# SWINGING CONTROL OF ROTATING PENDULUM

A.L. FRADKOV
Institute for Problems of Mechanical Engineering,
Academy of Sciences of Russia
61 Bolshoy V.O., St.Petersburg, 199178, RUSSIA.
E-mail: alf@alf.stu.spb.su FAX: +7(812)217-8614.

O.P.TOMCHINA, O.L.NAGIBINA
St.Petersburg Machine-building Institute
14 Poliustrovskii av. St.Petersburg, RUSSIA

*Abstract.*The speed-gradient method of control design used previously for problems of regulation and tracking is extended to oscillating systems with energy-based objective functions. The concept of swinging control is introduced meaning achievement of arbitrary large level of the objective function by arbitrary small control level. The existence of swinging control for Hamiltonian systems is established. Simulation results for pendulum swinging problem are demonstrated.

## 1. INTRODUCTION.

Control theorists paid recently much attention to the control problems for nonlinear systems, particularly for nonlinear oscillatory systems arising in many fields of mechanics, electronics, medicine, etc. The conventional control objectives related to regulation or tracking can be described by specifying the desired plant trajectory $x_*(t)$ with the aim of making the real trajectory close to the desired one, e.g. :

$$x(t) - x_*(t) \to 0 \quad \text{when} \quad t \to \infty \qquad (1.1)$$

However some application problems related, e.g. to oscillations excitation (swinging) are not reducible immediately to standard problems of regulation and tracking and demand for new settings. The well known problem of swinging up and stabilizing a pendulum (see [1-4]) can be taken as example. The solutions to such problems are usually based upon energy considerations or specific tricks.

In this paper a new solution to the problem of swinging pendulum hanging from the rotating arm is suggested based on speed-gradient method [5-10].Simulation results confirming the validity of the method are presented. Section 2 gives a pendulum control problem statement. Speed-gradient algorithms for Hamiltonian systems and energy-based objective functions are described in section 3. In section 4 a few versions of speed-gradient algorithm for swinging the pendulum hanging from the rotating arm are considered. Simulation results are discussed in the section 5.

## 2. MATHEMATICAL MODEL OF ROTATING PENDULUM

This paper deals with the control of the pendulum having the weight m and length $2l_2$ ($l_2$ is the distance from its axis to the center of gravity) suspended from the horizontal beam having length $l_1$(see [1-3]). The beam may rotate in the horizontal plane moving by the DC drive. The Euler-Lagrange equations of the system are as follows:

$$\begin{bmatrix} J_1+m(l_1^2+l_2^2\sin^2 q_2) & -ml_1l_2\cos q_2 \\ -ml_1l_2\cos q_2 & J_2+ml_2^2 \end{bmatrix} \begin{bmatrix} \ddot q_1 \\ \ddot q_2 \end{bmatrix} + \begin{bmatrix} c_1+ml_2^2\sin(2q_2)\dot q_2 & ml_1l_2\cos(q_2)\dot q_2 \\ -\frac{m}{2}l_2^2\sin(2q_2)\dot q_1 & c_2 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mgl_2\sin q_2 \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix}, \qquad (2.1)$$

where $q_1$, $q_2$ are generalized angular coordinates $J_1$, $J_2$ are inertia of the arm and $c_1$, $c_2$ are friction coefficients , M is input torgue. The values of system parameters are as follows [2]:

$l_1$=0.215[m], $l_2$=0.113[m], $J_1 = 1.75\cdot 10^{-4}$[kg·m$^2$],

$J_2=1.98\cdot 10^{-4}$[kg·m$^2$], m=5.38 $10^{-2}$[kg],

$c_1$=0.118[N·m·s], $c_2=8.3\cdot 10^{-5}$[N·m·s].

