Stabilization of Linear Controllable Systems by Means of Bounded Continuous Nonlinear Feedback Control

Rodolfo Suárez, Julio Solís-Daun and Jesús Álvarez

División de Ciencias Básicas e Ingeniería,
Universidad Autónoma Metropolitana-Iztapalapa, Apdo. Postal 55-534
09340, México, D.F., México.
Phone (5) 7244654 Fax (5) 7244653

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Abstract:

Stabilization of linear systems with bounded continuous stabilizing feedback control is a problem which has received little attention, due to the difficulty of being a nonlinear problem. It is known that any open-loop unstable linear system cannot be globally stabilized by bounded continuous control, and that saturation induces periodic solutions even in the open-loop stable case (see [7]).

The bounded continuous feedback stabilization (BCFS) problem has been addressed by two approaches. One approach is the saturated linear feedback (SLF) : design a globally stabilizing linear feedback and then consider its saturation. The resulting feedback is continuous and piecewise linear. In [7] the problem of determining the set of initial conditions which can be driven to the origin, i.e, the stability region (SR), by means of a SLF control, is considered. The other approach is the bounded continuous nonlinear feedback (BCNF) : design a nonlinear bounded continuous control, which possible does not reach its limits, to solve the problem. In this direction, it has been proved in [6] that any system which eigenvalues have nonpositive real part, can be globally stabilized by a BCNF control. A technique to construct the feedback control has been proposed recently in [9] extending the "nested saturations" approach developed by [8] (specialized to the n-th order integrator).

There are examples of systems (specifically the n-th order integrator [2]) which can be globally stabilized by the BCNF approach but not by the SLF one. The use of a nonlinear feedback control design is suggested by the following argument : as gains of a system with a SLF control are decreased, the system SR is enlarged. So then, it is natural to propose that gains should be designed so that they are small when the system is far from the

origin, and increase when the system approaches the origin. The type of variable-gain control functions which we consider are of the form : $u(x) = k^T(x)\, x$ (where $(\cdot)^T$ denotes transpose).

The design method of a BCNF control proposed in this paper, is based in the approximation technique of controllable sets developed in [1,4]. The controllable sets are internally approximated by means of ellipsoids. The design idea is to define the control function in such a way that the closed-loop vector field points in the direction normal to the ellipsoids. The resulting feedback is bounded, continuous and drives the initial points to the origen in finite time [4]. A redesign of the finite-time stabilizing feedback in a neighborhood of the origen, yields a BCNF control which solves the BCFS problem. This procedure allows us to address the stabilization problem of open-loop unstable systems. The technique is applied to the two-dimensional case and to the n-th order integrator, for which the BCNF control function is globally defined and can be explicitly obtained. An application of this technique to a continuous stirred tank reactor has been presented in [5].

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