ITERATIVE DESIGN OF NONLINEAR ADAPTIVE STABILIZERS

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ABSTRACT

A new adaptive stabilizer design procedure is proposed for the case when the right hand side of the reduced system depends nonlinearly on additional nonlinear integrator variables. The global stability theorem and its corollary for the case of tracking problem are given. The iterative design procedure is illustrated by the model example.

1. INTRODUCTION

In recent years, we have seen intensive development of adaptive control theory for nonlinear plant models [1-3]. Several iterative design schemes were suggested (e.g. [4,5,6,7]) giving more simple tools for nonlinear feedback design with full-state measurement.

Many of the earlier results were used only when growth conditions on the nonlinearities or "extended matching conditions" on the location of unknown parameters were imposed [8]. Such conditions restricted the applicability of the corresponding schemes.

Investigations of the state-feedback problem resulted in a systematic design procedure called *adaptive backstepping* [4] which overcame some of the limitations. This design procedure when applied to so-called "parametric-strict-feedback" systems guarantees global stability for all types of nonlinearities. For a more general "parametric-pure -feedback" systems an estimate of the region of attraction can be provided. However, the resulting controllers employed multiple estimates for each unknown parameter. This overparametrization was reduced in half in [9] and then completely removed in [10] by the idea of adaptive backstepping with tuning functions.

Given any nonlinear system, in order to apply the above backstepping procedure, a parameter-independent diffeomorphism should be found to transform the system at hand into the "parametric-strict-feedback" or "parametric-pure-feedback" canonical form. However, the necessary and sufficient conditions for the existence of the diffeomorphism are formulated in [4] only as local. Therefore, there are no final results verifying the global validity of these transformations.

It is to be noted that the adaptive scheme proposed in [7] allows to enlarge the class of systems that can be stabilized using the approach of [4].

In this paper a new iterative design procedure is proposed to stabilize systems with nonlinear dependence of the reduced subsystem right hand sides on additional integrator variables.
The present design procedure provides global results for above mentioned systems without assuming the "parametric-strict-feedback" or "parametric-pure- feedback" canonical form. It based on the earlier results [11] using the ideas of backstepping and tuning functions and, therefore, removes the need for overparametrization. On the other hand, this adaptive control scheme extends the speed-difference algorithm [12,13] to the adaptive case.

The paper is organized as follows. In Section 2 the problem statement is given. The algorithm description and applicability conditions for the system with one input integrator are presented in Section 3. In Section 4 some extensions of the above results are discussed. Both stabilization and tracking problems are considered.In Section 5 we illustrate our procedure by example of system with two integrators. Finally, the conclusion is given in Section 6.

2. PROBLEM STATEMENT

Consider the following plant model

$$\dot{x}=f_0(x,v_1) + \theta f(x,v_1) \tag{2.1a}$$

$$\dot{v}_1=\Phi_1(x,v_1,v_2)$$

$$\dot{v}_2=\Phi_2(x,v_1,v_2,v_3)$$

$$\cdots\cdots$$

$$\dot{v}_r=\Phi_r(x,v_1,\dots,v_r,u) \tag{2.1b}$$

where $x\in R^N$ is the state, $u\in R^1$ is the input and $v_i\in R^1$, i=1...r are measurable variables; $\theta\in R^1$ is unknoun constant parameter.

The problem is to find the adaptive control algorithm

$$u=U(x,v_1,\dots,v_r,\hat{\theta},t) \tag{2.2}$$

$$\dot{\hat{\theta}}=\Theta(x,v_1,\dots,v_r,\hat{\theta},t) \tag{2.3}$$

where $\hat{\theta}$ is the estimate of the θ, which guarantees the boundedness of the system (2.1)-(2.3) trajectories and the achievement of the control objective

$$Q(x(t),\hat{\theta}(t),t)\to 0 \text{ as } t\to\infty \tag{2.4}$$

where $Q(x,\hat{\theta},t)\geq 0$ is the given objective function.

