Macroscopic Roadway Traffic Controller Design *

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Abstract

The purpose of this paper is to design, analyze and simulate a roadway controller for a single automated traffic lane that achieves desired traffic densities along the lane. A macroscopic traffic flow model that is modified for automated highway system (AHS) operation is used for control design and analysis. We have shown that the proposed roadway controller guarantees exponential convergence of the traffic density at each section of the lane to the desired density. Simulation results are used to illustrate the effectiveness of the proposed controller and the significant benefits AHS may bring to traffic flow.

1. Introduction

Urban freeway congestion is a growing problem that is demanding the attention of transportation authorities around the world. Solutions to this problem are actively sought not only to ensure shorter and more reliable travel times, but also to diminish its adverse effects on pollution and wasted fuel consumption.

A rudimentary approach to the congestion problem involves proper on-ramp metering at freeway entrance ramps to regulate the flow of incoming traffic to a freeway [5, 14, 15, 16, 19, 1]. Advisory message boards placed at various points along the freeway, advising freeway users about speed limits, traffic conditions and alternative routes have also been used as a means of controlling congestion. Current research on automated highway systems (AHS) which propose microscopic control through vehicle following or platooning [5, 14, 15, 16, 19, 1] or macroscopic control to homogenize traffic density [20, 3] allow a more sophisticated solution to this problem.

The goal of the macroscopic control approach is to prevent congestion or at least avoid its amplification caused by traffic inhomogeneities. This is addressed by seeking control strategies that help to achieve a homogeneous traffic density profile [20, 3]. However, due to the strong coupling and the nonlinear dynamics associated with the traffic flow model, no theoretical analysis of the efficacy of such controllers can be found in the literature. Rather, the validity of any such proposed control scheme is only justified through a simulation study or by experimental results on a particular freeway. The design of a theory supported macroscopic controller is an issue that is yet to be addressed.

In this paper we provide a solution that addresses the above concerns. More specifically, we propose a roadway traffic controller that operates on a macroscopic level to homogenize traffic density of a congested freeway. We support the analysis and design of our scheme using the theory of nonlinear integrator backstepping that has been developed recently. Simulations show that our proposed method drastically reduces congestion and helps to achieve a smooth traffic flow on a congested freeway.

This paper is organized as follows. In Sections 2 and 3, a discrete traffic flow model is presented and the problem statement is given. In Section 4, the details pertaining to the design and analysis of the proposed roadway controller are given. Simulation results which show the benefits of our scheme are presented in Section 5. Conclusions and future re-

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search directions are given in Section 6.

2. Traffic Flow Model

The analogy between traffic flow and fluid dynamics formed the basis for the first traffic flow model proposed by Lighthill and Witham [10]. Their model which had traffic density as its only state variable showed poor transient behavior. Payne [17, 18], Cremer and May [4] proposed several modifications to overcome this problem. A more sophisticated model was proposed by Papageorgiou in [11, 12, 13] which has been tested and validated using real traffic data from the Boulevard Peripherique in Paris. However, Karaaslan, Varaiya and Walrand [9] demonstrated several shortcomings of Papageorgiou's model, and proposed a more realistic model. This model forms the basis of our control design and its description follows below.

Consider a single freeway lane which is subdivided into N sections with lengths $L_i, (i = 1, 2, \ldots, N)$ as shown in Figure 1.

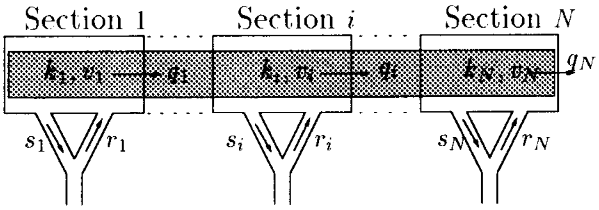

Figure 1: A freeway system subdivided into sections

The space and time discretized traffic flow model for a segment of the lane involves the following variables:

$k_i(n)$	:=	Density in section i at time nT (in $veh/km/lane$), where $n = 1, 2, \ldots$.
$v_i(n)$	:=	Space mean speed of vehicles in section i at time nT (in km/h).
$q_i(n)$	:=	Traffic volume entering section $i+1$ at time nT (in veh/h).
$r_i(n)$	:=	On-ramp traffic volume for section i (in veh/h).
$s_i(n)$	:=	Off-ramp traffic volume for section i (in veh/h).
L_i	:=	Length of the i-th section (in km).
T	:=	Time discretization step size (in h).
veh	:=	Number of vehicles.
h	:=	Hours.
km	:=	Kilometers.

