# Model Reduction for Stability and Bifurcation Calculations for Nonlinear PDEs

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## Abstract

Many physical and engineering systems are spatially distributed, described by partial differential equations, and hence characterized by a very large or infinite number of degrees of freedom. Simulation of such processes can be prohibitively expensive, and even if the simulation can be performed, the results may be difficult to interpret and exploit. It is therefore important to develop techniques to accurately reduce the governing equations (and high-dimensional data sets resulting from them, or from experiments) to forms which are more easily simulated and understood. Such accurate, low-dimensional models of distributed systems can also be used in conjunction with traditional linear of nonlinear control methodologies.

There are many observations and theoretical results suggesting that this reduction goal is attainable in a number of interesting cases. Many fluid flows and spatially distributed chemically reacting systems, for example, show behavior characteristic of systems with only a few degrees of freedom (low-dimensional attractors, universal routes to chaotic dynamics) even though the processes are formally modeled by infinite dimensional systems (PDEs). Rigorous results exist for many of these models showing that the long-term dynamics are indeed governed by a finite number of "determining modes." An important facet of the rigorous theory is the idea of the Approximate Inertial Manifold (AIM). An AIM is a finite-dimensional, smooth, exponentially attracting manifold which contains the long-time dynamics of the PDE. The dynamics on this surface are described by a finite number of ordinary differential equations (an Inertial Form). The practical question we address here is how these results can be exploited to determine the important degrees of freedom, and how to efficiently model the long-term dynamics of the PDEs. We discuss computational tools for this task and present some illustrative examples from applications.

One of the approaches we use is based on the approximation of Inertial Manifolds, and the subsequent construction of reduced Galerkin schemes on these manifolds (so-called nonlinear Galerkin schemes). Several such approximations are discussed, along with issues regarding their implementation. They are illustrated using amplitude equations for interfacial instabilities.