

# Robust Backstepping Control of Induction Motors Using Neural Networks

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Research supported by NSF grant IRI-9216545

## Nomenclature

$R_s$	stator resistance
$R_r$	rotor resistance
$i_s$	stator current
$\psi_s$	stator flux linkage
$i_r$	rotor current
$\psi_r$	rotor flux linkage
$\psi_d$	magnitude of rotor flux linkage
$u$	voltage input
$\omega$	angular speed
$\rho$	angle between the flux linkages in the rotor
$n_p$	number of pole pairs
$\delta$	angle of rotation
$L_s$	stator inductance
$L_r$	rotor inductance
$M$	mutual inductance
$\sigma$	$1 - M^2/(L_s L_r)$
$J$	rotor inertia
$T_L$	load torque
$T$	electric motor torque
$(\bullet)_d, (\bullet)_q$	$(\bullet)$ in the (d,q) frame
$(\bullet)_a, (\bullet)_b$	$(\bullet)$ in the (a,b) frame
$(\bullet)^f$	reference trajectory
$\hat{(\bullet)}$	estimate of $(\bullet)$
$\sigma_1, \sigma_2$	sliding variables
$(\bullet)_N$	nominal values of $(\bullet)$
$\alpha$	$R_r/L_r$
$\beta$	$M/((\sigma L_s L_r)$
$\gamma$	$M^2 R_r / (\sigma L_s L_r^2) + R_s / (\sigma L_s)$
$\mu$	$n_p M / (J L_r)$

## Abstract

In this paper, we present a new robust control technique for induction motors using neural networks (NN). New tuning schemes are proposed which *can guarantee the boundedness of tracking error and weight updates*. A main advantage of our method is that we *do not require the regression matrix*, so that no preliminary dynamical analysis is needed. Another salient feature of our NN approach is that *no off-line learning phase is needed*. Full state feedback is needed for implementation. Load torque and rotor resistance can be unknown but bounded.

## 1. Introduction

The induction motor is quite popular for fixed-speed applications. Since rotor currents are induced, no brushes and slip rings are needed. It is maintenance free, simple in operation, rugged and generally less expensive than either DC or synchronous motors [2]. On the other hand its model is much more complicated than other machines and because of this, it is considered as "**the benchmark problem in nonlinear systems**" by the Editorial Board of IEEE Transactions on Automatic Control in comments on a recent paper [16].

There are many approaches to induction motor control such as [1] [16] [21] and references therein.

Neural networks (NN) have been applied to system identification [4] or identification-based control [3] [18], little about the use of NN in direct closed-loop controllers that yield guaranteed performance. Some results on the application of NN to robot are presented in [9] [12] [14] [15]. Recently many NN controllers have been proposed for various control applications [19] [20].

Problems that remain to be addressed in NN research include ad hoc controller structures and the inability to guarantee satisfactory performance of the system in terms of small tracking errors and bounded NN weights. Uncertainty on how to initialize the NN weights leads to the necessity for "preliminary off-line tuning" [3] [5].

In the recent adaptive and robust control literature there has been a tremendous amount of activity on a special control scheme known as "backstepping" [10] [11]. When used under some mild assumptions, many existing robust and adaptive control techniques can be extended to wide classes of applications. Dawson *et al.* [7] have applied such techniques to various kind of robotic control schemes with the inclusion of motor dynamics. A major problem with backstepping approaches is that certain functions must be "linear in the unknown parameters", and some very tedious analysis is needed to determine a "regression matrix".

In this paper, we will use neural nets at each stage of the backstepping procedure to estimate certain nonlinear functions. This means that *linearity in the parameters is not needed, and no regression matrix need be found*. Thus, a major problem with backstepping is cured. Compared with other NN approaches, the NN weights here are tuned on-line, with *no learning phase required*. Most importantly, we can *guarantee the boundedness of tracking error and weight updates*. When compared with

adaptive controllers, we do not require persistent excitation conditions.

The paper is organized as follows. We will first describe a fifth-order model in Marino's paper [16] and then an equivalent field oriented model in Section 2. In Section 3 we will develop a robust NN control scheme without the PE requirement. We first treat certain signals in the system as fictitious control signals to a simpler subsystem. Two-layer NN is used in this stage to design the fictitious controllers. Then we apply a second two-layer NN to robustly realize the fictitious NN signals. Our method is modular in nature and hence can be applied to other nonlinear systems with similar structures. Load torque and rotor resistance can be unknown but bounded. Full state is needed for implementation. Theory as well as simulations in Section 3.2 and 3.3 show that our NN control scheme works very well.