The modified freeway traffic flow model given in [9] is in the following form:

$$
\begin{aligned}
q_i(n) &= \alpha k_i(n) v_i(n) \\
&\quad + (1-\alpha) k_{i+1}(n) v_{i+1}(n) && (2.1) \\
k_i(n+1) &= k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n) \\
&\quad + r_i(n) - s_i(n)] && (2.2) \\
v_i(n+1) &= v_i(n) + \frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\} \\
&\quad + \frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa'} v_{i-1}(n) \\
&\quad \times [\sqrt{v_{i-1}(n) v_i(n)} - v_i(n)] \\
&\quad - \frac{\mu(n) T}{\tau L_i} w_i(n) && (2.3)
\end{aligned}
$$

where

$$
\mu(n) = \begin{cases} \mu_1 \frac{\rho}{k_{jam} - k_{i+1}(n) + \sigma} & \textit{if } k_{i+1}(n) > k_i(n) \\ \mu_2 & \textit{otherwise}; \end{cases}
$$

Here $\alpha, \rho, \sigma, \kappa', \tau, \mu_1, \mu_2$ are positive constants with $0 \le \alpha \le 1$, and k_{jam} is the maximum possible density. The variable $V_e[k_i(\cdot)]$ in equation (2.3) represents the density dependent equilibrium speed. In a manual operating environment with homogeneous traffic conditions, this relationship has been characterized in [12, 13] as

$$
V_e(k_i) = v_f \left(1 - \left(\frac{k_i}{k_{jam}}\right)^l\right)^m \qquad (2.4)
$$

where $l > 0$ and $m > 1$ are real-valued parameters and v_f is the free speed that can be estimated from traffic data.

The term $w_i(n)$ in equation (2.3) under manual operation depends on the downstream density and can be expressed as

$$
w_i(n) = \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \kappa} \qquad (2.5)
$$

where the positive constant κ is introduced to prevent abnormal growth of the velocity for section i when its density is very low.

Typical parameter values associated with the above model can be found in [9].

Boundary conditions

We assume that the traffic flow rate entering section 1 during the time period nT and $(n+1)T$ is $q_0(n)$. In addition, we also assume that

$$
\begin{aligned}
k_0(n) &= \frac{\left[\frac{q_0(n)}{v_1(n)} - (1-\alpha) k_1(n)\right]}{\alpha} && (2.6) \\
v_0(n) &= v_1(n) && (2.7) \\
k_{N+1}(n) &= k_N(n) && (2.8) \\
v_{N+1}(n) &= v_N(n) \quad \forall n. && (2.9)
\end{aligned}
$$

The physical meaning of each term of equation (2.3) which influences the mean speed of a section can be interpreted as follows [11, 9].
The second term $\frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\}$ is the relaxation term which accounts for the evolution of the mean speed $v_i(n)$ towards its density dependent equilibrium speed $V_e[k_i(n)]$ with a time constant τ. The dependence of $V_e[k_i(n)]$ on the density is influenced by the environment in which the traffic flow is operating. For manual operation, this relationship is governed by (2.4) as reported in [12, 13]. For a fully automated highway system (AHS) operating under homogeneous heavy traffic conditions, the adopted safety policy for vehicles defines this relationship. For instance, if the desired safety distance between two vehicles S_d is made to depend on the equilibrium velocity V_e as

$$S_d = \varphi(V_e)$$

then the density-equilibrium speed relationship can be characterized by,

$$V_e(k_i) = \varphi^{-1}\left(\frac{1}{k_i}\right) \tag{2.10}$$

where $\varphi^{-1}(\cdot)$ is the inverse function of $\varphi(\cdot)$, i.e., $x = \varphi^{-1}(y)$ satisfies the equation $\varphi(x) = y$.
The third term $\frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa'}v_{i-1}(n)[\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)]$ in equation (2.3) is the convection term. It represents the influence of the incoming traffic on the mean speed evolution in segment i.
The last term $-\frac{\mu(n)}{\tau}\frac{T}{L_i}w_i(n)$ in equation (2.3) is the anticipation term. It reflects the effect of downstream traffic density on the mean speed evolution in section i at sampling time nT. For instance, if the density downstream is lower, this term reflects the tendency of human drivers to increase vehicle speed.
For an automated highway system (AHS), the anticipation term is no longer under human control. It depends on the control law used by AHS to control traffic flow.
The dynamics of an automated highway system are described by (2.1), (2.2), (2.3), but the adopted safety policy that defines the equilibrium speed-density relationship (2.10) replaces (2.4). Furthermore, the designed control law $u(n)$ replaces the last term in (2.3). The complete behaviour is governed by these equations repeated below for convenience.

$$\begin{aligned} q_i(n) &= \alpha k_i(n)v_i(n) \\ &\quad +(1-\alpha)k_{i+1}(n)v_{i+1}(n) \qquad (2.11) \\ k_i(n+1) &= k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n) \\ &\quad +r_i(n) - s_i(n)] \qquad (2.12) \\ v_i(n+1) &= v_i(n) + \frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\} \\ &\quad +\frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa'}v_{i-1}(n) \\ &\quad \times[\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)] \\ &\quad -u_i(n) \qquad (2.13) \\ V_e(k_i) &= \varphi^{-1}\left(\frac{1}{k_i}\right) \qquad (2.14) \end{aligned}$$

Here α, κ', τ are positive constants with $0 \leq \alpha \leq 1$ and $\varphi^{-1}(\cdot)$ is the inverse function of the adopted safety policy $\varphi(\cdot)$.

