A New Model used for Suboptimal Solutions in Control of Manufacturing Processes

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Abstract

In this paper, a constrained discrete-time nonlinear state space model is proposed for modeling the dynamics of general manufacturing system. The model is an extension/modification of the one proposed by Rovithakis and Christodoulou [RC94]. The proposed system possesses certain advantages over the existing ones, such as generallity and ability to solve optimal policy tasks using optimal control methods as well as ability to model both deterministic and stochastic processing times; moreover its stability and robustness properties can be easily identified. The optimal control of such a system is shown to be equivalent to the solution of a system of static nonlinear equations. Despite the fact that no close form solution is obtained, it is believed that the resulted control policy obtained from the solution of the aforementioned system of equations is locally optimal. Simulations performed on a very simple manufacturing process are also presented.

I Introduction

In this paper[1] we are dealing with the problem of modeling and control of manufacturing processes. The manufacturing processes that we are dealing with are composed of production machines and stroing units (buffers); several types of objects are processed by the nachines and are stored - temporarily - to the buffers. Once the processing of an object has been finished, the object is transfered to the buffer and the machine recieves the next object. Despite the fact that there exist many methods for dealing with such manufacturing systems (e.g. Discrete Event Systems, Petri Nets, Queueing Systems, etc), the fact is that there exists no general and effective method for dealing with such systems. One of the most important problems is that there exists no general mathematical model for modeling the dynamics taking place in arbitrary manufactur-

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ing systems; for instance discrete event systems fail when the system processing times are modeled as random variables, etc. A second system that arises is the system stability and robustness: the optimal control policies computed may fail when the system dynamics are modified a little bit, or when the system configuration is slightly modified. On the other hand, state space models (used mainly to describe continuous state dynamics) are known to overcome the two aforementioned problems since they can model both deterministic and stochastic processes and moreover their stability and robustness can be identified usign e.g. Lyapunov stability methods.

In this paper, a constrained discrete-time nonlinear state space model is proposed for modeling the dynamics of general manufacturing system. The model is an extension/modification of the one proposed by Rovithakis and Christodoulou [RC94]. The proposed system possesses certain advantages over the existing ones, such as generallity and ability to solve optimal policy tasks using optimal control methods as well as ability to model both deterministic and stochastic processing times; moreover its stability and robustness properties can be easily identified. The optimal control of such a system is shown to be equivalent to the solution of a system of static nonlinear equations. Despite the fact that no close form solution is obtained, it is believed that the resulted control policy obtained from the solution of the aforementioned system of equations is locally optimal. Simulations performed on a very simple manufacturing process are also presented.

II The Mathematical Model

Let us consider a manufacturing system composed of N machines and M buffers; there are K different types of objects that are processed by the machines and stored - temporarily - to the buffers. Every machine is connected to one *input buffer* and to one or more *output buffers.* The input buffer feeds feeds the objects to to the machine, and the machine feeds the - processed - objects to the output buffers. It is no loss of generallity to assume that a machine is not capable of processing more than one objects in parallel. Also, since a machine(say the i-th one) is capable of processing more than one different types of objects, we decompose such a machine into K_i distinct sub-machines each of whom is responsible of processing one and only one type of objects; here $K_i \leq K$ denotes the total number of different objects that can be processed by the i-th machine. Of course, if one of these sub=machines is processing an object the rest $K_i - 1$ must be idle. Thus, we have transformed the manufacturing system into an equivalent one with $Q = \sum_{i=1}^{N} K_i$ machines. In this equivalent system each submachine has one and only one output buffer, although there maybe buffers that are output buffers for more than one sub-machines. Let us consider the i-th actual machine and suppose that O_i denote the subset of $\{1, 2, \ldots, K\}$ satisfying the following condition: the integer j belongs to O_i if and only if the i-th machine is permitted to process the j-th type of object. Obviously the cardinality of O_i is equal to K_i.

