Control of a Mobile Robot with an On-board Arm Using Neural Networks

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Abstract

Navigation and control of autonomous mobile vehicles in uncertain manufacturing scenarios with on-board manipulator systems are currently being investigated. A systematic approach for modeling and cartesian motion control of a mobile vehicle with on-board arm is presented. A neural network based controller which feedback linearizes the composite system after the incorporation of non-holonomic constraints is considered. The feedback linearization provides an inner loop that accounts for possible motion of the on-board arm. This neural network controller exhibits learning-while functioning features instead of the traditional learning-then-control training approach. Therefore, the control action is immediate with no off line learning phase needed. The cases of maintaining a desired course and speed, following a desired cartesian trajectory and that of achieving a desired final orientation (docking angle) as the on-board arm moves to its desired orientation are considered. Simulation results are presented in order to justify the theoretical conclusion.

1. Introduction

In order to confront modern technological problems that require systems with intelligent functions such as simultaneous utilization of memory, learning, or high-level decision making in response to "fuzzy" or qualitative commands, intelligent controls is being investigated. Intelligent control should utilize cognitive theory effectively with various mathematical programming techniques. Learning is a first step toward intelligent control and would replace the human operator by making intelligent choices whenever the environment does not allow or justify the presence of a human operator. Learning has the capability of reducing the uncertainties affecting the performance of a dynamical system through on-line modeling (system identification), thereby improving the knowledge about the system so that it can be controlled more effectively.

Considerable effort is being devoted to the synthesis and analysis of control techniques for nonlinear systems that are subjected to non-holonomic constraints. Examples of such systems are wheeled mobile robots, free space manipulators [1], redundant manipulators, and robotic fingers [2]. Numerous papers have been reported in recent years on the control of mechanical systems with non-holonomic constraints [3-7]. Several papers [3-5,7] examine the control theoretic issues which pertain to both holonomic and non-holonomic constraints in a very general manner. From the theoretical point of view, the interest in the control of such systems stems from the fact that the Jacobian linearization around an equilibrium point does not yield useful results and the controllability of such systems does not imply stabilizability by using smooth feedback [4].

Motion planning and feedback control of non-holonomic systems, especially wheeled mobile robots have been extensively addressed in the past few years [3-5, 8, 9]. Studies show that despite the controllability of the mobile robot system, pure static state-feedback stabilization of the cart around a given terminal configuration which includes both position <u>and</u> orientation is impossible [4]. Therefore, feedback stabilization of only the position of the cart by employing static feedback, and/or the conditions for smooth stabilization by using such a static feedback is presented, to an m-dimensional manifold where m represents the number of non-holonomic constraints [4]. In addition, a non-holonomic system when linearized around an equilibrium point contains uncontrollable purely imaginary eigenvalues whose number is equal to the number of non-integrable constraints. However, the equilibrium solution is shown to be strongly accessible and small time locally controllable [4].

In recent years, there has been a general interest of using neural networks (NN) for designing controllers for nonlinear dynamical systems. The NN do not need the apriori knowledge of the dynamics of the system to be controlled. In addition, the parallel computing abilities of these networks naturally lend them to be a powerful tool for real time applications. Mathematical foundations of NN came through the famous Stone-Weierstrass theorem, which showed that the NN can approximate any measurable function over a compact set arbitrary well [10].

To confront all these issues head on in this paper, a novel learning scheme is investigated for a multilayer NN whose weights are tuned on-line with <u>*no explicit learning*</u>

phase needed. The weight tuning mechanisms guarantee convergence of the NN weights when initialized at nominal values even though there do not exist "ideal" weights such that the NN perfectly reconstructs a certain required function. The controller structure ensures good tracking performance, as shown through a Lyapunov's approach, so that the convergence to a stable solution is guaranteed. Finally, in contrast to adaptive control, it is not necessary to have *a priori* knowledge of the structure of the plant; this structural information is instead inferred on-line by the NN.

A mobile robot with an on-board robotic arm is very useful in uncertain manufacturing scenarios. Basic objectives for the vehicle are following a desired trajectory, a desired speed, a desired docking angle and at the same time move the on-board arm to a desired orientation while the vehicle is in motion. Even though numerous papers address the control of mobile base robots, most of them neglect the path planning, dynamics, and non-holonomic aspects of the robotic system.

In this paper we investigate the cases of maintaining a desired course and speed, following a desired Cartesian trajectory and that of achieving a desired final orientation (docking angle). An outer control loop is designed with the objective of homing the mobile base at the correct location and orientation. It gives capabilities to these autonomous vehicles to maintain a desired course and speed or track a Cartesian trajectory and home at the desired orientation at the end of the motion.

