Dead-Beat Response of SISO Systems to Parabolic Inputs with Optimum Step and Ramp Responses

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ABSTRACT

The design of SISO control systems that exhibit dead-beat response to parabolic inputs with minimum squared-error restrictions on step and ramp responses was introduced by J. L. Pokoski and D. A. Pierre [1]. The performance measure they considered was

$$I=\sum_{k=0}^{\infty} e_s(kT)^2 + h e_r(kT)^2$$

where $e_s(kT)$ and $e_r(kT)$ are the unit step and ramp errors at the kth sampling instant and h is a weighting factor. For the system to exhibit dead-beat response to a parabolic input, they also considered the two well known conditions for the overall transfer function $M(z)$ of the system, that is

$$M(z)=b_1 z^{-1}+b_2 z^{-2}+\cdots+b_n z^{-n}$$

and

$$1-M(z)=(1-z^{-1})^3\left(1+a_1 z^{-1}+\cdots+a_{n-3} z^{-n+3}\right)$$

Stability considerations impose that all unstable (or critically stable) poles of the plant must be included in 1-*M(z)* as zeros, and all zeros of the plant that lie on or outside the unit circle must be included in *M(z)* as zeros [2]. The equations resulting in the α_k which minimize I may be found by setting

$$\frac{\partial I}{\partial a_k}=0 \qquad for\ k=1,2,\ldots,n-3$$

This results in n-3 equations which may be solved for the n-3 values of α_k. But, according to the authors, since it is difficult to solve the resulting equations for the α_k as functions of both h and n, they are solved in [1] for the limiting cases, $h=\infty$ (only ramp error considered) and $h=0$ (only step error considered). The responses for intermediate values of h are generally expected to lie between those of the preceding cases.

In this work is proposed a new approach to the design of optimum dead-beat response systems to parabolic inputs based on the same performance criterion. Suppose there are n steps before the settling of the output signal when the system is forced by a step, a ramp or a parabolic input, and denote as α_k, b_k and c_k the terms of the error sequences in response to a step, a ramp and a parabolic input, respectively. These error sequences are not independent to each other, but are related through [3]

$$b_k=T\sum_{i=0}^{k-1} a_i \qquad for\ all\ k$$

and

$$c_k=T^2\sum_{i=0}^{k-1}\left(a_i+2\sum_{l=0}^{i-1} a_l\right) \qquad for\ all\ k$$

where T is the sampling period. From these relations are easily derived the two necessary and sufficient error conditions for dead-beat response to parabolic inputs, namely the

$$\sum_{i=0}^{n} a_i = 0$$

and

$$\sum_{i=0}^{n} \sum_{l=0}^{i-1} a_l = 0$$

The optimization is performed by the minimization of the sum of the squared step and ramp response error sequence coefficients of the system, in which each error sequence is regarded with a different weighting factor, that is as an objective function is selected the

$$J = \sum_{k=0}^{n} \left(s a_k^2 + r b_k^2 \right)$$

and the optimization problem is solved using the Lagrange method of undetermined multipliers, thus obtaining a general solution. Special cases are considered in detail, and response characteristics are illustrated through diagrams with typical prototype responses, and normalized overshoot and cost function curves. The response to complex inputs is also explored.

REFERENCES

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