## 2. Models of Induction Motor

### 2.1 Induction Motor Model

A fifth-order model, which includes rotor dynamics, under the assumptions of equal mutual inductance and linear magnetic circuit, is given by Marino *et al.* [16]

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{n_p M}{J L_r} (\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) - \frac{T_L}{J}, \\ \frac{d\psi_{ra}}{dt} &= -\frac{R_r}{L_r} \psi_{ra} - n_p \omega \psi_{rb} + \frac{R_r}{L_r} M i_{sa}, \\ \frac{d\psi_{rb}}{dt} &= -\frac{R_r}{L_r} \psi_{rb} + n_p \omega \psi_{ra} + \frac{R_r}{L_r} M i_{sb}, \\ \frac{di_{sa}}{dt} &= \frac{M R_r}{\sigma L_s L_r^2} \psi_{ra} + \frac{n_p M}{\sigma L_s L_r} \omega \psi_{rb} \left( \frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \right) i_{sa} + \frac{1}{\sigma L_s} u_{sa}, \\ \frac{di_{sb}}{dt} &= \frac{M R_r}{\sigma L_s L_r^2} \psi_{rb} - \frac{n_p M}{\sigma L_s L_r} \omega \psi_{ra} \left( \frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \right) i_{sb} + \frac{1}{\sigma L_s} u_{sb}. \end{aligned} \quad (2.1)$$

$i$ ,  $\psi$ ,  $u_s$  denote current, flux linkage and stator voltage input to the machine. The meaning of other symbols in (2.1) are listed in the Nomenclature.

### 2.2 Field Oriented Model

This technique was introduced by Blaschke [1]. It involves a transformation from the stator fixed frame (a,b) to a frame (d,q), which rotates along the flux vector ( $\psi_a, \psi_b$ ). The transformations between currents and flux magnitudes in different frames are given by

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \rho & \sin \rho \\ -\sin \rho & \cos \rho \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}, \quad (2.2)$$

$$\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} = \begin{bmatrix} \cos \rho & \sin \rho \\ -\sin \rho & \cos \rho \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \end{bmatrix} \quad (2.3)$$

where

$$\rho = \tan^{-1} \left( \frac{\psi_b}{\psi_a} \right).$$

A state transformation was suggested by Blaschke [1] as

$$\begin{aligned} \omega &= \omega, & \psi_d &= \sqrt{\psi_a^2 + \psi_b^2}, \\ \rho &= \tan^{-1} \left( \frac{\psi_b}{\psi_a} \right), & i_d &= \frac{\psi_a i_a + \psi_b i_b}{\psi_d}, \\ i_q &= \frac{\psi_a i_b - \psi_b i_a}{\psi_d}. \end{aligned} \quad (2.4)$$

Substituting (2.2), (2.3) and (2.4) into (2.1) yields the field oriented model

$$\frac{d\omega}{dt} = -\frac{T_L}{J} + \mu \psi_d i_q, \quad (2.5a)$$

$$\frac{d\psi_d}{dt} = -\alpha \psi_d + \alpha M i_d, \quad (2.5b)$$

$$\frac{di_d}{dt} = -\gamma i_d + \alpha \beta \psi_d + n_p \omega i_q + \alpha M \frac{i_q^2}{\psi_d} + \frac{1}{\sigma L_s} u_d, \quad (2.5c)$$

$$\frac{di_q}{dt} = -\gamma i_q - n_p \omega \beta \psi_d - n_p \omega i_d - \alpha M \frac{i_d i_q}{\psi_d} + \frac{1}{\sigma L_s} u_q, \quad (2.5d)$$

$$\frac{d\rho}{dt} = n_p \omega + \alpha M \frac{i_q}{\psi_d} \quad (2.5e)$$

with

$$\begin{aligned} \alpha &= R_r / L_r, & \gamma &= M^2 R_r / (\sigma L_s L_r^2) + R_s / (\sigma L_s), \\ \beta &= M / (\sigma L_s L_r), & \mu &= n_p M / (J L_r). \end{aligned}$$

Since  $R_r$  is unknown,  $\alpha$ ,  $\gamma$  are also unknown. However,  $\beta$  and  $\mu$  are known. Blaschke [1] also developed a feedback linearization plus PI controller to control (2.5). Marino *et al.* [16] went a step further to use adaptive input-output decoupling technique to tackle the control problem. In the next section, we will make use of a special structure of the above model to perform our NN controller design.

## 3. Robust Control of Induction Motor Using Neural Networks

By looking at (2.5), one will notice that there exists a very special structure in it. (2.5a) to (2.5d) can be considered as two nonlinear systems in cascade.  $i_d$ ,  $i_q$  can be treated as the outputs of subsystem (2.5c)-(2.5d). At the same time, they can also be treated as fictitious inputs to the subsystem (2.5a)-(2.5b). It is this special structure that we will exploit in our NN controller design. Before we go into the details, two general assumptions and some mathematical preliminaries are needed:

*Assumption 1:* The reference trajectories  $\omega^r$  and  $\psi_d^r$  are differentiable and bounded.

*Assumption 2:* The load torque  $T_L$  and rotor resistance  $R_r$  are unknown but bounded.

### 3.1 Preliminaries

#### Neural Networks

Given an input vector  $x$  in  $R^{N_1}$ , a three-layer neural net (NN) has an output given by

$$y_i = \sum_{j=1}^{N_2} \left[ w_{ij} \sigma \left[ \sum_{k=1}^{N_1} v_{jk} x_k + \theta_{vj} \right] + \theta_{wi} \right] \quad i = 1, \dots, N_3 \quad (3.1)$$

with  $\sigma(\cdot)$  the activation function,  $v_{jk}$  the first-to-second layer interconnection weights, and  $w_{ij}$  the second-to-third layer interconnection weights.  $\theta_{vm}$ ,  $\theta_{wm}$ ,  $m = 1, 2, \dots$ , are called the threshold offsets and the number of neurons in layer  $l$  is  $N_l$ , with  $N_2$  the number of hidden-layer neurons. A three layer neural network is shown in Fig. 3.1.