In the first part of this paper, the complete dynamics of a mobile robot with an on-board arm as presented in [6] are considered, including non-holonomic constraints. A suitable coordinate system is chosen so that the dynamics enjoy some important structural properties useful in the control of standard serial-link robot arms. Jagannathan et. al. [6] performed a feedback linearization to transform this system into a linear point-mass system in the coordinates corresponding to the control objectives under the assumption that the dynamics of the mobile base with an on-board arm system is completely known. This restrictive assumption is often violated in physical systems as the dynamics of the physical systems are not known accurately (e.g. friction). In addition, when the environment around the system is unknown (e.g. payload), the design of a feedback controller using conventional feedback linearization techniques becomes very difficult. Therefore, one of the objective of this paper is to employ neural networks for feedback linearization of the mobile base with an on-board arm as these networks do not need to know the dynamics of the nonlinear system to be controlled. The uniform ultimate boundedness of all the signals for this system is demonstrated by appropriately choosing the update laws of the neural networks. The update laws have been obtained from [11].

Traditional approaches to the control of a mobile base plus on-board arm system require the arm to be fixed during base motion, and the base to be anchored during arm motion. These requirements are unreasonable from the point of view of practical scenarios, wherein the mobile robot may transport parts automatically from one manufacturing cell to another, moving along a desired trajectory for the vehicle that may vary with the task. These trajectory errors should be small even if the arm is required to move during the base motion.

Thus, in the second part of this paper we deal with the base motion, confronting the issues of following a desired Cartesian trajectory and of homing on a desired orientation even if the on-board manipulator is moving. The complete mobile base controller has a feedback linearization inner loop using neural networks. The nonlinear neural network inner loop takes into account the non-holonomic constraints and compensates for a possibly moving on-board arm, thus achieving coordinated vehicle/arm motion.

2. Dynamical Equations of the Mobile Robot System

In order to derive the dynamical equations of the mobile robot with an on-board arm system shown in Figure 1a, we make the rolling-without-slipping assumption according to which the contact point of each wheel with the ground has zero velocity. In addition, we also consider that there is no skidding. Consider an inertial reference frame (world frame) (X_1, Y_1) as shown in Figure 1b and choose a point P along the axis of the rear driving wheels on the mobile base whose basis is (x_1, x_2). The mobile base in the (x, y) plane at point P can be described by the three variables (x, y, θ), where (x, y) denote the Cartesian position and θ describes the heading angle measured between X_1 and x_1 (see Figure 1b) respectively in the world frame. For definiteness the arm is assumed to have three links; let (q_4, q_5, q_6) be the generalized coordinates (e.g. joint angles) of the arm.

Therefore, the generalized coordinates for the composite base/robot arm system are denoted by $(x, y, \theta, q_4, q_5, q_6)$, where (x, y) describe the position and θ the heading angle of the base. In other words, the generalized coordinates in Cartesian space for the vehicle and the joint space for the arm can be represented as $(x, y, \theta, q_4, q_5, q_6)$. Note that the position (x, y) and the heading angle θ of the mobile base are not independent of each other due to the no slipping assumption. Actually, they are connected by a constraint equation.

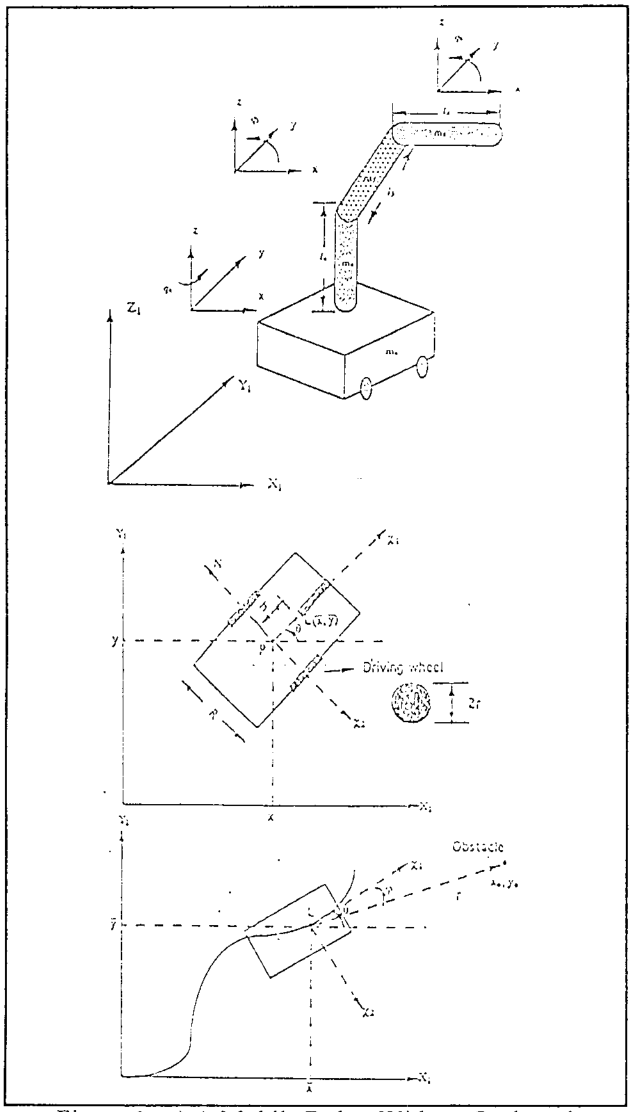

Figure 1. a) A Mobile Robot With an On-board Manipulator System and
b) Top View of Mobile Base.