3. Problem Statement

Traffic congestion in urban freeways is caused by adhoc velocities and headways that humans choose when they operate their vehicles. Therefore, any strategy that hopes to reduce congestion needs to remove this human subjective element and replace it with a method that directly controls the density and speed of vehicles by prescribing speed commands to individual vehicles.
Assume that the roadway has the capability of measuring mean speeds and traffic densities at each section of a lane. The goal of traffic management then is to assess the state of the traffic and provide appropriate speed commands to the vehicles at various sections of the lane in order to maintain a desired traffic density profile which under the current traffic conditions corresponds to some optimum traffic flow situation. A roadway controller can be designed to perform this task. The speed commands should be generated so that the desired traffic flow rate can be achieved and the density distribution along the lane leads to a homogeneous traffic flow.
Consider a lane which is subdivided into N sections with lengths $L_i, (i = 1, \ldots, N)$ as shown in Figure 1. The traffic flow rate entering section 1 at sampling time nT is $q_0(n)$ veh/h. The desired traffic density for section i of a single lane is assumed to be $k_{d_i}(n)$. Our objective is to choose a proper value of $u_i(n)$ for section i such that the traffic density of section i converges to the desired traffic density $k_{d_i}(n)$ exponentially fast. i.e.

$$k_i(n) \rightarrow k_{d_i}(n) \text{ as } n \rightarrow \infty.$$

4. Roadway Traffic Density Controller

In this section, we propose a macroscopic roadway traffic density controller for a single lane of a freeway with no on-ramp/ off-ramp traffic so that $r_i(n) = s_i(n) = 0$. Our design uses integrator backstepping to realize the control law needed to track a desired density profile. We shall repeatedly use the following lemma for this purpose.

Lemma 4.1 *Consider the following discrete time system*

$$z(n+1) = cz(n) + u(n), \quad z(0) = z_0$$

where c is a constant and $|c| < 1$.
Then $u(n) \to 0$ exponentially implies $z(n) \to 0$ exponentially.

Proof: The proof is trivial and is omitted. ∎

Our control design consists of three steps.

Step 1

We begin our design by defining the tracking error for section i as

$$\xi_i(n) := k_i(n) - k_{d_i}(n)$$

Then with (2.2) it follows that

$$\begin{aligned} \xi_i(n+1) &= k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n)] \\ &\quad - k_{d_i}(n+1) && (4.1) \\ &= c_\xi \xi_i(n) + \eta_i(n) && (4.2) \end{aligned}$$

where

$$\begin{aligned} \eta_i(n) &:= \frac{T}{L_i}[q_{i-1}(n) - q_i(n)] - k_{d_i}(n+1) \\ &\quad + k_i(n) - c_\xi \xi_i(n) \end{aligned}$$

From Lemma 4.1, we have $\xi_i(n) \to 0$ as $n \to \infty$ if $|c_\xi| < 1$ and $\eta_i(n) \to 0$ as $n \to \infty$. The goal of our next step is to choose the control input $u_i(n)$ that guarantees $\eta_i(n) \to 0$ as $n \to \infty$.

Step 2

From the definition of $\eta_i(n)$ and (2.1), we have

$$\begin{aligned} \eta_i(n) &= \frac{T}{L_i}[\alpha k_{i-1}(n) v_{i-1}(n) \\ &\quad + (1 - 2\alpha) k_i(n) v_i(n) \\ &\quad - (1-\alpha) k_{i+1}(n) v_{i+1}(n)] - k_{d_i}(n+1) \\ &\quad + k_i(n) - c_\xi \xi_i(n) && (4.3) \end{aligned}$$

To simplify the notation, we define

$$\begin{aligned} a_i(n) &:= \frac{T}{L_i} \alpha k_{i-1}(n) && (4.4) \\ b_i(n) &:= \frac{T}{L_i}(1 - 2\alpha) k_i(n) && (4.5) \\ c_i(n) &:= \frac{-T}{L_i}(1-\alpha) k_{i+1}(n) && (4.6) \\ d_i(n) &:= k_i(n) - c_\xi \xi_i(n) - k_{d_i}(n+1) && (4.7) \end{aligned}$$

Note from (2.2) and (4.1) that the quantities $a_i(n+1), b_i(n+1), c_i(n+1), d_i(n+1)$ are available at sampling time nT. To stress this fact, we define

$$\left.\begin{aligned} \bar{a}_i(n) &:= a_i(n+1) \\ \bar{b}_i(n) &:= b_i(n+1) \\ \bar{c}_i(n) &:= c_i(n+1) \\ \bar{d}_i(n) &:= d_i(n+1) \end{aligned}\right\} \quad (4.8)$$

Substituting our definitions (4.4)-(4.7) in equation(4.3) we obtain the compact form

$$\begin{aligned} \eta_i(n) &= a_i(n) v_{i-1}(n) + b_i(n) v_i(n) \\ &\quad + c_i(n) v_{i+1}(n) + d_i(n) && (4.9) \end{aligned}$$

Then with (4.8) it follows that

$$\begin{aligned} \eta_i(n+1) &= \bar{a}_i(n) v_{i-1}(n+1) + \bar{b}_i(n) v_i(n+1) \\ &\quad + \bar{c}_i(n) v_{i+1}(n+1) + \bar{d}_i(n) && (4.10) \end{aligned}$$

