Neural Networks for Learning of Robot Contact Surfaces and Forces

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I Introduction

When it is desired to design robot manipulators that will operate in uncertain and unknown environments, it is necessary to provide them with appropriate devices and algorithms that are capable of estimating and learning unknown surfaces and objects that are in contact with the robots' end-effectors. In many robotic applications, e.g. grasping, assembly, griding, deburring, cutting, inspection tasks, etc, the end-effector of the robot performs constraint motions on unknown surfaces; in order to be capable to successfully control such constraint motions it is necessary to know the characteristics of the unknown surfaces.

One solution to the surface estimation problem, is to use computer vision systems that are capable of estimating the unknown surface characteristics via appropriate techniques. However, the usage of vision systems increases considerably the cost and the complexity of the application; on the other hand, the current vision systems do not perform efficiently when either the environment where they operate is very noisy or the unknown surfaces are complex enough. Another possible solution is to use force or tactile sensors to estimate the contact force that is applied between the end-effector and the constraint surface. In fact, it can be easily shown - see [11, 12]; see also section III of this paper - that the contact force F is given by

$$F = \Lambda(\theta, \dot{\theta}, \tau, \vartheta)$$

where θ, $\dot{\theta}$ are the vectors of robot joint angular positions and accelerations, respectively, τ is the vector of joint torques, ϑ is the vector of the unknown surface characteristics, and $\Lambda(\cdot)$ is a nonlinear function. Therefore, if $\theta, \dot{\theta}, \tau$ and F are available for measurement, we can apply an extended Kalman filter or any other nonlinear parameter estimation method to estimate the unknown surface parameters ϑ. In the simplest case, the parameter vector ϑ can be estimated through the following gradient estimation algorithm

$$\dot{\hat{\vartheta}} = -\Pi \frac{\partial \Lambda}{\partial \vartheta}\Big|_{\vartheta=\hat{\vartheta}} \left(F - \Lambda(\theta, \dot{\theta}, \tau, \hat{\vartheta})\right)$$

where $\hat{\vartheta}$ denotes the estimated value of ϑ and Π is a positive definite - possibly time-varying - matrix. The above methodology, slightly modified, has been followed by Bay and Hemami [1] in order to solve the unknown surface estimation problem. However, it is a well-known fact, that nonlinear parameter estimation algorithms do not guarantee the convergence of $\hat{\vartheta}$ to the actual parameter vector ϑ; this is since, due to the nonlinear dependence of the function $\Lambda(\cdot)$ on the parameter vector ϑ, there are many local minima of the error functional that the parameter estimation algorithm minimizes. Therefore the nonlinear parameter estimation algorithm might get trapped into a local minimum, and thus, the estimation procedure will fail. Even worse, nonlinear parameter estimation algorithms may become unstable, which means that the estimated values $\hat{\vartheta}$ will reach unacceptable large values (theoretically infinite).

In this paper, we show that the unknown surface estimation problem can be formulated as a <u>linear</u> parameter estimation problem, and therefore a linear parameter estimation algorithm can be applied. The linear parameter estimation algorithms, contrary to nonlinear ones, ensure stability and convergence. Furthermore, we propose a new learning architecture that is capable of estimating the unknown surface characteristics even in the case where the contact force F is not available for measurement. This learning architecture consists of a linear parameter estimation algorithm as in the case where the contact force is available for measurement, and an appropriate approximator, that approximates (estimates) the unmeasured force. As we show, the whole scheme is globally stable and convergent. In both the unknown surface estimation and the force approximation problems, we make use of high order neural network approximators, which are nonlinear functions but they are linear with respect to their adjustable parameters. However, any other (either neural or not) linear-with-respect-to-parameters approximator can be applied as well.

The key idea used in this paper, is to use the fact that during the constrained motion, the second time-derivative of the constraint equation is zero. In other words, if $\phi(x) = const$ is the mathematical description of the constraint surface, then during the constrained motion the following relation must be valid

$$\ddot{\phi}(x) = 0$$

If we linearly parameterize - using, e.g. a neural network approximator - the constraint surface $\phi(x)$, and if we assume that the constraint forces are available for measurement, then the above differential equation, reduces to an algebraic equation which is linear with respect to the (unknown) parameters of the surface. Therefore, a linear parameter estimation algorithm can be directly applied. When the contact forces are not available for measurement, the robot dynamics (which are assumed to be completely known) are utilized in order to obtain - via another neural network approximator - a reliable estimation of the constraint forces, which in turn is used in the linear parameter estimation algorithm.

The complete proofs of the theoretical results of this paper can be found in [10].

