TRELLIS CODED DPSK MODULATION IN TIME-SELECTIVE FADING CHANNELS

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Abstract: It is well known that trellis-coded modulation (TCM) gives good error performance in fading channels. In this paper two types of fading are analysed: flat (non-selective) and time-selective. DPSK modulation with differential detection combined with trellis-coding is considered. An adequate closed form expression for probability of error in flat fading channel is derived. Then, with some transformations in receiver's matched filter design, the case of time-selective fading channel is studied. Absolutely original results for bit error rate showing efficiency of TCM scheme are obtained and discussed.

1. INTRODUCTION

It is well known that trellis-coded modulation (TCM) is a class of signaling techniques which achieves good power and bandwidth efficiency [1]. At the beginning it was developed for telephone channels while in recent years it becomes very attractive for use in multipath fading channels [2]. In this paper TCM performance evaluation for two different types of fading channels is performed. For both considered cases: non-selective, and time-selective fading, adequate receivers structures are described. Apart from that, DPSK modulation of transmitted signal is assumed as it is modulation technique frequently applied in communications systems where multipath fading is a predominant problem .

In the paper, we first analyse the effects of TCM signaling scheme on error performance of channel with non-selective (flat) fading. An appropriate relation for bit error rate is derived considering fading with Rayleigh statistics. It is obtained in closed analytical form not as upper bound as it could be found in literature for DPSK and other modulation techniques. Then, using the obtained relation, error performance is analysed in the case when the channel is characterized with time-selective multipath fading. To our knowledge the results presented in this paper give absolutely new facts about the TCM method efficiency in such communication systems and are certain contribution to mobile air radio systems performance improvement.

2. SYSTEM MODEL

Basic block diagram of the system considered is given in Fig.1. The input of the trellis encoder is a sequence of binary digit $\mathbf{a}=(a_1,\ldots,a_n,\ldots)$. The encoder output is a sequence of PSK symbols $\mathbf{C}_i=(c_{i1},\ldots,c_{ik},\ldots)$, where $\mathbf{C}_i$ denotes ith code word of the TCM scheme. In complex notation, c_{ik} is a point in the complex plane. Normalizing every symbol of the signal constellation, mean value becomes $E\{|c_{ik}|^2\}=1$.

Then the baseband equivalent of the transmitted signal becomes:

$$s(t) = A\sum_{k=-\infty}^{\infty} c'_{ik}\, p(t-kT) \qquad (1)$$

where c'_{ik} are symbols that appear at the interleaver output. p(t) is complex impulse response of a pulse shaping filter satisfying Nyquist's criterion for zero intersymbol interference, T is symbol duration and A is a constant.

Symbol energy is normalized so that

$$\int_{-\infty}^{\infty} |p(t)|^2 dt = 1$$

The baseband equivalent of the received signal is:

$$r(t)=g(t)s(t)+n_w(t) \qquad (2)$$

where g(t) is a zero mean, complex Gaussian fading process with variance σ_g^2. $n_w(t)$ is complex envelope of the channel white Gaussian noise with double-sided power spectral density N_0. The average received symbol energy is $E_s=A^2\sigma_g^2$. If each modulation symbol contains n bits of information, the average received energy per bit is $E_b=E_s/n$.

Output of the matched filter is sampled every T seconds and assuming an ideal interleaver at its output sample value can be expressed with:

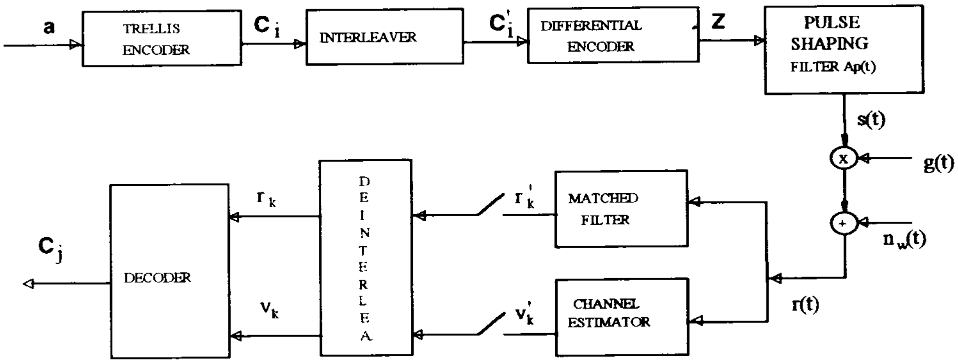


Fig. 1. Block diagram of the trellis coded DPSK system

$$r_k = \frac{Ag_k}{\sqrt{N_0}} c_{ik} + n_k = u_k c_{ik} + n_k \tag{3}$$

Known statistics of g_k's implies that u_k's are also statistically independent and indentically distributed complex Gaussian random variables with zero mean and variance $\sigma_u^2 = A^2\sigma_g^2/N_0 = E_s/N_0$. The noise samples n_k in (3) are also iid complex Gaussian with variance $\sigma_n^2=1$. Thus, if c_{ik} is transmitted then r_k is Gaussian random variable with zero mean and variance $\sigma^2_{r|cik}=1+|c_{ik}|^2E_s/N_0$.

At the output of the channel estimator/deinterleaver appears the sequence $v=(v_1,...v_k,...)$ where v_k is the estimate of u_k. u_k and v_k are independent, identically distributed variables, with zero mean while their covariance is $\sigma^2_{uv}=0{,}5E\{u_k v_k^*\}$. The corresponding correlation coefficient is $\mu=\sigma^2_{uv}/(\sigma_u\sigma_v)$, where σ_u^2 and σ_v^2 are variances of u_k and v_k, respectively.

In the receiver using Viterbi algorithm, MLS estimation is performed for two received sequences $r=(r_1,...,r_k,...)$ and $v=(v_1,...,v_k,...)$. The output of the decoder is the sequence $\mathbf{C_j}=(c_{j1},...,c_{jk},...)$ which is an estimate of the input sequence $\mathbf{C_i}=(c_{i1},...,c_{ik},...)$.

Assuming optimum decoder, the codeword $\mathbf{C_j}$ with the largest a posteriori probability $P(\mathbf{C_j}|\mathbf{r},\mathbf{v})$ is selected. In other words, the codeword $\mathbf{C_j}$ with the smallest metric is selected:

$$M_j = -\sum_k \ln\{p(r_k|c_{jk},v_k)\} \tag{4}$$

The above relation could be transformed in an appropriate form knowing the conditional probability density function $p(r_k|c_{jk},v_k)$. As u_k and v_k are correlated complex Gaussian random variables with known $p(u_k,v_k)$ [3] and $p(v_k)$, it could be found that:

$$p(u_k|v_k) = \frac{p(u_k,v_k)}{p(v_k)} =$$

$$= \frac{1}{\sqrt{2\pi}\sigma_u\sqrt{1-|\mu|^2}} e^{-\frac{1}{2}\frac{(u_k-\mu\sigma_u v_k/\sigma_v)^2}{\sigma_u^2(1-|\mu|^2)}} \tag{5}$$

It also means that for given v_k and c_{jk}, the random variable r_k is Gaussian with the following mean value and variance

$$E\{r_k|v_k,c_{jk}\} = \eta_{jk} = \mu v_k c_{jk}\sigma_u/\sigma_v;$$

$$\sigma_{jk}^2 = (1-|\mu|^2)\sigma_u^2|c_{jk}|^2+1; \tag{6}$$

Substituting (5) and (6) into (4), the final expression for metric is obtained:

$$M_j = \sum_k \left[\frac{|r_k - \beta c_{jk}v_k|^2}{2(|c_{jk}|^2\sigma_e^2+1)} + \ln\{|c_{jk}|^2\sigma_e^2+1\} \right] \tag{7}$$

where $\beta=\mu\sigma_u/\sigma_v$ and $\sigma_e^2=(1-|\mu|^2)\sigma_u^2$.