Parameters $c_1$, $c_2$ determine the dissipation . In case when dissipation is absent ($c_1 = c_2 = 0$) the system is conservative and may be described by Hamiltonian equations. The sensors (potentiometer and resolver) can measure coordinates $q_1$, $q_2$. The system has kinetic energy T, potential energy Π and Rayleigh dissipation function R (dissipation energy) as follows.

$$T = \frac{1}{2}\Big[J_1\dot q_1^2 + m(l_1^2+l_2^2\sin^2 q_2)\dot q_1^2 + ml_2^2\dot q_2^2 - 2ml_1l_2\cos(q_2)\dot q_1\dot q_2 + J_2\dot q_2^2\Big],$$

$$\Pi=mgl_2(1-\cos q_2), \qquad (2.2)$$

$$R = \frac{1}{2}\left(c_1\dot q_1^2 + c_2\dot q_2^2\right).$$

The main control aim is stabilizing the total energy of the system $H(p,q) = T(p,q) + \Pi(p,q)$ at the prespecified level $H_*$. The additional control goal is stabilizing the arm position $q_1$ at the prespecified value $q_1^*$:

$$q_1(t) \to q_1^* \text{ when } t \to \infty. \qquad (2.3)$$

## 3. SPEED-GRADIENT ALGORITHMS FOR MECHANICAL SYSTEMS

The convenient mathematical description for controlled oscillating system is Hamiltonian form. It allows for explicit describing surfaces of constant energy which unforced motions belong to. The Hamiltonian form of controlled plant equations is as follows:

$$\dot{p}=-\frac{\partial H(p,q,u)}{\partial q}, \quad \dot{q}=\frac{\partial H(p,q,u)}{\partial p}, \qquad (3.1)$$

where $p,q \in R^n$ are generalized coordinates and momenta; $H=H(p,q,u)$ is Hamiltonian function (energy of the system); $u=u(t) \in R^m$ is input (generalized force). Formalize the control aim as approaching the given energy surface of unforced system:

$$S = \{ (p,q) : H_0(p,q)=H_* \} \qquad (3.2)$$

where $H_0(p,q)=H(p,q,0)$. The objective (3.2) can be reformulated as

$$H_0(p(t),q(t)) \to H_* \text{ when } t \to \infty, \qquad (3.3)$$

and expressed in the form (2.2), where $x=col(p,q)$, and

$$Q(x) = \frac{1}{2} [ H_0(p,q) - H_*]^2 \qquad (3.4)$$

In what follows we assume the Hamiltonian is linear in control:

$$H(p,q,u)=H_0(p,q)+H_1(p,q)^T u$$

where $H_0(p,q)$ is internal Hamiltonian describing unforced motions of the system and $H_1(p,q)$ is a vector of interaction Hamiltonians [11]. The Speed-Gradient (SG) method [8-10] offers the general form algorithm:

$$u=-\gamma\nabla_u\dot{Q} \qquad (3.5)$$

where $\dot{Q}$ is derivative of Q with respect to the system (3.1), $\gamma>0$.

To design SG-algorithm for the problem (3.1), (3.4) calculate $\dot{Q}$ :

$$\dot{Q} = (H_0-H_*)[H_0,H_1]u , \qquad (3.6)$$

where $[H_0,H_1]=\left[\frac{\partial H_0}{\partial p}\frac{\partial H_1^T}{\partial q} - \frac{\partial H_0}{\partial q}\frac{\partial H_1^T}{\partial p}\right]$ is Poisson bracket (differentiation of vectors is componentwise).

The finite forms of SG-algorithms look as follows:

$$u = - \gamma (H_0-H_*)[H_0,H_1] \qquad (3.7)$$

$$u = - \gamma \text{ sign } \{(H_0-H_*)[H_0,H_1]\} \qquad (3.8)$$

where $\gamma>0$ is gain coefficient.

Let us formulate the result allowing to analyze the behavior of systems with algorithms (3.7),(3.8). To give precise formulations define recursive Poisson bracket:

$$ap_{H_0}^1 H_1=[H_0,H_1]; \; ap_{H_0}^{k+1}H_1=[H_0,ap_{H_0}^k H_1], \; k=1,2,\ldots (3.9)$$

Functions $H_0(p,q)$, $H_1(p,q)$ should be sufficiently smooth. In case when $H_1$ is vector-function the above definition is component-wise.

Theorem [7]. Consider system (3.1) together with the control algorithm

$$u(t)=-\gamma\psi((H_0-H_*)[H_0,H_1](t)), \qquad (3.10)$$

where $\gamma>0$, $\psi(z,t)^T z\ge\beta\|z\|^\delta$ for $\beta>0$, $\delta>0$ and for all $z\in R^m$; and

$\lim_{z\to 0}\psi(z,t)=0$ uniformly in $t\ge0$.

