Modelling and controlling two-degree-of-freedom impacts*

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Abstract

In this paper, it is considered the problem of modelling and controlling the impact of a two-degree-of-freedom body with an infinitely rigid and massive surface. Basic equations describing the motion of the body are derived, both in condition of non-contact and in condition of contact. A control scheme is proposed on the basis of an observer that is able to asymptotically estimate the impact-induced forces and to allow their asymptotic compensation when the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface. The paper is completed by a simulation test.

1 Introduction

The control of impact requires the study of the basic physical phenomena that happen when bodies collide under the action of external forces (including the control forces) and/or due to non-zero relative velocity. Several books (see, e.g., [1]-[10]) considered in detail, with a rigorous and extensive treatment, the study of the basic physical phenomena that attend the collision of bodies. The study of these phenomena is especially important in robotics [11, 12] because, each time a robotic manipulator interacts with the external world, the highest forces and greatest stresses (often, undesired) arise as a consequence of impact. Many robotic systems can fail their tasks if the impact forces are not properly recognised and taken under control. Numerous attempts have been made in the recent years to properly model the impact in robotics (see, e.g., [13]-[16]). At the author's knowledge, while the problem of modelling the impact has been widely studied, the problem of controlling the impact is still an open problem, due to the sudden change of the motion equations that happens when the bodies involved in the impact swish sharply from a condition of non-contact to a condition of contact.

In this paper (which is an extension of [17], where one-degree-of-freedom impacts are analysed), we consider not only the problem of modelling the impact of bodies, but also the problem of controlling the collision before, after and during the period of impact.

The outline of the paper is as follows. The classical theory of impact is briefly recalled in Section 2, with special enphasis on two-degree-of-freedom impacts. Section 3 is devoted to obtain the motion equations describing the two-degree-of-freedom impact that is schematically represented in Figures 1 and 2. The control design is discussed in Section 4, while the proposed control scheme is given in Section 5. The effectiveness of the proposed controller is tested in simulation in Section 6, while Section 7 draws the conclusions.

2 The classical theory of impact

The classical and simplest methods of the impact analysis are based on the law of conservation of momentum and on the law of conservation of energy (the latter law is valid under the assumption that the coefficient of restitution is equal to 1). These laws can be used to study the change in velocity of the centre of mass of each body involved in the collision before and after the impact, and the exchange of energy during the period of impact. This approach can be used only when the external forces that are exerted on each body involved in the collision are either null or negligible with respect to the impact-induced forces. Then, the classical methods cannot be used for control purposes, since they provide no information about the transient forces and about the duration of the impact, and since the external forces (which play often the role of input variables) will be never negligible with respect to the impact-induced forces, which constitute the output variables during the period of impact. Whence, it is necessary to use more basic equations for control purposes, which, due to their complexity, are to be properly approximated. In addition, since the type of the equations to be used for

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adequately describing the impact of two or more bodies depends largely upon the geometry of the bodies and upon the type of impact, suitable simplifications are also needed to state general results.

In this paper, impacts will be considered under the following two simplifications. The first simplification consists in assuming that all the stresses induced by the impact are well below the elastic limit, i.e., the plastic deformations are negligible. The second assumption consists in assuming that the centre of the radius of curvature of the surface of impact lies on the line connecting the centres of mass of the two bodies involved in the impact, so that the sliding at the surface of impact is avoided, together with its friction and rotational effects.

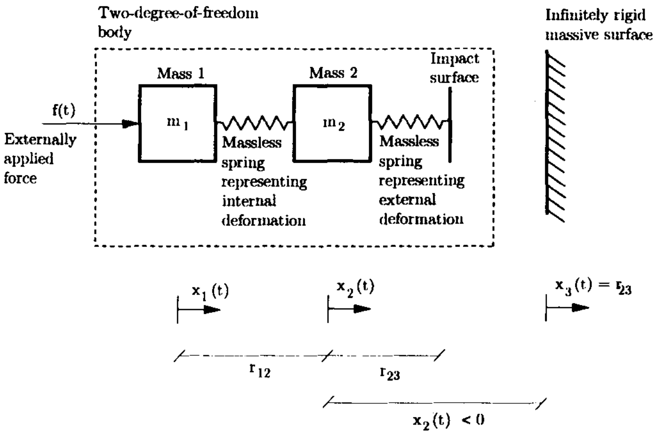

Figure 1: Schematic representation of collision between a two-degree-of-freedom body and an infinitely rigid and massive surface, before and after the impact ($x_2(t) < 0$).

