Neural Adaptive Control: The Presence of Modeling Error with Unknown Growth

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Abstract

A direct nonlinear adaptive controller, to solve the regulation problem for unknown dynamical systems that are modeled by recurrent neural networks is discussed. The behaviour of the closed loop system is analyzed for the case in which the true system differs from the recurrent neural network due to the presence of a modeling error term. Convergence of the state to zero plus boundedness of all signals in the closed loop is guaranteed provided that a complete matching at zero property is satisfied. However, if the above assumption is no longer valid, our adaptive regulator can still guarantee uniform bounedness with the addition of appropriately modified update laws. Furthermore, the magnitude of the growth of the modeling error is considered unknown.

1 Introduction

The application of artificial neural networks to control a variety of systems, has already gained considerable attention within the control systems community, mainly due to their massive parallelism, very fast adaptability and inherent approximation capabilities. In the past four-five years the field has experienced a great amount of research activity, which lead to numerous applications and furthermore, to the development of certain control architectures, based on neural network models.

The most significant problem in generalizing the application of neural networks in control, is the fact that the very interesting simulation results that are provided, lack theoretical verification. Crucial properties like stability, convergence and robustness of the overall system must be developed and/or verified. The main reason for the existence of the above mentioned problem, is the mathematical difficulties associated with nonlinear systems controlled by highly nonlinear neural network controllers. In view of the mathematical difficulties encountered in the past in the adaptive control of linear systems, (which remained as an active problem until the early 1980's [5], [17], [18], [6], it is hardly surprising that the analytical study of nonlinear adaptive control using neural networks, is a difficult problem indeed. However, progress has been made in this area and certain important results have begun to emerge, aiming to bridging the gap between theory and applications.

The problem of controlling an unknown nonlinear dynamical system, has been attacked from various angles using both direct and indirect adaptive control structures and employing different neural network models. A beautiful survey of the above mentioned techniques, can be found in a paper by Hunt et. al. [9] in which links between the field of control science and neural networks were explored and key areas for future research were proposed. However, all works share the following key idea:
Since neural networks can approximate arbitrarily well static and dynamic highly nonlinear systems, substitute the unknown system by a neural network model, which is of known structure but contains a number of unknown parameters, (synaptic weights), plus a modeling error term. The unknown parameters may appear both linearly or nonlinearly with respect to the network nonlinearities, thus transforming the original problem into a nonlinear robust adaptive control problem.

Recent advances in nonlinear control theory and in particular feedback linearization techniques, [11], [21] created a new and challenging problem, which came to be known as adaptive nonlinear control. It was formulated to deal with the control of systems containing both unknown parameters and known nonlinearities. Several answers to this problem have been proposed in the literature with typical examples [3], [12], [13], [15], [25], [29], [30]. A common assumption made in the above works is that of linear parameterization. Although sometimes it is quite realistic, it constraints considerably the application field. An attempt to relax this assumption and provide global adaptive output feedback control for a class of nonlinear systems, determined by specific geometric conditions, is given by Marino and Tomei in their recent paper [16].

The above discussion makes apparent that adaptive control research, thus far has been directed towards systems with special class of parametric uncertainties. The need to deal with increasingly complex systems, to accomplish increasingly demanding design requirements and the need to attain these requirements with less precise advanced knowledge of the plant and its environment, inspired many works that came mostly from the area of neural networks but with obvious and strong relation to the adaptive control field.

The use of dynamical neural networks for identification and more recently for control, was first introduced by Narendra and Parthasarathy [19]. They proposed dynamic backpropagation schemes, which are static backpropagation neural networks, connected either in series or in parallel with linear dynamical systems.

The main problem with the recurrent neural networks that are based on the static multilayer networks is that the synaptic weights appear nonlinearly in the mathematical representation that governs their evolution. This leads to a number of significant drawbacks. First, the learning laws that are used, require a high amount of computational time. Second, since the synaptic weights are adjusted to minimize a functional of the approximation error and the weights appear nonlinearly, the functional possesses many local minima. Therefore, there is no way to ensure the convergence of the weights to the global minimum. Moreover, due to the highly nonlinear nature of the neural network architecture, basic properties like stability, convergence and robustness, are very difficult to be verified. The fact that even for linear systems such adaptation methods can lead to instability was also shown in [1], [22]. On the other hand, the recurrent networks possess a linear in the weights property, thus making the issues of proving stability and convergence feasible and their incorporation into a control loop promising.

