MINIMUM ENERGY CONTROL PROBLEM FOR GENERAL LINEAR 2-D SYSTEMS IN HILBERT SPACES

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Two-dimensional discrete systems have enjoyed a great deal of research interest over the past several years [1], [10]. This has be motivated, on the one hand by a number of applications of 2-D model in various areas of science and engineering and, on the other hand, interesting and difficult theoretical problems posed by such syste [1], [10]. Recently, many results have been presented on controllabil and so called minimum energy control problem for various kinds of 2-D systems [2 - 9] and [11 - 12]. It is well known [1] , [10], that minimum energy control problems are strongly related to various ty of controllability of 2-D systems.

In the papers [2], [3], [5], and [12] local controllability and m nimum energy control problems for different kinds of regular finit dimensional 2-D systems have been extensively considered using the classical methods of linear algebra. Moreover, in the publications [[7], [8] and [9] some results concerning controllability and minimum energy control problems for singular 2-D systems have been formula Finally, the paper [4] contains controllability conditions and the lution of minimum energy control problem for infinite-dimensional systems defined in general Banach spaces.

The main purpose of this paper is to present a complete solut of the minimum energy control problem for the general linear 2-D s tems with constant coefficients, defined in infinite-dimensional Hi bert spaces. The obtained results are generalizations of the result given in the papers [3] and [12] to infinite-dimensional case.

Let us consider the general model of linear 2-D system with constant coefficents, defined in infinite dimensional Hilbert spac

$$x(i+1,j+1) = A_0x(i,j) + A_1x(i+1,j) + A_2x(i,j+1) + Bu(i,j) \qquad /1/$$

where $x(i,j) \in X$ is the local state , X is a Hilbert space ,

$u(i,j) \in U$ is the control , U is a Hilbert space ,

$(i,j) \in Z \times Z$, Z is the set of non-negative integers ,

A_k , $k=0,1,2$. are linear and bounded operators from X to X

B is linear and bounded operator from U to X .

Boundary conditions for the bstract equation /1/ are given by

$$x(i,0) = x_{io} \in X \quad \text{for } i \in Z$$

$$x(0,j) = x_{oj} \in X \quad \text{for } j \in Z \qquad /2/$$

where x_{io} and x_{oj} are known elements of the Hilbert space X .

Similarly as in finite-dimensional case [1], [10], let us de so called transition operator $A^{i,j}$ as follows :

$A^{0,0} = I$, identity operator on the Hilbert space X ,

$A^{i,j} = A_0A^{i-1,j-1} + A_1A^{i,j-1} + A_2A^{i-1,j}$ for $i,j=0,1,$

$A^{i,j} = 0$, for $i<0$ or $j<0$.

It should be pointed out ,that the transition operator $A^{i,j}$ is th linear and bounded operator from the space X to the space X . The transition operator is used to derive the compact form of the sol tion of the abstfact difference equation /1/ .For simplicity of n tations,we may assume without loss of generality,that the boundar conditions /2/ are zero.

Using the method presented in the paper [3] we may prove,that solution of the equation /1/ with zero boundary conditions /2/ ha the following form :

$$x(i,j) = \sum_{p=0}^{p=i-1} \sum_{q=0}^{q=j-1} A^{i-p-1,j-q-1} Bu(p,q) \qquad /3/$$

Since, in the sequel we shall consider 2-D system /1/ in a give finite rectangle $[(0,0),(r,s)]$, then it is possible to express the solution /3/ in a more convenient and compact form . In order to do that, let us introduce the following notations :

$$V_{rs} = \underbrace{U \times U \times \dots \times U}_{rs\text{-times}} \qquad /4/$$

V_{rs} is the Hilbert space of all possible input sequences $u(i,j) \in U$ for $(i,j) \in [(0,0),(r,s))$.

$$W_{rs}u_{rs} = \sum_{p=0}^{p=r-1} \sum_{q=0}^{q=s-1} A^{r-p-1,s-q-1} B\, u(p,q) \qquad /5/$$

