An adaptive guidance technique for the aeroassisted re-entry for a space vehicle

G. Ambrosino
Computer Science and System Department - University of Naples "Federico II" - via Claudio 21, 80125 Napoli, Italy
U. Ciniglio, F. Ferrara, E. Filippone, L. Verde
C.I.R.A. (Italian Aerospace Research Center) - Via Maiorise - 81043 Capua (CE), Italy

Abstract

In this work a new technique for the determination of the optimal re-entry trajectory of a space vehicle is proposed. In the proposed approach both conventional optimization techniques and modern "artificial intelligence" techniques are used in order to meet a satisfactory compromise between optimality of the solution and moderateness of the computational burden. Such a technique can then be used for on-line guidance algorithms.

Simulation results showing the performances of the proposed approach are presented with reference to the optimal reentry guidance problem of a space vehicle with aerodynamic capabilities.

1 Introduction

In the recent past, different powerful guidance and control techniques for moderate lift/drag vehicles have been investigated in order to perform aeroassisted orbit transfer or re-entry manoeuvres.

For both tasks, the problem can be set up in the following general form: to find the best way to drive the vehicle from the current position to a the desired one, minimizing or not exceeding the structural and thermal loads. Manoeuvre limitations are additional constraints.

To solve this problem, the first is to find an "optimal" guidance law, i.e. the commanded angle af attack and bank angle which drive the vehicle along the "optimal" trajectory.

Due to the atmosphere perturbations and to the high aerodynamic non-linearities, both the guidance and control laws have to provide a high degree of adaptiveness, i.e. an on-line computation of the guidance and control commands is required.

As far as the guidance problem is concerned, the most common approach used to solve this kind of problem is the so called Calculus of Variation (COV). The main difficulties arising in trying to solve the optimal guidance problem with the COV techniques are essentially:

- flight path constraints are not easy to implement;
- the computational load for the numeric treatment of the analytic necessary conditions is very heavy.

There are alternative approaches that partially alleviate some of these problems. Some of them (like Non Linear Programming (NLP) and Predictor Corrector) are based on a crude numerical approach and make assumptions on the structure of the state and of the control in order to relieve the computational burden. Some others (like Asymptotic Expansion and Singular Perturbation Techniques) are based on a mixed analytical numerical approach. They take advantage from the peculiarities of the problems (the presence of two time scale variables) and allow an analytical or greatly reduced numeric computation in obtaining the solutions.

In this work, the authors propose a new method for the solution of the adaptive guidance problem based both on classical optimization techniques and on artificial intelligence techniques, the latter utilising the so called "neural networks". The proposed approach is based on the Bellmann's principle and on "learning" capabilities of a neural network in approximating non linear functions. A well compromise between the optimality of the solution and the computational burden is reached.

In §2, the proposed method with reference to a generic optimal control problem is described; in §3 basic concepts on neural networks are quoted. In §4 the problem of the determination of optimal re-entry trajectory of a space vehicle is formulated as an optimal control problem. Finally, in §5, simulation results showing the performances of the proposed approach are presented.

2 Mathematical formulation of the problem and proposed solution technique

Many guidance problems, such as optimal re-entry trajectory determination or optimal orbit transfer, can be expressed in terms of an optimal control problem, stated as follows:

$$\begin{aligned} \min J(x_o, t_o, u(\cdot)) &= m(x(t_f), t_f) + \quad (1a) \\ &+ \int_{t_o}^{t_f} L(x(t), u(t), t)dt \\ \text{s.t.:} & \\ \dot{x}(t) &= f(x(t), u(t), t) \\ & \quad x(t_o) = x_o \quad (1b) \\ g(x(t), u(t), t) &\leq 0 \quad \forall t \in [t_o, t_f] \quad (1c) \\ h(x(t), u(t), t) &= 0 \quad \forall t \in [t_o, t_f] \quad (1d) \end{aligned}$$

where

x is the vector of state variables,

u is the vector of control variables.

