "0". It is very important to be aware of this when comparing results. Comparative analyses show that the sidelobe suppression with The R-G filters is not so attractive as it seems in the first moment.

Hua and Oksman [6] have chosen the reduced version of the R-G filter as a subject of optimization. The reduced filter is based on only first four elements of the transfer function developed in convergent exponential series. The authors designated such filters as the $(R\text{-}G)_{opt}$. The results obtained with such filters were compared to results of minimax filters designed by linear programming [7]. Such comparison is not correct, because it suggests that the $(R\text{-}G)_{opt}$ filters obtain quoted results with the basic filter length equal N, which is not true. Which realization of the R-G or the $(R\text{-}G)_{opt}$ filter is chosen - either direct, canonical or reduced - such filters are either longer or more complicated for realization.

III. MATCHED FILTER MISMATCHING OBJECTIVES

In the previous section we have presented the most important ways of sidelobes suppression based on receiver mismatching. Such a way does not only complicate the realization and compound the receiver, but at the same time makes the equipment more expensive and enlarge its dimensions. Here the need for mismatched filter design which could make the compression filter simpler and less expensive has been recognized. So, the objective is to design a filter which simultaneously performs compression and mismatching according to the given criteria. Such solutions are not only economical, but also give better overall results.

The ideas for mismatched filters design have come from the field of signal deconvolution. The presentation of some works which treat this fruitful approach follows.

Deconvolution method

Acroyd and Ghani in 1973 published the paper [8] which was the turning point in the field of sidelobes suppression in radars with pulse compression. It turned to be the most quoted paper in works concerning mismatched filters up to that time. The authors applied geophysical methods [9,10], which were not familiar enough to researchers in the field of radars and telecommunications. This was correctly pointed out by the authors. We think their merit for the perception of the solutions from a completely different field at first glance is indisputable. This case shows that similar and different fields hide questions and potential answers which are nearly the same.

In seismic and medical ultrasonic, sound signals propagation is used in order to determine a media structure by signal analyzing. Mostly, the objective is the estimation of media pulse response. Geophysicists **Treitel and Robinson** [9] have analyzed non recursive filters design in a way that the pulse response of the given input sequence approximates the desired output satisfying the criterion of the least mean square (LMS) error. **Cavin at all.** [10] have developed optimal convolution filters design for the geophysical Ricker wave using linear programming. This idea was later implemented on the radar problem by **Zoraster** [7].

Besides the mentioned geophysical methods, the application of methods from the field of automatics mismatched filters design is also very interesting. This approach has been suggested by **Mese and Giuli** [12] from the Italian school of automatics. The proposed method can be applied to complex, i.e. polyphase sequences. It must be pointed out that this approach had nonsignificant response. Anyway, this method has to be mentioned, because it could not be reduced to the other methods. The main disadvantage is related to the method complexity and the absence of advantages in relation to other known methods.

ECF as mismatched filter

The analogy between the antenna sidelobe suppression and the matched filter suppression has been used from the very beginning. It achieved the most complete and fruitful form in works of **Evans and Fortman** [11]. The problem could be defined as time invariant FIR filter design.

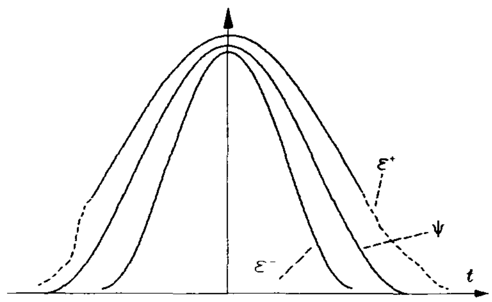

Fig.5: Envelope constraint.

The output signal $\psi = s * x$, where "*" means convolution, has to be put in the pulse envelope with the upper and lower bounds ϵ^+ and ϵ^- (see Fig.5). Optimal Envelope Constrained Filter (ECF) is defined as a filter which minimize output noise power keeping acceptable pulse shape.

