Systems Identification using Recurrent Asymptotically Stable Neural Networks

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Abstract— **A training procedure for a class of neural networks that are asymptotically stable is presented. The training procedure is a gradient method which adapts the interconnection weights as well as the relaxation constants and the slopes of the activation functions used so as the error between the expected and obtained responses is minimized. A method for assuring that stability is maintained throughout the training procedure is also given. Such a network was used to identify the dynamic behavior of several nonlinear dynamical systems which included a PUMA 560 robot and a boat based on collected rudder/heading data.**

I. Introduction

This paper is a summary of some recent work done in the area of identification of nonlinear systems using neural networks. The main purpose of this work is to provide a way of establishing models of complex nonlinear systems that can be used in controllers. Neural networks are selected as a potentially effective way of modeling these systems, since a trained neural network is fast and easy to implement, properties that are desirable in real controllers.

One problem with using a system as complex as a nonlinear neural network in a control or identification setting is that they are often too complex to analyze fully; in particular, their stability can not be assured. When dealing with real systems, stability is the single most important property of a controller or model. Fortunately, a class of neural networks exists which is known to be asymptotically stable. This class of neural networks is used here, and the work done on identification pertains to this class of dynamic neural networks.

II. Background

It has been shown [1] that asymptotic stability is ensured for neural networks described by the differential equation

$$\dot{O} = -TO + Wf(O) + b \tag{1}$$

In (1), there are N neurons divided into k classes, and

$$O = \begin{bmatrix} O_1 & O_2 & \cdots & O_k \end{bmatrix} = \begin{bmatrix} o_1 & o_2 & \cdots & o_N \end{bmatrix} \tag{2}$$

is the state of the neural network,

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1k} \\ \vdots & & & \\ W_{k1} & W_{k2} & \cdots & W_{kk} \end{bmatrix} \tag{3}$$

is the network connectivity matrix, $T = \text{diag}(\tau_i)$ is the diagonal matrix of neural relaxation constants, b is the input to the neural network, and $f(O)$ belongs to the class of so-called neuromime functions, which are essentially positive and monotonically non-decreasing. The condition on W that guarantees asymptotic behavior is that it must contain all of its positive entries on one side of the main diagonal [1]. This gives an easy way to check whether a neural network is stable. For instance, the neural network shown in Figure 1 is stable provided that the connection weights in submatrices W_{23} and W_{34} are non-positive (i.e., inhibitory). This result is extremely useful in the area of identification and control. The most important feature of a controller or model is that it must be stable. By starting with a model as defined by (1), stability is ensured.

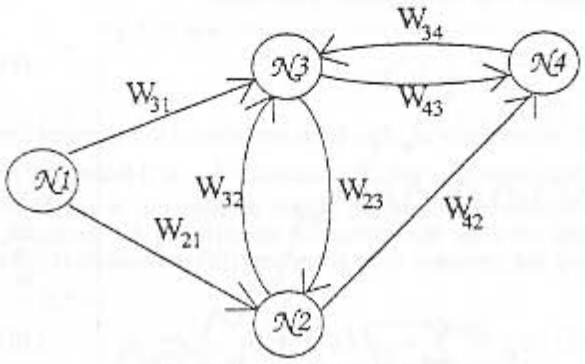

Fig. 1. Sample Neural Network

III. Parameter Adjustment In Stable Neural Networks

This section discusses a method for adjusting the weights and other parameters of neural networks that are stable in the sense described in Section 2. The general approach that is used here is to define some a criterion and then adjust the parameters in a direction that will decrease this cost. In this sense the technique is similar to linear recursive adaptive methods [4] and to classical back propagation [5]. However, since the stable neural networks described in section 2. have certain restrictions on the polarity of the connection of classes, a straightforward gradient adjustment is not possible. A solution for this is also presented here.