So, if the transmitted codeword is $\mathbf{C_i}=(c_{i1},...,c_{ik},...)$, the received codeword $\mathbf{C_j}$ will be erroneous if $M_j<M_i$, or:

$$D = \sum_k D_k \le \delta \tag{8}$$

In (8) D_k's are given by [4]:

$$D_k = A_k|r_k|^2 + B_k|v_k|^2 + C_k r_k v_k^* + C_k^* r_k^* v_k \tag{9}$$

with:

$$A_k = \frac{1}{2}\left(\frac{1}{\sigma_{jk}^2} - \frac{1}{\sigma_{ik}^2}\right),$$

$$B_k = \frac{|\beta|^2}{2}\left(\frac{|c_{jk}|^2}{\sigma_{jk}^2} - \frac{|c_{ik}|^2}{\sigma_{ik}^2}\right),$$

$$C_k = \frac{\beta^*}{2}\left(\frac{c_{ik}^*}{\sigma_{ik}^2} - \frac{c_{jk}^*}{\sigma_{jk}^2}\right),$$

$$\sigma_{ik}^2 = |c_{ik}|^2\sigma_e^2+1,$$

$$\sigma_{jk}^2 = |c_{jk}|^2\sigma_e^2+1, \tag{10}$$

and

$$\delta = \ln\left(\prod_k \frac{|c_{ik}|^2\sigma_e^2+1}{|c_{jk}|^2\sigma_e^2+1}\right) = \ln\left(\prod_k \frac{\sigma_{ik}^2}{\sigma_{jk}^2}\right) \tag{11}$$

For the considered case of DPSK the above relations are greatly simplified since A_k's, B_k's and δ are all zero.

The probability of confusing the codeword $\mathbf{C}_i$ with the codeword $\mathbf{C}_j$, $P(\mathbf{C}_i \rightarrow \mathbf{C}_j)$, is actually the probability that random variable D is less than δ. The final expression for $P(\mathbf{C}_i \rightarrow \mathbf{C}_j)$ could be derived in the form [5]:

$$P(C_i \rightarrow C_j) = \begin{cases} -\sum \mathrm{Re}\,sidue\left[e^{s\delta}\dfrac{\Phi_D(s)}{s}\right]_{RP}, & \delta \le 0 \\ \sum \mathrm{Re}\,sidue\left[e^{s\delta}\dfrac{\Phi_D(s)}{s}\right]_{LP}, & \delta > 0 \end{cases} \tag{12}$$

where $\Phi_D(s)$ is two-sided Laplace transform of the probability density function od D [5]:

$$\Phi_D(s) = \left(\prod_k \frac{\{1+|c_{jk}|^2(1-|\mu|^2)\dfrac{E_s}{N_0}}{|\mu|^2 d_k^2 \dfrac{E_s}{N_0}} \right) \cdot \left(\prod_k \frac{-1}{(s-p_{1k})(s-p_{2k})} \right) \tag{13}$$

$d^2_k=|c_{ik}-c_{jk}|^2$ is the squared Euclidean distance between c_{ik} and c_{jk} in the signal space, while p_{1k} and p_{2k} are the left plane and right plane poles, respectively.

3. PROBABILITY OF ERROR IN NON-SELECTIVE FADING CHANNEL

Assuming trellis-coded DPSK transmission via flat (non-selective) fading channel, block diagram of the system is that given in Fig.1 with matched filter having an impulse response $p^*(-t)/(N_0)^{1/2}$.