Sanner and Slotine [28], incorporate Gaussian radial-basis-function neural networks with sliding mode control and linear feedback to formulate a direct adaptive tracking control architecture, for a class of continuous time nonlinear dynamic systems. However, the use of sliding mode, which is a discontinuous control law, generally creates various problems, such as existence and uniqueness of solutions [23], introduction of chattering phenomena [31], and possibly excitation of high frequency unmodeled dynamics [32].

Polycarpou and Ioannou [24], employed Lyapunov stability theory to develop stable adaptive laws for identification and control of dynamical systems with unknown nonlinearities, using various neural network architectures. Their control results were restricted to SISO feedback linearizable systems. Moreover, they also use a sliding mode control.

More recently, Rovithakis [26] and Rovithakis and Christodoulou [27], considered affine in the control nonlinear dynamical systems. Since they assumed that no *a priori* information is available for the nonlinearities, they proposed a two step algorithm. In step one "black box" identification is performed around a known operational point, using a recurrent neural network identifier. Stable learning laws were developed with the aid of Lyapunov stability theory. In step two, an indirect adaptive control architecture is developed, employing all information obtained previously, to guarantee stability of the closed loop system and convergence of the control error to zero, provided that certain assumptions on the nonlinearities are satisfied. However, although not all the plant states were assumed to be available for measurement, the restrictions imposed on the system need to be relaxed, in order to be more widely applied.

From the above discussion becomes obvious that all theoretical works in control using neural networks have up to now restrictive applicability, even though they employ Lyapunov theory to establish stability results. The use of sliding mode control can, (as already mentioned), introduce significant theoretical and practical problems. Furthermore, the two step, indirect adaptive control architecture first proposed in [26], [27], introduces the extra complexity of having a recurrent neural network identifier working in parallel and in real time with the unknown system. Moreover, the way the modeling error was treated, leaves interesting theoretical problems open for further research.

Thus the present paper aims first of all to relax the restrictive assumptions made in [27], and simultaneously to avoid the use of sliding mode control. In this way we broaden the applications field. Second, the implementation of the proposed control scheme becomes less complex, since there is no need of using a neural network identifier to work in parallel with the actual system. Furthermore, the issues of stability and robustness are carefully examined and rigorously analyzed. However, in this paper we investigate the state regulation problem only, which is known to be as the basic control problem, where the state of a given plant is to be reduced to zero, from an arbitrary initial value, by applying feedback control to the plant input. This problem arises, for example, when a plant is to be operated at a desired, constant set point and hence deviations from this set point are to be regulated to zero. It is possible though, the unknown system not to possess any equilibrium points. Thus generally, the design objective will be defined as the approximate regulation of the state as follows:

For any given $\epsilon > 0$, design a control law which renders the system globally uniformly ultimately bounded with respect to an arbitrary small set $\mathcal{A} \subset \Re^n$ satisfying

$$\mathcal{A} = \{x : |x| \leq \epsilon\}$$

In other words we want the state to enter $\mathcal{A}$ after a finite time and never leave it, for all time thereafter.

Thus generally, the proposed regulator guarantee the uniform ultimate boundedness of the state and the boundedness of all other signals in the closed loop, even in the presence of modeling errors which are not assumed to be *a priori* bounded. Moreover, the magnitude of the growth of the modeling error is assumed unknown.

2 Problem Formulation & the Recurrent Neural Network

We consider affine in the control, nonlinear dynamical systems of the form

$$\dot{x} = f(x) + G(x)u \tag{2.1}$$

where the state x, living in a n-dimensional smooth manifold $\mathcal{M}$, is assumed to be completely measured, the control u is in $\Re^n$, f is an unknown smooth vector field called the drift term and G is a matrix with columns the unknown smooth controlled vector fields g_i , $i = 1, 2, \ldots, n$ $G = [g_1\ g_2\ \ldots\ g_n]$.

The state regulation problem is known as to reduce the state to zero from an arbitrary initial value, by applying feedback control to the plant input. However, the problem, as it is stated above for the system (2.1), is very difficult or even impossible to be solved since the vector fields f , g_i $i = 1, 2, \ldots, n$ are assumed to be completely unknown. Therefore, it is obvious that in order to provide a solution to our problem, it is necessary to have a more accurate model for the unknown plant. For that purpose we apply recurrent neural networks.

