TIME-DOMAIN H_∞ IDENTIFICATION

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ABSTRACT

The problem of parameter identification, for single-input, single-output, ARX systems, is considered. Recent results in H_∞- nonlinear filtering are used to formulate, for the first time, a nonlinear H_∞ time-domain prediction-error-modelling (PEM) identification method. The performance of the new method is guaranteed by a preassigned bound on the ratio between the energy of the prediction error of the obtained model, and the energy of the exogenous disturbances. The potential usefulness of the H_∞ time-domain identification method is illustrated by a numerical example.

1. INTRODUCTION

The problem of identifying the parameters of a dynamic linear system is of prime importance to engineering practice. A vast amount of research have been devoted to linear system identification in the last decades [1],[2]. Most of the conventional identification methods, for example the well known least-squares and extended least-squares methods, are time-domain algorithms, which can be classified as prediction error modelling methods [1]. These time domain methods are intimately related to well known results in the theory of linear and extended filtering [3].

In the last decade, many authors (see e.g. [4],[5]) have dealt with the problem of frequency-domain H_∞ -identification. These frequency-domain H_∞ methods are very different from the time-domain methods They are basically methods to fit a stable transfer function, of a given order, to a finite sample of the frequency response of a system.

At the same time, many published works have been devoted to the topic of H_∞-filtering (See [6] and the references therein). The application of the time-domain H_∞-theory to system identification is attractive, since such methods guarantee a bound on the ratio between the energies of the prediction error of the obtained model, and the exogenous signals (measurement noises and driving disturbances), even if the statistics of these signals are completely unknown. The question has thus naturally arised whether the new H_∞ filtering results can be applied successfully in the related field of time-domain system identification.

A first attempt in this direction was made by [7], where the parameters of the identified system were treated as the states of a discrete-time trivial system. The same approach was used by [8], to formulate the first linear PEM H_∞ identification method. It was shown there, that for long identification intervals, the H_∞-optimal linear PEM identification method tends to the well known LMS algorithm. The theory of continuous-time H_∞ identification was recently applied in [9] to identify the parameters of continuous-time systems. The approach there is not limited to linear systems.

The approach of [7] and [9] is limited to the use of linear H_∞-filtering theory, while the problem of identifying the parameters of a system on the basis of noise-contaminated measurements is basically nonlinear. In this paper, we use the emerging nonlinear H_∞ -filtering methods [10][11],[12], to extend the work of [8], and obtain, for the first time, a nonlinear PEM H_∞-identification method.

The paper is organized as follows: In Section 2, we introduce the time-domain formulation of an H_∞-identification problem. The problem is analyzed and solved in Section 3. Section 4 demonstrates the application of the new method by a numerical example.

We use the following standard notation: The Euclidean norm of a vector d is denoted by $||d||^2 \triangleq d^t d$. The L_2 -norm of a vector signal $\{d_k\} \in \mathcal{R}^n \times L_2[0, N-1]$ is denoted by $||d_k||_2^2 \triangleq \sum_{i=0}^{N-1} d_k^t d_k$. The maximal and the minimal singular values of a matrix A are denoted by $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$, respectively.

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2. PROBLEM FORMULATION

We consider the following linear, discrete-time, ARX process:

$$z_k = -\sum_{i=1}^{n} a_i z_{k-i} + \sum_{i=1}^{m} b_i u_{k-i} + c_0 w_k \ , \quad k \in [-n, N] \tag{2.1}$$

$$y_k = z_k + \nu_k \tag{2.2}$$

where $\{a_i\}$ and $\{c_i\}$ are unknown fixed scalar parameter sequences. The signal $\{w_k\}$ is an unknown driving disturbance, and the signal $\{\nu_k\}$ is the measurement noise. We do not assume any statistical information concerning $\{w_k\}$ or $\{\nu_k\}$. The known constant c_0 is used to weight the disturbances. In order to identify the parameters of the system, we apply to its input a known signal $\{u_k\}$, and read the noisy measurements $\{y_k\}$.