Then $u(t)\to0$, i.e. trajectories of (3.1), (3.10) approach the trajectories of unforced system. Let also the unforced Hamiltonian $H_0$ have only isolated equilibria and $\dim D=2n$, where $D=\text{span}\{ap_{H_0}^k H_1, \; k=1,2\ldots\}$. Then for any trajectory there is an alternative: either a)the control aim (3.3) is achieved; or b)the trajectory tends to an equilibrium of the unforced system. The set of initial conditions from which trajectory tends to the unstable equilibrium is contained in a countable set of manifolds having dimension not greater then 2n-1, i.e. has Lebesgue measure zero and its supplement is an open dense set.

The proof of the theorem can be found in [7].

Remark 1. Suppose that the system (3.1) is Lagrange system,i.e.

$$H_0(p,q)=p^T A^{-1}(q)p+\Pi(q) \qquad (3.11)$$

where $A(q)$ is positive-definite matrix of kinetic energy and $\Pi(q)$ is potential energy. In this case equilibria of unforced system have the form $(0,\hat{q})$, where $\hat{q}$ is a stationary point of potential $\Pi(q)$. Suppose that all the stationary points of $\Pi(q)$ are isolated. Then it follows from theorem that almost all the trajectories of the control system either achieve the goal set (3.2) or converge to some local minimum of potential $\Pi(q)$.

Remark 2. The condition $\dim D=2n$ is a kind of observability condition because it means that the identity $y(t)\equiv0$ implies $p\equiv0,q\equiv0$, where $y=H_1(p,q)$ is vector of "natural" outputs (see[11]).

## 4. ALGORITHMS OF SWINGING ROTATING PENDULUM.

First let us write the algorithm for reduced problem supposing that dissipative forces are absent. Since $B=[1 \; 0]^T$ the speed-gradient algorithm in final form is as follows:

$$u = -\gamma\Big[(J_1\dot{q}_1 + m(l_1^2+l_2^2\sin^2q_2)\dot{q}_1 + ml_2^2\dot{q}_2^2 - 2ml_1l_2\cos(q_2)\dot{q}_1\dot{q}_2 + J_2\dot{q}_2^2) + 2mgl_2(1-\cos q_2) - 2H_*\Big]\dot{q}_1, \qquad (4.1)$$

where H is expressed in (2.2). The theorem 1 implies that $H \to H_*$ is achieved for almost all initial conditions. In case when dissipative forces are taken into account the control algorithm preserves the form (4.1) but according to theorem 2 it guarantees convergence not to point, but to domain which size decreases when $\gamma$ increases.

To achieve the additional aim (2.3) the additional objective function is introduced

$Q_2 = (q_1 - q_1^*)^2$ and the trick is applied

consisting in interlacing speed-gradient motions along Q and along $Q_2$. It seems reasonable to move along $\dot{Q}_1$ near the region of a phase space where $\dot{q}_1$ is close to maximum and along $\dot{Q}_2$ when $\dot{q}_1$ is close to zero (more exactly, when $|\dot{q}_1|<\varepsilon$ ). In the other time we take u = 0, i.e. we use the dead zone.

The problem of convergence without measurement of $\dot{q}_1$ and $\dot{q}_2$ seems interesting. The observer equation for estimation of vector $x=[q_1,\dot{q}_1,q_2,\dot{q}_2]^T$ is as follows.

$$\dot{\hat{x}}=F(\hat{x})+K(q-\hat{q})+D(\hat{x})u, \qquad (4.2)$$

where K is the gain matrix of appropriate dimension and D(x) has the structure corresponding to equation (2.1) resolved for highest derivatives.

It can be shown that the observer works for the proper choice of K if the plant (2.1) is hyper-minimum-phase (HMP) [7]. If the plant is not HMP, it is possible to transform it into augmented HMP form introducing some shunt (parallel feedforward compensator).

The described algorithms were analyzed by computer simulation.

## 5. SIMULATION RESULTS

The simulation results for system (2.1) affected with control signal (4.1) without dissipative forces ($c_1=c_2=0$) are illustrated by Fig.1 (the system energy plot) and Fig.2 (phase plot). The value $H_*$ was calculated for the maximum deviation $q_2 = 45$ , $\gamma = 100$, while the initial deviation is 5 . Similar plots in presence of dissipative forces for $c_1 = 0.006[N\cdot m\cdot s]$, $c_2 = 8.3\cdot 10^{-5}[N\cdot m\cdot s]$ are shown at Fig.3,4. The other pictures demonstrate the results for systems with the observer (Fig.5,6 - without dissipation; Fig.7,8 - with dissipation).