Consider now the i-th actual machine and its input buffer and let I_i be a subset of $\{1, 2, \ldots, N\}$ such that the integer j belongs to I_i if and only if the input buffer to the i-th actual machine is an output buffer for the j-th actual machine. It is not difficult for someone to see that the subsets O_i and I_i describe completely the topology of the manufacturing process. Let now $u_{ij}(t)$ be equal to 1 if the object of j-th type is processed by the i-th actual machine at the t-th time-instant and be zero, otherwise. Let also $\chi_{ij}(t)$ be equal to 1 if, at the t-th time-instant, the i-th machine outputs an object of the j-th type and be zero, otherwise. It is not difficult to see that the following conditions must be satisfied

$$\sum_{j \in O_i} \chi_{ij}(t) \in \{0, 1\} \; \forall t \tag{1}$$

$$\sum_{j \in O_i} u_{ij}(t) \in \{0, 1\} \; \forall t \tag{2}$$

Let now $y_{ij}(t)$ denote the total number of objects of the j-th type that are stored in the input buffer of the i-th actual machine. Then, we can easily see that $y_{ij}(t)$ satisfies the following difference equation

$$y_{ij}(t+1) = y_{ij}(t) + \sum_{k \in I_i} \chi_{kj}(t) - u_{ij}(t) \tag{3}$$

Note that a machine requires a time-interval in order to process an object. Following the methodology of Rovithakis and Christodoulou [RC94] we model the process taking place in a machine by the following linear difference equation

$$x_{ij}(t+1) = (1 - a_{ij})x_{ij}(t) + a_{ij}u_{ij}(t) \tag{4}$$

where a_{ij} is a positive scalar. Since u_{ij} belongs to $\{0, 1\}$, it can be easily verified that x_{ij} converges to $\{0, 1\}$. Following the methodology of [RC94] the state x_{ij} models the state of the j-th submachinbe of the i-th actual machine; when x_{ij} is equal (or very close) to zero then the submachine is idle, while when it is equal (or very close) to one, the submachine has completed the processing of an object. Note that since (4) is a linear difference equation x_{ij} requires infinite time in order to converge to $\{0, 1\}$. However, if we define the function $H(\cdot)$ to be such that $H(x) = 0$ iff $x \leq 1 - \epsilon$ and $H(x) = 1$ otherwise, we can easily see that $H(x_{ij})$ converges to $\{0, 1\}$ in finite time. The scalar a_{ij} controls the time required for a submachine to process an object, which is assumed to be constant; however, if the processing time is modeled as a stochastic process then a_{ij} may represent the statistical average of the actual a_{ij}. Letting now $\chi_{ij} = H(x_{ij})$ we may *rewrite (3) as follows:*

$$y_{ij}(t+1) = y_{ij}(t) + \sum_{k \in I_i} H(x_{kj}(t)) - u_{ij}(t) \quad (3)$$

If z denotes the vector whose entries are the states x_{ij} and y_{ij} and u denotes the vector whose entries are the u_{ij} then the difference equations (3) and (4) can be rewritten into the following compact form

$$z(t+1) = Az(t) + FH(z(t)) + Bu(t) \qquad (5)$$

where A, F, B are matrices of appropriate dimensions; the entries of the matrix A are 0, 1, or $1 + a_{ij}$, the entries of F are 0 or 1, and the entries of the matrix B are 0, 1 or a_{ij}; the i-th entry of the $(Q + M)$-dimensional vector $H(z)$ is equal to $H(z_i)$.

Although system (5) describes completely the evolution of the state vector z of the manufacturing process, there are physical constraints that are not involved in the state space equations (5). A first family of constraints have to do with the fact that u_i must be either zero or one and the fact that at most one of the sub-machines that correspond to an actual machine must be in operation. These constraints can be written as

$$u_{ij} \in \{0, 1\} \qquad (6)$$

together with the constraints (1) and (2). A second family of constraints arises by the fact that in most practical cases, the buffers have limited capacity; if C_i denotes the maximum capacity of *the input buffer of the i-th actual machine, then* we have that

$$0 \leq \sum_{j \in I_i} y_{ij} \leq C_i \qquad (7)$$

A third family of constraints arises due to the fact that the control input cannot be equal to one if there is no object of the j-th type in the input buffer. Such a constraint can be described as follows

$$u_{ij}(t) \leq y_{ij}(t) \qquad (8)$$

At last, we have to introduce a constraint that restricts the control $u_{ij}(t)$ to be zero whenever the i-th machine is processing an object. Such a constraint is described by

$$u_{ij}(t) \leq \prod_{j \in O_i} (1 - G(x_{ij}(t))) \qquad (9)$$

where $G(x) = 1$ if x is greater than a small positive ϵ^*, and $G(x) = 0$ otherwise.