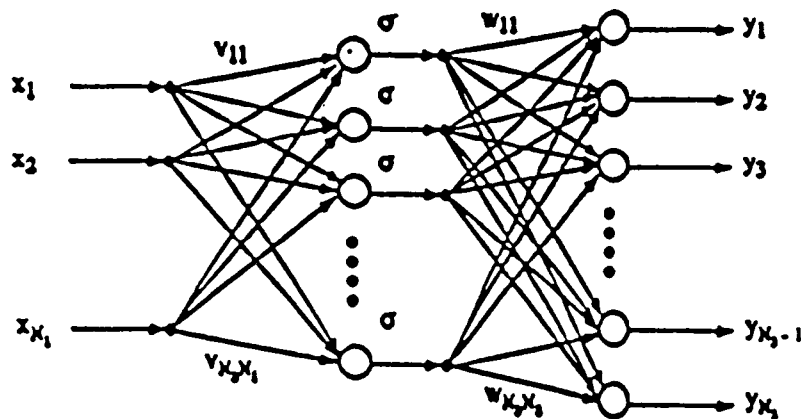


Fig. 3.1 Three layer Neural Network

The NN equation may be conveniently expressed in matrix format by defining  $x = [x_0 \ x_1 \ \dots \ x_{N_1}]^T$ ,  $y = [y_1 \ y_2 \ \dots \ y_{N_2}]^T$ , and weight matrices  $W^T = [w_{ij}]$ ,  $V^T = [v_{jk}]$ . Including  $x_0 = 1$  in  $x$  allows one to include the threshold vector  $[\theta_{v1} \ \theta_{v2} \ \dots \ \theta_{vN_1}]$  as the first column of  $V^T$ . Hence the NN outputs can be compactly written as

$$y = W^T \sigma(V^T x) \quad (3.2)$$

where, if  $z = [z_1 \ z_2 \ \dots]^T$  is a vector we define  $\sigma(z) = [\sigma(z_1) \ \sigma(z_2) \ \dots]^T$ . Including 1 as a first term in the vector  $\sigma(V^T x)$  allows one to incorporate the thresholds  $\theta_{w_j}$  as the first column of  $W^T$ . Any tuning of  $W$  and  $V$  then includes tuning of the thresholds as well. Basically the functional approximation property of NN is used in this paper [6] [8].

### Stability of Systems

Consider the following nonlinear system

$$\dot{x} = f(x, u, t), \quad y = h(x, t). \quad (3.3)$$

with state  $x(t) \in \mathbb{R}^n$ . We say the solution is uniformly ultimately bounded (UUB) if there exists a compact set  $U \subset \mathbb{R}^n$  such that for all  $x(t_0) = x_0 \in U$ , there exists an  $\varepsilon > 0$  and a number  $T(\varepsilon, x_0)$  such that  $\|x(t)\| < \varepsilon$  for all  $t \geq t_0 + T$ .

### 3.2 Robust Backstepping Controller Design

In the remainder of the paper we consider the NN for the case of fixed first-layer weights  $V$ . The use of two-layer NN with no backstepping has been applied to several occasions [15] [20]. Here, we consider general basis functions  $\phi(x)$  and propose various weight tuning algorithms with and without the requirement of persistent excitation (PE) on certain signals in the system. The tuning algorithms generally need persistent excitation for suitable performance. A modified tuning algorithm is then proposed to make the NN controller *robust* so that PE is not needed.

Define  $\phi(x) = \sigma(V^T x)$  so that the net output is

$$y = W^T \phi(x). \quad (3.4)$$

Then, for suitable NN approximation properties,  $\phi(x)$  must satisfy some conditions. Take  $N_1 = n$ ,  $N_2 = m$ .

#### Definition 3.1 [20]

Let  $S$  be a compact simply connected set of  $\mathbb{R}^n$ , and  $\phi(x): S \rightarrow \mathbb{R}^{N_2}$  be integrable and bounded. Then  $\phi$  is said to provide a **basis** for  $C^m(s)$  if

1. A constant function on  $S$  can be expressed as (3.4) for finite  $N_2$ .
2. The functional range of NN (3.4) is dense in  $C^m(s)$  for countable  $N_2$ .

The issue of selecting  $\sigma$  and  $V$  so that  $\phi$  provides a basis, as well as the further issue of selecting  $N_2$  for a given  $S \subset \mathbb{R}^n$  and  $\varepsilon_N$ , are topics of current research. One possibility is to use the radial basis functions.

#### 3.2.1 Controller Structure

We first treat  $i_d, i_q$  as the ideal fictitious control signals for a subsystem consisting of (2.5a) - (2.5b). Then we use a second 2-layer NN to realize these fictitious signals.