I.1 Notations & Preliminaries

Our notations are quite standard. I denotes the identity matrix; $\mathrm{tr}\{A\}$ denotes the trace of the matrix A; A^T denotes the transpose of the matrix (vector) A. If x is a vector then $|x|$ denotes the usual Euclidean norm of x. In the case where x is a scalar $|x|$ denotes its absolute value. If A is a matrix, then $|A|$ denotes the Frobenious norm of this matrix. Let now $f(t)$ be a vector function of time. Then

$$||f||_2 \triangleq \left(\int_0^\infty |f(\tau)|^2 d\tau\right)^{\frac{1}{2}}$$

and

$$||f||_\infty \triangleq \sup_{t\geq 0} |f(t)|$$

We will say that $f \in \mathcal{L}_2$ when $||f||_2$ is finite; similarly we will say that $f \in \mathcal{L}_\infty$ when $||f||_\infty$ is finite. If $f : \Re^n \mapsto \Re^m$ is an at least twice differentiable vector function, then $\nabla f(x)$ and $\nabla^2 f(x)$ denote the gradient and the Hessian of $f(\cdot)$, respectively.

Let $f(t)$ and $g(t)$ be two vector functions of time. We will say that $f \in \mathcal{S}(g)$ if there are two positive constants α_1 and α_2 such that

$$\int_0^t |f(s)|^2 ds \leq \alpha_1 + \alpha_2 \int_0^t |g(s)|^2 ds, \quad \forall t \geq 0$$

Similarly, if $f(t), g_1(t), g_2(t), \ldots, g_n(t)$ are vector functions of time, we will say that $f \in \mathcal{S}(g_1, g_2, \ldots, g_n)$ if there are positive constants α_i, $i = 0, 1, \ldots, n$ such that

$$\int_0^t |f(s)|^2 ds \leq \alpha_0 + \alpha_1 \int_0^t |g_1(s)|^2 ds + \ldots + \alpha_n \int_0^t |g_n(s)|^2 ds \quad \forall t \geq 0$$

Note that if $f \in \mathcal{S}(g_1, g_2)$ and $g_1 \in \mathcal{S}(g_2)$ then $f \in \mathcal{S}(g_2)$. Also, we will say that the function $f(\cdot)$ is *persistently exciting* (symbolically $f \in PE$) if there are positive constants β_1, β_2, δ such that

$$\beta_1 I \leq \int_t^{t+\delta} f(s) f^T(s) ds \leq \beta_2 I < \infty, \qquad \forall t \geq 0$$

II High Order Neural Network Approximators

In this section, we briefly describe the mathematical representation and the approximation properties of the high order neural network (HONN) approximators. For more details and for applications of HONNs to various engineering problems the reader is referred to [6, 7, 8, 9].

In general the HONN approximator is described by a nonlinear function of the from

$$y_i = \sum_{k=1}^{L} w_{ik} \prod_{j \in I_k} S(x_j)^{d_j(k)} \tag{2.1}$$

where y_i is the i-th output of the HONN; $y \triangleq [y_1, \ldots, y_m]^T$, x_j is the j-th input; $x \triangleq [x_1, \ldots, x_n]^T$, w_{ik} denote the *synaptic weights* of the HONN, $\{I_1, \ldots, I_L\}$ is a collection of not-ordered subsets of $\{1, \ldots, n\}$ and $d_j(k)$ are nonnegative integers. The function $S(\cdot)$ is either a linear function of the form $S(z) = \alpha z$ or a sigmoidal function of the form[1]

$$S(z) \triangleq \alpha \frac{1}{1 + \exp(-\beta z)} - \gamma \tag{2.2}$$

[1] Although, it looks simpler to use linear functions in place of $S(\cdot)$, simulation results indicate us that the usage of a sigmoid-type $S(\cdot)$ improves considerably the HONN approximation capabilities.

where α, β, γ are positive constants. It is not difficult someone to see that if we define $\zeta_k(x) \triangleq \prod_{j \in I_k} S(x_j)^{d_j(k)}$, $\zeta(x) \triangleq [\zeta_1(x), \ldots, \zeta_L(x)]^T$, and W the $m \times L$ matrix whose ik-th entry equals w_{ik}, then the HONN operations can be described by

$$y = W\zeta(x) \tag{2.3}$$

Although it is not explicitly written in (2.3), the HONN input/output function depends on the particular selection of the integer L; in the sequel we will call this integer the *number of the high order terms.*

The next proposition (see e.g. [3]) demonstrates the approximation capabilities of the HONN.

Proposition 1 *For any continuous function $f : \Re^n \mapsto \Re^m$, for any compact subset $\mathcal{X}$ of $\Re^n$, and for every $\epsilon > 0$, there is a number of high order terms L and a vector W^* such that the HONN with L high order terms and $W = W^*$ satisfies*

$$\sup_{x \in \mathcal{X}} |f(x) - W^*\zeta(x)| < \epsilon$$

Remark 1 The proof of the above proposition is based on the well-known Stone-Weierstrass theorem [13]. In fact, it can be easily shown that the family of all the HONN approximators of the form (2.3) is *dense* in the space of continuous functions $f(\cdot)$ with compact domain, which practically means that for every such function $f(\cdot)$ there is a HONN of the form (2.3) that is very "close" to it. The measure of "closeness in proposition 1 is the metric $\sup_{x \in \mathcal{X}} |f(x) - g(x)|$. However, other distortion measures can be applied as well. For instance, proposition 1 is also valid if we replace the distortion measure $\sup_{x \in \mathcal{X}} |f(x) - g(x)|$ by the following "Sobolev distance"

$$\max \left\{ \sup_{x \in \mathcal{X}} |f(x) - g(x)|, \sup_{x \in \mathcal{X}} |\nabla f(x) - \nabla g(x)|, \right.$$
$$\left. \sup_{x \in \mathcal{X}} |\nabla^2 f(x) - \nabla^2 g(x)| \right.$$