We look for a predictive model of the form

$$\hat{z}_k = -\sum_{i=1}^{n} \hat{a}_i \left(y_{k-i} - \hat{\nu}_{k-i}\right) + \sum_{i=1}^{m} \hat{b}_i u_{k-i} \tag{2.3}$$

that will provide a good prediction $\hat{z}_k$ of the next output z_k of the system, on the basis of the available measurements $Y^k = \{y_i | i \in [0, k-1]\}$ and the known signal sequence $\{u_k\}$. We say that the predictive model of (2.3) achieves an H_∞ performance level of (P_0, γ) if:

$$\|\varepsilon_{i+1}\|_2^2 \le \gamma^2 \left[\theta_0^t P_0 \theta_0 + \|w_i\|_2^2 + \|\nu_i\|_2^2\right] \ ,$$

$$\forall \ \{w_i\} \ , \ \{\nu_i\} \in l_2[0, N-1] \tag{2.4}$$

where ε_k is the following prediction error:

$$\varepsilon_k = z_k - c_0 w_k - \hat{z}_k \ ,$$

and

$$\theta_k^t = [-a_1 \cdots - a_n \ , \ b_1 \cdots b_m \ , \ \nu_{k-1} \cdots \nu_{k-n}] \, . \tag{2.5}$$

We can obtain an approximate solution for the problem of achieving the above performance level, by using the nonlinear H_∞ filtering theory of [12]. We define

$$d_k = \left[\ w_k \quad \nu_k \ \right]^t \ , \tag{2.6}$$

and readily find that

$$\theta_{k+1} = A\theta_k + Bd_k \tag{2.7}$$

$$z_k = C_k(\theta_k)\theta_k + c_0 w_k \tag{2.8}$$

and

$$y_k = C_k(\theta_k)\theta_k + Dd_k \tag{2.9}$$

where

$$A = \begin{bmatrix} I_{m+n} & 0 \\ 0 & \begin{bmatrix} 0 & 0_{1\times 1} \\ I_{n-1} & 0 \end{bmatrix} \end{bmatrix} \ ,$$

$$B = \begin{bmatrix} 0_{2n+m} & \begin{matrix} 0_{n+m} \\ 1 \\ 0_{n-1} \end{matrix} \end{bmatrix}$$

$$C_k(\theta_k) = [y_{k-1} \cdots y_{k-n} \ , \ u_{k-1} \cdots u_{k-m} \ , \ a_1 \cdots a_n] \ ,$$

and

$$D = [\ c_0 \quad 1 \ ] \ .$$

Remark: Many common system identification methods assume an ARMAX model structure, where the term $C_0 w_k$ in (2.1) is replaced by $\sum_{i=0}^{l} c_i w_{k-i}$. The additional terms in the ARMAX model serve as a coloring filter for the unknown driving signal $\{w_k\}$. It is not difficult to apply the theory of the present paper to develop an ARMAX H_∞ -model identification scheme. The motivation for doing so is however dubious, since H_∞-theory does not assume any specific color for the driving signal.

3. THE IDENTIFICATION PROCESS

We consider the following estimation scheme:

$$\hat{\theta}_{k+1} = A\hat{\theta}_k + G_k(\hat{\theta}_k)(y_k - \hat{z}_k) \ , \quad \hat{\theta}_0 = 0 \tag{3.1a}$$

$$\hat{z}_k = C_k(\hat{\theta}_k)\hat{\theta}_k \tag{3.1b}$$

where $G_k \in \mathcal{R}^{2n+m}$. The estimation error $e_k = \theta_k - \hat{\theta}_k$ thus satisfies the following:

$$e_{k+1} = F_k(e_k, \hat{\theta}_k, G_k) + \tilde{g}_k d_k \ , \quad e_0 = \theta_0 \tag{3.2}$$

