Stabilization of non linear uncertain systems by dynamical closed loops

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Abstract

This paper deals with controlled systems with bounded dynamics uncertainty, in other words, systems which evolution is governed by a differential inclusion. Our aim is to derive a control law such that the system is exponentially stable, in the sense of a given candidate Lyapunov function. We show that under regularity assumptions, a set-valued evolution law of controls that guarantee the exponential stability of the system can be found explicitely. This enable us to select the minimal norm velocity of control satisfying the stability condition, and finally, to propose a single valued explicit control scheme devoted to robust stablization of non linear systems.

Introduction

This paper focuses on the control of a class of systems, whose uncertainty is deterministic and known by its bounds.

Several approaches has been proposed in the field of control of uncertain systems. Among them is the sliding mode control, introduced by Soviet mathematicians in the seventies [15], which is now used in applications [14]. Lyapunov stability theory has also been used for deriving stable feedback controllers of uncertain systems (see [13], [8], [4], [11], [10]). Knowing bounds of the system uncertainty, or the range of dynamics parameters, feedback are constructed in order to meet a Lyapunov stability condition. In the last past years, the problem of robust controller design has been addressed in considering the uncertain system dynamics as a differential inclusion (see [9]).

Our approach follows both of these ideas: We consider the evolution of the system through a set of feasible velocities (the right hand side of a differential inclusion), and we construct a feedback that guarantee the closed loop system to meet a Lyapunov stability condition. It is also important to mention that the synthesis of the control law is non parametric, in the sense that we obtain the control input value at each time, as the solution of a differential equation. The determination of the dynamics of control that allow to reject some bounded disturbances is the main purpose of this paper.

Let us consider the following uncertain system:

$$x'(t) \in F(x(t), u(t)) = f(x(t), u(t)) + \phi(x(t))B \quad (1)$$

The state of the system is denoted by $x(\cdot)$, and the control input by $u(\cdot)$. In the following, X denotes the state space, Z, the control space (X and Z are finite dimensionnal vector spaces). The unit ball of X is denoted by B.

The uncertainty around $f(x(\cdot), u(\cdot))$ is supposed to be bounded [1] by a $\mathcal{C}^1$ real valued positive function $\phi(x(\cdot))$. In the following, f will be supposed to be a $\mathcal{C}^1$ map. Under these assumptions, existence of solutions to (1) is guaranteed[2].

We also suppose that 0 is an equilibrium of inclusion (1) (i.e. $0 \in F(0,0)$). Let us consider a $\mathcal{C}^2$ candidate Lyapunov function $V : X \to \mathbf{R}+$ satisfying $V(0) = 0$, and $\forall x \neq 0$, $V(x) > 0$. Our goal is to design a robust control law that can stabilize system (1) around zero, in the sense that we want V to satisfy the following property: For all solutions $(x(\cdot), x(0) = x_0)$ of inclusion (1):

$$\forall t \geq 0, \qquad V(x(t)) \leq w(t)$$

where $w(\cdot)$ is the solution of $w' = \Psi(w)$, $w(0) = V(x(0))$, and Ψ is a strictely increasing function. An instance of such function is $\Psi(w) = aw - b$ with $a > 0$ and $b > 0$, which leads to ultimate boundedness condition to the ball of center 0 and radius $\frac{b}{a}$ (in setting $V(x) = \|x\|^2$):

$$\|x(t)\|^2 - \frac{b}{a} \leq e^{-at}(\|x_0\|^2 - \frac{b}{a})$$

[1] Let us point out that the control enter the dynamics of the system with no uncertainty.

[2] Let us mention that $x(\cdot)$ is a solution of (1) if it is an absolutely continuous function satisfying (1) almost everywhere. Solutions to $x'(t) \in F(x(t))$ exists if F is upper semicontinous with compact and convex values.

