ROBUST CONTROL OF COMPLEX SYSTEMS WITH JUMPS – DECENTRALIZED VERSUS CENTRALIZED JLQ APPROACH *

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Abstract. We compare decentralized and centralized JLQ approaches combined with additional nonlinear control design algorithm used by some authors to ensure practical stability of uncertain systems. Simply we decompose the system into subsystems and consider two level control structure. In decentralized structure decision makers of the lower level have only nominal linear models of their subsystems neglecting interconnections between subsystems. A local controller is found using the quadratic criterion for the subsystem and incorporates the information about its local state. The role of the coordinator (upper level decision maker) is to ensure robust stability and guaranteed cost in spite of uncertainties represented by interconnections among subsystems and deviation of parameters. In the centralized structure the roles of decision makers of the two levels are inverted.

Key Words. Piecewise deterministic process, large system, hierarchical control, guaranteed cost, robust stability

1 Introduction

Design of feedback controllers for systems with uncertain parameters has been a topic of interest of system designers for many years. Parameter uncertainty can be dealt with in variety of ways. One possibility is constant parameter estimation through extensive testing or through use of real - time or nonreal time system identification. Alternatively, parameters may be accepted at their a priori levels, and a control should be designed so as to be, in some sense, robust or insensitive to their variations. It is the latter approach that is applied in this paper.

In many practical situations, the natural state space is hybrid: to the usual plant state in $\mathbf{R^n}$ we append a discrete variable taking values in $\mathcal{B} = \{1, 2, ...s\}$ called the mode that describes sudden changes in the plant characteristics. It is typical case in the complex large scale systems, such as manufacturing systems (see for example [2]), power systems (see for example [11]) or redundant multiplex control systems ([9]).

In this paper we consider systems which are linear in the continuous plant state and whose mode dynamics is described via random jumps modelled by a discrete-state Markov chain. One way of stabilizing the linear stochastically stabilizable system with markovian jumps is to solve the JLQ problem (see for example [5], [13], [8]). However the optimality of the solution as well as the stability

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of the system is guaranteed only for the perfectly measurable state variables and complete information about the system parameters. Moreover an optimal controller uses all the state variables to construct a control vector. This is an overidealization especially in the case of a complex system containing many subsystems interconnected by incompletely known crosscouplings. The situation becomes especially complex for the piecewise deterministic processes when the controller is designed under the assumption of the complete access to the mode i.e. discrete random state variables representing the form of the process.

To overcome at least a part of these difficulties we propose to combine decentralized jump linear quadratic (DJLQ) approach with nonlinear control design method used by some authors (see for example [1], [6], [7]) to ensure practical stability of uncertain systems. Simply we decompose the system into subsystems and consider two level control structure. Decision makers of the lower level have only linear models of their subsystems neglecting interconnections between subsystems. A local controller is found using the quadratic criterion for the subsystem and incorporates the information about its local state. The role of the coordinator (upper level decision maker) is to ensure robust stability and guaranteed cost in spite of uncertainties represented by interconnections among subsystems and deviation of parameters. The uncertainty is described by deterministic inequality model and the main assumption is the well-known matching conditions. The coordinator uses the information about local states and bounds for uncertainties to design the robust control actions which are transmitted to the local decision makers and added to the local control variables. This control is nonlinear but it is bounded by the constraints imposed on the uncertainties. Yet another possibility is to design the JLQ controller for the overall system described by a model without uncertainty and to robustify this strategy by local nonlinear law based on the local estimation of bounds for uncertain variables. We call this approach a centralized one.

The paper is organized as follows. In section 2, we establish a model of the system, a model of the uncertainty and a nominal model used by the local decision makers in the decentralized structure. Then, we state the control problem and we describe an information structure in the system. In section 3, we construct the control laws of the local decision makers and the coordinator and we give the main results of this paper in the form of two theorems dealing with robust stochastic stability and guaranteed control property of the system. In section 4 we compare the results with those obtained for the centralized structure and in section 5, we present some concluding remarks.