The classical assumption of a *quasi-static* behaviour [17] during the impact is not made in this paper. The *quasi-static* behaviour assumption would imply the assumption that all the stresses are transmitted instantaneously to all the points of the bodies involved in the impact: this is not certainly satisfied when the bodies involved in the collision have non-uniform geometry and non-linear properties. Impacts involving these bodies can be conveniently described by multi-mass lumped systems, such as the one described in Figures 1 and 2, which is the simplest multi-mass structure representing the impact of a two-degree-of-freedom body against an infinitely rigid and massive surface. During the impact only the two-degree-of-freedom body will be deformed. The two-degree-of-freedom body will be adequately described as composed by four bodies, as depicted in Figures 1 and 2: two of these bodies are rigid and have masses m_1 and m_2, respectively, while the other two bodies are flexible and have negligible masses (the total mass of the two-degree-of-freedom body is $m_1 + m_2$). Then, the motion of the two-degree-of-freedom body is well described in terms of the positions $x_1(t)$ and $x_2(t)$ of the centres of mass of the two bodies m_1 and m_2, respectively. The position $x_3(t)$ of the centre of mass of the infinitely rigid and massive surface can be assumed to be in a steady-state and equal to $x_3(t) = c$, for some constant c. The two-degree-of-freedom body is assumed to be internally deformed if the centres of mass of bodies m_1 and m_2 are closer or farther than r_{12} (i.e., if $x_2(t) - x_1(t) > r_{12}$ or $x_2(t) - x_1(t) < r_{12}$), otherwise (i.e., if $x_2(t) - x_1(t) = r_{12}$) the two-degree-of-freedom body is assumed to be internally non-deformed, where r_{12} is some positive constant. The two-degree-of-freedom body and the infinitely rigid and massive surface are assumed to be in contact if the centre of mass of body m_2 and the centre of mass of the infinitely rigid and massive surface are closer than r_{23} (i.e., if $x_3(t) - x_2(t) \leq r_{23}$), while not in contact if these centres of mass are farther than r_{23} (i.e., if $x_3(t) - x_2(t) > r_{23}$), for some positive r_{23}. For convenience, one can take $c = r_{23}$ so that the inequalities $x_2(t) < 0$ and $x_2(t) \geq 0$ discriminate the condition of non-contact from the condition of contact.

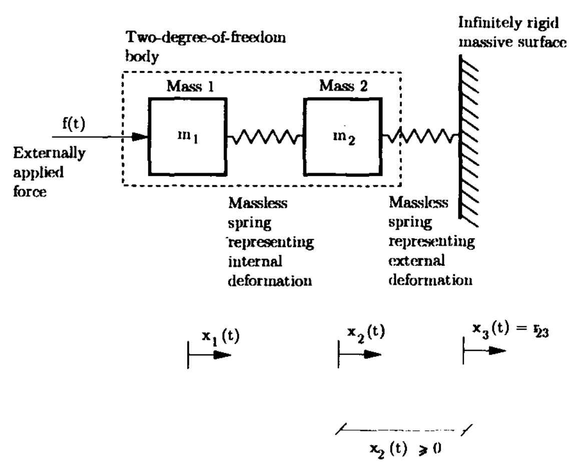

Figure 2: Schematic representation of collision between a two-degree-of-freedom body and an infinitely rigid and massive surface, during the period of impact ($x_2(t) \geq 0$).

3 Motion equations

As shown in Figures 1 and 2, the motion is assumed to be positive toward right and is controlled by force $f(t)$, which is exerted on mass body m_1 and is assumed to be positive toward right. Before and after the impact (i.e., when $x_2(t) < 0$), the equations of motion of the two-degree-of-freedom body are

$$
\begin{aligned}
m_1 \ddot{x}_1(t) + \phi_{12}(x_1(t) - x_2(t) + r_{12}) &= f(t), && \text{(1a)}\\
m_2 \ddot{x}_2(t) - \phi_{12}(x_1(t) - x_2(t) + r_{12}) &= 0, && \text{(1b)}
\end{aligned}
$$

where $\phi_{12}(\alpha)$ represents the impact-induced force as a function of the internal deformation α.

During the period of impact (i.e., when $x_2(t) \geq 0$), the equations of motion of the two-degree-of-freedom body are

$$m_1 \ddot{x}_1(t) + \phi_{12}(x_1(t) - x_2(t) + r_{12}) = f(t), \quad (2a)$$
$$m_2 \ddot{x}_2(t) - \phi_{12}(x_1(t) - x_2(t) + r_{12}) + \phi_{23}(x_2(t)) = 0, \quad (2b)$$

where $\phi_{23}(\beta)$ represents the impact-induced force produced at the surface of impact as a function of the external deformation $\beta \geq 0$. Even for perfectly elastic bodies, the impact-induced forces $\phi_{12}(\alpha), \phi_{23}(\beta)$ may not be linear functions of α and $\beta \geq 0$, respectively; linear elastic reaction forces happen only for small deformations α and $\beta \geq 0$. General forms of the reaction forces $\phi_{12}(\alpha), \phi_{23}(\beta)$ as functions of the deformations α and $\beta \geq 0$, which are useful for representing many collisions, are