**Step 1:** Selection of desired  $i_d$  and  $i_q$  to control subsystem (2.5a) and (2.5b)

Our control objective is to regulate the rotor speed and the magnetic flux magnitude. Denote  $\omega^r$  and  $\psi_d^r$  as the desired reference levels of  $\omega$  and  $\psi_d$  respectively. First, we rewrite (2.5a-b) as

$$\frac{de_1}{dt} = -\frac{T_L}{J} + \mu \psi_d i_q, \quad (3.5a)$$

$$\frac{de_2}{dt} = -\alpha \psi_d + \alpha M i_d \quad (3.5b)$$

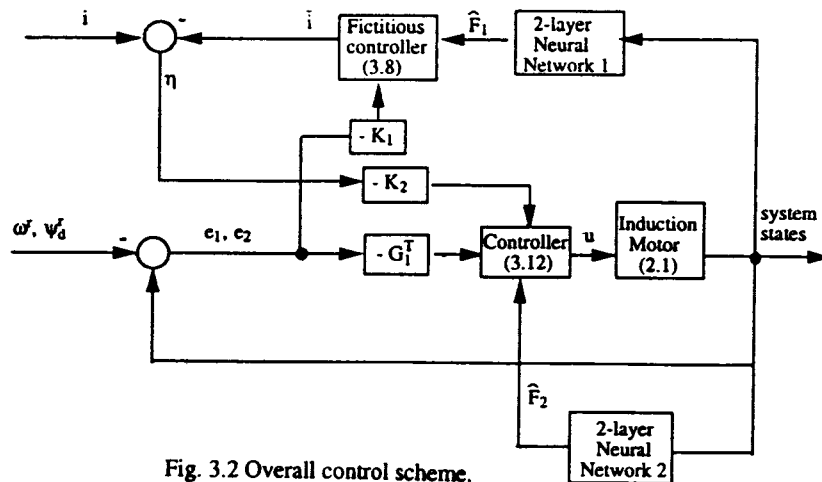
where

$$e_1 = \omega - \omega^r, \quad e_2 = \psi_d - \psi_d^r.$$

Since  $\alpha$  in (3.5b) is unknown, this will cause some difficulties of using the results of backstepping NN [13]. The difficulty is due to that fact that the stability analysis will be much more complicated if  $\alpha$  is unknown. To alleviate this difficulty, we apply the following trick. Dividing both sides of (3.5b) by  $\alpha M$  yields

$$\frac{1}{\alpha M} \frac{de_2}{dt} = -\frac{1}{M} \psi_d + i_d. \quad (3.6)$$

Now the coefficient of  $i_d$  is unity and known. Then we can express (3.5) as



$$\dot{D}e = F_1 + G_1 i \quad (3.7)$$

where

$$\begin{aligned} \mathbf{e} &= \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}, & \mathbf{i} &= \begin{bmatrix} \mathbf{i}_q \\ \mathbf{i}_d \end{bmatrix}, & \mathbf{D} &= \begin{bmatrix} 1 & 0 \\ 0 & 1/(\alpha M) \end{bmatrix}, \\ \mathbf{F}_1 &= \begin{bmatrix} -\mathbf{T}_L/J \\ 0 \end{bmatrix}, & \mathbf{G}_1 &= \begin{bmatrix} \mu \Psi_d & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned}$$

It should be noted that  $G_1$  is exactly known and invertible. By treating  $\bar{i}$  as a fictitious input, we design a controller for the ideal  $\bar{i}$  as

$$\bar{i} = G_1^{-1}[-\hat{F}_1 - E - K_1 e], \quad K_1 > 0 \quad (3.8)$$

with  $K_1$  a design parameter,  $E = [0 \quad \psi_d/M]^T$ ,  $\hat{F}_1$  the estimate of  $F_1$ . Substituting (3.8) into (3.7) gives

$$De = F_1 - \hat{F}_1 - K_1 e + G_1 \eta \quad (3.9)$$

where  $\eta = \hat{F}_1 - F_1$ . The form of  $\hat{F}_1$  will be discussed in the next section. The usual approach is by assuming  $F_1$  to be linear parametrizable (LP) so that standard adaptive control can be used in this stage [10] [11]. As we will see in a moment, we will use a two-layer NN method to approximate  $F_1$ . The advantage is that no linearity in the unknown parameters is needed and no regression matrix need be found.

**Step 2:** Realization of the desired signals in (3.8).

In order to achieve the desirable result in Step 1, i.e. the ideal fictitious control signal in (3.8), we need to find the error dynamics of  $\eta$  which is defined as

$$\eta = i - \bar{i}. \quad (3.10)$$

Differentiating (3.10) and using the dynamics in (2.5) yields

$$\dot{\eta} = F_2 + F_2^{\text{known}} + G_2 u \quad (3.11)$$

where

$$F_2^{\text{known}} = \begin{bmatrix} -\gamma_{iq} - n_p \omega \beta \psi_d - n_p \omega i_d \\ -\gamma_{id} + n_p \omega i_q \end{bmatrix} + G_1^{-1} \hat{F}_1,$$

$$F_2 = \begin{bmatrix} -\alpha M \frac{i_q i_d}{\psi_d} \\ \alpha \beta \psi_d + \alpha M \frac{i_q^2}{\psi_d} \end{bmatrix} + \ddot{G}_1^{-1} (\hat{F}_1 + K_1 e) + G_1^{-1} K_1 D^{-1} [F_1 - \hat{F}_1 - K_1 e + G_1 \eta],$$

$$G_2 = \frac{1}{\sigma I} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

To make  $\eta$  as small as possible, the following control  $u$  is chosen as

$$\mathbf{u} = \mathbf{G}_2^{-1}[-\mathbf{F}_2^{\text{known}} - \hat{\mathbf{F}}_2 - \mathbf{K}_2\boldsymbol{\eta} - \mathbf{G}_1^T\mathbf{e}]. \quad (3.12)$$