The kinetic energy of the system is given by

$$K = \frac{1}{2}\sum_{i=1}^{4}\left(m_i v_{gi}^2 + I\omega_i^2\right) \tag{1}$$

where m_i is the mass of each link (vehicle is considered to be one link of mass), v_{gi} denotes the linear velocity of the center of mass, I is rotational inertia of the center of mass, and ω_i represents the angular velocity of the center of mass of each link. The dynamical equations of the composite mobile robot arm system are obtained from Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau \tag{2}$$

where $L = K - P$, is referred to as the Lagrangian, P is the potential energy, and τ represents a vector of generalized forces. Also, a no slipping constraint of the form $\psi(q, \dot{q}, t) = 0$ has to be incorporated during the development of the dynamics.

After performing analytical calculations for the specific base and arm configuration, the moment of inertia matrix $\bar{M}(q) \in R^{6x6}$ is obtained. The nonlinear coriolis and centripetal interactive (k,j) element for the m link can then be computed by using

$$\bar{V}_{mkj} = \frac{1}{2}C_{ijk}\dot{q}_i \tag{3}$$

where C_{ijk} is the Christoffel symbol [12] given by

$$C_{ijk} = \sum_{i=1}^{6}\left(\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ki}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k}\right) \tag{4}$$

where m_{kj} represents the (k,j) element of the inertia matrix. Similarly, the corresponding gravitational terms are obtained from the potential energy P as

$$\bar{G}(q) = \frac{\partial P}{\partial q} \tag{5}$$

By employing equations (1) through (5), the dynamical equations of the mobile robot system can be expressed as

$$\bar{M}(q)\ddot{q} + \bar{V}_m(q, \dot{q}) + \bar{G}(q) = b\tau + N(\psi) \tag{6}$$

where N is the normal or centripetal force (see Figure 1b) and ψ is the non-holonomic (no slipping) constraint. The complete dynamical equations are obtained by solving equation (6) with the non-holonomic constraint, where the non-holonomic constraint for the mobile robot system is derived from Figure 1b as

$$\psi = -\dot{x}sin\theta + \dot{y}cos\theta = 0 \tag{7}$$

Considering first the mobile base alone with no on-board arm, and referring to Figure 1b, the dynamical equations written in Cartesian coordinates, after the inclusion of the forces due to the non-holonomic constraint, are

$$\begin{aligned} m_v\ddot{x} &= \frac{cos\theta}{r}(\tau_1 + \tau_2) - Nsin\theta \ldots (a) \\ m_v\ddot{y} &= \frac{sin\theta}{r}(\tau_1 + \tau_2) + Nsin\theta \ldots (b) \\ I\ddot{\theta} &= \frac{R}{r}(\tau_1 - \tau_2) \ldots (c) \end{aligned} \tag{8}$$

where (τ_1, τ_2) denote the external torque inputs, r is the radius of the wheel, R is the width of the mobile base, and N is the centripetal force.

Differentiating equation (7), multiplying equations (8a) and (8b) by $-sin\theta$ and $cos\theta$ respectively, and adding, we obtain

$$N = m_v(\dot{x}cos\theta + \dot{y}sin\theta) \tag{9}$$

Rewriting equation (8) in the form given by equation (6) with the constraint included, the following equation is obtained for the mobile base with no arm:

$$\begin{bmatrix} m_v & 0 & 0 \\ 0 & m_v & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{Bmatrix} m_v sin\theta (\dot{x}cos\theta + \dot{y}sin\theta)\dot{\theta} \\ -m_v cos\theta (\dot{x}cos\theta + \dot{y}sin\theta)\dot{\theta} \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{cos\theta}{r} & \frac{cos\theta}{r} \\ \frac{sin\theta}{r} & \frac{sin\theta}{r} \\ \frac{R}{r} & -\frac{R}{r} \end{bmatrix} \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \tag{10}$$

Thus, the $\bar{b}$ matrix is found to be not a square matrix but of rank 2. However, the inertia matrix is found to be positive definite and symmetric. In order to make the $\bar{b}$ matrix square and of full rank, parametrization of x, y has to be conducted. In other words, the dynamical equations should be expressed in space coordinates (s, θ) where $s(t)$ denotes the arc length traced by the mobile base from t_o to t which results from the parametrization of x and y, that is

$$s(t) = \int_0^t |(\dot{x}(\rho)cos\theta(\rho) + \dot{y}(\rho)sin\theta(\rho))| d\rho \tag{11}$$

The resulting dynamical equations for the mobile base in space coordinates (s, θ) are given by

$$\begin{bmatrix} m_v & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{s} \\ \ddot{\theta} \end{Bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{1}{r} \\ \frac{R}{r} & -\frac{R}{r} \end{bmatrix} \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \tag{12}$$

For the case of the composite system which includes the on-board arm, the dynamical equations are obtained in a similar fashion. The centripetal force, N, due to turning for the mobile robot with an on-board manipulator is given by Equation (13).