Recurrent neural networks are fully interconnected nets, containing dynamical elements in their neurons. Therefore, they are described by the following set of differential equations

$$\dot{\hat{x}} = -A\hat{x} + WS(x) + W_{n+1}S'(x)u \tag{2.2}$$

where $\hat{x} \in \mathcal{M}$, the inputs $u \in \Re^n$, W is a $n \times L$ matrix of adjustable synaptic weights, W_{n+1} is a $n \times n$ upper triangular matrix of adjustable synaptic weights and A is a $n \times n$ matrix with positive eigenvalues which for simplicity can be taken diagonal. $S(x)$ is a L-dimensional vector with elements $S_i(x)$, $i = 1, 2, \ldots, L$ of the form

$$S_i(x) = \prod_{j \in I_i} [s(x_j)]^{d_j(i)} \tag{2.3}$$

where I_i, $i = 1, 2, \ldots, L$ are collections of L not ordered subsets of $\{1, 2, \ldots, n\}$ and $d_j(i)$ are non-negative integers. Similarly, $S'(x)$ is a $n \times n$ upper triangular matrix with elements $s'_{lk}(x)$ of the form

$$s'_{lk}(x) = \prod_{j \in I_{lk}} [s(x_j)]^{d_j(l,k)} \tag{2.4}$$

for all $l, k = 1, 2, \ldots, n$ and $l \leq k$ where I_{lk} are a collections of n^2 not-ordered subsets of $\{1, 2, \ldots, n\}$ and $d_j(l, k)$ are non-negative integers. In both (2.3) and (2.4), $s(x_j)$ is a monotone increasing, smooth, continuous, function, which is usually represented by sigmoidals of the form

$$s(x_j) = \frac{\mu}{1 + e^{-l_0 x_j}} + \lambda \tag{2.5}$$

for all $j = 1, 2, \ldots, n$, with the parameters μ, l_0 to represent the bound and slope of sigmoid's curvature and λ a bias constant.

Clearly the recurrent neural network model described above, can be viewed as an extension to the Hopfield [8], and Cohen [2] models, that permit higher order connections between neurons. For the above neural network model, there exists the following approximation theorem [14]

Theorem 2.1 *Suppose that the system (2.1) and the model (2.2) are initially at the same state $x(0) = \hat{x}(0)$. Then for any $\varepsilon > 0$ and any finite $T > 0$, there exists an integer L and matrices $W^\star, W^\star_{n+1}$, such that the state $\hat{x}(t)$ of the recurrent neural network model (2.2) with weight values $W = W^\star$, $W_{in} = W^\star_{in}$ satisfies*

$$\sup_{0 \leq t \leq T} |\hat{x}(t) - x(t)| \leq \varepsilon$$

□

Stability of higher order neural networks in the case of fixed weight values, has been investigated in [4].

Due to the approximation capabilities of the recurrent neural networks, we can assume, with no loss of generality, that the unknown system (2.1) can be completely described by a recurrent neural network plus a modeling error term $\omega(x, u)$. In other words, there exists weight values $W^\star$ and $W^\star_{n+1}$ such that the system (2.1) can be written as

$$\dot{x} = -Ax + W^\star S(x) + W^\star_{n+1}S'(x)u + \omega(x, u) \tag{2.6}$$

Where for the modeling error term, we make the following assumption:

Assumption (A.1) We assume that there exists positive constants k'_i, $i = 0, 1, 2$ such that the following relation is satisfied by the modeling error term

$$|\omega(x, u)| \leq k'_0 + k'_1|x| + k'_2|u|$$

with k'_1 considered unknown.

□

Therefore, the state regulation problem is analyzed for the system (2.2) instead of (2.1). Since $W^\star$ and $W^\star_{n+1}$ are unknown, our solution consists of designing a control law $u(W, W_{n+1}, x)$ and appropriate update laws for the weight estimates W and W_{n+1} to guarantee at least uniform ultimate boundedness of x and boundedness of all signals in the closed loop.