where

$$\tilde{g}_k = B - G_k D \tag{3.3}$$

$$F_k = \left[A - G_k \tilde{C}_k(\theta_k)\right] e_k \tag{3.4}$$

$$\tilde{C}_k(\theta_k) = C_k(\theta_k) + \hat{\theta}_k^t E_1 \tag{3.5}$$

and

$$E_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} . \tag{3.6}$$

We define the following Hamiltonian:

$$\mathcal{H}(e_k, G_k, d_k, \gamma) \triangleq V_{k+1}\left(F_k(e_k, \hat{\theta}_k, G_k) + \tilde{g}_k d_k\right)$$

$$- \gamma^2 \|d_k\|^2 + \|\varepsilon_{k+1}\|^2 \tag{3.7}$$

where

$$V_k(e_k) = \gamma^2 e_k^t Q_k e_k \ , \ Q_0 = P_0 \ , \ Q_k \geq 0 \ , \ \forall k \in [0, N] \, .$$

We then obtain the following result from [12]:

Lemma 3.1 *Let d_k^* denote the maximizing disturbance signal*

$$d_k^* = arg \max_{d_k} \mathcal{H}(e_k, G_k, d_k, \gamma)$$

Then, if

$$\mathcal{H}(e_k, G_k, d_k^*, \gamma) - V_k(e_k) \leq 0 \ , \ \forall k \in [0, N-1] \ , \tag{3.8}$$

the estimator of (3.1a,b) achieves the performance level (P_0, γ).

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Lemma 3.2 *The estimator of (3.1a,b) achieves the performance level of (P_0, γ) if the following conditions*

$$\begin{aligned} \bar{Q}_{k+1}^{-1} \ \geq \ & (G_k - G_k^*)\, \phi_k(\theta_k)\, (G_k - G_k^*)^t + A Q_k^{-1} A^t \\ & - G_k^* \phi_k(\theta_k) G_k^{*t} + B B^t && (3.9) \\ & I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k > 0 && (3.10) \\ & Q_k > 0 && (3.11) \end{aligned}$$

are satisfied for all $k \in [0, N-1]$, where

$$\phi_k(\theta_k) = DD^t + \tilde{C}_k(\theta_k) Q_k^{-1} \tilde{C}_k{}^t(\theta_k) \tag{3.12}$$

$$G_k^* = \left(A Q_k^{-1} \tilde{C}_k{}^t(\theta_k) + B D^t\right) \phi_k{}^{-1}(\theta_k) \tag{3.13}$$

and

$$\bar{Q}_k \triangleq Q_k + \gamma^{-2} \tilde{C}_k{}^t(\theta_k) \tilde{C}_k(\theta_k) \, . \tag{3.14}$$

Proof:
If follows by (3.1a,b),(3.2) and (3.7), that

$$\begin{aligned} & \gamma^{-2} \mathcal{H}(e_k, G_k, d_k, \gamma) \\ & = \left[F_k^t + d_k^t \tilde{g}_k^t\right] \bar{Q}_{k+1} \left[F_k + \tilde{g}_k d_k\right] - d_k^t d_k \end{aligned}$$

Completing to squares, we obtain that

$$\begin{aligned} & \gamma^{-2} \mathcal{H}(e_k, G_k, d_k, \gamma) = \\ & - \left\{ d_k^t - F_k^t \bar{Q}_{k+1} \tilde{g}_k \left(I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k\right)^{-1} \right\} \left(I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k\right) \\ & \times \left\{ d_k - \left(I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k\right)^{-1} \tilde{g}_k^t \bar{Q}_{k+1} F_k \right\} \\ & + F_k^t \left[\bar{Q}_{k+1} + \bar{Q}_{k+1} \tilde{g}_k (I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k)^{-1} \tilde{g}_k^t \bar{Q}_{k+1}\right] F_k \end{aligned} \tag{3.15}$$

Hence, by (3.10), a maximizing disturbance exists, and is given by

$$d_k^* = \left(I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k\right)^{-1} \tilde{g}_k^t \bar{Q}_{k+1} F_k \, .$$

