MODELING OF CONVERSION OF ELECTROMAGNETIC ENERGY TO ACOUSTIC ENERGY IN TISSUE

Yun Wang
Mathematical Products Division
Armstrong Laboratory,OES
Brooks AFB, TX 78235

Biological effects of pulse modulated microwave radiation has been studied by many scientists and engineers in life and physical sciences. It has been observed experimentally that microwaves interact with biological systems ([2]). Some effects induced by pulse modulated microwaves are a) hearing, b) the sensitivity of laboratory animals to certain drugs, and c) the permeability in the brain tissues. The interaction of microwaves with biological systems can be harmful, as well as beneficial to the human being. However, the mechanism of the interaction is not well understood yet. One hypothesis is the direct conversion of electromagnetic energy to acoustic energy in tissues.

In our preliminary model of energy conversion, we focus on the processing based on the thermoelastic theory. According to the theory, during microwave absorption by tissue materials a portion of electromagnetic energy is converted to heat which in turn generates a spatial temperature gradient normal to the surface. This temperature gradient generates strains in the dielectric media, hence produces an acoustic stress wave. Mathematically, this process is governed by the following differential equations ([1, 2, 3, 4, 5])

$$\begin{aligned} &\rho \frac{\partial^2 u}{\partial t^2} - 2\gamma \frac{\partial^3 u}{\partial z^2 \partial t} - (\lambda + 2\mu) \frac{\partial^2 u}{\partial z^2} = -\beta \frac{\partial v}{\partial z} \\ &\rho c_h \frac{\partial v}{\partial t} - K \frac{\partial^2 v}{\partial z^2} = \sigma E^2, \end{aligned} \tag{1}$$

where

$u(t,z)$	=	displacement of particles,
$v(t,z)$	=	temperature distribution,
c_h	=	specific heat,
λ, μ	=	Lamé constants,
γ	=	medium mechanical damping coefficient,
ρ	=	mass density,
β	=	$\alpha(3\lambda + 2\mu)$,
α	=	coefficient of linear thermal expansion,
K	=	thermal conductivity,
σ	=	dielectric conductivity,
E	=	electric field intensity.

Furthermore, all coefficients are assumed to be position dependent. The acoustic pressure distribution $p(t,z)$ is evaluated by

$$p(t,z) = (\lambda + 2\mu) \frac{\partial u}{\partial z} + 2\gamma \frac{\partial^2 u}{\partial z \partial t} - \beta v.$$

In the above formulation, we assumed a one-dimensional geometry in which a plane wave $E(t,z)$ impinges normally on the interface of air and tissue.

The validity of the model will be tested against experimental data. As a preliminary investigation, we will carry out simulation studies. The problem of model validation is formulated as parameter estimation problems. For given incident (pulse modulated microwave signals) electric field and measured pressure, the parameter estimation problem is to find a set of parameters such that the estimated acoustic pressure matches the measured data optimally under some criterion. Let the collection of unknown parameters be denoted by a vector $q = (\rho, c_h, \lambda, \mu, \gamma, \alpha, K, \sigma)$. For given observations $\{\tilde{p}_{i,j}\}$ corresponding to measurements at times t_i and position z_j, we consider the least squares estimation problem of minimizing over $q \in Q$ the least squares functional

$$J(\tilde{p}, p; q) = |\{p(t_i, z_j; q)\} - \{\tilde{p}_{i,j}\}|^2, \tag{2}$$

where $\{p(t_i, z_j; q)\}$ are the parameter dependent weak solutions of (1) evaluated at each time t_i, $i = 1, 2, \cdots, N_t$ and each position z_j, $j = 1, \cdots, N_z$, and $|\cdot|$ is an appropriately chosen Euclidean norm. The set Q is some admissible parameter set.

The minimization in our parameter estimation problem involves an infinite dimensional state space and an infinite dimensional admissible parameter set. We thus consider Galerkin type approximations in the context of the variational formulation of (1). Solving the approximate estimation problems, we obtain a sequence of estimates $\{\bar{q}^{N,M}\}$. Parameter estimate convergence and continuous dependence (with respect to the observations $\{\tilde{p}_{i,j}\}$) results can be obtained under certain assumptions.

The convergence results allowing approximation of the parameter set provides a sound necessary basis for the estimation of parameter vector q.

References

[1] R. A. Albanese, Mathematical Products Division, Armstrong Laboratory, Brooks AFB, TX, personal conversations.

[2] J. C. Lin, *Microwave Auditory Effects and Applications*, Charles C Thomas, Springfield, 1978.

[3] A. E. H. Love, *Mathematical Theory of Elasticity*, Cambridge University Press, New York, 1927.

[4] I. S. Sokolnikoff, *Mathematical Theory of Elasticity*, McGraw-Hill, New York, 1956.

[5] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.