We now consider the dynamics of this equation for the entrance ($i = 1$), exit ($i = N$) and for the intermediate sections ($1 < i < N$) of the freeway lane. Let us first define

$$\begin{aligned} f_i(n) &:= v_i(n) + \frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\} \\ &\quad + \frac{T}{L_i} \frac{k_{i-1}(n)}{k_i(n) + \kappa} v_{i-1}(n) \\ &\quad \times [\sqrt{v_{i-1}(n) v_i(n)} - v_i(n)]. && (4.11) \end{aligned}$$

Then, using equations (2.13) and (4.11), we have

$$v_i(n+1) = f_i(n) - u_i(n) \quad (4.12)$$

Case i: $1 < i < N$.

Using equations (4.10) and (4.12), we have

$$\begin{aligned} \eta_i(n+1) &= c_\eta \eta_i(n) + e_i(n) \\ &\quad - [\bar{a}_i(n) u_{i-1}(n) + \bar{b}_i(n) u_i(n) \\ &\quad + \bar{c}_i(n) u_{i+1}(n)] && (4.13) \end{aligned}$$

where

$$\begin{aligned} e_i(n) &:= -c_\eta \eta_i(n) + [\bar{a}_i(n) f_{i-1}(n) + \bar{b}_i(n) f_i(n) \\ &\quad + \bar{c}_i(n) f_{i+1}(n) + \bar{d}_i(n)]. && (4.14) \end{aligned}$$

Therefore, if we choose the control signal $u_i(n)$, $1 < i < N$ to satisfy

$$\bar{a}_i(n)u_{i-1}(n) + \bar{b}_i(n)u_i(n) + \bar{c}_i(n)u_{i+1}(n) = e_i(n) \tag{4.15}$$

and choose c_η so that $|c_\eta| < 1$, then by application of lemma 4.1 to equation (4.13) we have $\eta_i(n) \to 0$ as $n \to \infty$.

Case ii: $i = 1$.

Using boundary condition (2.7) in equation (4.10) we have

$$\begin{aligned} \eta_1(n+1) &= (\bar{a}_1(n) + \bar{b}_1(n))v_1(n+1) \\ &\quad + \bar{c}_1(n)v_2(n+1) + \bar{d}_1(n) \end{aligned}$$

Substituting for $\bar{a}_1(n)$ and $\bar{b}_1(n)$ from (4.8) and (4.4)-(4.5) in this equation and using boundary condition (2.6) with some manipulations it is easy to show that

$$\begin{aligned} \eta_1(n+1) &= c_\eta\eta_1(n) + e_1(n) \\ &\quad - \left[\frac{\alpha}{2\alpha-1}\bar{b}_1(n)u_1(n) \right. \\ &\quad \left. + \bar{c}_1(n)u_2(n)\right] \end{aligned} \tag{4.16}$$

where

$$\begin{aligned} e_1(n) &:= -c_\eta\eta_1(n) + \left[\frac{\alpha}{2\alpha-1}\bar{b}_1(n)f_1(n) \right. \\ &\quad + \bar{c}_1(n)f_2(n) + \bar{d}_1(n) \\ &\quad \left. + \frac{T}{L_1}q_0(n+1)\right]. \end{aligned} \tag{4.17}$$

Therefore, if we choose control $u_1(n)$ to satisfy

$$\frac{\alpha}{2\alpha-1}\bar{b}_1(n)u_1(n) + \bar{c}_1(n)u_2(n) = e_1(n) \tag{4.18}$$

then by application of lemma 4.1 to equation (4.16) we have $\eta_1(n) \to 0$ as $n \to \infty$.

Case iii: $i = N$.

Using boundary condition (2.9) in (4.10) we have

$$\begin{aligned} \eta_N(n+1) &= \bar{a}_N(n)v_{N-1}(n+1) + (\bar{b}_N(n) \\ &\quad + \bar{c}_N(n))v_N(n+1) + \bar{d}_N(n). \end{aligned}$$

Substituting for $\bar{b}_N(n)$ and $\bar{c}_N(n)$ from (4.8) and (4.5)-(4.6) in this equation and using boundary condition (2.8) we have

$$\begin{aligned} \eta_N(n+1) &= c_\eta\eta_N(n) + e_N(n) - \left[a_N(n)u_{N-1}(n) \right. \\ &\quad \left. + \frac{\alpha}{2\alpha-1}b_N(n)u_N(n)\right] \end{aligned} \tag{4.19}$$

where

$$\begin{aligned} e_N(n) &:= -c_\eta\eta_N(n) + [\bar{a}_N(n)f_{N-1}(n) \\ &\quad + \frac{\alpha}{2\alpha-1}\bar{b}_N(n)f_N(n) \\ &\quad + \bar{d}_N(n)]. \end{aligned} \tag{4.20}$$

Therefore, if we choose control $u_N(n)$ to satisfy

$$\bar{a}_N(n)u_{N-1}(n) + \frac{\alpha}{2\alpha-1}\bar{b}_N(n)u_N(n) = e_N(n) \tag{4.21}$$

then by application of lemma 4.1 to equation (4.19) we have $\eta_N(n) \to 0$ as $n \to \infty$.