3 Robust Adaptive Regulation

Let us assume that the true plant is of known order n and can be modeled exactly by the dynamic neural network (2.1) plus a modeling error term $\omega(x,u)$.

$$\dot{x} = -Ax + W^{\star}S(x) + W_{n+1}^{\star}S'(x)u + \omega(x,u) \quad (3.1)$$

with the modeling error term satisfying Assumption (A.1). Observe that the modeling error is not restricted to be *a priori* bounded. The above together with the fact the actual parameter k_1 is considered unknown, makes assumption (A.1) quite general and therefore valid in many applications.

Let us take a function $h(x)$ of class C^2 from $\mathcal{M}$ to $\Re^+$ whose derivative with respect to time is

$$\dot{h} = \frac{\partial h}{\partial x}\left[-Ax + W^{\star}S(x) + W_{n+1}^{\star}S'(x)u + \omega(x,u)\right] \quad (3.2)$$

Equation (3.2) can also be written

$$\begin{aligned} \dot{h} + \frac{\partial h}{\partial x}Ax - \frac{\partial h}{\partial x}\omega(x,u) &= \frac{\partial h}{\partial x}W^{\star}S(x) \\ &\quad + \frac{\partial h}{\partial x}W_{n+1}^{\star}S'(x)u \end{aligned} \quad (3.3)$$

Define

$$\begin{aligned} \nu &\triangleq \frac{\partial h}{\partial x}WS(x) + \frac{\partial h}{\partial x}W_{n+1}S'(x)u - \dot{h} \\ &\quad - \frac{\partial h}{\partial x}Ax - \frac{\partial h}{\partial x}sgn(e)\hat{k}_1 x \end{aligned}$$

where W, W_{n+1} and $\hat{k}_1$ are the estimates of the unknown parameters $W^{\star}, W_{n+1}^{\star}$ and k_1 respectively, obtained by update laws which are to be designed later. Moreover, the function $sgn(e)$ is defined as follows:

$$sgn(e) = \begin{cases} 1 & \text{if } e > 0 \\ -1 & \text{otherwise} \end{cases}$$

Furthermore, observe that the signal ν can not be measured since $\dot{h}$ is unknown. To round this problem we use the following filtered version of ν.

$$\begin{aligned} \dot{e} + re &= \nu \\ &= -\dot{h} + \frac{\partial h}{\partial x}\left[-Ax + WS(x) + W_{n+1}S'(x)u\right] \\ &\quad - \frac{\partial h}{\partial x}\left[sgn(e)\hat{k}_1 x\right] \end{aligned} \quad (3.4)$$

with r a strictly positive constant. To implement (3.4) we take

$$e \triangleq \eta - h \quad (3.5)$$

Employing (3.5), equation (3.4) can be written as

$$\begin{aligned} \dot{\eta} + r\eta &= rh + \frac{\partial h}{\partial x}\left[-Ax + WS(x) + W_{n+1}S'(x)u\right] \\ &\quad - \frac{\partial h}{\partial x}\left[sgn(e)\hat{k}_1 x\right] \end{aligned} \quad (3.6)$$

with the state $\eta \in \Re$. This method is referred to as error filtering. Furthermore, we choose $h(x)$ to be

$$h(x) = \frac{1}{2}|x|^2$$

Hence (3.6) becomes

$$\begin{aligned} \dot{\eta} &= -r\eta + \frac{1}{2}r|x|^2 - x^TAx + x^TWS(x) \\ &\quad + x^TW_{n+1}S'(x)u - sgn(e)\hat{k}_1|x|^2 \end{aligned} \quad (3.7)$$

To continue, consider the Lyapunov like function

$$\mathcal{L} = \frac{1}{2}e^2 + \frac{1}{2}tr\{\tilde{W}^T\tilde{W}\} + \frac{1}{2}tr\{\tilde{W}_{n+1}^T\tilde{W}_{n+1}\} + \frac{1}{2}\tilde{k}_1^2 \quad (3.8)$$

where

$$\begin{aligned} \tilde{W} &= W - W^{\star} \\ \tilde{W}_{n+1} &= W_{n+1} - W_{n+1}^{\star} \\ \tilde{k}_1 &= \hat{k}_1 - k_1 \end{aligned}$$

If we take the derivative of (3.8) with respect to time we obtain

$$\dot{\mathcal{L}} = e\dot{e} + tr\{\dot{\tilde{W}}^T\tilde{W}\} + tr\{\dot{\tilde{W}}_{n+1}^T\tilde{W}_{n+1}\} + \tilde{k}_1\dot{\tilde{k}}_1 \quad (3.9)$$

