Optimal Samplers and Optimal Hold Functions in Sampled-Data Problems

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Abstract

We consider the design of controllers in the context of the ℓ^1 sampled-data problem. Given a continuous-time plant, with continuous-time performance objectives, expressed in terms of the L^∞-induced norm, we consider two possible controller configurations. The first is a controller with sampled measurements and continuous-time control signals, and the second is a controller with continuous-time measurements but with fixed hold functions for the controls. There is a further restriction on the structure of the controller that leads to optimal "hold functions", and optimal sampling operations respectively. We show that these two problems are in some sense dual problems. These problems differs from standard discrete-time methods in that it takes into consideration the inter-sample behavior of the closed loop system. The resulting closed loop system dynamics consist of both continuous-time and discrete-time dynamics and thus such systems are known as *hybrid* systems. These problems further differ from standard so-called sampled-data problems in that the sampler and hold operations are not both fixed, but are allowed to be part of the design process. We show that optimal controllers (in the sense of induced norms) have an appropriately defined shift invariance property. We also present a an approximation procedure for designing almost optimal controllers in the case of the ℓ^1 problem.

1 Introduction

This paper is concerned with designing certain types of sampled-data controllers for continuous-time systems to optimaly achieve certain performance specifications in the presence of uncertainty. Contrary to discrete time designs, such controllers are designed taking into consideration the inter-sample behavior of the system. Such hybrid systems are generally known as sampled-data systems, and have recently received renewed interest by the control community.

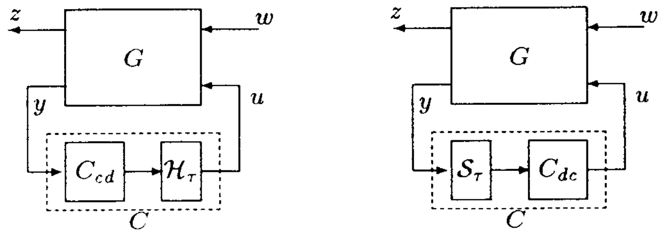

Figure 1: The two types of controllers

Much work has been devoted recently to these sampled-data problems with fixed hold and sampler operations. Relatively much less work has concentrated on the extra design freedom possible in the design of the sampler and hold operations (the notable exceptions being [1,2], in the context of the $\mathcal{H}^\infty$ problem). In this paper we will explicitly address the design of optimal hold functions and optimal sampler by posing the problems of design of controllers which are discrete-input/continuous-output and continuous-output/discrete-input systems respectively. The resulting controllers will have incorporated in them the optimal hold functions and optimal sampling operations respectively.

The difficulty in considering the continuous-time behavior of sampled-data systems, is that it is time varying, even when the plant and the controller are both time-invariant. We consider in this paper two versions of the so-called *standard problem* shown in figure 1. The continuous time controller C is constrained to be of either of the two forms $C_{dc}\mathcal{S}_\tau$ or $\mathcal{H}_\tau C_{dc}$, where $\mathcal{S}_\tau$ is an ideal sampler, $\mathcal{H}_\tau$ is a zero-order hold (both of period τ), and C_{dc}, C_{cd} are discrete-input/continuous-output and

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continuous-input/discrete-output systems respectively. All the developments in this paper apply equaly to the case when $\mathcal{S}_\tau$ and $\mathcal{H}_\tau$ are generalized sample and hold operations, as long as they are fixed *apriori*. The problems thus become those of optimizing over the controllers C_{cd} and C_{dc} respectively. Because of the assumed structure, the optimal C_{cd} will essentially have an optimal sampler imbedded in it, and the optimal C_{dc} will have an optimal hold function imbedded in it.

Since the closed loop mapping from exogenous input w to regulated output z could be time-varying or periodically time-varying, we will use the L^∞-induced norm as measure of performance of the closed loop system. In this paper we will use the framework developed in [5,6,7] to study these ℓ^1 sampled-data problem. Specificaly, our objective is to design stabilizing controllers C_{cd} and C_{dc} that minimize (or bound) the closed loop L^∞-induced norm. This minimization results from posing time domain specifications and design constraints, which is quite natural for control system design. To emphasize the point made earlier, the inputs are continuous time inputs, the errors are continuous time errors (see figure 1), however the system is a hybrid system with a continuous-time plant and a hybrid controller. The discrete time methods for ℓ^1 designs (e.g. [8,10,9]), cannot handle this problem directly, and is only concerned with the performance at the sampling instants. Furthermore, the setup in [7,11] is not immediately applicable since there, a fixed sampler and hold devices are assumed. In contrast, here we do not assume any previous structure on the sampling or hold devices.

