SELF-TUNING FUZZY LOGIC CONTROL WITH A SINGLE FUNCTION

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ABSTRACT

A general fuzzy logic controller (FLC) with a filtered tracking error $r = \dot{e} + \Lambda e$ is proposed to control a class of dynamical systems with unknown dynamics. Consider a nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2, t, u) \\ y = g(x_1, x_2) \end{cases} \tag{1}$$

where $x_1, x_2 \in R^n$ is the state variables; $u \in R^r$, $y \in R^m$ are system input and output, respectively. We assume the input/output performance of system (1) is governed by the relationship $u \geq 0 \Rightarrow \Delta x_2 > 0$ for some $x_1 \in [x_1(t_{i-1}), x_1(t_i)]$, $x_2 \in [x_2(t_{i-1}), x_2(t_i)]$, $u \geq 0 \Rightarrow \Delta x_2 < 0$ for some other $x_1 \in [x_1(t_i), x_1(t_{i+1})]$, $x_2 \in [x_2(t_i), x_2(t_{i+1})]$, and vice versa. If the input/output relationship of system (1) is piecewise monotonic, then this kind of dynamic system is called a "Newton's-law" or "Newton's-law-like" system. As a matter of fact, almost all of the practical systems fall into this category, for instance, the robot, aircraft, space shuttle, automobile, and so on. A common feature of this class of systems is that even if one does not know exactly how much control energy must be put into the system to track the desired trajectory, one does know that what kind of control actions (i.e. increase or decrease the control energy, change a little or change a large amount) should be taken to make the system output follow the desired trajectory.

If there is perfect knowledge of the system dynamics and all the state variables are observable, then some kind of conventional controller, say full-state feedback linearization plus PD control, may handle this problem very well. Otherwise, new control technique must be involved and an estimator might be built to deal with unknown dynamics and unobservable states. On the other hand, the "Newton's-law-like" system is easily controlled by using fuzzy logic control (FLC) even if the precise system dynamics are unknown. The reason is evident. First, FLC is basically an error-based control. Its performance does not depend on, or depends very slightly on, the detailed dynamics of the plant. This property makes it suitable to control uncertain, or ill-understood dynamic system. Secondly, the information needed by FLC is the external behavior of the system, i.e. the output signals. FLC treats any dynamic system as a black box with input/output channels which are used to communicate to the outside world. It does not care what is inside the box. If the internal state variables are measurable, it is fine. If they are not measurable, that is still suitable as long as the output signal is measurable (of course!).

Usually the condition-action rules in FLC are designed based on a heuristic of the form

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Change input according to the extent of the tracking error and its rate of change.

That is, the fuzzy control signal is a nonlinear function of the tracking error and its derivative, $u_{FLC} = \phi(e, \dot{e})$. A complete rule set in the $e - \dot{e}$ plane consists of 49 condition-action rules. If some other rules based on the information of measurable internal state variables are added to the rule base (note that the controlled system may be piecewise monotonic), the whole rule base may become huge. Observing the rule-matrix in the $e - \dot{e}$ plane reveals that the rule base can be built on a one-dimensional line instead of in a two-dimensional plane, as follows.

Define a filtered tracking error $r = \dot{e} + \Lambda e$ where Λ is a positive definite constant matrix. It is known that if the filtered tracking error r is bounded, then e is bounded. Especially if r goes to zero then e would go to zero too. If a rule base is designed based on the information in r, then one complete rule set contains only 7 rules. In addition to the reduction in the size of the rule base, automatically adjusting (or tuning) the controller parameters (i.e. the definitions of membership functions) becomes easier and more possible.

An interesting point is that if the L-R fuzzy number r_i, i=1,2,...,7, is represented by a symmetric triplet (r_{iL}, r_{iM}, r_{iR}) (a symmetric triangle) with $r_{iR}-r_{iL}$=constant, and if a centroid formula is applied in defuzzification procedure, then it is proved in this paper that the FLC is nothing but a PD controller with constant PD gains. Though this result gives an insight into the relationship between the fuzzy control and the PD control, a question may arise: if FLC is equivalent to D, why should we use fuzzy control instead of conventional PD control? As mentioned before, a PD controller can provide a good result if the system dynamics are known and all the state variables are measurable. However, even in that case, there is still a conflict of interests between the requirements of fast-response and minimum-overshoot. It is very difficult to design a PD controller with constant PD gains to meet both the response and overshoot requirements. On the other hand, if the membership functions are appropriately designed for the filtered tracking error r, then the fuzzy controller works like a PD controller with variable PD gains. The value of the variable PD gains depends on the magnitude of r. It is the variable gains that make the closed-loop system respond quickly with almost-zero-overshoot even though exact knowledge of the system dynamics is not available.

Another benefit may be gained from the result is that some sophisticated techniques behind PD controller design can be used in the design of FLC. For instance, the stability analysis of a FLC system has given designers bad headaches for a long time. Because it is proved here that a FLC based on the one-dimensional filtered tracking error r is equivalent to a conventional PD controller with variable PD gains, the Lyapunov's direct method, which is a very powerful tool in nonlinear control system analysis and design, can be eventually used in fuzzy logic controller design.

In this paper, we use Lyapunov techniques to design FLC for unknown dynamic systems. The tracking errors are expressed in terms of the control parameters, which gives us the direction in which to automatically adjust the parameters on-line. This FLC is used, associated with a reduced-order-computed-torque controller (ROCT), in the control of a two-link flexible robot arm, to command the rigid modes to track the desired trajectories while maintaining the residual vibration close to zero.