AUTOMATIC TUNING OF DECENTRALIZED PID CONTROLLERS FOR MIMO PROCESSES

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Abstract

A new algorithm for automatic tuning of Decentralized PID Control for multi input - multi output (MIMO) plants is presented. The algorithm consists of two stages. In the first, the desired critical point, which consists of the critical gains of all the loops and a critical frequency, is identified. The auto-tuner identifies the desired critical point with almost no *a priori* information of the process. During the identification phase all controllers are replaced by relays, thus generating limit cycles with the same period in all loops. It is shown that each limit cycle corresponds to a single critical point of the process. By varying the relays parameters different points can be determined. The auto-tuner contains a procedure which converges rapidly to the desired critical point while maintaining the amplitudes of the process variables as well as of the manipulated variables within prespecified ranges. In the second stage, the data of the desired critical point is used to tune the PID controllers by the Ziegler-Nichols rules or its modifications. This paper focuses on the first stage. The steady-state process gains, which are required for the appropriate choice of the desired critical point, are determined by the auto-tuner in closed-loop fashion simultaneously with the identification of the critical points. The identification of the process gains is achieved at no extra plant time. Based upon a large number of simulated cases, the proposed auto-tuner seems to be efficient and robust. The paper discusses the underlying principles of the auto-tuner and its properties and capabilities demonstrated via an example.

1. INTRODUCTION

Decentralized PID control is one of the most common control schemes for interacting multiple-input multiple-output (MIMO) plants in the chemical and process industries. The main reason for this is its relatively simple structure which is easy to understand and to implement. The number of tuning parameters is 3n, where n is the number of inputs and outputs, while in full matrix PID control there are $3n^2$ parameters. In case of actuator or sensor failure, it is relatively easy to stabilize manually because only one loop is directly affected by the failure. Despite its simple structure, decentralized PID control has a long record of satisfactory performance.

It is quite surprising that despite the wide popularity of the decentralized PID control, even the number of applicable manual tuning methods is extremely limited. It is assumed throughout this paper that an analytic model of the process is not available and that the tuning procedure is based on experimental data. Even for single-input single-output (SISO) systems the tuning of a PID controller is not an easy task. the most common design procedure is the Ziegler-Nichols (ZN) method [14]. The fundamental step in that method is the identification of the critical gain and critical frequency of the plant, which together are commonly called the *critical point*. Based on these values, the controller gain and the integral and derivative coefficients are calculated. In MIMO systems the tuning problem is many times more complicated due to interactions between loops. A change of a single parameter affects, in general, all other loops as well in a way which is hard to predict. Only a limited number of works addressed the tuning of decentralized PID controllers. The method of Niederlinski [7] is a natural extension of the ZN tuning procedure to the MIMO case. It is based on replacing the controllers by gains and identifying a critical point consisting of n scalar critical gains and the critical frequency. The main departure from the SISO case is that MIMO systems have infinitely many critical points. The collection of these points defines a hypersurface in the gains space which is called the *stability limit*. Consequently one has to prespecify the desired critical point, e.g. equal loop gains. The choice of the desired critical point depends on the relative importance of the various loops, which commonly is expressed through weighting factors [7]. Once the parameters of the critical point are determined, the controllers are tuned in a fashion similar to the classical ZN rules, with possibly some modifications.

The direct application of the ZN method in the SISO case has some practical shortcomings. The procedure involves trial and error experiments to identify the critical point. During these experiments the system might be unstable for a period of time, which is risky. When the closed loop plant is brought to the verge of instability, there is no control over the amplitude of oscillations of the process variable. In addition the procedure requires an experienced operator and is time consuming. Clearly, the above problems are more severe in the MIMO case. Aström and Hagglund [1, 2] suggested the use of a relay in the identification phase for a SISO system. Instead of a system on the verge of instability, the critical point is identified from a stable limit cycle. This is also very convenient for auto-tuning where by setting a tuning mode the PID controller is replaced by a relay, the critical point is identified and the parameters are updated. due to its simplicity and efficiency, the relay based auto-tuner for single loop PID control can be found in many commercial process control products.

An extremely limited number of attempts to extend the single loop relay based auto-tuner to the MIMO cases were recently reported. However, these extensions are only partial extensions as they remain within the framework of the single relay concept. Zgorzelski et al. [13] proposed replacing one controller by a relay and the rest of the controllers by proportional controllers whose values are adjusted in each of the experiments in an iterative fashion until the desired critical point is found. To initiate this auto-tuner, a knowledge of the process steady state gains, as well as of the independent critical gains of the various loops, is required. Hence a large number (2n where n is the number of loops) of experiments needs to be carried out before the search for the desired critical point can be initiated. Half of the initial experiments involve open loop step responses