If v_1 were the plant input, the Speed-Gradient algorithm [3,14] would achieve the goal (2.4) under some stabilizability conditions. The presence of the input nonlinear integrators chain in (2.1b) does not allow to apply the general methods used in the compensation and feedback-linearization approaches.

We present a new design procedure to stabilize (2.1) based, on the one hand, on the ideas of *backstepping* and *tuning* functions [11] and, on the other hand, on the *speed-difference* algorithm [12,13].

3. ADAPTIVE STABILIZATION ALGORITHM FOR NONLINEAR SYSTEM WITH ONE INTEGRATOR

Consider a single input nonlinear system

$$\dot{x}=f_0(x,u) + \theta f(x,u) \tag{3.1}$$

where $x\in R^N$ is the state, $u\in R^1$ is the input and $\theta\in R^1$ is an unknoun constant parameter.

Let us assume that we have designed an adaptive control law

$$u=\alpha_0(x,\hat{\theta},t) \tag{3.2a}$$

$$\dot{\hat{\theta}}=\tau_0(x,\hat{\theta},t) \tag{3.2b}$$

where $\hat{\theta}$ is the estimate of the θ,

which guarantees the boundedness of the system (3.1)-(3.2) trajectories and the achievement of the control objective

$$Q(x(t),\hat{\theta}(t),t)\to 0 \text{ as } t\to\infty$$

for the given objective function $Q(x,\hat{\theta},t)\geq 0$.
These properties are ascertained by the inequality

$$\frac{\partial Q}{\partial x}[f_0(x,\alpha_0)+\theta f(x,\alpha_0)]+\frac{\partial V}{\partial\hat{\theta}}\tau_0+\frac{\partial Q}{\partial t}\leq -\rho(Q(x,\hat{\theta},t)) \tag{3.3}$$

for all $x\in R^N$, $\hat{\theta}\in R^1$
where

$$V(x,\hat{\theta},t)=Q(x,\hat{\theta},t)+\frac{1}{2\gamma}(\theta-\hat{\theta})^2\geq 0,\quad \gamma>0 \tag{3.4}$$

is the Lyapunov function for (3.1);
$\rho(q)$ is a positive for $q>0$, continuous function.

Let add one input integrator to the system (3.1):

$$\dot{x}=f_0(x,v) + \theta f(x,v) \tag{3.5a}$$

$$\dot{v}=\Phi(x,v,u) \tag{3.5b}$$

where $x\in R^N$, u, v are scalar;

Definition . A function $G(z,t)$ is called *locally bounded uniformly in* $t\geq 0$ if for any $\beta>0$ there exists $C(\beta)$ such as $\|G(z,t)\|\leq C(\beta)$ for $\|z\|\leq\beta$; $t\geq 0$.

Theorem. Consider the system (3.5) with the following assumptions:

A). $Q(x,\hat{\theta},t)$ satisfies *the growth condition*:
$Q(x,\hat{\theta},t)\geq\alpha(\|x\|+|\hat{\theta}|)$ for some $\alpha(\mu)\geq 0$ such that $\alpha(\mu)\to\infty$ as $\mu\to\infty$;

B). There exists the function $\Psi(x,v,z)$ such that $\Phi(x,v,\Psi(x,v,z))=z$ for all $x\in R^N$, $v,z\in R^1$; *(solvability condition for the subsystem (3.5b))*

C). There exist the smooth function $\alpha_0(x,\hat{\theta},t)$ and $\tau_0(x,\hat{\theta},t)$ which satisfy the inequality (3.3); *(stabilizability condition for the subsystem (3.5a))*

D). $Q(\cdot)$, $\Phi(\cdot)$, $f_0(\cdot)$, $f(\cdot)$ are the smooth functions; α_0, τ_0, $\partial Q/\partial x$, $\partial Q/\partial\hat{\theta}$, $\partial\alpha_0/\partial t$, $\partial\alpha_0/\partial\hat{\theta}$, $\partial\tau_0/\partial x$, $\partial\tau_0/\partial\hat{\theta}$, $\partial\alpha_0/\partial x$ are locally bounded uniformly in $t\geq 0$ *(smoothness condition)*;