We will begin without any assumptions on the structure of the systems C_{cd} and C_{dc}, beyond that they be linear (not necessarly stable) operators between signals over the appropriate time axes. We will then show that optimal C_{cd} and C_{dc} exist that have a certain shift-invariant structure if the original plant G is time invariant. These result is elucidated by appropriately lifting the systems and considering them as purely discrete-time systems with infinite-dimensional input and output spaces as in [5,6,7]. We then use arguments about the performance of time-varying versus time-invariant controllers when the plant in question is time-invariant (similar to the averaging arguments in [3,4]), in order to show that the optimal controllers C_{cd} and C_{dc} can be chosen to be shift-invariant (in the sense that their liftings are shift invariant).

The above arguments settle the issue of which class of controllers one should look for in these problems. We then address the issue of design in the context of the ℓ^1 problem. The resulting problems are ℓ^1 problems but with a controller of either infinite-dimensional input or output space. We use an approximation procedure to tackle this problem and show that in the limit, it is very intuitively related to the problem of designing multirate controllers in which the measurements are sampled much faster than the control or visa versa.

The remainder of this paper is organized as follows; in the next section we collect some facts about the lifting technique in continuous time that will be used in this paper. Section 3 then applies this to the problem at hand to elucidate the structure of the controllers in terms of their liftings. We then outline the averaging argument that shows that one can always find optimal controllers whose liftings are shift invariant. The last section illustrates an approximation procedure for reducing these two problems to standard discrete-time ℓ^1 problems. The relations of these approximations to certain multirate configurations in the limit are outlined, and the convergence of the approximation procedure is investigated.

2 The Lifting Technique in Continuous Time

In this section we briefly summarize the lifting technique for continuous-time periodic systems and apply it to the two problems at hand (for the details, see [5,6]). The idea of the lifting technique is to put a periodic continuous-time system in a strong correspondence with a shift-invariant (i.e. discrete-time time-invariant) system, which amounts to rearranging the original system so that its periodicity can be viewed as shift invariance. To accomplish this, we first define the lifting for signals, for which the appropriate signal spaces need to be established.

For continuous time signals, we consider the usual $L^\infty[0,\infty]$ space of essentially bounded functions [12], and it's extended version $L_e^\infty[0,\infty]$. We will also need to consider discrete time signals that take values in a function space, for this, we define ℓ_X to be the space of all X-valued sequences, where X is some Banach space. We define ℓ_X^∞ as the subspace of ℓ_X with bounded norm sequences, i.e. where for $\{f_i\} \in \ell_X$, the norm $\|\{f_i\}\|_{\ell_X^\infty} := \sup_i \|f_i\|_X < \infty$. Given any $f \in L_e^\infty[0,\infty]$, we define it's *lifting* $\hat{f} \in \ell_{L^\infty[0,\tau]}$, as follows: $\hat{f}$ is an $L^\infty[0,\tau]$-valued sequence, we denote it by $\{\hat{f}_i\}$, and for each i,

$$\hat{f}_i(t) \;:=\; f(t+\tau i) \qquad 0 \le t \le \tau.$$

The lifting can be visualized as taking a continuous time signal and breaking it up into a sequence of 'pieces' each corresponding to the function over an interval of length τ (see figure 2). Let us denote this lifting by $W_\tau \;:\; L_e^\infty[0,\infty] \longrightarrow \ell_{L^\infty[0,\tau]}$. W_τ is a linear isomorphism, furthermore, if restricted to $L^\infty[0,\infty]$, then W_τ :

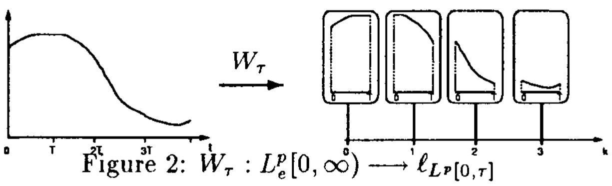

Figure 2: $W_\tau : L^p_e[0,\infty) \longrightarrow \ell_{L^p[0,\tau]}$

$L^\infty[0,\infty] \longrightarrow \ell^\infty_{L^\infty[0,\tau]}$ is an isometry, i.e. it preserves norms.