1 Exponential stabilization of uncertain systems

Definition 1 *We shall say that $V : X \to \mathbf{R}^+$ is a* Lyapunov candidate function *if it is a $\mathcal{C}^2$-map and satisfy:*

- *$V(0) = 0$, and $\quad \forall x \neq 0$, $V(x) > 0$.*
- *There exist two class $\mathcal{K}$ functions[3] $a(.)$ and $b(.)$ such that*

$$b(\|x\|) \leq V(x) \leq a(\|x\|)$$

Definition 2 *Let be $V : X \to \mathbf{R}^+$ a Lyapunov candidate function, G a upper semicontinuous set-valued map with convex and compact values, and Ψ, a strictely increasing function.*

*V is said to be a Ψ-*universal Lyapunov function *associated to the differential inclusion $x'(t) \in G(x(t))$ if its satisfies*

$$\operatorname*{Sup}_{w \in G(x)} \{\langle V'(x), w\rangle\} + \Psi(V(x)) \leq 0$$

We know that if G is a Lipschitz set valued map, defining a differential inclusion

$$x'(t) \in G(x(t))$$

satisfying $0 \in G(0)$), and if V is an universal lyapunov function associated to the set-valued map G, then all solutions starting at $x(0) = x_0$ satisfy $lim_{t \to +\infty} x(t) = 0$.

Let us consider the set valued map $U(x) \subset Z$ associated to system (1):

$$U(x) := \{v, \sup_{w \in F(x,v)} \langle V'(x), w\rangle + \Psi(V(x)) \leq 0\} \quad (2)$$

[3] A function ϕ is said to be a class $\mathcal{K}$ function if it is continous, non decreasing, and such that $\phi(0) = 0$.

which is nothing else than the set of controls enabling system (1) to admit the function V as an universal Lyapunov function.

To find controls such that V be a universal Lyapunov function associated to system (1) amounts to look for solutions of the following system under constraints:

$$\begin{cases} x'(t) & \in & F(x(t), u(t) \\ u(t) & \in & U(x(t)) \end{cases} \tag{3}$$

Robustness of the control law will be guaranteed if *all solutions* of the differential inclusion (1) satisfy the constraint $u(t) \in U(x(t))$.

Definition 3 *Let X be a finite dimensional vector space and K a subset of X. We shall say that a solution $x(\cdot)$, $x(0) = x_0 \in K$ of the differential inclusion $x'(t) \in G(x(t))$ is* viable *in the subset $K \subset X$ if for all time $t \geq t_0$, $x(t) \in K$.*

A subset $K \subset X$ will be said to be invariant *under the map G if all solutions of the differential inclusion $x'(t) \in G(x(t))$ starting from $x_0 \in K$ are viable in the set K.*

Actually, invariance of a set K under a set valued feedback map can be characterized by a geometrical condition, thanks to the invariance theorem[1]. This geometrical condition involve the Bouligand's tangent cone which is defined hereafter.

Definition 4 *Let be $K \in X$, a subset of a normed vector space X, and $x \in \overline{K}$. The* Bouligand's tangent cone *of K at point x, denoted as $\mathrm{T}_K(x)$ is:*

$$\left\{ v \in X \mid \liminf_{h \to 0^+} \frac{d_K(x + hv)}{h} = 0 \right\}$$

where d_K is the distance to K function:

$$d_K(x) = \inf_{y \in K} \|x - y\|$$

Theorem 1 (Invariance theorem) *Let us suppose that $G : X \rightarrow X$ is a Lipschitz map, with compact values, and let us consider a closed subset $K \subset X$. Then the two following statements are equivalent:*

1. *For all $x_0 \in K$ all solutions of the differential inclusion*

$$x'(t) \in G(x(t))$$

are viable in K.

2. $\forall x \in K, \quad F(x) \subset \mathrm{T}_K(x)$

In order to apply the invariance theorem to the system (3) (for the set Graph(U)), we introduce a auxilliary evolution law of the controls which amount to *set a bound on the velocities of the controls*:

$$u'(t) \in \rho(x(t), u(t))B$$

where ρ is a non negative Lipschitz function, and let us consider the new system of differential inclusions:

$$\begin{cases} x'(t) & \in & F(x(t), u(t) \\ u'(t) & \in & \rho(x(t), u(t))B \end{cases} \tag{4}$$

submitted to the constraint:

$$(x(t), u(t)) \in \text{Graph}(U)$$

2 Robust dynamical closed loops

The problem is now to derive a control law such that Graph(U) be invariant under the map

$$F \times \rho B : (x, u) \rightsquigarrow (F(x, u), \rho(x, u)B)$$

This can be done by applying the invariance theorem to the system (4).

As f and ϕ are $\mathcal{C}^1$ maps and ρ is Lipschitz, the set salued map $F(x,u) = f(x,u) + \phi(x)B$ is Lipschitz. Moreover, it has compact values.

