A SIMPLE REALIZATION OF SEPARABLE 2-D SYSTEMS

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Summary. The realization problem for separable 2-D linear digital system is considered. A simple solution to the problem is proposed in the form of 2-D state-space Roesser model. The solution is minimal.

1. INTRODUCTION.

The problem of realization is very important in control theory. This problem is effectively solved for 1-D systems. For 2-D systems, however, there is no a unique effective solution. The first methods for realization of 2-D systems were presented in 1972-1975, eg. [6,3]. There was considered a problem of realization for a general as well for separable 2-D systems, eg. [2]. Recently papers were published where some aspects of minimal realization for 2-D systems had been considered, e.g. systems whose transfer functions can be expanded to a continued fraction [4] or systems whose transfer functions have separable numerators [1].

In this note a realization problem for separable 2-D system, i.e. system whose transfer function has separable denominator, is considered. Based on circuit techniques a simple solution to the problem is proposed.

2. PROBLEM FORMULATION.

Given SISO linear shift invariant separable 2-D system described by the transfer function

$$G(w,z)=\frac{b(w,z)}{a(w,z)}=\frac{\sum\limits_{i=0}^{f}\sum\limits_{j=0}^{h} b_{ij}w^{f-i}z^{h-j}}{\left(w^{f}+\sum\limits_{i=0}^{f} a_{i0}w^{f-i}\right)\left(z^{h}+\sum\limits_{j=0}^{h} a_{0j}z^{h-j}\right)} \tag{1}$$

find matrices A,B,C,D of the 2-D Roesser model (2-DRM) [5]

$$\begin{bmatrix} x_1(k+1,t)\\ x_2(k,t+1)\end{bmatrix}=Ax(k,t)+Bu(k,t) \tag{2}$$

$$y(k,t)=Cx(k,t)+Du(k,t)$$

where $x=\begin{bmatrix} x_1\\ x_2\end{bmatrix}\in R^n$ is a local state vector, $x_1\in R^{n_1}$, $x_2\in R^{n_2}$, $n=n_1+n_2$, $u\in R^m$ is an input vector, $y\in R^p$ is an output vector, and

$$A=\begin{bmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},\ B=\begin{bmatrix} B_1\\ B_2\end{bmatrix},\ C=[C_1\ \ C_2] \text{ and } D$$

are real matrices of appropriate dimensions, such that

$$C[I(w,z)-A]^{-1}B+D = G(w,z)$$

where $I(w,z)=\text{block diag}(wI_{n_1}, zI_{n_2})$ and I_n is identity matrix $n\times n$.

3. PROBLEM SOLUTION

Transfer function (1) may be rewritten in the form

$$G(w,z)=G_1(w^{-1},z^{-1})G_2(z^{-1}) \tag{3}$$

where

$$G_1(w^{-1},z^{-1}) = \frac{\sum_{i=0}^{f}\sum_{j=0}^{h} b_{ij}w^{-i}z^{-j}}{1+\sum_{i=1}^{f} a_{i0}w^{-i}} = \frac{\sum_{i=0}^{f} b_i(z^{-1})w^{-i}}{1+\sum_{i=0}^{f} a_{i0}w^{-i}}$$

and

$$G_2(z^{-1})= \frac{1}{1+\sum_{j=1}^{h} a_{0j}z^{-j}}$$

Thus

$$y(w,z)=G(w,z)u(w,z)=G_1(w^{-1},z^{-1})u_1(w,z) \tag{4}$$

where

$$u_1(w,z)=G_2(z^{-1})u(w,z) \tag{5}$$

From (4) and (5) the following shift equations follows

$$y(k,t)= -\sum_{i=1}^{f} a_{i0}y(k-i,t)+ \sum_{i=0}^{f}\sum_{j=0}^{h} b_{ij}u_1(k-i,t-j) \tag{6}$$

and

$$u_1(k,t)= -\sum_{j=1}^{h} a_{0j}u_1(k,t-j)+u(k,t) \tag{7}$$

Then, using circuit techniques and realizing transfer functions $G_2(z^{-1})$ and $G_1(w^{-1},z^{-1})$ according to 1-D canonical controller and observer forms one can obtain realization for 2-D system (1) presented on fig. 1.