By using the Bellmann's Principle of Optimality, one can state that both the optimal control segment $u^*(\cdot)_{[t_o,t_f)}$ and the optimal value of the performance index J depend only on initial conditions $(x(t_o), t_o)$. Then we can write:

$$V(x,t) = J^*(x,t) = J(x, t, u^*(\cdot)_{[t,t_f)}) \qquad (2)$$

with $V(x,t)$ solution of Hamilton-Jacobi equation for the problem (1) [11].

If we indicate with $\varphi(t, t_o, x(t_o), u(\cdot))$ the state transition function for the system (1 b), the Bellmann's principle yields:

$$\begin{aligned} V(x,t) = \min\nolimits_{u(\cdot)_{[t,t+\Delta t)}} \{ & J(x, t, u(\cdot)_{[t,t+\Delta t)}) + \\ & V(\varphi(t+\Delta t, t, x, u(\cdot)_{[t,t+\Delta t)}), t+\Delta t) \\ & \mid g(x(\tau), u(\tau), \tau) \leq 0, \\ & h(x(\tau), u(\tau), \tau) = 0, \forall \tau \in [t, t+\Delta t)\} \qquad (3) \end{aligned}$$

Equation (3) points out that it is possible to determine the optimal control in the time interval $[t, t+\Delta t)$ once the function $V(x,t)$ is known for each value of x and t. Under this hypothesis it is possible to find the solution of the problem (1) in a closed loop form. In such a case, in fact, by approximating the control function $u(\cdot)$ in the time interval $[t, t+\Delta t)$ with a function belonging to a predefined family $u_A(p, \cdot)_{[t,t+\Delta t)}$, completely characterized by a parameter vector p, a

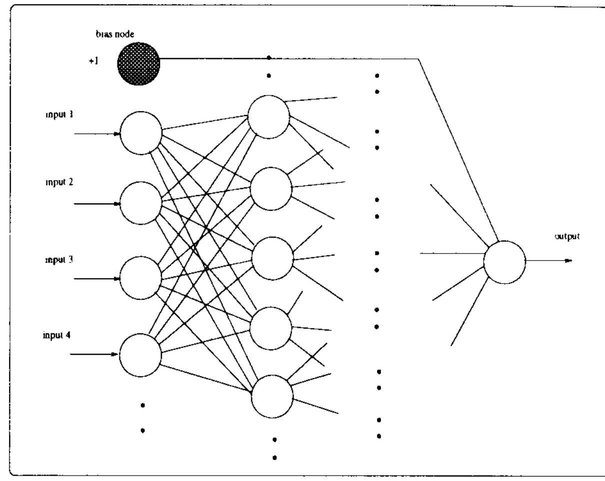

Figure 1: A schematic neural network

sub-optimal control segment $u^*(\cdot)_{[t,t+\Delta t)}$ could be obtained by solving the following parametric optimization problem:

$$\begin{aligned} \min_p \quad \{ & J(x, t, u_A(p, \cdot)_{[t,t+\Delta t)}) + \\ & V(\varphi(t+\Delta t, t, x, u_A(p, \cdot)_{[t,t+\Delta t)}), t+\Delta t) \\ & \mid g(x(\tau), u_A(p,\tau), \tau) \leq 0, \\ & h(x(\tau), u_A(p,\tau), \tau) = 0, \forall \tau \in [t, t+\Delta t)\} \quad (4) \end{aligned}$$

Unfortunately, the determination of $V(x,t)$ as a solution of the Hamilton-Jacobi equation is almost impossible. To overcome this drawback, we propose to properly train a neural net in such a manner to obtain a well approximated description of the function $V(x,t)$ needed to solve problem (4).

3 The neural networks

Multilayer feedforward neural networks (MFNN) are tipically used to approximate complex non linear functions; to this aim the neural net must be "trained" by means of an adeguate number of samples of the functions to be learned [12]. The approximation is obtained by superposition of suitable non linear multi input-single output functions, organized in a multilayer, multielement structure (see fig. 1).

Recently, it has been proved that, under wide hypotheses, such nets are "universal approximators" [13]. There are two kinds of problems to deal with in using neural net:

1) representation problem, consisting in the choice of the kind of non linear function to be implemented in elementary units of the net, of the number of

layers and of the number of units per layer. As far as the elementary non linear functions are concerned, they can be global (sigmoid, hyperbolic tangent, etc.) or local (i.e. $y = exp(-||x - c||)$.

