SELF-BOUNDED CONTROLLED INVARIANT SUBSPACES FOR SINGULAR SYSTEMS

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ABSTRACT Geometric theory of singular systems has been greatly developed in the last years by several authors [1] [2] [3] [4]. From a methodological point of view, the key points in this processes are represented by extending, from the classical case, the definition of the basic geometric objects, like the Controlled Invariant Subspace and the Conditioned Invariant Subspace and in describing suitable algorithms for their construction. Various applicationg of geometric methods to sinthesys and design problems have then been developed (see the collection of papers in [5] for an updated account).

For nonsingular systems, a number of recent results in the geometric framework have been obtained using the notion of self-bounded controlled invariant subspace and of self-hidden conditioned invariant subspace introduced in [6]. In particular such concept have been proved to be useful in the solution of noninteracting control problems with stability requirements (see [7]).

The aim of this paper is to extend to the framework of singular systems the notion of self-bounded controlled invariant subspace mentioned above, to investigate its dynamical properties and to study its possible applications to noninteracting control problems.

The systems we consider are discrete-time linear systems described by equations of the form

$$\Sigma = \begin{cases} Ex(t+1) = Ax(t) + Bu(t) \\ y = Cx(t) \end{cases}$$

where $x \in X = R^n$, $u \in U = R^m$, $y \in Y = R^p$, and A, B,C, E are matrices of suitabl dimensions, E is singular and such that rank(sE - A) = n, i.e. the system is solvable anc conditionable. For a system of the above kind, we introduce the following Definition.

<u>DEFINITION</u> Let $V \subset L$ be twosubspaces of the generalized state space X of Σ.Then, V i called a *self-bounded controlled invariant subspace with respect to* L if and only if
i) V is a controlled invariant subspace for Σ, that is $AV \subset EV + \mathrm{Im}B$, and
ii) $EL \cap \mathrm{Im}\, B \subset EV$.

Under the hypothesis that $EL \cap \mathrm{Ker}\, E = \{0\}$, the basic property of the self-boundec controlled invariant (briefly SBCI) subspaces with respect to L that we prove is the fact tha they are closed with respect to trajectories starting in them and evolving in L. For noninteractin control problems, it is therefore of interest to investigated the class of the SBCI subspaces wit respect to the maximum controlled invariant subspace V^* of Ker C, that will simply be callec SBCI subspaces. The basic result we obtain is given by the following Proposition.

<u>PROPOSITION 1</u> Assume that the condition $EV^* \cap \mathrm{Ker}\, E = \{0\}$ holds. Then, the class o SBCI subspaces is closed with respect to subspace intersection.

An obvious consequence of the above Proposition is the fact that the class of SBC subspaces that contain a given subspace S has a mimimum element. One can then ask for suitable geometric procedure for constructing such minimum element. Differently from wha happens in the nonsingular case (compare with [6]), it does not seem useful in this case t employ a combination of the Controlled Invariant Subspace Algorithm and of the Conditionec Invariant Subspace Algorithm in their generalized form. The methods we employ consists firs in showing that every feedback F that makes V^* (A + BF, E)-invariant, i.e. such that (+BF)$V^* \subset EV^*$, has the same property also with respect to any SBCI subspace. Then we hav the following Proposition.

<u>PROPOSITION 2</u> Given $S \subset V^*$, let F be such that $(A + BF)V^* \subset EV^*$. Then, the minimun SBCI subspace containing S coincides with the minimum (A + BF, E)-invariant subspace of X

Since the minimum (A + BF, E)-invariant subspace of X can be found as the limit of th sequence $S_{i+1} = S_i + (E^{-1}(A + BF)S_i \cap V^*)$; $S_0 = S$, the above Proposition does provide procedure for determining the minimum SBCI subspace containing S.

The case in which $EV^* \cap \mathrm{Im}\, B = \{0\}$ is of particular interest, since such conditio guarantees, for any $S \subset V^*$, the existence of a minimum controlled invariant subspace contain S. In the design of noninteracting control strategies, the use of such minimum subspace ma provide additional degrees of freedom. In the disturbance decoupling problem, for instance, a straighforward geometric approach gives as solution a feedback that makes the compensated system as much unobservable as possible. Actually, a smaller modification of the system dynamics may suffices to solve the problem, since what is really needed is to make unobservable the miminum controlled invariant subspace containg the image of the disturbance. In the paper, the possible role of SBCI subspaces for singular systems in this and in similar noninteracting control problems will be investigated, with particular regard to problem characterized by stability requirements.

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