Employing (3.4), equation (3.9) becomes

$$\begin{aligned} \dot{\mathcal{L}} &= -re^2 + e\left[-\dot{h} - x^TAx + x^TWS(x)\right] \\ &\quad + e\left[x^TW_{n+1}S'(x)u - sgn(e)\hat{k}_1|x|^2\right] \\ &\quad + tr\{\dot{\tilde{W}}^T\tilde{W}\} + tr\{\dot{\tilde{W}}_{n+1}^T\tilde{W}_{n+1}\} + \tilde{k}_1\dot{\tilde{k}}_1 \end{aligned} \quad (3.10)$$

which together with (3.3) gives

$$\begin{aligned} \dot{\mathcal{L}} &= -re^2 + e\left[-x^TW^{\star}S(x)\right] \\ &\quad + e\left[-x^TW_{n+1}^{\star}S'(x)u + x^TWS(x)\right] \\ &\quad + e\left[x^TW_{n+1}S'(x)u - x^T\omega(x,u)\right] \\ &\quad - e\, sgn(e)\hat{k}_1|x|^2 + tr\{\dot{\tilde{W}}^T\tilde{W}\} \\ &\quad + tr\{\dot{\tilde{W}}_{n+1}^T\tilde{W}_{n+1}\} + \tilde{k}_1\dot{\tilde{k}}_1 \end{aligned}$$

or equivalently

$$\begin{aligned} \dot{\mathcal{L}} &= -re^2 + ex^T\tilde{W}S(x) + ex^T\tilde{W}_{n+1}S'(x)u \\ &\quad - x^T\omega(x,u) - |e|\hat{k}_1|x|^2 \\ &\quad + tr\{\dot{\tilde{W}}^T\tilde{W}\} + tr\{\dot{\tilde{W}}_{n+1}^T\tilde{W}_{n+1}\} + \tilde{k}_1\dot{\tilde{k}}_1 \end{aligned}$$

Hence, if we choose

$$tr\{\dot{\tilde{W}}^T\tilde{W}\} = -ex^T\tilde{W}S(x) \quad (3.11)$$

$$tr\{\dot{\tilde{W}}_{n+1}^T\tilde{W}_{n+1}\} = -ex^T\tilde{W}_{n+1}S'(x)u \quad (3.12)$$

$\dot{\mathcal{L}}$ becomes

$$\dot{\mathcal{L}} = -re^2 - ex^T\omega(x,u) - |e|\hat{k}_1|x|^2 + \tilde{k}_1\dot{\tilde{k}}_1 \quad (3.13)$$

It can be easily verified that (3.11), (3.12) can be written element wise as

$$\dot{w}_{ij} = -ex_is(x_j) \quad (3.14)$$

$$\dot{w}_{in+1} = -ex_is'(x_i)u_i \quad (3.15)$$

for all $i, j = 1, 2, \ldots, n$
and in matrix form as

$$\begin{aligned} \dot{W} &= -exS^T(x) & (3.16) \\ \dot{W}_{n+1} &= -ex'S'(x)U & (3.17) \end{aligned}$$

where

$$\begin{aligned} x' &= diag[x_1, x_2, \ldots, x_n] \\ U &= diag[u_1, u_2, \ldots, u_n] \end{aligned}$$

To continue observe that

$$\begin{aligned} \dot{\mathcal{L}} &\leq -re^2 + |e||x||\omega(x,u)| - |e|\hat{k}_1|x|^2 + \tilde{k}_1\dot{\hat{k}}_1 \\ &\leq -re^2 + |e||x||k_0' + k_1'|x| + k_2'|u|| - |e|\hat{k}_1|x|^2 + \tilde{k}_1\dot{\hat{k}}_1 \end{aligned}$$

To proceed we make the following claim:

Claim (C.1): The control law is such that the following inequality holds for all $x \in \mathcal{M}$.