Using the lifting of signals, one can define a lifting on systems. Let G be a linear continuous time system on $L^\infty_e[0,\infty]$, then it's *lifting* $\hat{G}$ is the discrete time system $\hat{G} := W_\tau G W_\tau^{-1}$, this is illustrated in the commutative diagram below:

$$\begin{tikzcd} \ell_{L^\infty[0,\tau]} \arrow[r, "\hat{G}"] \arrow[d, "W_\tau^{-1}"'] & \ell_{L^\infty[0,\tau]} \\ L^\infty_e[0,\infty] \arrow[r, "G"] & L^\infty_e[0,\infty] \arrow[u, "W_\tau"'] \end{tikzcd}$$

Thus $\hat{G}$ is a system that operates on Banach space ($L^\infty[0,\tau]$) valued signals, we will call such systems infinite dimensional. Note that since W_τ is an isometry, if G is stable, i.e. a bounded linear map on L^∞ then $\hat{G}$ is also stable, and furthermore, their respective induced norms are equal, $||\hat{G}|| = ||G||$. The correspondence between a system and it's lifting also preserves algebraic system properties such as addition, cascade decomposition and feedback (see [5] for the details).

The usefulness of the lifting in the sampled-data problem is the fact that if G is a τ-periodic system, then $\hat{G}$ commutes with the shift on $\ell_{L^\infty[0,\tau]}$, that is, $\hat{G}$ is shift-invariant. This basic fact allows us to treat continuous time periodic systems as discrete-time time-invariant, albeit infinite dimensional systems.

State space models can be found for the lifted systems. For the details, see [5].

Note that the state space of $\hat{G}$ is finite dimensional (the n_x in $\mathbb{R}^{n_x}$ refers to the dimension of the state space of G), while it's input and output spaces are infinite dimensional. This fact is significant in that, although lifted systems have infinite dimensional input and output spaces, they can be realized with a state space of dimension no larger than the dimension of the original continuous-time state space model.

3 Lifting the Sampled-Data Problems

To apply the lifting to the two sampled-data problems in figure 1, note that what we are ultimately after is a closed loop mapping between $\hat{w}$ and $\hat{z}$, the liftings of the signals w and z. Let us denote the closed loop mapping formed between a generalized plant G and feedback element K by $\mathcal{F}(G,K)$ (i.e. the linear fractional transformation of G with K). For the two systems in figure 1 we can write the lifting of the closed loop mapping in the first system as:

$$\begin{aligned} & W_\tau \mathcal{F}(G, \mathcal{H}_\tau C_{cd}) W_\tau^{-1} \\ & \quad = \mathcal{F}(\begin{bmatrix} W_\tau & 0 \\ 0 & I \end{bmatrix} G \begin{bmatrix} W_\tau^{-1} & 0 \\ 0 & I \end{bmatrix}, \mathcal{H}_\tau C_{cd}) \\ & \quad = \mathcal{F}(\begin{bmatrix} W_\tau & 0 \\ 0 & W \end{bmatrix} G \begin{bmatrix} W_\tau^{-1} & 0 \\ 0 & \mathcal{H}_\tau \end{bmatrix}, C_{cd} W_\tau^{-1}) \\ & \quad = \mathcal{F}(\tilde{G}^1, \tilde{C}_{cd}), \end{aligned}$$

where the last equality are the definitions of the systems $\tilde{G}^1$ and $\tilde{C}_{cd}$. Note that $\tilde{G}^1$ is a system with three infinite-dimensional input and output spaces, and one finite dimensional input space, namely that of the control. This occurs because the original plant is a continuous-time system, and $\tilde{G}^1$ is its lifting with the hold device "absorbed" in it. The system $\tilde{C}_{cd} := C_{cd} W_\tau^{-1}$ has an infinite-dimensional input space and finite dimensional output space, reflecting the fact that the original system C_{cd} has continuous-time inputs and discrete-time outputs. The advantage of this reformulation is that the feedback system $\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})$ operates completely in discrete-time, and has the same closed loop norm as the original hybrid system.