Step 3

To obtain the control law $u_i(n), i = 1, 2, \ldots, N$ from equations (4.15), (4.18), and (4.21), we need to solve the algebraic equation,

$$P_1(n)U(n) = E(n) \tag{4.22}$$

where

$$\begin{aligned} P_1(n) &:= \begin{bmatrix} \frac{\alpha}{2\alpha-1}\bar{b}_1 & \bar{c}_1 & 0 & \cdot & \cdot & \cdot & 0 \\ \bar{a}_2 & \bar{b}_2 & \bar{c}_2 & 0 & \cdot & \cdot & 0 \\ 0 & \bar{a}_3 & \bar{b}_3 & \bar{c}_3 & 0 & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & & & 0 & \cdot & \cdot & \bar{c}_{N-1} \\ 0 & 0 & \cdot & \cdot & 0 & \bar{a}_N & \frac{\alpha}{2\alpha-1}\bar{b}_N \end{bmatrix} \\ U(n) &:= \left[u_1(n), u_2(n), \ldots, u_N(n)\right]^T \\ E(n) &:= \left[e_1(n), e_2(n), \ldots, e_N(n)\right]^T \end{aligned} \tag{4.23}$$

and

$$\begin{aligned} e_1(n) &= -c_\eta\eta_1(n) + \frac{\alpha}{2\alpha-1}\bar{b}_1(n)f_1(n) \\ &\quad + \bar{c}_1(n)f_2(n) + \bar{d}_1(n) + \frac{T}{L_1}q_0(n+1) \\ e_i(n) &= -c_\eta\eta_i(n) + \bar{a}_i(n)f_{i-1}(n) + \bar{b}_i(n)f_i(n) \\ &\quad + \bar{c}_i(n)f_{i+1}(n) + \bar{d}_i(n) \;\; \forall i = 2, \ldots, N-1 \\ e_N(n) &= -c_\eta\eta_N(n) + \bar{a}_N(n)f_{N-1}(n) \\ &\quad + \frac{\alpha}{2\alpha-1}\bar{b}_N(n)f_N(n) + \bar{d}_N(n) \end{aligned}$$

We define two constants ρ_i, β_i where

$$\rho_i = \frac{L_i}{L_{i+1}}\frac{\alpha}{1-2\alpha} < 0 \tag{4.24}$$

$$\beta_i = \frac{-L_i}{L_{i-1}}\frac{1-\alpha}{1-2\alpha} > 0 \tag{4.25}$$

Substituting equations (4.24)-(4.25) to the tridiagonal matrix equation (4.22) yields

$$P(n)U(n) = E(n) \tag{4.26}$$

where

$$P(n) = \begin{bmatrix} \frac{\alpha}{2\alpha-1}\bar{b}_1 & \beta_1\bar{b}_2 & 0 & \cdot & \cdot & \cdot & 0 \\ \rho_2\bar{b}_1 & \bar{b}_2 & \beta_2\bar{b}_3 & 0 & \cdot & \cdot & 0 \\ 0 & \rho_3\bar{b}_2 & \bar{b}_3 & \beta_3\bar{b}_3 & 0 & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & & & 0 & \cdot & \cdot & \beta_{N-1}\bar{b}_N \\ 0 & 0 & \cdot & \cdot & 0 & \rho_N\bar{b}_{N-1} & \frac{\alpha}{2\alpha-1}\bar{b}_N \end{bmatrix} \tag{4.27}$$

The uniqueness of the solution of (4.26) depends on the non-singularity of $P(n)$, which in turn can be guaranteed if

$$\bar{b}_i(n) \geq \delta > 0, \ \text{for all } i, n.$$

To ensure satisfaction of this condition we make the following assumption for each section of a freeway lane at any sampling time.

Assumption 4.1 Traffic Flow Controllability

There exists a small positive constant δ^* such that

$$\begin{aligned} k_i(n+1) &= k_i(n) + \frac{T}{L_i}[\alpha k_{i-1}(n)v_{i-1}(n) \\ &\quad + (1-2\alpha)k_i(n)v_i(n) \\ &\quad - (1-\alpha)k_{i+1}(n)v_{i+1}(n)] \\ &\geq \delta^* > 0 \quad \forall i, n. \end{aligned}$$

Remark 4.1 *If assumption (4.1) is violated, it implies quantitatively that the solution to equation (4.26) is not unique. It also implies qualitatively that the density at the next sampling instant is not sufficient to warrant control action. Hence if this condition occurs, then the control law for this section can be switched off. The requirement in assumption (4.1) is therefore intuitively reasonable for applying control.*

Our design is summarized in the following theorem.

Theorem 4.1 *Assume that the traffic flow controllability stated in Assumption 4.1 is satisfied for each section at any sampling time. Then there exists a control input $u_i(n)$ which satisfies*

$$P(n)U(n) = E(n) \tag{4.28}$$

where $P(n)$, $U(n)$ and $E(n)$ are defined in Step 3, and furthermore, drives the traffic density $k_i(n)$ for section i, $i = 1, 2, \ldots, N$ to the desired traffic density $k_{d_i}(n)$ exponentially fast.