A. *Gradient of Cost Function*

The general equation for calculating the behavior of the class of neural networks of interest here is

$$\dot{O} = -TO + Wf(O) + b \tag{4}$$

One possible criterion for measuring the performance is the quadratic cost function

$$J(e) = 1/2(O - O_d)^T A(O - O_d) = 1/2e^T Ae \tag{5}$$

where O_d is the desired state of the neural network. A is a diagonal matrix with ones corresponding to output neurons

and zero's elsewhere. As in other recursive adaptive methods [2,4], parameters θ in the neural network are adjusted along the negative gradient of this cost, i.e.,

$$\frac{d\theta}{dt} = -\eta \frac{\partial J}{\partial \theta} \tag{6}$$

The chain rule for differentiation is used to allow for the calculation of this gradient for parameters associated with neuron j:

$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial o_j}\frac{\partial o_j}{\partial \theta} = \gamma_j \frac{\partial o_j}{\partial \theta} \tag{7}$$

The notation γ_j is used to denote the derivative of the cost with respect to the activation of neuron j. If neuron j is an output neuron, this derivative is simply given by

$$\gamma_j = o_j - o_{d_j} \tag{8}$$

In a manner analogous to traditional back propagation of the error [5], this gradient may be calculated for units that are not output neurons by using the values of the gradient in all the neurons k that have neuron j as inputs:

$$\gamma_j = \sum_k \gamma_k \frac{\partial o_k}{\partial o_j} = \sum_k \gamma_k \Delta_{kj} \tag{9}$$

Here, the notation Δ_{kj} has been introduced to represent the partial derivative $\partial o_k / \partial o_j$. To calculate Δ_{kj}, it is necessary to use the differential equation which defines the behavior of the neural network. Rewriting (4) specifically for neuron k, and using the operator D to represent differentiation results in

$$(\tau_k + D)\, o_k = \sum_j w_{kj} f(o_j) + b_k \tag{10}$$

Differentiating (10) with respect to o_j results in

$$\dot{\Delta}_{kj} = (-\tau_k)\, \Delta_{kj} + w_{kj} f'(o_j)\,. \tag{11}$$

All the derivatives required in (7) to adjust a parameter θ have now been obtained, except for the derivative $\partial o_j / \partial \theta$.

B. Input Weight Adjustment

Let θ represent a connecting weight w_{ji} which connects neuron i (input) to neuron j. Use the notation $\xi_{ji} = \partial o_j / \partial w_{ji}$. Differentiating (10) with respect to w_{ji}, the differential equation for ξ_{ji} is obtained:

$$\dot{\xi}_{ji} = -\tau_j \xi_{ji} + f(o_i) \tag{12}$$

Using this equation and the results of the previous section, equation (7) may now be written as

$$\frac{dw_{ji}}{dt} = -\eta \gamma_j \xi_{ji} \tag{13}$$

with γ_j calculated using (8) or (9) as appropriate.

C. Adjustment of Structural Parameters

The same analysis that was used to determine how to adjust the connection weights can also be used to obtain a formula for adjusting any of the other variables that parameterize the neural network [6].

D. Weight Clamping

Section 2 describes a class of neural networks that are asymptotically stable. This condition is guaranteed provided that the connectivity matrix W has all of its positive entries on one side of the diagonal [1]. However, (13) gives a formula for adjusting the connection weights that may violate this condition. To combat this, it is necessary to check the polarity of certain crucial weights after each weight adjustment. For instance, as discussed in section 2, if the weights labeled W_{23} in Figure 1 are guaranteed to be non-positive, then the neural network will be stable. Thus after any weight in W_{23} is adjusted using (13), the weight should be checked to ensure that it is not positive. If it is, then it should be clamped at 0. This technique ensures that inhibitory weights stay inhibitory throughout the training procedure.

IV. Identification

The term identification is used in this section to refer to the process of developing a model of an unknown system by observing its input/output behaviour.[2,4].

In this section, we propose a suitable neural network architecture and use it to identify the dynamic behavior of a PUMA-560 two-link robot and that of a boat.

A. Identification Architecture

Consider the nonlinear system described by the relation

$$\dot{y} = u - \frac{y}{1 + 4y^2} \tag{14}$$

If y remains relatively constant near some value y_{ss}, then this system can be approximated by a first order linear system with a pole at $(1 + 4y_{ss}^2)^{-1}$. If y varies from this value, then the pole can be thought of as "roving" in some sense. Although this is not an exact description of the behavior of the system, it does illustrate one of the more common types of nonlinearity which is encountered in real systems such as valve flows and airplanes cruising at various velocities.