$$\begin{aligned} \phi_{12}(\alpha) &= a_n \alpha^n, & (3a) \\ \phi_{23}(\beta) &= b_m \beta^m, & (3b) \end{aligned}$$

where n, a_n and m, b_m are constants. Constants $n = 1$ and $m = 1$ characterise *linear elastic collisions*, while $n = 3/2$ and $m = 3/2$ characterise *Hertz elastic collisions.* Good values for constants a_1, b_1 and $a_{3/2}, b_{3/2}$ are $a_1 = E_{12}\left(\frac{r_{12}}{2.46}\right)^{3/2}$, $b_1 = E_{23}\left(\frac{r_{23}}{2.46}\right)^{3/2}$, $a_{3/2} = E_{23}\sqrt{\frac{r_{12}}{2.46}\frac{1}{1.23}}$, $b_{3/2} = E_{23}\sqrt{\frac{r_{23}}{2.46}\frac{1}{1.23}}$, where E_{12} and E_{23} are the internal and external modulii of elasticity, respectively.

4 Discussion on control design

In this section, the control of the body before, during and after the impact is discussed, under the assumption of linear elastic deformations (i.e., under the assumption that $n = 1, m = 1$ in (3)).

4.1 Control design under the condition of non-contact

When the two-degree-of-freedom body is not in contact with the infinitely rigid and massive surface, it is possible to define the following state-space transformation:

$$\begin{aligned} z_1(t) &= x_2(t), & (4a) \\ z_2(t) &= \dot{x}_2(t), & (4b) \\ z_3(t) &= \frac{a_1}{m_2}\left(x_1(t) - x_2(t) + r_{12}\right), & (4c) \\ z_4(t) &= \frac{a_1}{m_2}\left(\dot{x}_1(t) - \dot{x}_2(t)\right), & (4d) \end{aligned}$$

which is non-singular with the following inverse transformation

$$\begin{aligned} x_1(t) &= \frac{m_2}{a_1} z_3(t) + z_1(t) - r_{12}, & (5a) \\ \dot{x}_1(t) &= \frac{m_2}{a_1} z_4(t) + z_2(t), & (5b) \\ x_2(t) &= z_1(t), & (5c) \\ \dot{x}_2(t) &= z_2(t). & (5d) \end{aligned}$$

In the z-coordinates, motion equations (1) become

$$\begin{aligned} \dot{z}_1(t) &= z_2(t), & (6a) \\ \dot{z}_2(t) &= z_3(t), & (6b) \\ \dot{z}_3(t) &= z_4(t), & (6c) \\ \dot{z}_4(t) &= -\frac{m_1 + m_2}{m_1 m_2} a_1 z_3(t) + \frac{a_1}{m_1 m_2} f(t). & (6d) \end{aligned}$$

It is easy to see that system (6) is in the controller canonical form, whence that the design of a suitable feedback control law for the motion equations is a straightforward task before and after the impact.

4.2 Control design under the condition of contact

When the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface, it is possible to define the following state-space transformation:

$$\begin{aligned} z_1(t) &= x_2(t), & (7a) \\ z_2(t) &= \dot{x}_2(t), & (7b) \\ z_3(t) &= \frac{a_1}{m_2}\left(x_1(t) - x_2(t) + r_{12}\right) - \frac{b_1}{m_2} x_2(t), & (7c) \\ z_4(t) &= \frac{a_1}{m_2}\left(\dot{x}_1(t) - \dot{x}_2(t)\right) - \frac{b_1}{m_2}\dot{x}_2(t), & (7d) \end{aligned}$$

which is non-singular with the following inverse transformation

$$\begin{aligned} x_1(t) &= \frac{m_2}{a_1} z_3(t) + z_1(t) - r_{12} + \frac{b_1}{a_1} z_1(t), & (8a) \\ \dot{x}_1(t) &= \frac{m_2}{a_1} z_4(t) + z_2(t) + \frac{b_1}{a_1} z_2(t), & (8b) \\ x_2(t) &= z_1(t), & (8c) \\ \dot{x}_2(t) &= z_2(t). & (8d) \end{aligned}$$

In the z-coordinates, motion equations (2) become

$$\begin{aligned} \dot{z}_1(t) &= z_2(t), & (9a) \\ \dot{z}_2(t) &= z_3(t), & (9b) \\ \dot{z}_3(t) &= z_4(t), & (9c) \\ \dot{z}_4(t) &= -\frac{m_1 + m_2}{m_1 m_2} a_1 z_3(t) + \frac{a_1}{m_1 m_2} f(t) \\ &\quad -\frac{a_1 b_1}{m_1 m_2} z_1(t) - \frac{b_1}{m_2} z_3(t). & (9d) \end{aligned}$$

It is easy to see that system (9) is in the controller canonical form, whence that the design of a suitable feedback control law for the motion equations is a straightforward task also during the period of impact.