The set valued map U can be computed explicitely:

$$U(x) = \{v \in Z, \quad a(x,u) \leq 0\}$$

where

$$a(x,u) = \langle V'(x), f(x,u)\rangle + \phi(x)\,\|V'(x)\| + \Psi(V(x))$$

We observe that the graph of U is defined by inequality constraints and that $a(\cdot,\cdot)$ is a $\mathcal{C}^1$ map. Moreover, as f is continuous, Graph (U) is a closed subset of $X \times Z$. Then, the invariance theorem applies: Graph (U) enjoys the invariance property (with regards to the set valued map $F \times \rho B$) if and only if $\forall (x,u) \in$ Graph (U)

$$(F(x,u), \rho(x,u)B) \subset \mathrm{T}_{\mathrm{Graph}(U)}(x,u)$$

Let us recall that the tangent cone to the graph of a set valued map is the graph of a set valued map, called the contingent derivative (see [3] for details) defined as follows:

Definition 5 *Let be X and Y, two normed spaces. Let be $F : X \rightarrow Y$, a set valued map, and $y \in F(x)$. The* contingent derivative *of F at point (x,y) is the set valued map denoted by $\mathcal{D}F(x,y) : X \rightarrow Y$ and defined by:*

$$\mathrm{Graph}\,(\mathcal{D}F(x,y)) = \mathrm{T}_{\mathrm{Graph}(F)}(x,y)$$

If $F = f$ is a single valued map, we set

$$\mathcal{D}f(x) = \mathcal{D}f(x, f(x))$$

Actually, the graph of the contingent derivative of a map F at point (x,y) is the tangent cone to the graph of F at point (x,y).

According to the definition of contingent derivatives, the invariance condition becomes: $\forall (x,u) \in$ Graph (U),

$$\rho(x,u)B \subset \bigcap_{y \in F(x,u)} \mathcal{D}U(x,u)(y) \qquad (5)$$

Under condition (5), all solutions of the system of differential inclusions (4) are viable in Graph (U).

Therefore, the couple $(x(t), u(t))$ remains viable in Graph (U) whenever it satisfies the following system of differential inclusions[4]

$$\begin{array}{lcl} x'(t) & \in & F(x(t), u(t)) \\ u'(t) & \in & \bigcap_{y \in F(x(t),u(t))} \mathcal{D}U(x(t), u(t))(y) \end{array} \qquad (6)$$

3 Explicit robust dynamical closed loop

In order to implement the robust dynamical closed loop, we need to find an explicit formulation of the set valued right hand side of inclusion (6). Let us recall at the onset the following results about tangent cones of sets defined by inequality constraints:

Proposition 1 *Let us consider a $\mathcal{C}^1$ map $g : X \rightarrow \mathbf{R}$, and suppose that*

$$K = \{x \in X, \quad g(x) \leq 0\}$$

Then $\mathrm{T}_K(x)$ is:

$$\begin{cases} \{v \in X, \quad \langle g'(x), v\rangle \leq 0\} & \text{if} \quad g(x) = 0 \\ X \quad \text{if} \quad g(x) < 0 \\ \emptyset \quad \text{if} \quad g(x) > 0 \end{cases}$$

We observe that the graph of $\mathrm{T}_K(\cdot)$ may not be closed. Moreover, if we compute the tangent cone to Graph (U) at point (x,u), and use it in inclusion (6) a discontinous right hand side will then appear, which is non desirable in our framework. In order to avoid this kind of non regularity of the feedback map, we prefer

[4] This scheme is nothing else than a dynamical closed loop (according to the terminology adopted in [1]).

to consider the subset of $\mathrm{T}_K(x)$ denoted by $\mathrm{T}_K^{\diamond}(x)$, and defined as follows:

$$\mathrm{T}_K^{\diamond}(x) = \{v \in X, \quad \alpha g(x) + \langle g'(x), v\rangle \leq 0\}$$

where α is a positive constant. We know that

$$\mathrm{T}_K^{\diamond}(x) \subset \mathrm{T}_K(x)$$

Moreover, when g is a $\mathcal{C}^1$ map, the graph of $\mathrm{T}_K^{\diamond}(\cdot)$ is closed. We can then consider the subset of the contingent derivative of the set valued map U denoted by: $\mathcal{D}U^{\diamond}(x,u)(y)$ and defined by:

$$\{v \in X, \alpha a(x,u) + \langle a'(x,u), (y,v)\rangle \leq 0\}$$

and replace the dynamical closed loop defined by the second inclusion of (6) by

$$u'(t) \in \bigcap_{y \in F(x(t),u(t))} \mathcal{D}U^{\diamond}(x(t), u(t))(y)$$