We substitute (3.4) in (3.15) and obtain that

$$\begin{aligned} & \gamma^{-2} \mathcal{H}_k(e_k, G_k, d_k^*, \gamma) = \\ & e_k^t \left[A - G_k \tilde{C}_k(\theta_k)\right]^t \\ & \left[\bar{Q}_{k+1} + \bar{Q}_{k+1} \tilde{g}_k (I - \tilde{g}_k^t \bar{Q}_{k+1} \tilde{g}_k)^{-1} \tilde{g}_k^t \bar{Q}_{k+1}\right] \\ & \left[A - G_k \tilde{C}_k(\theta_k)\right] e_k \, . \end{aligned} \tag{3.16}$$

Using the identity

$$\begin{aligned} & \bar{Q}_{k+1} \left[I + \tilde{g}_k \left(I - \tilde{g}^t \bar{Q}_{k+1} \tilde{g}_k\right)^{-1} \tilde{g}_k^t \bar{Q}_{k+1}\right] = \\ & \left(\bar{Q}_{k+1}^{-1} - \tilde{g}_k \tilde{g}_k^t\right)^{-1} \end{aligned}$$

and (3.16), we obtain that (3.8) is equivalent to

$$\bar{Q}_{k+1}^{-1} \geq \tilde{g}_k \tilde{g}_k^t + \left(A - G_k \tilde{C}_k(\theta_k)\right) Q_k^{-1} \left(A - G_k \tilde{C}_k(\theta_k)\right)^t \tag{3.17}$$

We obtain the condition of (3.9) by substituting (3.3) in (3.17), and completing to squares, as follows:

$$\begin{aligned} & \tilde{g}_k \tilde{g}_k^t + \left(A - G_k \tilde{C}_k(\theta_k)\right) Q_k^{-1} \left(A - G_k \tilde{C}_k(\theta_k)\right)^t = \\ & A Q_k^{-1} A^t + B B^t \\ & + \left[G - \left(A Q_k^{-1} \tilde{C}_k{}^t(\theta_k) + B D^t\right) \phi_k{}^{-1}(\theta_k)\right] \phi_k(\theta_k) \\ & \times \left[G - \left(A Q_k^{-1} \tilde{C}_k{}^t(\theta_k) + B D^t\right) \phi_k{}^{-1}(\theta_k)\right]^t \\ & - \left(A Q_k^{-1} \tilde{C}_k{}^t(\theta_k) + B D^t\right) \phi_k{}^{-1}(\theta_k) \\ & \times \left(A Q_k^{-1} \tilde{C}_k{}^t(\theta_k) + B D^t\right)^t \end{aligned}$$

$\square$

We want to make (3.9) as non-restrictive as possible. Our policy is thus to select G_k so as to minimize the right hand side of (3.9). Note that such policy is only sub-optimal, since, by (3.14) and (3.5), $\bar{Q}_{k+1}$ depends on G_k.

It follows from lemma 3.2, and the fact that $\phi_k(\theta_k) > 0$, that it would be best to take $G_k = G_k^*$. However, G_k^* is not available. The best approximation we know for G_k^* is

$$\hat{G}_k = \left(A Q_k^{-1} \tilde{C}_k{}^t(\hat{\theta}_k) + B D^t\right) \phi_k{}^{-1}(\hat{\theta}_k) \, . \tag{3.18}$$

For $\hat{G}_k$ of (3.18) we have the following result:

Lemma 3.3 *The identification algorithm of (3.1a,b), with the gain $G_k = \hat{G}_k$, achieves the performance level of (P_0, γ) if, for all $k \in [0, N-1]$, conditions (3.11) and (3.10) are satisfied, and*

$$\begin{aligned} \bar{Q}_{k+1}^{-1} \geq \ & A Q_k^{-1} A^t - \hat{G}_k \phi_k(\hat{\theta}_k) \hat{G}_k^t + B B^t + \hat{G}_k \Delta_k \hat{G}_k^t \\ & - A Q_k^{-1} \delta_k \hat{G}_k^t - \hat{G}_k \delta_k^t Q_k^{-1} A^t \, , \end{aligned} \tag{3.19}$$