$$\begin{aligned} N = {} & m(\dot{x}cos\theta + \dot{y}sin\theta)\dot{\theta} + C_{m1}(\dot{q}_3 + \dot{q}_4)\, cos(q_4) \\ & + (C_{m2}\dot{q}_5 + C_{m3}\dot{q}_6)\, sin(q_4) - C_{m1}(\dot{q}_3 + \dot{q}_4)^2 \\ & - C_{m1}\ddot{q}_5 - C_{m4}\ddot{q}_5\dot{q}_6 - C_{m4}(\dot{q}_5 + \dot{q}_6)\dot{q}_6 sin(q_4) \end{aligned} \tag{13}$$

where $cq_{ij} = cos(q_i + q_j)$ and $sq_{ij} = sin(q_i + q_j)$,

$$\begin{aligned} C_{m1} &= m_5\frac{l_5}{2}cq_5 + m_6\frac{l_6}{2}cq_{56} + m_6 l_5 cq_5 \\ C_{m2} &= m_5\frac{l_5}{2}sq_5 + m_6\frac{l_6}{2}sq_{56} + m_6 l_5 sq_5 \\ C_{m3} &= m_6\frac{l_6}{2}sq_{56} \\ C_{m4} &= m_6\frac{l_6}{2}cq_{56} \end{aligned} \tag{14}$$

and, m_i, l_i represent the mass and length of the i link respectively.

Equation (13) contains not only terms involving velocities of the base and the arm, but also includes the effects of link accelerations of the on-board manipulator. From equation (13) it is clear that the interactive forces between the mobile base and the arm cannot be neglected. Rearrangement of the terms after the inclusion of the centripetal force N, in the form of equation (6) and rewriting, we obtain

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) = B\tau \tag{15}$$

Equation (15) results in a non-positive definite asymmetric singular matrix $M(q)$. The implication being that when the arm effects are included in the dynamical equations of the mobile base, then these equations are no longer suitable for designing a controller using standard feedback linearization techniques because of the poor properties of $M(q)$, which can no longer be considered as an inertia matrix. Therefore, a transformation into space coordinates $(s, \theta, q_4, q_5, q_6)$ is again conducted to express the dynamics in the form of equation (15), and the following theorem results.

Theorem 2.1:
When the dynamics of the composite mobile base with an on-board arm system are expressed in space coordinates $q^T = (s, \theta, q_4, q_5, q_6)$, then the dynamics are of the form of equation (15) and the following properties are satisfied:

• Inertia Matrix:
$M(q)$ is symmetric and positive definite, that is $\mu_1 I \le M(q) \le \mu_2 I$

• Coriolis/Centripetal Matrix:
$S(q, \dot{q}) \equiv \dot{M}(q) - 2V_m(q, \dot{q})$ is a skew-symmetric matrix.

Remark: Note that the inertia matrix does not contain terms involving the arc length $s(t)$, and the dynamical equations expressed in the form of equation (15) will be considered hereafter for the control of the composite mobile base with an on-board robot system. The proof of this theorem can be found in Jagannathan et. al. [6].

3. Neural Network Controller

The neural network controller that is used in this work assumes no apriori knowledge of the dynamics of the system as well as no apriori definition of the weights. Instead, the weights are initialized to either zero or a nominal value and updated on-line dynamically. This neural network controller will feedback linearize the system to be controlled. Characteristics of this NN are that linearity in the parameters of the system and certainty equivalence is not used. This overcomes several limitations of adaptive control. The weights update is performed on-line and there is no knowledge or need for apriori tuning (training) [11].

The class of non-linear systems which can be controlled using this approach can be presented in the general form of

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\dots \\ \dot{x}_n &= f(x) + g(x)u + d \\ y &= x_1 \end{aligned} \tag{16}$$

where d is an unknown disturbance and $g(x)$ must have a lower bound for all x, i.e.

$$|g(x)| \geq \underline{g} \geq 0 \tag{17}$$

where $\underline{g}$ is a known lower bound. The mobile robot with onboard arm system is a system that can be presented in the general form of equation (16).

The neural network used has three layers (input-hidden-output), and employs a sigmoid squashing function. The approximate function reconstruction and the weight update laws for the $f(x)$ and $g(x)$ used in this NN controller are given by equations (18) and (19) respectively.

For a more thorough explanation of the variables, stability analysis and details about the specifics of this NN controller the interested reader is referred to [11].

$$\begin{aligned} \hat{f}(x) &= \hat{W}_f^T \sigma\left(\hat{V}_f^T x\right) \\ \dot{\hat{W}}_f &= M_f\left(\hat{\sigma}_f - \hat{\sigma}'_f \hat{V}_f^T x\right) r - \kappa |r| M_f \hat{W}_f \\ \dot{\hat{V}}_f &= N_f r x \hat{W}_f^T \hat{\sigma}'_f - \kappa |r| N_f \hat{V}_f \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{g}(x) &= \hat{W}_g^T \sigma\left(\hat{V}_g^T x\right) \\ \dot{\hat{W}}_g &= \begin{cases} M_g\left[\left(\hat{\sigma}_g - \hat{\sigma}'_g \hat{V}_g^T x\right) u_c r - \kappa |r| \left|u_c\right| \hat{W}_g\right] \dots I=1 \\ 0 \end{cases} \\ \dot{\hat{V}}_g &= \begin{cases} N_g u_c r x \hat{W}_g^T \hat{\sigma}'_g - \kappa |r| \left|u_c\right| N_g \hat{V}_g \dots I=1 \\ 0 \end{cases} \end{aligned} \tag{19}$$