Then the control law

$$u=\Psi(x,v,\alpha_1(x,v,\hat{\theta},t)) \tag{3.6}$$

and the parameter update law

$$\dot{\hat{\theta}}=\tau_1(x,v,\hat{\theta},t) \tag{3.7}$$

where

$$\alpha_1= -\gamma_0(v-\alpha_0) - \gamma_1\frac{\frac{\partial Q}{\partial x}[f_0(x,v)-f_0(x,\alpha_0)]}{v-\alpha_0} - \gamma_1\frac{\frac{\partial Q}{\partial x}[f(x,v)-f(x,\alpha_0)]}{v-\alpha_0}\left(\hat{\theta}+\frac{\partial Q}{\partial\hat{\theta}}\gamma\right) + \gamma\frac{\partial Q}{\partial\hat{\theta}}\frac{\partial\alpha_0}{\partial x}f(x,v)+\frac{\partial\alpha_0}{\partial\hat{\theta}}\tau_1 + \frac{\partial\alpha_0}{\partial x}[f_0(x,v)+\hat{\theta}f(x,v)]+\frac{\partial\alpha_0}{\partial t} \quad \text{for } v\neq\alpha_0$$

and

$$\alpha_1= -\gamma_0(v-\alpha_0)-\gamma_1\frac{\partial Q}{\partial x}\frac{\partial f_0(x,\alpha_0)}{\partial v}-\gamma_1\frac{\partial Q}{\partial x}\frac{\partial f(x,\alpha_0)}{\partial v}\left(\hat{\theta}+\frac{\partial Q}{\partial\hat{\theta}}\gamma\right)+\gamma\frac{\partial Q}{\partial\hat{\theta}}\frac{\partial\alpha_0}{\partial x}f(x,v)+\frac{\partial\alpha_0}{\partial\hat{\theta}}\tau_1 + \frac{\partial\alpha_0}{\partial x}[f_0(x,v)+\hat{\theta}f(x,v)]+\frac{\partial\alpha_0}{\partial t} \quad \text{for } v=\alpha_0$$

$$\tau_1=\tau_0-\frac{\gamma}{\gamma_1}\frac{\partial\alpha_0}{\partial x}f(x,v)(v-\alpha_0)+\gamma\frac{\partial Q}{\partial x}[f_0(x,v)-f_0(x,\alpha_0)]$$

where γ, γ_0, $\gamma_1>0$ are constant design parameters, guarantees the boundedness of the system (3.5)-(3.7) trajectories $x(t)$, $\hat\theta(t)$, $v(t)$ and the achievement of the control objective

$Q(x(t),\hat\theta(t),t)\to 0$; $v(t)-\alpha_0(x(t),\hat\theta(t),t)\to 0$ as $t\to\infty$.

Proof: The augmented Lyapunov function candidate for the system (3.5)-(3.6) is defined as follows:

$$V_1(x,\hat\theta,v,t)= Q(x,\hat\theta,t)+\frac{1}{2\gamma}(\theta-\hat\theta)^2+\frac{1}{2\gamma_1}(v-\alpha_0)^2 =$$

$$=V(x,\hat\theta,t)+\frac{1}{2\gamma_1}(v-\alpha_0)^2\ge 0$$

Now calculate its time-derivative with respect to (3.5),(3.6):

$$\dot V_1=\frac{\partial Q}{\partial x}\dot x+\frac{\partial Q}{\partial\hat\theta}\dot{\hat\theta}+\frac{\partial Q}{\partial t}-\frac{1}{\gamma}(\theta-\hat\theta)\dot{\hat\theta}+\frac{1}{\gamma_1}(v-\alpha_0)[\Phi(x,v,u)-$$