where

$$\Delta_k = \phi_k(\theta_k) - \phi_k(\hat\theta_k) = \tilde C_k(\theta_k) Q_k^{-1} \tilde C_k{}^t(\theta_k) - \tilde C_k(\hat\theta_k) Q_k^{-1} \tilde C_k{}^t(\hat\theta_k) \quad (3.20)$$

and

$$\delta_k^t = \tilde C_k(\theta_k) - \tilde C_k(\hat\theta_k) = [\ 0\ldots 0,\ \ 0\ldots 0,\ \ -(a_1-\hat a_1)\ldots -(a_n-\hat a_n)\ ] \quad (3.21)$$

Proof:
Substituting $\hat G_k$ for G_k in (3.9), we obtain the following condition:

$$\begin{aligned}\bar Q_{k+1}^{-1} \ \geq\ & (\hat G_k - G_k^*)\phi_k(\theta_k)\hat G_k^t + \hat G_k \phi_k(\theta_k)(\hat G_k - G_k^*) \\ & -\hat G_k\phi_k(\theta_k)\hat G_k^t + AQ_k^{-1}A^t + BB^t \qquad (3.22)\end{aligned}$$

We evaluate

$$\begin{aligned}&(\hat G_k - G_k^*)\phi_k(\theta_k) \\ &= \left(AQ_k^{-1}\tilde C_k{}^t(\hat\theta_k) + BD^t\right)\left[I + \phi_k{}^{-1}(\hat\theta_k)\Delta_k\right] \\ &- \left(AQ_k^{-1}\tilde C_k{}^t(\hat\theta_k) + BD^t\right) - AQ_k^{-1}\delta_k \\ &= \hat G_k\Delta_k - AQ_k^{-1}\delta_k\ .\end{aligned}$$

The result of the lemma follows by substituting the above result in (3.22), and using(3.20).

□

The values of Δ_k and δ_k are not known. However, if we have the following estimate of the current identification errors

$$\delta_{0,k} \geq \|\delta_k\| = \left[\sum_{i=1}^{n}(a_i - \hat a_{i,k})^2\right]^{1/2} \qquad (3.23)$$

we can easily establish the following bounds:

$$-AQ_k^{-1}\delta_k\hat G_k^t - \hat G_k\delta_k^t Q_k^{-1}A^t \leq \bar\delta_k\ \ ,\ \ \Delta_k \leq \bar\Delta_k$$

and

$$\tilde C_{k+1}^t(\theta_{k+1})\tilde C_{k+1}(\theta_{k+1}) - \tilde C_{k+1}^t(\hat\theta_{k+1})\tilde C_{k+1}(\hat\theta_{k+1}) \leq \tilde\delta_k\ ,$$

where

$$\bar\delta_k = \bar\sigma\left(AQ_k^{-1}[\ 0\ \ 0\ \ I_n\ ]^t\right)\delta_{0,k}\|G_k\| I_{2n+m}\ ,$$

$$\begin{aligned}\Delta_k \ =\ & 2\delta_{0,k}\left\|[\ 0\ \ 0\ \ I_n\ ]\,Q_k^{-1}\tilde C^t{}_k(\hat\theta_k)\right\| \\ & +\delta_{0,k}^2\bar\sigma\left([\ 0\ \ 0\ \ I_n\ ]\,Q_k^{-1}[\ 0\ \ 0\ \ I_n\ ]^t\right)\end{aligned}$$

and

$$\begin{aligned}\tilde\delta_k \ =\ & \frac{\delta_{0,k}}{\left\|\tilde C_k(\hat\theta_k)\right\|}\tilde C_k{}^t(\hat\theta_k)\tilde C_k(\hat\theta_k) \\ & + \left(1 + \frac{\left\|\tilde C_k(\hat\theta_k)\right\|}{\delta_{0,k}}\right)\delta_{0,k}^2\begin{bmatrix} 0_{n+m\times n+m} & 0 \\ 0 & I_n\end{bmatrix}.\end{aligned}$$