Note that  $\hat{F}_2$  is the estimate of the unknown function  $F_2$ . Similar to Step 1, the usual way of design is to assume  $F_2$  to be LP. However, in our controller design here, we will use another two-layer NN to approximate  $F_2$  which means no regression matrix requirement is needed. Also note that a term  $-G_1^T \epsilon$  is added in (3.12) which is necessary to cancel the effect of  $G_1 \eta$  in (3.9) so that we will be able to prove the closed-loop stability.

### Step 3: Closed-loop Stability Analysis and Weight Tuning Algorithms

We will perform a detailed treatment of stability and performance analysis of two weight tuning algorithms in the next few sections. The overall control scheme is shown in Fig. 3.2.

Finally  $u = [u_q \ u_d]^T$  is related to the actual control  $u_a, u_b$  in (2.1) through the following relation

$$\begin{bmatrix} u_a \\ u_b \end{bmatrix} = \frac{1}{\psi_d} \begin{bmatrix} \psi_a & -\psi_b \\ \psi_b & \psi_a \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix}. \quad (3.13)$$

### 3.2.2 Bounding Assumptions

Assume that the nonlinear functions  $F_1, F_2$  in (3.7) and (3.11) can be represented as two-layer neural nets for some constant "ideal" weights  $W$ , i.e.

$$\mathbf{F}_1 = \mathbf{W}_1^T \phi_1 + \varepsilon_1, \quad \|\varepsilon_1\| < \varepsilon_{1N} = \text{constant}$$

$$\mathbf{F}_2 = \mathbf{W}_2^T \phi_2 + \varepsilon_2, \quad \|\varepsilon_2\| < \varepsilon_{2N} = \text{constant} \quad (3.14)$$

where  $\phi_1(x)$ ,  $\phi_2(x)$ , provide suitable basis functions. The net reconstruction error  $\varepsilon_i(x)$  is bounded by a known constant  $\varepsilon_{iN}$ ,  $i=1, 2$ .

Define the NN functional estimate of  $F_1$  in (3.7) by

$$\hat{\mathbf{F}}_1 = \hat{\mathbf{W}}_1^T \phi_1, \quad (3.15)$$

Then the error dynamics (3.9) becomes

$$\dot{\mathbf{D}}\mathbf{e} = \tilde{\mathbf{W}}_1^T \phi_1 - \mathbf{K}_1 \mathbf{e} + \mathbf{G}_1 \eta + \varepsilon_1. \quad (3.16)$$

Similarly, define the NN functional estimate of  $F_2$  in (3.11) by

$$\hat{F}_2 = \hat{W}_2^T \phi_2. \quad (3.17)$$

The error dynamics of  $\eta$  in (3.11) is then given by

$$\dot{\eta} = \tilde{W}_2^T \phi_2 - K_2 \eta - G_1^T e + \varepsilon_2. \quad (3.18)$$

Note that there is a term  $G_1\eta$  in (3.16) and a term  $-G_1^T e$  in (3.18). This means there are couplings between the error dynamics (3.16) and (3.18). The closed-loop stability analysis and the weight tuning algorithms will be discussed in next section.

An additional standard assumption, which is quite common in the neural networks literature [14] [15], is stated next.

**Assumption 3:** The ideal weights are bounded by known positive values so that

$$\|W_1\|_F \leq W_{1M}, \|W_2\|_F \leq W_{2M}, \text{ or } \|Z\|_F \leq Z_M$$

where  $Z = \text{diag}\{W_1, W_2\}$ .

### 3.2.3 Weight Updates

The result is summarized in the following theorem.

**Theorem 3.1:** Let the desired trajectories be bounded. Take the control input (3.12) with NN weight tuning be provided by

$$\dot{\hat{W}}_1 = \Gamma_1 \phi_1 e^T - k_v \Gamma_1 \|\zeta\| \hat{W}_1 \quad (3.19a)$$

$$\dot{\hat{W}}_2 = \Gamma_2 \phi_2 \eta^T - k_v \Gamma_2 \|\zeta\| \hat{W}_2 \quad (3.19b)$$

with any constant matrices  $\Gamma_1 = \Gamma_1^T > 0$ ,  $\Gamma_2 = \Gamma_2^T > 0$ , and scalar positive constant  $k_v$ . Then the errors  $e(t)$ ,  $\eta(t)$  are UUB. NN weight estimates are bounded. The errors  $e(t)$ ,  $\eta(t)$  can be kept as small as desired by increasing gains  $K$  in (3.12).

