NOTIONS OF OUTPUT EQUIVALENCE FOR NONLINEAR SYSTEMS

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EXTENDED ABSTRACT

Inferential control is a popular design method in linear process control [1] and is currently being expanded to nonlinear process control ([2], [3], [4], and [5]). The key idea behind inferential control is to define an auxiliary output to the system that has the following properties:

- It can be measured or estimated on-line
- It goes to its set point if and only if the actual process output goes to its set point

A controller can then be designed for the auxiliary output and this will also bring the actual output to its set point.

The popularity of the inferential control design method in linear process control as well as its recent promising extensions to nonlinear process control necessitate a rigorous mathematical formulation of the method in a general nonlinear setting. The first step in this direction is the precise formulation of the requirement that the auxiliary output "goes to its set point if and only if the actual process output goes to its set point" by introducing notions of output equivalence. Once this is done, one can proceed to precisely formulate the design algorithm in a general setting and investigate issues of stability, tracking, etc. The purpose of the present work is to undertake the first step of this theoretical effort, i.e. rigorously formulate notions of output equivalence for nonlinear systems for the first time.

It is well known that by changing the output, the process zeros (or zero dynamics) and static gain can be affected. Therefore, it is natural to consider two cases:

1) Require the equivalent output to have the same static gain as the primary output, but with possibly different zero dynamics (Statically Equivalent Outputs).
2) Require the equivalent output to have the same zero dynamics as the primary output, but with possibly different static gain (Dynamically Equivalent Outputs).

These two notions have been key in designing controllers for several classes of difficult nonlinear control problems. However, it is possible through the use of either type of equivalent output that the relative order and disturbance decoupling properties of the system with the auxiliary output may be different from those of the system with the primary output. This area has not been previously addressed.

Throughout this work, the system under consideration will be a SISO nonlinear system with two different outputs of the form

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\,u \\
y &= h(x) \\
y^* &= h^*(x)
\end{aligned}
\tag{1}
$$

where $u \in \mathbf{R}$, $x \in X$, with X an open subset of $\mathbf{R}^n$, $y \in \mathbf{R}$ is the process output, and $y^* \in \mathbf{R}$ is an auxiliary output. The vector fields $f(x)$ and $g(x)$ are smooth vector fields on X, and $h(x)$ and $h^*(x)$ are smooth scalar functions on X. Additionally, it is assumed that the input $u:[0,\infty) \rightarrow \mathbf{R}$ is integrable over any finite interval.

<u>Statically Equivalent Outputs</u>

<u>Definition #1</u>: Consider the system (1) and denote by E its equilibrium set:

$$E = \left\{ x \in X \mid \exists\, \lambda \in \mathbf{R} : f(x) + \lambda\, g(x) = 0 \right\} \tag{2}$$

The outputs y and y^* are called <u>statically equivalent</u> if

$$h(x) = h^*(x) \qquad \forall\, x \in E \tag{3}$$

Since the equilibrium set E is unaffected by the choice of output, an immediate consequence of the definition is that locally around every equilibrium point with nonzero gradient, the input / output relationships at equilibrium of the two statically equivalent outputs are the same. Consequently, the static gains of the two statically equivalent outputs are the same.

The general concept of statically equivalent outputs is central to the controllers designed for nonminimum phase compensation for nonlinear processes [4] and deadtime compensation for nonlinear processes [3].

Dynamically Equivalent Outputs

Requiring the same zero dynamics may be enforced in either of two ways. The first way is to force the equivalent output to have the same zero dynamics at every set point.

Definition #2: Consider the system (1). The outputs y and y^* are said to be globally dynamically equivalent if h(x) and $h^*(x)$ have the same level sets, l_{y_o} and $l_{y_o^*}$ respectively, i.e.

$$\forall y_o \in h(X)\ \exists\, y_o^* \in h^*(X) \text{ such that } l_{y_o} = l_{y_o^*}$$

and

$$\forall y_o^* \in h^*(X)\ \exists\, y_o \in h(X) \text{ such that } l_{y_o} = l_{y_o^*}$$

An immediate consequence of the definition is that there is a real function of a real variable φ such that

$$h^*(x) = \varphi(h(x)) \quad \forall x \in X$$

By definition, the zero dynamics is the dynamics of the system when the output is constrained to a constant value [6]. When $y=y_o$ or $y^*=y_o^*$, the states of the system will evolve on the same manifold. Since the choice of output does not affect the state equations, the u-y and u-y^* systems clearly have the same local zero dynamics for every $y_0 \in h(X)$.

The other method by which a dynamically equivalent output may be defined is to require the equivalent output to have the same zero dynamics only at the set point value of the process output. Thus, for each value of the set point, it is possible to have a different equivalent output defined.

Definition #3: Consider the system (1). Given $y_0 \in h(X)$ and $y_0^* \in h^*(X)$, the outputs y and y^* are said to be pointwise dynamically equivalent at y_0 and y_0^*, respectively, if the corresponding level sets are equal, $l_{y_o} = l_{y_o^*}$.

An immediate consequence of this definition is that the local zero dynamics of the u-y and u-y^* systems are guaranteed to be the same at the particular points y_0 and y_0^*.

The notion of dynamic output equivalence allows a nonlinear controller to be designed to provide a nonlinear input/output response in the primary output [7] This is achieved by requiring a linear input/output response in the auxiliary output. For involutive nonlinear systems a coordinate transformation and state feedback may be used to linearize the state equations [8], but the output map will usually remain nonlinear. Under rigorously derived conditions, a linear dynamically equivalent output may exist. In this a case a linear controller may be used in terms of the auxiliary output and the linearized state equations. It is also possible to use standard input / output linearizing state feedback [9] in terms of a dynamically equivalent output for a general nonlinear system.

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