$$|u| \leq \bar{k}|x| + \bar{k}_0$$

□

Using claim (C.1), $\dot{\mathcal{L}}$ becomes

$$\dot{\mathcal{L}} \leq -re^2 + |e||x||k_0 + k_1|x|| - |e|\hat{k}_1|x|^2 + \tilde{k}_1\dot{\hat{k}}_1 \quad (3.18)$$

where $k_0 = \bar{k}_0 k_2' + k_0'$ and $k_1 = k_2'\bar{k} + k_1'$. Furthermore, (3.18) becomes

$$\begin{aligned} \dot{\mathcal{L}} &\leq -re^2 + k_0|e||x| + k_1|e||x|^2 \\ &\quad -|e|\hat{k}_1|x|^2 + \tilde{k}_1\dot{\hat{k}}_1 \\ &= -re^2 + k_0|e||x| - |e|\tilde{k}_1|x|^2 + \tilde{k}_1\dot{\hat{k}}_1 \quad (3.19) \end{aligned}$$

Therefore, if we choose

$$\dot{\hat{k}}_1 = |e||x|^2 \quad (3.20)$$

(3.19) becomes

$$\dot{\mathcal{L}} \leq -r|e|^2 + k_0|e||x| \quad (3.21)$$

To continue we need the following Lemma

Lemma 3.1 *The control law*

$$\begin{aligned} u &= -[W_{n+1}S'(x)]^{-1}[WS(x) + v] & (3.22) \\ v &= \frac{1}{2}rx - Ax - sgn(e)\hat{k}_1 x & (3.23) \end{aligned}$$

where the synaptic weight estimates W and W_{n+1} are adjusted according to (3.14) and (3.15) respectively and $\hat{k}_1$ according to (3.20), guarantees the following

- $\eta(t) \leq 0 \quad \forall t \geq 0$
- $lim_{t \to \infty}\eta(t) = 0$ *exponentially fast*

provided that $\eta(0) < 0$, where $\eta(0)$ the initial value of $\eta(t)$.

□

However in order to apply the control law (3.22), (3.23) we have to assure the existence of $[W_{n+1}S'(x)]^{-1}$. Since, W_{n+1} and $S'(x)$ are upper triangular matrices, all we need to establish is that their diagonal entries be different from zero $\forall t \geq 0$. The diagonal entries of $S'(x)$ can be different from zero by an appropriate selection of the bias term λ. Let w_{in+1} $i = 1, 2, \ldots, n$ denote the diagonal entries of W_{n+1}. Hence, $w_{in+1}(t)$ $i = 1, 2, \ldots, n$ should be confined through the use of a projection algorithm [20], [7], [10], to the set $\mathcal{W}' = \{w_{in+1} : 0 < \epsilon \leq w_{in+1} \leq w^m\}$ where ϵ, w^m are appropriately chosen positive constants[1]. In particular, the standard update law defined by (3.15) is modified to

$$\dot{w}_{in+1} = \begin{cases} -ex_i s'(x_i)u_i & \text{if } w_{in+1} \in \mathcal{W}' \\ & \text{or } w_{in+1}sgn(w^\star_{in+1}) = \epsilon \\ & \text{and } ex_i s'(x_i)u_i sgn(w^\star_{in+1}) \leq 0 \\ 0 & \text{if } w_{in+1}sgn(w^\star_{in+1}) = \epsilon \text{ and} \\ & ex_i s'(x_i)u_i sgn(w^\star_{in+1}) > 0 \\ -ex_i s'(x_i)u_i & \text{if } w_{n+1} \in \mathcal{W}' \\ & \text{or } w_{in+1}sgn(w^\star_{in+1}) = w^m \\ & \text{and } ex_i s'(x_i)u_i sgn(w^\star_{in+1}) \geq 0 \\ 0 & \text{if } w_{in+1}sgn(w^\star_{in+1}) = w^m \text{ and} \\ & ex_i s'(x_i)u_i sgn(w^\star_{in+1}) < 0 \end{cases} \quad (3.24)$$

for all $i, j = 1, 2, \ldots, n$.
where the update law is written element wise for easier understanding. It can be easily verified that the update law (3.15) with the projection modification (3.24), can only make $\dot{\mathcal{L}}$ more negative and in addition guarantee that $w_{in+1} \in \mathcal{W}'$ for all $i = 1, 2, \ldots, n$, provided that $w_{in+1}(0) \in \mathcal{W}'$ and $w^\star_{in+1} \in \mathcal{W}'$. In principal, the projection modification does not alter $\dot{w}_{in+1}$ given by (3.15) if w_{in+1} is in the interior $\mathcal{W}'_{in}$ of $\mathcal{W}'$ or if w_{in+1} is at the boundary $\delta(\mathcal{W}')$ of $\mathcal{W}'$ and has the tendency to move inward. Otherwise, it subtracts a vector normal to the boundary so that we get a smooth transformation from the original vector field, to an inward or tangent to the boundary, vector field.