$$-\frac{\partial\alpha_0}{\partial x}\dot x-\frac{\partial\alpha_0}{\partial\hat\theta}\dot{\hat\theta}-\frac{\partial\alpha_0}{\partial t}]=\frac{\partial Q}{\partial x}f_0(x,v)+\frac{\partial Q}{\partial x}\theta f(x,v)+$$

$$+\frac{\partial Q}{\partial\hat\theta}\dot{\hat\theta}+\frac{\partial Q}{\partial t}-\frac{1}{\gamma}(\theta-\hat\theta)\dot{\hat\theta}+\frac{1}{\gamma_1}(v-\alpha_0)[\Phi-\frac{\partial\alpha_0}{\partial\hat\theta}\dot{\hat\theta}-$$

$$-\frac{\partial\alpha_0}{\partial x}f_0(x,v)-\frac{\partial\alpha_0}{\partial x}\theta f(x,v)-\frac{\partial\alpha_0}{\partial t}]$$

Then

$$\dot V_1=\frac{\partial Q}{\partial x}f_0(x,v)+\frac{\partial Q}{\partial x}\theta f(x,\alpha_0)+\frac{\partial V}{\partial\hat\theta}\tau_0+\frac{\partial Q}{\partial\hat\theta}[\dot{\hat\theta}-\tau_0]+$$

$$+\frac{\partial Q}{\partial t}+\frac{1}{\gamma}(\theta-\hat\theta)[\tau_0-\dot{\hat\theta}+\frac{\partial Q}{\partial x}[f(x,v)-f(x,\alpha_0)]+$$

$$\gamma\frac{\partial Q}{\partial x}[f(x,v)-f(x,\alpha_0)]-\frac{\gamma}{\gamma_1}\frac{\partial\alpha_0}{\partial x}f(x,v)(v-\alpha_0)]$$

$$+\frac{\partial Q}{\partial x}\hat\theta[f(x,v)-f(x,\alpha_0)]+\frac{1}{\gamma_1}(v-\alpha_0)[\Phi(x,v,u)$$

$$-\frac{\partial\alpha_0}{\partial\hat\theta}\dot{\hat\theta}-\frac{\partial\alpha_0}{\partial x}f_0(x,v)-\frac{\partial\alpha_0}{\partial x}\hat\theta f(x,v)-\frac{\partial\alpha_0}{\partial t}]$$

Eliminate $(\theta-\hat\theta)$ from the last expression with the choice $\dot{\hat\theta}=\tau_1(x,v,\hat\theta,t)$ where τ_1 is defined in (3.7). Then, noting that

$$\dot{\hat\theta}\ \tau_0=\tau_1-\tau_0=$$

$$=-\frac{\gamma}{\gamma_1}\frac{\partial\alpha_0}{\partial x}f(x,v)(v-\alpha_0)+\gamma\frac{\partial Q}{\partial x}[f_0(x,v)-f_0(x,\alpha_0)],$$

obtain:

$$\dot V_1=\frac{\partial Q}{\partial x}f_0(x,v)+\frac{\partial Q}{\partial x}\theta f(x,\alpha_0)+\frac{\partial V}{\partial\hat\theta}\tau_0+\frac{\partial Q}{\partial t}+$$

$$+\frac{\partial Q}{\partial x}[f(x,v)-f(x,\alpha_0)](\hat\theta+\frac{\partial Q}{\partial\hat\theta}\gamma)+$$

$$+\frac{1}{\gamma_1}(v-\alpha_0)[\Phi-\gamma\frac{\partial Q}{\partial\hat\theta}\frac{\partial\alpha_0}{\partial x}f(x,v)-\frac{\partial\alpha_0}{\partial\hat\theta}\tau_1-$$

$$-\frac{\partial\alpha_0}{\partial x}(f_0(x,v)+\hat\theta f(x,v))-\frac{\partial\alpha_0}{\partial t}]$$

Replacing $u=\Psi(x,v,\alpha_1(x,v,\hat\theta,t))$ from (3.6) and using the stabilizability condition for the subsystem (3.5a) we have

$$\dot V_1\le-(v-\alpha_0)^2\gamma_0-\rho(Q(x,\hat\theta,t)) \tag{3.8}$$

Thus, (3.6)-(3.7) is designed to make the time derivative of V_1 nonpositive.