2) training problem, consisting in the choice of the training algorithm for the calibration of the weights of the net. The algorithm used in our work is the classic *back propagation* algorithm [14]. This technique implement the so called *delta rule*, in which the weights are determined so as to minimize the error function:

$$E = \frac{1}{2}(y - y^{NN})^2 \tag{5}$$

where y is the desired value or function value and y^{NN} is the neural net output.

4 Aeroassisted re-entry problem as an optimal control problem

From a mathematical point of view, a guidance problem of a vehicle is completely defined when the model of the vehicle is specified together with the constraints on the path and the cost function.

In this work we examine the re-entry problem of an aerodynamic manoeuvrable space vehicle, which formulation is stated below.

The vehicle model

The equations of motion are described with reference to the point mass hypothesis in terms of the following state variables (see fig.2):

- distance r to the center of the Earth
- longitude θ
- latitude ϕ
- velocity V relative to the airspeed
- flight path angle γ
- heading angle ψ.

The point mass motion is then described by the equations:

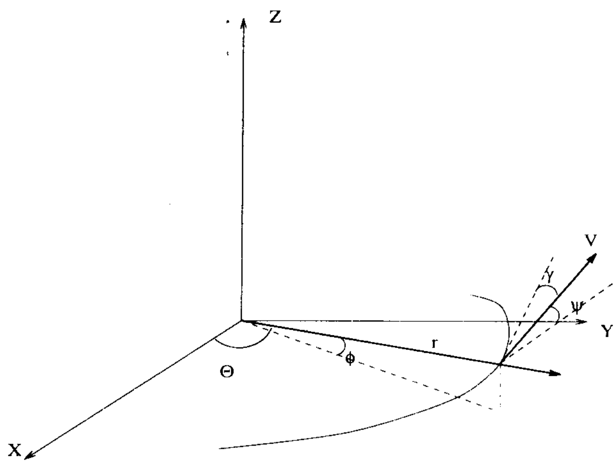

Figure 2: Reference axes

$$\begin{aligned}
\dot{r} &= V \sin\gamma && \text{(6a)}\\
\dot{\theta} &= \frac{V\cos\gamma\cos\psi}{r\cos\phi} && \text{(6b)}\\
\dot{\phi} &= \frac{V\cos\gamma\sin\psi}{r} && \text{(6c)}\\
\dot{V} &= -\frac{D}{m} - \frac{\mu_g \sin\gamma}{r^2} + \omega_E^2 r\cos\phi(\sin\gamma\cos\phi - \\
&\quad \cos\gamma\sin\phi\sin\psi) && \text{(6d)}\\
\dot{\gamma} &= \frac{L\cos\sigma}{mV} - (\frac{\mu_g}{r^2} - \frac{V^2}{r})\frac{\cos\gamma}{V} + 2\omega_E\cos\phi\cos\psi\\
&\quad + \frac{\omega_E^2 r}{V}\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi) && \text{(6e)}\\
\dot{\psi} &= \frac{L\sin\sigma}{mV\cos\gamma} - \frac{V}{r}\cos\gamma\cos\psi\tan\phi\\
&\quad + 2\omega_E(\tan\gamma\cos\phi\sin\psi - \sin\phi)\\
&\quad - \frac{\omega_E^2 r}{V\cos\gamma}\sin\phi\cos\phi\cos\psi && \text{(6f)}
\end{aligned}$$

In the model we explicity take into account the aerodynamic forces of lift (L) and drag (D), expressed as:

$$\begin{aligned}
L &= \frac{1}{2}\rho V^2 S C_L && \text{(7a)}\\
D &= \frac{1}{2}\rho V^2 S C_D && \text{(7b)}
\end{aligned}$$

where C_L e C_D are nondimensional aerodynamic coefficients, assumed known.

The gravitational field is hypothesized to be a newtonian field with:

$$g = \frac{\mu_g}{r^2} \tag{8}$$

Finally, for the computation of air density ρ the US76 Atmosphere model has been utilized.

