A MATHEMATICAL PROGRAMMING METHOD FOR DESIGNING F.I.R. DIGITAL FILTERS WITH NONUNIFORM FREQUENCY SAMPLES

by

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The purpose of this project is to develop a method for the design of 1-D, causal, F.I.R. digital filters, the transfer function of which is polynomial with real co-efficients and with no further restraints concerning the form of the amplitude and phase functions. The data of our problem are the desired values of the transfer function (real and imaginary part or phase and amplitude) corresponding to a specific set of frequency values, which belong to the space $[0,\pi]$. The co-efficients of the tranfer function are the unknown values. A basic demand of the problem is the coincidence of the resulting transfer function at the sampled values for a given subset of frequencies (subset of the initial above set). At the beginning we will assume an upper bound for the rank of the polynomial at which the design process will result; nevertheless, this rank will always allow to more than one polynomials to fulfil the interpolation conditions, as discussed above. From all these polynomials we shall choose the one that best approximates the desired transfer function. The choice will be based on the minimization of an "error function". The "error function" derives from the implementation of Euclidean norm between the values of the transfer function and the sampled ones. In essence, we may say that we have to deal with a linear programming problem that consists of a cost function, the minimization of which is our aim, subject to a set of constraints. The unknown co-efficients are involved in both the cost function and the constraints. The fulfilment of the constraints guarantees the interpolation at the critical points, whereas the minimization of the cost function guarantees the best fitting of the filter to the data of the problem. It is also obvious that by increasing the rank of the polynomial successively, we gradually improve the adjustment of the filter to the desired values.

Let us consider $(k+1)$ not uniformly distributed points -frequencies at space $[0,\pi]$ - including zero and the corresponding sampled values. It is obvious that for the above sampled values we have available $(k+1)$ real parts and k imaginary parts. With these frequency-points we create the set Ω. We also create the set Ω_I (subset of Ω with $(l+1)$ elements). The zero frequency is included in Ω_I . Moreover, we must have $l<(k-1)$. We now consider the polynomial transfer function of a 1-D, causal, F.I.R. digital filter with rank 2m. The $(2m+1)$ co-efficients of the polynomial are the unknown values and are assumed real numbers. We should also mention that $l<m<k$. First of all, for every frequency-member of Ω_I the corresponding value of the polynomial should coincide with the sampled value. Thus, if we examine the interpolation problem separately from anything else we conclude to a linear system with $(2m+1)$ variables and $(2l+1)$ equations. But $(2l+1)<(2m+1)$. Generally, this linear system determines a convex set of solutions. We shall choose those solutions that best fit to the sampled data. Therefore, we have to create a cost function, whose minimization we will pursue inside the boundaries of the above set of convex solutions. Thus, we can deal with the minimization of the maximum distance between the values of the polynomial and the sampled ones, corresponding to the elements of $(\Omega-\Omega_I)$. This maximum derives from a set of $(k-1)$ distances , each one of which is a function of the unknown co-efficients. It should be plain by now that the entire problem is a mathematical programming problem and especially an optimization under constraints one. This project suggests an iterative method approximating the optimal point with its convergence being assured.

We observe that by rotating the sampled data by a suitable angle for each case we transform the problem to the design of a non causal, 1-D, F.I.R. digital filter whose co-efficients are identical with the unknown ones. This transformation facilitates the development of the algorithm and reduces its complexity. The transfer function of the non causal filter can be considered as the superposition of an even (smoothing) and an odd (differentiating) component. In this way we create an equivalent problem, where the real components have been disengaged from the imaginary ones. We now have a linear system with $(l+1)$ equations and $(m+1)$ variables for the real parts and another one with l equations and m variables for the imaginary parts. In a given cycle of the algorithm we first minimize for the real parts, assuming that the imaginary parts are constant. It can be proved that the above minimization results in the solution of successive typical linear problems. The next step is to minimize for the imaginary parts, assuming that the real parts are constant. With the completion of each cycle of the algorithm the maximum distance (the cost of the problem) is reduced. Finally, there must be a check for the fulfilment of the termination criteria, which signals the end of the procedure.

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