Integrating (3.8) over interval $[0,t]$ gives

$$0\le Q(x(t),\hat\theta(t),t)\le V_1^t\le V_1^0-\int_0^t\rho(Q(x(s),\hat\theta(s),s)ds-$$

$$-\gamma_0\int_0^t(v(s)-\alpha_0(x(s),\hat\theta(s),s)^2ds \tag{3.9}$$

It follows from (3.9), the growth condition A) and the smoothness condition D) that the boundedness of trajectories of the system (3.5)-(3.7) and the achievement of the control objective

$Q(x(t),\hat\theta(t),t)\to 0$; $v(t)-\alpha_0(x(t),\hat\theta(t),t)\to 0$ as $t\to\infty$

can be established according lines of the proof of the Theorem 1 in [12]. ∎

4. DISCUSSION

The theorem of Section 3 gives the following procedure for the control algorithm design: first to view v as the control and to stabilize the reduced system (3.5a) by the control and parameter update laws defined as $u=\alpha_0(x,\hat\theta,t)$ and $\dot{\hat\theta}=\tau_0(x,\hat\theta,t)$ i.e. to satisfy the inequality (3.3) in the stabilizability condition C). Then augmenting the subsystem (3.5a) by the integrator (3.5b) and accounting for the fact that v is not the actual control to use (3.6)-(3.7) to describe the control and update laws which guarantees the achievement of the control objective for augmented system (3.5).

The above iterative design procedure can be extended to the system (2.1a) with $r\ge 1$ integrators (2.1b) when (2.1) satisfies the assumptions A)-D) of the Theorem and, moreover,the functions $Q(\cdot)$, $\alpha_0(\cdot)$, $f_0(\cdot)$, $f(\cdot)$ are r times continuously differentiable (i.e. belong to C^r) and all partial derivatives of $Q(\cdot)$, $\alpha_0(\cdot)$, $\tau_0(\cdot)$ for the order not more then r are locally bounded uniformly in $t\ge 0$.

This design procedure is iterative. At its i-th step the subsystem with i integrators, i=1...r, is stabilized with respect to a Lyapunov function

$$V_i(x,v_1,..v_i,\hat\theta,t)=Q(x,\hat\theta,t)+\frac{1}{2\gamma}(\theta-\hat\theta)^2+$$

$$+\frac{1}{2}(v_1-\alpha_0)^2+....+\frac{1}{2}(v_i-\alpha_{i-1})^2,\ i=1..r$$

by the design of a control law $\alpha_i(x,v_1,..v_i,\hat{\theta},t)$ and a update law $\tau_i(x,v_1,..v_i,\hat{\theta},t)$, i=1..r., called in [10] as *stabilizing* and *tuning* functions. The update law for the parametric estimate and the feedback control u are designed at the final step.

Corollary 1 *(tracking problem).*The special case of above theorem covers the case of tracking problem using the objective function

$Q(x,\hat{\theta},t)=|h(x)-r(t)|^2$, where h(x), r(t) are known scalar functions. It follows from the theorem that the algorithm (3.6)-(3.7) can be applied and ensures the tracking goal $|h(x)-r(t)|^2\to 0$ as $t\to\infty$, if h(x), $r(t)\in C^r$, and, moreover, the first r derivatives of r(t) are bounded and known.