which are difficult to automate, are time-consuming and are known to be highly sensitive to disturbances. In certain cases the overall system can go unstable during the tuning session [11, 12].
Another partial extension of the single loop relay auto-tuner to the MIMO case is proposed in [6], and is a combination of sequential loop closing and single loop relay tuning. The method will be referred to as "sequential tuning" (ST). In the ST method, the decentralized PID's are tuned sequentially, loop by loop, closing each loop once it is tuned, until all loops are tuned. To tune each loop, a single relay is used to determine the corresponding critical point and the ZN settings are then employed. n sequential limit cycle experiments are required in the ST method to tune the decentralized PID controllers for $n \times n$ plant. While the ST method is a simple method, it has several shortcomings [12]. First, roughly speaking, the ST method identifies an arbitrary critical point. In that case only a single experiment is required in the method developed in this paper in order to obtain similar tunings in the MIMO case. Second, in the ST method a large number of loops are open during the tuning session. Such uncontrolled situations are likely to introduce disturbances.
Recently the authors, [11, 12] have presented an auto-tuner for a couple of PIDs in two-inputs two-outputs (TITO) plants. In this paper that algorithm is generalized to any number of loops. It fully extends the single loop auto-tuner to the MIMO case by simultaneous replacement of all controllers by relays. Under relatively mild condition on the process, each such experiment identifies a critical point. By varying the magnitudes of the relays, the identified critical point moves along the stability limits. The algorithm changes the magnitudes of the relays such that convergence to the desired critical point is obtained within a small number of experiments. Another novel component of the algorithm is the identification of the process steady state gains which are required for the proper definition of the desired critical point. The steady state gains are identified in a robust fashion in closed loop simultaneously with the identification of the critical points at no extra plant time and without any separate experiments. The data of the desired critical point is then used to calculate the settings for the various controllers.
The paper is organized as follows: Preliminaries, terminology and assumptions are given in section 2. Some results form the theory of decentralized relay control systems which are utilized in the auto-tuner are discussed in section 3. Section 4 describes the suggested auto-tuning algorithm. An example demonstrating various properties and the performance of the auto-tuner is presented in section 5. The results are summarized and discussed in section 6.

2. PRELIMINARIES AND ASSUMPTIONS.

In Fig. 1 a block diagram of decentralized control system for a MIMO process is depicted. $y(t) \in R^n$ is the vector of process outputs, $u(t) \in R^n$ is the vector of manipulated variables (or control signals) and $r(t) \in R^n$ is the vector of the reference signals. Similarly $e(t) \in R^n$ and $d(t) \in R^n$ are the vectors of loop errors and input disturbances respectively. In decentralized control the control matrix $C(s)$ is a diagonal matrix given by:

$$C(s) = \begin{bmatrix} C_1(s) & & 0 \\ & \ddots & \\ 0 & & C_n(s) \end{bmatrix} \tag{2.1}$$

and in the case considered here each of the $C_i(s)$ is a PID controller. It is clearly seen that the control signal u_i in loop i ($i = 1..n$) depends just on the error, e_i, in the same loop which is the characteristic of decentralized control. The process transfer matrix $P(s)$ is:

$$P(s) = \begin{bmatrix} P_{11}(s) & \cdots & P_{12}(s) \\ \vdots & & \vdots \\ P_{n1}(s) & \cdots & P_{nn}(s) \end{bmatrix} \tag{2.2}$$

In SISO systems there is only a finite number of critical gains which bring the system to the verge of instability. In most cases there is only one such gain. In the nxn MIMO system in Fig. 1, on the other hand, there is an infinite number of sets of gains ($K_{1cr}, K_{2cr} \ldots K_{ncr}$) that, when replacing ($C_1, C_2, \ldots C_n$) lead to neutral stability, i.e. poles on the imaginary axis. The collection of all these gains is called the stability limits of the system. Three typical cases of the stability limits for 2×2 systems are shown in Fig. 2. Since the significant performance parameter is the loop gain, the axes are $K_iP_{ii}(0)$. Each point on the stability limit corresponds to a pair of gains (K_{1cr}, K_{2cr}) and a critical frequency ω_{cr}. In the sequel we refer to it as a critical point. The points on the axes represent the situation where one loop is open ($K_i = 0$) hence the other gain is the SISO critical gain of the other loop. If the system does not have full interaction, i.e. in the 2x2 case, either $P_{12}(s)$ or $P_{21}(s)$ or both are zero, the stability limits take the rectangular form (curve 1 in Fig. 2). In that case the two critical gains are independent of each other and the system becomes unstable when either one of the gains exceeds its SISO critical value. The other two curves (2 and 3 in Fig. 2) represent two typical cases of systems with interactions. In the nxn MIMO case the stability limit is a hypersurface of order n-1 and each point on the stability limit consists of n normalized gains, $K_{icr}P_{ii}(0)$ ($i = 1,\ldots n$) and on a critical frequency, ω_{cr}. On the basis of a (for the time being arbitrary) critical point, one can tune the PID controllers either via the ZN method or its modifications [7].
Clearly, different critical points lead to different settings, hence different performance. Therefore, one needs to specify on which critical point the tuning should be based. The latter will be referred to as the desired critical point (DCP). The choice of the DCP depends on the relative importance of the various loops, which is commonly expressed through the weighting factors [7]. The relative importance of the loops corresponding to a particular point can be conveniently described by the ratios:

$$\frac{K_{icr}P_{ii}(0)}{K_{1cr}P_{11}(0)} = w_i \qquad i = 2,\ldots,n \tag{2.3}$$

where w_i is the weighting factor of loop i relative to loop 1. w_1 is taken to be 1. For example, if $w_i/w_j > 1$ $(i \neq j)$ it means that loop i is required to be under tighter control relative to loop j. We denote the vector of the weighting factors by $W \in R^n$, that is:

$$W^T = (1, w_1, ..., w_n) \tag{2.4}$$

and the vector from the origin of the normalized gain space to a critical point on the stability limit by $K \in R^n$:

$$K^T = (K_{1cr}P_{11}(0), \cdots, K_{ncr}P_{nn}(0)) \tag{2.5}$$

The intersection of W with the stability limit defines the DCP. Thus, the angle ϕ defined by:

$$\phi = \cos^{-1}\left\{\frac{K \cdot W}{|K||W|}\right\} \tag{2.6}$$

is a measure of how close is a critical point to the desired one. The vectors W, K and the angle ϕ are depicted in Fig. 2 for the 2×2 case. Given W and assuming that the steady state gains $P_{ii}(0)$ $(i = 1,...,n)$ are known, Niederlinski [7] proposed a manual method for identifying the DCP. In this method, which extends the ZN method to the MIMO cases, one increases all gains simultaneously, while keeping the ratios in (2.3), until all loops in the MIMO system oscillate at a constant amplitude each. This procedure clearly suffers from even more severe shortcomings than its SISO counterpart. The motivation for using relays is therefore clear.

As mentioned previously, it is assumed throughout the paper that no analytical model of the process is available. In fact the only process information required by the auto-tuner of this paper is the signs of the steady-state process gains of $P_{ii}(s)$ $(i = 1,...,n)$. That is, $\mathrm{sgn}(P_{ii}(0))(i = 1,...,n)$. This information is usually available.

For the results to follow, the following assumptions about the process are made:

(i) Process is open loop stable.

(ii) Interactions are significant. If the process is decoupled or weakly coupled, then the multi-loop tuner is not needed. However, the auto-tuner is capable of handling such situations as will be discussed later.

(iii) Process has low pass characteristics. Fortunately, most processes in the chemical and process industries satisfy this assumption.

In addition we will naturally assume that the decentralized PID control can stabilize and provide adequate control for the plant.

3. DECENTRALIZED CONTROL SYSTEMS

During he tuning session, all controllers are replaced by relays. The situation in the MIMO case is shown in Fig. 3. The magnitude of the relay in the ith loop is denoted $m_i(i = 1,...,n)$. The relays may contain hystereses which are convenient in coping with noisy signals as will be discussed later. The hysteresis in the ith relay is denoted $b_i(i = 1,...,n)$. Under the assumptions stated in the previous section, it is most likely that stable limit cycles in all loops will be reached. Unfortunately, no closed theory to characterize the exact conditions for the development of limit cycles in decentralized relay control system exist. Available are necessary conditions and conditions for checking the stability of such limit cycles which only recently have been formulated in closed form [9, 10]. However, numerous simulated examples have indicated that this is indeed the case if the above assumptions are satisfied. The limit cycles that are generated have the following properties: (i) A common time period T. (ii) Different amplitudes, which are denoted by a_i $(i = 1,...,n)$. (iii) Time shifts between the cycles in the various loops.

An important result which is utilized in the design procedure in the next section is as follows:

Theorem 3.2. [9,10]: Consider the system in fig. 3. If the system reaches a limit cycle then the cycle period, T, the time shifts between the cycles in the various loops, and the ratios between the amplitudes: a_1: a_2, ...: a_n, are invariant for all m_i $(i = 1, ..., n)$ and $b_i(i = 1,... n)$ satisfying:

$$m_1: m_2: .....: m_n = \alpha_1: \alpha_2: ....: \alpha_n \tag{3.1}$$

and

$$m_i/b_i = \beta_i \qquad i = 1, ..., n \tag{3.2}$$

where α_i, β_i are nonnegative constants.

Remark: The Theorem has significant practical implications. In practice, the manipulated variables are always constrained. The Theorem makes it possible for the auto-tuner to handle such constraints, in almost a trivial manner.

Using the multi-variable describing function (DF), [3], it is possible to relate the parameters of a limit cycle developed in the decentralized relay control system and a critical point as indicated by the following result:

Theorem 3.2 [4]: Consider the MIMO decentralized relay system in Fig. 3. If the system reaches a stable limit cycle with a single period T and the usual DF assumptions are met, then the parameters of the critical point are given by:

$$K_{icr} = \frac{4m_i}{\pi a_i} \qquad i = 1,... n \tag{3.3}$$

$$\omega_{cr} = \frac{2\pi}{T} \tag{3.4}$$

Relations (3.3) - (3.4) are seen to extend the well known corresponding relations in the SISO [1, 2] and the TITO [11, 12] cases. As in the SISO case, the accuracy of these relations depends on how well the assumptions of the DF are satisfied. In addition to the process low pass characteristics, it is required in the multi-variable case to have a low pass transfer function between any two non-linearities. Assumptions (ii) and (iii) and the decentralized

relay structure guarantee the satisfaction of the DF assumptions. An extensive number of various simulated examples was carried out to test the accuracy of relations (3.3) - (3.4). For all practical purposes, the results obtained were accurate. Note that the DF is not used here to predict possible limit cycles and to estimate their characteristics, but rather to estimate the critical point from experimentally found frequency and amplitudes of the limit cycles.
The preceding discussion indicates that: 1) by using n relays a critical point is identified. 2) All m_i $(i = 1,\dots,n)$ satisfying (3.2) correspond to one critical point. 3) It is possible to "move" along the stability boundary by varying the ratios between the amplitudes of the relays.

