An Application of Robust Control to Nonlinear Distributed Parameter Systems

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ABSTRACT

In this paper we present several results concerning the construction of low-order robust feedback control laws for certain classes of nonlinear distributed parameter systems. As a first step we use a linearization of the nonlinear system to define various optimal regulator problems leading to optimal feedback control laws. In particular, the standard weighted LQR problem and the robust control problem based on the differential game formulation of H^∞ control will be considered. Although the resulting control laws are optimal and robust for the linearized model, there are a number of issues that should be considered before selecting and applying a feedback law to the nonlinear system. It is important to analyze and compare the resulting closed loop systems. We shall carry out this program for a general nonlinear system, apply the results to a nonlinear shock wave problem and use finite elements/reduction of order to construct feedback control laws of low order. These laws will be tested on specific examples to illustrate the basic idea.

Let Z, U, W and Y be real Hilbert spaces. We consider the system

$$\dot{z}(t) = Az(t) + F(z(t)) + Bu(t) + Gw(t) \tag{1}$$

with initial data

$$z(0) = z_0 \in Z \tag{2}$$

and output

$$y(t) = Cz(t). \tag{3}$$

Here we assume that A is self-adjoint and generates a stable analytic semigroup $S(t)$ on H, $B : U \to Z, G : W \to Z$ and $C : Z \to Y$ are continuous linear operators. The nonlinear operator $F : D(F) \subseteq Z \to Z$ is assumed to be locally Lipschitzian on the space Z_μ defined for $0 \le \mu \le 1$ by $Z_\mu = D((-A)^\mu)$ with $\|z\|_\mu = \|z\| + \|(-A)^\mu z\|$. Also, we assume that $F(0) = 0$.

Consider the "linearized system"

$$\dot{z}(t) = Az(t) + Bu(t) + Gw(t) \tag{4}$$

$$y(t) = Cz(t) \tag{5}$$

and the "disturbance-augmented" cost functional

$$J = \frac{1}{2}\int_0^\infty e^{\alpha t}\{\langle Qz(t), z(t)\rangle + \langle Ru(t), u(t)\rangle - \frac{1}{\gamma^2}\langle Mw(t), w(t)\rangle\}dt \tag{6}$$

where $\gamma > 0, \alpha \geq 0, Q = C^*C, R : U \to U$ and $M : W \to W$ are bounded and self-adjoint. Note that if $\alpha = 0$ and $G = M = 0$, then (4) - (6) define the standard LQR problem. If $\alpha = 0$ and $G \neq 0$, then (4) - (6) define the so called soft-constrained differential game that (under certain conditions) is equivalent to the H^∞ control problem for (4) - (5).

In the second case, if the operators R and M are positive definite and $BR^{-1}B^* - \gamma^2 GM^{-1}G^* \geq 0$, then there exists a unique pair of functions $u^{opt}(\cdot), w^{opt}(\cdot)$ that provide a saddle point for the differential game. Moreover,

$$u^{opt}(t) = -R^{-1}B^*\Pi z^{opt}(t) = -Kz^{opt}(t) \tag{7}$$

$$w^{opt}(t) = \gamma^2 M^{-1}G^*\Pi z^{opt}(t) = Lz^{opt}(t) \tag{8}$$

where Π solves the algebraic Riccati equation

$$\langle \Pi z, Ay\rangle + \langle Az, \Pi y\rangle + \langle Qz, y\rangle - \langle \Omega\Pi z, \Omega\Pi z\rangle = 0 \tag{9}$$

and

$$\Omega = [BR^{-1}B^* - \gamma^2 GM^{-1}G^*]^{1/2} \geq 0$$

In any case, the feedback control law has the form

$$u(t) = -Kz(t) \tag{10}$$

where K can be computed by solving a Riccato equation. Consequently, numerical methods developed over the past ten years can be applied to construct approximations K^N to K. These approximations are finite dimensional and hence can be implemented. In particular, it is important to know that if the control law

$$u^N(t) = -K^N z(t) \tag{11}$$

is applied to the full nonlinear system, then the resulting closed-loop distributed parameter system achieves performance and robustness.

We shall formulate a shock wave control problem as a system of the form (1) - (3) and use approximation theory to compute the corresponding sub-optimal robust gain operators. This application is used to illustrate the convergence of the computational algorithm, to study the robustness properties of various control laws obtained by this method and to motivate a few open problems.