DECENTRALIZED MODEL BASED PREDICTIVE CONTROL OF LARGE SCALE SYSTEMS

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Abstract **This paper proposes an approach for the design of discrete-time decentralized control systems based on model-based predictive control (MBPC). The class of interconnected large scale systems is considered and a model is used at each control station to predict the corresponding subsystem output over a long-range time period. The interaction trajectories are estimated, and an on-line improvement of them is performed. In the case of subsystems with m-step delay information patterns this estimation is based on a linear model for the interconnections with on-line updating of its parameters. For all other cases an on-line discrete-time decentralized model following controller is obtained. Representative computer simulation results are provided and compared for three nontrivial example systems.**

I. INTRODUCTION

The decentralized control of interconnected dynamical systems has received an increasing interest in recent years ([1], [2]). One of the benefits of decentralized control is that the large scale systems can be decomposed into many subsystems [4],[7], and the control, design and implementation of each one of them can be performed independently. This simplifies the overall control problem. Moreover the computational burden can be shared by all the control stations involved. The main difficulty in designing decentralized control systems is the limited information available for the control [3],[5],[8]. The set of the control stations is constrained to have access only to local information, i.e. only measurements of local outputs and states (and no communication among them are permitted [9]). This is exactly what characterises a non-classical information pattern. Finally, the local control stations cannot take into account the interactions with the other subsystems, and therefore techniques to deal with this problem must be developed. Recently, much work has been conducted aiming at the analysis of stability of decentralized control problems and the development of decentralized stabilizers using output or state feedback [6].

Moreover optimal decentralized control algorithms have been developed for dealing with such problems. But it is well known that optimal control is not very suitable for complex industrial systems or general large-scale systems since it needs an accurate model of the controlled system and it is sensitive to parameter variations and to the existence of stochastic disturbances.

A fundamentally new control appproach that does not have the drawbacks of pure optimal control and it is suitable for complex large scale systems is the so called Model-Based Predictive Control (MBPC) approach [12-14]. This approach was developed in the mid-seventies, and allows for model uncertainties, it updates the output of the model by closed-loop corrections, and optimizes the control law on a moving horizon. In this paper the decentralized control approach is combined with the model-based predictive control approach. The result is a unique control scheme called *Decentralized Model Based Predictive Control* (DMBPC). This is achieved by a suitable estimation of the interactions at each control station [10].

Section II presents the problem formulation, Section III introduces the concept of MBPC and deals with the computation of the centralized part of the control algorithm. Section IV proposes two techniques for approximating the decentralized part of the control. The first concerns the case of m-step delay sharing pattern and a linear model for the interconnections. The second is based on an on-line improvement for the predictions of the interconnections which is based on a model of them. Finally, Section V presents a set of representative simulation results for three particular examples including some comparisons, and Section VI gives some concluding remarks.

II. FORMULATION OF THE PROBLEM

Consider a discrete time, linear, possibly time-varying large scale system which consists of N-interconnected subsystems, each of which has the following state space description:

$$x_i(t+1)=A_i x_i(t)+B_i u_i(t)+E_i z_i(t) \tag{1a}$$
$$x_i(0)=x_{i_0} \tag{1b}$$
$$y_{m_i}(t)=C_i x_i(t) \tag{1c}$$

for $i=1,2,...,N$, where

$x_i(t)$: is the n_i-dimensional state vector of the i-th subsystem at time t,

$u_i(t)$: is the r_i-dimensional control vector of the i-th subsystem at time t,

$y_{m_i}(t)$: is the p_i-dimensional output vector of the model of the i-th subsystem at time t,

$z_i(t)$: is the q_i-dimensional interconnection vector which describes the influence of all other subsystems upon the i-th one .

The vector $z_i(t)$ is considered to be a linear combination of the states of all other subsystems, i.e

$$z_i(t)=\sum_{i=1}^{N} L_{ij} x_j(t), \quad i=1,2,\dots,N \quad \text{with } i \neq j \tag{2}$$

where L_{ij} are matrices of proper dimensions assumed to be known to every control station.

It is accentuated that the output $y_{m_i}(t)$ of the model may generally have a small difference from the real output $y_i(t)$ because of modelling errors or noise which affect the whole system or parts of it.

Finally the matrices A_i, B_i, E_i, C_i are of proper dimensions, i.e. $A_i \in M_{n_i x n_i}$, $B_i \in M_{n_i x r_i}$, $E_i \in M_{n_i x q_i}$, $C_i \in M_{p_i x n_i}$ where M_{kxs} is the set of matrices of kxs dimensions.

The problem is *to find at every time t the best control $u_i(t)$ for the i-th subsystem, which leads the output current value $y_i(t)$ to its set-point $w_i(t)$.* The control must be "the best" in the sense of minimizing a cost function which will be described later.

Moreover the control laws must be specified in a decentralized way, i.e. the control laws are assumed to be of the form $u_i(t)=\gamma_i(I_i(t))$, $i=1,2,...,N$, where $\gamma_i(t)$ is a function of the available information set $I_i(t)$ of the i-th subsystem defined as follows :

$$I_i(t)=\{ y_i(1), y_i(2),..., y_i(t) \;;\; u_i(1), u_i(2),...,u_i(t-1) \;;\; x_i(0), x_i(1),......, x_i(t)\}$$
$$I_i(0)=\emptyset$$

This means that $I_i(t)$ consists not only of the measurement of the current output $y_i(t)$ but also of the past outputs $y_i(r)$, $u_i(r)$ $r<t$. The information set $I_i(t)$ does not contain $z_i(t)$ which is necessary for the computation of $u_i(t)$ but not available to the i-th control station because it depends on the non-available states of the other subsystems $x_j(t)$, $j \neq i$.

