A Parallel LU-Decomposition Algorithm for Modeling Discrete Signals by Polynomial or Exponential Interpolation

E. Angelidis and M. Kazantzakis

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ABSTRACT. *A parallel LU-decomposition algorithm for modeling polynomial or exponential signals using Newton interpolation is presented. The three solution stages required by the LU-decomposition procedure-(1) LU decomposition (2) forward substitution and (3) backward substitution-are performed in one stage with optimum speedup and efficiency. Constructing parallel graphs and task schedules we can obtain high performance in SIMD or MIMD computer architectures. The proposed algorithm is based on selecting properly triangular subsystems and deriving first-order linear recursive relations for the parallel evaluation of their elements we can also use array processors. Given n discrete sample points the modeling problem is solved in 2n computational levels if n processors are available. The obtained efficiency $E_n(n)$ and the speedup $S_n(n)$, for large n, take the ideal values of 1 and n, respectively. Increasing the number of computational levels we can restrict the number of processors in the range of 2 to O(n). In the progressive case when a new n+1 sample appears, the proposed method requires only 4n time units to update the design parameters. The method can be extended to develop parallel algorithms for Fourier matrix factorization, DFT calculation, design of FIR digital filters, 2-D applications, etc.*

I. INTRODUCTION

Interpolation methods have been used extensively in numerical mathematics, electrical engineering and digital signal processing [1],[2],[5]-[8]. The emergence of fast and efficient parallel algorithms particular in digital signal processing has meant that the general LU-method [9],[11],[15]-[17] and modifications [3],[12],[13] applied to polynomial or exponential signal modeling and FIR filter design have to be improved in order to obtain better efficiency and speedup with minimum computational requirements and economically number of processors. This can be achieved by selecting properly triangular subsystems in the LU-method and deriving simple recursive relations for their parallel computation. We implement this idea using lower triangular matrices with elements that can be obtained from the Newton interpolating polynomials and inverted upper triangular matrices with elements that can be calculated from simple recursive relation. Considering a time unit to be a floating point add, a floating point multiply, and some subscripting, the general parallel algorithm in [11] requires $T_p=n^2-1$ computational levels using [n/2] processors with an efficiency of $E_p=2/3$ for large n. The tridiagonal and bidiagonal factorization methods [3] use systolic arrays of n^2 processors to compute sequences of k-points DFT's. The Quadrant Interlocking Factorization methods [12],[13] need either a maximum number of $2(n-1)^2$ or $(n-1)^2$ processors working in parallel.

The serial algorithm described in [15],18] needs $3n^2/2$ time units in $T_n=3n$ levels with an efficiency, $E_n(n)=0.5$ and a speedup, $S_n(n)=0.5n$.

The proposed algorithm provides the solution of the modeling problem in $2n^2$ time units in only $T_n=2n$ levels, with an efficiency, $E_n(n)=1$, and a speedup, $S_n(n)=n$, for large values of n. Thus the n processors are fully employed and allow to speed up the computation by a factor of n. Increasing the number of computational levels we can also restrict the number of the required processors in the range of 2 to n. Using the first-order linear recurrence relations derived for the solution of the forward and backward substitution subsystems we can also develop cyclic reduction parallel algorithms as described in [16], [17], suitable for processor arrays. In the progressive case when a new sample is added, we do not have to evaluate the unknown coefficients from the beginning, but only updating them with correction terms which depend on the last sample. In this case only 4n time units are required.

The proposed method can be extended to develop efficient parallel algorithms for Fourier matrix factorization, calculation of DFT, design of Finite Impulse Response (FIR) digital filters, 2-D applications etc.

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MAILING ADDRESS

Dr. E. Angelidis
Marathonos 22
Agia Paraskevi
GR-15343
Athens-GREECE