Similarly for the second system in figure 1, we can write:

$$\begin{aligned} & W_\tau \mathcal{F}(G, C_{dc} \mathcal{S}_\tau) W_\tau^{-1} \\ & \quad = \mathcal{F}(\begin{bmatrix} W_\tau & 0 \\ 0 & \mathcal{S}_\tau \end{bmatrix} G \begin{bmatrix} W_\tau^{-1} & 0 \\ 0 & W_\tau^{-1} \end{bmatrix}, W_\tau C_{dc}) \\ & \quad = \mathcal{F}(\tilde{G}^2, \tilde{C}_{dc}). \end{aligned}$$

We similarly note that $\tilde{G}^2$ has one finite dimensional output space, namely that of the measurement. This occurs because $\tilde{G}^2$ is its lifting of G with the sampler "absorbed" in it. The system $\tilde{C}_{dc}$ has an infinite-dimensional ouput space and finite dimensional input space, reflecting the fact that the original system C_{dc} has continuous-time outputs and discrete-time input.

With the above transformations, the two problems now seems very similar, the only difference being that in the first problem the control space is finite-dimensional, while in the second problem the measurement space is.

Since the lifting W_τ is an isometry, we have then characterized the L^∞ induced norm of the hybrid systems as the $\ell^\infty_{L^\infty[0,\tau]}$ induced norm of the lifted system $\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})$ or $\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})$. The conclusion is that the problem of minimizing the L^∞ induced norm of the sampled-data system, is equivalent to that of minimizing the closed loop induced norm for the standard problem with the *partly* infinite dimensional generalized plants $\tilde{G}^1$ or $\tilde{G}^2$. The previous discussion together with a slight generalization of the internal stability results for hybrid systems in [13] (conditions for non-pathological sampling) yield the following theorem:

Theorem 1 *Consider the feedback systems in figure 1, and let $\tilde{G}^1$, $\tilde{G}^2$, $\tilde{C}_{cd}$, and $\tilde{C}_{dc}$ be defined as above, then*

(i) $\mathcal{F}(G, \mathcal{H}_\tau C_{cd})$ *is internally stable if and only if* $\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})$ *is.*
$\mathcal{F}(G, C_{dc}\mathcal{S}_\tau)$ *is internally stable if and only if* $\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})$ *is.*

(ii) $\|\mathcal{F}(G, \mathcal{H}_\tau C_{cd})\| = \|\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})\|$.
$\|\mathcal{F}(G, C_{dc}\mathcal{S}_\tau)\| = \|\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})\|$.

The following remarks about this reformulation of the problem follow easily from the properties of the lifting:

- If the original continuous-time generalized plant G is time invariant, then the lifted plants $\tilde{G}^1$ and $\tilde{G}^2$ will be shift invariant.
- Since both C_{cd} and C_{dc} are linear, then $\tilde{G}_{cd}$ and $\tilde{G}dc$ are also linear. If C_{cd}, C_{dc} are arbitrary, then $\tilde{C}_{cd}$, $\tilde{C}_{dc}$ are arbitrary shift-varying operators. If $\tilde{C}_{cd}$, $\tilde{C}_{dc}$ are shift invariant, then the original systems are time-invariant in the following sense:

  $$C_{cd}D_\tau = S\, C_{cd} \qquad ; \qquad D_\tau C_{dc} = C_{dc}S,$$

  where S is the right shift operator on sequences, and D_τ is the "delay by τ" operator on continuous-time signals.
- The lifted controllers have no "structural constraints" as the original controllers C (figure 1) do.

The advantage of the lifting technique is that one can essentialy view the original sampled-data problems as the so-called standard problem, albeit with the added complication of the infinite dimensionality of some of the input and output spaces. One rather immediate advantage of looking at the lifted problem is that it allows us to naturaly conjecture that since the plants $\tilde{G}^i$ are shift-invariant, then one cannot do better with shift-varying controllers versus shift-invariant ones. This result follows by arguments very similar to the averaging arguments in [3,4]. To state the result, let us denote by the classes LSV and LSI, the Linear Shift Varying and Linear Shift Invariant operators respectively, corresponding to $\tilde{C}_{cd}$ or $\tilde{C}_{dc}$ (i.e. operators from $\ell_{L^\infty[0,\tau]} \longrightarrow \ell_{R^n}$ or $\ell_{R^n} \longrightarrow \ell_{L^\infty[0,\tau]}$).

To capture the idea that systems with mixed discrete/continuous inputs and outputs have a shift invariance property, we make the following two definitions:

Definition 1 *A continuous-time input, discrete-time output (discrete-time input, continuous-time output) system such as C_{cd} (C_{dc}) is called time invariant if*

$$C_{cd}D_\tau = S\, C_{cd} \qquad ; \qquad (D_\tau C_{dc} = C_{dc}S),$$

Or in other words, if the liftings of these systems are shift invariant.