The reader is referred to [4] and the references therein for further details. ◇

III Manipulator Dynamics in Constraint Motion

Consider a robot manipulator consisting of $n+1$ links connected serially by n joints (either translational or rotational), and suppose that there are no external forces that are applied on the end-effector of the manipulator. In this case, the closed-form dynamic model of the manipulator is described by the following nonlinear differential equation [14]

$$\tau(t) = M(\theta(t), p)\ddot{\theta}(t) + C(\theta(t), \dot{\theta}(t), p)\dot{\theta}(t) + G(\theta(t), p) \tag{3.1}$$

where

- $\tau(t)$ is an $n \times 1$ vector of joint torques,
- $\theta(t)$ is an $n \times 1$ vector containing the joint angular positions,
- $\dot{\theta}$ is an $n \times 1$ vector containing the joint angular velocities,
- $\ddot{\theta}$ is an $n \times 1$ vector containing the joint angular accelerations,
- $M(\theta(t), p)$ is an $n \times n$ positive definite matrix representing the contribution of the inertial forces to the dynamical equation, hence the matrix M represents the inertia matrix of the manipulator ($M = M^T$)
- $C(\theta(t), \dot{\theta}(t), p)$ is an $n \times n$ matrix representing the Coriolis , centripetal, and frictional forces
- $G(\theta(t), p)$ is an $n \times 1$ matrix representing the gravitational forces
- p is a parameter vector whose elements are functions of the geometric and inertial characteristics of the manipulator links and the payload, i.e., p depends on the lengths and moments of inertia of each individual link and the payload.

An introduction on the derivation of the dynamical model of a robotic manipulator can be found in [14].

In the case where external forces are applied to the end-effector of the robot, the closed-form dynamic model of the manipulator becomes [14, 12]

$$\tau(t) + J^T(\theta, p)F = M(\theta(t), p)\ddot{\theta}(t)$$
$$+ C(\theta(t), \dot{\theta}(t), p)\dot{\theta}(t) + G(\theta(t), p) \tag{3.2}$$

where, $J(\cdot)$ is the $3 \times n$ manipulator Jacobian and F is the 3×1 vector of external forces. In this paper we will treat the case where the robot's end-effector is constrained to be in contact with a surface. We assume that contact occurs at a point. Let x denote the 3-dimensional position vector from a fixed reference frame to the constraint frame. The constraint surface is assumed to satisfy the following scalar relation

$$\phi(x) = const \tag{3.3}$$

where $\phi(\cdot)$ is a smooth function. In this case the contact force, or *workless* force, is given by the following (McClamroch [11])

$$F = D^T(x)\lambda \tag{3.4}$$

where

$$D(x) = \frac{\partial \phi(x)}{\partial x} \tag{3.5}$$

and λ is the Lagrange multiplier and its value is given by the solution of the following equation (see Mills and Goldenberg [12])

$$DJM^{-1}(-C\dot{\theta}-G+\tau)+DJM^{-1}J^TF+\Gamma = 0 \tag{3.6}$$

where

$$\Gamma(\theta) = \frac{d}{dt}(DJ)\dot{\theta}$$

The algebraic equation (3.6) is obtained by differentiating twice the constraint (3.3), i.e. (3.6) is obtained from

$$\ddot{\phi}(x) = 0$$

We mention here that the term $DJM^{-1}J^TD^T$ is always invertible [5]. Thus the parameter λ is given by

$$\lambda = \frac{DJM^{-1}(C\dot{\theta}+G-\tau)-\Gamma}{DJM^{-1}J^TD^T} \tag{3.7}$$

and therefore the external forces vector is given by

$$F = D^T\frac{DJM^{-1}(C\dot{\theta}+G-\tau)-\Gamma}{DJM^{-1}J^TD^T} \triangleq \Lambda(\theta,\dot{\theta},\tau) \tag{3.8}$$

Although it is not explicitly written, the function Λ depends on the particular contact surface.

IV Estimating the Unknown Surface: Case 1 — The Contact Forces are Available for Measurement

Suppose now that the robot manipulator is moving on a unknown surface $\phi(x) = const$. In this section, we will assume that the manipulator is provided with appropriate devices (encoders, tachometers, force sensors) that measure the joint angular positions and velocities θ and $\dot{\theta}$, the joint torques τ and the contact forces F. Also, in order to have a well-posed parameter estimation problem, we will assume that all the above quantities are bounded. In other words, we assume that

(A1) $\theta, \dot{\theta}, \tau, F \in \mathcal{L}_\infty$, $\dot{\tau} \in \mathcal{L}_\infty$.