Let us now give an explicit form of this inclusion. As $a(\cdot,\cdot)$ is not differentiable at $x=0$, we overcome this difficulty in considering a set-valued feedback map whose graph is contained in Graph (U):

$$U^1 := \{u \in Z, \quad a^1(x,u) \leq 0\}$$

with

$$\begin{array}{rl} a^1(x,u) = & \langle V'(x), f(x,u)\rangle + \Phi(x) N(\|V'(x)\|) \\ & +\Psi(V(x)) \end{array}$$

The function N is supposed to verify $N(\|V'(x)\|) > \|V'(x)\|$ and to be such that $x \rightsquigarrow N(\|V'(x)\|)$ be a $\mathcal{C}^1$ function[5].

Let us set

$$\begin{array}{rcl} a_x^1(x,u) & = & \frac{\partial a^1(x,u)}{\partial x} \\ a_u^1(x,u) & = & \frac{\partial a^1(x,u)}{\partial u} \end{array}$$

[5]For instance the function $N(s) = 1 + \frac{1+s^2}{1+s}$ do satisfy these requirements.

Then, according to proposition (1) we have

$$\begin{array}{rl} \mathcal{D}U^{1\diamond}(x,u)(y) = & \{v \in Z, \quad \langle a_u^1(x,u), v\rangle \leq \\ & \langle a_x^1(x,u), y\rangle - \alpha a^1(x,u)\} \end{array}$$

So, we have the following equivalent statements:

$$v \in \bigcap_{r \in B} \mathcal{D}U^{1\diamond}(x,u)(f(x,u) + r\phi(x))$$

$$\Updownarrow$$

$$\langle a_u^1(x,u), v\rangle \leq \begin{array}{l} \inf_{r \in B} \langle a_x^1(x,u), f(x,u) + \phi(x) r\rangle \\ -\alpha a^1(x,u) \end{array}$$

$$\Updownarrow$$

$$\begin{array}{rl} \langle a_u^1(x,u), v\rangle \leq & \langle a_x^1(x,u), f(x,u)\rangle \\ & -\phi(x)\left\|a_u^1(x,u)\right\| - \alpha a^1(x,u) \end{array}$$

Finally, the evolution law of controls providing invariant state-control solutions to system (4) is

$$\left\{\begin{array}{lcl} x' & \in & f(x,u) + \phi(x) B \\ \langle u', a_u^1(x,u)\rangle & \leq & \langle a_x^1(x,u), f(x,u)\rangle \\ & & -\phi(x)\left\|a_x^1(x,u)\right\| \\ & & -\alpha a^1(x,u) \end{array}\right. \qquad (7)$$

We can now state the following theorem:

Theorem 2 *Let us consider $f : X \times Z \to X$ a $\mathcal{C}^1$-map and a control system defined by*

$$x'(t) \in f(x(t), u(t)) + \phi(x(t)) B$$

where $\phi(\cdot) : x \to \mathbf{R}$ is a $\mathcal{C}^1$ non-negative map. Let us suppose that $0 \in f(0,0) + \phi(0) B$. Let us consider $V : X \to \mathbf{R}$ a Lyapunov candidate function.

Then, whenever there exists solutions to the system of differential inclusions (7), they verify the universal Lyapunov property:

$$\sup_{w \in f(x(t),u(t)) + \phi(x(t))B} \langle V'(x(t)), w\rangle \leq -\Psi(V(x(t)))$$

In other words, whenever the controls are solutions to the differential inclusion (7), the solution $u(\cdot)$ is a robust stabilizing control map of system (1).

In order to implement this scheme, we can select a solution among all the controls providing the robust stabilization of the system, through a selection procedure. This is explained in the next section.

4 Heavy robust dynamical closed loops

In order to implement the robust dynamical closed loop, one can select at each time, the minimal value of the norm of $u'(t)$ (following an idea proposed in [2]). This is consistent with the aim of *minimizing chattering*[6] of the control system. The selection process can be done explicitely.

Let us set

$$d(x,u) = \begin{array}{l}\left\langle a_x^1(x,u), f(x,u)\right\rangle - \phi(x)\left\|a_x^1(x,u)\right\| \\ -\alpha a^1(x,u)\end{array}$$

Consider the minimization problem

$$\begin{array}{l}\text{Min} \ \ \frac{1}{2}\|w\|^2 \\ \left\langle w, a_u^1(x,u\right\rangle \leq d(x,u)\end{array}$$

which is nothing else than a quadratic minimisation problem under linear inequalities constraints. Its solution can be found thanks to Kuhn and Tucker optimality conditions[7]

$$\begin{array}{lll}\bar{w} = \frac{a_u^1(x,u)}{\|a_u^1(x,u)\|^2}d(x,u) & \text{if} & d(x,u) < 0 \\ \bar{w} = 0 & \text{if} & d(x,u) \geq 0\end{array} \quad (8)$$

[6]As it is done using the so-called suction control technique in [14].