In the following, predictive control techniques will be used to satisfy the problem requirements. It will be shown that the control laws are of the form :

$$u_i(t)=u_i^c(t)+u_i^d(t) \quad , \; i=1,2,...,N \tag{3}$$

where $u_i^c(t)$ is the centralised part of the control, i.e. the part that is available to the i-th control station, and $u_i^d(t)$ is the decentralized part of the control law which depends on information not available to the i-th control station, and more especially depends on $z_i(t)$ and predictions of it.

III. MBPC AND COMPUTATION OF THE CENTRALIZED PART OF THE CONTROL

Model-Based Predictive Control is a control algorithm which uses a model for open-loop predictions, optimizing the control inputs on a moving horizon and updating the outputs of the model by closed loop predictions.

Especially, at each time t, the output $y_i(t+k)$ is predicted over a future period of time $k=1,2,...,L_y$ where L_y is the prediction horizon. The predictions are symbolised by $y_{p_i}(t+k/t)$ and are determined by means of a model, for example a state space model like (1a-1c). The predictions $y_{p_i}(t+k/t)$, $k=1,2,...,L_y$ depend on future control values $u_i(t+k/t)$, $k=0,1,...,L_u$, where L_u is the control horizon ($L_u \leq L_y$). In the control horizon we have:

$$u_i(t+L_u+k/t)=u_i(t+L_u-1) \quad , \quad k \geq 0$$

The i-th subsystem output predictions can be calculated as :

$$y_{p_i}(t+k/t)=y_{m_i}(t+k/t)+q_i(t+k/t) \tag{4}$$

where, by (1a, b),

$$\begin{aligned} y_{m_i}(t+k/t) &= C_i x_i(t+k/t) \\ &= C_i [A_i^k x_i(t) + \sum_{j=1}^{k} A_i^{j-1} B_i u_i(t+k-j/t) \\ &\quad + \sum_{j=1}^{k} A_i^{j-1} E_i z_i(t+k-j/t)], \\ &\qquad k=1,2,\dots,L_y \end{aligned} \tag{5}$$

and $q_i(t+k/t)$ is the cloded-loop correction vector based on the available information set at time t. A recommended form for $q_i(t+k/t)$ is the following:

$$q_i(t+k/t)=y_i(t)-y_{mi}(t) \tag{6}$$

where $y_i(t)$ is the measured value of the output vector at time t.

A reference trajectory $r_i(t+k/t)$, $k=1,2,...,L_y$ is defined over the prediction horizon, which describes how we want to guide the output vector $y_i(t)$ to its set-point $w_i(t)$, i.e.

$$r_i(t+k/t)=w_i(t+k/t)+\upsilon_i(t+k/t) \quad (7)$$

where $\upsilon_i(t+k)$ is a correction vector based on the previous error information set $\{w_i(t)-y_i(t), w_i(t-1)-y_i(t-1),..., w_i(1)-y_i(1)\}$.

A simple form which gives good results is the following:

$$\upsilon_i(t+k/t)= a^k[w_i(t)-y_i(t)] \quad (8)$$

where $0<a\leq 1$ is a tuning parameter that specifies the desired closed-loop dynamic ($a\rightarrow 0$:fast control; $a\rightarrow 1$:slow control).

The reference trajectory is initiated at the current measured output i.e. $r_i(t/t)=y_i(t)$.

It is noted that if the future set-point values $w_i(t+k/t)$, $k=1,2,...,L_y$ are unknown at time t, one can assume that :

$$w_i(t+k/t) = w_i(t) \quad , \quad k=1,2,...,L_y$$

All the above are illustrated in Fig.1.

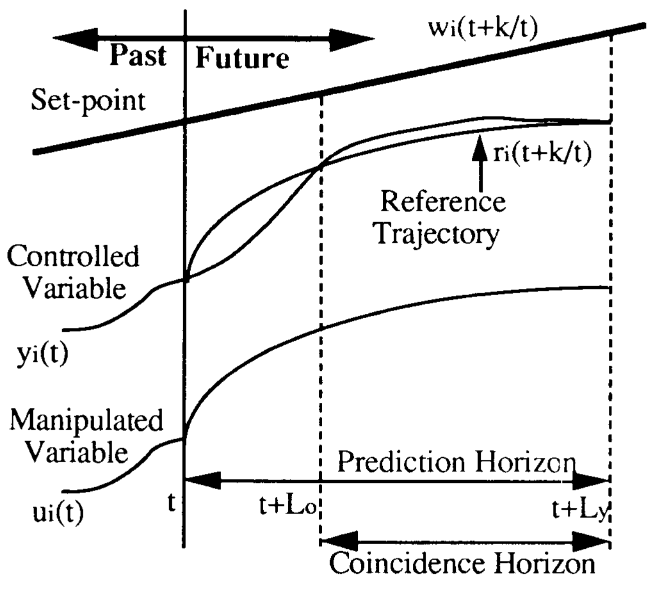


Fig.1. The reference trajectory, the set-point trajectory, the prediction horizon and the coincidence horizon

The cost function of the i-th control station has the form:

$$\min J_i(t) = \frac{1}{2}\Big[\sum_{k=L_o}^{L_y} \left\| r_i(t+k/t) - y_{p_i}(t+k/t) \right\|^2_{Q_i(k)} + \sum_{k=0}^{L_u-1} \left\| u_i(t+k/t) \right\|^2_{R_i(k)} \Big] \quad (9)$$

where $Q_i(k) \geq 0$ for $k=L_o,...,L_y$ and $R_i(k) > 0$ for $k=0,...,L_u-1$.

Since $J_i(t)$ varies with time t and has a moving optimisation horizon, only the first term in the optimal solution is implemented to control the i-th subsystem. The optimization parameter L_o determines, togerther with L_y, the "coincidence horizon" we want the predicted output to follow the reference trajectory over the time interval $[t+L_o,...,t+L_y]$. The basic concepts of the MBPC structure as it is applied to the i-th subsystem is shown in the Fig.2.