$W_{rs} : V_{rs} \rightarrow X$ is the bounded linear operator and u_{rs} is the input sequence defined as follows

$$u_{rs} = \{u(0,0),\ u(1,0), \dots, u(i,0), \dots, u(r-1,0),\ u(0,1),\ u(1,1), \dots$$
$$\dots,\ u(i,1), \dots, u(r-1,1), \dots, u(0,j),\ u(1,j), \dots,\ u(i,j), \dots$$
$$\dots, u(r-1,j), \dots, u(0,s-1), u(1,s-1), \dots,\ u(r-1,s-1)\} \qquad /6/$$

Therefore, tyking into account the equality /3/ the solution o the equation /1/ can be expressed in the following very simple for

$$x(r,s) = W_{rs}u_{rs} \qquad /7/$$

However, it should be stressed, that formula /7/ represents the solu- tion of the equation /1/ for fixed point (r,s) and zero boundary c ditions.

Now, let us concentrate on the controllability problem for 2 system /1/. It is well known [10], that for infinite-dimensional ca it is necessary to introduce two different notions of controllabil exact controllability and approximate controllability.

<u>Definition 1.</u> Dynamical system /1/ is said to be exactly co trollable in a given rectangle $[(0,0),(r,s)]$ if for each $x_{rs} \in X$, th exists a sequence of controls $u_{rs} \in V_{rs}$ such that

$$x(r,s) = x_{rs} \qquad /8/$$

<u>Definition 2.</u> Dynamical system /1/ is said to be approximat controllable in a given rectangle $[(0,0),(r,s)]$ if for every $\varepsilon > 0$ and each $x_{rs} \in X$, there exists a sequence of controls $u_{rs} \in V_{rs}$ such th

$$\left\| x(r,s) - x_{rs} \right\|_X \leqslant \varepsilon$$

In other words, approximate controllability means, that we can steer our 2-D system to every neighbourhood of the arbitrary point $x_{rs} \in X$.

Theorem 1. The following statements are equivalent :

i/ 2-D system /1/ is exactly controllable in rectangle $[(0,0),(r,s)]$

ii/ The image of the operator W_{rs} is the whole space X ,i.e. $\mathrm{Im} W_{rs}$

iii/ The selfadjoint controllability operator $W_{rs}W^*_{rs}$ has bounded inverse.

Theorem 2. The following statements are equiavlent :

i/ 2-D system /1/ is approximately controllable in rectangle $[(0,0)$

ii/ The image of the operator W_{rs} is ecerywhere dense in the space i.e. $\overline{\mathrm{Im}\, W_{rs}} = X$

iii/ The selfadjoint controllability operator $W_{rs}W^*_{rs}$ has an inverse / not necessarily bounded/.

In the proofs of the Theorems 1 and 2 the methods of function analysis are extensively used.

Corollary 1. If the spaces X and U are finite-dimensional, the exact controllability is equivalent to approximate controllability in every rectangle .

Several computable conditions for controllability of 2-D syst /1/ in finite-dimensional case are formulated and proved in [1], [5], [10] and [12].

Now, let us consider the minimum energy control problem for 2- system /1/. We generally assume, that our 2-D system is exactly controllable in a given rectangle $[(0,0),(r,s)]$. We want to steer 2-D system /1/ from the zero boundary conditions /2/ to the desired final state $x_{rs} \in X$,using controls $u_{rs} \in V_{rs}$. Since generally, there exist different such controls, we shall look for one, which minimizes the following performace index

$$J(u_{rs}) = \sum_{p=0}^{p=r-1} \sum_{q=0}^{q=s-1} \left\langle u^*(p,q),\ u(p,q) \right\rangle_U = \left\langle u^*_{rs},\ u_{rs} \right\rangle_{V_{rs}} \qquad /10/$$

Now, we shall give the explicite solution to our minimum energy control problem for 2-D system /1/ .