Theorem 4.1 provides the control strategy needed for tracking of a desired density profile. According to this theorem, if the velocity command that is sent to vehicles in each section i of the lane at sampling time $(n+1)T$ is chosen as

$$\begin{aligned} v_{command} &= v_i(n) + \frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\} \\ &\quad + \frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa'}v_{i-1}(n) \\ &\quad \times[\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)] - u_i(n) \end{aligned}$$

where $u_i(n)$ is chosen as (4.28), then the traffic density at section i converges to the desired traffic density k_{d_i} exponentially.

5. Simulation Studies

Consider a long segment of freeway with only one lane which is divided into 12 sections. The length of each section is $500m$. The initial traffic volume entering section 1 is assumed to be 1500 veh/h. Initial density and mean speed of each section are as follows:

- The initial density for sections 1 to 12 are: 18, 18, 18, 18, 18, 52, 52, 52, 18, 18, 18, 18 $veh/km/lane$ respectively.
- The initial velocity for sections 1 to 12 are: 81, 81, 81, 81, 81, 29, 29, 29, 81, 81, 81, 81 km/h respectively.

Three cases are considered. In the first case, we show in Figures 2 and 3 the situation when no control is applied. From these figures we see the propagation of congestion upstream due to the initial traffic congestion in sections $6-8$ which eventually causes a traffic jam.
In the second case, we use our proposed controller to achieve a desired traffic density of 23 veh/km. As evidenced by our simulation results shown in Figures 4-7, the initial congested conditions are quickly dampened out by our controller and traffic flow is regulated to achieve the desired traffic densities.
In the third case, we assume that the input traffic flow rate in section 1 increases exponentially from 1500 veh/h to 2000 veh/h as shown in Figure 8. We set the desired traffic density for this case to 23 veh/km. From our simulation results shown in Figure 9 we see that the desired traffic density is exponentially achieved.

6. Conclusion

In this paper, we presented the theory and design of a macroscopic traffic controller to track a desired density profile. Our simulations demonstrated the effectiveness of our controller in reducing traffic congestion and exponentially tracking desired densities. However, due to the highly nonlinear and strong coupling characteristics of the freeway model, computation of the control required for density tracking is not straightforward. First, it needs density and velocity data of *all* sections of the freeway. The cost of data acquisition for this purpose increases with the freeway length and can be substantial. Second, computation of the control involves a recursive type algebraic equation formed by (4.15), (4.18) and (4.21) which leads to a large scale matrix equation (4.26) that must be solved. The solution of this equation yields the control that must be applied for *all* sections of the freeway under consideration. Computational costs associated with solving such large scale systems can be considerable and should be avoided for quick and effective control. Due to these concerns, it is desirable to move from a centralized controller to a decentralized one. Such a controller should apply control on a particular section based only on the information from its neighboring sections. Furthermore, any computations necessary in determining this control should not be unnecessarily costly. The design of decentralized controllers that achieve these objectives is a topic of current research.

References

[1] L. S. Yuanand J. B.Kreer. An optimal control algorithm for ramp metering of urban freeways. *Proc. 6th IEEE Annual Allerton Conf. Circuit and System Theory*, University of Illinois, Allerton, IL, 1968.

[2] C. C. Chien and P. Ioannou. Automatic vehicle following. *Proc. American Control Conference*, 1992.

[3] C. C. Chien and P. Ioannou. Fuzzy traffic density homogenizer for automated highway systems. *Pre-print*, 1993.

[4] M. Cremer and A. D. May. An extended traffic model for freeway control. *Research Report UCB-ITS-RR-85-7*, 1985.

[5] T. L. Greenlee and H. J. Payne. Freeway ramp metering strategies for responding to incidents. *Proc. IEEE Conf. Decision and Control*, pages 982–7, 1977.

[6] J. K. Hedrick, D. McMahon, V. Narendran, and D. Swaroop. Longitudinal vehicle controller design for ivhs systems. *Proc. American Control Conference*, pages 3107–3112, 1991.

[7] P. Ioannou, F. Ahmed-Zaid, and Durk Wuh. A time headway autonomous intelligent cruise controller : Design and simulation. *Technical Report. USC-SCT 92-11-01*, 1992.

[8] P. Ioannou and C. C. Chien. Autonomous intelligent cruise control. *IEEE Transactions on Vehicular Technology*, 42:657–672, Nov., 1993.

[9] U. Karaaslan, P. Varaiya, and J. Walrand. Two proposals to improve freeway traffic-flow. *Preprint*, 1990.

[10] J. J. Lighthill and G. B. Witham. On kinematic waves ii: A theory of traffic flow on long crowded roads. *Proc. of the Royal Society of London, Series A*, 229:317–345, 1955.

[11] M. Papageorgiou. Applications of automatic control concepts to traffic flow modeling and control. *Springer-Verlag*, 1983.

[12] M. Papageorgiou, J. M. Blosseville, and H. Hadj-Salem. Macroscopic modelling of traffic flow on the boulevard periherique in paris.