*Proof:* Define

$$\begin{aligned} \zeta &= [e^T \ \eta^T]^T, & \tilde{W}_i &= W_i - \hat{W}_i, \\ Z &= \text{diag}\{\tilde{W}_1 \ \tilde{W}_2\}, & \Gamma &= \text{diag}\{\Gamma_1, \Gamma_2\}. \end{aligned}$$

Let the NN approximation property (i.e. equations (3.14) holds with given accuracy  $\varepsilon_{iN}$ 's for all  $\zeta$  in the compact set  $U_\zeta \equiv \{\zeta \mid \|\zeta\| \leq b_\zeta\}$  with  $b_\zeta$  a positive constant. Let  $\zeta(0) \in U_\zeta$ . Now consider the following Lyapunov function candidate for error systems (3.9) and (3.11)

$$V = \frac{1}{2} \zeta^T D \zeta + \frac{1}{2} \text{tr}(\tilde{Z}^T \Gamma^{-1} \tilde{Z}) \quad (3.20)$$

where  $D > 0$  is defined in (3.7). Differentiating (3.20) and using (3.12), (3.16), (3.18), (3.19) we have

$$\dot{V} = -\zeta^T K \zeta + k_v \|\zeta\| \text{tr}\{\tilde{Z}^T (Z - \tilde{Z})\} + \zeta^T \varepsilon. \quad (3.21)$$

Since  $\text{tr}\{\tilde{Z}^T (Z - \tilde{Z})\} = \langle \tilde{Z}, Z \rangle_F - \|\tilde{Z}\|_F^2 \leq \|\tilde{Z}\|_F \|Z\|_F - \|\tilde{Z}\|_F^2$ , there results

$$\begin{aligned} \dot{V} &\leq -K_{\min} \|\zeta\|^2 + k_v \|\zeta\| \|\tilde{Z}\|_F (Z_M - \|\tilde{Z}\|_F) + \varepsilon_N \|\zeta\| \\ &= -\|\zeta\| [K_{\min} \|\zeta\| + k_v \|\tilde{Z}\|_F (\|\tilde{Z}\|_F - Z_M) - \varepsilon_N] \end{aligned} \quad (3.22)$$

which is negative as long as the term in square bracket is positive. Completing the square yields

$$\begin{aligned} &K_{\min} \|\zeta\| + k_v \|\tilde{Z}\|_F (\|\tilde{Z}\|_F - Z_M) - \varepsilon_N \\ &= k_v (\|\tilde{Z}\|_F - Z_M/2)^2 - k_v Z_M^2/4 + K_{\min} \|\zeta\| - \varepsilon_N \end{aligned}$$

which is guaranteed positive as long as

$$\|\zeta\| > [k_v Z_M^2/4 + \varepsilon_N]/K_{\min} \quad (3.23)$$

or

$$\|\tilde{Z}\|_F > Z_M/2 + \sqrt{Z_M^2/4 + \varepsilon_N/K_v}. \quad (3.24)$$

Thus,  $\dot{V}$  is negative outside a compact set. The form of the right-hand side of (3.23) shows that the control gain  $K$  can be selected large enough so that

$$[k_v Z_M^2/4 + \varepsilon_N]/K_{\min} < b_\zeta.$$

Therefore, any trajectory  $\zeta(t)$  beginning in  $U_\zeta$  evolves completely within  $U_\zeta$ . According to a standard Lyapunov theorem extension [17], this demonstrates the UUB of both  $\zeta$  and  $\tilde{Z}$ .

Q.E.D.

Note also that the problem of **neural net weight initialization** occurring in other approaches in the literature does not arise, since if  $\varepsilon$  is taken as zero the terms  $-K_1 e$ ,  $-K_2 \eta$  stabilizes the plant on an interim basis. A formal proof reveals that  $K$  or  $K_1$ ,  $K_2$  should be large enough and the initial error  $\zeta(0)$  small enough [15].

A comparison with the results of Narendra *et al.* [17] shows that the NN reconstruction error  $\varepsilon_N$  increases the bounds on  $\|\zeta\|$  and  $\|\tilde{Z}\|_F$  in a very interesting way. Note, however, that arbitrarily small tracking error bounds may be achieved by selecting large control gains  $K$ . On the other hand, the NN weight error is fundamentally bounded by  $Z_M$ , the known bound on the ideal weights  $Z$ . The parameter  $k_v$  offers a design tradeoff between the relative eventual magnitudes of  $\|\zeta\|$  and  $\|\tilde{Z}\|_F$ ; a smaller  $k_v$  yields a smaller  $\|\zeta\|$  and a larger  $\|\tilde{Z}\|_F$ , and vice versa.

Note that PE is not needed to establish the bounds on  $\tilde{Z}$  with the modified weight tuning algorithm (3.31).