4.3 Control design under the condition of non-contact/contact

It is readily seen from the above subsections that the control of the body is an easy task, provided that we are able to recognise the condition of contact from the condition on non-contact (which is certainly a unrealistic assumption, as discussed later on). As a matter of fact, denoting by $d(t)$ the reaction force induced by the impact, i.e.

$$d(t) = \begin{cases} 0, & \text{in condition of non-contact} \\ -b_1 x_2(t), & \text{in condition of contact,} \end{cases} \tag{10}$$

the motion equations can be rewritten as follows both in condition of contact and in condition of non-contact

$$m_1\,\ddot{x}_1(t) + a_1(x_1(t) - x_2(t) + r_{12}) = f(t), \tag{11a}$$
$$m_2\,\ddot{x}_2(t) - a_1(x_1(t) - x_2(t) + r_{12}) + d(t) = 0. \tag{11b}$$

In the same manner, the z-coordinates can be defined as follows both in condition of contact and in condition of non-contact

$$\begin{aligned}
z_1(t) &= x_2(t), && \text{(12a)}\\
z_2(t) &= \dot{x}_2(t), && \text{(12b)}\\
z_3(t) &= \frac{a_1}{m_2}(x_1(t) - x_2(t) + r_{12}) + \frac{1}{m_2}d(t), && \text{(12c)}\\
z_4(t) &= \frac{a_1}{m_2}(\dot{x}_1(t) - \dot{x}_2(t)) + \frac{1}{m_2}\dot{d}(t). && \text{(12d)}
\end{aligned}$$

In the z-coordinates given by (12), motion equations (11) become

$$\begin{aligned}
\dot{z}_1(t) &= z_2(t), && \text{(13a)}\\
\dot{z}_2(t) &= z_3(t), && \text{(13b)}\\
\dot{z}_3(t) &= z_4(t), && \text{(13c)}\\
\dot{z}_4(t) &= -\frac{m_1 + m_2}{m_1 m_2} a_1 z_3(t) + \frac{a_1}{m_1 m_2} d(t) \\
&\quad + \frac{a_1}{m_1 m_2} f(t) + \frac{1}{m_2}\ddot{d}(t). && \text{(13d)}
\end{aligned}$$

Denoting by $h_i, i = 1,2,3,4$, the suitable positive real numbers such that the polynomial

$$h(s) := s^4 + h_4 s^3 + h_3 s^2 + h_2 s + h_1 \tag{14}$$

is Hurwitz with the desired spectrum, and denoting by $x_{R,2}(t)$ the desired reference signal (which is assumed to be differentiable the needed number of times) to be tracked by $x_2(t)$, under the (unrealistic) assumption that the $d(t)$ term is measurable, as well as its time derivatives $\dot{d}(t)$ and $\ddot{d}(t)$, a feedback control law that ensures the asymptotic tracking of $x_{R,2}(t)$ by $x_2(t)$ with the desired transient behaviour expressed by (14), is

$$\begin{aligned}
f(t) &= (m_1 + m_2) z_3(t) - d(t) - \frac{m_1}{a_1}\ddot{d}(t) + \\
&- \frac{m_1 m_2}{a_1}\Big(h_1(z_1(t) - x_{R,2}^{(0)}(t)) + h_2(z_2(t) - x_{R,2}^{(1)}(t)) \\
&+ h_3(z_3(t) - x_{R,2}^{(2)}(t)) + h_4(z_4(t) - x_{R,2}^{(3)}) - x_{R,2}^{(4)}(t)\Big), \quad (15)
\end{aligned}$$

where $x_{R,2}^{(i)}(t)$ denotes the i-th time derivative of $x_{R,2}(t)$, $i = 0, ..., 4$.