It can be seen that the dissipative forces produce oscillatory transient processes for total energy. However the desired energy level $H_*$ is achieved in all cases. In observer-based system the transient time increases from 0.03[s] to 0.08[s] but can be reduced by the proper choice of K (we took gain coefficients $K_{ij}=100$ ).

## 6. CONCLUSIONS

The applicability of the speed-gradient method to the problem of swinging the two-degree-of-freedom mechanical system has been demonstrated. The resulting algorithms are more simple compared with the exisiting ones [1-4]. The proposed approach may be applied in robotics, e.g. for design of brachiation robot [12] or legged valking robots [13-14].

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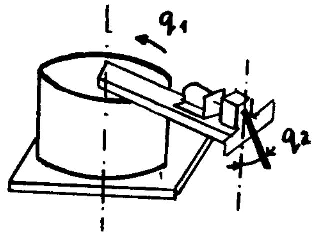

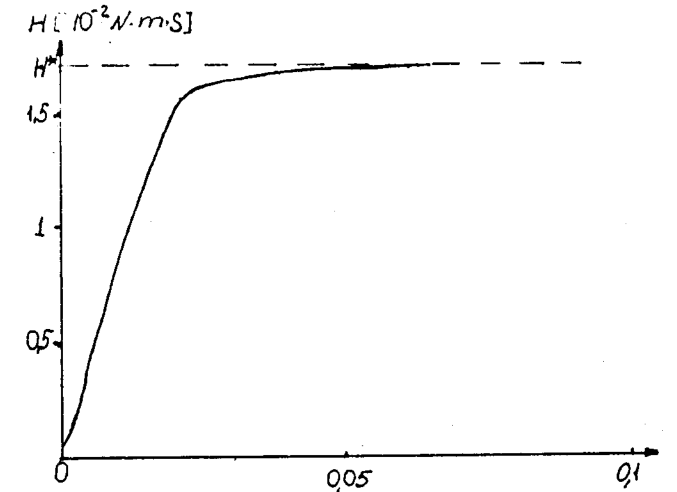


Fig. 1

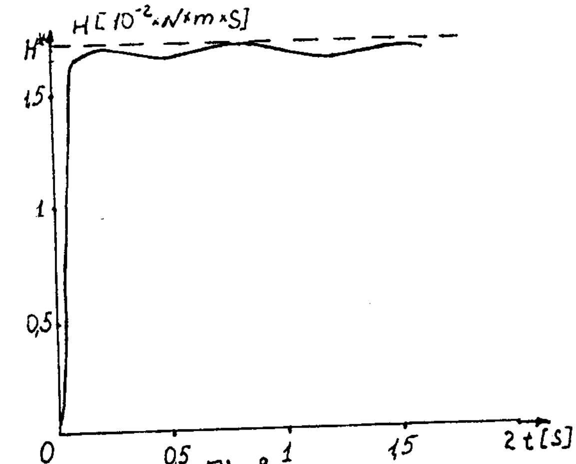


Fig.3

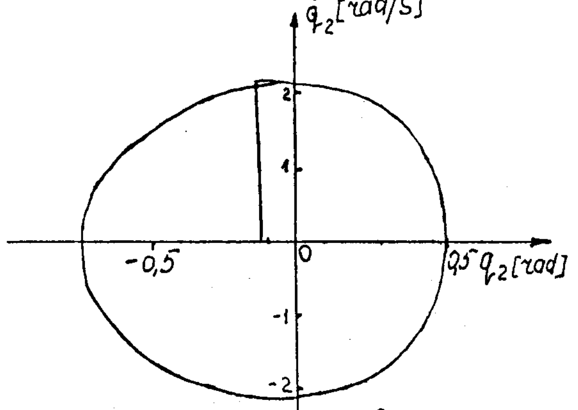


Fig.2.