This kind of problems is usually solved by minimizing the mean square difference between ψ and the desired pulse shape. However, in a number of applications, a "soft" LS approach is unacceptable because there could appear narrow but high jumpings out of the desired pulse shape. The solution could be sensitive to the structure of desired pulse. Moreover, it is not often evident which shape could lead to solution.

The optimal filter can be defined by $\hat{x} = -(S\hat{\lambda})/2$, where λ is the vector of Lagrange multipliers, $\hat{}$ designates approximation and $\hat{\lambda} = \hat{\lambda}^{+} - \hat{\lambda}^{-}$.

Testing this method on several sequences has shown that for some sequences very good results and for some very bad results are obtained. Thus, we can conclude that, in spite of an attractive idea and very simple realization, the ECF method does not give acceptable results for radar applications. But we can not exclude a possibility that a modification of this method could give better results.

In [19] **Teo, Cantoni and Lin** have introduced anew method for the optimization of envelope - constrained filters with uncertain input (ECUI). This problem has been examined elsewhere, but the proposed algorithm for its solution has, in general, inferior convergence characteristics.

IV. LS ALGORITHM APPLICATION

In the previous section we have mentioned LS filters which minimize the square error of filter response. In this section, LS algorithm will be explained in more details. Most of good results in sidelobe suppression has been obtained with LS filtering.

Definition:

We have the sequence $\mathbf{s} = (s_1, s_2, \ldots, s_N)^T$ which can be complex, and have to find the coefficients of the desired FIR filter $\mathbf{x} = (x_1, x_2, \ldots, x_M)^T$. Then the expression for the filter response or convolution $\psi = (\psi_1, \psi_2, \ldots, \psi_{N+M-1})^T$ forms the set of linear equations which may be presented in matrix form:

$$\mathbf{S}\mathbf{x} = \psi \,, \tag{3}$$

where

$$\mathbf{S}_{f_i} = \begin{bmatrix} s_1 & 0 & 0 & \ldots & 0 & \ldots & 0 \\ s_2 & s_1 & 0 & \ldots & 0 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_N & s_{N-1} & \ldots & s_2 & s_1 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & s_N & \ldots & 0 \end{bmatrix}_{(N+M-1)\times M} \tag{4}$$

where N is the sequence length and M is the filter length ($M \geq N$).

If it is chosen that the filter response ψ is equal to the pulse sequence δ then the set of $(N+M-1)$ linear equations with N unknown x_i will form

$$\mathbf{S}\mathbf{x} = \delta \tag{5}$$

The relation for LS filter coefficients estimation can be found which approximates the filter with an ideal response:

$$\hat{x} = (S^T S)^{-1} S^T \delta \tag{6}$$

B-C optimal mismatched filter

Looking for perfect binary sequences whose ACF-s have zero sidelobe level, Marvin N. Cohen and Philip E. Cohen have searched all possible binary sequences with the length of 14 to 48 which give the minimal level of maximal sidelobes. For those sequences, the authors have analyzed sidelobe suppression. **Baden and Cohen** in [13] have described a method which gives excellent results for biphase sequences. The description of Baden - Cohen (B-C) method follows.

For the input sequence $\{s_i\}$ the authors search for filter coefficients $\{x_i\}$ which minimize Integrated Side Lobe (ISL) levels. They named such filters the optimal ISL filters. In case when maximal sidelobes level is minimized, they use the term of optimal Peak Side Lobe (PSL) filters. ISL is related to sidelobe energy, so its minimization is related to LS. Thus, the ISL filter is adequate to the LS filter and the PSL filter is equivalent to the minimax filter.

Optimal ISL filter:

Let $s = \{s_i\}$ be a vector which describes the known input sequence, and let $x = \{x_i\}$ be a vector which describes the unknown filter's coefficients. CCF, i.e. filter response is given by the vector $\psi = \{\psi_i\}$, and $w = \{w_i\}$ is the weighting vector. The problem solution is similar to Evans's one [11].