Corollary 2 *(convex case).*In the case when the functions f(x,v) and fo(x,v) defined in (3.5) are convex in v the adaptive controller can be simplified. Indeed, let the inequalities

$$f_0(x,v')-f_0(x,v) \ge (v-v')\frac{\partial f_0(x,v)}{\partial v},$$

$$f(x,v')-f(x,v)\ge (v-v')\frac{\partial f(x,v)}{\partial v},$$

be satisfied for all v, $v'\in R^1$, $x\in R^N$.
Then the adaptive control law

$$u=\Psi(x,v,\alpha_1(x,v,\hat{\theta},t)) \qquad (4.1a)$$

$$\dot{\hat{\theta}}=\tau_1(x,v,\hat{\theta},t) \qquad (4.1b)$$

where

$$\alpha_1(x,v,\hat{\theta},t)= -\gamma_0(v-\alpha_0)-\gamma_1\frac{\partial Q}{\partial x}\frac{\partial f_0(x,v)}{\partial v} -$$

$$-\gamma_1\frac{\partial Q}{\partial x}\frac{\partial f(x,v)}{\partial v}(\hat{\theta}+\frac{\partial Q}{\partial\hat{\theta}}\gamma) + \gamma\frac{\partial Q}{\partial\hat{\theta}}\frac{\partial\alpha_0}{\partial x}f(x,v)+$$

$$+\frac{\partial\alpha_0}{\partial\hat{\theta}}\tau_1 +\frac{\partial\alpha_0}{\partial x}[f_0(x,v)+\hat{\theta}f(x,v)] +\frac{\partial\alpha_0}{\partial t},$$

$\tau_1(x,v,\hat{\theta},t)$ is defined in (3.7),

guarantees the boundedness of the system (3.5), (4.1) trajectories x(t), $\hat{\theta}(t)$, v(t) and the achievement of the control objective

$Q(x(t),\hat{\theta}(t),t)\to 0$; $v(t)-\alpha_0(x(t),\hat{\theta}(t),t)\to 0$ as $t\to\infty$.

Remark 1.
The present adaptive stabilization algorithm (3.6), (3.7) has the least possible number of parameter estimates, i.e. it is not overparametrized [10]. Thus, the addition of a new integrator to the system (2.1a) does not produce new estimates of the unknown parameter θ.

Remark 2.
The stabilizability condition for the subsystem (3.5a) is just the so-called "soft matching condition" [15].

Remark 3.
Applying to systems with the linear in v1 right hand side and standard integrators, namely

$$\dot{x}=f_0(x) + \theta f(x) + g(x)v$$

$$\dot{v}=u$$

the adaptive stabilization algorithm (3.6)-(3.7) coincides with [11], namely

$u= \alpha_1(x,v,\hat{\theta})=$

$$-\gamma_0(v-\alpha_0)+ \frac{\partial\alpha_0}{\partial x}[f_0(x) + \hat{\theta}f(x) + g(x)v] +\frac{\partial\alpha_0}{\partial\hat{\theta}}\tau_1+$$

$$+ \gamma\frac{\partial Q}{\partial\hat{\theta}}\frac{\partial\alpha_0}{\partial x} f(x) -\frac{\partial Q}{\partial x}g(x)$$

$$\dot{\hat{\theta}} = \tau_1(x,v,\hat{\theta})= \tau_0 - \frac{\gamma}{\gamma_1}\frac{\partial\alpha_0}{\partial x} f(x)(v-\alpha_0) \qquad (4.2)$$

5. EXAMPLE

Consider the nonlinear system with two integrators

$$\dot{x}= - \frac{xv_1^2}{2} + \theta x^2 \qquad (5.1)$$

$$\dot{v}_1=v_2 \qquad (5.2)$$

$$\dot{v}_2=u \qquad (5.3)$$

where $x,v_1,v_2 \in R$, $u\in R^1$ is the input and $\theta\in R^1$ is an unknoun constant parameter.

This system violates the geometric "extended matching" conditions and not in the "parametric-pure-feedback" form [4]. However, the system satisfies the conditions of theorem in Section 3 and, hence, the iterative design procedure, applyed to (5.1)-(5.3) with the objective function $Q(x)=\frac{1}{2}x^2\ge 0$, is as follows.