The control variable assumed in our example is the bank angle σ.

The constraints

Two constraints has been considered in solving our optimal guidance problem: a first one on the structural limit of the vehicle and expressed as:

$$n = \frac{\sqrt{L^2 + D^2}}{g} \leq n_{max}, \quad \forall t. \tag{9}$$

A second one defines the height to which the re-entry phase finishes and is expressed as:

$$r(t_f) - R_E = h_f \tag{10}$$

The cost function

A typical objective of a re-entry problem is the maximization of the cross-range, defined in our work as the distance of the final position from the initial great circle, centered in the origin of the inertial axes and tangent to the initial heading of the vehicle. With reference to fig.3, in mathematical terms we can write:

$$CR = r(t_f) \arcsin(\sin LF \sin \xi) \tag{11}$$

where

$$\xi = \zeta - \sigma_o \tag{12a}$$

$$\zeta = \arctan\left(\frac{\sin(\phi(t_f) - \phi_o)\cos\phi(t_f)\cos\phi_o}{\sin\phi(t_f) - \sin\phi_o\cos LF}\right) \tag{12b}$$

$$LF = \arccos(\sin\phi(t_f)\sin\phi_o + \cos\phi(t_f)\cos\phi_o\cos(\theta(t_f) - \theta_o)) \tag{12c}$$

5 Case study

The proposed approach has been applied to the problem of the computation of optimal re-entry trajectoriy of a space vehicle as formulated in §4.

The geometric and aerodynamic characteristics of the vehicle utilized in our study, identified as "non winged return vehicle", are described in table 1. Table 2 details the initial conditions of the vehicle at the start of the re-entry manoeuvre.

The first step of the proposed approach consists in training a neural net about the function $V(x, t)$ defined by eqn.(2).

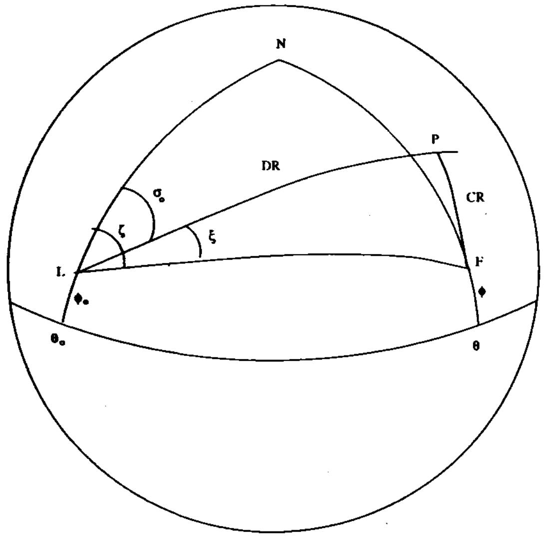

Figure 3: Cross-range definition

mass	8000 Kg
reference area	16.296 m^2
C_L	0.51594 ($\alpha = 20^o$)
C_D	0.4733 ($\alpha = 20^o$)
L/D max	1.09 ($\alpha = 20^o$)

Table 1: Vehicle data

r	6.453628E+06 m
θ	0.3415080515E+02 deg
ϕ	-6.3852272e+00 deg
V	6.7691442e+03 m/s
γ	-5.3452177e-01 deg
ψ	-2.3985529e+01 deg

Table 2: Initial conditions of the re-entry

	cross-range	time computation
off-line optimization	$\sim$ 1524 Km	127 sec.
on-line optimization	$\sim$ 1495 Km	5 sec.

Table 3: Comparison between off-line and on-line optimization

In order to obtain the samples of this function needed for training of the net, the optimization problem defined by eqn.(1) must be solved for various initial events (x, t). To determine the set of the initial events, we assumed that, because of variations of environmental conditions with respect to their nominal values, the vehicle trajectory will have a small deviation with respect to the nominal one (i.e. the one obtained in the presence of nominal environment conditions). With this assumption, we selected 1000 initial events (x, t) uniformly distributed in a range of $\pm 10\%$ around the nominal state trajectory. These values are shown in fig. 4.