The filter coefficients could be obtained by

$$X = \frac{\lambda}{2} Y^{-1} S \,, \tag{7}$$

where $Y = [Y_{k,l}]$ whose elements are defined by:

$$Y_{k,l} = \sum_{i=-p}^{p} (w_i^2 s_{l+i} s_{k+i}) - w_0^2 s_l s_k \,,$$

$$1 \leq k, l, < p. \tag{8}$$

S is the signal matrix defined by (4), and the constant λ is the Lagrange multiplier. In (7) $\lambda/2$ is a scaling factor which does not influence the filter structure. So, it could take an arbitrary value.

Optimal PSL filter:

That filter minimizes maximal sidelobes. The iterative procedure is used to adopt the weighting function w in (8). For binary sequences this method produces nearly the same results as the method described in the following subsection.

Iterative Reweighted Least Square (IRLS)

To avoid nonlinear problems in minimax filter design, **P. Rapajić** and **A. Zejak** proposed the IRLS procedure [14,15] which can be used for real as well as complex codes. Using this procedure we can obtain minimax filter coefficients:

$$\hat{x}(n)=[S^H(0)R(n-1)S(0)]^{-1}\cdot$$

$$\cdot S^H(0)R(n-1)d^*(0) \qquad (9)$$

where S(0) is a signal matrix which is constant and therefore it has the index *"0"*. *R(n)=diag(r(n))*, where *r(n)* is the weighting vector.

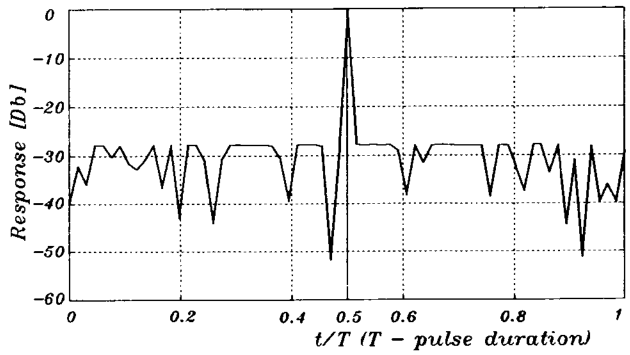

Fig.6: Sidelobe suppression with IRLS for 34-element B-C sequence. Maximal SL=-27.81dB; suppression =-6.72dB.

The main advantage of this method is that it solves a very important problem of sidelobe suppression for complex sequences. At the same time, it produces equally good results for particular sequences as the methods which solve only those sequences. For example, B-C optimal filter, which is especially projected for B-C 'perfect' binary sequences, produces identical results [13] as the IRLS procedure (see Fig.6).

V. MINIMAX FILTERING

The methods considered till now are based on the minimization of total energy distributed in sidelobes of compression filter response. Such filters, being based on the minimization of total energy of sidelobes, allow for a small number of very high jumpings of sidelobes. Although those singular jumpings contain low energy, they could exceed detection threshold and cause false alarms. Because of that, compression filters which minimize maximums of sidelobes (called minimax filters) are more acceptable in radar systems. Design methods of minimax filters are based on linear programming algorithms [7,10]. As it seems, they are applicable to binary phase coded signals only.

Let us formulate the problem of filter design in a way which allows the linear programming approach. The transmitted binary coded signal could be presented using real elements s_i such that $s_i = \pm 1$ and $1 \le i \le N$. There are M filter coefficients and they are represented with x_i such that $M \ge N$. Using this notation, let us consider the following problem:

$$\textit{maximize} \quad \sum_{i=1}^{M} x_i s_i^{-(M-N)/2}$$

subject to

$$\sum_{i=1}^{M} x_i s_i^{(-k)} < 1 \qquad (10)$$

$$-\sum_{i=1}^{M} x_i s_i^{(-k)} \le 1, \quad 1-N \le k \le M-1, \quad k \ne (M-N)/2.$$

In (10) we have a linear objective function with *M* variables and *2(M+N-2)* linear inequality constraints. This type of optimization problem is classified as a linear programing problem. Any linear programing problem which has a finite solution can be solved in finite number of steps by an iterative and monotonic "simplex" algorithm. **Zoraster** [7] applied linear programing for minimax filter design.