### 3.3 Simulation Results

Using the controller described in Section 3.2, we performed some simulation studies. In these simulations, we assume states are available. Using the data in [16], we simulate our robust NN controller without PE in Section 3.2. Fig. 3.3 shows the performance of PD control. The system goes unstable when there is a change in reference. There also exists steady-state error when the load disturbance is on. Fig. 3.4 shows the performance of PD + NN 1. The load disturbance is removed. However, due to the lack of compensation for the nonlinear dynamics by NN 2, the system becomes unstable when there is sudden change in speed reference. Fig. 3.5 shows the performance of PD + NN 2. Although the system is stable now, the load disturbance effects still exists. Fig. 3.6 shows the performance of the complete controller. Now the system is both stable and clear of load disturbance. We used 4 and 10 units in the two NN's which approximate  $F_1$ ,  $F_2$ , respectively. The reference trajectories are the same as those in Marino's paper [16].  $\omega^r$  is zero from 0 to 0.3 s., 220 r/s from 0.3 to 5 s., and 350 r/s from 5 sec. onwards.  $\psi_d^r$  is 1.3 Wb from 0 to 5 s. and 0.8 Wb after 5 s. The discontinuities are smoothed by linear interpolations. A load disturbance of 40 Nm is added at  $t = 2$  s. We set  $R_{rN} = 0.15$ ,  $T_{LN} = 0$ ,  $K_1 = \text{diag}\{35, 25\}$ ,  $K_2 = \text{diag}\{25, 25\}$ ,  $k_v = 1$ ,  $\Gamma_i = 10 I$ ,  $i = 1, 2$ . The applied voltage  $u_a$  has the same magnitude as that of Marino's paper [16] and is well within inverter limits.

It should be noted that the plots of  $u_a$ ,  $u_b$  are not due to switching in sliding mode control as it appears to be. Similar waveforms have also been observed in Marino's adaptive input-output feedback method [16].

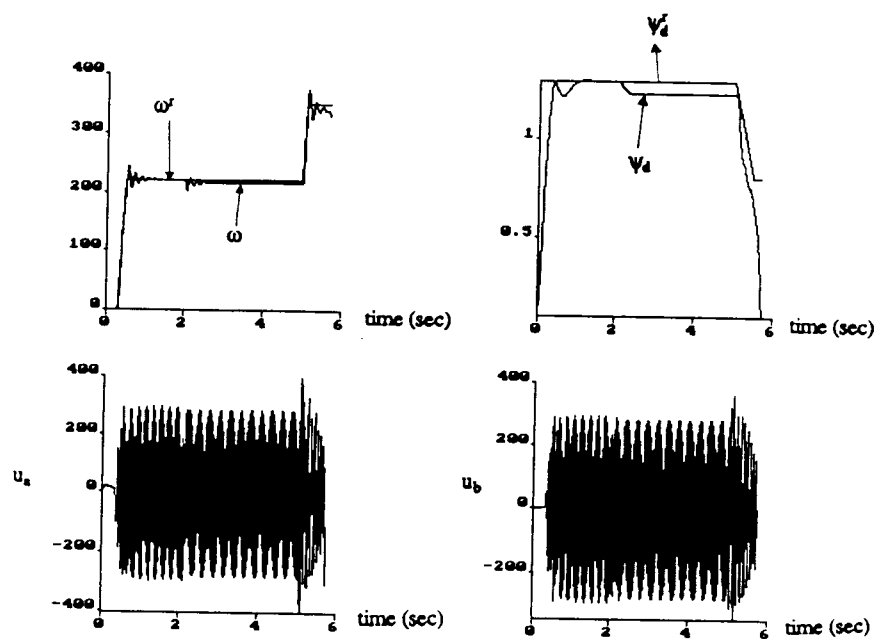


Fig. 3.3 Performance of PD controller.

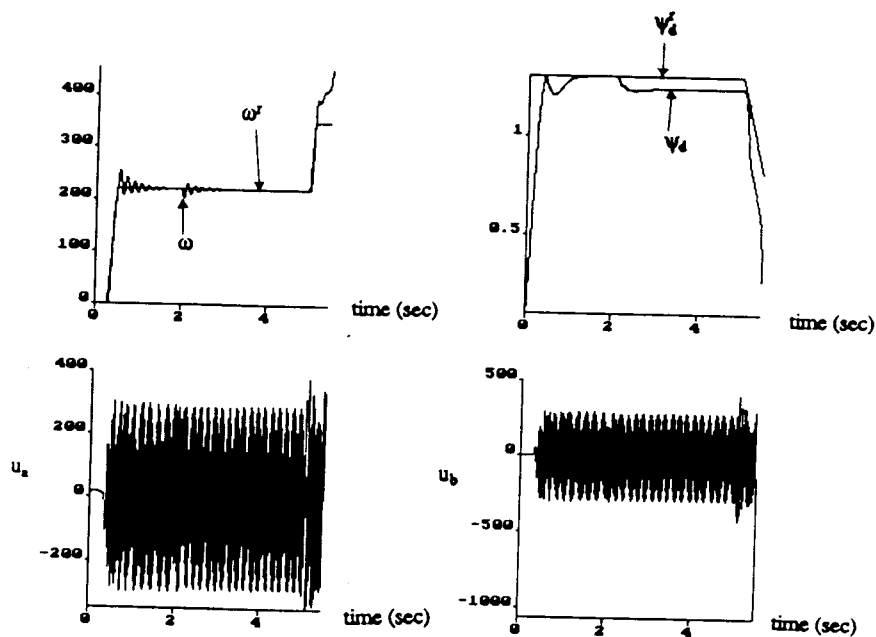


Fig. 3.4 Performance of controller (PD + NN 1).