From assumption (A1) we readily obtain that there exists a compact subset Ω of $\Re^{3n+3}$ such that $(\theta,\dot{\theta},\tau,F) \in \Omega$. Note now that since the cartesian vector x is related with the joint angular positions θ via a nonlinear kinematic transformation $x = T(\theta)$, we can write the constraint surface as

$$\psi(\theta) = const \tag{4.1}$$

where $\psi(\theta) = \phi(T(\theta))$. Although the surface is unknown, we can approximate it by a HONN approximator. To be more precise let us select a HONN with L_ψ high order terms of the form

$$y = W_\psi^T\zeta_\psi(\theta) \tag{4.2}$$

where $y \in \Re^1$, $W_\psi \in \Re^{L_\psi}$. Define now the *optimal weight vector* W_ψ^* as follows

$$W_\psi^* \triangleq \arg\min_{|W_\psi|\le\mathcal{M}_\psi}\left\{\sup_{\theta\in\Theta}\left|\psi(\theta) - W_\psi^T\zeta_\psi(\theta)\right|\right\} \tag{4.3}$$

where Θ is a compact subset of $\Re^n$ where θ belongs[2], and $\mathcal{M}_\psi$ is a sufficiently large positive constant; the role that $\mathcal{M}_\psi$ plays will become clear later. Let also define the *modeling error* ν_ψ as follows

$$\nu_\psi(\theta) \triangleq \psi(\theta) - W_\psi^{*T}\zeta_\psi(\theta) \tag{4.4}$$

We now return to relation (3.6); using the identity $\psi = W_\psi^{*T}\zeta_\psi + \nu_\psi$ and, after some algebraic manipulations, relation (3.6) becomes

$$\begin{aligned} 0 &= \left(W_\psi^{*T}\nabla\zeta_\psi + \nabla\nu_\psi\right)\times \\ &\quad \left(M^{-1}(-C\dot{\theta}-G+\tau)+M^{-1}J^TF\right) \\ &\quad +W_\psi^{*T}\frac{d}{dt}\left(\nabla\zeta_\psi\right)\dot{\theta} + \dot{\theta}^T\nabla^2\nu_\psi\dot{\theta} \qquad (4.5) \\ &= W_\psi^{*T}\varphi_\psi(\theta,\dot{\theta},\tau,F) + \mu_\psi(\theta,\dot{\theta},\tau,F) \qquad (4.6) \end{aligned}$$

where $\varphi_\psi \triangleq \nabla\zeta_\psi(M^{-1}(-C\dot{\theta}-G+\tau)+M^{-1}J^TF) + \frac{d}{dt}(\nabla\zeta_\psi)\dot{\theta}$, $\mu_\psi \triangleq \nabla\nu_\psi(M^{-1}(-C\dot{\theta}-G+\tau)+M^{-1}J^TF)+\dot{\theta}^T\nabla^2\nu_\psi\dot{\theta}$. Subtracting from both sides of (4.6) the term $W_\psi^T\varphi_\psi$ and denoting $\tilde{W}_\psi \triangleq W_\psi^* - W_\psi$, we obtain that

$$-W_\psi^T\varphi_\psi(\theta,\dot{\theta},\tau,F) = \tilde{W}_\psi^T\varphi_\psi(\theta,\dot{\theta},\tau,F)+\mu_\psi(\theta,\dot{\theta},\tau,F) \tag{4.7}$$

Since $\theta, \dot{\theta}, \tau, F$ are available for measurement and the robot dynamics are assumed to be exactly known, the function $\varphi_\psi(\theta,\dot{\theta},\tau,F)$ can be computed on-line during the application. Also, since W_ψ denotes the current estimate of W_ψ^*, we can also compute the LHS of (4.7). Note now that the smaller is the term ν_ψ, the smaller is the term μ_ψ. Moreover, since from proposition 1 we have that ν_ψ can be made arbitrarily small, we have the following result.

[2]Note that the implementation of the proposed method does not require the knowledge of the subsets Ω and Θ.

Result 1 *For any* $\varepsilon_1 > 0$ *there is a number* L_ψ *of high order terms such that*

$$\sup_{(\theta,\dot{\theta},\tau,F)\in\Omega} |\mu_\psi(\theta,\dot{\theta},\tau,F)| < \varepsilon_1$$

The main problem therefore is to find the appropriate synaptic weights W_ψ^*, the collections I_k and the powers $d_i(k)$ such that $\sup|\mu_\psi| \leq \varepsilon_1$, where ε_1 denotes the desired degree of accuracy. If this is achieved then obviously the problem of unknown surface estimation has been solved. In this paper, we will concentrate our attention in the optimal selection of the synaptic weights W_ψ^*. The selection of the optimal collections I_k and the powers $d_i(k)$ will not be treated in this paper since, in general, the more collections I_k we have the better the approximation is. Hence we assume that the number of collections I_k is sufficiently large in order to make the neural network able to approximate the unknown surface sufficiently close.

