Adaptive Estimation of a Flexible Beam via Piezoceramic Actuation

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ABSTRACT

In this presentation we present and discuss a modification of the adaptive identification theory for second order infinite dimensional systems that we developed in [2] which allows for the identification of an unknown input operator in the model. This extension of our earlier treatment was motivated by a laboratory experiment that we are in the process of designing and carrying out. This experiment, which involves a cantilevered aluminum beam, is being built and performed in collaboration with staff members at the Phillips Laboratory at Edwards Air Force Base in California. The beam will be set to vibrating via moments generated by piezoceramic patches attached to the test article at the root.

In mathematically modeling our flexible beam, we assume that it is of length L (which, without loss of generality, we assume to be unity) with uniform rectangular cross section of height h and width b. We let $u(t,x)$ denote the transverse displacement of the beam at position x along its span at each time t. This is measured relative to the x-axis in the coordinate frame determined by the longitudinal axis of the beam in its undeformed state with origin located at the beam's fixed end. We assume a cantilevered Euler Bernoulli beam with Kelvin-Voigt damping for the modeling of the dynamics and dissipation, see [1]. It is assumed that the beam undergoes only small deformations (i.e. $|u(t,x)| << L$, and $|(\partial u/\partial x)(t,x)| << 1$). The Euler-Bernoulli theory including Kelvin-Voigt viscoelastic damping yields the partial differential equation

$$\rho D_t^2 u + D_x^2 M(t,x) = f(t,x), \qquad 0 < x < 1, \qquad t > 0, \tag{1}$$

with the cantilevered boundary conditions

$$u(t,0) = D_x u(t,0) = D_x^2 u(t,1) = D_x^3 u(t,1) = 0, \qquad t > 0, \tag{2}$$

where ρ is the linear mass density, $M(t,x)$ is the internal moment and f is the external applied force. For an uncontrolled beam with Kelvin-Voigt damping, the moment M in (1) is given by (see [1])

$$M(t,x) = EID_x^2 u + c_D I D_x^2 D_t u,$$

where E is Young's modulus, I is the cross sectional moment of inertia, and c_D is the damping modulus. For actuation, a piezoceramic patch is attached to the beam. This patch is excited in such a way so as to produce a pure bending moment, see [1]. If H_0 is used to denote the Heavyside function with unit step at $x = 0$, the model for the beam is given by

$$\rho D_t^2 u + D_x^2 \left\{ EID_x^2 u + c_D I D_x^2 u_t \right\} = D_x^2 \left\{ EI \frac{K^B d_{31}}{\tau} g(t)[H_0(x - a_1) - H_0(x - a_2)] \right\}, \tag{3}$$

where $g(t)$ is the voltage applied to the patch. K^B is a parameter which depends on the geometry and piezoceramic materal properties, τ is the patch thickness, a_1 and a_2 denote the length and position of the patch and d_{31} is the piezoceramic strain constant (see [1]).

We let H be $L_2(0,1)$, $V = H_L^2(0,1) = \{\varphi \in L_2(0,1) : D_x\varphi$ absolutely continuous with $D_x^2\varphi \in L_2(0,1)$, $\varphi(0) = D_x\varphi(0) = 0\}$, and $Q = R^3$. We then rewrite (3) together with the boundary conditions (2) in weak form as

$$\langle D_t^2 u(t), \varphi\rangle + b(q; D_t u(t), \varphi) + a(q; u(t), \varphi) = c(q; f(t), \varphi), \qquad \varphi \in V, t > 0, \tag{4}$$

$$u(0) = u_0, \qquad D_t u(0) = v_0, \tag{5}$$

where $q \in Q$, $u_0 \in V$, $v_0 \in H$, and $f \in L_2(0,T;H)$, for all $T > 0$. The forms $a(\cdot;\cdot,\cdot)$, $b(\cdot;\cdot,\cdot)$ and $c(\cdot;\cdot,\cdot)$ in (4) are given by

$$a(q;\varphi,\psi) = \int_0^1 q_1 D_x^2\varphi D_x^2\psi dx, \quad \varphi,\psi \in V,$$

$$b(q;\varphi,\psi) = \int_0^1 q_2 D_x^2\varphi D_x^2\psi dx, \quad \varphi,\psi \in V,$$

$$c(q;\varphi,\psi) = \int_0^1 q_3\varphi D_x^2\psi dx, \quad \varphi \in H, \quad \psi \in V,$$

where $q_1 = \frac{EI}{\rho}$, $q_2 = \frac{c_D I}{\rho}$, and $q_3 = \frac{EIK^B d_{31}}{\tau\rho}$. It is our intention to use the modified on-line identification scheme to be outlined below to estimate the parameters q_1, q_2, and q_3 in (4).