4. THE AUTO-TUNER

The main objective of the auto-tuner is to identify the DCP. Based upon the DCP the setting for the PID controllers can be determined. For a proper definition of the DCP, the steady state gains of the process need to be identified, as the important factor, as was mentioned earlier, is the relative magnitude of the loop gains and not that of the controller gains as such. Only the gains of the diagonal elements of the process transfer matrix, i.e. $P_{ii}(0)(i = 1,\dots n)$ are actually sought. These three items, which together with the results of the previous section constitute the core of the auto-tuning algorithm, are discussed next.

4.1 Determination of Steady State Gains.

The traditional methods for identifying steady state gains usually involve open loop step or pulse response tests. Such methods, however, are quite difficult to automate, are highly sensitive to disturbances and are time-consuming. When incorporated in auto-tuners it requires separate and dedicated experiments [13] extending substantially the overall tuning time. The novel method developed here identifies steady state gains in closed loop fashion. However, the identification is done with no extra experiments and therefore at no additional plant time. It uses the relays setup utilizing the existing limit cycles such that the steady state gains are identified simultaneously with the critical points.
Consider the system in fig. 3 with $r_1(t)\dots, r_n(t)$ which are not all zero mean. Then the error signals e_i and the controls u_i are also nonzero mean. A DC balance, or in mathematical terms comparing the constant terms in the Fourier series, yields,

$$\bar{y} = P(0)\bar{u} \tag{4.1}$$

where

$$\bar{y} = \begin{bmatrix} \frac{1}{T}\int_0^T y_1(t)dt \\ \vdots \\ \frac{1}{T}\int_0^T y_n(t)dt \end{bmatrix}; \quad \bar{u} = \begin{bmatrix} \frac{1}{T}\int_0^T u_1(t)dt \\ \vdots \\ \frac{1}{T}\int_o^T u_n(t)dt \end{bmatrix} \tag{4.2}$$

and T is the time period of one cycle. Repeating the experiment n times, with different reference signals leads to:

$$\overline{Y} \stackrel{\Delta}{=} \left[\bar{y}^1,\dots,\bar{y}^n\right] = P(0)\left[\bar{u}^1,\dots,\bar{u}^n\right] = P(0)\overline{U} \tag{4.3}$$

where the superscripts denote the experiment number. If the reference signals are selected appropriately, e.g.:

$$r^1 = \begin{pmatrix} r_1^1 \\ r_2^1 \\ \vdots \\ r_n^1 \end{pmatrix} = \begin{pmatrix} \beta_1 \cdot 1(t) \\ 0 \\ \vdots \\ 0 \end{pmatrix}; \quad r^n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \beta_n \cdot 1(t) \end{pmatrix} \tag{4.4}$$

then the matrix $\overline{U}$ in (4.3) is diagonally dominant and the $n \times n$ matrix of the steady state gains, $P(0)$, can be easily solved for. Hence, after n limit cycle experiments, all steady state gains are identified and n critical points found. It is worthy of noting that even the computational effort involved in the determination of the steady state gains is very small as the algorithm follows the signals used in the summations in equation (4.2) anyway. Based on many simulations, it appears that the method is robust and yields accurate results as demonstrated in the example in section 5.

4.2 A method for convergence to the DCP.

In order to identify the DCP, which is defined by (2.3) the corresponding relay ratios need to be known. If the approximate relations for the critical gains in equation (3.3) are substituted into equation (2.3), the desired relay ratios, that is, the ones that correspond to the DCP, are obtained:

$$\frac{m_1}{m_i} = \frac{1}{w_i}\frac{a_1}{a_i}\left|\frac{P_{ii}(0)}{P_{11}(0)}\right| \quad i = 2,\dots,n \tag{4.5}$$

Using the following quantities:

$$\overline{P}(0) \stackrel{\Delta}{=} \mathrm{diag}\left[\frac{|P_{ii}(0)|}{w_i|P_{11}(0)|}\right]; \quad \overline{P}(0) \in R^{(n-1)\times(n-1)} \tag{4.6}$$

$$M^T \stackrel{\Delta}{=} \left[\frac{m_1}{m_2}, \frac{m_1}{m_3}\dots, \frac{m_1}{m_n}\right]; \quad M \in R^{(n-1)} \tag{4.7}$$

and

$$\Omega^T \stackrel{\Delta}{=} \left[\frac{a_1}{a_2},\dots,\frac{a_1}{a_n}\right]; \quad \Omega \in R^{(n-1)}$$

equation (4.5) may be put in a matrix form:

$$M = \overline{P}(0)\Omega \tag{4.8}$$

Eq. (4.8) cannot be used to determine the desired M (which leads to the DCP) a priori because of two reasons. First, M and Ω are not independent, this will be discussed in the sequel. Furthermore, $\overline{P}(0)$ is unknown at the initial time.

