Hybrid feedback linearizing control for bounded bilinear dyadic plants

L. del Re
Automatic Control Laboratory – ETH Zürich
CH-8092 Zürich – Switzerland
e-mail: delre@aut.ee.ethz.ch

1. Many physical processes with industrial relevance can be described by bilinear processes [1]. Many more processes can be suitably approximated by bilinear models, using the approximation properties of bilinear series [2]. A generic bilinear model is given as
$$\dot{x} = \mathbf{A}x + bu + \mathbf{N}xu, \quad y = c^{\mathrm{T}}x \tag{1}$$
A special kind of bilinear plants are the so called dyadic bilinear plants, defined as
$$\dot{x} = \mathbf{A}x + b\left(1 + n^{\mathrm{T}}x\right)u, \quad y = c^{\mathrm{T}}x \tag{2}$$
Bilinear dyadic plants are usually obtained as the approximation of a modulated input function, *e.g.* as the flow through throttles:
$$\dot{V} = k_v x_{SV}\sqrt{1 - \frac{\Delta P_L}{p_s}} \approx c_0 \cdot x_{SV}(1 - a_1 x_1) \tag{3}$$

2. Many real plants are characterized by hard bounds on states and or inputs. From the industrial practice point of view, it is of interest to exploit these bounds, as this often means exploiting better the installed power. This means that the input to the system u will have to remain inside a compact set $\Omega_u \subset \mathbb{R}^1$.

3. A similar limitation exists for the state vector x, as normally only a limited region of the state space $\Omega_x \subset \mathbb{R}^n$ is attainable. In the case of bilinear dyadic plants, this region should exclude the manifold defined by $1 + n^{\mathrm{T}}x = 0$. Happily this is often the case with man-made systems, as otherwise the system would lose its controllability in that manifold. It may be less easy if biological or chemical systems are considered.

4. Much work has been done on the stabilization of bilinear systems[1]. If precise dynamic requirements are to be met, like shape of the step answer, overshooting *etc.*, then forcing the plant to behave like a linear one can greatly simplify the controller design task. Feedback linearization [3] is a well established technique to achieve this goal. Unfortunately, feedback linearization requires the nonlinear model to contain only smooth nonlinearities. Bounds clearly do not belong in this category, and, if feedback linearization has to be used, we have to account separately for them.

5. Feedback linearization of a bilinear plant using a linear ficitious output is always possible [4] iff the plant is dyadic. The feedback - including a pole placement part -can be computed by the classical formula

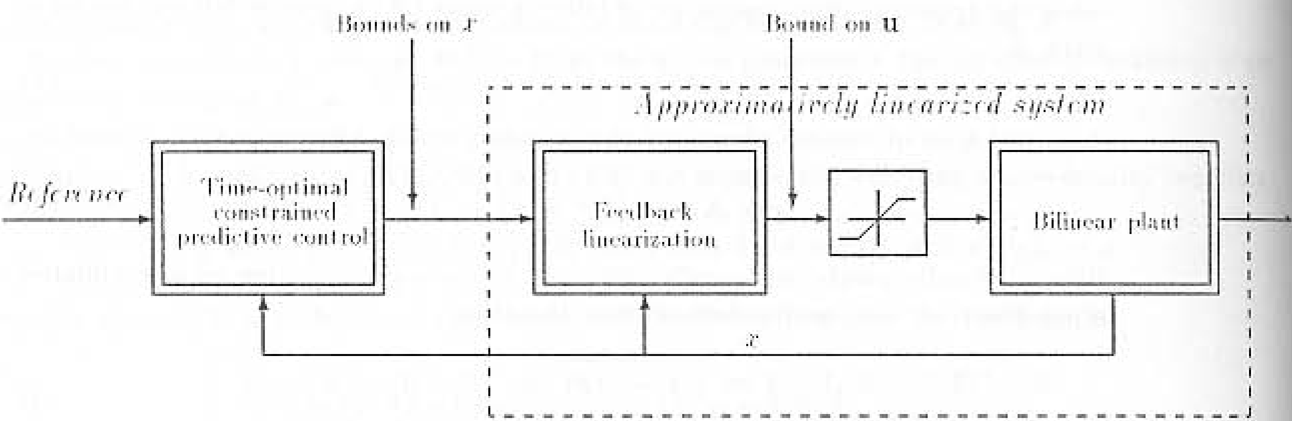
$$u = \frac{r - L_f^n h(x) - c_0 h(x) - c_1 L_f h(x) - c_1 L_f^2 h(x) - \ldots - c_{n-1} L_f^{n-1} h(x)}{L_g L_f^{n-1} h(x)} \tag{4}$$

where $c_0, \ldots, c_{n-1}$ are the roots of the characteristic polynomial of the wished A matrix of the linearized system.

Unfortunately, the hard bounds may even cause the system to become unstable [5].

6. The treatment of the bounds of the composed system is not very easy, as the limits on the reference input applied to the system are a function of the hard bounds but also of the state and of the state feedback applied to the linearized system. In the field of predictive control, however, very efficient, alas time-consuming, algortihms have been developed.

7. This paper proposed a mixed approach as shown in the following picture



Our approach to retain both the linearity induced by the feedback linearization but to avoid saturation consists in using locally a feedback linearization scheme and designing a time optimal control for the linearized system with its global constraints, which are a (linear) function of the state and the original constraints on the physical input to the actual plant.

8. In order to limit the actual activity of the predictive control to the necessary cases, it is designed as a time-optimal control, which delivers a one-step solution each time this does not conflict with the bounds and the most relevant condition on $n^T x + 1$, which is also accounted for explicitely.

9. The basic approach taken is a slight modification of the procedure of [6], exploiting the knowledge on the special structure of the boundary conditions and the system nature to reduce the computing burden.

10. As any such law must be implemented digitally. If a continuous implementation of the feedback linearization is wished, some more assumptions on the kind of the local feedback are necessary to insure respect of the tresholds. On the other side, if the feedback linearization is implemented in an approximative way in the form of a discrete controller, only approximately linear behaviour is obtained, but the limits to the basic plant can be conserved.

11. Appropiare choice of pole placement inside the loop may greatly simplify the design task of the predictive control. Corresponding conditions on the feedback linearized system are given.

12. A simulation study is presented to analyze the performance of the closed loop system. Conclusions and an outlook close the paper.

References

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