[13] M. Papageorgiou, J. M. Blosseville, and H. Hadj-Salem. Modelling and real time control of traffic flow on the southern part of boulevard periherique in paris i. modelling. *Transpotation Research*, A 24:345–359, 1990.

[14] M. Papageorgiou. Applications of automatic control concepts to traffic flow modeling and control. *Springer, Berlin*, 1983.

[15] M. Papageorgiou. Freeway on ramp control strategies: Overview, discussion and possible application to boulevard peripherique de paris. *Internal Report, INRETS, Arcueil, France*, 1986.

[16] M. Papageorgiou, J. M. Blosseville, and H. Hadj-Salem H. Modelling and real-time control of traffic flow on the southern part of boulevard peripherique in paris, pt 2 - coordinated on-ramp metering. *Transp. Res. B (in press)*, 1990.

[17] H. J. Payne. Models of freeway traffic and control. *Mathematical Models of Public Systems, Simulation Council Proceedings*, pages 51–61, 1971.

[18] H. J. Payne. A critical review of a macroscopic freeway model. *Engineering Foundation Conference on Research Directions in Computer Control of Urban Traffic Systems*, pages 251–265, 1979.

[19] H. J. Payne, W. S. Meisel, and M. D. Teener. Ramp control to relieve freeway congestion caused by traffic disturbances. *Int. J. Control*, 469:52–64, 1973.

[20] B. S. Y. Rao, P. Varaiya, and F. Eskafi. Investigations into achievable capacities and stream stability with coordinated intelligent vehicles. *PATH, Technical Report*, 1992.

[21] S. Sheikholelslam and C. A. Desoer. A system level study of the longitudinal control of a platoon of vehicles. *ASME Journal of Dynamic System, Measurement and Control*, 1991.

[22] Shahab Sheikholeslam. Control of a class of interconnected nonlinear dynamical system: the platoon problem. *Ph. D. Dissertation, University of California, Berkeley*, 1991.

[23] S. Shladover. Operation of automated guideway transit vehicles in dynamically reconfigured trains and platoons. *UMTA-MA-06-0085-79*, 1979.

[24] S. E. Shladover. Longitudinal control of automotive vehicles in close-formation platoons. *ASME Journal of Dynamic System, Measurement and Control*, 113:231–241, 1991.

[25] S. E. Shladover, C. A. Desoer, J. K. Hedrick, M. Tomizuka, J. Walrand, W. B. Zhang, D. McMahon, S. Sheikholeslam H. Peng, and N. McKeown. Automatic vehicle control developments in the path program. *IEEE Transactions on Vehicular Technology*, 40:114–130, 1991.

[26] A. Stotsky, C. C. Chien, and P. Ioannou. Robust platoon-stable controller design for autonomous intelligent vehicles. *Proceedings of the 33rd Conference on Decision and Control*, Florida, December 1994.

[27] D. Swaroop, C. C. Chien, P. Ioannou, and J. K. Hedrick. A comparision of spacing and headway control laws for automatically controlled vehicle. *to appear in Journal of Vehicle System Dynamics*, 1993.

