A SINGULARITY-ROBUST KINEMATIC CONTROL STRATEGY FOR AN OPTIMUM DECOUPLED GEOMETRY

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An inherent problem in the control of robotic manipulators is the existence of *singular* positions inside the workspace of the manipulator. The singular positions of *ideal* robot geometries are always known *a priori*. Since the presence of singularities complicates robot control, singularity-robust algorithms that avoid the neighborhood of singularities have been developed. In this paper, a resolved-motion rate kinematic control strategy for decoupled robot geometries that guarantees smooth motion of the manipulator even at the neighborhood of singular points, is presented. For expository convenience, the examples in this paper deal with *ideal* dual-elbow robots.

Dual-elbow manipulators are articulated robots consisting of six rotational joints. It has been demonstrated that the dual-elbow configuration is a non-anthropomorphic geometry, alternative to the elbow geometry, that shares the basic properties of the elbow configuration and that the kinematic characteristics of the ideal dual-elbow geometry mirror those of the ideal elbow geometry [1].

The dual-elbow manipulator has a decoupled robot geometry and thus guarantees *singularity decoupling*; that is, the singularities of the entire manipulator consist of exactly the positional *and* the orientational singularities. As a result, near singular positions singularity-robust algorithms need to be employed only for the distinct subsystem that produces the singularity [2].

Since the presence of singularities complicates robot control, kinematic control strategies have traditionally focused on singularity avoidance. Yoshikawa developed the *manipulability measure* as a criterion to avoid singularities at the *task-planning level* for redundant manipulators. The manipulability measure prevailed among other criteria proposed to measure the distance of a robot configuration from a singularity, because it is *not* frame-dependent and because it uses the minimum number of kinematic parameters [3,4]. Typically, the manipulability measure $w(\mathbf{q})$, is defined as a continuous positive function obtaining values between zero and one, that depend solely on the joint displacements:

$$w(\mathbf{q})=\sqrt{\det[\mathbf{J}(\mathbf{q})\mathbf{J}(\mathbf{q})^{T}]} \tag{1}$$

where $\mathbf{J}(\mathbf{q})$ is the configuration dependent 6xN Jacobian matrix of the manipulator and $\mathbf{q}$ is the Nx1 joint coordinate vector. For a six-axis manipulator, $w(\mathbf{q})$ reduces to $|\det[\mathbf{J}(\mathbf{q})]|$. A zero manipulability measure thus indicates a singular configuration, while a unit value indicates a configuration for which the robot's ability to move is maximized [5,6].

The differential motion equation for a manipulator with N degrees of freedom is:

$$\delta\mathbf{x}=\mathbf{J}(\mathbf{q})\delta\mathbf{q} \tag{2}$$

where $\mathbf{x}$ is the 6x1 pose vector. The inverse of the mapping in (2) gives rise to the following equation:

$$\delta\mathbf{q}=[\mathbf{J}(\mathbf{q})]^{-1}\,\delta\mathbf{x} \tag{3}$$

which exists as long as configuration points for which $\det[\mathbf{J}(\mathbf{q})]=0$ are avoided. The zeros of this equation correspond to the singular configurations of a robotic manipulator.

Equation (3) is at the core of resolved motion rate kinematic control algorithms for industrial robots since it provides the necessary joint displacements needed to produce a desired differential motion of the end-effector. In the context of Equation (3), *resolved motion rate control* means that the motions of the various joint motors are combined and resolved simultaneously at different rates in order to achieve desired end-effector motions along any coordinate axis.

For the dual-elbow manipulator, which has a decoupled geometry, Equation (2) can be decomposed into the following equations:

$$\delta\mathbf{x}_A=\mathbf{J}_A\delta\mathbf{q}_A \tag{4}$$

$$\delta\mathbf{x}_S=\mathbf{J}_S\delta\mathbf{q}_S+\mathbf{C}\delta\mathbf{q}_A \tag{5}$$

where the subscripts A and S denote the arm and shoulder subsystems, respectively, and $\mathbf{C}$ is a coupling matrix.

Nakamura extended Yoshikawa's method and utilized the manipulability measure to solve inverse kinematics while simultaneously evaluating the feasibility of joint motion; the resulting *singularity-robust* control scheme offers a feasible motion close to the desired trajectory of the end-effector even in the neighborhood of singularities. In his implementation, Nakamura introduced the *singularity-robust* inverse Jacobian matrix as an alternative to the inverse Jacobian matrix in Equation (3) for use in the neighborhood of singularities, and clarified its properties by comparing it with the actual inverse and the pseudoinverse.