3.1 Feedback Linearization Inner Control Loop

The main objective of this section is to appropriately design a control structure with a suitable set of generalized coordinates in order to perform a specified task. The control structure should account for the interactions between the moving base and the on-board arm. If the aim is to maintain a prescribed vehicle course and speed, then the control structure should yield space coordinates (s, θ). On the other hand, if the mobile base has to follow a prescribed path, then Cartesian coordinates (x, y) are selected for the mobile base so that path planning in Cartesian space can be performed easily. Path planning for non-holonomic systems is more involved than for the holonomic systems since the prescribed trajectory must satisfy the no-slip non-holonomic constraint. Therefore, the design of the controller in a specific coordinate system will be dependent upon the type of task to be performed.

3.2 Inner Loop Design for Maintaining Prescribed Course and Speed

The dynamic equations expressed in (15) can be written as

$$\dot{q} = f(x) + g(x)u \tag{20}$$

with $q^T = (s, \theta, q_4, q_5, q_6)$ and

$$\begin{aligned} f(x) &= -M^{-1}(q) V_m(q, \dot{q}) \dot{q} - M^{-1}(q) G(q) \\ g(x) &= M^{-1}(q) B \end{aligned} \tag{21}$$

Our objective is to determine the control torque inputs $\tau(t)$ that guarantee suitable performance in terms of the motion of the mobile base, while compensating for the interactive forces arising from a possibly moving on-board arm. There are two ways to specify a desired base motion; one may specify a desired Cartesian position $(x_d(t), y_d(t))$ or a a desired course $\theta_d(t)$ and speed $\dot{s}_d(t)$. In the case where it is desired to follow a prescribed course and speed, one may define an auxiliary input $u(t)$ using input/output feedback linearization under the assumption that the dynamics are accurately known according to

$$\tau = b^{-1}(u - f(x)) \tag{22}$$

This cancels the non-linearities to obtain the simple input-output relation of a Newtonian system where u is an input. After performing the feedback linearization, the dynamical equations of the mobile base can be expressed as

$$\dot{z} = Az + Bu \tag{23}$$

where $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $z = [\dot{s}, \theta, \dot{\theta}]^T$.

To complete the design of the base steering control law, it remains evidently to select $u(t)$ to stabilize the trajectory following error system. In this paper, we assume that the dynamics of the system are completely unknown.

4. Mobile-Base with an On-board Arm Control for Desired Course and Speed

After performing the feedback linearization, the dynamics of the composite mobile robot with on-board arm system are expressed in Newtonian form by (23), depending upon the performance objectives. It is important to realize that the dynamical effects of a possibly moving arm are automatically compensated for by the inner feedback linearization control loops. Space coordinates are employed for maintaining a desired course and speed. Therefore, this section will discuss the choice of an auxiliary outer-loop control input so that the tracking error will be asymptotically stable.

4.1 Outer Loop Design For Trajectory Following and Desired Course and Speed

A standard choice for the trajectory-following control is the PD control law

$$v = \ddot{q}_d + \bar{v} \tag{24}$$

where $\bar{v}$ denotes another auxiliary input, $[x_d, y_d]$ is the desired trajectory, and $\ddot{q}_d = [\ddot{x}_d, \ddot{y}_d]^T$ is the desired acceleration. Then, using equation (24), the error system for trajectory following is represented in the Brunosky canonical form [13] as

$$\dot{\bar{z}} = A\bar{z} + B\bar{v} \tag{25}$$

with A, B given in equation (17), and the states of the error system given by $\bar{z} = [e, \dot{e}]^T$ where $e = [\bar{x} - \bar{x}_d, \bar{y} - \bar{y}_d]^T$.

In the case of prescribing a course and speed, the error system is (25), with A and B given by equation (23) and $\bar{z} = [\dot{s} - \dot{s}_d, \theta - \theta_d, \dot{\theta} - \dot{\theta}_d]^T$ where the desired course and speed are $\theta_d, \dot{s}_d$, and $\ddot{q}_d = [\ddot{s}_d, \ddot{\theta}_d]^T$ the desired acceleration, is considered to be zero. The controller should yield suitable stability properties of the tracking error given by equation (25) in both cases.

5. Cartesian Path Planning

In this section the cartesian path planning procedure for mobile bases when the non-holonomic constraints are included is presented. The non-holonomic constraint imposes constraints on the velocities which can not be integrated. These constraints are exclusively a function of the configuration variables. This motion planning problem has been formulated by many researchers. In a control terminology this problem is reworded as finding a set of control inputs to steer a system along a trajectory from an initial to a target location. The path planning for a non-holonomic system is quite different and more complicated than that of a classical holonomic system.

The approach employed in this work is the one presented by Jagannathan et. al. [6, 14]. This approach generates an explicit representation of the path and a set of control inputs to steer the system along the path. This algorithm is a simple and a computationally efficient solution to the motion planning problem for mobile vehicles with non-holonomic constraints.

