Analysis and optimization of discrete event systems modelling manufacturing processes

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Current research on Discrete Event Dynamic Systems (DEDS) is based on a number of methods among which there are the logical approach to automata [8], the perturbation analysis of trajectories [7], and the max-plus algebra approach [3],[4]. This work refers to the last approach.

Starting from basic works such as [3],[4] and [5], the authors proposed in [1],[2] a workable approach to represent and analyse the behaviour of a DEDS modelling a manufacturing process. This was achieved by exploiting the analytic tools of max-plus algebra theory to take into account all at once several alternatives for decisions about the assignment and the sequencing of tasks.

This paper focuses on this topic. A class of discrete event dynamic systems is taken into account, modelling the execution of a set of jobs by a set of resources (machines). Each job is made up by a number of tasks (operations) organized in an oriented precedence-relation graph. Each task can be executed by any machine 'compatible' with it, and it is characterized by a release time, a deadline, and a due-date. Execution times are deterministic and known and no preemption is allowed. Each machine can process a single task at a time. All the numeric quantities introduced are assumed to have integer values, without significant loss of generality.

In connection with an instance of the model considered, the solution of any optimization problem, can be carried out through three conceptually distinct phases, which are: i) the assignment of the tasks to the machines, whenever multiple choices are possible for the execution of a single task; ii) the sequencing of the executions of the assigned tasks on every machine; iii) the timetabling of the task executions (i.e., the choice of the actual activation time for each task).

Any decisional problem consisting in the minimization of a significant cost functional in connection with the model considered, leads to a combinatorial optimization problem which is a generalization of well known NP-hard problems. This is the reason why one is led to consider a simplified model where only a limited number of the decision variables are to be fixed. In this paper, a model of a DEDS will be proposed where some decisions are fixed and others are to be taken regarding the assignment of the tasks to the machines and the sequencing of such tasks on the machines. This model can be thought of as a 'family' of discrete event patterns, 'perturbed' with respect to a 'nominal' feasible pattern. Each possible perturbation is associated with a boolean decision variable, whose specification indicates whether the relevant perturbation is implemented or not. Using this representation and standard mathematical results of max-plus algebra theory, one is able to write the earliest and the latest completion times of all the tasks considered as analytic functions of the set of perturbation variables considered. Moreover, whenever non-regular cost functionals (i.e., cost functionals which are not monotonically non-decreasing as the task completion times increase) are considered, the attention cannot be limited to semi-active schedules [6]. Then, it becomes necessary to consider, as well as the perturbation variables mentioned, an additional set of degrees of freedom, namely, the task activation delays with respect to the earliest activation times. Then, one is able to write the completion times of the tasks as analytic functions of the two mentioned set of variables, which also makes it possible to write an analytic expression of any performance index uniquely related to task completion times.

The possibility of finding analytic expressions of the class of performance indexes considered appears to be even more interesting since, as it will be clear in the final version of the paper, it does not require the explicit analysis of every possible perturbed discrete event pattern. In other words, it is not necessary to investigate in detail the effects of any allowed combination of perturbations.

The procedure that will be proposed in this paper is not stricly intended to optimization purposes. Indeed, it is clear that any optimization problem regarding a performance index of the class considered turns out to be of the mixed-integer type, and so, usually difficult to solve. In this connection, the use of max-plus algebra formalism helps in providing a quite compact procedure to write a performance index as a function of the two sets of decision variables, together with the set of constraints affecting the choice of such variables. Then, this approach is primarily intended to analysis purposes, even though in the paper the structure of the functional dependence mentioned above will be given for some special case, regarding the choice of the objective function, with the aim of determining the peculiar structure of the mixed-integer programming problem in such cases.

The model and the approach presented in this paper seem to be specially suitable for analysis and design purposes relevant to the class of manufacturing processes where production is completely based on orders, each one having its own individuality (i.e., each order is different from any other one for type or quantity of products, constraints, and so on). So, the manufacturing model considered does not foresee a continuous production of identical items. This means that the production planning only considers a finite number of tasks (i.e., those relevant to the orders already got).

Indeed, whenever the dynamics of order (job) arrivals is quite 'fast', it seems realistic to assume that periodic re-scheduling procedures do not attempt at re-organizing the whole production process whenever a new job arrives or a significant variation (e.g., a machine failure) is recognized in the production plant facilities. Instead, one can suppose that most of the previously taken decisions about the assignment and the sequencing of tasks on machines are retained, whereas the remaining ones can be changed. In this sense, the re-scheduling procedure may be thought of as performing a perturbation analysis/optimization with respect to both the decisions regarding previously assigned tasks whose assignment/sequencing may be changed and the ones about the tasks relevant, e.g., to the last arrived job.

In the final version of the paper, we will present some experimental results relevant to manufacturing processes involving hundreds of tasks. Such results have been obtained by means of a dedicated simulation tool that, given the instance of a model, produces the statement of the relevant optimization problem in a compact, algebraic fashion.

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