Probability that codeword is erroneously interpreted at the system output is given with (12). For standard signal constellation given with Ungerboeck's 8 state PSK code [6], probability of bit error rate could be found using the relation:

$$P_b \approx \frac{1}{n}\sum_j m_{ij} P(C_i \rightarrow C_j) \tag{14}$$

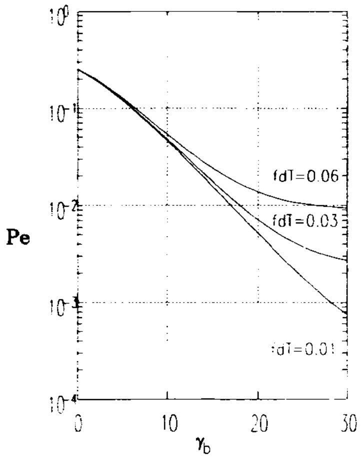
where n is the number of input bits per encoding interval, m_{ij} is the number of bit - errors associated with each error event. In (14) dominant error events are taken into account, i.e. events that are characterized with:

$$L(C_i, C_j) \le N \tag{15}$$

where $L(\mathbf{C}_i,\mathbf{C}_j)$ denotes the number of encoding intervals it takes for $\mathbf{C}_j$ to merge with $\mathbf{C}_i$ in the trellis diagram. In this P_b computation dominant error events with N=4 are considered.

Now, it is possible to find probability of error for the analysed case of differential encoder at the interleaver output and conventional differential detection at the receiver. Assuming fading with Rayleigh statistics it could be found: $\sigma_v^2=1+E_s/N_0$ and $\sigma^2_{uv}=(A^2/N_0)R_g(T)z^*_{l-1}$ where $R_g(\tau)=\sigma_g^2 J_0(2\pi f_d \tau)$ is autocorrelation function of the fading process with f_d being the maximum Doppler frequency. $z=(z_1,\ldots,z_k,\ldots)$ denotes the sequence after differentially encoding and $J_0(\cdot)$ is the Bessel function of order zero. Also, variable δ in (8) for above mentioned conditiones becomes zero, while $|c_{ik}|=1$ for every k. Thus, it can be shown that:

$$|\mu|^2 = \frac{J_0^2(2\pi f_d T)}{1+\Gamma^{-1}}; \qquad \Gamma = \frac{E_s}{N_0} \tag{16}$$



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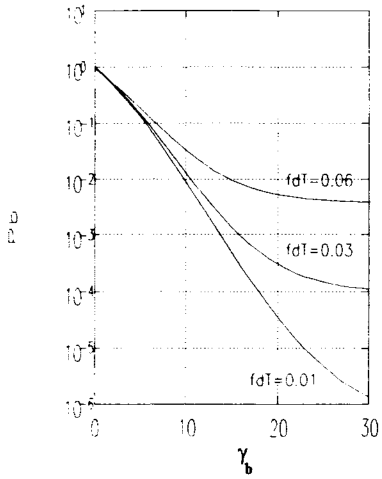


b)

Fig.2. Bit error rate for non-selective fading channel:
a) DPSK modulation
b) Trellis-coded DPSK modulation

Using (12) and (13), probabilities of particular error events could be determined. For the shortest error event the probability of erroneous reception of codeword is given with:

$$P(C_i \to C_j) = \frac{1}{\Gamma_e^2} \times$$

$$\times \frac{1+\Gamma_e(\sqrt{1+2\Gamma_e^{-1}}-\sqrt{1+\Gamma_e^{-1}})}{(1+\sqrt{1+2\Gamma_e^{-1}})(1+\sqrt{1+\Gamma_e^{-1}})(\sqrt{1+2\Gamma_e^{-1}})(\sqrt{1+\Gamma_e^{-1}})} \quad (17)$$

where Γ_e is equivalent signal-to-noise ratio:

$$\Gamma_e = \frac{|\mu|^2\Gamma}{1+(1-|\mu|^2)\Gamma} \quad (18)$$

Substituting (17) into (14), bit error probability as a function of $\gamma_b=\Gamma/n$ could be easily calculated for different fading rate f_dT. Results obtained, as well as for DPSK system without TCM, are given in Fig.2. For DPSK scheme without TCM, well known expression for bit error rate in flat fading environment is given by [4]:

$$P_e = \frac{1}{2}\left(1-\sqrt{\frac{|\mu|^2\Gamma}{1+\Gamma}}\right) \quad (19)$$

Efficiency of trellis coded modulation is obvious. Also, better effects are performed for smaller fading rates and for signal-to-noise ratio above 10dB.