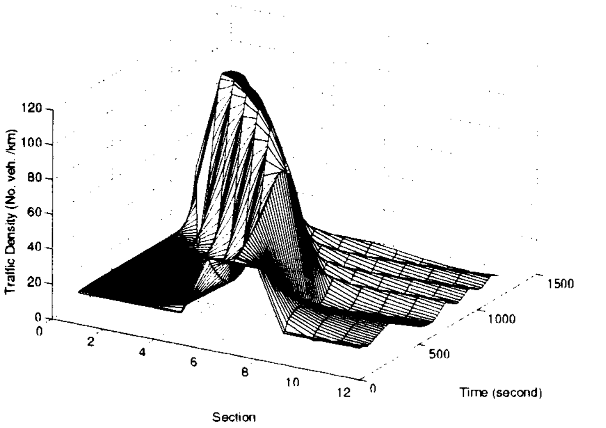

Figure 2: Density Profile without control

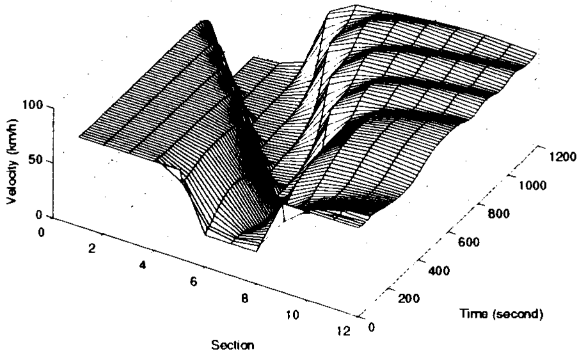

Figure 3: Velocity Profile without control

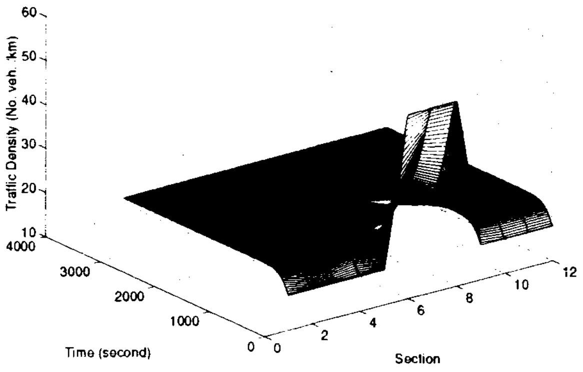

Figure 4: Density Profile: desired density is $23No.veh/km$

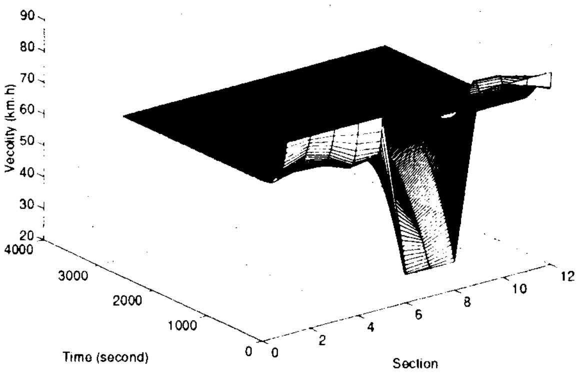

Figure 5: Velocity Profile: desired density is $23No.veh/km$

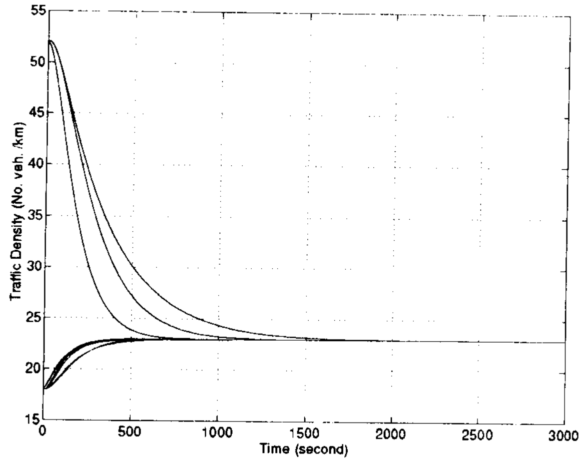

Figure 6: Density in each section: desired density is $23 No.veh/km$

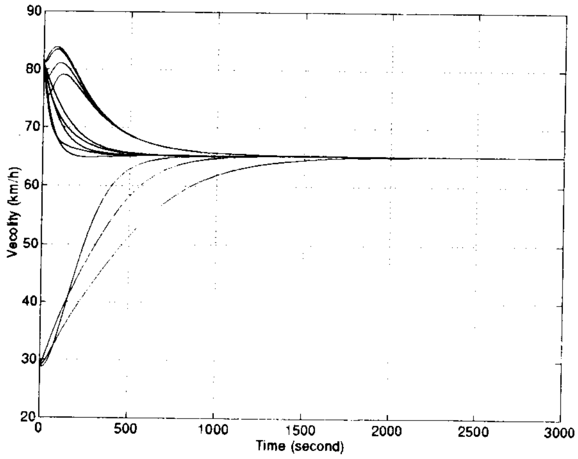

Figure 7: Velocity in each section: desired density is $23 No.veh/km$

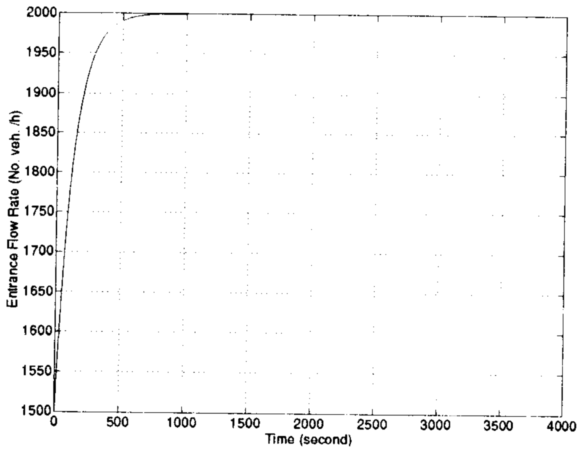

Figure 8: Increasing Entrance Flow Rate

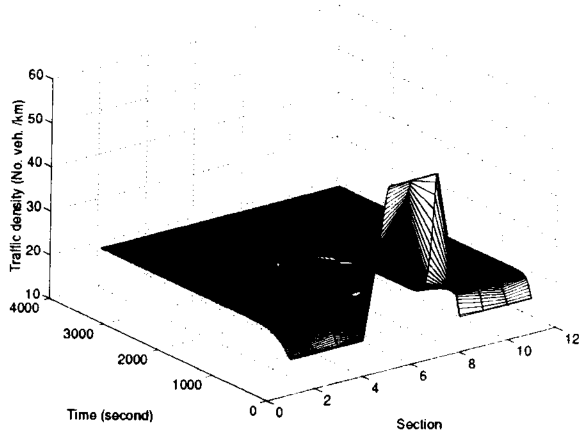

Figure 9: Density Profile with Increasing Entrance Flow Rate