We therefore, perform n initial experiments that are needed to define the stability limit which is a hypersurface of order n-1. In each of the first n experiments, say experiment i, the relays amplitudes are chosen such that $m_i/m_j >> 1$ (i, j = 1,2...,n; $i \neq j$). Such a choice corresponds to the identification of a critical point located close to the axis $K_iP_{ii}(0)$ in the normalized gain space. That is to say, a critical point which is close to the SISO critical point of loop i. As will be elaborated upon in the next section, the selection of the relay amplitude ratios in the first n experiments is usually determined by operational considerations. Thus after the initial experiments, n critical points on the stability limit situated close to axes of the normalized gain space, and all the steady-state gains are identified by the auto-tuner. Having identified n critical points, the question now is how to determine M, the vector of the relay ratios, for the next experiments in a systematic fashion which will lead to a rapid convergence to the DCP. Recalling from Theorem (3.1) that on the stability limit:

$$M = F(\Omega) \tag{4.9}$$

via some unknown implicit function F, the problem becomes that of finding M such that (4.5) or (4.8) hold. In other words, M which leads to the DCP. To this end the function F in (4.9) is approximated by the linear relation:

$$M \cong A\Omega + B \tag{4.10}$$

Equating (4.8) and (4.10) and solving for M yields:

$$M_{n+1} = \bar{P}(0)[\bar{P}(0) - A]^{-1}B \tag{4.11}$$

where M_{n+1} is the vector of relay amplitudes to be used in experiment n + 1. $\bar{P}(0)$ is known from the first n experiments as discussed in section 4.1. $A \in R^{(n-1)(n-1)}$ and $B \in R^{n-1}$ are also determined from the data of the first n experiments as follows: In each of the first n experiments we have M_i and Ω_i, where the subscripts denote the experiment number. Define the vectors $\tilde{M} \in R^{n-1}$ and $\tilde{\Omega} \in R^{n-1}$ as follows:

$$\tilde{M}_i \overset{\Delta}{=} M_i - M_{i+1}\,; \quad \tilde{\Omega}_i \overset{\Delta}{=} \Omega_1 - \Omega_{i+1} \quad i = 1,\ldots,n-1 \tag{4.12}$$

then it is easy to verify that A and B in (4.11) are given by:

$$A = L\,\Gamma^{-1} \quad ;B = M_1 - L\Gamma^{-1}\Omega_1 \tag{4.13}$$

where $L \in R^{(n-1)\times(n-1)}$ and $\Gamma \in R^{(n-1)\times(n-1)}$ are:

$$L \overset{\Delta}{=} (\tilde{M}_1, \tilde{M}_2, \ldots \tilde{M}_{n-1}); \; \Gamma \overset{\Delta}{=} (\tilde{\Omega}_1, \ldots \tilde{\Omega}_{n-1}) \tag{4.14}$$

substituting (4.13) into (4.11) gives:

$$M_{n+1} = \bar{P}(0)[\bar{P}(0) - L\Gamma^{-1}]^{-1}(M_1 - L\Gamma^{-1}\Omega_1) \tag{4.15}$$

If the approximation (4.10) is perfectly accurate then M_{n+1} leads exactly to the desired critical point. Since it is not, there will be some error. A tolerance ε is defined and the algorithm is stopped if:

$$|\phi| < \varepsilon \tag{4.16}$$

where ϕ is defined in (2.6). Otherwise we continue in the same fashion as in experiment n+1 where the best n experiments, i.e. those with the smallest ϕ_i's play the role of the n experiments in equations (4.13) and (4.15). This simple algorithm shows excellent convergence properties as is demonstrated in the example in section 5.

4.3 Initialization issues.

In the first n experiments one needs to determine the relays' amplitudes beforehand. In the previous section it was recommended to set those amplitudes in experiment i (i = 1, ..., n) such that $m_i/m_j >> 1$ (i,j = 1,...,n; $i \neq j$). This point is elaborated on next.

From Theorem 3.1 it follows that in the relay set-up, the critical points depend only on the ratios of the relays amplitude, that is, on M. Though theoretically the results are independent of the absolute magnitudes of m_i, those values do have practical significance. To reduce the effect of noise one needs a certain amount of hysteresis and to keep the identified points close to their real values the hysteresis should be small with respect to m_i. In addition m_i must be large enough to cause noticeable changes in y_i. Hence there is a lower bound on m_i which is denoted by $\underline{m}_i$. On the other hand too large m_is cannot be used either as a result of saturation or because the allowed change in y_i is restricted. Hence there is an upper bound on m_i which is denoted by $\overline{m}_i$.

Thus it is seen that $\overline{m}_i$ and $\underline{m}_i$ are usually dictated by operational considerations. Since no information about the process is assumed to be available, it is recommended to use in the first experiment $\overline{m}_1 / \underline{m}_j$ (j = 2,...,n), in the second one: $\overline{m}_2 / \underline{m}_j$ (j = 1,3,... n) and so on. These relay ratios will lead to n critical points, each close to one of the loop gain axes in the normalized gain plane. Those n first critical points provide good starting points for the next step in which the convergence algorithm is active and are also close to the n independent single loop critical points. Under certain circumstances, the latter information may be helpful in determining the settings, as will be discussed later. Upon the completion of the n first automatic experiments, the relays' ratios are determined automatically via relation (4.15). The actual relays' amplitudes are set such that at least one amplitude is at its upper or lower bound. In this fashion the relay set-up fully utilizes the allowed ranges. This is possible due to theorem 3.1.

4.4 Tuning considerations

Once the DCP is found, the settings of the PID controllers can be determined in a straight forward manner. A simple choice is to use the ZN rules. This gives reasonable settings which provide a good starting point for further tuning. As in the SISO case, no claim is made that such settings are optimal. Another possibility is to use Niederlinski's [7] "improved" settings which are the ZN rules with one modification. That is, gains are increased or decreased relative to the ZN gains depending upon the ratios between the frequency of the DCP and those of the individual loops. Note that frequencies relatively close to the individual critical frequencies are available to the auto-tuner from the data of the first n experiments as explained above.

