Optimal ℓ^∞ to ℓ^∞ Estimation: a Model Matching Approach

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ABSTRACT

Worst case estimation is an alternative approach to stochastic estimation when statistical information about the uncertainty is not available. The subject of worst case estimation for linear systems has been treated by several researchers and is often related to the advances in robust control. The reader is referred to [1,2,11,13] and references therein where, the subject of worst case estimation is treated in the presence of energy (ℓ^2 or $\mathcal{L}^2$) bounded input uncertainty with the objective to minimize the worst case energy of the estimation error. Also, the case where the noise is magnitude bounded and the objective is to minimize the worst case magnitude of the error, is treated in [3,9,12,10] and references therein. In particular, in [3] Euclidean norms for the magnitude are considered and the authors present a recursive algorithm (not necessarily optimal) with similar structure to Kalman filters. In [9, 12] optimal algorithms are presented for pointwise estimation problems where the uncertainty is magnitude (ℓ^∞) bounded. More specifically, these algorithms are obtained by solving finite dimensional linear programs; also, time varying bounds on the magnitude of the noise can be handled. However, these algorithms are not recursive and cannot be easily implemented when the ammount of data is large, and in particular, for infinite horizon problems.

In this paper we consider first the infinite horizon problems of optimal filtering, smoothing and prediction in discrete-time, linear-time-invariant systems (LTI), stable or unstable, when the sources of uncertainty are ℓ^∞-bounded process and observation noise together with uknown (but bounded) initial conditions. We set up these problems as (2-block) model

matching problems [8] over ℓ^∞-bounded operators. In the case where the initial condition is known, the resulting model matching problem involves time invariant operators. Hence, a recursive suboptimal (arbitrarily close to optimal) estimator can be produced by solving a standard ℓ^1-optimization [4]. In the case where the initial condition is not known the resulting model matching problem is time varying. Yet, these time varying operators have a specific structure that is being exploited. A suboptimal solution consists of utilizing the suboptimal known-inital-condition (KIC) estimator after some *apriori* computable time index N which depends on the KIC solution, while up to time N the solution of $N+1$ finite dimensional linear programs is required to construct the suboptimal estimates. This time index N, represents the time that takes the suboptimal KIC estimator to make the error that is due only to initial conditions very small. Also, conditions are given under which the suboptimal KIC filter is also suboptimal in the presence of unknown initial conditions; this, of course, would be the case whenever the initial condition is relatively small so that it does not affect the worst case estimation error. Simulation results are also presented which compare the performance of these estimators with that of Kalman filters.

Also in this paper, we consider the problems of optimal ℓ^∞ to ℓ^∞ filtering, smoothing and prediction in the case of discrete-time periodic systems. Via the use of lifting transformations it is shown how to transform these problems to equivalent LTI. The so-obtained model matching problem however, entails additional causality constraints [5] on the estimator which complicate substantially the problem. It is shown how to appropriately integrate these constraints to the previous (unconstrained) problem and thus obtain the solution.

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