4. PROBABILITY OF ERROR IN TIME-SELECTIVE FADING CHANNEL

Trellis-coded DPSK system for the case when transmitted signal is corrupted by time-selective fading process and additive white Gaussian noise, is given in Fig.1. The multiplicative gain factor modelling time non-flat fading g(t) is 1+m(t), with m(t) a complex Gaussian process. In this type of fading channel, matched filer is realized so that its output is defined with:

$$\frac{1}{T\sqrt{S}}\int_0^T r(t)s(t)dt \quad (20)$$

where S is received signal power.

Non-flat fading characteristics m(t) could be expressed in terms of its average component and a zero-mean fluctuating component. The fluctuations about the average value give rise to a performance degradation which is directly attributed to the Doppler spread effect. Actually, the decorrelation from symbol to symbol in DPSK signal induced by channel fluctuations causes certain number of errors since the average phase in one DPSK symbol is used as the demodulation reference for the next symbol.

For the purpose of this calculations, it is assumed that the spectrum of m(t) is second-order Butterworth. Thus:

$$M(\omega) = \frac{1}{1+\frac{\omega^4}{4a^4}} \quad (21)$$

where 3dB bandwidth of the process is $f_{3dB}=2^{1/2}a/2\pi$ [Hz]. The Doppler spread of the proces is then $D=2^{1/2}a/\pi=2f_{3db}$ [Hz]. The autocorrelation of m(t) is:

$$K_m(\tau)=K_m(0)\exp(-a|\tau|)(\cos a\tau+\sin a|\tau|) \quad (22)$$

where $K_m(0)=I$ is the multipath fading process power.

Bit error rate could be found by using (14) after defining an equivalent received signal-to-noise ratio which expresses the effect of time-selective fading process. With the characteristics of the matched filter given by (20), after some mathematical transformations it could be shown that:

$$\Gamma_{eq} = \frac{\Gamma}{1+\Gamma\cdot\Psi} \quad (23)$$

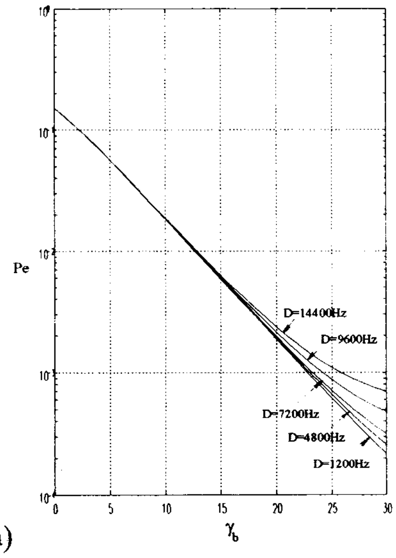
In (23) Ψ denotes the following expression:

$$K_m(0)\{\frac{1}{aT}\left[\frac{e^{-aT}}{aT}(\cos aT-\sin aT)-\frac{1}{aT}+2\right]-$$

$$-\frac{1}{(aT)^2}\left[\frac{e^{-2aT}}{aT}\sin 2aT-\frac{2e^{-aT}}{aT}\sin aT+2aT\right]+$$

$$+K_m(0)\{\frac{1}{(aT)^3}[\frac{4(aT)^2}{3}+\frac{2e^{-aT}}{aT}(\cos aT+\sin aT)-\frac{3}{2aT}-$$

