Control of Smart Material Structures

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ABSTRACT

In this presentation we will give an outline of recent advances in the control of smart material structures. We focus on the use of embedded or bonded piezoceramic patches as actuators and sensors. Models for identification and control of cylindrical thin shells, plates and beams with piezoceramic patch actuators have been derived in [1]. These models lead to new theoretical and computational challenges for control theorists. To illustrate with a specific example, consider a beam of length ℓ, thickness h and width 1.

We assume that patch pairs are bonded to the beam at $x_1 \le x \le x_2$ along the beam which is cantilevered with fixed end at $x = 0$ and free end at $x = \ell$. Each piezoceramic patch is inherently an electro-mechanical transducer which, when excited by an electric field, induces a strain in the material in the axial direction of the beam. We further assume that we have two patches with identical polarization, each of which can be excited independently with an applied voltage to produce elongation or contraction.

Quantitative expressions for excitation in varied ways (in-phase, out-of-phase, etc.) can be derived as special cases of the general patch/thin shell interaction models developed in [1]. If we consider transverse and axial displacements, denoted by $w(t,x)$ and $u(t,x)$, respectively, of the beam, we find that for an undamped (assumed here for simplicity; internal damping can be readily added) beam we have the dynamic equations of motion given by

$$\begin{aligned} \rho h \frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(Eh\frac{\partial u}{\partial x}\right) &= -S_{1,2}(x)\frac{\partial}{\partial x}[N_x]_{pe} \\ \rho h \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) &= \hat{q}_n + \frac{\partial}{\partial x}\left(-\frac{\partial}{\partial x}[M_x]_{pe}\right) \end{aligned} \tag{1}$$

for $0 \le x \le \ell$, $t > 0$. Here ρ is the mass density (in mass per unit volume) of the beam, E is the Young's modulus, $I = h^3/12$ is the moment of inertia (of the beam of thickness h and width 1), and $S_{1,2}(x)$ is the indicator function having the values 1 for $x < (x_1+x_2)/2$, 0 for $x = (x_1+x_2)/2$, and -1 for $x > (x_1+x_2)/2$. The force components are given in terms of $\hat{q}_n$, the total external normal (transverse) surface load, and $[N_x]_{pe}$, the piezoceramic induced line forces in the x-direction. The quantity $[M_x]_{pe}$ represents

the piezoceramic induced line moments (about the neutral axis) about the x-axis and has units of moment. These dynamic equations must be coupled with appropriate boundary conditions which, in the cantilevered configuration, are given by

$$u(t,0) = 0 \ , \quad \frac{\partial u}{\partial x} = 0$$

$$w(t,0) = \frac{\partial w}{\partial x}(t,0) = 0 \ , \quad \frac{\partial^2 w}{\partial x^2}(t,\ell) = \frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)(t,\ell) = 0,$$

for $t \geq 0$, as well as initial conditions $w(0,x) = \theta(x)$, $\frac{\partial w}{\partial t}(0,x) = \psi(x)$, $u(0,x) = \eta(x)$ and $\frac{\partial u}{\partial t}(0,x) = \gamma(x)$.

It can be shown [1] that the forces and moments in (1) are given by

$$\begin{aligned} [N_x]_{pe} &= -Eh[H(x-x_1) - H(x-x_2)]S_{1,2}(x)\frac{d_{31}}{E2+2T}\{V_1 + V_2\} \\ [M_x]_{pe} &= -Eh[H(x-x_1) - H(x-x_2)]\frac{a_3 d_{31}}{E1+2a_2}\{V_1 - V_2\}, \end{aligned} \tag{2}$$

where H is the Heaviside function. Substituting the expressions from (2) into the system (1) leads to a control system which contains δ and δ' (δ is the Dirac delta function) as coefficients in the control input terms.

More generally, such problems are a special case of general control problems for distributed parameter systems which can be abstractly written as

$$\dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}u(t) \qquad \text{in } \mathcal{V}^*$$

where $\mathcal{A}$ generates a semigroup in abstract spaces $\mathcal{V} \to \mathcal{H} \to \mathcal{V}^*$ forming a Gelfand triple and $\mathcal{B} \in \mathcal{L}(\mathcal{V}, \mathcal{V}^*)$. An abstract LQR theory along with an approximation framework leading to computational methods has been developed recently [2]. This theoretical framework as applied to control of smart structures will be explained.

The above ideas have been tested computationally in two areas of applications. Time permitting, we will present some of our experiences in computing feedback gains for (1) nonlinear fluid/structure interactions arising in an active control noise suppression system for aircraft and (2) vibration suppression in linear and nonlinear beams.

Since control via piezoceramic actuators as outlined above permit one to apply structure borne shear forces to a flow field (as well as the usual transverse wall forces through bending moments), we shall discuss potential applications of smart material walls in controlling flow in chambers as well as flow over airfoils.

REFERENCES

[1] H.T. Banks and R.C. Smith, The modeling of piezoceramic patch interactions with shells, plates and beams, *Quart. Appl. Math.*, to appear.

[2] H.T. Banks and K. Ito, Approximation in LQR problems for infinite dimensional systems with unbounded input operators, to appear.