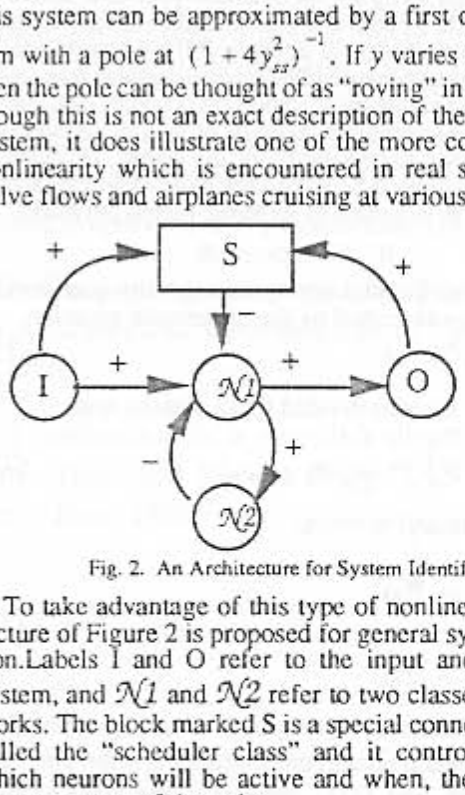

Fig. 2. An Architecture for System Identification

To take advantage of this type of nonlinearity, the architecture of Figure 2 is proposed for general system identification.Labels I and O refer to the input and output of the system, and $\mathcal{N}1$ and $\mathcal{N}2$ refer to two classes of neural networks. The block marked S is a special connection of classes called the "scheduler class" and it controls or schedules which neurons will be active and when, thereby emulating the movement of the pole.

Neurons in the scheduler class have a peaked response as

shown in Figure 4. (This class of neurons is not a single class as governed by (4). However, the response shown in Figure 4 was generated using a combination of 4 standard classes in a configuration shown in Figure 3. For clarity, the scheduler neurons are discussed as a single class).

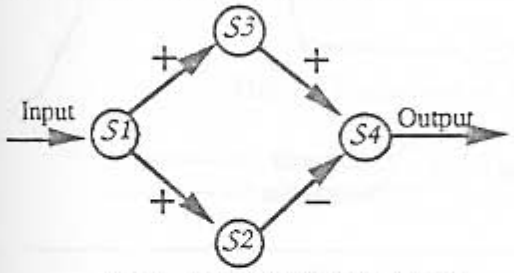

Fig. 3. Architecture for Scheduler class

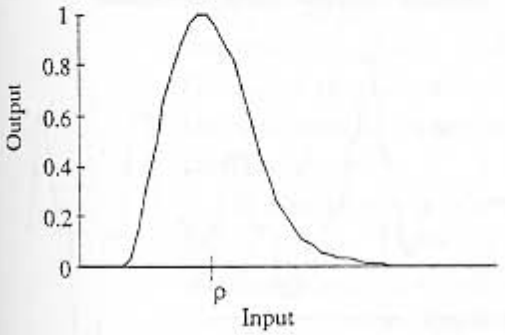

Fig. 4. Response of Scheduler Neurons

Each neuron in the class has a peak ρ that occurs at a different value. Figure 2 shows that the scheduler class receives input from I and O. Thus, depending on the value of the input and output, different neurons in $\mathcal{N}1$ and $\mathcal{N}2$ will be active.