We will not elaborate on the analytical equations since they are presented in [14] except to mention the presented definition for the path planning problem. The path planning problem is defined as finding three functions $x_d(t)$, $y_d(t)$ and $\theta_d(t)$ which satisfy the following conditions:

• Non-holonomic constraint

$$-\dot{x}_d \sin(\theta_d(t)) + \dot{y}_d \sin(\theta_d(t)) = 0.0$$

• Initial conditions

$x_d(t_0) = x_{d0}, y_d(t_0) = y_{d0}$, and $\theta_d(t_0) = \theta_{d0}$

• Final (Terminal) conditions

$x_d(t_f) = x_{df}$, $y_d(t\ ) = y_{df}$, and $\theta_d(t_f) = \theta_{df}$ for a specified final time t_f.

The three functions $x_d(t)$, $y_d(t)$ and $\theta_d(t)$ are given by the equations (26 - 28) where r_{11} and r_{22} are functions of the initial and final conditions [14].

$$\theta_d(t) = \frac{t_f\theta_{d0} - t_0\theta_{df}}{t_f - t_0} + \frac{\theta_{df} - \theta_{d0}}{t_f - t_0} t \tag{26}$$

$$x_d(t) = x_{d0} + (t - t_0)\left[-r_{11}\frac{1}{4}\frac{\cos(2\theta_d(t)) - \cos(2\theta_{d0}(t))}{\theta_d(t) - \theta_{d0}} + r_{22}\left(\frac{1}{2} + \frac{1}{4}\frac{\sin(2\theta_d(t)) - \sin(2\theta_{d0}(t))}{\theta_d(t) - \theta_{d0}}\right)\right] \tag{27}$$

$$y_d(t) = y_{d0} + (t - t_0)\left[r_{11}\left(\frac{1}{2} - \frac{1}{4}\frac{\sin(2\theta_d(t)) - \sin(2\theta_{d0}(t))}{\theta_d(t) - \theta_{d0}}\right) - r_{22}\frac{1}{4}\frac{\cos(2\theta_d(t)) - \cos(2\theta_{d0}(t))}{\theta_d(t) - \theta_{d0}}\right] \tag{28}$$

6. Simulation Results

The main thrust of this work is to employ a mobile base with the on-board manipulator for uncertain manufacturing applications. Therefore, uncertainty management is very important issue. The mobile base should adapt with uncertainty in its environment without any deterioration in its performance. That is, for intelligent manufacturing applications, autonomous vehicles that travel for pick-and-place operations encounter other vehicles or objects during their course of travel. In such situations off-line path planning is doomed to fail due to the dynamical nature of the environment.

Simulation results have been conducted by employing the dynamical equations of the composite mobile robot system for both maintaining a desired course and speed, and tracking a planned Cartesian trajectory. The simulations also illustrate the ability of the algorithm to reject/compensate due to disturbances arising from the interactive forces between the mobile base and the on-board manipulator. Usually, the arm is held at the home position till the mobile base reaches the goal position. However, motion of the arm while the mobile base in operation is allowed because the feedback linearization will automatically compensate any interactive forces between the base and the arm. A typical case study of the end-effector extension while the base is in motion is also illustrated during trajectory following.

6.1 Following a Desired Course and Speed

A desired course and speed is planned for the mobile base. The mobile base was requested to move with a linear velocity of 0.5 units and a heading angle of $\pi/3$ units. During the motion, at $t = 10$ units the three degrees-of-freedom (dof) of the on-board arm were requested to move from their current positions. Also, at the same time interval the assumption that the end effector has picked up an object was included by doubling the mass of the last link.

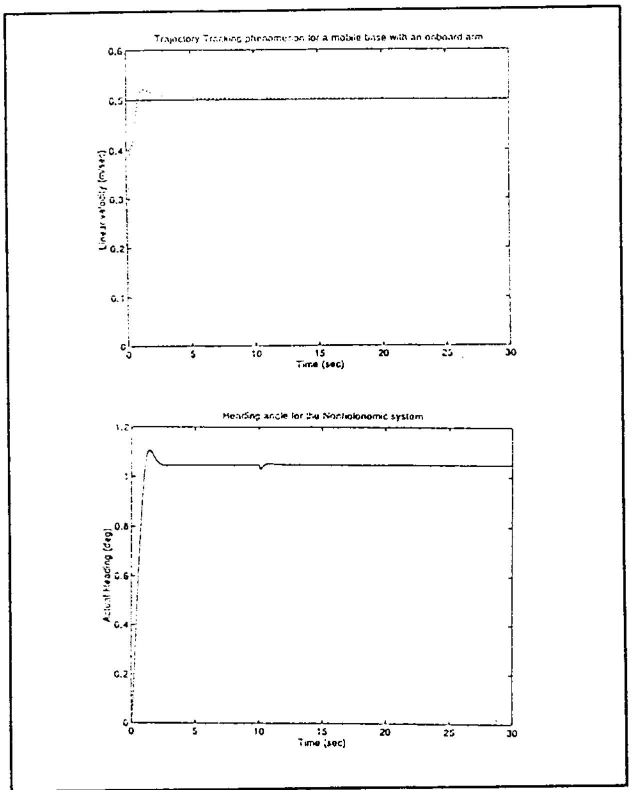

Figure 2. Mobile base motion and effect of on-board arm a) desired speed and b) heading angle