Theorem 3. Let us assume, that 2-D system /1/ is exactly contro able in a prescribed rectangle $[(0,0),(r,s)]$ and that $x_{rs} \in X$ is given Then. the control sequence

$$u^o(p,q) = \left(A^{r-p-1,s-q-1}\right)^* \left(W_{rs}W^*_{rs}\right)^{-1} x_{rs} \quad , \quad (p,q) \in [(0,0),(r,s)) \qquad /11/$$

transfers dynamical system /1/ to the desired final state $x_{rs} \in X$,a minimizes the performance index /10/ .Moreover, the minimum value o /10/ is given by the following formula

$$J(u^o_{rs}) = x^*_{rs}\left(W_{rs}W^*_{rs}\right)^{-1} x_{rs} \qquad /12/$$

Remark 1. The solution of minimum control problem given in the Theorem 3 extends to the case of infinite-dimensional 2-D systems t previous results concerning finite-dimensional cases [2], [3], [7], [8 [10], and [12].

Remark 2. It should be stressed, that the assumption about exa controllability is essential in our considerations. Without this as ption, Theorem 3 is no longer true. However, under the weaker assump about approximate controllability, we construct the sequence of con trolswhich minimizes /10/ and steer 2-D system to the arbitrary ne bourhood of the given final state $x_{rs} \in X$.

Remark 3. The method of proof for Theorem 3 is similar to tho which are valid for finite-dimensional cases [2], [3], [7], [8], [10], However, it should be mentioned, that it is necessary also to use so general theorems taken from the functional analysis.

Remark 4. It should be pointed out, that there are many possi extensions of the presented results. We may consider more general quadratic performance index or to introduce 2-D infinite-dimension systems with variable coefficients [2]. The next possibility is to c sider the so called straightline controllability and to reformulat our minimum energy control problem [3].

References

[1] Kaczorek T.,"Two-Dimensional Linear Systems",Springer-Verlag, New York,Berlin, 1985.

[2] Kaczorek T.,and Klamka J.,"Minimum energy control of 2-D linea systems with variable coefficients",International Journal of Control,vol.44,no.3,1986,pp.645-650.

[3] Kaczorek T.,and Klamka J.,"Minimum energy control for general of 2-D linear systems",International Journal of Control,vol.47 no.5,1988,pp.1555-1562.

[4] Klamka J.,"M-dimensional nonstationary linear discrete system Banach spaces",Proceedings of 12 World IMACS Congress, Paris,1 vol.4,pp31-33.

[5] Klamka J.,"Constrained controllability of 2-D linear systems", Procedings of 12 World IMACS Congress,Paris,1988,vol.2,pp.166-

[6] Klamka J.,"Controllability of singular 2-D systems",Preprints IFAC Workshop on"System Structure and Control:State -Space and Polynomials Methods",Praga,1989,pp.245-248.

[7] Klamka J.,"Optimal control problems for singular 2-D systems", Preprints of IMACS Symposium on "Mathematical and Intelligent Models in System Simulation",Brussels,1990,pp.VII.A.3.1-3.4.

[8] Klamka J.,"Minimum energy control of singular 2-D linear syste with variable coefficients",Preprints of IMACS Symposium on "Modelling and Control of Technological Systems",Lille,1991, vol.2,pp.155-159.

[9] Klamka J.,"Complete controllability of singular 2-D systems", Proceedings of 13 Imacs World Congress ,Dublin,1991,pp.1839-18

[10] Klamka J.,"Controllability of Dynamical Systems", Kluwer Acade Publishers,Dordrecht,The Netherland,1991,

[11] Klamka J.,"Controllability of nonlinear 2-D systems",Bulletin the Polish Academy of Sciences,Technical Sciences,vol.40,no.2, 1992,pp.127-133.

[12] Klamka J.,and Kaczorek T.,"Local controllability and minimum e gy control of n-D linear systems",Bulletin of the Polish Acade of Sciences,Technical Sciences,vol.35,no.11-12,1987,pp.679-685