B. Identification of a Two-Link Robot Arm

A two-link robot arm is known to have its dynamic response governed by the differential equation

$$H(q)\ddot{q} + h(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \Phi v \qquad (15)$$

where the state q contains the angle θ_1 that the first link makes with the vertical and the angle θ_2 that is formed between the two links; $H(q)$ is the 2x2 inertial matrix; $h(q, \dot{q})$ models the Coriolis and centripetal forces; F is the friction matrix; $g(q)$ represents the gravitational torque; and Φ is the voltage-to-torque conversion matrix [8]. All of these variables rely on many machine-specific factors, such as dimensions, weights, inertia, and joint friction. To obtain an accurate model, one measures directly as many variables as possible. This was done for a PUMA-560 robot. Lengths, masses, and inertias were obtained through direct measurement [9]. Variables which could not be easily directly measured were the matrices F and Φ. This represented four unknown scalars in total. Classical RLS parameter estimation was used to identify these four variables, and the final response to the input vector shown in Figure 7 is shown in Figure 8. Although Figure 8 shows that there was some prediction error, it was found [8] that this model was accurate enough to allow for an extremely accurate controller design when this model was used for closed loop control. The fact that the model deviates from the actual response underlies the difficulty in identifying complex systems using traditional model based methods.

A neural network was trained to identify the dynamic response for θ_1, the angle that the first link makes with the vertical. Both $v_1(t)$ and $v_2(t)$ were used as inputs to the system. The neural network had an architecture similar to that shown in Figure 2, except that $\mathcal{N}2$ was not included. Class $\mathcal{N}1$ contained 5 neurons as did the scheduler class.

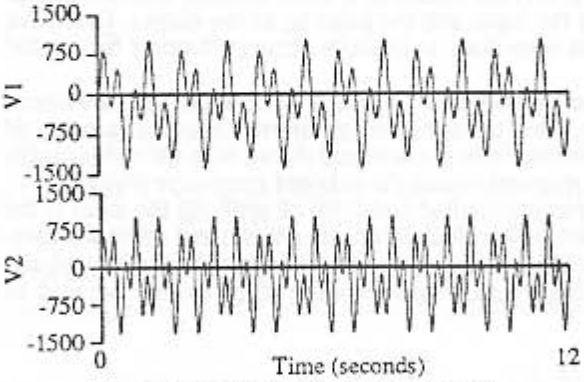

Fig. 5. Control Voltage for Link 1 and Link 2

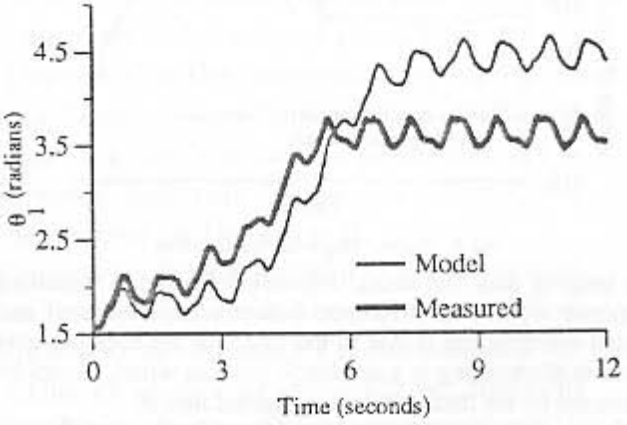

Fig. 6. Classical Model (Equation 15) and Measured Response to Input

Convergence of the response was rather slow; several million training iterations were required. Good results were obtained after two days of training on a Sparcstation. After training was completed, the neural network followed the model closely.The response is shown in Figure 7.

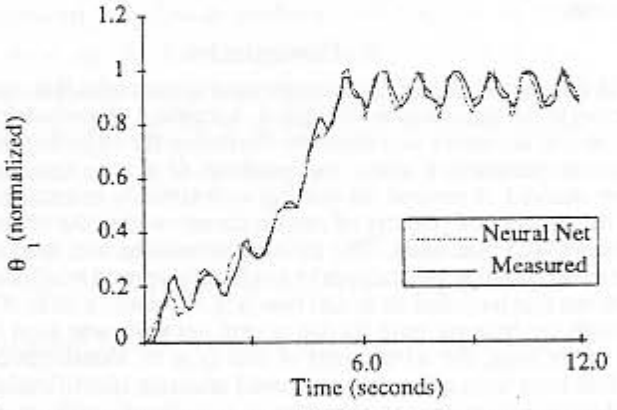

Fig. 7. Measured and Neural Net Response to Input shown in Figure 5

This reveals that the neural network tracks this particular input better than the classical model obtained using least squares parameter estimation and empirical measurement. The advantage of the neural network is that due to its small size and simple calculation procedure, it is ideally suited to use in real hardware controllers driven by chips such as the HC-11. Furthermore, the laborious process of physical parameter measurement is avoided

C. Identification of a Boat

A boat may be treated as a SISO system, with the rudder angle as the input and the heading as the output. Extensive work has been done to produce accurate models for marine craft[3].