We summarize our findings in the following theorem:

Theorem 3.1 *Suppose that the following recursion*

$$\begin{aligned}&Q_{k+1} = \\ &\left\{AQ_k^{-1}A^t - \hat G_k\left(\bar\Delta_k + \phi_k(\hat\theta_k)\right)\hat G_k^t + BB^t + \bar\delta_k\right\}^{-1} \\ &-\gamma^{-2}\left[\tilde C_{k+1}{}^t(\hat\theta_{k+1})\tilde C_{k+1}(\hat\theta_{k+1}) + \tilde\delta_k\right] \qquad (3.24)\end{aligned}$$

is solved by a sequence $\{Q_k\}$ of matrices which satisfy, for $k \in [0, N-1]$, (3.11) and

$$\bar g_k^t\left(Q_{k+1} + \gamma^{-2}\left[\tilde C_{k+1}{}^t(\hat\theta_{k+1})\tilde C_{k+1}(\hat\theta_{k+1}) + \tilde\delta_{k+1}\right]\right)\bar g_k < I\ , \qquad (3.25)$$

Then, the estimator (3.1a,b) with $G_k = \hat G_k$, where $\hat G_k$ is given by (3.18) and (3.12), achieves the performance level of (P_0, γ).

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Remarks:

- If the identification process converges, then we may expect that for large values of k, $\delta_{0,k}$ will become negligibly small. In this case, the identification algorithm of Theorem 3.1 tends to the extended H_∞ estimation algorithm of [12].
- If we assume that $\delta_{0,k}$ is negligibly small, and if we let $\gamma \to \infty$, then our results reduce to the well known extended-least-squares (ELS) identification scheme [1].

4. EXAMPLE

Consider the following system:

$$\begin{aligned} z_k &= 1.7z_{k-1} - 0.85z_{k-2} + u_{k-1} + 4u_{k-2} + w_k\ . \\ y_k &= z_k + \nu_k \end{aligned} \qquad (4.1)$$

We simulated the system of (4.1), with $\{w_k\}$ and $\{\nu_k\}$ that are random white sequences of unit intensity. The known signal $\{u_k\}$ was also chosen as a standard white signal of unit intensity. We applied the method of this paper, with $\gamma = 2.5$, to evaluate the parameters of

	H_∞	Least-squares	LMS	True
$-a_1$	1.6926	1.6588	1.9170	1.7000
$-a_2$	-0.8423	-0.8086	-1.0562	-0.8500
b_1	1.2846	1.2782	0.7800	1.0000
b_2	3.7953	3.8862	3.1869	4.0000

Table 4.1: Comparison between the identification results of the H_∞ and the least-squares algorithms, and the true parameters of the system.

the system. We assumed that for large k, the identification error will be negligible. We have thus taken $\delta_{0,k} = 0$. With this choice for $\delta_{0,k}$, the obtained H_∞ performance bound is meaningful only asymptotically, for large values of k. For comparison, we have also evaluated the parameters of the system using the standard least-squares identification, and the LMS [8] algorithm. In all the algorithms, we assumed the correct order of the system.

The identified parameters are summarized, after 300 simulation steps, in Table 4.1. The time-evolution of the identified parameters is depicted in Figure 4.1 for the nonlinear H_∞ and for the LS methods, and in Figure 4.2 for the LMS. In Figures 4.3 and 4.4 we compare the frequency responses of the models obtained by the nonlinear H_∞ identification, and LS identification. It is seen that the H_∞ identification algorithm yields very accurate results, while the results of the least-squares algorithm are heavily biased, due to the high measurement noise that was present in our simulation. The convergence of the LMS algorithm is poor.