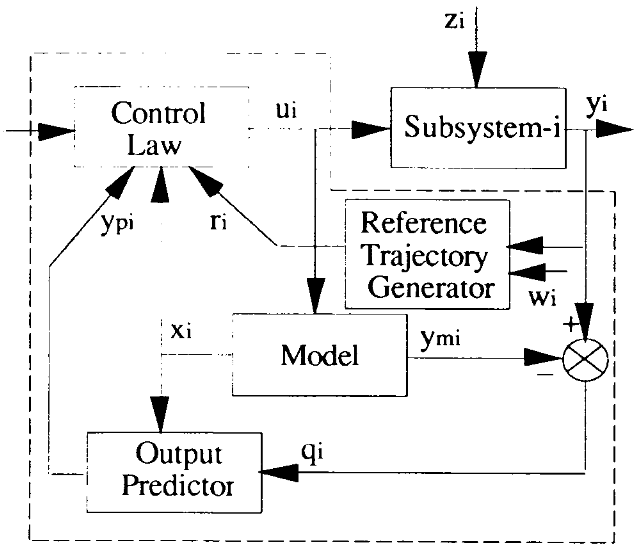


Fig.2. The basic MBPC structure

Let us now proceed to the computation of the optimal control in the i-th control station. To this end, the following notation is introduced:

$$\hat{y}_{p_i} = [y_{p_i}(t+1/t),\dots,y_{p_i}(t+L_y/t)]^T \quad (10)$$

$$\hat{y}_{m_i} = [y_{m_i}(t+1/t),\dots,y_{m_i}(t+L_y/t)]^T \quad (11)$$

$$\hat{r}_i = [r_i(t+1/t),\dots,r_i(t+L_y/t)]^T \quad (12)$$

$$\hat{q}_i = [q_i(t+1/t),\dots,q_i(t+L_y/t)]^T \quad (13)$$

$$\hat{u}_i = [u_i(t/t), u_i(t+1/t), \ldots, u_i(t+L_u-1/t)]^T \quad (14)$$

$$\hat{w}_i = [w_i(t+1/t), \ldots, w_i(t+L_y/t)]^T \quad (15)$$

$$\hat{\upsilon}_i = [\upsilon_i(t+1/t), \ldots, \upsilon_i(t+L_y/t)]^T \quad (16)$$

$$\hat{d}_i = [C_i A_i x_i,\ C_i A_i^2 x_i, \ldots, C_i A_i^{L_y} x_i]^T \quad (17)$$

$$\hat{z}_i = [z_i(t/t), z_i(t+1/t), \ldots, z_i(t+L_y-1/t)]^T \quad (18)$$

Then, from equation (5) one obtains:

$$\hat{y}_{m_i}(t) = \hat{d}_i(t) + S_i^* \hat{u}_i(t) + S_i \hat{z}_i(t) \quad (19)$$

where

$$S_i^* = \begin{bmatrix} L \\ \cdots\cdots\cdots \\ F \ \vdots \ V \end{bmatrix} \quad (20$$

and L, F, V are the following block-matrices:

$$L = \begin{bmatrix} C_i A^0 B_i & 0 \ldots\ldots\ldots\ldots\ldots 0 \\ C_i A^1 B_i & C_i A^0 B_i \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots 0 \\ C_i A^{L_u-1} B_i & C_i A^{L_u-2} B_i \ldots\ldots C_i A^0 B_i \end{bmatrix}$$

$$F = \begin{bmatrix} C_i A^{L_u} B_i & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots C_i A^2 B_i \\ C_i A^{L_u+1} B_i & C_i A^{L_u} B_i \ldots\ldots\ldots C_i A^3 B_i \\ \ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_i A^{L_y-1} B_i & C_i A^{L_y-2} B_i \ldots C_i A^{L_y-L_u+1} B_i \end{bmatrix}$$

$$V = [C_i(\sum_{k=0}^{1} A_i^k)B_i, \ldots C_i(\sum_{k=0}^{L_y-L_u} A_i^k)B_i]^T \quad (21)$$

and

$$S_i(t) = \begin{bmatrix} C_i A_i^0 E_i & 0 \ \ldots\ldots\ldots\ldots\ldots \ 0 \\ C_i A_i^1 E_i & C_i A_i^0 E_i \ \ldots\ldots\ldots\ldots \ 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots \ 0 \\ C_i A_i^{L_y-1} E_i & C_i A_i^{Ly-2} E_i \ldots C_i A_i^0 E_i \end{bmatrix} \quad (22)$$

Moreover

$$\hat{y}_{p_i}(t) = \hat{y}_{m_i}(t) + \hat{q}_i(t) \quad (23)$$

$$\hat{r}_i(t) = \hat{w}_i(t) + \hat{\upsilon}_i(t) \quad (24)$$

and so the cost function (9) can be rewritten as:

$$J_i(t) = \frac{1}{2}\{\|\hat{y}_{p_i}(t) - \hat{r}_i(t)\|^2_{\hat{Q}_i} + \|\hat{u}_i(t)\|^2_{\hat{R}_i}\} \quad (25)$$

where

$$\hat{Q}_i = \mathrm{diag}[0,0,\ldots,0,Q_i(L_0),Q_i(L_0+1),\ldots,Q_i(L_y)] \quad (26)$$

$$\hat{R}_i = \mathrm{diag}[R_i(0),R_i(1),\ldots,R_i(L_u-1)] \quad (27)$$

Optimizing the cost function defined by (25) together with (19), (23) and (24) one gets:

$$\begin{aligned} 0 &= \frac{\partial J_i(t)}{\partial \hat{u}_i(t)} \\ &= \frac{\partial}{\partial \hat{u}_i(t)} \{\frac{1}{2}[(S_i^* \hat{u}_i(t) + K_i(t))^T \hat{Q}_i(t)(S_i^* \hat{u}_i(t) \\ &\quad + K_i(t)) + \hat{u}_i^T(t)\hat{R}_i(t)\hat{u}_i(t)]\} \\ &= \frac{\partial}{\partial \hat{u}_i(t)} \{\frac{1}{2}[(S_i^* \hat{u}_i(t)]^T \hat{Q}_i(t) S_i^* \hat{u}_i(t)\} \\ &\quad + \frac{1}{2} S_i^{*T} \hat{Q}_i(t) K_i(t) + \frac{1}{2} S_i^{*T} \hat{Q}_i(t) K_i(t) \\ &\quad + \hat{R}_i(t)\hat{u}_i(t) \\ &= S_i^{*T} \hat{Q}_i(t) S_i^* \hat{u}_i(t) + S_i^{*T} \hat{Q}_i(t) K_i(t) \\ &\quad + \hat{R}_i(t)\hat{u}_i(t) \\ &= [S_i^{*T} \hat{Q}_i(t) S_i^* + \hat{R}_i(t)]\hat{u}_i(t) + S_i^{*T} \hat{Q}_i(t) K_i(t) \end{aligned}$$

whence

$$\hat{u}_i(t) = -[S_i^{*T} \hat{Q}_i(t) S_i^* + \hat{R}_i(t)]^{-1} S_i^{*T} \hat{Q}_i(t) K_i(t) \quad (28)$$

where

$$K_i(t) = \hat{d}_i(t) + \hat{q}_i(t) - \hat{r}_i(t) + S_i\hat{z}_i(t) \qquad (29)$$

or

$$\hat{u}_i(t) = [\hat{R}_i(t) + S_i^{*T}\hat{Q}_i(t)S_i^*]^{-1}S_i^{*T}\hat{Q}_i(t)[\hat{w}_i(t) + \hat{b}_i(t) - \hat{q}_i(t) - \hat{d}_i(t) - S_i\hat{z}_i(t)]$$

i.e.

$$\hat{u}_i(t) = \hat{D}_i(t)[\hat{w}_i(t) + \hat{b}_i(t) - \hat{q}_i(t) - \hat{d}_i(t)] - \hat{D}_i(t)S_i\hat{z}_i(t) \qquad (30)$$

with

$$\hat{D}_i(t) = [\hat{R}_i(t) + S_i^{*T}\hat{Q}_i(t)S_i^*]^{-1}S_i^{*T}\hat{Q}_i(t) \qquad (31)$$

Note that the assumptions made imply that

$$\frac{\partial^2 J_i(t)}{[\partial\hat{u}_i(t)]^2} = \hat{R}_i(t) + S_i^{*T}\hat{Q}_i(t)S_i^* > 0$$

and so the problem is always solvable with the criterion being minimized.

The first term of the optimal control in equation (30) is the centralised part of the solution i.e.

$$\hat{u}_i^c(t) = \hat{D}_i(t)[\hat{w}_i(t) + \hat{b}_i(t) - \hat{q}_i(t) - \hat{d}_i(t)] \qquad (32)$$

which is realizable in a decentralized way because it depends on information which is locally available to the i-th control station.

On the other hand the second term of equation (30), i.e.

$$\hat{u}_i^d(t) = -\hat{D}_iS_i\hat{z}_i(t) \qquad (33)$$

depends on information which is not available to the i-th control station. This information is the extended interaction vector $\hat{z}_i(t)$ which consists of present and future values of the interaction vectors $z_i(t+k/t)$, $k=0,1,\ldots,L_y-1$. None of the previous vectors are available to the i-th control station because they all depend on the states (present and future) of the other subsystems. So we need to approximate or to predict them using interaction models. This is done in the next section.

IV. APPROXIMATIONS FOR THE DECENTRALIZED PART OF THE CONTROL

It is well-known that MBPC techniques can manage very well systems with unusual dynamical behaviour and introduce in a natural way feedforward control action for compensating the disturbances. If we consider $z_i(t)$ in the dynamical equation (1a) as a disturbance for the i-th subsystem, then for the case of weakly coupled systems in which the elements of the matrices E_i, $i=1,2,\ldots,N$ are small enough we can make the assumption that $u^d_i(t)\approx 0$, and let the centralised part $u^c_i(t)$ alone to control the system. Two different techniques for the case where this is not true are the following.

(a) The case of problems with m-step delay sharing pattern and a linear model for the interconnections.

A decentralized control problem is said to have an *m-step delay sharing pattern* when it permits the spreading of its information through the subsystems but with delay of m time steps. Clearly each control station obtains *instantaneously* all information about its associated subsystem, and after a delay of m time steps all the information available to *all* control stations.

For our problem this means that at time t in the i-th control station the vectors $x_j(t-m)$, $j\neq i$ and all the past values $x_j(t-m-k)$, $k>0$ are known. Then one can calculate $z_i(t-m)$ using equation (2) as:

$$z_i(t-m) = \sum_{j=1}^{N} L_{ij}x_j(t-m) \quad , \quad j\neq i \qquad (34)$$

i.e. $z_i(t-m)$ is well known to the i-th control station at time t. The preassumption that the vector $z_i(t-m)$ is the output of a linear model having order p and coefficients a_{ij}, $j=1,2,\ldots,p$, i.e.