2 Decentralized information structure

We consider a decentralized system composed of L interconnected subsystems described in the state form by the following differential equation:

$$\dot{x}^i(t) = A^{ii}(\xi^i(t))x^i(t) + B^i(\xi^i(t))[u^i(t) + v^i(t) + e^i(\xi(t), x(t), t)] + \sum_{j=1, j\neq i}^{L} A^{ij}(\xi(t))x^j(t)$$
$$x^i(0) = x^i_0;\ i = 1, 2, \cdots, L \tag{1}$$

where x^i is a local state vector of the ith subsystem, $x^i(t) \in \mathbf{R}^{n_i}$,

u^i is a local control, $u^i(t) \in \mathbf{R}^{m_i}$,

v^i is a coordinator control for the ith subsystem, $v^i(t) \in \mathbf{R}^{m_i}$,

$A^{ii}(\xi^i(t)), B^i(\xi^i(t))$ are local system and input matrices respectively,

$A^{ij}(\xi(t))$ represents crosscoupling between the ith and the jth subsystems,

$e^i(\xi(t), x(t), t)$ are model uncertainties resulting from parameter deviations and bounded nonlinearities acting in the range of the local input for the ith subsystem, $e^i(\xi(t), x(t), t) \in \mathbf{R}^{m_i}$.

$\xi^i(t)$ is an irreducible and continuous time discrete state Markov process representing a local mode of the ith subsystem and taking values in a finite set $\mathcal{B}^i = \{1, 2, ..., s^i\}$ with transition probability matrix $P = \{p_{\alpha^i\beta^i}\}$ from mode α^i to mode β^i during the time interval $[t, t + dt]$, given by:

$$p_{\alpha^i\beta^i} = Pr\{\xi^i(t+\delta t) = \beta^i | \xi^i(t) = \alpha^i\} = \\ = \begin{cases} q^i_{\alpha^i\beta^i}\delta t + o(\delta t), & \text{if } \alpha^i \neq \beta^i \\ 1 + q^i_{\alpha^i\alpha^i}\delta t + o(\delta t), & \text{if } \alpha^i = \beta^i \end{cases} \tag{2}$$

In this relation, $q^i_{\alpha^i\beta^i}$ stands for the transition probability rate from mode α^i to mode β^i and satisfies the following relations:

$$q^i_{\alpha^i\beta^i} \geq 0 \tag{3}$$

$$q^i_{\alpha^i\alpha^i} = -\sum_{\beta^i \in \mathcal{B}^i, \alpha^i \neq \beta^i} q^i_{\alpha^i\beta^i} \tag{4}$$

$x(t)$ is an overall system state vector composed of the subsystem state vectors $x^i(t), i = 1, 2, ..., L$ taking values in $\mathbf{R}^{n_1} \times \mathbf{R}^{n_2} \times \ldots \times \mathbf{R}^{n_L}$, while the mode $\xi(t)$ of the overall system contains modes of the subsystems $\xi^i(t)$ and takes values in the product set $\mathcal{B} = \mathcal{B}^1 \times \mathcal{B}^2 \times \ldots \times \mathcal{B}^L$.

It is assumed that the unknown cross-couplings satisfy the following matching conditions (see for example [6], [1])

$$A^{ij}(\xi(t)) = B^i(\xi^i(t))D^{ij}(\xi(t)) \tag{5}$$

where the matrix $D^{ij}(\xi(t))$ for each $\xi(t) = \alpha = [\alpha^1, \ldots, \alpha^L]'$ satisfies the following relation:

$$||D^{ij}(\alpha)|| \leq d^{ij}(\alpha) \tag{6}$$

and $d^{ij}(\alpha)$ is a known scalar.

Uncertainty $e^i(\xi(t), x(t), t)$ is assumed to be bounded for each $\xi(t) = \alpha = [\alpha^1, \ldots, \alpha^L]'$ (′ represents the transpose operation; this notation will be used subsequently in text) by:

$$||e^i(\alpha, x(t), t)|| \leq f^i(\alpha)||x(t)|| \tag{7}$$

where $f^i(\alpha)$ is a known scalar.

