ADAPTIVE CONTROL FOR A CERTAIN CLASS OF NONLINEAR SYSTEMS.

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ABSTRACT

In this paper an adaptive controller based on model matching problem for a specific class of nonlinear systems, is developed.

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1. THE ADAPTIVE CONTROLLER.

We shall be concerned with the adaptive control of nonlinear systems of the form:

$$y(t) + \sum a_i y(t-i) + \sum a_{ij} y(t-i)y(t-j) + \cdots + \sum a_{i_1,\ldots,i_n} y(t-i_1)\cdots y(t-i_n) = \\ = \sum b_i u(t-i) \tag{1.1}$$

We define the operator $\delta_{\mathrm{i}} = \delta_{\underline{i_m}}$ as follows:

$$\delta_{\underline{i_m}}\{y(t)\} = \delta_{i_1}\delta_{i_2}\delta_{i_3}\ldots\delta_{i_m}\{y(t)\} = y(t-i_1)y(t-i_2)\ldots y(t-i_m) \tag{1.2}$$

where a multindex $\mathrm{i} = \underline{i_m}$ of order m is an element $(i_1, i_2, \ldots, i_m)$ of the set $Z^m = I_m$. We write the above equation (1.1) in an algebraic fashion as in [3]: $A(\delta)y(t) = B(\delta)u(t)$ where

$$A(\delta) = \sum_{n=1}^{k} \sum_{\mathrm{i}\in \mathbf{I_n}} a_{\mathrm{i}}\delta_{\mathrm{i}} \quad , \quad B(\delta) = \sum_{n=1}^{m} \sum_{i\in J_1} a_i\delta_i$$

and $\mathbf{I_n}$, J_1 finite sets of integers.

The design philosophy of the adaptive controller relies on the model reference approach [1]. The controller is specified by the equation: $R(\delta)u(t) = T(\delta)u_c(t) - S(\delta)y(t)$ where R, S, T are δ-polynomials and $u_c(t)$ is the input signal to a desired system. We shall assume that the desired system is linear: $A_d(\delta)y(t) = B_d(\delta)u_c(t)$ The controller aims to reconfigure the plant so that it matches the above desired model.

Let us assume that the unknown systems parameters are the coefficients of $A(\delta)$ and $B(\delta)$ collected in the vector θ_0. Then it is easy to see that it takes the linear regression form: $y(t) = \varphi^T(t-1)\theta_0$ where the regressor vector has the form:

$$\varphi(t-1) = [Y(t-1), \quad U(t-1)]$$

$$Y(t-1) = [y(t-1), \ldots, y(t-k), y^2(t-1), \ldots, y(t-l_1)y(t-l_2)\cdots y(t-l_k)]$$

$$U(t-1) = [u(t-1), \ldots, u(t-m)]$$

Once an estimate of the nominal parameters is available the corresponding control polynomials $R(t), S(t), T(t)$ are determined from a suitable nonlinear Diophantin equation. To define these equations we need the concept of the star product [3]. Given two series(polynomials) their star product is the δ-series:(δ-polynomial)

$$C(\delta) = A(\delta) * B(\delta) = \sum_{m=1}^{\infty} \sum_{\mathrm{k} \in \mathrm{K}_m} \sum_{\mathrm{j} \in (\cup_n \mathrm{I}_n)^m} a_{\mathrm{k}} b_{j_1} b_{j_2} \cdots b_{j_m} \delta_{\mathrm{j} \oplus \mathrm{k}} \tag{1.3}$$

where $j = (j_1, j_2, \ldots, j_m) \in (\cup_n I_n)^m = \cup_n I_n \times \cup_n I_n \times \cdots \times \cup_n I_n$ m-times. Furthermore the pointwise sum $j \oplus \underline{i_m}$ is defined as follows: Let $j = (j_1, j_2, \ldots, j_m) \in I_{k_1} \times I_{k_2} \times \ldots \times I_{k_m} \subset Z^{k_1+k_2+\ldots k_m}$, we extend $\underline{i_m}$

to the vector: $\underline{i_m}' = (\underbrace{i_1, \ldots, i_1}_{k_1 times}, \underbrace{i_2, \ldots, i_2}_{k_2 times}, \ldots, \underbrace{i_m, \ldots, i_m}_{k_m times})$ and we set: $j \oplus \underline{i_m} = j + \underline{i_m}'$