Following [2], for $q \in Q$, let $A(q) : V \to V'$, $B(q) : V \to V'$, and $C(q) : H \to V^*$ be the linear operators defined by the bilinear forms $a(q;\cdot,\cdot)$, $b(q;\cdot,\cdot)$, and $c(q;\cdot,\cdot)$, respectively, where V' denotes the algebraic dual of V. That is, for $q \in Q$, $\langle A(q)\varphi,\psi\rangle = a(q;\varphi,\psi)$ for $\varphi,\psi \in V$, $\langle B(q)\varphi,\psi\rangle = b(q;\varphi,\psi)$ for $\varphi,\psi \in V$, and $\langle C(q)\varphi,\psi\rangle = c(q;\varphi,\psi)$ for $\varphi \in H$ and $\psi \in V$. We also identify f via $f(t) = g(t)\{H_0(\cdot - a_1) - H_0(\cdot - a_2)\}$.

Definition 1 A *plant* is a triple $(\overline{q}, \overline{u}, f)$ for which there exists a constant $\lambda > 0$ such that

$$|\langle B(p)D_t\overline{u}(t)+A(p)\overline{u}(t)-C(p)f(t),\varphi\rangle| \leq \lambda|p|_Q\|\varphi\|_W, \qquad \text{for all } t>0, p\in Q \text{ and } \varphi\in V,$$

and $\overline{u}$ satisfies the initial value problem (4), (5) with $q=\overline{q}$.

Given a plant $(\overline{q}, \overline{u}, f)$ we define our estimator for $\overline{q}$ and $\overline{u}$ in the form of the initial value problem

$$\begin{aligned} &\langle D_t^2u(t),\varphi\rangle + b(q^*;D_tu(t),\varphi) + a(q^*;u(t),\varphi) + b(q(t);D_t\overline{u}(t),\varphi) \\ &\quad + a(q(t);\overline{u}(t),\varphi) - c(q(t);f(t),\varphi) \\ &\quad = b(q^*;D_t\overline{u}(t),\varphi) + a(q^*;\overline{u}(t),\varphi), \quad \varphi\in V, t>0, \end{aligned} \tag{6}$$

$$\begin{aligned} &\langle D_tq(t),p\rangle_Q + b(p;D_t\overline{u}(t),\overline{u}(t)-u(t)) + a(p;\overline{u}(t),\overline{u}(t)-u(t)) \\ &\quad -c(p;f(t),\overline{u}(t)-u(t)) + \gamma\{b(p;D_t\overline{u}(t),D_t\overline{u}(t)-D_tu(t)) \\ &\quad +a(p;\overline{u}(t),D_t\overline{u}(t)-D_tu(t)) - c(p;f(t),D_t\overline{u}(t)-D_tu(t))\} = 0, \ p\in Q, t>0, \end{aligned} \tag{7}$$

$$u(0)\in V, \qquad D_tu(0)\in H, \qquad q(0)\in Q, \tag{8}$$

where γ is an appropriately chosen gain.

Setting $\epsilon(t) = u(t) - \overline{u}(t)$ and $r(t) = q(t) - \overline{q}$ we use (4) - (8) to obtain the error equations

$$\begin{aligned} &\langle D_t^2\epsilon(t),\varphi\rangle + b(q^*;D_t\epsilon(t),\varphi) + a(q^*;\epsilon(t),\varphi) + b(r(t);D_t\overline{u}(t),\varphi) \\ &\quad + a(r(t);\overline{u}(t),\varphi) - c(r(t);f(t),\varphi) = 0, \ \varphi\in V, t>0, \end{aligned} \tag{9}$$