For binary sequences, according to published papers, the best sidelobe suppression is achieved using minimax filters produced using linear programming algorithms. Using them as reference, we shall compare several concurrent methods for mismatched filters design on 13-element Barker sequence.

Let us compare minimax filters to ILRS filters, proposed by Rapajić and Zejak [14,15], and to LS filters. For 13-element Barker sequence, the coefficients of MX and ILRS filters are nearly identical. This is very important for the verification of ILRS method. It shows that ILRS method efficiently replaces complicated algorithm based on linear programming. What is more important, ILRS method solves sidelobe suppression of complex sequences.

Comparative characteristics of mismatched filters for zero Doppler of 13-element Barker sequence are given in Tab.1. It could be seen that LS filter has excellent suppression of sidelobes, which is its main intention.

As conclusion, we could say that ***IRLS filters, according to minimizing maximal sidelobes criteria, are minimax filters.***

Tab.1: Comparative characteristics of mismatched filters for zero Doppler of 13-element Barker sequence.

Filter type	max SL (dB)	RMS SL (dB)	suppress SL (dB) MX	suppress SL (dB) RMS	SNR loss (dB)
MF	*-22.279*	*-39.091*	...	...	...
MX	*-25.691*	*-41.245*	*3.412*	*2.154*	*0.222*
IRLS	*-25.762*	*-41.243*	*3.483*	*2.152*	*0.223*
LS	*-24.005*	*-43.286*	*1.726*	*4.195*	*0.143*

VI. DOPPLER OPTIMIZED IRLS ALGORITHM

The optimization procedure for the mismatched filter in a given Doppler band can be defined as a procedure of forming a suitable shape of the ambiguity function. Unlike standard filters, where the object of shaping is the correlation function, in this type of filters the object is the ambiguity function, or rather its sector.

Assuming that s_f is a column vector which describes the signal sequence for the particular Doppler shift of frequency f, then

$$s_f = [s_{1,f} \cdot s_{i,f} \cdot s_{N,f}]^T \;;$$

$$s_{i,f} = s_{i,f=0} e^{(j2\pi f i/N)}, \qquad (11)$$

where $i=1, 2, ..., N$, N is the sequence length, and $[\cdot]^T$ stands for transpose. The relation (11) gives the generalization of the signal sequence. In a similar way, the filter response can be described as:

$$\psi_f = (\psi_{1,f}, \dots, \psi_{i,f}, \dots, \psi_{(N+M-1),f})^T, \qquad (12)$$

where ψ_f is the matched filter response for a particular Doppler shifted frequency and M is the filter length.

If a matrix is formed in such a way that its rows are the filter response as in (11),

$$\Psi_\Phi = (\psi_{f_1}, \dots, \psi_{f_i}, \dots, \psi_{f_P})^T, \qquad (13)$$

where $f_i \in \Phi$, $i = 1, 2, \dots, P$, P is the number of a particular Doppler shifts, then Ψ_Φ will represent a sector of the digitized ambiguity function. The desired ambiguity function corresponds to the desired filter response,

$$\Delta_\Phi = (d_{f_1}, \dots, d_{f_i}, \dots, d_{f_P})^T, \qquad (14)$$

where d_{f_i} is the desired filter response for a particular Doppler shifted frequency.