Figure 8 shows the data which was used to train the neural network. This is equivalent to approximately 2 minutes of data collected from the boat and shows both the rudder angle and the response which the onboard gyroscope yielded.

The training method consisted of applying the input to the neural network, calculating its response, and using the measured heading to generate an error signal for weight adjustment. Figure 9 shows the response of the trained network to

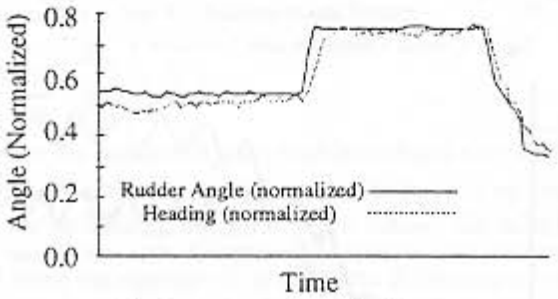

Fig. 8. Rudder Angle for Training Run

the training data.The neural network follows the measured response well. The difference between the measured and neural net response is due to the fact that the measurement noise in the heading is a stochastic process which cannot be predicted by the deterministic neural net model.

To test if the neural network had completely identified the boat at this particular speed through the water, a longer run of data was used. This consisted of approximately 12 minutes of collected data. The weights which had been developed on the shorter training run were used. Figure 10 shows that the neural network had indeed developed a good model of the boat since good tracking was obtained throughout the longer test run.

V. Conclusions

In this paper, a class of recurrent neural networks that was known to be stable was investigated. A training procedure for the neural networks was obtained Formulae for adjusting the network parameters along the gradient of a cost function were derived. A method for dealing with stability restrictions on the connection polarity of certain classes within the neural network was discussed. This training procedure was used to train very small neural networks to identify several nonlinear systems that included an actual two-link robot and a boat. Although the training time for the neural network was seen to be rather long, the advantages of this type of identification model were seen to be that it allowed accurate identification and produced an easy to calculate, a guaranteed stable models of very complex systems.

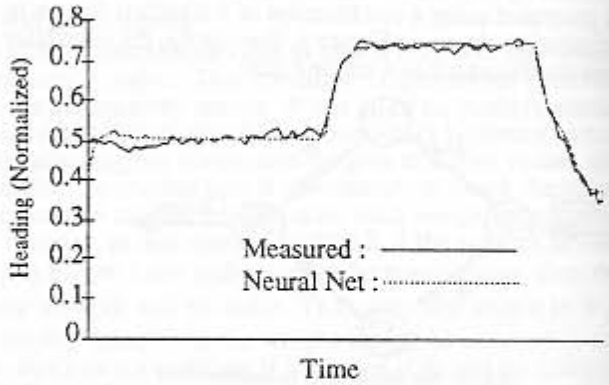

Fig. 9. Measured and Trained Neural Net Response

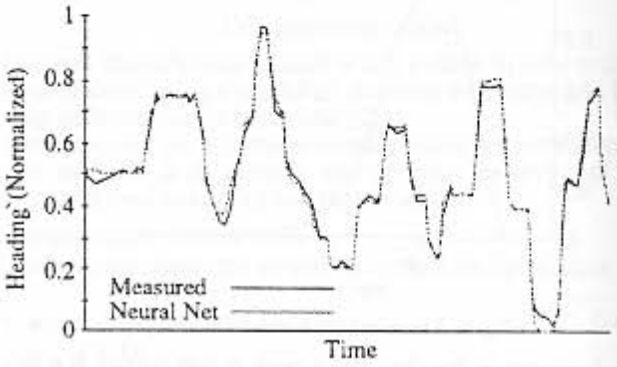

Fig. 10. Measured and Neural Net Response to 12 Minute Test Run

Acknowledgement

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