$$\bar{z}_i(t-m) = \sum_{j=1}^{p} a_{ij}z_i(t-m-j) \qquad (35)$$

where $\bar{z}_i(t-m)$ is an estimate of $z_i(t-m)$, is now made. The coefficients a_{ij} of the linear model (35) can be calculated on-line at every time t in the least squares sense by minimizing the norm of the error vector:

$$e_{z_i}(t) = z_i(t-m) - \bar{z}_i(t-m) \qquad (36)$$

After having calculated the coefficients a_{ij} one can use the model (35) to produce the predictions $z_i(t+k/t)$, $k=0,1,\ldots,L_y-1$ that are needed in equation (33) in order to approximate the decentralized part of the control $u^d_i(t)$. The predictions are:

$$\bar{z}_i(t-m+k/t) = \sum_{j=1}^{p} a_{ij}\bar{z}_i(t-m+k-j/t),$$
$$k = 1,2,\ldots m, m+1,\ldots, L_y+m-1 \qquad (37)$$

Then $\hat{z}_i(t) \approx \hat{z}'_i(t)$ implies

$$[z_i(t/t), z_i(t+1/t), \ldots, z_i(t+L_y-1/t)]$$
$$= [\bar{z}_i(t/t), \bar{z}_i(t+1/t), \ldots, \bar{z}_i(t+L_y-1/t)] \qquad (38)$$

and so

$$\hat{u}'^{d}_i(t) = -\hat{D}_i S_i \hat{z}'_i(t).$$

(b) On-line improvement for the predictions of the interconnections, based on a model for them.

In this approximation a method will be suggested which provides on-line improvement for the predictions of the interconnections. This method will reduce the dependence of the predictions from the original model used for them. Moreover, the case of problems that do not have m-step delay sharing patterns will be covered, where one cannot use a linear model for the interconnections with on-line computation of its parameters because of the non-spreading of the information.

For this case one can use a model of the form

$$z_i(t) = A_{z_i} z_i(t-1) \qquad (39)$$

where A_{z_i} is the part of the whole system matrix $A_g = A + EL$ ($A = \mathrm{diag}[A_i]$, $E = \mathrm{diag}[E_i]$, $L = [L_{ij}]$) that corresponds to the elements of the vector z_i as it is suggested by Singh ([6] *Chapter 6)*. The predictions provided by (39):

$$z'_i(t+k/t) = A_{z_i} z'_i(t+k-1/t),$$
$$k = 0, 1, \ldots, L_y - 1 \qquad (40)$$

are recognised as "crude" ones and therefore they need improvement. To this end, using the extended vector

$$\hat{z}'_i(t) = [z'_i(t/t), z'_i(t+1/t), \ldots, z'_i(t+L_y-1/t)]^T \qquad (41)$$

one can produce a first approximation

$$\hat{u}'_i(t) = -D_i S_i \hat{z}'_i(t) \qquad (42)$$

for the decentralized part of the control which will feed only the model of the i-th subsystem.

Then, suboptimal state space trajectories are produced as:

$$x'_i(t+1) = A_i x'_i(t) + B_i u'_i(t) + E_i z_i(t) \qquad (43)$$

where $u'_i(t)$ is the first block vector of the extended control vector

$$\hat{u}'_i(t) = \hat{u}^{c}_i(t) + \hat{u}'^{d}_i(t)$$

An observation for $x'_i(t)$ is provided by:

$$\bar{x}'_i(t+1) = A_i \bar{x}'_i(t) + B_i u'_i(t) + E_i z'_i(t) \qquad (44)$$

Substracting (44) from (43) gives:

$$x'_i(t+1) - \bar{x}'_i(t+1) = A_i[x'_i(t) - \bar{x}'_i(t)]$$
$$+ E_i[z_i(t) - z'_i(t)] \qquad (45)$$

Now, the problem is to minimize the norms of the error vectors $x'_i(t) - \bar{x}'_i(t)$ and $z_i(t) - z'_i(t)$. To this end, an additional optimization problem (at each control station) is constructed, namely:

$$\min J_i(t) = \frac{1}{2}\{[x'_i(t_f) - \bar{x}'_i(t_f)]^T G_i [x'_i(t_f) - \bar{x}'_i(t_f)]$$
$$+ \sum_{k=t}^{t_f-1} \{\|x'_i(k) - \bar{x}'_i(k)\|^2_{M_i} + \|z_i(k) - z'_i(k)\|^2_{T_i}\} \qquad (46)$$

where $G_i \geq 0$, $M_i \geq 0$, $T_i > 0$. For computational simplicity it is assumed that $t_f = t+1$ (one step ahead optimization problem), so that the solution for $z_i(t) - z'_i(t)$ will not depend on future values $x'_i(k)$, $\bar{x}'_i(k)$, $k > t$. The optimization problem (46) with the constraint equation (45) is a linear regulation problem where $z_i(t) - z'_i(t)$ plays the role of control vector. Its solution is well-known [7]:

$$z_i(t) - z'_i(t) = K_i(t)[x'_i(t) - \bar{x}'_i(t)]$$

i.e.

$$z_i(t) = z'_i(t) + K_i(t)[x'_i(t) - \bar{x}'_i(t)] \qquad (47)$$

Here $K_i(t)$ is the discrete time Kalman-matrix obtained by:

$$K_i(t) = -T_i^{-1} E_i^T [A_i^T]^{-1} [P_i(t) - M_i] \qquad (48)$$

where $P_i(t)$ is the positive definite solution of the Riccati equation:

$$P_i(t) - A_i^T P_i(t+1)[I_i + E_i T_i^{-1} E_i^T P_i(t+1)]^{-1} A_i = M_i \qquad (49a)$$

$$P_i(t_f) = G_i \qquad (49b)$$

Equation (47) introduces an improvement $z_i(t)$ for the "crude" estimation of the interconnection vector $z'_i(t)$ obtained by the original model (39). So one can make an improvement in the extended interconnection vector as follows:

$$\hat{z}_i(t) \approx \hat{z}'_i(t) \quad \text{with}$$

$$\hat{z}'_i(t) = [z'_i(t/t) + \Delta_i(t),\ z'_i(t+1/t) + \Delta_i(t), \ldots$$
$$\ldots, z'_i(t + L_y - 1/t) + \Delta_i(t)]^T \qquad (50)$$

where $\Delta_i(t)$ is the second term in equation (47), i.e.

$$\Delta_i(t) = K_i(t)[x'_i(t) - \bar{x}'_i(t)] \qquad (51)$$

Thus an improved approximation for the decentralized part of the control is obtained as:

$$\hat{u}^d_i(t) = -D_i S_i \hat{z}_i(t)$$

which will be the control for the i-th subsystem.