Since the controllers C_{cd}, C_{dc} are of a somewhat unusual type, it is not *apriori* clear to what class optimal controllers should belong to. The next theorem states the intuitive results that for time-invariant plants, one need not look any further than for time-invariant controllers in the above sense.

Theorem 2 *Consider the systems in figure 1, with G a linear time-invariant finite-dimensional plant, then one cannot improve the closed loop norm by using time-varying controllers C_{cd}, C_{dc}, over time-invariant ones. In other words, in terms of the lifted systems, we have*

$$\inf_{\substack{\tilde{C}_{cd}\ stabl.\\ \tilde{C}_{cd}\in\ LSV}} \|\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})\| = \inf_{\substack{\tilde{C}_{cd}\ stabl.\\ \tilde{C}_{cd}\in\ LSI}} \|\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})\|.$$

$$\inf_{\substack{\tilde{C}_{dc}\ stabl.\\ \tilde{C}_{dc}\in\ LSV}} \|\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})\| = \inf_{\substack{\tilde{C}_{dc}\ stabl.\\ \tilde{C}_{dc}\in\ LSI}} \|\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})\|,$$

where "stabl." stands for "stablizing" in the above theorem.

The above theorem then implies that one can always find optimal or almost optimal controllers whose liftings have shift invariant structure. An interesting implication of the above result is that the optimal hold and sampling operations which are imbedded in the the controllers C_{cd} and C_{dc} are actualy time-invariant, i.e. that they do not change from one sampling interval to the next. This fact is certainly intuitively appealing, but by no means immediate from the setup of the problem. We note that a similar observation was made in the work of [2] for the case of infinite-horizon sampled-data filtering and control in the $\mathcal{H}^\infty$ norm. It should also be clear from the general nature of the results in [4], that the above theorem applies to all L^P-induced norms as well, and to the $\mathcal{H}^\infty$ problem in particular.

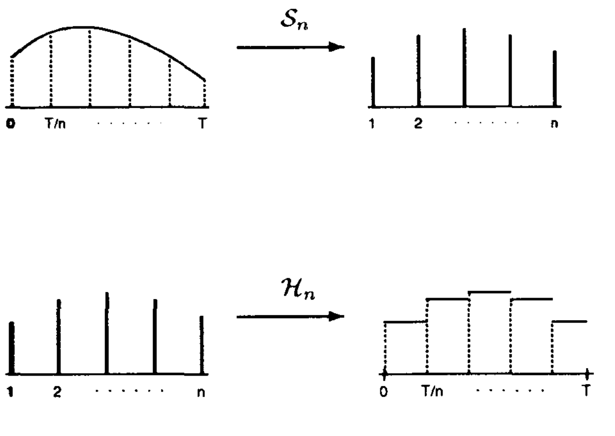

Figure 3: The operators $\mathcal{S}_n$ and $\mathcal{H}_n$

4 Reduction to a Standard ℓ^1 Problem

The infinite-dimensional problem obtained so far will now be solved by an approximation procedure through solving a standard MIMO ℓ^1 problem. The idea we use is similar to that in [11,14] where multirate sampling is used to obtain discrete-time systems that approximate the continuous time behavior of hybrid systems. We adopt a slightly different (but essentialy equivalent) point of view, and treat the approximation as that of the infinite-dimensional input and output spaces $L^\infty[0,\tau]$ by finite dimensional spaces. These ideas were used in [7] to obtain bounds on the approximation procedure that are in terms of the inter-sample dynamics of the plant. The difference in the problem at hand is that there is one more infinite-dimensional input or output space, and it involves the continuous-time dynamics of the partly continuous-time controller C_{cd} (or C_{dc}). This latter fact makes the convergence analysis even more delicate than that in [11,7].

We now describe the approximation procedure. Let $\mathcal{H}_n$ and $\mathcal{S}_n$ be the following operators defined between $L_q^\infty[0,\tau]$ and $\ell_q^\infty(n)$ ($\ell_q^\infty(n)$ is $R^{n\times q}$ with the maximum norm),

$$\begin{aligned}(\mathcal{S}_n u)(i) &= u(\frac{\tau}{n}i) \ ; \ u \in L_q^\infty[0,\tau]\\ (\mathcal{H}_n u)(t) &= u(\lfloor \frac{tn}{\tau} \rfloor) \ ; \ \{u(i)\} \in \ell_q^\infty(n),\end{aligned}$$

(strictly speaking, $\mathcal{S}_n$ is not an operator on L_q^∞ but on the subspace of left and right continuous functions, this distinction is irrelevant here since in our setting, assumptions are made to guarantee that $\mathcal{S}_n$ operates only on continuous signals), the above operators can be thought of as 'fast' sample and hold operators (see figure 3). For simplicity of notation we will suppress the dimension q in the sequel.