[7]When $d(x,u) < 0$ and $\frac{|d(x,u)|}{\|a_u^1(x,u)\|} \geq \rho(x,u)$, no absolutely continuous solution to differential inclusion (7) exists.

Finally, the heavy robust dynamical closed loop control scheme is:

$$\left\{\begin{array}{lll} x'(t) & \in & f(x(t),u(t)) + \phi(x(t))B_X \\ u'(t) & = & u'_\star \end{array}\right. \quad (9)$$

where

$$u'_\star = \left\{\begin{array}{ll} \frac{a_u^1(x,u)}{\|a_u^1(x,u)\|^2}d(x,u) & \text{if} \ \ d(x,u) < 0 \ \ \text{and} \\ & \frac{|d(x,u)|}{\|a_u^1(x,u)\|} \geq \rho(x,u) \\ 0 & \text{if} \ \ d(x,u) \geq 0 \end{array}\right.$$

Actually, this control scheme can be viewed as a closed loop feedback, in the sense that the control law is given by a time varying function of the system state.

Under the assuptions of theorem (2), the solution of (9) is asymptotically and exponentially stable, whenever the perturbations remains bounded by the state function $\phi(x)$.

An illustrative example Consider the system $x'(t) = x(t) - u(t)$, where both x and u are scalar functions of time. No uncertainty is considered, for sake of clarity. Let us choose as Lyapunov candidate function $V(x) := 1/2x^2$, and $\Psi(s) = s$ (which means that we want the system to be exponentially stable at zero). In that case, Graph $(U) = \{u, \ \ \sigma x^2 - ux \leq 0\}$ (see figure (1)) where $\sigma = 3/2$. First, we observe that *slow solutions*, constructed in picking the minimal norm control in Graph (U), i.e. the solution of

$$\underset{u \in \text{Graph}(U)}{\text{Min}} \quad \{\|u\|\}$$

is the linear control law $u(t) = \sigma x(t)$.

Let us now look for *dynamical feedbacks*, i.e. absolutely continous feedbacks, solution of a differential equation with measurable right hand side. The scheme presented above aims to keep the couple (x,u) in the set Graph (U)

(for exponential stability in our framework). The derivative of control is chosen in such a way that invariance conditions (which involves the contingent cone to Graph (U) at each point (x, u) of Graph (U)) holds true. Unfortunately, when we consider the tangential condition at $x = 0$, for $u > 0$, we observe that $x' \geq 0 \Longrightarrow u \leq 0$. Therefore *all solutions starting from the set* $\{(0, u), u > 0\}$ *leaves* Graph (U), *whatever the control value is*, and whatever the derivative of control u' is[8]. Let us now consider the system:

$$\left\{ \begin{array}{lcl} x' & = & x - u \\ |u'| & \leq & \rho \end{array} \right. \tag{10}$$

One way to avoid the emptyness of the set of *absolutely continuous* feedback regulating exponentially stable solutions is to consider the *viability kernel* (see [1], [12]) of Graph (U), which is the set of initial conditions (x_0, u_0) of system (10), such that there exist at least one solution viable in Graph (U). For our simple example, this set can be explicitely computed. In case $x \geq 0$, it is the set of (x, u) satisfying $u \geq \sigma x$ and $u \leq g_\rho(x)$ where $u = g_\rho(x) \Longleftrightarrow x = \rho/(e^{-u/\rho} + u/\rho - 1) \approx u^2/2\rho$. All solutions starting from Graph (U), but outside the viability kernel leave Graph (U) in finite time.

In figure (1), we consider the system $x' = x - u$ with $V(x) = x^2/2$, and $\Psi(s) = s$. We bound the chaterring of control by $\rho = 1$ (i.e. $|u'| \leq 1$), and we approximate the viability kernel of Graph (U) with respect to the dynamics $x - u, [-1, +1]$ (denoted by $\mathrm{Viab}_{x-u,[-1,+1]}$ (Graph (U))) by: $u \geq 3x/2$ and $u \leq \sqrt{2x}$.