Setting now $e_\psi \triangleq -W_\psi^T \varphi_\psi(\theta,\dot{\theta},\tau,F)$ and omitting the arguments for simplicity, we rewrite (4.7) as follows

$$e_\psi(t) = \tilde{W}_\psi \varphi_\psi(t) + \mu_\psi(t) \tag{4.8}$$

Consider now the following learning law for adjusting the W_ψ's

$$\dot{\tilde{W}}_\psi = -\Pi e_\psi(t)\varphi_\psi(t) - \sigma_\psi(t)\Pi W_\psi \tag{4.9}$$

where

$$\sigma_\psi = \begin{cases} 0 & \text{if} \quad |W_\psi| \leq \mathcal{M}_\psi \\ \left(\frac{|W_\psi|}{\mathcal{M}_\psi} - 1\right)\sigma_0 & \text{if} \quad \mathcal{M}_\psi < |W_\psi| \leq 2\mathcal{M}_\psi \\ \sigma_0 & \text{if} \quad |W_\psi| > 2\mathcal{M}_\psi \end{cases}$$

The following Result summarizes the properties of the proposed scheme.

Result 2 *Consider the error equation* (4.8) *and the parameter estimation algorithm* (4.9). *Then the following statements hold:*

(a) $e_\psi, W_\psi, \tilde{W}_\psi \in \mathcal{L}_\infty$.

(b) $e_\psi \in \mathcal{S}(\mu_\psi)$. *Moreover, in the case where* $\mu_\psi = 0$, *we have that*

$$\lim_{t\to\infty} e_\psi(t) = 0$$

(c) *If* $|W_\psi^*| < \mathcal{M}_\psi$ *and* $\varphi_\psi \in PE$, *then parameter estimation error* $\tilde{W}_\psi$ *converges exponentially fast to the residual set*

$$\mathcal{D}_\psi \triangleq \{\tilde{W}_\psi : |\tilde{W}_\psi| \leq c_\psi \bar{\mu}_\psi\}$$

where c_ψ *is a positive constant and* $\bar{\mu}_\psi \triangleq \sup_t |\mu_\psi(t)|^2$.

V Estimating the Unknown Surface: Case 2 — The Contact Forces are not Available for Measurement

Suppose now that all the assumptions of the previous section hold, with the difference that the constraint forces vector F is not available for measurement anymore. However, since $F = \Lambda(\theta,\dot{\theta},\tau)$ where $\Lambda(\cdot)$ is an unknown function - which depends on the constraint surface $\phi(\cdot)$ - we may approximate $\Lambda(\cdot)$ with a HONN approximator. More precisely, we consider a HONN with L_F high order terms of the form

$$\hat{F} = W_F \zeta_F(\theta,\dot{\theta},\tau) \tag{5.1}$$

where $\hat{F}$ denotes the estimation of F, the $3 \times L_F$ matrix W_F denotes the set of adjustable parameters (synaptic weights) and ζ_F is an L_F-dimensional vector function. Similar to the previous section, we define W_F^* and ν_F, respectively, as follows

$$W_F^* \triangleq \arg\min_{|W_F|\leq\mathcal{M}_F} \left\{ \sup_{(\theta,\dot{\theta},\dot{\tau}\in\Xi} \left|\Lambda(\theta,\dot{\theta},\tau) - W_F\zeta_F(\theta,\dot{\theta},\tau)\right| \right\} \tag{5.2}$$

and

$$\nu_F(\theta,\dot{\theta},\tau) \triangleq \Lambda(\theta,\dot{\theta},\tau) - W_F^*\zeta_F(\theta,\dot{\theta},\tau) \tag{5.3}$$

where Ξ is a compact subset of $\Re^{3n}$ where $(\theta,\dot{\theta},\tau)$ belongs and $\mathcal{M}_F$ is a sufficiently large positive design constant which is defined similarly to $\mathcal{M}_\psi$.

Let us now define $\hat{\chi}$ as the solution of the following differential equation

$$\dot{\hat{\chi}}(t) = A\tilde{\chi}(t) + M^{-1}(\theta(t),p)\times$$

$$\left(C(\theta(t),\dot{\theta}(t),p)\dot{\theta}(t) - G(\theta(t),p) + \tau(t) + J^T(\theta,p)\hat{F}\right) \tag{5.4}$$

where A is an $n \times n$ constant negative definite matrix, and $\tilde{\chi} \triangleq \hat{\chi} - \dot{\theta}$. Note now that from (5.2) and (5.3) the constraint forces vector F can be written as

$$F = W_F^*\zeta_F + \nu_F \tag{5.5}$$

By substituting the above identity into (III), we can easily see that the following relation holds

$$\begin{aligned} \dot{\tilde{\chi}} &= A\tilde{\chi} - M^{-1}J^T\tilde{W}_F\zeta_F - M^{-1}J^T\nu_F \\ &= A\tilde{\chi} + \tilde{W}_F\bar{\zeta}_F + \mu_F \end{aligned} \tag{5.6}$$

where $\bar{\zeta}_F \triangleq \zeta_F J M^{-1}$ and $\mu_F \triangleq -M^{-1}J^T\nu_F$.