Also, the block matrix corresponds to the signal matrix,

$$S_\Phi = (S_{f_1}, \dots, S_{f_i}, \dots, S_{f_P})^T, \qquad (15)$$

where S_{f_i}(16) is the signal matrix for a particular Doppler shifted frequency, N is the sequence length and M is the filter length ($M \geq N$)

The IRLS algorithm can be generalized in the following way:

$$\hat{x}(n) = [S_\Phi^H(0) W_\Phi(n-1) S_\Phi(0)]^{-1} \cdot$$

$$\cdot S_\Phi^H(0) W_\Phi(n-1) \Delta_\Phi(n-1), \qquad (16)$$

where $\hat{x}$ is estimated filter coefficients, the upper script H stands for the Hermitian matrix. $W(n)$ is the block matrix, made of diagonal matrices $R(n)=diag(r(n))$, where $r(n)$ is the weighting vector. The window function which is included in the matrix **W** can be understood as a corrective factor of the LS algorithm.

The analysis of implementation of the proposed algorithm is carried out also for the most significant and well known phase coded signals in radar systems: such as Frank polyphase, P1, P2, P3, P4, Barker's binary and others.

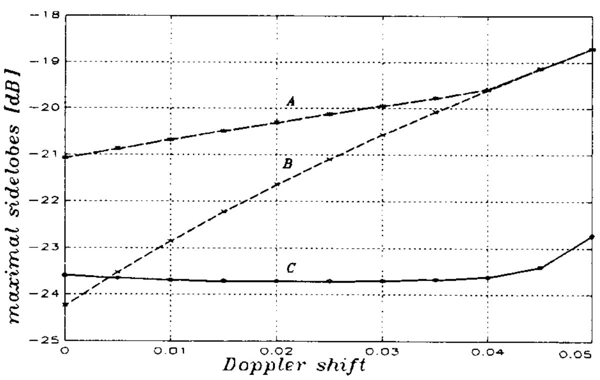

***Fig.7** The maximal SL levels as the function of the Doppler shift. Comparative characteristics for A - Matched, B - IRLS and C - DIRLS filter for the 16-element aperiodical P2 sequence. The Doppler shift normalized as $f_d T$, where T is pulse duration. The SL normalized as 10log(sidelobes/mainlob).*

It can be noticed that the DIRLS filter can suppress very successfully the maximal levels of the SL. Among all the sequences which have been analyzed, the highest suppression has been achieved for the P2 sequence (length 16) with normalized Doppler shifted frequency

$f_d = 0.05$. The amount of suppression was 4.019 dB. In Fig.7, the Doppler characteristic of the maximal SL levels is presented.

VII. DIRLS ALGORITHM APPLICATION IN SS COMMUNICATION SYSTEMS

Dealing with the optimization of mismatched compression filters in the SS radar systems, the possibilities to apply developed algorithms for solving some problems in other types of SS communication systems appeared [17,18].

The channel capacity of the SS CDMA communications systems is directly affected by the SL level of ACF and by the maximal level of interfering signal crosscorrelation functions (CCF). The channel capacity is defined as the permitted number of SS CDMA signals in the same frequency range and depends in every case directly on the level of correlation functions. On the other hand, the SLs increase drastically with the increase of the Doppler shifts of frequency. From the analysis of these facts came the idea to apply the Doppler optimized mismatched filters to the suppression of SL in the SS CDMA systems.

The response of the matched filter with the periodical orthogonal sequence for the zero Doppler shifted frequency does not have sidelobes. More precisely, the periodical ACF of orthogonal sequences have SLs only on their edges. The practice often imposes the use of non ideal periodical sequences, i.e. sequences whose SLs of a periodical ACF are not equal to zero. That is to say, practically there are not binary periodical sequences (except the 4-bit Barker) with an ideal periodical ACF, and on the other hand devices with binary sequences are the easiest to be realized in practice.