In both the ZN tuning rules and the "improved" ones, the frequency of he DCP, which directly reflects the interactions in the system, is used to set all integral and all derivative (if derivative is used) times. This means that the same integral and derivative times are used for each loop. No claim is made that this is an optimal or a general result. In some cases it might be preferable to use the individual loop frequencies or some combinations of the latter and the frequency of the DCP to set the integral and the derivative times. This problem is treated in [5], and is beyond the scope of this paper.

5. EXAMPLE

A multi-product pilot plant distillation column was modelled experimentally in [8]. The transfer matrix of the 3×3 binary ethanol-water system is given by:

$$P(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.6s+1} & \dfrac{-0.005e^{-1s}}{9.1s+1} \\ \dfrac{1.11e^{-6.5s}}{3.2s+1} & \dfrac{-2.36e^{-3s}}{5.0s+1} & \dfrac{-0.012e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.7e^{-9.2s}}{8.1s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.6s+1)e^{-1s}}{(3.9s+1)(18.8s+1)} \end{bmatrix} \quad (5.1)$$

The three outputs and the three inputs and physical details of the column can be found in [8]. The autotuning algorithm was applied to the column model in (5.1). The complete time history of the simulated tuning session is shown in Fig. 4 and the data collected are summarized in Table 1. Fig. 4 depicts both the reference and the error signals in the three loops during the autotuning session. Note the rapid development of steady limit cycles in all loops in the four experiments.

As can be seen from Table 1 the autotuner converged to within 2 spacial degrees of the desired critical point in just four experiments . Note that the rapid convergence and the excellent accuracy of the results is achieved despite the fact that in this case assumption (iii) is not satisfied as the model in this example provides limited filtering as it consists of just first order filters plus dead times. This is clearly reflected in the wave form of the limit cycles in Fig. 4. The steady state gains identified by the auto-tuner during the tuning session simultaneously with the 3 first critical points are given in table 2. The identified s.s. gains are remarkably close to their true values.

Next the performance of the algorithm in the presence of measurement noise was checked. The three outputs were

ϕ	50.44	43.59	16.93	1.94
r_1	0.01	0	0	0
r_2	0	0.01	0	0
r_3	0	0	-0.5	0
m_1	0.088	0.01	0.02	0.0342
m_2	0.01	0.0465	0.02	0.0465
m_3	0.01	0.01	1.0	1.1564
k_{1cr}	6.467	1.264	3.06	2.751
k_{2cr}	-0.192	-1.16	-0.638	-0.745
k_{3cr}	0.015	0.021	3.59	2.03
ω_{cr}	0.686	0.603	0.498	0.499

Table 1: Data and results of the DCP identification.

contaminated with uniformally distributed Gaussian noises with zero average and the following variances:

$$\mathrm{Var}(v)^T = (0.001, 0.004, 0.5) \quad (5.2)$$

where $v \in R^3$ is the vector of measurement noises. Note that the above noises are quite significant relative to amplitudes of the error signals shown in Fig. 4. Due to the noisy measurements the relays operate with hystereses, each with amplitude, b_i, larger than the corresponding noise variance. In order to keep the identified critical points close to their values in the noise free situation, the relays amplitudes, m_is, are forced by the algorithm not to go below lower bounds, $\underline{m}_i$s, which are set to four times the corresponding hysteresis size, b_i, each. Such restrictions are easily handled by the algorithm as discussed previously. The time history of the complete auto-tuning session in this case is shown in Fig. 5. The algorithm converged in four experiments to within $\phi = 4^o$ and to $\phi = 1.5^o$ in the fifth one. The critical point and the s.s. gains identified are summarized in table 2. Despite the noisy measurements, all identified values are within several percents of the corresponding values identified in the noise-free case. This clearly demonstrates the robustness of the algorithm.

	No noise	With noise
K_{1cr}	2.751	2.55
K_{2cr}	-0.745	-0.7
K_{3cr}	2.03	2.0
ω_{cr}	0.499	0.489
$P_{11}(0)$	0.66	0.71
$P_{22}(0)$	-2.35	-2.24
$P_{33}(0)$	0.84	0.83

Table 2. Identified parameters with and without noise.

6. SUMMARY AND DISCUSSION

It is surprising that although decentralized PID control is the most common multi-variable control in industry, very little effort has been devoted by researchers to the development of simple, practical and reliable tuning methods which require limited process information for such control schemes. In this paper, a new algorithm for autotuning of decentralized PID controllers, which fully generalizes the single-loop SISO relay auto-tuner to MIMO systems, was presented. In the tuning mode, all controllers are replaced by relays and a critical point is identified from the limit cycles reached in all the loops via relations derived form the theory of decentralized relay control systems. An algorithm for changing the relays amplitudes in order to obtain the desired critical point was presented and was shown to be very efficient with excellent convergence properties. Except for the signs of the n principal steady state gains, the auto-tuner does not require any information about the process. The auto-tuner was tested on a large number of cases and was found to be highly efficient and robust.

Another novelty of he proposed algorithm is the steady state gains identifier. These gains are identified from the same experiments used to find the critical points. Hence, the gains identification is achieved without any dedicated experiments and at no extra plant time.