For each initial event, the sample value $V(x, t)$ was obtained by solving the optimization problem defined in §4. The solution was obtained by using numerical algorithm described in [15] and assuming $u(t)$ stepwise constant.

For what concerns the so called "representation" problem, a rough analisys lefts to the following net topology:

- 6 nodes in input layer, one of these are a bias unit;
- 11 nodes in a first hidden layer;
- 44 nodes in a second hidden layer;
- only one unit (depending on the problem) for the output layer.

In fig. 5 the normalized output error of the net vs number of cycles of learning (epochs) is shown.

This neural net has been utilized for the adaptive computation of the optimal trajectory appliyng the approach proposed in §2. With reference to the problem (4), we assumed $u(t)$ constant in the time interval $[t, t + \Delta t)$, with $\Delta t = 100sec$. The optimal trajectory was the updated every $10sec$.

Fig.6 and fig.7 show the time histories of control input and of the state variables obtained assuming the initial conditions given in table 2, and then solving the optimization problem first with the algorithm proposed in [15], and then with the proposed technique using the neural network. It is evident that, against little worsening of the cross-range there is a remarcable reduction of time computation (see table 3).

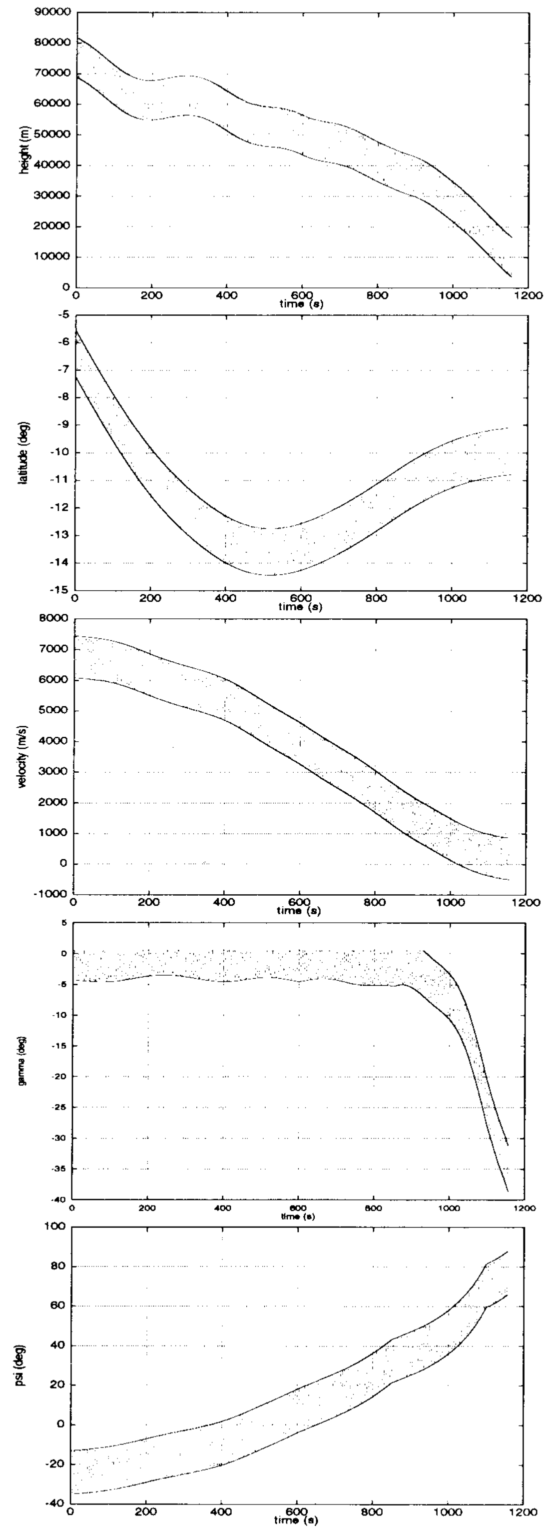

Figure 4: Description of input patterns of the neural net