Now to approximate the infinite dimensional problems, we define the finite-dimensional plants $\bar{G}_n^1$ and $\bar{G}_n^2$ by:

$$\begin{aligned}\bar{G}_n^1 &:= \begin{bmatrix} \mathcal{S}_n & 0 \\ 0 & \mathcal{S}_n \end{bmatrix} \tilde{G}^1 \begin{bmatrix} \mathcal{H}_n & 0 \\ 0 & I \end{bmatrix}\\ \bar{G}_n^2 &:= \begin{bmatrix} \mathcal{S}_n & 0 \\ 0 & I \end{bmatrix} \tilde{G}^1 \begin{bmatrix} \mathcal{H}_n & 0 \\ 0 & \mathcal{H}_n \end{bmatrix}.\end{aligned}$$

Note that both $\bar{G}_n^1$ and $\bar{G}_n^2$ have finite-dimensional input and output spaces (whose dimension is proportional to n, though the state space dimension is constant) and are shift invariant. The idea behind using this approximation is first to put $\mathcal{H}_n$ and $\mathcal{S}_n$ at the exogenous input and regulated output of the lifted plant, and second to put the operator $\mathcal{H}_n\mathcal{S}_n$ in the infinite-dimensional signal path between the plant and the controller. The operator $\mathcal{H}_n\mathcal{S}_n$ in some sense approximates the identity operator as n is large. In fact, one can show that the following limits are obtained:

$$\begin{aligned}\lim_{n\longrightarrow\infty} \left\| \mathcal{F}(\bar{G}_n^1, \tilde{C}_{cd}\mathcal{H}_n) \right\|, &= \|\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd})\|\\ \lim_{n\longrightarrow\infty} \left\| \mathcal{F}(\bar{G}_n^2, \mathcal{S}_\tau \tilde{C}_{dc}) \right\|. &= \|\mathcal{F}(\tilde{G}^2, \tilde{C}_{dc})\|\end{aligned}$$

Note that since $\bar{G}_n^1$, and $\tilde{C}_{cd}\mathcal{H}_n$ are both finite dimensional, then the quantity $\|\mathcal{F}(\bar{G}_n^1, \tilde{C}_{cd}\mathcal{H}_n)\|$ is the ℓ^1 norm of a standard system (similar remarks hold for the case of $\tilde{G}^2$).

The above convergence statment can be used to compute the norm of the hybrid system by choosing n large enough. It also suggests a synthesis procedure, since the system $\tilde{C}_{cd}\mathcal{H}_n$ is finite dimensional. We illustrate this for the case of $\tilde{G}^1$ only, a very similar procedure works for the other case. For a fixed n, we define the following standard ℓ^1 problem:

$$\mu_n := \inf_{\hat{C} \ stabilizing} \left\| \mathcal{F}(\bar{G}_n^1, \hat{C}) \right\|,$$

where $\hat{C}$ has compatible dimensions with $\bar{G}_n^1$. One would expect that as n is large, μ_n approaches the optimal performance limit for the real hybrid problem. Furthermore, for any given n, and the resulting controller $\hat{C}_n^{opt}$ that solves the above problem, we can construct a controller $\tilde{C}_{cd_n}$ with "comparable" performance. Namely we can construct

$$\tilde{C}_{cd_n} := \hat{C}_n^{opt} \mathcal{J}_n,$$

where the operator $\mathcal{J}_n$ (the normalized integration operator, see [7]) $\mathcal{J}_n : L^{\infty}[0,\tau] \longrightarrow \ell^{\infty}(n)$ is defined by $(\mathcal{J}_n(u))(i) := \frac{n}{\tau}\int_{i\tau/n}^{(i+1)\tau/n} u(t)\, dt$.

One can show that for the sequence of controllers thus constructed $\tilde{C}_{cd_n}$, the limit

$$\lim_{n\longrightarrow\infty} \|\mathcal{F}(\tilde{G}^1, \tilde{C}_{cd_n})\|$$

does actualy converge to the limit of performance for the real hybrid problem. The details of these convergence arguments will be presented in the final version of this paper.

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