As a matter of fact, the closed loop system (13), (15) takes the following form:

$$\begin{aligned}
\dot{\tilde{z}}_1(t) &= \tilde{z}_2(t), && \text{(16a)}\\
\dot{\tilde{z}}_2(t) &= \tilde{z}_3(t), && \text{(16b)}\\
\dot{\tilde{z}}_3(t) &= \tilde{z}_4(t), && \text{(16c)}\\
\dot{\tilde{z}}_4(t) &= -h_1\tilde{z}_1(t) - h_2\tilde{z}_2(t) - h_3\tilde{z}_3(t) \\
&\quad - h_4\tilde{z}_4(t), && \text{(16d)}
\end{aligned}$$

(where $\tilde{z}_i(t) := z_i(t) - x_{R,2}^{(i-1)}, i = 1,2,3,4$), independently of the fact that the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface or not. This nice behaviour is certainly difficult to be actually realised, since the $d(t)$ term will be in general not measurable. In practice, we will not be able to recognise by measurements if two bodies are in contact or not: it is very difficult to distinguish a sequence of impacts of small period of duration, one close to the other one, from one impact of long duration. This motivates the introduction of the state feedback control law proposed in the following section, which compensates the reaction force (if it is present) on the basis of an estimate supplied by a reduced order observer.

5 The proposed control law

Under the assumption of linear elastic deformations, the proposed feedback control law takes the following form

$$\begin{aligned}
\dot{\xi}_1(t) &= -3\mu\xi_1(t) + \xi_2(t) \\
&\quad - 6\mu^2(m_1\dot{x}_1(t) + m_2\dot{x}_2(t)) - 3\mu f(t), && \text{(17a)}\\
\dot{\xi}_2(t) &= -3\mu^2\xi_1(t) + \xi_3(t) \\
&\quad - 8\mu^3(m_1\dot{x}_1(t) + m_2\dot{x}_2(t)) - 3\mu^2 f(t), && \text{(17b)}\\
\dot{\xi}_3(t) &= -\mu^3\xi_1(t) - 3\mu^4(m_1\dot{x}_1(t) + m_2\dot{x}_2(t))
\end{aligned}$$

$$
\begin{aligned}
&- \mu^3 f(t), && (17c)\\
\hat{d}(t) &= \xi_1(t) + 3\mu(m_1\dot{x}_1(t) + m_2\dot{x}_2(t)), && (17d)\\
\hat{\dot{d}}(t) &= \xi_2(t) + 3\mu^2(m_1\dot{x}_1(t) + m_2\dot{x}_2(t)), && (17e)\\
\hat{\ddot{d}}(t) &= \xi_3(t) + \mu^3(m_1\dot{x}_1(t) + m_2\dot{x}_2(t)), && (17f)
\end{aligned}
$$

$$
\begin{aligned}
z_1(t) &= x_2(t), && (18a)\\
z_2(t) &= \dot{x}_2(t), && (18b)\\
z_3(t) &= \frac{a_1}{m_2}(x_1(t) - x_2(t) + r_{12}) + \frac{1}{m_2}\hat{d}, && (18c)\\
z_4(t) &= \frac{a_1}{m_2}(\dot{x}_1(t) - \dot{x}_2(t)) + \frac{1}{m_2}\hat{\dot{d}}(t), && (18d)\\
f(t) &= (m_1 + m_2)z_3(t) - \hat{d}(t) - \frac{m_1}{a_1}\hat{\ddot{d}}(t)\\
&- \frac{m_1 m_2}{a_1}\big(h_1(z_1(t) - x_{R,2}(t))\\
&+ h_2(z_2(t) - x_{R,2}^{(1)}(t))\\
&+ h_3(z_3(t) - x_{R,2}^{(2)}(t))\\
&+ h_4(z_4(t) - x_{R,2}^{(3)}(t)) - x_{R,2}^{(4)}(t),\big), && (18e)
\end{aligned}
$$

where (17) is a reduced-order asymptotic observer for the estimation of the $d(t)$ term, as well as of its time derivatives $\dot{d}(t)$ and $\ddot{d}(t)$, and (18) is the feedback control law (12), (15) with the terms $d(t), \dot{d}(t), \ddot{d}(t)$ substituted by their estimates $\hat{d}(t), \hat{\dot{d}}(t), \hat{\ddot{d}}(t)$ supplied by (17).

In the z-coordinates given by (18a)-(18d), motion equations (11) can be rewritten as follows:

$$
\begin{aligned}
\dot{z}_1(t) &= z_2(t), && (19a)\\
\dot{z}_2(t) &= z_3(t) + \frac{1}{m_2}(d(t) - \hat{d}(t)), && (19b)\\
\dot{z}_3(t) &= z_4(t), && (19c)\\
\dot{z}_4(t) &= -\frac{m_1 + m_2}{m_1 m_2}a_1 z_3(t) + \frac{a_1}{m_1 m_2}\hat{d}(t)\\
&+ \frac{a_1}{m_1 m_2}f(t) + \frac{1}{m_2}\dot{\hat{\dot{d}}}(t)\\
&+ \frac{a_1}{m_2^2}(d(t) - \hat{d}(t)). && (19d)
\end{aligned}
$$