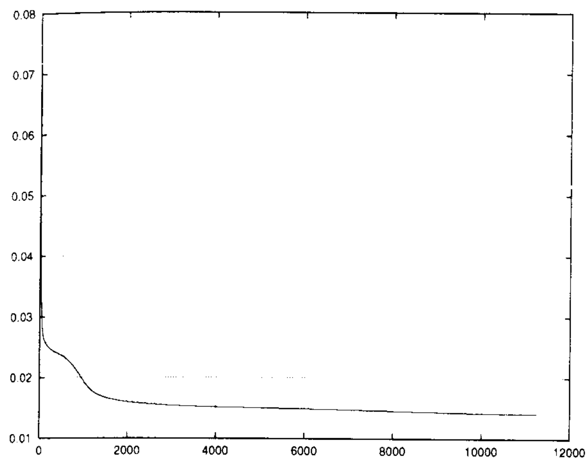

Figure 5: Error function of the net

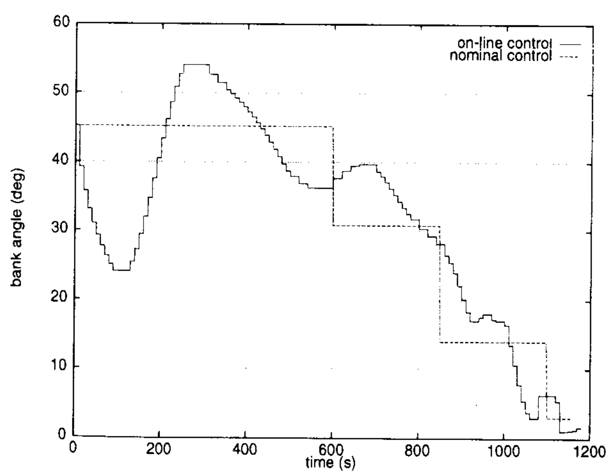

Figure 6: Time histories of off-line and adaptive on-line guidance laws

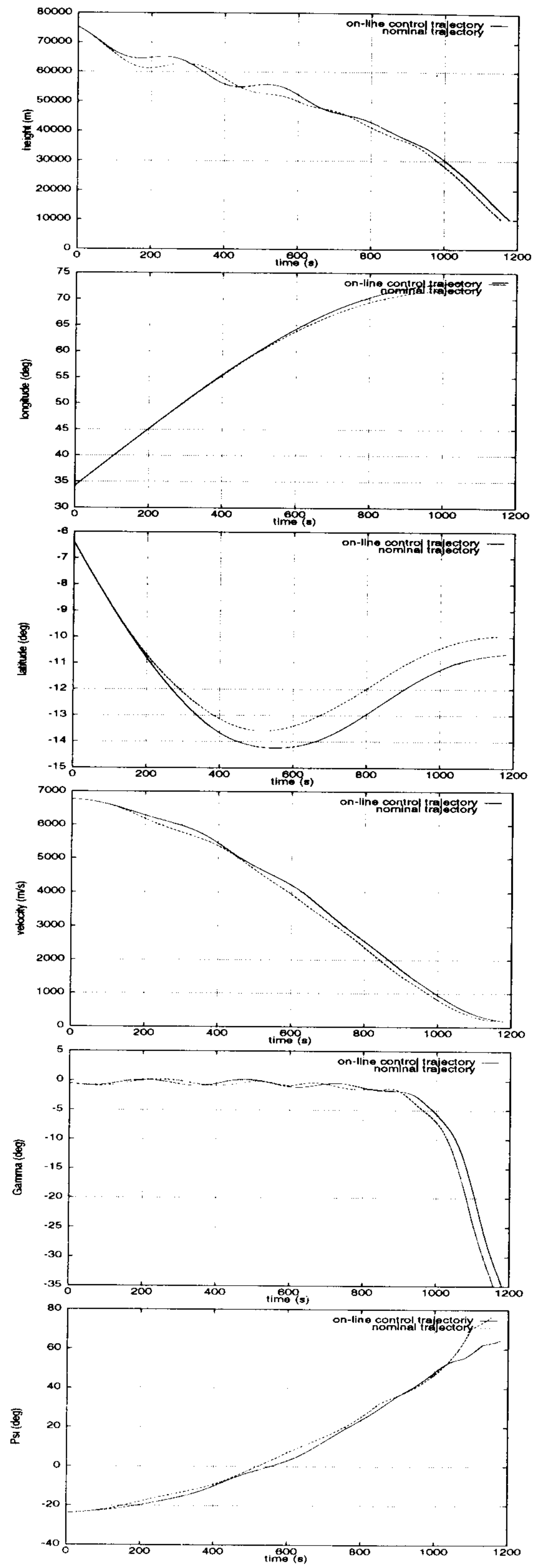

Figure 7: Comparison of results obtained with off-line and on-line optimization

List of simbols

C_L	=	lift coefficient
C_D	=	drag coefficient
CR	=	crossrange
D	=	aerodynamic drag
DR	=	downrange
h	=	height
J	=	performance index
L	=	aerodynamic lift
m	=	mass
n_T	=	load factor
r	=	distance from the center of the Earth
R_E	=	radius of the Earth
S	=	reference area
V	=	relative speed
γ	=	flight path angle
θ	=	longitude
μ_g	=	gravitational constant of the Earth
ρ	=	atmospheric density
σ	=	bank angle
ϕ	=	latitude
ψ	=	heading angle
ω_E	=	rotational velocity of the Earth

References

[1] M. C. Bartholomew-Biggs, L. C. Dixon, S. E. Hersem, Z. A. Maany, W. Flury, M. He chler, *From High Thrust to Low Thrust; Application of Advanced Optimization Methods to Mission Analysis*, ESA Journal, Vol.11, 1987.

[2] C. D. Baker, W. E. Causey, H. L. Ingram, *Mathematical Concepts and Historic al Development of the MASCOT Guidance Technique for Space Vehicles*, NASA Techni cal Memorandum, NASA TM X-44408, 1971.

[3] J. C. Harpold, C. A. Jr. Graves, *Shuttle Entry Guidance*, The Journal of th e Astronautical Sciences, Vol. 27, No. 3, 1979.

[4] D. G. Hull, J. L. Speyer, *Optimal Reentry and Plane Change Trajectories*, The Journal of the Astronautical Sciences, Vol. 30, No. 2, 1982.