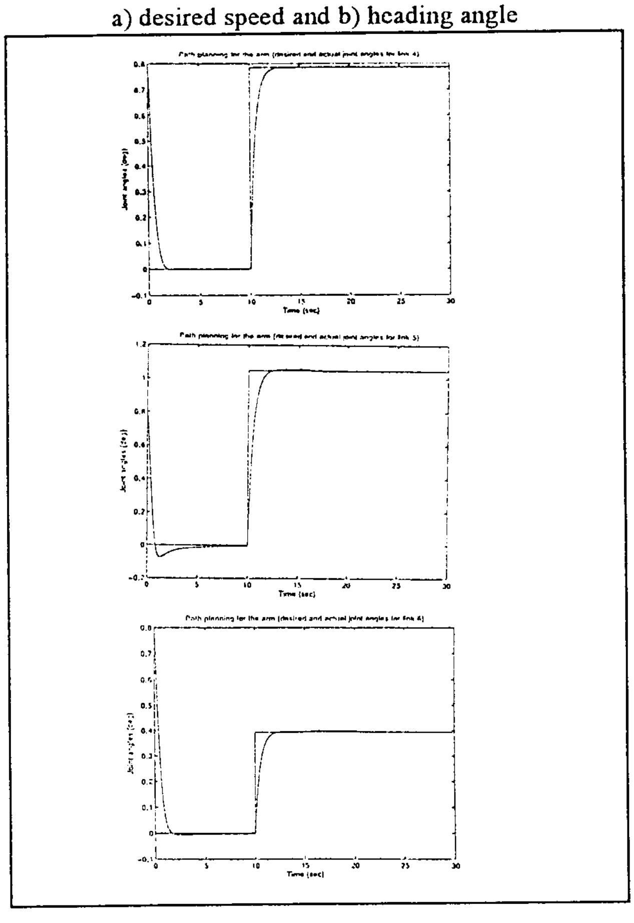

Figure 3. Motion of Links of On-board Arm a) Link 4, b) Link 5 and c) Link 6

The simulation results corroborate that the mobile base cruises with the desired speed and attains the desired heading angle, see Figure 2. In addition, when the on-board arm moved during the motion of the mobile base the interactive forces/disturbances introduced were compensated for and the base continue its motion with the desired speed and heading angle. The simulation indicates that the inner feedback linearization loop automatically compensates for the dynamical effects of the moving on-board arm and returns the mobile base to the desired heading angle.

6.2 Tracking a Desired Cartesian Trajectory

To start with, a straight line path in Cartesian coordinates is planned for the mobile base. The mobile base was initially at $(x_0, y_0) = (0, 0)$ and requested to go to the target poitn $(x_f, y_f) = (5, 5)$ with a desired heading angle of $\theta_f = \pi/4$ at the final location. The path traversed by the mobile base is illustrated with a dashed line in Figure 4a. As expected, the mobile base starts from the origin and reaches the target at (5, 5). The response in both x and y directions is given in Figure 4a. The heading angle during the motion is shown in Figure 4b. The discontinuity at $t = 10$ occurs due to the interactive forces arising from the motion of the on-board arm. This simulation result indicates that the motion of the arm, acting as a disturbance to the mobile base, is automatically compensated by the inner feedback linearization loop.

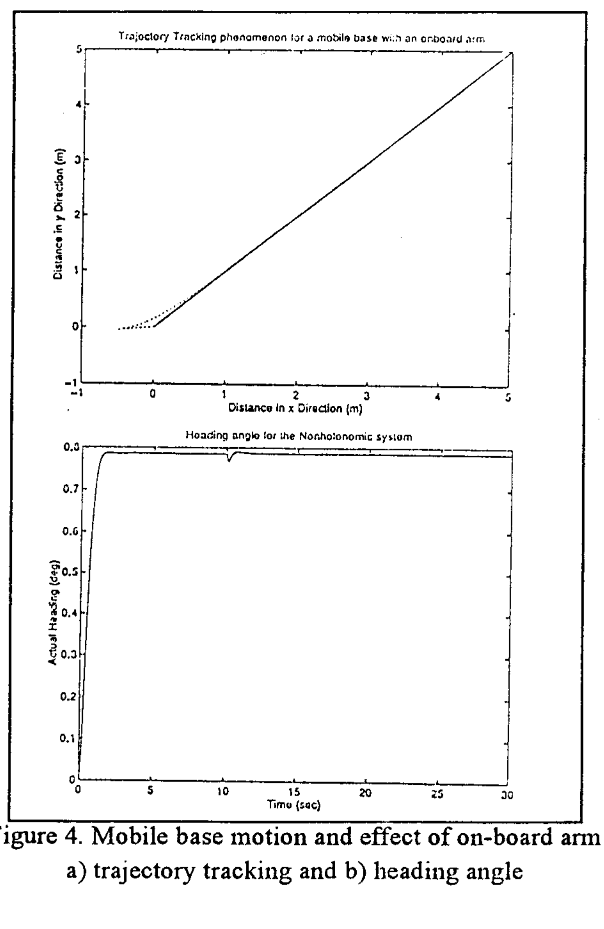

Figure 4. Mobile base motion and effect of on-board arm a) trajectory tracking and b) heading angle

The on-board arm moves from its current position to the desired location while the base is moving.