Taking into account that

$$
m_1\ddot{x}_1(t) + m_2\ddot{x}_2(t) = d(t) + f(t), \qquad (20)
$$

under the action of the feedback control law (18e), equations (19) take the following form when the body is not in contact with the surface

$$
\begin{aligned}
\dot{\tilde{z}}_1(t) &= \tilde{z}_2(t), && (21a)\\
\dot{\tilde{z}}_2(t) &= \tilde{z}_3(t) - \frac{1}{m_2}\hat{d}(t), && (21b)\\
\dot{\tilde{z}}_3(t) &= \tilde{z}_4(t), && (21c)\\
\dot{\tilde{z}}_4(t) &= -h_1\tilde{z}_1(t) - h_2\tilde{z}_2(t) - h_3\tilde{z}_3(t)\\
&- h_4\tilde{z}_4(t) - \left(\frac{3\mu^2}{m_2} + \frac{a_1}{m_2^2}\right)\hat{d}(t), && (21d)
\end{aligned}
$$

where

$$
\begin{aligned}
\dot{\hat{d}}(t) &= -3\mu\hat{d}(t) + \hat{\dot{d}}(t), && (22a)\\
\dot{\hat{\dot{d}}}(t) &= -3\mu^2\hat{d}(t) + \hat{\ddot{d}}(t), && (22b)\\
\dot{\hat{\ddot{d}}}(t) &= -\mu^3\hat{d}(t), && (22c)
\end{aligned}
$$

$\tilde{z}_i(t) := z_i(t) - x_{R,2}^{(i-1)}, i = 1,2,3,4$, and μ is a suitable positive real.

Taking into account (20), under the action of the feedback control law (18e), equations (19) take the following form when the body is in contact with the surface

$$
\begin{aligned}
\dot{\tilde{z}}_1(t) &= \tilde{z}_2(t), && (23a)\\
\dot{\tilde{z}}_2(t) &= \tilde{z}_3(t) + \frac{1}{m_2}(d(t) - \hat{d}(t)), && (23b)\\
\dot{\tilde{z}}_3(t) &= \tilde{z}_4(t), && (23c)\\
\dot{\tilde{z}}_4(t) &= -h_1\tilde{z}_1(t) - h_2\tilde{z}_2(t)\\
&- h_3\tilde{z}_3(t) - h_4\tilde{z}_4(t)\\
&+ \left(\frac{3\mu^2}{m_2} + \frac{a_1}{m_2^2}\right)(d(t) - \hat{d}(t)), && (23d)
\end{aligned}
$$

where

$$
\begin{aligned}
\dot{\hat{d}}(t) &= 3\mu(d(t) - \hat{d}(t)) + \hat{\dot{d}}(t), && (24a)\\
\dot{\hat{\dot{d}}}(t) &= 3\mu^2(d(t) - \hat{d}(t)) + \hat{\ddot{d}}(t), && (24b)\\
\dot{\hat{\ddot{d}}}(t) &= \mu^3(d(t) - \hat{d}(t)), && (24c)
\end{aligned}
$$

$\tilde{z}_i(t) := z_i(t) - x_{R,2}^{(i-1)}, i = 1,2,3,4$, and μ is a suitable positive real.

As one can see, system (21),(22) is asymptotically stable for all positive μ, i.e., the closed loop system (11), (17), (18) is asymptotically stable when the body is not in contact with the surface, for all positive μ. If $\hat{d}(0)$, $\hat{\dot{d}}(0)$, $\hat{\ddot{d}}(0)$ are chosen equal to 0 and the body is (at the initial time $t = 0$) not in contact with the surface, then $\hat{d}(t) = 0$, $\hat{\dot{d}}(t) = 0$, $\hat{\ddot{d}}(t) = 0$ for all times t up to the time of the first contact between the body and the surface, and, in addition, in such a period the behaviour of the $\tilde{z}_i(t)$'s will be exactly the desired one (16). On the contrary, the asymptotic stability of (22), (23) depends strongly on the value of μ. It can be easily recognised from (24) that the estimates $\hat{d}(t)$, $\hat{\dot{d}}(t)$, $\hat{\ddot{d}}(t)$ of $d(t)$, $\dot{d}(t)$, $\ddot{d}(t)$ are designed so to negatively react against variations of the estimation error $d(t) - \hat{d}(t)$, and that such an effect can be increased by making greater the value of

μ. In addition, for small values of the estimation error $d(t) - \hat{d}(t)$, it is noted that the behaviour of the $\tilde{z}_i(t)$'s will be very close to the desired one (16).

The following theorem, which ensures the desired properties of asymptotic stability with a prescribed rate of convergence and of steady-state response, can be proved as Theorem 2 of [18], where it is also shown how an estimate of the region of attraction of the closed loop system (11), (17), (18) can be obtained in case of non-linearity (induced, in this case, by the non-linear elastic deformations if they are present); the proof is omitted because it is very similar to the one of Theorem 2 of [18].