Remark 3.1 Observe that in order to apply the projection algorithm (3.24) we need to know the sign of the unknown parameter $w^\star_{in+1}$.

□

Moreover, it is true that

$$h = \eta - e$$

Hence, since $h(x) \geq 0$, we have that

$$\eta(t) \geq e(t)\ ,\ \forall t \geq 0 \quad (3.25)$$

However

$$\eta(t) \leq 0\ ,\ \forall t \geq 0$$

[1]Observe that w_{in+1} can be also confined to be negative. However, the above choice does not harm the generality.

which implies that

$$|\eta(t)| \le |e(t)| \ , \ \forall t \ge 0 \tag{3.26}$$

Furthermore,

$$\begin{aligned} |h(x)| &\le |\eta(t) - e(t)| \\ &\le |\eta(t)| + |e(t)| \\ &\le 2|e(t)| \end{aligned}$$

Therefore

$$|x|^2 \le 4|e|, \ \forall t \ge 0 \tag{3.27}$$

or

$$|x| \le 2\sqrt{|e|}, \ \forall t \ge 0 \tag{3.28}$$

Remark 3.2 Observe that the proposed control law (3.22), (3.23) is in first sight discontinuous since it includes the function $sgn(e)$. However, from Lemma 3.1 and inequality (3.25) we conclude that $e(t) \le 0$, $\forall t \ge 0$. Therefore $sgn(e)$ does not switch from -1 to $+1$ as time passes and admits the constant value of -1 $\forall t \ge 0$. In this way problems concerning the existence and uniqueness of solutions, chattering phenomena and/or excitation of possibly present unmodeled dynamics, are avoided.

□

Substituting (3.28) into (3.21), $\dot{\mathcal{L}}$ becomes

$$\begin{aligned} \dot{\mathcal{L}} &\le -r|e|^2 + 2k_0|e|\sqrt{|e|} \\ &= -\left[r\sqrt{|e|} - 2k_0\right]|e|\sqrt{|e|} \end{aligned} \tag{3.29}$$

From (3.29) we have that $\dot{\mathcal{L}} \le 0$ as long as

$$\sqrt{|e|} > \frac{2k_0}{r}$$

or equivalently when

$$|e| > \frac{4k_0^2}{r^2}$$

with $r > 0$. The above analysis together with (3.28) demonstrate that the trajectories of $e(t)$ and $x(t)$ are uniformly ultimately bounded with respect to the arbitrary small, (since r can be chosen sufficiently large), sets $\mathcal{E}$, $\mathcal{X}$ shown below.

$$\mathcal{E} = \left\{e(t) : |e(t)| \le \frac{4k_0^2}{r^2}\right\}$$

and

$$\mathcal{X} = \left\{x(t) : |x(t)| \le \frac{4k_0}{r}\right\}$$

Thus we have the following theorem.

Theorem 3.1 *Consider the system*

$$\begin{aligned} \dot{x} &= -Ax + W^\star S(x) + W_{n+1}^\star S'(x)u + \omega(x,u) \\ \dot{\eta} &= -r\eta \\ u &= -[W_{n+1}S'(x)]^{-1}[WS(x) + v] \\ v &= \frac{1}{2}rx - Ax - sgn(e)\hat{k}_1 x \\ e &= \eta - h \\ h &= \frac{1}{2}|x|^2 \\ r &> 0 \end{aligned}$$