6.3 Tracking a Desired Cartesian Trajectory and Heading Angle

Another simulation run was performed with the base starting initially at $(x_0, y_0) = (0, 0)$ and finally reaching $(x_f, y_f) = (0, 5)$ with a desired heading angle of $\pi/2$ (more severe situation than that discussed in Section 6.2) while commanding the arm to move from the home position to a target location specified in Cartesian coordinates with respect to the local reference coordinates. Using the inverse kinematics transformation, the desired joint angles were calculated for the three joints of the arm as $\pi/4$ each and the arm was allowed to extend to this amount while the base was still in motion.

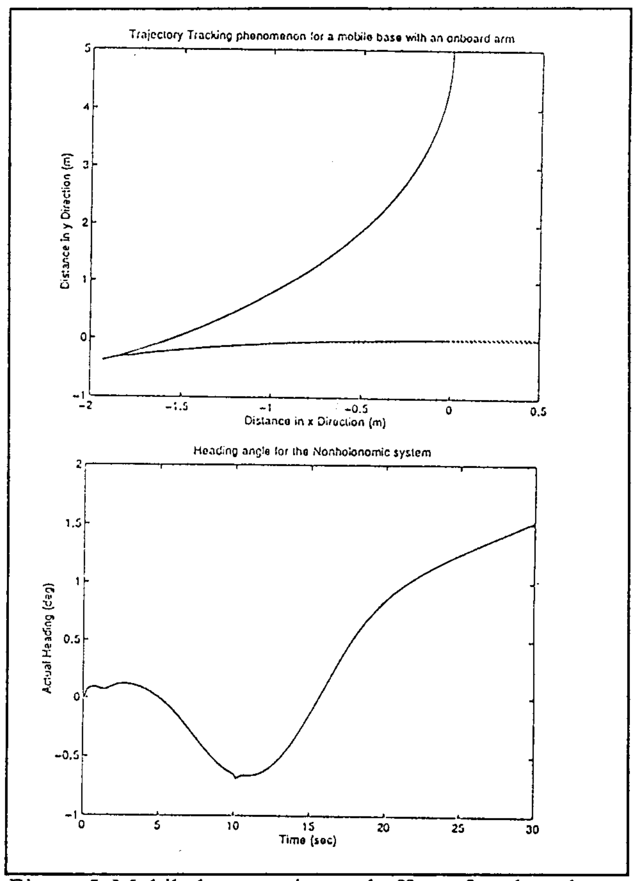

Figure 5. Mobile base motion and effect of on-board arm a) trajectory tracking and b) heading angle

Figure 5a indicates the projection in $x - y$ plane of the path traveled by the mobile base with an on-board manipulator, while holding the manipulator in the home position and with an initial orientation of the mobile base of -2 radians. Note due to the awkward initial orientation, the mobile base tries to reorient itself in the initial stages and when it achieves a satisfactory orientation, the mobile base traverses the path and reaches the goal position and orientation. The time history of the heading angle of the mobile base is shown in Figure 5b. The discontinuity observed at $t = 10$ is due to the motion of the on-board arm. This simulation result once again indicates that the motion of the arm, acting as a disturbance to the mobile base, is automatically compensated by the inner feedback linearization loop.

7. Conclusions

Navigation and control of autonomous mobile vehicles with on-board arms are currently being investigated for intelligent manufacturing applications. The problem of control for cartesian path planning, trajectory tracking and homing angle requirements of the mobile base with an on-board manipulator system, such as might be employed for pick-and-place operations has been systematically formulated. The dynamical equations of the composite mobile base and on-board robot arm system, with the inclusion of the interactive forces between the base and the arm, have been developed. In addition it was demonstrated that with a suitable choice of a coordinate system, the dynamics have some important properties useful in the control of standard serial-link arms. Furthermore, the nonlinear effects of a possibly moving arm are compensated automatically by employing a feedback linearization inner loop design.

Simulation results are then presented for both maintaining a desired course and speed and tracking a desired trajectory. The desired homing (heading) angles are substantially different between simulations in order to validate the intelligent capabilities of the mobile robot arm system. The results corroborate that the mobile base properly maintains the desired course and speed and cruises through the target location in the case of tracking a desired trajectory. Also, the results indicate that the mobile base can successfully navigate and reorient itself in order to reach the target position with the desired homing angle. The motion of the on-board manipulator, while the mobile base is moving, introduces disturbances to the mobile base. These disturbances are promptly rejected / compensated. This is an important result since it allows for the simultaneous motion of both the mobile base and the on-board manipulator.

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