Theorem 1 *Under the assumption that the $h_i, i = 1,2,3,4$, are positive and chosen so that the polynomial (14) is Hurwitz (with the desired spectrum),*

(i) the closed loop (11), (17), (18) is asymptotically stable for all positive μ and the $\tilde{z}_i(t)$, $i = 1,2,3,4$, asymptotically converge to 0 for all $x_{R,2}(t)$, when the two-degree-of-freedom body is not in contact with the infinitely rigid and massive surface;

(ii) there exists a positive μ^* such that if $\mu > \mu^*$ then the closed loop system (11), (17), (18) is asymptotically stable also when the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface; in addition, if $x_{R,2}(t)$ is constant, then the tracking errors $\tilde{z}_i(t)$, $i = 1,2,3,4$, asymptotically go to zero also when the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface;

(iv) if $x_{R,2}(t)$ and its time derivatives are bounded, then for any $\varepsilon > 0$ there exists a positive $\mu^*_\varepsilon \geq \mu^*$ such that if $\mu > \mu^*_\varepsilon$ then the tracking errors $\tilde{z}_i(t)$, $i = 1,2,3,4$, asymptotically become smaller than ε when the two-degree-of-freedom body is in contact with the infinitely rigid and massive surface.

The values of the lower bounds μ^*, μ^*_ε are cumbersome functions of the system parameters and of the reference signal: they can be easily computed following the same reasoning used in the proof of Theorem 2 of [18]. In practice, parameter μ can be chosen by increasing its value up to the accomplishment of the desired properties, which can be simply tested by simulation.

6 Simulation test

A simulation test has been carried out to confirm the properties of the closed loop system (11), (17), (18) that are stated in Theorem 1. For the sake of simplicity, only elastic linear deformations are considered. The masses m_1, m_2 and the constants a_1, b_1 have been chosen as follows:

$$m_1 = 0.1 \text{ kg}, \quad m_2 = 0.1 \text{ kg},$$

$$a_1 = 10^5 \text{ N/m}, \quad b_1 = 10^5 \text{ N/m}.$$

The two-degree-of-freedom body has been assumed to be spherical with a radius $r = 0.01$ m; whence, r can be taken as the distance r_{12} between the masses m_1 and m_2, discriminating the condition of internal deformation from the condition of non-deformation. Since the period of impact under the action of constant external forces is of the order of some milliseconds, the positive real numbers h_i's have been chosen as follows $h_1 = 6.25 \cdot 10^{10}$, $h_2 = 5 \cdot 10^8$, $h_3 = 1.5 \cdot 10^6$, $h_4 = 2 \cdot 10^3$, which correspond to time constants equal to 2 ms. The reference signal $x_{R,2}(t)$ has been taken constant so that the impact-induced force is regulated to 1 N: i.e., $x_{R,2}(t) = 1/a_1$. Few simulative attempts have shown that a good value of parameter μ is $\mu = 2000$.

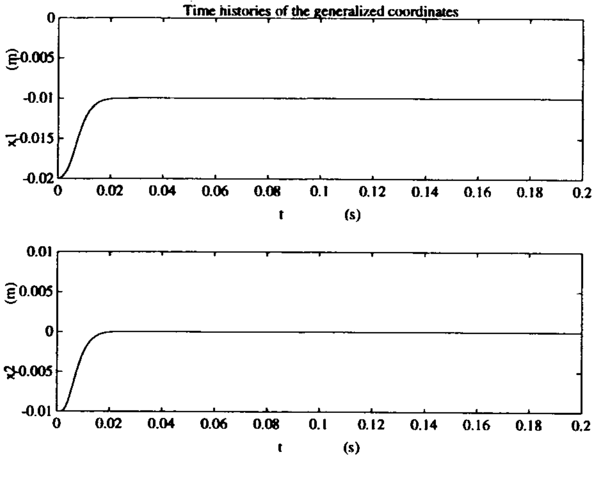

Figure 3: Time histories of the generalised coordinates.

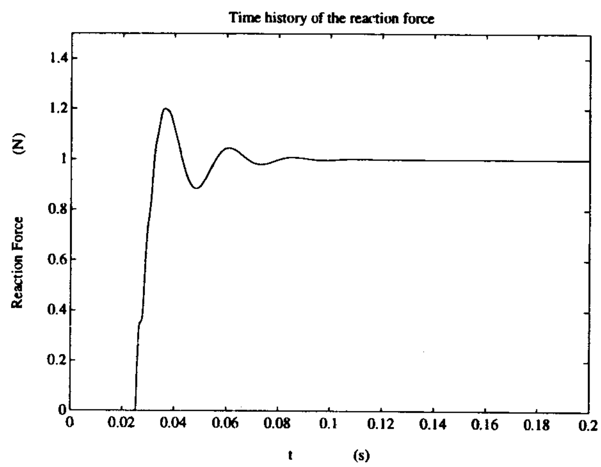

Figure 4: Time history of the impact-induced force.