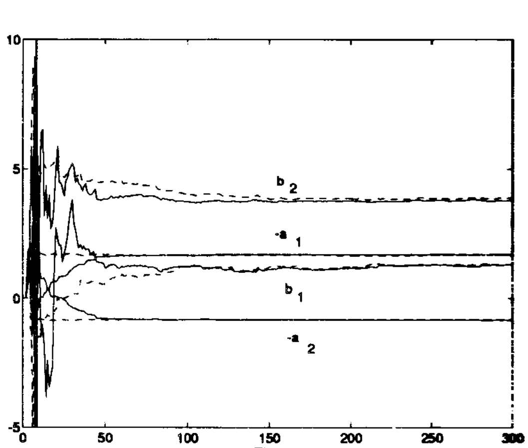

Figure 4.1: The time evolution of the identified parameters, for the nonlinear H_∞ and for the LS methods.

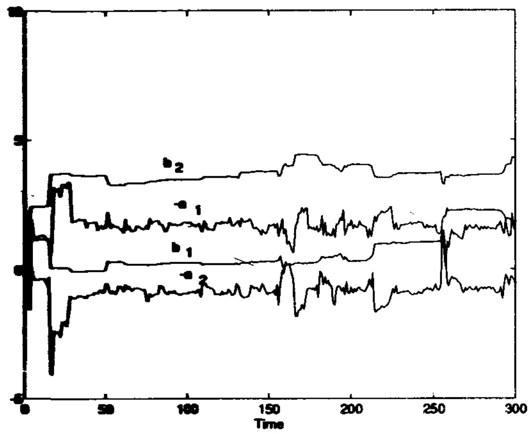

Figure 4.2: The time evolution of the identified parameters, for the LMS method.

5. CONCLUSIONS

In this paper we have presented a nonlinear H_∞ time-domain prediction-error-modelling system identification method. The new method can guarantee H_∞ performance, i.e. guarantee that the ratio between the energy of the prediction error, and the energy of the disturbances present in the identification experiment, will be bounded by a prescribed positive number γ, if the bound (3.23) on the identification errors can be estimated, and if the conditions of Theorem 3.1 are met. The H_∞ performance of the identified model can be guaranteed asymptotically, even if we do not have an a-priori bound on the identification errors, if we assume that after enough time have elapsed, the results of the identification become precise.

Many aspects of H_∞ identification are left for further research. For example, the convergence properties of the H_∞ identification process, and identification problems where the order of the system is unknown.

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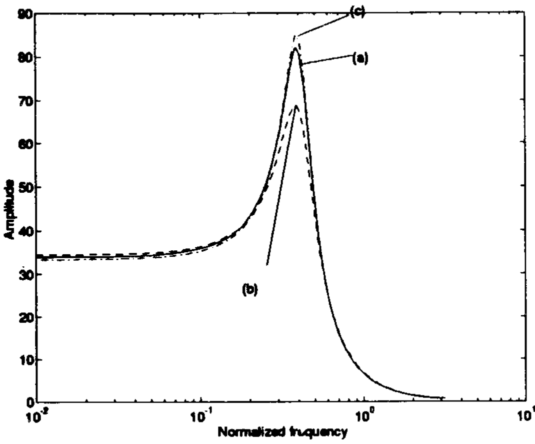

Figure 4.3: Frequency plots of (a) the H_∞ identified system, (b) the LS identified system, and (c) the true system.

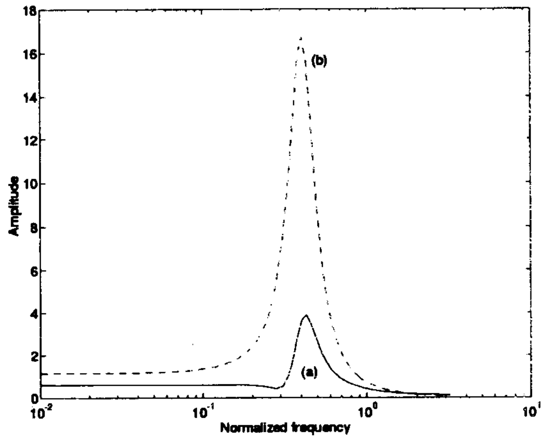

Figure 4.4: Frequency plots of the identification error of (a) the H_∞ identified system, and (b) the LS identified system.

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