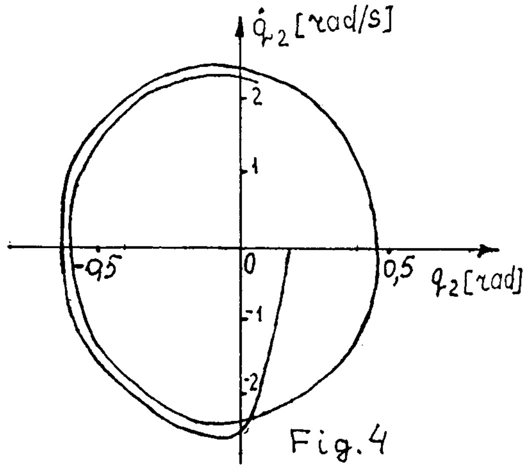


Fig.4

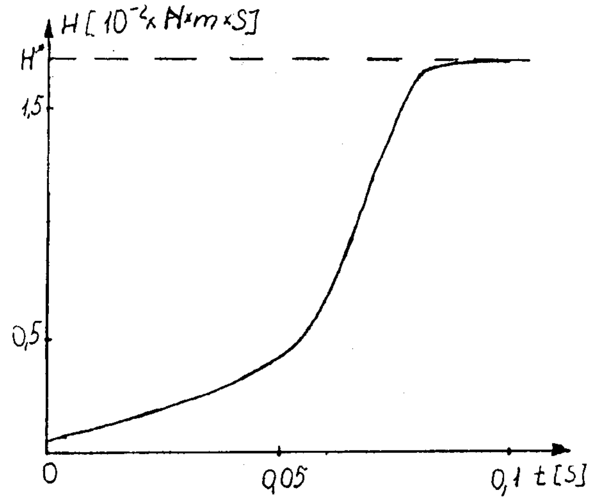

Fig.5

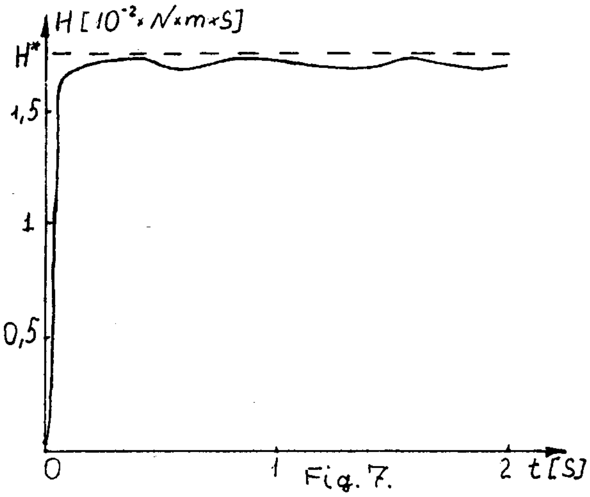

Fig.7.

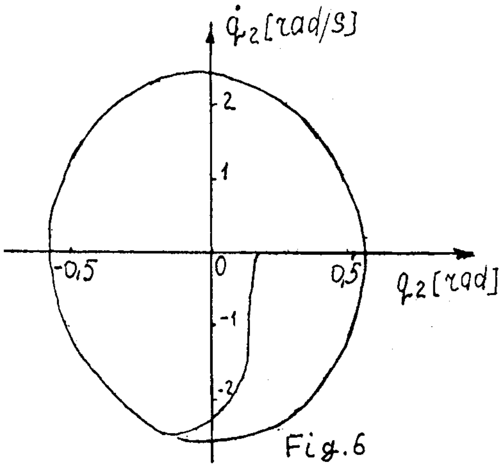

Fig.6

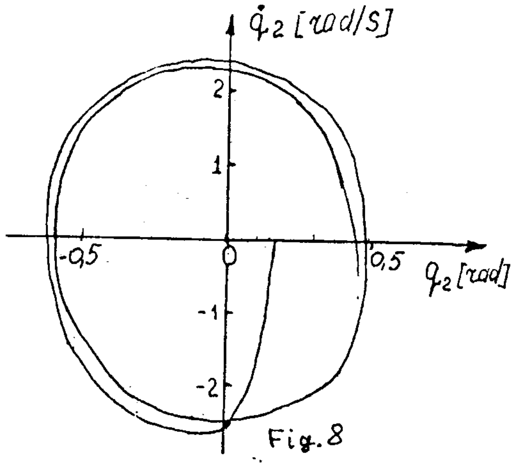

Fig.8