APPROXIMATIONS FOR CHAOTIC SYSTEMS VIA STOCHASTIC LINEAR SYSTEMS–EXPERIMENTAL RESULTS

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EXTENDED ABSTRACT

Chaotic systems receive increasing attention in the literature. Their simplicity of structure combined with the richness of their output will continue to inspire their use in modeling efforts for many years to come. On the other hand, the difficulty of their analysis warrants approximation methods, especially since the absence, by definition, of well-defined limit sets prohibits a meaningful linearization. The purpose of this work is to present some experimental results which will hopefully support a methodology for approximating chaotic systems via stochastic linear systems. The approximations are mostly based on spectral analysis. By experimental we do not refer to a physical setup but to the use of computer-based tools such as Matlab.

The idea to work mostly in frequency domain came from the fact that the chaotic signal looks very much like noise. The use of stochastic linear systems as an approximation followed naturally, given that the relevant theory is well developed and known. The main novelty is to use spectral moments as a central measure of the quality of the approximation.

The methodology we follow can be referred to as another type of shaping filter design. It simply means that the colored noise (non-flat spectrum), or "noise" in the case of chaotic signals, is approximated by filtering white noise through a linear system of appropriate order and parameters. To identify the stochastic linear system whose output approximates the chaotic signal we use the following four approaches:

- Spectral moment matching
- Spectral factorization/stochastic realization approach
- ARMA identification
- Heuristic identification of a second order linear system

which are briefly described below.

Spectral moment matching. Spectral moments characterize the spectrum of a signal in the same way probabilistic moments can summarize the corresponding probability density [1]. To use this concept, one has to calculate the spectral moments of a linear (or nonlinear, for that matter) system and match them to those of the chaotic signal. This is straightforward for low-order linear systems but for higher orders analytic calculation becomes extremely tedious. The complexity of the inverse problem, namely determining the parameters of a linear system from its spectral moments, increases at the same rate. Here, a second order linear system is used.

Spectral factorization/Stochastic realization approach. Based on the theoretical results of [2] and the algorithms of [3, Ch. 9], it is possible to match a linear system structure to the chaotic signal. We determine the covariance coefficients of the latter and we form the succession of Hankel matrices of increasing order and check their rank. If the rank remains unchanged through a step the order of the system equals this rank. For "noisy" signals, such as in our case, this is realized by judging when a singular value is much smaller than its preceding one. Then simple formulae [3] yield the system matrices.

ARMA identification. The signal is considered as a time series and an input-output model is matched to it in the classical way. The order of the model is determined in the same way as for the stochastic realization. Other options include Akaike's criterion.

Heuristic identification of a second order linear system. It is observed that the spectra of many chaotic signals contain a peak. This is also true for the frequency response of a second order linear system (driven by white noise). Thus, the parameters of a second order linear system are determined so that its resonant frequency coincides with the chaotic peak.

The quality of the approximation depends on the form of the chaotic signal. From the above methods the most consistently successful is the ARMA approach, especially if the quality is judged by the first few spectral moments only.

REFERENCES

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