Step 0:
Let $\hat{\theta}$ be an estimate of θ. In order to stabilize the first subsystem (5.1), the following control and parameter update laws can be selected:

$$u=\alpha_0(x,\hat{\theta})= x+\hat{\theta}$$

$$\dot{\hat{\theta}}=\tau_0(x,\hat{\theta})=x^3 \qquad (5.4)$$

and let the Lyapunov function be $V=\frac{1}{2}x^2+\frac{1}{2}(\theta-\hat{\theta})^2$
It is easy to check that (5.4) stabilizes (5.1).

Because $\alpha_0(x,\hat{\theta})$ is not the actual control, consider the subsystem (5.1) augmented by the integrator (5.2).

Step 1:
Considering v1 as the integrator variable and v2 as the control we use the algorithm (3.6),(3.7) to describe the control and update laws:

$$u=\alpha_1(x,v_1,\hat{\theta})=-(v_1-x-\hat{\theta})+\frac{x^2}{2}(x+v_1+\hat{\theta})+\tau_1-\frac{xv_1^2}{2}+\hat{\theta}x^2$$

$$\dot{\hat{\theta}}=\tau_1(x,v_1,\hat{\theta})=x^3-(v_1-x-\hat{\theta})x^2 \qquad (5.5)$$

to stabilize the first two subsystems (5.1),(5.2) with respect to the Lyapunov function

$V_1(x,v_1,\hat{\theta})=\frac{1}{2}x^2+\frac{1}{2}(\theta-\hat{\theta})^2+\frac{1}{2}(v_1-x-\hat{\theta})^2$

Because $\alpha_1(x,v1,\hat{\theta})$ is not the actual control, consider the augmented system (5.1)-(5.3).

Step 2:
Considering the vector $[x,v1]^T$ as the state, v2 as the integrator variable and u as the control, we have

$f=[x^2\ 0]^T$; $f_0=[\ -xv1^2/2\ \ v2]^T$.

Then we use the algorithm (3.6),(3.7) for the system (5.1)-(5.3) with $\alpha_1(x,v1,\hat{\theta})$, $\tau_1(x,v1,\hat{\theta})$ from (5.5) and $Q1=Q+\frac{1}{2}(v1-\alpha0)^2$ to find the control and update laws:

$$u=\alpha2(x,v1,v2,\hat{\theta})=-(v2-\alpha1)-[\frac{\partial Q1}{\partial x}\ \ \frac{\partial Q1}{\partial v1}][0\ 1]^T +$$

$$+\frac{\partial Q1}{\partial \hat{\theta}}[\frac{\partial \alpha1}{\partial x}\ \ \frac{\partial \alpha1}{\partial v1}]f+\frac{\partial \alpha1}{\partial \hat{\theta}}\tau2+[\frac{\partial \alpha1}{\partial x}\ \ \frac{\partial \alpha1}{\partial v1}](f0+\hat{\theta}f)$$

$$\dot{\hat{\theta}}=\tau_2(x,v1,v2,\hat{\theta})=\tau_1-(v2-\alpha1)[\frac{\partial \alpha1}{\partial x}\ \ \frac{\partial \alpha1}{\partial v1}]f \qquad (5.6)$$

The laws (5.6) are actual and stabilize the closed-loop system (5.1)-(5.3) with respect to the Lyapunov function

$$V_2(x,v1,v2,\hat{\theta})=Q1+\frac{1}{2}(\theta-\hat{\theta})^2+\frac{1}{2}(v2-\alpha1)^2 =$$

$$= Q+\frac{1}{2}(\theta-\hat{\theta})^2+\frac{1}{2}(v1-\alpha0)^2+\frac{1}{2}(v2-\alpha1)^2$$

Thus, the given objective function $Q(x(t),\hat{\theta}(t))\to 0$ as $t\to\infty$.

6. CONCLUSION

A new adaptive stabilizer design procedure is proposed for the case when the right hand side of the reduced system depends nonlinearly on additional nonlinear integrator variables. The design procedure is iterative and does not involve overparametrization. For the case when the above dependence is convex the adaptive contol algorithm can be simplified. The obtained stabilization and tracking results extend the existing ones (e.g.[4, 11]) to the wider class of nonlinear controlled systems.

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