Throughout the entire tuning sessions, the process is under tight closed loop control. Furthermore, it was shown via Theorem 3.1 that the use of multiple relays provides convenient control over the limit cycle amplitudes and copes easily with practical constraints on manipulated variables.

Due to lack of space only one example which highlighted various properties of the algorithm, was shown. Among other things, the example demonstrated that the algorithm performs well in cases where the plants do not provide sufficient filtering and that it has low sensitivity to noisy measurements.

The paper focused on the identification of the desired critical point which is the crucial part of the tuning process. Having the data of the desired critical point, the PID settings via the ZN rules can be performed. It was pointed out, however, that additional information becomes available to the proposed auto-tuner during the identification stage. Consequently a variety of modifications on the ZN rules can be easily incorporated in the auto-tuner.

Acknowledgement

this work was partially supported by ABB Kent-Taylor Inc. and ABB Simcon Inc. This support is gratefully acknowledged.

References

[1] Aström, K.J. and T. Hagglund, Automatic tuning of simple regulators. Proc. IFAC, 9th World Congress, Budapest, 1984.

[2] Aström, K.J. and T. Hagglund, Automatic tuning of PID controllers, ISA, 1988.

[3] Atherton, D.P. Non-linear Control Engineering. Van Nostrand - Reinholt, London, 1975.

[4] Efrati, T. Automatic tuning of decentralized controllers for MIMO systems. M.Sc. thesis. Faculty of Mechanical Engineering, Technion - Israel Institute of Technology, 1993.

[5] Halevi, Y., Z.J. Palmor and T. Mazor. Tuning decentralized PID Controllers in MIMO processes. In this proceedings, 1995.

[6] Loh, Ai, P., C.C. Hang, C.K. Quek and V.U. Vasnani, Auto-tune of Multi-loop proportional-integral controllers using relay feedback. Ind. Eng. Chem. Res., 32, 1102-1107, 1993.

[7] Niederlinski, A., A heuristic approach to the design of linear multivariable interacting control systems. Automatica, 7, pp. 691-701, 1971.

[8] Ogunnaike, B.A., J.P. Lemaire, M. Morari and W.H. Ray. Advanced multivariable control of a pilot-plant distillation column. AIChE Journal, 29, 4, 632, 1983.

[9] Palmor, Z.J., Y. Halevi and T. Efrati. Limit cycles in decentralized relay systems. Int. J. control, 56, 4, 755-765, 1992.

[10] Palmor, Z.J., Y. Halevi and T. Efrati. A general and exact method for determining limit cycles in decentralized relay systems. Automatica, to appear, 1995.

[11] Palmor, Z.J., Y. Halevi and N. Krasney, Automatic tuning of decentralized PID controllers for TITO processes. Proceedings of IFAC's 12th World congress, volume 2, 311-314, 1993.

[12] —— Automatic tuning of decentralized PID controllers for TITO processes. Automatica, to appear, 1995.

[13] Zgorzelski, P.H. Unbehauen and A. Niederlinski. A new simple decentralized adaptive multivariable regulator and its application to multivariable plants. Proc. IFAC 11th World Congress, Tallinn, 4, 226-231, 1990.

[14] Ziegler, J.G. and N.B. Nichols, Optimum settings for automatic controllers. Trans. ASME, 64, 759-768, 1942.

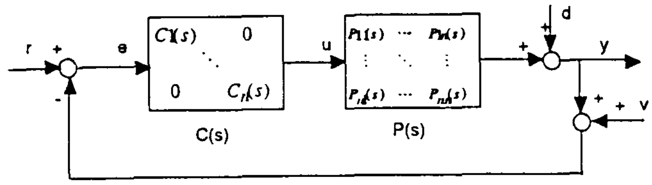

Fig. 1 A MIMO decentralized control system.

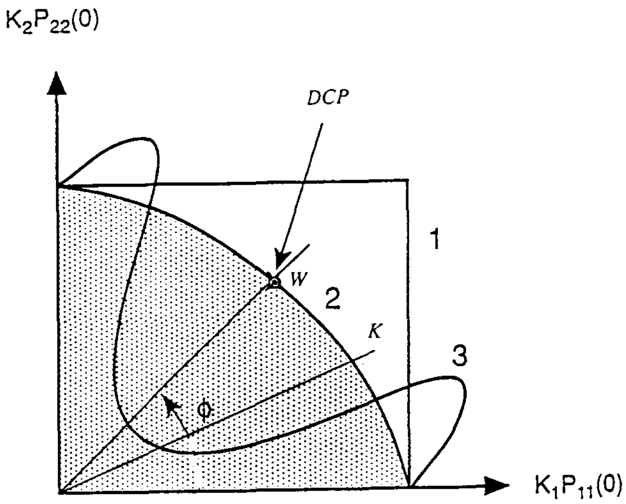

Fig. 2 Stability limits - three typical cases.