together with the update laws

$$\begin{aligned} \dot{\hat{k}}_1 &= |e||x|^2 \\ \dot{w}_{ij} &= -ex_i s(x_j) \\ \dot{w}_{in+1} &= \begin{cases} -ex_i s'(x_i)u_i & \text{if } w_{in+1} \in \mathcal{W}' \\ & \text{or } w_{in+1}sgn(w_{in+1}^\star) = \varepsilon \\ & \text{and} \\ & ex_i s'(x_i)u_i sgn(w_{in+1}^\star) \le 0 \\ 0 & \text{if } w_{in+1}sgn(w_{in+1}^\star) = \varepsilon \\ & \text{and} \\ & ex_i s'(x_i)u_i sgn(w_{in+1}^\star) > 0 \\ -ex_i s'(x_i)u_i & \text{if } w_{n+1} \in \mathcal{W}' \\ & \text{or } w_{in+1}sgn(w_{in+1}^\star) = w^m \\ & \text{and} \\ & ex_i s'(x_i)u_i sgn(w_{in+1}^\star) \ge 0 \\ 0 & \text{if } w_{in+1}sgn(w_{in+1}^\star) = w^m \\ & \text{and} \\ & ex_i s'(x_i)u_i sgn(w_{in+1}^\star) < 0 \end{cases} \end{aligned}$$

for all $i,j = 1,2,\ldots,n$ and the modeling error term to satisfy (A.1), guarantee the uniform ultimate boundedness of $e(t)$, $x(t)$ with respect to the sets

- $\mathcal{E} = \left\{e(t) : |e(t)| \le \frac{4k_0^2}{r^2}\right\}$
- $\mathcal{X} = \left\{x(t) : |x(t)| \le \frac{4k_0}{r}\right\}$

□

Boundedness of W_{n+1} is achieved through the use of the projection modification. However, theorem 3.1 does not tell us anything about the boundedness of W and $\hat{k}_1$. To achieve such a goal $W(t)$ and $\hat{k}_1(t)$ are confined through the use of a projection algorithm to the sets $\mathcal{W}$ and $\mathcal{K}$ respectively, where

$$\mathcal{W} = \{W(t) : \|W(t)\| \le w_m\}$$

and

$$\mathcal{K} = \left\{\hat{k}_1(t) : 0 \le \hat{k}_1(t) \le k_u\right\}$$

In particular, the standard update laws (3.14) and (3.20) are modified to

$$\dot{W} = \begin{cases} -exS^T(x) & \text{if } W \in \mathcal{W} \\ & \text{or } \|W\| = w_m \\ & \text{and } tr\{exS^T(x)W\} \ge 0 \\ -exS^T(x) + P & \text{if } \|W\| = w_m \\ & \text{and } tr\{exS^T(x)W\} < 0 \end{cases} \tag{3.30}$$

and

$$\dot{\hat{k}}_1 = \begin{cases} 0 & \text{if } \hat{k}_1 = k_u \\ |e||x|^2 & \text{otherwise} \end{cases} \tag{3.31}$$

where $P = tr\{exS^T(x)W\}(\frac{1+\|W\|}{w_m})^2 W$. Therefore, if the initial weights are chosen such that $\|W(0)\| \le w_m$, then we have $\|W\| \leq w_m$ for all $t \geq 0$. This can be readily established by noting that whenever $\|W\| = w_m$ then

$$\frac{d}{dt}(\|W\|^2) \leq 0 \tag{3.32}$$

which implies that the weights W, are directed towards the inside of the ball $\{W : \|W\| \leq w_m\}$. Furthermore, based on the adaptive law (3.30), the additional terms introduced in the expression for $\dot{\mathcal{L}}$, can only make $\dot{\mathcal{L}}$ more negative, provided that $W(0), W^* \in \mathcal{W}$. Similarly, we have that the update law (3.20) with the projection modification (3.31) can only make $\dot{\mathcal{L}}$ more negative and in addition guarantee that $\hat{k}_1 \in \mathcal{K}$, $\forall t \geq 0$, provided that $\hat{k}_1(0), k_1 \in \mathcal{K}$.

4 Conclusions

In this paper we investigate the possibilities of exploiting the approximation capabilities of recurrent neural networks, in order to develop a direct adaptive state regulator for unknown nonlinear dynamical systems.

The key idea in our analysis is the following:
Since the system under consideration is assumed unknown, use a recurrent neural network, which is of known structure but contains a number of unknown parameters, to model its behavior and then develop a control architecture based on the recurrent neural network model. Thus the problem is actually transformed into a nonlinear robust adaptive control problem.

The problems arising from the presence of a modeling error term, which is unavoidable when we employ models to develop control algorithms, are rigorously analyzed. One important aspect of our analysis is that the modeling error is not assumed to be *a priori* bounded and moreover, the gain of the growth term is considered unknown. In this way many more interesting cases are included, leading to a natural extension of the application field.

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