The singularity-robust inverse Jacobian $\mathbf{J}^*$ of an n x n Jacobian matrix $\mathbf{J}$ is an n x n matrix defined as:

$$\mathbf{J}^* = (\mathbf{J}^{\mathrm{T}}\mathbf{J}+k\mathbf{I})^{-1}\mathbf{J}^{\mathrm{T}}=\mathbf{J}^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}}+k\mathbf{I})^{-1} \tag{6}$$

where $\mathbf{J}^{\mathrm{T}}$ is the transpose of the Jacobian matrix, $\mathbf{I}$ is the n x n identity matrix and k is a variable positive scale factor which determines the relative weight between the exactness and the feasibility of the end-effector motion. As k decreases in value, the exactness of the end-effector motion prevails over the feasibility of the joint motion; as a result, the magnitude of the joint displacement vector $|\delta\mathbf{q}|$ is decreasing in value while the magnitude of the posing error vector $|\delta\mathbf{r}|$ is decreasing as the singularity is approached. The matrices $(\mathbf{J}^{\mathrm{T}}\mathbf{J}+k\mathbf{I})$ and $(\mathbf{J}\mathbf{J}^{\mathrm{T}}+k\mathbf{I})$ are always positive definite and thus nonsingular. In Nakamura's implementation, k is a function of the robot's manipulability measure $w(\mathbf{q})$, in order to ensure the continuous transition from $\mathbf{J}^{-1}$ to $\mathbf{J}^*$ in the neighborhood of singularities; the point of transition is defined by the scale factor k. It should be pointed out that the determinant $\det(\mathbf{J}^*)$ is independent of the coordinate frame it is expressed in.

An efficient way to guarantee singularity avoidance for the dual-elbow geometry at the *control-level* is to utilize Nakamura's singularity-robustness approach along with the singularity decoupling principle; the singularity-robust inverse Jacobian matrix of the arm or shoulder subsystem needs to replace the corresponding inverse Jacobian matrix in the control hierarchy whenever an arm or shoulder singularity, respectively, is approached. The singularity-robust inverse Jacobian matrices of the arm or shoulder subsystems are:

$$\mathbf{J}_A^* = \mathbf{J}_A^{\mathrm{T}}(\mathbf{J}_A\mathbf{J}_A^{\mathrm{T}}+k_A\mathbf{I})^{-1} \tag{7}$$

$$\mathbf{J}_S^* = \mathbf{J}_S^{\mathrm{T}}(\mathbf{J}_S\mathbf{J}_S^{\mathrm{T}}+k_S\mathbf{I})^{-1} \tag{8}$$

where k_A and k_S are variable scale factors equal to:

$$k_A = \begin{cases} k_{A_0}(1-\frac{w_A}{w_{A_0}})^2 & \text{for } w_A < w_{A_0} \\ 0 & \text{for } w_A \geq w_{A_0} \end{cases} \quad \text{and} \quad k_S = \begin{cases} k_{S_0}(1-\frac{w_S}{w_{S_0}})^2 & \text{for } w_S < w_{S_0} \\ 0 & \text{for } w_S \geq w_{S_0} \end{cases} \quad (9)$$

respectively. In Equation (9), w_{A_0} and w_{S_0} are thresholds that define the neighborhood of singular points and also the transition point at which the singularity-robust Jacobian replaces the inverse Jacobian in the control hierarchy.

The decoupled control strategy for the dual-elbow geometry using the singularity-robust inverse Jacobian matrices of the shoulder and the arm subsystems, offers a considerable computational advantage as compared to the singularity-robust control scheme for the entire manipulator. Furthermore, the use of the singularity-robust inverses increases the computational load compared to the simple inverses, but guarantees the stability of each distinct subsystem.

A resolved-motion rate kinematic control strategy for decoupled robot geometries, that guarantees smooth motion of the manipulator even at the neighborhood of singular positions, has been developed. This approach has then be customized fro dual-elbow manipulators to perform singularity avoidance and to provide feasible solutions for the end-effector motion at the neighborhood of singularities. To accomplish this objective, the singularity-robust Jacobian matrices of the shoulder and arm subsystems have been utilized. It has been shown that the computational cost associated with this control scheme is considerably lower than that of existing control algorithms. Finally, a set of examples has been used to illustrate the smooth behavior of the control strategy in terms of the accuracy of the end--effector motion and of the feasibility of the joint motion.

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