V. SIMULATION RESULTS

System (i): The present system consists of three subsystems with state-space matrices:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.3 & 0.2 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 0.1 & -0.4 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ -0.2 & -0.5 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0.3 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix} \qquad B_3 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$$

$$C_1 = [2\ \ 1] \qquad C_2 = [-3\ \ 1.5] \qquad C_3 = [1\ \ 4]$$

$$L_{12} = [2\ \ 1] \qquad L_{21} = [-1\ \ 1] \qquad L_{31} = [0\ \ 2]$$

$$L_{13} = [-1\ \ 0] \qquad L_{23} = [0\ \ -2] \qquad L_{32} = [-2\ \ 1]$$

and 3-step delay sharing pattern. The proposed contoller is applied to the following cases:

a) controller with no on-line improvement for the predictions of the interconnections (see fig. 1.1-1.3), and

b) controller with on-line improvement for the predictions of the interconnections (see fig. 1.4-1.6).

In both cases the predictions for the interconnections are produced from a linear model for them since the system allows the spreading of its information (3-step delay sharing pattern). For the controller the parameters we used are $L_y=10$, $L_u=4$, $\alpha=0.1$, $p=2$, $Q_i=I_i$ and $R_i=10^{-3}I_i$ (I_i is the identity matrix of proper dimensions). The improvement of the results is obvious in case (b) for both the transient response and steady state response.

System (ii): The system consists of two subsystems with state-space matrices:

$$A_1 = \begin{bmatrix} 0.1 & 1 & -0.5 \\ 0.25 & -0.1 & 0.9 \\ -0.2 & -0.3 & 0.4 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 1 \\ -0.2 & 0.3 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ -0.5 & 0.4 & 0.9 \\ 0.5 & 0 & 0.3 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -0.5 \\ 0 & 2 & -1 \end{bmatrix} \qquad C_2 = [3\ \ -1]$$

$$L_{12} = [-3\ \ 1]$$
$$L_{21} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

and does not allow the spreading of its information. Thus its control will be based on the algorithm of section (IV-b) and on a model of the form $z_i(t+1)=A_{zi}z_i(t)$. For the controller we used the parameters $L_y=10$, $L_u=4$, $L_o=1$, $\alpha=0.1$, $Q_i=I_i$ and $R_i=10^{-3}I_i$. Figures 2.1-2.4 show the results which verify that the control of systems without m-step delay sharing pattern is more difficult.

System (iii): This system consists of three subsystems with state-space matrices:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.2 & -0.4 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.4 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ -0.1 & 0.3 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & 0.4 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \qquad B_3 = \begin{bmatrix} 0.3 & 0.1 \\ -0.2 & 0.3 \end{bmatrix}$$

$$C_1=[2\ \ 1] \qquad C_2=[3\ \ -2] \qquad C_3=[1\ \ 2]$$

$$L_{12}=[-1\ \ 2] \qquad L_{21}=[-1\ \ 2] \qquad L_{31}=[3\ \ -2]$$

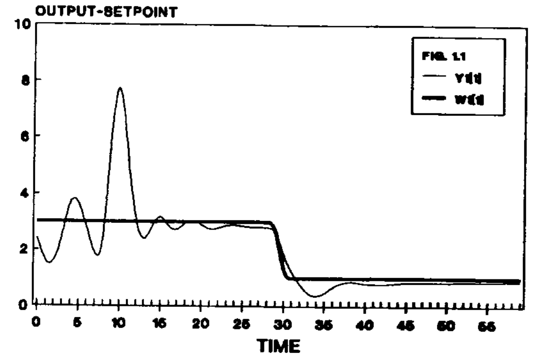
$$L_{13}=[1\ \ 1.5] \qquad L_{23}=[0\ \ 1] \qquad L_{32}=[0\ \ 1]$$

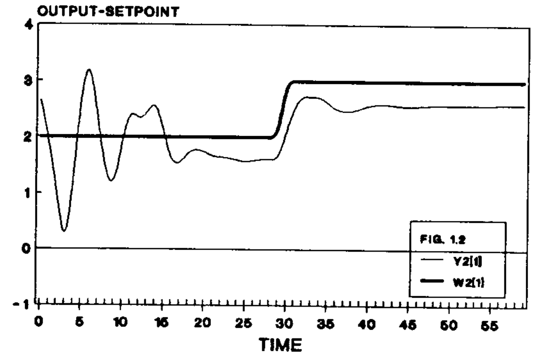
and m-step delay sharing pattern. We apply the suggested controller in the following cases:

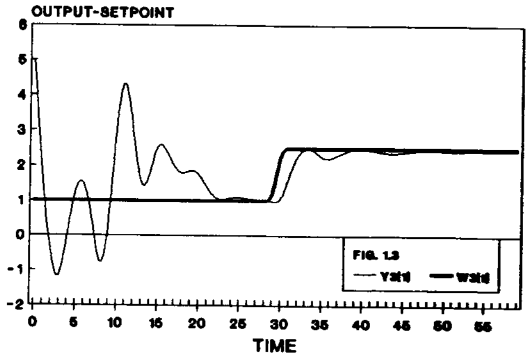
a) controller with m=1 (see fig. 3.1-3.3), and

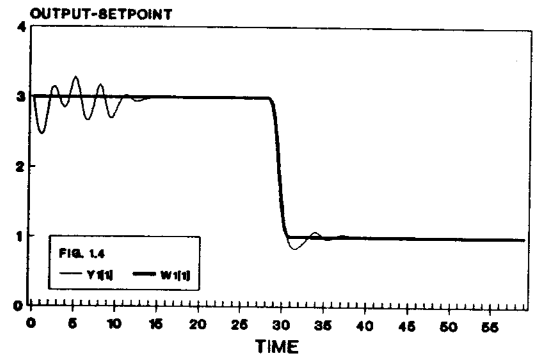
b) controller with m=3 (see fig. 3.4-3.6).

