An Optimization Approach for Robust Stability Analysis of Linear and Nonlinear Systems with Parametric Uncertainty

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Keywords: Parametric Uncertainty, Robust Stability, Optimization, Nonlinear Systems

January 12, 1993

Abstract

The primary motivation for the use of feedback control is to achieve stabilization, disturbance attenuation and sensitivity reduction in the phase of variations in the plant parameters. The design of the feedback controller is based on a mathematical model of the process which attempts to mimic the dynamic behavior of the physical system. Unfortunately, no matter how sophisticated the model might be there will always be uncertainty associated with the modeling effort.

This suggests that the model of the process should be accompanied by an appropriate uncertainty description in order to address a realistic feedback synthesis problem. Then, the feedback controller should be designed to satisfy stability and performance requirements for the nominal plant, as well as for the family of plants that is generated by the accompanied uncertainty description. The mathematical description of the uncertainty has a profound effect in this task. Two different kinds of perturbations have been considered in the literature:

- Real parameter uncertainty, expressed as bounds on the real parameters of the model.
- Norm bounded complex uncertainty in the inputs and outputs of the process (sensor or actuator uncertainty).

Real parameter uncertainty assumes that the order of the plant is known. This means that the differential equations that describe the dynamics of the system are known, but some of their coefficients can vary. It is a very appealing to practicing engineers because the bounds on the real uncertain parameters can be interpreted intuitively. Complex uncertainty treats effectively unmodeled dynamics. However, the intuition on the selection of the complex bounds can be very limited in certain applications.

For this class of uncertainties the stability and performance analysis of the feedback loop is then carried out with the concepts of the *multivariable stability margin.* or the *structured singular value* (SSV, or μ function). These quantities made possible the derivation of necessary and sufficient conditions for robust stability for the

uncertainty description considered. These conditions are usually checked in terms of an upper bound on the μ function, which is computed from the solution of a convex optimization problem. For purely complex uncertainty numerical evidence suggests that this bound is tight (*exact* for *three* blocks or less). This means that the upper bound provides tight sufficient conditions for robust stability in the presence of complex uncertainty. Unfortunately, in problems involving real parameter uncertainty, the upper bound on μ can be arbitrarily conservative because the real nature of the parameters is ignored.

For nonlinear systems there exist some results on robust stability in the presence of unmodeled dynamics. However, nonlinear systems with parametric uncertainty has been studied much less.

The first part of this paper focuses on linear systems and presents a nonlinear programming approach for the exact computation of the stability margin (or equivalently the SSV) for *real* parameter uncertainty. The problem is formulated with the aid of the *zero exclusion condition* as this is interpreted in the Hurwitz domain. Since the resulting nonlinear program is nonconvex, it admits several local minima. A decomposition approach is proposed for its efficient solution. The approach derives an upper and a lower bound on the stability margin from the solution of a *primal* and a *dual* problem. These problems are derived from an appropriate partitioning of the variable set and nonlinear duality theory. The algorithm iterates between the primal and the dual problems and tightens the bounds at each iteration. It is guaranteed to converge to the global minimum in a finite number of iterations. An advantage of the method over existing domain splitting algorithms is that it explores the structure of the nonconvexities of the problem. As a result computational complexity does not explode with the number of the uncertain parameters of the system, but rather with the parameters responsible for the nonconvexities of the related optimization problem. An example is presented to illustrate the approach.

Aside from possible computational advantages this method provides an attractive framework for the extension of the analysis to a class of nonlinear systems with parametric uncertainty. This is a very difficult task because in nonlinear systems the parameter variations affect not only the stability of a steady state, but also the number of steady states of the system. We assume that the nominal system operates at a single stable steady state. Then, a *parametric local stability analysis* is performed with Liapunov's indirect method. The basic idea is to treat the linearization point (the system's steady state) as a variable which depends implicitly on the uncertain parameters of the system through the steady state equations. This allows for the definition of a nonlinear stability margin. With the aid of this quantity we prove a theorem of necessary and sufficient conditions for both robust stability and uniqueness of the steady state of the system. The nonlinear stability margin is computed from the solution of a nonlinear programming problem with similar structure to the one of the linear case. This provides unity with the corresponding linear theory. Although the results can not cover the whole phase plane of uncertain parameters, they are quite useful for the region of the phase plane that there is no multiplicity in the steady states. A case study of a nonisothermal stirred tank reactor is presented to illustrate the analysis.