We applied the control scheme proposed in theorem (2), but in replacing Graph (U) by $\mathrm{Viab}_{x-u,[-1,+1]}$ (Graph (U)). Indeed, doing so insure existence of absolutely continous controls regulating the system exponentially towards zero (this is also motivated by the fact that some trajectories starting in Graph (U)) leaves Graph (U) whatever the control is). Therefore, existence of solution to equation (9) is guaranteed in this case.

Computer simulation results are plotted in figure (1). The viability kernel is plotted (area between the curve $x \rightsquigarrow 3x/2$ and $x \rightsquigarrow \sqrt{2x}$. A trajectory starting from $\mathrm{Viab}_{x-u,[-1,+1]}$ (Graph (U)) (initial condition is $(x(0), u(0)) = (0.6, 0.9)$) is plotted when u' is computed using equation (9). We observe that this trajectory is viable within Graph (U), and that the constraint $|u'| \leq 1$ is respected (see (1)).

We plotted in figure (1) a solution starting *outside* the viability kernel $\mathrm{Viab}_{x-u,[-1,+1]}$ (Graph (U)) (initial conditions are $(x(0), u(0)) = (1, 3/2)$). We can check that the solution to equation (9) starting from that point, is such that $|u'(t_0)| > 1$. Therefore the scheme (9) do not admit any solution starting from that point.

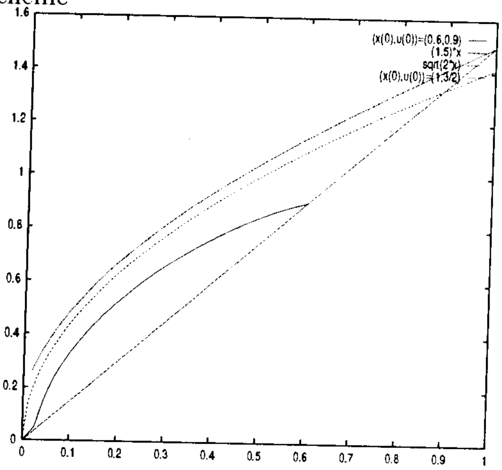
Conclusion

The method we proposed consist to keep the couple (state, control) within the set of stabilizing controls Graph (U) in minimizing chaterring. The theorem we proposed provides viable evolution laws of the control within the set Graph (U), whenever they exist. In fact, the ilustrative example shows that there is no reasons for Graph (U) to be a viability kernel with respect to the state dynamics and the proposed control dynamics (in the sense that some initial conditions may lead to solution which leaves Graph (U) whatever the control action is).

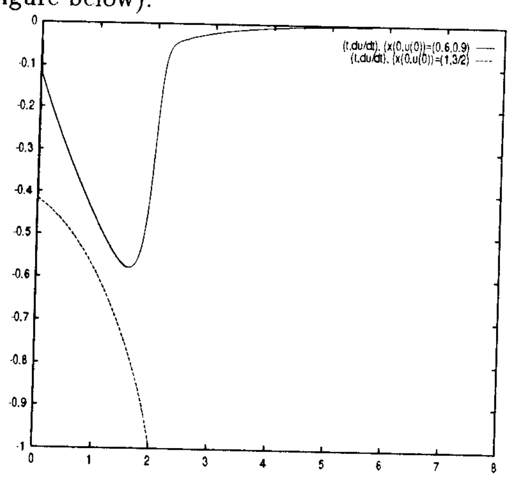
Therefore, in order to insure existence of at least one absolutely continous solution (x, u) to inclusion (7), it necessary to use the viability kernel $\mathrm{Viab}_{F \times \rho B_Z}$ (Graph (U)) as set

[8] Actually, the solution leaves Graph (U) because we do not accept discontinous feebacks

Figure 1: Simulation of the proposed control scheme



The first trajectory (originating from $(0.6, 0.9)$) remains viable within $\mathrm{Viab}_{x-u,[-1,+1]}(\mathrm{Graph}\,(U))$ when scheme (9) is applied. The second trajectory (starting from $(1, 3/2)$, which do not belong to the viability kernel of Graph (U) is not viable, because u' do not remains bounded by 1 (see figure below).



One can check on this plot that the solution starting from $(1, 3/2)$ requires derivative of control greater than the specified chattering bound.

of constraints, rather than the set Graph (U). This is also means that we need to know the viability kernel of Graph (U). For non linear systems this set can be only computed by numerical approximations [12].

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