$$\begin{aligned} &\langle D_tr(t),p\rangle_Q - b(p;D_t\overline{u}(t),\epsilon(t)) - a(p;\overline{u}(t),\epsilon(t)) - c(r(t);f(t),\epsilon(t)) \\ &\quad -\gamma\{b(p;D_t\overline{u}(t),D_t\epsilon(t)) - a(p;\overline{u}(t),D_t\epsilon(t)) - c(r(t);f(t),D_t\epsilon(t))\} = 0, \quad p\in Q, t>0, \end{aligned} \tag{10}$$

$$\epsilon(0)\in V, \quad D_t\epsilon(0)\in H, \quad r(0)\in Q. \tag{11}$$

We establish convergence of the state estimate (i.e. $\lim_{t\to\infty}\|\epsilon(t)\|_V = 0$ and $\lim_{t\to\infty}|D_t\epsilon(t)| = 0$) and, with the additional assumption of *Persistence of Excitation*, parameter convergence. That is, $\lim_{t\to\infty}|r(t)|_Q = \lim_{t\to\infty}|q(t)-\overline{q}|_Q = 0$. We assume throughout that $(\overline{q}, \overline{u}, f)$ is a plant.

The convergence of the state estimator is established via a Lyapunov-like estimate for the system (9) - (11).

Lemma 2 *If $\gamma > \max\{K_V, K_V/\alpha_0(q^*), K_W^2/\beta_0(q^*)\}$, then there exist constants ρ, $\sigma > 0$ such that for all $t>0$*

$$\|e(t)\|_V^2 + |D_te(t)|^2 + |r(t)|_Q^2 + \rho\int_0^t\left\{\|\epsilon(s)\|_V^2 + \|D_se(s)\|_W^2\right\}ds \leq \xi,$$

where $\xi = \sigma\left\{\|e(0)\|_V^2 + |D_te(0)|^2 + |r(0)|_Q^2\right\}$.

The next theorem establishes the convergence of the state estimate. The proof is in the spirit of the arguments used to verify an analogous result in [2] and [3]. We define what we shall refer to as an energy functional, $E : [0, \infty) \to R^1$, for the system (9), (10) by

$$E(t) = \gamma \left\{ a(q^*; e(t), e(t)) + |D_t e(t)|^2 \right\} + 2(e(t), D_t e(t)) + b(q^*; e(t), e(t)) + |r(t)|_Q^2. \quad (12)$$

Theorem 3 *If* $\gamma > \max\{K_V, K_V/\alpha_0(q^*), K_W^2/\beta_0(q^*)\}$, *then the energy functional* E *given by* (12) *is nonincreasing,* $\lim_{t\to\infty} \|e(t)\|_V = 0$ *and* $\lim_{t\to\infty} |D_t e(t)| = 0$.

In order to establish parameter convergence we require the notion of *Persistence of Excitation.*

Definition 4 A plant $(\overline{q}, \overline{u}, f)$ is said to be *persistently excited*, or the input f is called *persistently exciting* for the plant $(\overline{q}, \overline{u}, f)$, if there exists $T_0, \delta_0, \epsilon_0 > 0$ such that for each $p \in Q$ with $|p|_Q = 1$ and each $t_1 > 0$ sufficiently large, there exists a $\bar{t} \in [t_1, t_1 + T_0]$ such that

$$\left\| \int_{\bar{t}}^{\bar{t}+\delta_0} B(p) D_\tau \overline{u}(\tau) - A(p)\overline{u}(\tau) - C(p) f(\tau) d\tau \right\|_{V^*} \geq \epsilon_0.$$

Theorem 5 *If the plant* $(\overline{q}, \overline{u}, f)$ *is persistently excited then*

$$\lim_{t\to\infty} |r(t)|_Q = 0.$$

REFERENCES

[1] H.T. Banks, R.J. Silcox, and R.C. Smith. The modeling and control of acoustic/structure interaction problems via piezoceramic actuators: 2-D numerical examples. *ICASE* Report No. 92-17. April 1992. NASA Langley Research Center, Hampton, VA.

[2] M.A. Demetriou and I.G. Rosen, Adaptive identification of second order distributed parameter systems, *Inverse Problems*, submitted.

[3] I.G. Rosen and M.A. Demetriou, An on-line parameter estimation scheme for flexible structures, *Proceedings of Conference on Sensing, Identification, and control of Flexible Structures*, Fields Institute, University of Waterloo, Waterloo, Canada, June 28-30, 1992.