Consider now that the W_F is adjusted according to the following learning law.

$$\dot{W}_F = -\Pi \tilde{\chi}(t)\bar{\zeta}_F^T(t) - \sigma_F(t)\Pi W_F \quad (5.7)$$

where

$$\sigma_F = \begin{cases} 0 & \text{if } |W_F| \le \mathcal{M}_F \\ \left(\frac{|W_F|}{\mathcal{M}_F} - 1\right)\sigma_0 & \text{if } \mathcal{M}_F < |W_F| \le 2\mathcal{M}_F \\ \sigma_0 & \text{if } |W_F| > 2\mathcal{M}_F \end{cases}$$

We have the following result.

Result 3 *Consider the error equation (5.6) and the parameter estimation algorithm (5.7). Then the following statements hold:*

(a) $\tilde{\chi}, W_F, \tilde{W}_F \in \mathcal{L}_\infty$.

(b) $\tilde{\chi} \in \mathcal{S}(\mu_F)$. *Moreover, in the case where* $\mu_F = 0$, *we have that*

$$\lim_{t\to\infty} \tilde{\chi}(t) = 0$$

(c) *If* $|W_F^*| < \mathcal{M}_F$ *and* $\bar{\zeta}_F \in PE$, *then parameter estimation error* $\tilde{W}_F$ *converges exponentially fast to the residual set*

$$\mathcal{D}_F \triangleq \{\tilde{W}_F : |\tilde{W}_F| \le c_F \bar{\mu}_F\}$$

where c_F *is a nonnegative constant and* $\bar{\mu}_F \triangleq \sup_t |\mu_F(t)|^2$.

Let us now return to relation (4.5), i.e. to relation

$$\left(W_\psi^{*T}\nabla\zeta_\psi + \nabla\nu_\psi\right)\left(M^{-1}(-C\dot{\theta} - G + \tau) + M^{-1}J^T F\right)$$
$$+W_\psi^{*T}\frac{d}{dt}(\nabla\zeta_\psi) + \dot{\theta}^T\nabla^2\nu_\psi\dot{\theta} = 0$$

Using now relations (5.5), (5.6), we can easily see that (4.5) can be rewritten as

$$\begin{aligned} 0 &= \left(W_\psi^{*T}\nabla\zeta_\psi + \nabla\nu_\psi\right)\times \\ &\quad \left(M^{-1}(-C\dot{\theta} - G + \tau) + W_F^*\bar{\zeta}_F + \mu_F\right) \\ &\quad +W_\psi^{*T}\frac{d}{dt}(\nabla\zeta_\psi) + \dot{\theta}^T\nabla^2\nu_\psi\dot{\theta} \qquad (5.8) \\ &= \left(W_\psi^{*T}\nabla\zeta_\psi + \nabla\nu_\psi\right)\times \\ &\quad \left(M^{-1}(-C\dot{\theta} - G + \tau) + W_F\bar{\zeta}_F + \tilde{W}_F\bar{\zeta}_F + \mu_F\right) \\ &\quad +W_\psi^{*T}\frac{d}{dt}(\nabla\zeta_\psi) + \dot{\theta}^T\nabla^2\nu_\psi\dot{\theta} \qquad (5.9) \\ &= W_\psi^{*T}\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F) + \tilde{W}_F\bar{\zeta}_F\xi_1(\theta,W_\psi^*) \\ &\quad +\xi_2(W_\psi^*,\theta,\dot{\theta},\tau,F) \qquad (5.10) \end{aligned}$$

where $\tilde{W}_F \triangleq W_F^* - W_F$ denotes the parameter estimation error, and φ, ξ_1, ξ_2 are given by

$$\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F) \triangleq \nabla\zeta_\psi\left(M^{-1}(-C\dot{\theta} - G + \tau) + W_F\bar{\zeta}_F\right)$$
$$+\frac{d}{dt}(\nabla\zeta_\psi)$$

$$\xi_1(\theta,W_\psi^*) \triangleq W_\psi^{*T}\nabla\zeta_\psi + \nabla\nu_\psi \triangleq \frac{\partial\psi}{\partial\theta}$$

$$\begin{aligned} \xi_2(W_\psi^*,\theta,\dot{\theta},\tau,F) &\triangleq \nabla\nu_\psi \times \\ &\quad \left(M^{-1}(-C\dot{\theta} - G + \tau) + W_F^*\bar{\zeta}_F + \mu_F\right) \\ &\quad +W_\psi^{*T}\nabla\zeta_\psi\mu_F + \dot{\theta}^T\nabla^2\nu_\psi\dot{\theta} \end{aligned}$$

