Parameteridentification for state dependent delays originating from threshold conditions

F. Kappel
Institute for Mathematics
University of Graz
Heinrichstraße 36
A-8010 GRAZ (Austria)
e-mail:kappel@edvz.uni-graz.ada.at

Delay systems with state dependent delays occur as models for the dynamics of diseases when the mechanism of infection is such that the infectious dosage received by an individual has to reach a threshold value before the resistance of the individual is broken down and as a result the individual becomes infectious. A prototype of such a model was proposed in [1]. we refer to this paper for basic assumptions on the mechanisms governing the evolution of the disease and for the derivation of the model from these assumptions.

The process starts at $t = 0$ when a number I_0 of infectious individuals is introduced into a population of S_0 susceptible individuals. Without restriction we assume $S_0 + I_0 = 1$. Let $I(t)$ resp. $S(t)$ denote the number of infectious resp. susceptible individuals at time t. The number t_0 is defined by

$$\int_0^{t_0} \rho(x) I_0 \, dx = m, \tag{1}$$

where m is the threshold value for the infectious dosage accumulated by an individual. The total infectious dosage received up to time $t_2 > t_1$ by an individual which belongs to the class of susceptibles at time t_1 is given by

$$\int_{t_1}^{t_2} \rho(x - t_1) I(x) \, dx,$$

where ρ is a given positive continuous function describing the rate of breakdown of the resistance of an individual against the disease through contacts with infectious individuals. Since no individual becomes infectious during $0 \leq t \leq t_0$, we have

$$
\begin{aligned}
&I(t) = I_0, \\
&S'(t) = -r(t) I_0 S(t), \quad 0 \leq t \leq t_0,
\end{aligned}
\tag{2}
$$

where $r(t)$ is a positive continuous function. For $t \geq t_0$ we define $\tau(t)$ by

$$\int_{\tau(t)}^{t} \rho(x - \tau(t)) I(x)\, dx = m.$$

In this model it is assumed that an individual which becomes infectious at time t recovers at time $t + \sigma$, σ a positive constant, and belongs again to the susceptible class. The equations of the model on $t_0 \leq t \leq t_0 + \sigma$ are

$$
\begin{aligned}
&S'(t) = -r(t) S(t)(1 - S(\tau(t))), \\
&I(t) = 1 - S(\tau(t)).
\end{aligned}
\tag{3}
$$

For $t > t_0 + \sigma$ we also have recovery of individuals and the equations are

$$
\begin{aligned}
&S'(t) = -r(t) S(t)(I_0 - S(\tau(t)) + S(\tau(t - \sigma))), \\
&I(t) = I_0 - S(\tau(t)) + S(\tau(t - \sigma)), \quad t > t_0 + \sigma.
\end{aligned}
\tag{4}
$$

Based on a numerical scheme developed in [2] for equations (ref6.2) – (4) we present results on identification of the threshold level m and on the threshold kernel ρ in (1) from data on $S(t)$ and $I(t)$ at times $t_i \geq t_0 + \sigma$.

References

[1] K. L. Cooke: Functional-differential equations: Some models and perturbation problems, in "Differential Equations and Dynamical Systems" (J. K. Hale and J. P. LaSalle, Eds.). Academic Press, New York 1967, pp. 167–183.

[2] K. Ito and F. Kappel: Approximation o semilinear Cauchy problems, submitted to JMAA.