It is easy now to show that if the controller is connected to the plant by feedback, it will match the desired plant if the following equations hold:

$R * A + B * S = A_d$ $B * T = B_d$ Since B and B_d are linear, T is linear as well. We decompose A into its linear and nonlinear part, ie: $A = A_l + A_{nl}$ and $A_l = \sum_{i \in I_1} a_i \delta_i$. Likewise let S_l and S_{nl} be the linear and nonlinear parts of S. Then the initial equation is equivalent to: $R * A_l + B * S_l = A_d$, $R * A_{nl} + B * S_{nl} = 0$

Suppose now that the following assumptions hold for the nominal plant:

Assumption 1.1:

1. $m(A), m(B), d(A), d(A_l), d(B)$ are known.

1. The degrees of polynomials A and B are known.

3. $gcd(A_l, B)|A_d$, $B|B_d$, $d(A_{nl}) > d(B)$.

Then the following indirect adaptive algorithm applies.

Adaptive Controller 1.2:

STEP 1: We estimate the unknown parameters:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-2)\varphi(t-1)}{1+\varphi^T(t-1)P(t-2)\varphi(t-1)}[y(t) - \varphi^{\mathrm{T}}(t-1)\hat{\theta}(t-1)] \tag{1.4}$$

$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t)\varphi^{\mathrm{T}}(t)P(t-1)}{1+\varphi^T(t)P(t-1)\varphi(t)} \tag{1.5}$$

with $P(0) = \varepsilon \mathrm{I}, \varepsilon > 0$ and

$$\hat{\theta}^T(t) = (-a_1(t), -a_2(t), \ldots, -a_{ij}(t), \ldots, -a_{i_1 i_2 \cdots i_\lambda}(t), b_1(t), \ldots, b_r(t)] \tag{1.6}$$

STEP 2: We first factorize $\hat{A}_{nl}$ i.e. $\hat{A}_{nl} = \hat{L} * \overline{\hat{A}}_{nl}$. Using the previous results we calculate polynomials R, S_l from

$$\hat{R} * \hat{A}_l + \hat{S}_l * \hat{B} = A_d$$

where $\hat{A}_l, \hat{B}$ are the estimated polynomials at time instant t. let $\hat{G} = \gcd(\hat{B}, \hat{L})$, $\hat{B} = \hat{G} * \hat{\bar{B}}$ and $\hat{L} = \hat{G} * \hat{\bar{L}}$. We further compute

$$\hat{B} * \hat{S}_{nl} = -\hat{G} * \hat{R} * \hat{\bar{L}} * \overline{\hat{A}}_{nl}$$

Note that for proper initial conditions we have $\hat{A}_{nl} = \hat{L} * \overline{\hat{A}}_{nl}$ for each estimation. Controller is determined by the expression:

$$\hat{B} * \hat{R}u = B_d u_c - \hat{B} * \hat{S}_l y - \hat{B} * \hat{S}_{nl} y \tag{1.7}$$

STEP 3: We determine the input using the previous feedback formula.

STEP 4: GOTO STEP 1.

The next theorem describes the behaviour of the algorithm:

Theorem 1.3: Consider the system (1.1) and the desired system

$$A_d y^*(t) = B_d u_c(t) \tag{1.8}$$

Suppose the following assumptions hold:

(i) The polynomials A_d, B_d, B are stable and assumptions 1.1 hold.

(ii) We can find a feedback connection of (1.1) of the form $Ru = Tu_c - Sy$ such that R^{-1} is stable. (Its roots are inside the unit circle).

Then the algorithm 1.2 converges, in the sense that:

(i) $\{u(t)\}, \{y(t)\}$ are bounded.

(ii) $\lim_{t\to\infty}[y(t) - y^*(t)] = 0$.

Theorem 1.4: Under the assumptions of theorem 1.3, the linear part of $\hat{S}$ converges to the polynomial A_d, in the sense that:

$$|A_d y(t) - \hat{B} * \hat{S}_l y^*(t)| \to 0 \tag{1.9}$$

where $y^*(t)$ the output sequence of the desired system:

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