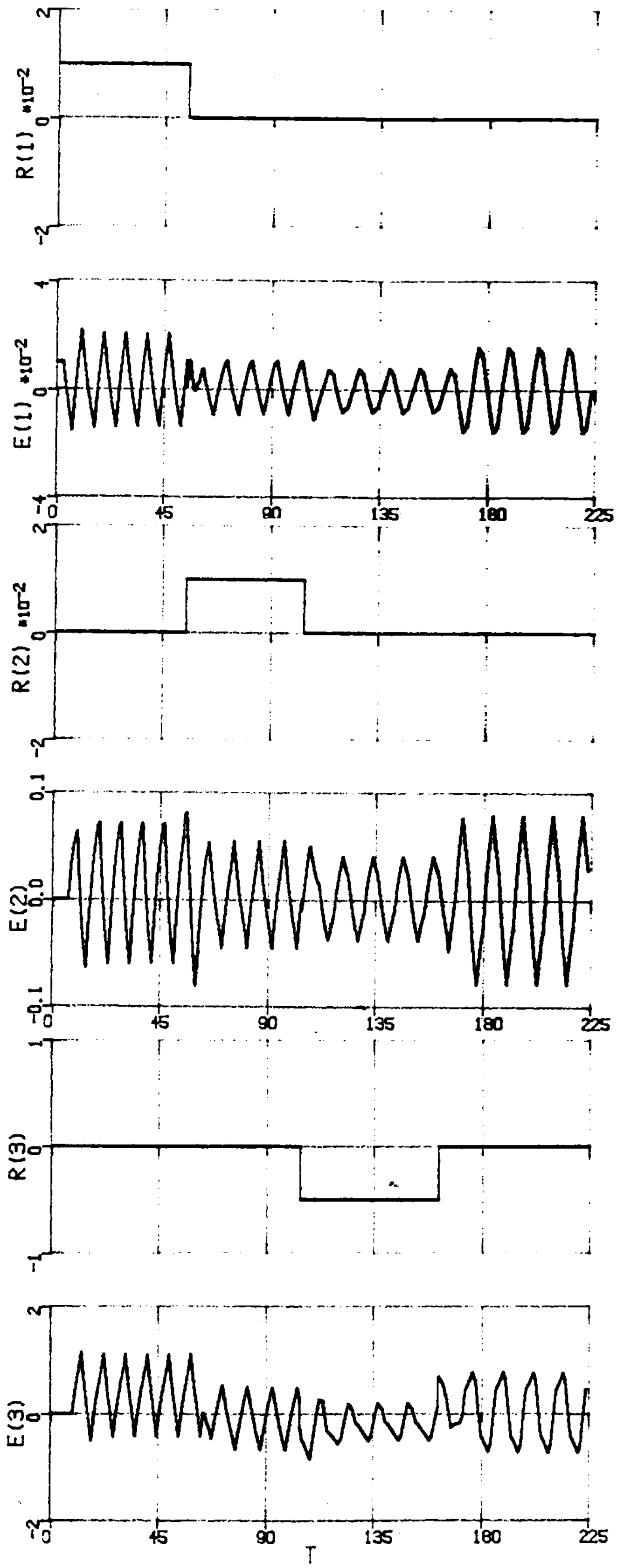

Fig. 4. Time history of the automatic experiments with noise free measurements.

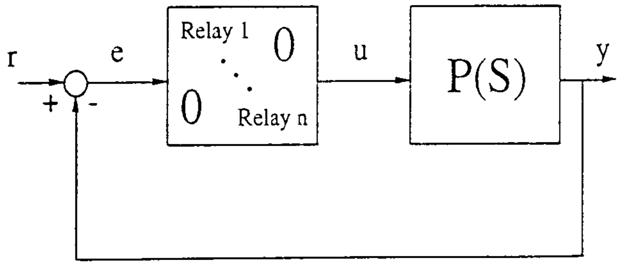

Fig. 3. A MIMO decentralized relay control system.

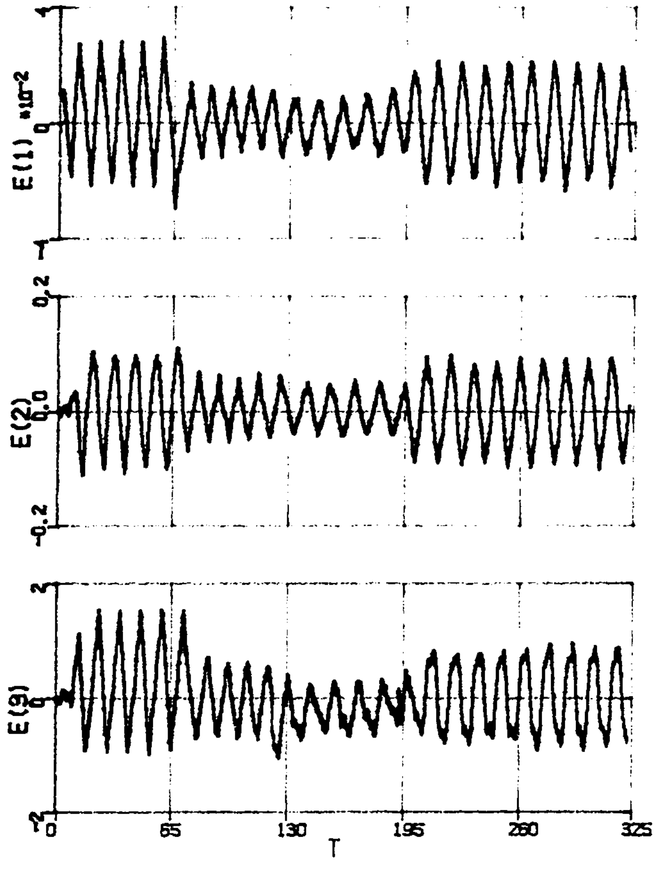

Fig. 5. Time history of the automatic experiments with noisy measurements.