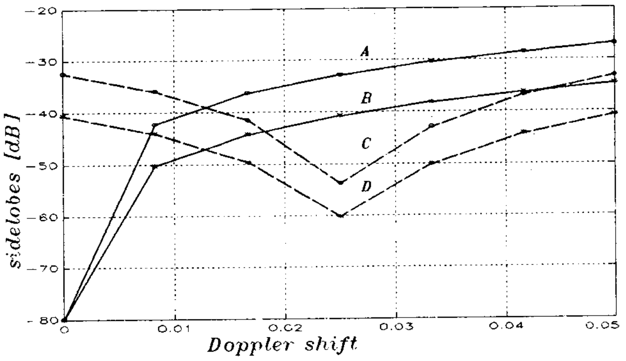

*Fig.8 SL of the periodical matched and DIRLS filters responses as the function of the normalized Doppler shift. **A** - maximal SL of MF, **B** - RMS SL of MF, **C** - maximal SL of the DIRLS filter and **D** - RMS SL of the DIRLS filter.*

The algorithm of periodical sequence mismatching is made basically of the algorithms of aperiodical sequences mismatching described by (16). Therefore their relations are similar or the same so there is no need to explain them again in detail. The speciality of the DIRLS algorithm for periodical sequences when compared to the algorithm for aperiodical sequences, apart from the signal matrix S, is to be found only in the corresponding periodical ACF. All expressions, thus, remain the same and the content of the signal matrix and ACF corresponds to the periodical sequence.

The DIRLS algorithm of filtering was applied to the set of orthogonal SS CDMA sequences, whose length is 25. The method is applicable to other classes of sequences and there is no essential difference in behavior of this method depending on the sequence class. It is important to notice the characteristic effect which we did not find in aperiodical sequences. In other words, periodical SLs of the mismatched filter symmetrically decrease towards the center of the Doppler range where the optimization was made. This unexpected effect is clearly seen in Fig.8 where the maximal and mean square periodical SLs are comparatively shown, for the matched and mismatched filters. It is important to notice that this phenomenon regards the maximal as well as the mean square SL levels.

VIII. CONCLUSION

In this paper the detailed investigation of the method of the matched filters sidelobes rejection in modern radar and spread spectrum communication systems are presented. The main results of the above analysis are as follows:

1. The method of the mismatched filter application in the process of the sidelobes rejection is very attractive and efficient.
2. It was showed that the new DIRLS algorithms can be the optimized mismatched filters for all kind of complex sequences, which is not the case with the other known algorithms.
3. The DIRLS filters suppress maximal sidelobes in accordance with the minimax criterion, so they can be classified as minimax filters although the initial criterion was the LS error criterion.

In special cases, the DIRLS algorithm gives the following mismatched filters:

- IRLS minimax filters which suppress the highest sidelobes, optimized for zero Doppler shifted frequency. It is important for the cases where the Doppler shifted frequency is near zero. In that case still even suppression of the maximal sidelobes can be achieved.
- LS filters which suppress RMS sidelobes level optimized for zero Doppler shifted frequencies.

- DLS filters which suppress RMS sidelobe level optimized for a given Doppler shifted frequency bend.

5. A variant of the DIRLS algorithm is developed for the mismatched filters for periodic sequences, which can be of wide application in SS communication.

6. The DIRLS algorithm can be used in designing matched filters. It was successfully used to design multilevel and periodic sequences, [15,18].

By applying the DIRLS algorithm for ***suppression of periodical*** sidelobes used in the SS CDMA communications, very significant results are obtained for the non - zero Doppler shifted frequency. For the investigated sequence with the normalized Doppler shifted frequency $f_D = 0.025$, the suppression of 20.34 dB is achieved. However, it should be emphasized that the DIRLS filter will have periodical sidelobes for the zero Doppler shifted frequency while the matched filter will not have them at all. Thus the DIRLS filter for the zero Doppler shifted frequency considerably worsens the self - clatter. This has to be taken into account in the application of the DIRLS filter in the SS CDMA systems. The suppression of the crosscorrelation interference is affected about 0.5 dB. But it should not be forget that the investigation was made for the orthogonal sequence which by its composition already has the minimal cross correlation.

The theoretical capacity of the SS CDMA channel is greatly affected by the interference between channels, i.e. crosscorelation sequences and the feature of the particular matched or mismatched filter influence considerably the level of sidelobes. That is why the existing theoretical apparatus for the analysis of the SS CDMA system with matched filters is to be generalized and made applicable for the analysis of the SS CDMA system with mismatched filter.

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