OUTPUT TRACKING FOR A FAMILY OF LINEAR SYSTEMS

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KEYWORDS Simultaneous design; Parameter dependent families of linear systems; Periodic controllers; Dead-beat controllers

ABSTRACT In this paper we consider continous-time, linear system Σ described by a set differential equations of the form

$$\Sigma = \begin{cases} \dot{x}(t) = A(\beta)(t)\, x(t) + B(\beta)(t)\, u(t) \\ y(t) = C(\beta)(t) \end{cases}$$

where the coefficient matrices depend on a vector of parameters β, taking value in a finite set $\Theta = \{\beta_1,.....,\beta_n\}$. The system Σ can be thought to model, for instance, a plant subject to sudden changes due to the failure of some components or it can represent the family of linearizations around different operating points of a single non linear plant. An important control problem for systems of this kind consists in designing regulators that perform a desired control action for all possible values of β. in particular, the Robust Output Tracking Problem, or ROTP, that is considered here consists in finding a controller for Σ that enables the compensated system to track a reference signal with zero steady-state error after a finite time, independently of the actual value of the parameter β.

SL-AMP-DC/OutputTrackIEEEMedConf(abs)/1/11/93

From a theoretic point of view, a solution for the ROTP has been proposed in [1], under the assumptions of existence of an internal model, and of controllability and dectectability of each $\Sigma(\beta_i)$. The method followed in [1] extends previus works by Kargonekar et al. ([2]) and Olbrot ([3]) and it produces a controller Σ_C in form of a periodic linear systems. More precisely, the controller Σ_C is constructed by means of a set of discrete-time, dead-beat, linear compensators Σ_C^i , one for each value of the parameter β in Θ, with Σ_C^i achieving the regulation of the system $\Sigma(\beta_i)$ in a finite time. The construction of Σ_C^i as a discrete-time, dead-beat controller is made possible by a suitable choice of the discretization step, in such a way that a ripple-free condition is satisfied. The control action consists in applying one after the other, over a sufficiently long finite time interval, each Σ_C^i. If the actual value of β coincides with β_i, in the time interval $k_{i-1} \leq k < k_i$, in which the compensator Σ_C coincides with the dead-beat controller Σ_C^i, the control task is achieved and the tracking error, as well as the state of Σ_C, go to zero. Hence, for every time instant greater than k_i, the control action of Σ_C is null and the tracking condition is satisfied.

The key point in implementing the above control procedure consists in constructing for each value β_i of the parameter a compensator Σ_C^i that, when connected with $\Sigma(\beta_i)$, brings the regulation error and its internal state to zero in a finite time. Essentially, this amounts to the problem of computing, for a given controllable pair (A,B), a feedback gain matrix K such that all the eigenvalues of the closed loop matrix (A + BK) are zero. Actually, since in general the number of eigenvalues exceeds the number of columns in B, this problem is numerically ill-conditioned. As a consequence, due to possible uncertainties in the coefficients of the matrices and to round-off errors, classical algorithms that compute the required feedback gain matrix K with an accuracy of ε (machine accuracy) can only guarantee an accuracy of $\varepsilon^{1/\nu}$, ν being the controllability index of the pair (A,B), on the closed-loop eigenvalues. This fact limits the practical applicability of the control procedure described above to the point that it may be of some usefulness only in cases in which the dimension of Σ and the cardinality of Θ are very small.

In this paper, we propose a way for overcoming these difficulties and for enhancing the practical applicability of the control procedure described above. The basic idea we develop consists in using periodic time-varying compensators Σ_C^i for achieving the regulation of the system $\Sigma(\beta_i)$ in a finite time interval. More precisely, the compensator Σ_C^i is designed as the connection of a dead-beat periodic observer with a dead-beat periodic state feedback. Practically, the construction of Σ_C^i requires to find a periodic feedback gain matrix K such that all the eigenvalues of the closed loop matrix (A + BK) are zero. As described in [4], using the degree of freedom offered by the periodicity of K, this can be done with ε-accuracy. In the paper we give an algorithm for the construction of Σ_C^i using two different strategies, namely constructing the periodic feedback from past state or, respectively, from present state, and we investigate the properties and the performances of such compensator. In particular, the periodic compensator is shown by simulation to achieve the regulation task with comparatively little control efforts.

To test the effectiveness of the control procedure described above in a real situation, we consider the ROTP in the case in which Σ represents a family of linearized models of an underwater vehicle (see [5]). In such case, the dimension of Σ is six and the cardinality of Θ is four. The hypothesis of existence of an internal model is satisfied by precompensating Σ in a suitable way, according to the chosen class of reference signals. Accurate simulations show that the controller Σ_C we obtain is capable of solving the ROTP with satisfactory performances for several classes of reference signals.

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