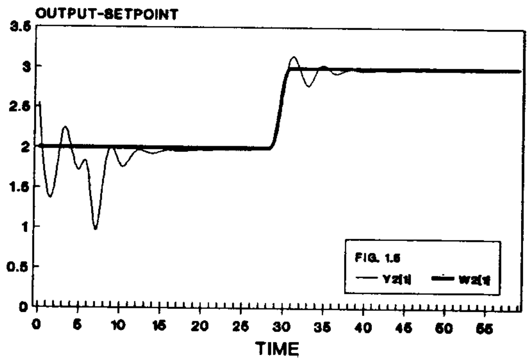
In both cases the predictions for the interconnections are produced from a linear model for them since the system allows the spreading of its information (1 or 3-step delay

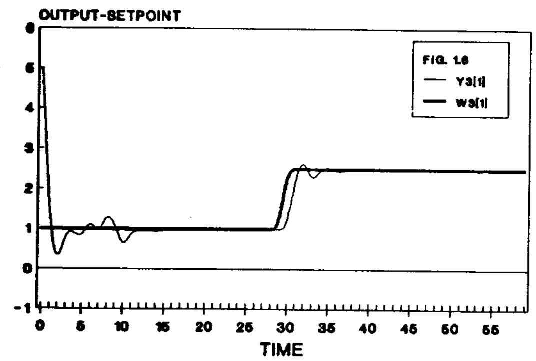

OUTPUT-SETPOINT
FIG. 1.1
Y1[1]
W1[1]
TIME

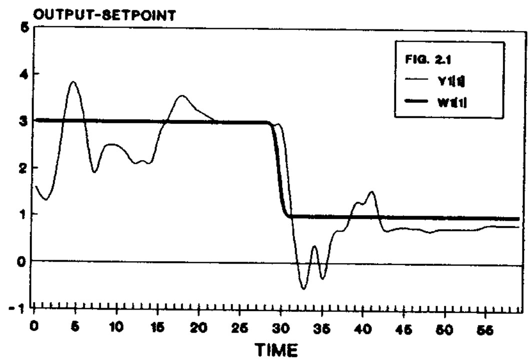

OUTPUT-SETPOINT
FIG. 1.2
Y2[1]
W2[1]
TIME

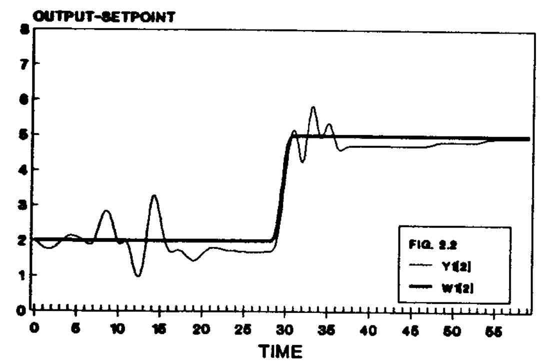

OUTPUT-SETPOINT
FIG. 1.3
TIME

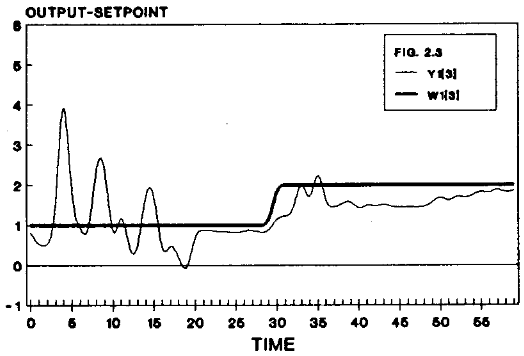

OUTPUT-SETPOINT
FIG. 1.4
Y1[1]
W1[1]
TIME

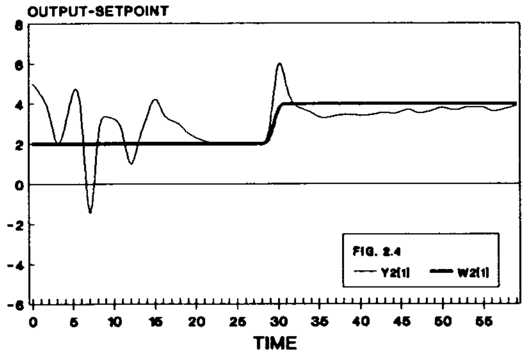

OUTPUT-SETPOINT
FIG. 1.5
Y2[1]
W2[1]
TIME

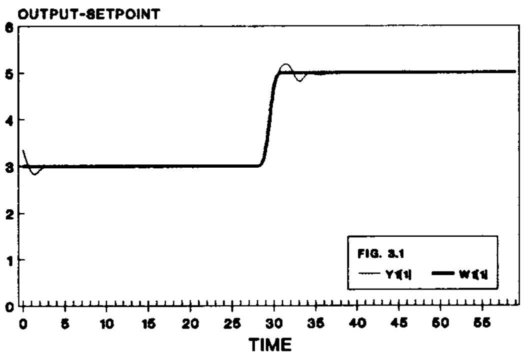

OUTPUT-SETPOINT
FIG. 1.6
Y3[1]
W3[1]
TIME

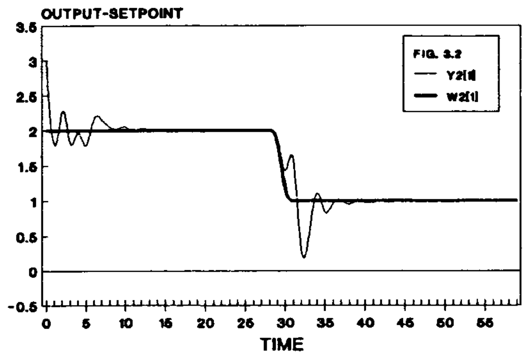

OUTPUT-SETPOINT
FIG. 2.1
Y1[1]
W1[1]
TIME

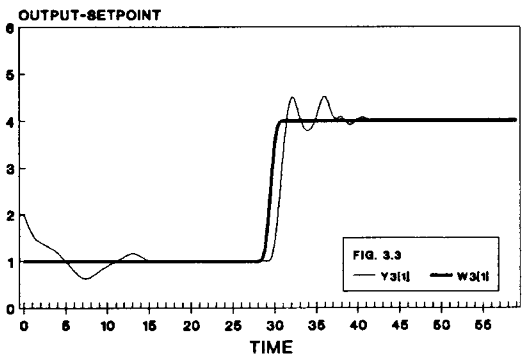

OUTPUT-SETPOINT
FIG. 2.2
TIME

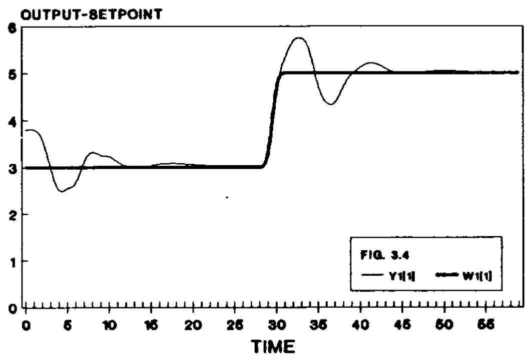

OUTPUT-SETPOINT
FIG. 2.3
Y1[3]
W1[3]
TIME

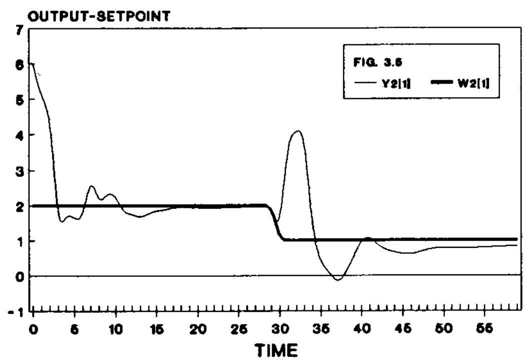

OUTPUT-SETPOINT
FIG. 2.4
Y2[1]
W2[1]
TIME

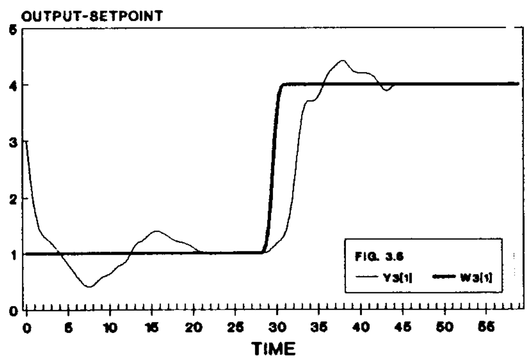

OUTPUT-SETPOINT
FIG. 3.1
Y1[1]
W1[1]
TIME


OUTPUT-SETPOINT
FIG. 3.2
Y2[1]
W2[1]
TIME


OUTPUT-SETPOINT
FIG. 3.3
Y3[1]
W3[1]
TIME


OUTPUT-SETPOINT
FIG. 3.4
Y1[1]
W1[1]
TIME


OUTPUT-SETPOINT
FIG. 3.5
Y2[1]
W2[1]
TIME


OUTPUT-SETPOINT
FIG. 3.6
Y3[1]
W3[1]
TIME

sharing pattern). For the controller the parameter values $L_y=10$, $L_u=3$, $L_0=1$, $\alpha=0.1$, $p=2$, $Q_i=I_i$ and $R_i=10^{-2}I_i$ were used. One can easily see the delay that exists in the results of case (b) in both the transient and steady state response.

Note that the symbol xi[j] in the diagrams means the j-component of the vector x of the i-th subsystem.

VI. CONCLUDING