[5] J. A. Calise, H. B. Gyoung, *A Near Optimal Guidance Algorithm for Aero-Assi sted Orbit Transfer*, AIAA Paper 88-4175-CP

[6] J. L. Speyer, E. Z. Crues, *Approximate Optimal Atmospheric Guidance Law for Aeroassisted Plane-Change Maneuvers*, J. of G.C.D., Vol. 13, No.5,1990.

[7] J. E. Bradt, M. V. Jessick, J. W. Hardtla, *Optimal Guidance for Future Spac e Application*, AIAA Paper 87-2401.

[8] C. R. Hardgraves, S. W. PAris *Direct Trajectory Optimization Using Non-Line ar Programming and Collocation*, J. of G.C.D., Vol.10,pp 338-342, 1986

[9] D. P. Bertsekas, *Dynamic Programming and Stochastic Control*, Academic Press, N. Y. 1976.

[10] S. Jallade, T. Duhamel, C. Champetier, *Trajectory Optimization and Guidance Strategies for Reentry*, AIAA Paper N92-24451.

[11] M. Athans, P. L. Falb, *Optimal Control*, McGraw-Hill, 1966.

[12] J. C. Mason, P. C. Parks, *Selection of Neural Networks Structure: Some Approximations Theory Guidelines*, Neural Networks for Control and Systems, by K. Warwick and others (eds.), P. Pereguinus, 1992.

[13] E. K. Blum, L. K. Li, *Approximation Theory and Feedforward Networks*, Neural Networks, Vol.4, N. 4, 1991.

[14] J. M. Zurada, *Introduction to Artificial Neural Systems*, West Publ. Co., 1992.

[15] A. Marzano, F. Ferrara, A. Capuano, G. Borriello, *Trajectory optimization and guidance strategies for reentry*, Atti del Congresso AIDAA '93