#### 4. Conclusion

There are four important contributions in this paper. The first one is that we have derived a novel robust NN

control scheme to a "nonlinear benchmark problem" known as induction motors. The scheme involves the backstepping technique. Our method has

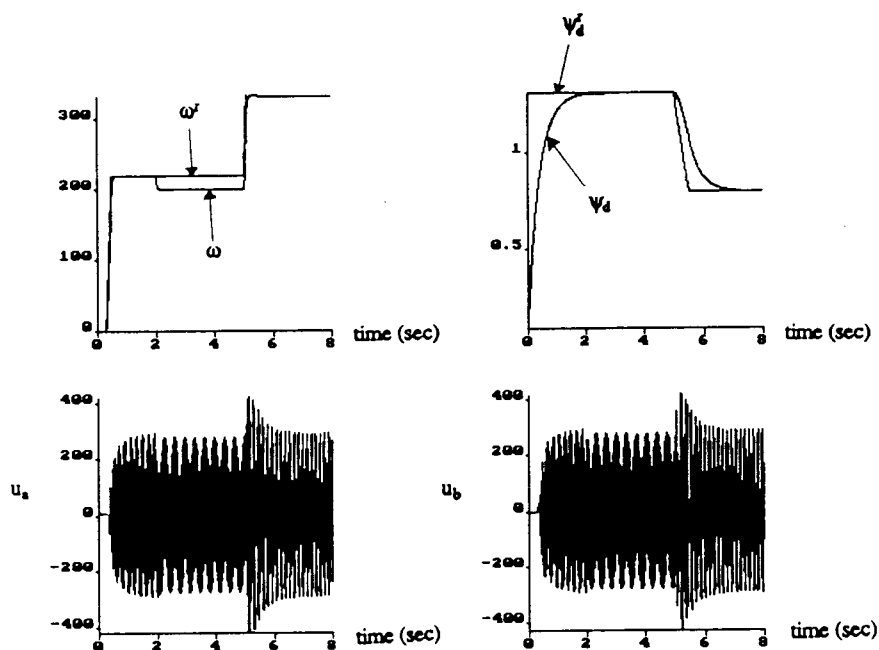


Fig. 3.5 Performance of controller (PD + NN 2).

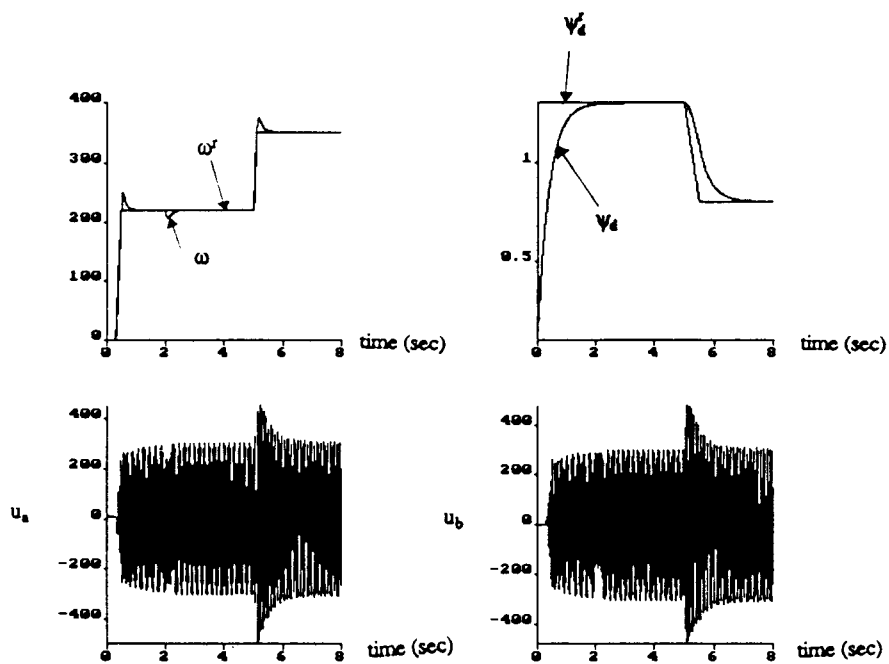


Fig. 3.6 Performance of complete NN controller.

achieved the same performance as that of Marino's adaptive input-output feedback method [16]. The design is modular and systematic and hence can be easily applied to nonlinear systems with similar structures. The second contribution is that our method does not require the linear parametrizability of the unknown parameters (LP). No

regression matrix is needed, so that no preliminary dynamical analysis is needed. We believe this advantage is very useful for some systems where the LP property does not hold. The third one is that new tuning schemes, which *do not require any persistent excitation conditions*, are proposed which *can guarantee the boundedness of*

tracking error and weight updates. The fourth one is that we do not need motor accelerations and the derivative of flux magnitude, as compared to some conventional sliding control schemes such as the works of Soto and Yeung [21].

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