The "optimal" control $u_i(t)=u_i^c(t)+u_i^d(t)$ is optimal in the sense of decentralisation, but it remains suboptimal in comparison with the solution that would be attainable if the whole information was available to every control station. The decentralized controller of section IV depends weakly on the initial conditions and strongly on the approximate model used for the interconnections. The last dependence seems to be reduced by the on line improvement for the predictions of the interconnections based on the approximate model for them. On the other hand the dependence on the linear model is reduced in the case of systems with m-step delay sharing patterns due to the on-line computation of its parameters in the least squares sense.

A comparison between the two approximations described in sections (IV-a) and (IV-b) reveals the following:

(i) The method of section (IV-a) can cover only problems with m-step delay sharing pattern.

(ii) If the problem has an m-step delay sharing pattern, then there is the possibility of selecting between the methods of (IV-a) and (IV-b).

The first method seems to be more suitable because in this kind of problems the linear model for the interconnections is obviously of less computational complexity and the dependence of the control on a model of the form $z_i(t)=A_{z_i}z_i(t-1)$ which sometimes may be unstable (depending on the choice of the matrix A_{z_i}) is avoided.

(iii) For problems with m-step delay sharing patterns and unusual dynamical behaviour one can make on-line improvement for the predictions of the interconnections on the basis of a linear model for them by the methods described in (IV-b). This may lead to a closed-loop system with better behaviour.

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