$$-\frac{e^{-2aT}}{2aT}(\sin 2aT+\cos 2aT)]\} \quad (24)$$



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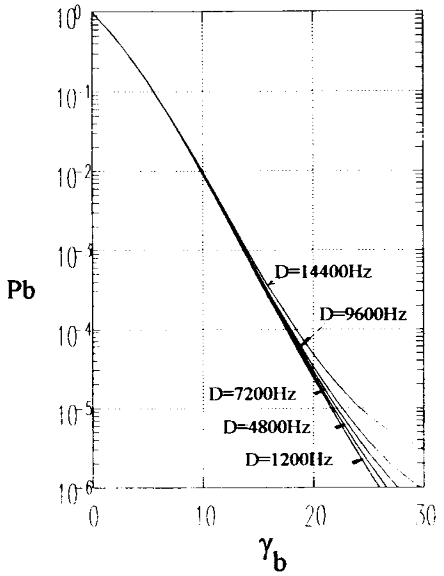

b)

Fig.3. Bit error rate for time-selective fading:
a) DPSK modulation
b) Trellis-coded DPSK modulation

Thus instead of Γ in all relevant relations for P_b calculations given for the previously analysed case of flat fading channel, Γ_{eq} is used.

Results for bit error rate for DPSK in time-selective fading channel are obtained and shown in Fig. (3.a) (without TCM) and Fig. (3.b) (with TCM). P_b is given as a function of γ_b for different values of Doppler spread and signalling rate of 9600b/s.

For DPSK scheme without TCM in time-selective fading, the following expression for bit error rate is used [7]:

$$P_e = \left(2 + 2\frac{\Gamma_{eq}}{S/I_{eq}}\right)^{-1} \exp\left[-\frac{\Gamma_{eq}}{1+\Gamma_{eq}/(S/I_{eq})}\right] \quad (25)$$

where I_{eq} is first term in (24).

It is evident that as the value of Doppler spread is of the order of bit rate and beyond a significant performance improvement is present. For Doppler spread above bit rate number of errors increases. The same trend could be observed for conventional DPSK as well as for the case where TCM is implemented. Comparing the obtained values for bit error rate a considerable performance improvement with trellis-coding is evident. It is achieved for every value of Doppler spread. Results presented in this paper could be used in obtaining better quality of digital signal transmission in mobile air radio channel of any kind.

5. CONCLUSION

In this paper trellis-coded DPSK modulation is analysed. Effects of such modulation scheme are evaluated through bit error rate calculations. Two types of mobile radio channels are considered: with non-selective fading and with time-selective fading. For both cases adequate receivers' structures are described. Actual difference between receivers is in matched filter realization. Considering first the channel with non-selective fading of Rayleigh statistics, the closed form expression for trellis-coded DPSK bit error rate is derived. Effects of trellis-coding are exactly determined. Then, characteristics of time-selective fading channel are described. For trellis-coded DPSK scheme an adequate relation for bit error rate is derived using the relation for the same modulation in the flat fading channel. Presented results are original and prove a significant performance improvement for this type of mobile radio channel.

LITERATURE

[1] E.Zehavi and J.K.Wolf, "On the performance evaluation of trellis codes", *IEEE Tans. Inform. Theory*, vol. IT-33, pp. 196-202, Mar. 1987.

[2] D.Divsalar and M.K.Simon, "The design of trellis coded MPSK for fading channels: Performance criteria," *IEEE Trans. Commun.*, vol. 36, pp. 1004-1012, Sept. 1988.

[3] A.Papulis, *Probability, Random Variables, and Stochastic Proceses*, New York: McGraw-Hill, 1984.

[4] J.G.Proakis, *Digital Communications*, New York:McGraw-Hill, 1989.

[5] J.K.Cavers and P.Ho, "Analysis of the error performance of trellis-coded modulation in Rayleigh fading channels", *IEEE Trans. Commun.*, vol. 40, pp. 74-83, January 1992.

[6] G.Ungerboeck, "Channel coding with multilevel/phase signal," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 55-66, January 1982.

[7] H.Salwen, "Differential phase-shift keying performance under time-selective multipath fading," *IEEE Trans. Commun.*, vol. 23, pp. 383-385, May 1975.