We close this section, by mentioning that the constraints (1), (2) and (6) can be rewritten as follows

$$\sum_{j \in O_i} \chi_{ij}(t)(\sum_{j \in O_i} \chi_{ij}(t) - 1) = 0 \qquad (10)$$

$$\sum_{j \in O_i} u_{ij}(t)(\sum_{j \in O_i} u_{ij}(t) - 1) = 0 \qquad (11)$$

$$u_{ij}(t)(1 - u_{ij}(t)) = 0 \qquad (12)$$

III Control of the Manufacturing Process

Once the manufacturing system has been designed, it is of primary interest to construct a control policy that optimizes the manufacturing system's performance. Usually the purpose of the control policy is to produce a certain number of objects in the minimum time. Let v_j denote the total number of objects of the j-th type we wish the system to produce. Also, let W_j denote a subset of $\{1,\ldots,N\}$ which satisfies $i \in W_j$ if the i-th machine is an output machine for the j-th type of object, i.e. after an object of the j-th type is processed by the i-th machine its production is supposed to be complete and the object leaves the system.

Using the above definition it can be easily seen that a control policy is optimal if it minimizes the following criterion

$$J = \sum_{i\in\{1,\ldots,K\}} \left(\sum_{j\in W_i} y_{ij}(T) - v_i)^2 \right) + \sum_{t=1}^{T} 1$$

where T denotes the final time, i.e., the time-instant at which the purpose has been achieved. By taking into account the dynamics (5) and the constraints described in the previous section, we see that the optimization problem is a *constraint optimal control* problem. Thus, optimal control strategies such a dynamic control, or optimal control using the Pontryagin's maximum principle may be used in order to construct the optimal control policy. Since, the dynamic programming suffers from the *curse of dimensionality* drawback, we prefer to use the Pontryagin's maximum principle (PMP). Before we proceed to the application of such a principle, we transform the constraint optimization problem into an unconstraint one. This can be done by modifying J as follows

$$\begin{aligned}
J' &= \sum_{i\in\{1,\ldots,K\}} \left(\sum_{j\in W_i} y_{ij}(T) - v_i)^2 \right) + \sum_{t=1}^{T} 1 \\
&\quad + \sum_{t=1}^{T} \Gamma \left[\sum_{j\in O_i} \chi_{ij}(t)(\sum_{j\in O_i} \chi_{ij}(t) - 1) \right] \\
&\quad + \sum_{t=1}^{T} \Gamma \left[\sum_{j\in O_i} u_{ij}(t)(\sum_{j\in O_i} u_{ij}(t) - 1) \right] \\
&\quad + \sum_{t=1}^{T} \Gamma \left[u_{ij}(t)(1 - u_{ij}(t)) \right] \\
&\quad + \sum_{t=1}^{T} R \left[\sum_{j\in I_i} y_{ij}(t) - C_i \right] \\
&\quad + \sum_{t=1}^{T} R \left[- \sum_{j\in I_i} y_{ij}(t) \right] \\
&\quad + \sum_{t=1}^{T} R \left[u_{ij}(t) - y_{ij}(t) \right] \\
&\quad + \sum_{t=1}^{T} R \left[u_{ij}(t) \prod_{j\in O_i} (1 - G(x_{ij}(t))) \right] \\
&= \sum_{i\in\{1,\ldots,K\}} \left(\sum_{j\in W_i} y_{ij}(T) - v_i)^2 \right) + \sum_{t=1}^{T} 1 \\
&\quad + \sum_{t=1}^{T} F(z(t), u(t))
\end{aligned}$$

where $R(\cdot)$ is a positive smooth function satisfying $R(x) = 0$ iff $x \leq 0$ and $R(x)$ is a rapidly increasing function for $x > 0$, and $\Gamma(\cdot)$ is a positive smooth function that satisfies $\Gamma(0) = 0$ and $\Gamma(x)$ rapidly grows to $+\infty$ whenever x goes far from zero.