Subtracting from both sides of (5.10) the quantity $W_\psi\varphi$, we obtain that

$$-W_\psi\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F) = \tilde{W}_\psi^T\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F)$$
$$+\tilde{W}_F\bar{\zeta}_F\xi_1(\theta,W_\psi^*) + \xi_2(W_\psi^*,\theta,\dot{\theta},\tau,F) \quad (5.11)$$

Note now that since $\theta, \dot{\theta}, \tau$ are available for measurement and $W_F, \bar{\zeta}_F$ are known quantities (in fact, W_F is computed from the adaptive law (5.7) and $\bar{\zeta}_F$ is a known function of the measurable quantities $\theta, \dot{\theta}, \tau$), the function $\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F)$ can be computed on line during the application. Also, we can easily see that the following result holds.

Result 4 *For any* $\varepsilon_2 > 0$ *there are numbers* L_ψ *and* L_F *of high order terms such that*

$$\sup_{(\theta,\dot{\theta},\tau,F)\in\Omega} |\xi_2(W_\psi^*,\theta,\dot{\theta},\tau,F)| < \varepsilon_2$$

$$\sup_{(\theta,\dot{\theta},\tau)\in\Upsilon} |\mu_F(\theta,\dot{\theta},\tau)| < \varepsilon_2$$

where Υ *is a compact subset of* $\Re^{3n}$ *where* $(\theta,\dot{\theta},\tau)$ *belongs.*

Setting now $e \triangleq -W_\psi\varphi(\theta,\dot{\theta},\tau,W_F,\bar{\zeta}_F)$ relation (V) takes the form

$$e(t) = \tilde{W}_\psi^T\varphi(t) + \tilde{W}_F\bar{\zeta}_F(t)\xi_1(t) + \xi_2(t) \quad (5.12)$$

or

$$e(t) = \tilde{W}_\psi^T\varphi(t) + \omega(t) \quad (5.13)$$

where $\omega(\cdot) \triangleq \tilde{W}_F\bar{\zeta}_F(\cdot)\xi_1(\cdot) + \xi_2(\cdot)$. Consider now the following learning law for adjusting the W_ψ.

$$\dot{W}_\psi = -\Pi e(t)\varphi(t) - \sigma_\psi(t)\Pi W_\psi \quad (5.14)$$

where

$$\sigma_\psi = \begin{cases} 0 & \text{if } |W_\psi| \le \mathcal{M}_\psi \\ \left(\frac{|W_\psi|}{\mathcal{M}_\psi} - 1\right)\sigma_0 & \text{if } \mathcal{M}_\psi < |W_\psi| \le 2\mathcal{M}_\psi \\ \sigma_0 & \text{if } |W_\psi| > 2\mathcal{M}_\psi \end{cases}$$

The following result demonstrates the stability and convergence capabilities of the whole scheme.

Result 5 *Consider the error equations* (5.6), (5.12) *and the parameter estimation algorithms* (5.7), (5.14). *Then the following statements hold:*

(a) $e, W_\psi, \tilde{W}_\psi \in \mathcal{L}_\infty$.

(b) $e \in \mathcal{S}(\mu_F, \xi_2)$. *Moreover, in the case where* $\mu_F = 0, \xi_2 = 0$, *we have that*

$$\lim_{t\to\infty} e(t) = 0$$

(c) *If* $|W_\psi^*| < \mathcal{M}_\psi$ *and* $\varphi \in PE$, *then parameter estimation error* $\tilde{W}_\psi$ *converges to the residual set*

$$\mathcal{D} \triangleq \{\tilde{W}_\psi : |\tilde{W}_\psi| \le \alpha_4\bar{\mu}_F + \alpha_5\bar{\xi}_2 + \alpha_6\sigma_0^2\}$$

where $\alpha_4, \alpha_5, \alpha_5$ *are positive constants and* $\bar{\mu}_F \triangleq \sup_t |\mu_F(t)|^2$, $\bar{\xi}_1 \triangleq \sup_t |\xi_1(t)|^2$.

VI Simulations

In order to test our theoretical results, we performed simulations of a robot manipulator moving on a constraint surface. The simulation programs were written in C, and they run in a SUN SPARC 1+ machine.

For simplicity we considered a manipulator that consists of $n = 2$ degrees of freedom and more especially of two revolute joints whose axes are parallel. The system matrices M and C can be written as:

$$M(\theta(t),p) = \begin{pmatrix} (1,0,2\cos\theta_2)p & (0,1,\cos\theta_2)p \\ (0,1,2\cos\theta_2)p & (0,0,0)p \end{pmatrix}$$

$$C(\dot\theta(t),\theta(t),p) = \begin{pmatrix} (0,0,-\dot\theta_2\sin\theta_2)p & (0,0,-(\dot\theta_1+\dot\theta_2)\sin\theta_2)p \\ (0,0,-\dot\theta_1\sin\theta_2)p & (0,0,0)p \end{pmatrix}$$

The above mathematical model and the particular numerical values of the robot parameters has been taken from [2]. Note that no gravitational forces affect the robot dynamics.