A nominal model of the ith local decision maker has a simplified form:

$$\dot{x}^i(t) = A^{ii}(\xi^i(t))x^i(t) + B^i(\xi^i(t))u^i(t) \tag{8}$$

and is used to find a control $u^i(t)$ minimizing a local quadratic performance index:

$$J^i = \mathbf{E}\left\{\int_0^\infty x^{i\prime}(t)Q^i(\xi^i(t))x^i(t) + u^{i\prime}(t)R^i(\xi^i(t))u^i(t)dt\right\} \tag{9}$$

where the cost weighting matrices $R^i(\xi^i(t))$ and $Q^i(\xi^i(t))$ are symmetric respectively positive definite and positive semidefinite for each $\xi(t)^i$.

Each ith nominal model is assumed to be stochastically stabilizable [5] and each pair $(A^{ii}(\alpha^i), C^i(\alpha^i))$ is observable for all $\alpha^i \in \mathcal{B}^i$ where $C^{i\prime}(\alpha^i)C^i(\alpha^i) = Q^i(\alpha^i)$. It is also assumed that all state variables of the ith subsystem are perfectly measurable. The information transmitted to the coordinator at each time t consists of values of the state vector norm $||x^i(t)||$, the mode $\xi^i(t)$ and the control $u^i(t)$ for each $i = 1, \ldots, L$. Based on these informations, the robustification control $v^i(t)$, $i = 1, \ldots, L$ is evaluated by the coordinator and then transmitted to the ith subsystem. The design objective is to find a feedback control law that guarantees robust stability of each subsystem. Moreover it will be shown that the control ensures robustness of the overall system in the sense of guaranteed cost property [4], [10] given by the inequality:

$$J = \sum_{i=1}^{L} J^i \leq \sum_{i=1}^{L} x^{i\prime}_0 K^i(\alpha^i)x^i_0 \tag{10}$$

where $K^i(\alpha^i)$, $(\alpha^i \in \mathcal{B}^i)$ is the set of the unique positive solutions of the coupled Riccati equations corresponding to the local JLQ problem (8)-(9) given by:

$$A^{ii\prime}(\alpha^i)K^i(\alpha^i) + K^i(\alpha^i)A^{ii}(\alpha^i) - K^i(\alpha^i)B^i(\alpha^i)R^{i^{-1}}(\alpha^i)B^{i\prime}(\alpha^i)K^i(\alpha^i) + \sum_{\beta^i=1}^{s^i} q_{\alpha^i\beta^i}K^i(\beta^i) + Q^i(\alpha^i) = 0$$
$$\alpha^i \in \mathcal{B}^i \tag{11}$$

The coordinator uses his own resources to realize his control policy $v^i(t)$ thus its cost is not included in the local performance index.

3 Control law design

The feedback control used in each subsystem is the sum of the local decision maker strategy and the coordinator's one. The local control law is found by minimizing (9) for the nominal model of the ith subsystem (8) and has for any given $\xi^i(t) = \alpha^i$ the following form [5]:

$$u^i(\alpha^i,t) = -R^{i^{-1}}(\alpha^i)B^{i'}(\alpha^i)K^i(\alpha^i)x^i(t) \quad (12)$$

The corresponding optimal cost for the nominal model is given by (for $\xi^i(0)=\alpha^i$):

$$J^{i^o} = x^{i'}_0 K^i(\alpha^i)x^i_0 \quad (13)$$

The coordinator has an information about the structure of the overall system including bounds on the incompletely known crosscouplings (5), (6) and uncertainties (7), and the actual information about the values of $u^i(\xi^i(t),t)$, $\|x^i(t)\|$ and $\xi^i(t)$ from all subsystems. This information is used to construct the control law v^i defined for each $\xi(t)=\alpha$ as follows:

$$v^i(\alpha,t) = \begin{cases} \frac{R^i(\alpha^i)u^i(\alpha^i,t)}{\|R^i(\alpha^i)u^i(\alpha^i,t)\|}\rho^i(\alpha,\|x(t)\|) & \text{if } u^i(\alpha^i,t)\neq 0 \\ 0 & \text{if } u^i(\alpha^i,t)=0 \end{cases} \quad (14)$$

where $\rho^i(\alpha,\|x(t)\|)$ is an upper bound on the entire uncertainty $\eta^i(\alpha,x(t),t)$ for the ith system, defined as:

$$\eta^i(\alpha,x(t),t) = \sum_{j=1,j\neq i}^{L} D^{ij}(\alpha)x^j(t) + e^i(\alpha,x(t),t)$$

where the matrice $D^{ij}(\alpha^i)$ comes from the use of the Eq. (5).

The upper bound $\rho^i(\alpha,\|x(t)\|)$ is defined by the following formula for any $\xi(t)=\alpha$:

$$\begin{aligned} \|\eta^i(\alpha,x(t),t)\| &= \Big\|\sum_{j=1,j\neq i}^{L} D^{ij}(\alpha)x^j(t) + e^i(\alpha,x(t),t)\Big\| \leq \\ &\leq \sum_{j=1,j\neq i}^{L} \|D^{ij}(\alpha)x^j(t)\| + \|e^i(\alpha,x(t),t)\| \leq \\ &\leq \sum_{j=1,j\neq i}^{L} d^{ij}(\alpha)\|x^j(t)\| + f^i(\alpha)\sum_{j=1}^{L}\|x^j(t)\| = \\ &= \rho^i(\alpha,\|x(t)\|) \end{aligned} \quad (15)$$

If we define $d^{ii}=0$, then

$$\rho^i(\alpha,\|x(t)\|) = \sum_{j=1}^{L}(d^{ij}(\alpha)+f^i(\alpha))\|x^j(t)\| \quad (16)$$

Thus the coordinator control law is also bounded.

To find the sufficient conditions for robust stochastic stability, let assume Lyapunov function candidate for each subsystem i in the form:

$$V(x^i,\alpha^i) = x^{i'}K^i(\alpha^i)x^i = S^i(x^i,\alpha^i) \quad (17)$$

where $S^i(x^i,\alpha^i)$ is the optimal cost to go for the nominal model of the ith subsystem (8) starting from $x^i_0, \xi^i(0)=\alpha^i$. Its expression is given by (13).

The following theorem gives the required conditions.

Theorem 3.1. Assume that the ith subsystem described by the state equation (1) meets the matching conditions (5)-(7) and is governed by the control law (12) and coordinator control (14). Then the overall system remains stochastically stable in the whole ranges of uncertainty.

The next theorem states the conditions for the guaranteed cost property.

Theorem 3.2. Assume that the ith subsystem described by the state equation (1) meets the matching conditions (5)-(7) and is governed by the control law (12) and coordinator control (14). Then the value of the performance index (9) for the subsystem does not exceed the optimal cost for the nominal model (8) given by the function:

$$S^i(x^i_0,\xi^i(0)=\alpha^i) = x^{i'}_0 K^i(\alpha^i)x^i_0 \quad (18)$$

The theorems may be proven using the same arguments as in [12] where however one level control structure has been used.

□

If the performance index of the overall system is defined as a sum of the local indices then the proposed control strategy guarantees the cost given by the formula (10).

4 Centralized JLQ problem and its robustification

In the centralized structure decision maker of the upper level has only linear models of the overall system neglecting uncertainties resulting from imprecisely known parameters of the subsystems and local environmental disturbances. A controller is found using the quadratic criterion for the system which is the sum of local cost functions and incorporates the information about its state. The role of the local decision maker is to ensure robust stability and guaranteed cost in spite of the uncertainties The uncertainty is described by deterministic inequality model and the main assumption is the matching condition. The local decision maker uses the information about local states and bounds for uncertainties to design the robust control actions which are added to the control variables transmitted by the coordinator to the subsystem. This control is nonlinear but it is bounded by the constraints imposed on the uncertainties. More specifically the coordinator control law for the ith subsystem is now given by:

$$\begin{aligned} &v^i(\alpha,t) = \\ &-R^{i^{-1}}(\alpha)B^{i'}(\alpha)K_c^i(\alpha)x(t) \end{aligned} \tag{19}$$

where $K_c^i(\alpha)$ is the ith block row of the solution $K_c(\alpha)$ to a coupled Riccati equation for the entire system:

$$\begin{aligned} &A'(\alpha)K_c(\alpha) + K_c(\alpha)A(\alpha) - \\ &K_c(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K_c(\alpha) \\ &+\sum_{\beta\in\mathcal{B}} q_{\alpha\beta}K_c(\beta) + Q(\alpha) = 0 \\ &\alpha \in \mathcal{B} \end{aligned} \tag{20}$$

$$K_c(\alpha) = [K_c^{1'}(\alpha), \cdots, K_c^{i'}(\alpha), \cdots, K_c^{L'}(\alpha)]' \tag{21}$$

x is the state of the system combined of the states of all subsystems while matrices A,B,R,Q are combined of the respective matrices for the subsystems.

On the other hand the local control law has now the following form:

$$u^i(\alpha^i,t) = \tag{22}$$
$$\begin{cases} \frac{R^i(\alpha^i)v^i(\alpha^i,t)}{\|R^i(\alpha^i)v^i(\alpha^i,t)\|}\rho^i(\alpha^i,\|x^i(t)\|) & \text{if } v^i(\alpha^i,t) \neq 0 \\ 0 & \text{if } v^i(\alpha^i,t) = 0 \end{cases}$$

where $\rho^i(\alpha^i,\|x^i(t)\|)$ is an upper bound on the local uncertainty $e^i(\alpha^i,x^i(t),t)$ for the ith subsystem, given by:

$$\|e^i(\alpha^i,x^i(t),t)\| \leq f^i(\alpha^i)\|x^i(t)\| \tag{23}$$

where $f^i(\alpha^i)$ is a known scalar.

The idea presented in this section is a complete contrast to the decentralized one described before nevertheless the same properties of the closed-loop system i.e. robust stochastic stability and guaranteed cost property are ensured (see [3] for proof). The main differences between the two approaches are in the complexity of computation performed by the specific levels and the amount of information required by decision makers in order to design their strategies. In the decentralized structure the coordinator should only be endowed in the sufficient amount of additional resources to support the action of local decision makers which should be able to solve quite complex local systems of the coupled Riccati equations.The information processed by the coordinator is very simple and could be easily obtained from the subsystems. In the centralized structure the complexity of the full order coupled Riccati equation solved by the coordinator may be really huge. On the other hand local decision makers have very simple task and should only care to compensate the effect of uncertainty disturbing locally their subsystems.

5 Conclusion

The main idea of the paper is to decompose the complex system into subsystems and to use hierarchical structure to ensure robustness in the sense of robust stability and guaranteed cost property. This purpose can be realized in two different structures. In the decentralized approach the control law minimizing the quadratic cost is decentralized while the effect of imprecisely known crosscouplings and uncertainties disturbing the subsystems is compensated by the coordinator. Although the control law depends on perfectly measurable state variables and modes but due to decentralization the local decision maker needs only to measure the local state variables. On the other hand the coordinator needs only an aggregated information from the subsystems in the form of the local control actions and the norm of the local state vectors.

In the case of MIMO large scale system decomposed into SISO subsystems it enables to transmit only three numbers from each subsystem to the coordinator at each time t and only one number from the coordinator to each subsystem. Moreover in this case the matching conditions imposed on the uncertain crosscoupling are not restricting at all. The centralized structure leads to the huge computational effort at the coordinator level where the coupled Riccati equation system should be solved. On the other hand the coordinator must only use the nominal model of the system while a knowledge of the uncertainties is used by local decision makers to design a simple local strategy which robustifies the closed-loop system.

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