We are now ready to apply PMP. According to PMP, we define the Hamiltonian

$$\mathcal{H}(t) = t + F(z(t), u(t) + \lambda^T(t+1) f(z(t), u(t))$$

where λ is a vector denoting the *costate* of the system and $f(\cdot)$ states for the RHS of (5). Then the optimal solution is obtained from the solution of the following system of equations

$$
\begin{aligned}
\frac{\partial \mathcal{H}}{\partial u}(t) &= 0 \\
\lambda^T(t) &= \frac{\partial \mathcal{H}}{\partial z}(t) \qquad (13) \\
\lambda^T(T) &= \frac{\partial \phi}{\partial z}(T)
\end{aligned}
$$

where $\phi(T) = \sum_{i\in\{1,\ldots,K\}} \left(\sum_{j\in W_i} y_{ij}(T) - v_i)^2\right)$. There are mainly two problems when one tries to solve (13). The first of them is that the function $H(\cdot)$ is not differentiable everywhere, and thus discontinuities will appear in the RHS of the first part of (13). However, such a problem can be easily overcame by replacing $H(\cdot)$ by a smooth function $h(\cdot)$ which approximates $H(\cdot)$. In a similar manner we may replace the discontinuous function $G(\cdot)$ by a smooth one. The second problem is that x_{ij} is a fictitius state and it does not correspond to a physical state; there $x_{ij}(t)$ is not observable to the system coordinator. Such a problem can be overcame by selecting appropriately $x_{ij}(0)$ and by estimating $x_{ij}(t)$ from the difference equation (4).

Despite the simplicity of the manufacturing system dynamics, it is very difficult to obtain the exact solution of the above optimization problem. In fact, if one tires to solve (13), he will observe that at each time step, a system of static nonlinear equations has to be solved; thus numerical methods have to be applied in order to solve such a system of equations.

IV Simulations

In order to test the applicability of our model, we performed simulations of a very simple manufacturing system. Such a manufacturing system consists of two machines connected in series, and three buffers, the first of whom is an input buffer to first machine, the second is an output buffer of the first machine and an input buffer for the second one, and the third is an output buffer for the second machine. The first and third buffers are assumed to have infinite capacity and the capacity of the second buffer was set equal to 2. There are three objects that are processed by the system. The times needed for the machines to process the objects were as follows: the first machine requires 5, 3 and 5 time units in order to process objects of type 1, 2, and 3 respectively, while the second machine requires 4, 5 and 7 time units in order to process objects of type 1, 2, and 3 respectively. The purpose of the system is to produce 10 parts of each type of object in minimum time. We have simulated the algorithm proposed in the previous section. The resulted policy is shown in figure 1; the first row corresponds to 1st machine, while the second row corresponds to the second one. The filled rectangles correspond to the objects that are processed by the machines and their length is proportional to the processing time required. Note that, although the proposed policy is not optimal, it is quite satisfactory and no constraint is violated. We believe that such a policy is locally optimal.

V Conclusions

In this paper, we have proposed a constrained nonlinear state space model for modeling the dynamics of a manufacturing system. The approach makes use of the idea proposed in [RC94], to model the machines as stable LTI filters. We have shown that the problem of optimal control of the manufacturing system is equivalent to an optimal control problem of a nonlinear state space system. The Pontryagin's maximum principle is then used in order to construct the optimal policy; unfortunately no closed form solution has been derived.

References

[RC94] . Rovithakis and M.A. Christodoulou, "A preliminary result on factory dynamics modeling," *IEEE CDC 95*, submitted.