The simulation policy was as follows: At first the unknown surface was selected. More precisely, we selected the unknown surface to be represented by a constraint equation of the form

$$\psi(\theta) = W_{\psi,1}^*S(\theta_1) + W_{\psi,2}^*S(\theta_2) + W_{\psi,3}^*S(\theta_1)^2 + W_{\psi,4}^*S(\theta_2)^2 + W_{\psi,5}^*S(\theta_1)S(\theta_2) + \epsilon\sin(\theta_1\theta_2) \quad (6.1)$$

where $W_{\psi,i}^*$ are the optimal values for $W_{\psi,i}$ - see e.g. (4.3) - and $\epsilon\sin(\theta_1,\theta_2)$ represents the modeling error term ν_ψ. The function $S(\cdot)$ is defined in (2.2); the values for α, β, γ were chosen to be equal to $2, 1$ and 1 respectively. The particular values of $W_{\psi,i}^*$ are shown in the first column of table 1. The constant ϵ was set equal to 0.01.

Once the constraint surface was selected, we created several constraint robot trajectories $(\theta(t), \dot\theta(t), \ddot\theta(t))$; this was done as follows: the joint position $\theta_1(t)$ was selected to be equal to a smooth function of time $f(t)$; substituting $\theta_1(t) = f(t)$ into $\psi(\theta(t)) = 0$ we solved this algebraic equation with respect to $\theta_2(t)$. Then by differentiating twice $\theta(t)$, we obtain a complete trajectory $(\theta(t), \dot\theta(t), \ddot\theta(t))$, which satisfies $\psi(t) = 0, \dot\psi(t) = 0, \ddot\psi(t) = 0$. Several such trajectories were created.

For each of the these trajectories, we computed the input torques and constraint forces as follows: by substituting (3.4) into (III) and (3.6) we obtain a system of three equations with three unknown variables, which are the input torques $\tau_1(t)$ and $\tau_2(t)$ and the Lagrange multiplier λ. By solving this system of equations for each time-instant we obtained the trajectories $\tau(t)$ and $\lambda(t)$; using now (3.4) we were capable of computing $F(t)$.

Thus using the above procedure, we were capable to compute all the simulated quantities for each of the constraint trajectories. The HONN (4.2) was selected as follows

$$y = W_{\psi,1}S(\theta_1) + W_{\psi,2}S(\theta_2) + W_{\psi,3}S(\theta_1)^2 + W_{\psi,4}S(\theta_2)^2 + W_{\psi,5}S(\theta_1)S(\theta_2) = 0.03$$

where $W_{\psi,i}$ denotes the estimate of $W_{\psi,i}^*$. The vector ζ_F of the HONN (5.1) used for estimation of the forces, was as follows

$$\zeta_F = [S(\theta_1), S(\theta_2), S(\dot\theta_1), \ldots, S(\tau_2), S(\theta_1)^2, S(\theta_1)S(\theta_2), \ldots, S(\tau_2)^2]^T$$

For each constraint trajectory we used the proposed learning laws for estimating $W_{\psi,i}^*$. Two different cases arise: in the first case the constraint forces were assumed to be known by the HONN estimator (see section IV) while in the second case they were assumed unknown (see section V). We employed the learning procedure for ten different constraint trajectories in both cases. In table 1, the reader can see the values for $W_{\psi,i}^*$ after the end of the learning procedure for both cases.

Table 1

i	1	2	3	4	5
$W^*_{\psi,i}$	0.1	0.1	0.01	0.01	0.01
$W_{\psi,i}(0)$	0.0	0.0	0.0	0.0	0.0
$W_{\psi,i}$ - Case 1	0.944	1.04	0.0096	0.0098	0.012
$W_{\psi,i}$ - Case 2	0.938	0.98	0.014	0.0096	0.0098

As we can see from the table 1, $W_{\psi,i}$ converged very close to the actual $W^*_{\psi,i}$, even in the case 2, where no force measurements were available. Of course, in the case where the forces were available for measurement, the convergence was better than the case where the forces were not available for measurement.

VII Conclusions

In this paper, we have designed appropriate neural network architectures which, when implemented in a robotic manipulator, make it capable of learning the characteristics of unknown constraint surfaces, even if the robot manipulator is not provided with any force sensor. The proposed architecture was shown to be stable and convergent; in fact, as **Results 1-6** state, we can always find appropriate HONN architectures that are capable of making the parameter estimation error as small as desired. We mention here that the parameter estimation error converges to - or very close to - zero, if some persistently of excitation conditions hold; however, it is well-known in parameter estimation and adaptive systems literature that such conditions are necessary and sufficient for the convergence of the parameter error. On the other hand, the practical meaning of such conditions, is that the neural architecture must be provided with sufficient information about the unknown surface in order to be capable to estimate it.

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