For this values, Figures 3 report the time-histories of the positions of masses m_1 and m_2, while Figure

4 reports the time-history of the impact-induced force starting from the following initial conditions $x_1(0) = -2r_{12}\,\mathrm{m}, x_2(0) = -r_{12}\,\mathrm{m}, \dot{x}_1(0) = 0\,\mathrm{m/s}, \dot{x}_2(0) = 0\,\mathrm{m/s}$: i.e., at the initial time the two-degree-of-freedom body is in a steady-state position and is not in contact with the surface; the initial condition of the $\xi_i(t)$'s have been chosen equal to 0. These time histories have been reported for an interval of 0.2 s. As one can see from Figure 3, the positions of the two masses m_1 and m_2 asymptotically go to constant values with a fast transient, while the contact between the body with the surface begins at time $t \approx 0.02$ s and continues for all successive times; whence, the impact-induced force asymptotically reaches the desired value of 1 N.

7 Conclusions

In this paper, it has been considered the problem of modelling and controlling the impact of a two-degree-of-freedom body with a infinitely rigid and massive surface. Basic equations describing the motion of the body have been derived, both in condition of non-contact and in condition of contact. A control scheme has been proposed on the basis of an observer that is able to asymptotically estimate the impact-induced forces and to allow their asymptotic compensation when the two-degree-of-freedom body is in contact with the infinitely rigid and massive. Simulation results have confirmed the effectiveness of the control scheme proposed here.

References

[1] J. A. Zukas, *Impact Dynamics.* New York: John Wiley and Sons, Inc., 1982.

[2] R. J. Wasley, *Stress wave propagation in solids, an introduction.* New York: Marcel Dekker, Inc., 1973.

[3] W. Goldsmith, *Impact.* London: Edward Arnold (Publisher) Ltd., 1960.

[4] J. S. Rhinehart and J. Pearson, *Behaviour of metals under impulsive loads.* Cleveland: American Society for Testing and Materials, 1954.

[5] J. D. Achenbach, *Wave propagation in elastic solids.* New York: American Elsevier Publishing Co., Inc., 1975.

[6] W. Johnson, *Impact strength of materials.* New York: Crane, Russak and Co., Inc., 1972.

[7] U. S. Lindholm, *Mechanical behaviour of materials under dynamic loads.* New York: Springer-Verlag, 1968.

[8] N. J. Huffinston, *Behaviour of materials under dynamic loading.* New York: American Society of Mechanical Engineering, 1965.

[9] R. Kinslolo, *High velocity impact phenomenon.* New York: Academic Press, Inc., 1970.

[10] H. Kosly, *Stress waves in solids.* Fair Lawn, NJ: Oxford University Press, 1953.

[11] B. Brogliato, P. Orhant, and R. Lozano, "On the transition phase in robotics - part 1: impact models and dynamics," in *IFAC Symposium on Robot Control '94*, (Capri, Italy), September 1994.

[12] B. Brogliato, P. Orhant, and R. Lozano, "On the transition phase in robotics - part 2: control," in *IFAC Symposium on Robot Control '94*, (Capri, Italy), September 1994.

[13] W. B. Gevarter, "Basic relations for control of flexible vehicles," *AIAA Journal*, vol. 8, pp. 666–672, April 1972.

[14] B. Chapnik, G. Heppler, and J. Aplevich, "Modelling impact on a one-link flexible robotic arm," *IEEE Trans. on Robotics and Automation*, vol. 7, pp. 479–488, August 1991.

[15] A. Yigit, A. Ulsoy, and R. Scott, "Dynamics of a radially rotating beam with impact," *ASME J. of Vibration and Acoustics*, vol. 112, pp. 515–525, January 1990.

[16] Q. F. Wei, P. S. Krishnaprasad, and W. P. Dayawansa, "Modeling of impact on a flexible beam," in *Proc of 32nd Conference on Decision and Control*, vol. 2, (San Antonio, Tx), pp. 1377–1382, December 1993.

[17] A. Tornambè, "Modelling and controlling two bodies before, after and during the period of a one-degree-of-freedom impact," in *7th International Conference on Advanced Robotics*, (Sant Feliu de Guixols, Spain), September 1995.

[18] A. Tornambè and P. Valigi, "A decentralized controller for the robust stabilization of a class of mimo dynamical systems," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 116, pp. 293–304, June 1994.