The realization can now be simply presented in the form of 2-DRM (2) where $x_1\in R^f$, $x_2\in R^h$ and

$$A=\left[\begin{array}{cccc|ccc} -a_{10} & 1 & \dots & 0 & \bar{b}_{11} & \dots & \bar{b}_{1h} \\ \vdots & & \ddots & & \vdots & & \\ \vdots & & & 1 & \vdots & & \\ -a_{f0} & & \dots & 0 & \bar{b}_{f1} & \dots & \bar{b}_{fh} \\ \hline 0 & \dots & & 0 & -a_{01} & \dots & -a_{0h} \\ \vdots & & & \vdots & 1 & & \vdots \\ \vdots & & & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & & 0 & 0 & 1 & 0 \end{array}\right], \quad B=\left[\begin{array}{c} \bar{b}_{10} \\ \vdots \\ \bar{b}_{f0} \\ \hline 1 \\ 0 \\ \vdots \\ 0 \end{array}\right]$$

$$C=\left[\begin{array}{cccc|ccc} 1 & 0 & \dots & 0 & b_{01} & \dots & b_{0h} \end{array}\right], \quad D=[\, b_{00} \,]$$

where $\bar{b}_{ij}=b_{ij}-a_{i0}b_{0j}$.

6. CONCLUDING REMARKS.

A simple realization of separable 2-D system has been presented. It is easy to see that the realization is minimal, the transfer function order is equal to the order of the realization - 2-D state-space Roesser model. One of the advantage of the realization is that the transfer function parameters are directly used in the given circuit of the state-space model.
Finally, let us note that any separable 2-D system can be simply presented in the form (1).

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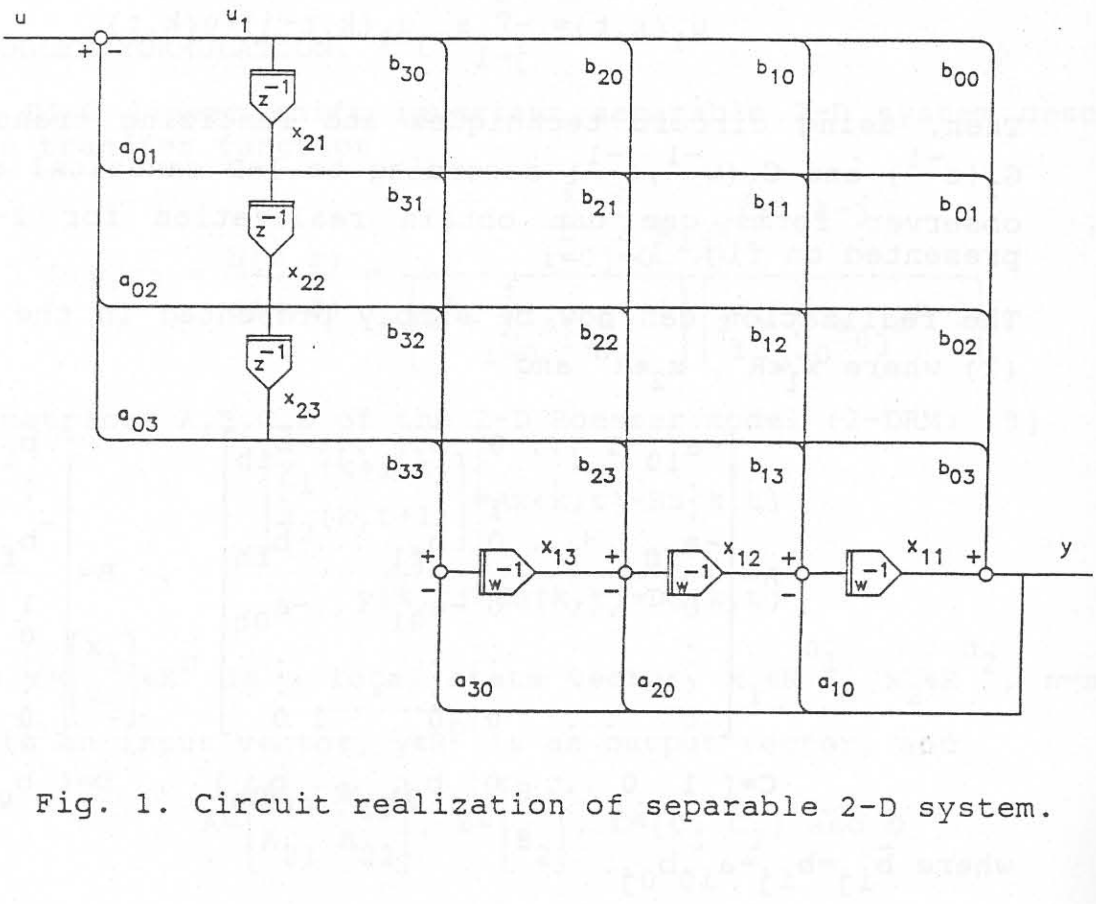

Fig. 1. Circuit realization of separable 2-D system.