COMPUTABLE PERFORAMNCE BOUNDS FOR LINEAR REPETITIVE PROCESSES

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Repetitive, or multipass, processes are class of 2D systems characterised by a recursive action over a finite fixed duration - the pass length α. On each sweep, or pass, through the dynamics an output - the pass profile - is produced which acts as a forcing function on, and hence contributes to, the next pass profile. Formally, the pass profile $Y_k(t)$, $0 \leq t \leq \alpha$, produced on the kth pass contributes explicitly to the new pass profile $Y_{k+1}(t), 0 \leq t \leq \alpha, k \geq 0$.

Industrial examples include coal mining operations and metal rolling and the inherent 2D structure is clear since two parameters are required to specify a variable, i.e. pass number and 'position' along a pass.

The essential unique control problem for these processes is the possible presence in the output sequence $\{Y_k\}_{k \geq 1}$ of oscillations which increase in amplitude from pass to pass. Further, it is known that attempting to control this behaviour using, in effect, standard, or conventional, techniques will almost always end in failure [1]. This, in turn, has led to the development of a rigorous stability theory for linear constant pass length processes based on an abstract model formulated in a Banach space setting [1].

Differential linear processes are a sub-class which are of particular interest in the modelling, at least for preliminary simulation and control related studies, of industrial examples such as bench mining systems. The state space model in this case is

$$\begin{aligned} \dot{X}_{k+1}(t) &= AX_{k+1}(t) + BU_{k+1}(t) + B_0 Y_k(t) \\ Y_{k+1}(t) &= CX_{k+1}(t), X_{k+1}(0) = 0 \ , \\ & 0 \leq t \leq \alpha < +\infty \ , \ k \geq 0 \end{aligned} \tag{1}$$

Here $Y_{k+1}(t)$ is the $m \times 1$ current pass profile vector, $X_{k+1}(t)$ is the $n \times 1$ current pass state vector, and $U_{k+1}(t)$ is the $\ell \times 1$ current pass input vector.

The abstract stability theory in this case translates to a set of necessary and sufficient conditions which can be tested using, in effect, standard linear systems tests. Suppose also that these conditions hold. Then the output sequence $\{Y_k\}_{k \geq 1}$ converges to a 'steady', or so-called limit, profile Y_∞ which is a stable standard linear system. This, in turn, suggests the following general control objective:

Drive $\{Y_k\}_{k \geq 1}$ to a limit profile Y_∞ with 'acceptable' dynamics in a 'reasonable' number of passes and, simultaneously, maintain a 'tolerable' error Y_k-Y_∞.

Here the quotation marks denote features whose precise interpretation is to be determined by knowledge of the particular application under consideration. A complete treatment of this control objective, including refinements, can be found in [2].

A major advantage of 'Bode and Nyquist like' tests for standard systems is that useful performance indicators, such as gain and phase margins in the SISO case, are immediately available. In the repetitive systems case, useful performance indicators, again see [2] for a complete treatment, clearly would be appropriately defined gain and phase margins plus computable information on
(i) the rate of approach of $\{Y_k\}_{k \geq 1}$ to Y_∞; and
(ii) the error Y_k-Y_∞ on any pass k.

The difficulty here is that the currently available tests are of little use in terms of (i) and (ii) and the only option is to undertake detailed simulation studies

which could, of course, be computationally very expensive.
This paper shows how this difficulty can be removed by the use of so-called simulation based stability tests. Essentially, these tests are based on the use of operator norms that can be evaluated from step response data using relatively simple computations. The underlying space here is $L_\infty(0,T)$, with T finite or infinite, and the calculations involved are the evaluation of the L_1 norm of the impulse response of a linear time invariant system in the form of the total variation of the step response. Examples will be given which illustrate the software, see [2] and [3], which has been written to implement these tests.

The basic result used here is the following (proved, for example, in [1]).

Lemma 1: Suppose that f is a scalar continuous function defined on $[0, +\infty]$ and of bounded variation on any finite interval $[0, T]$. Then the norm of f on $[0, T]$, denoted $N_T(f)$, is given by

$$N_T(f) = |f(0+)| + \sum_{j=1}^{k} |f(t_j) - f(t_{j-1})| + |f(T) - f(t_k)| \qquad (1)$$

where $0 = t_0 < t_1 < t_2 < \ldots$ are the local maxima and minima of f on $[0, +\infty]$ and $t_k \leq T < t_{k+1}$. In the case of $T = +\infty$,

$$N_\infty(f) := \sup_{T \geq 0} N_T(f) = \lim_{T \to +\infty} N_T(f) \qquad (2)$$

whenever the limit exists.

Some on-going research in this general area will also be briefly discussed.

REFERENCES

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