# OBSERVER BASED CONTROL OF CULTURE FERMENTATION PROCESSES

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*Abstract*:

This paper deals with the problem of controlling a nonlinear culture fermentation process by computer, with the knowledge of only the input and the output of the system. This becomes possible by using an observer in the closed-loop system and proving that the separation principle holds.

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# 1.Introduction

We consider here a culture fermentation process which takes place in a bioreactor where microorganisms grow by eating a substrate, the feed of the reactor. Our purpose is, to control this process such that the production of biomass reaches a prescribed level. However, the concentrations of the biomass and the substrate (which represent the state vector of the system), are not known exactly, thus preventing us from using a control law which is a function of these variables. We propose to construct an observer to overcome this problem and implement the control law using the output of the observer.

# 2. Model and control law

The mathematical model of the culture fermentation process is the following ([3]) :

$$\begin{aligned} \dot{s} &= -\frac{\mu(s)}{R}x + D(s_0 - s) \\ \dot{x} &= (\mu(s) - D)x \\ y &= s \end{aligned} \tag{2.1}$$

where s and x are the substrate and biomass concentrations respectively, D is the dilution rate, usually regarded as the input of the system, $s_0$ is the feed substrate concentration, R is the biomass yield, a parameter of the system and $\mu(s)$ is the cell growth. We assume that the cell growth is described by the law of Herbert:

$$\mu(s) = \overline{\mu}\frac{s}{k+s} - d \tag{2.2}$$

where $\overline{\mu}$ and k are characteristic parameters of the process and d is the cell death rate. Since the objective is to design a control law which will steer the biomass concentration to some reference value $x_r$, we make the following change of variables: we define $\tilde{x} = x - x_r$. Then our system becomes:

$$\dot{s} = \frac{\mu(s)}{R}(\tilde{x}+x_r) - D(s_0 - s) \tag{2.3}$$

$$\dot{\tilde{x}} = (\mu(s) - D)(\tilde{x}+x_r)$$

$$y = s$$

We also normalize x by R, the yield, in order to simplify the calculations, i.e. $x \equiv \frac{x}{R}$.

Note also, that since $u \equiv D$, the system (2.1) belongs to the general class of systems :

$$\dot{X} = A(u,y)X + B(u,y) \tag{2.4}$$

$$y = CX$$

where A(u,y) and B(u,y) are matrices with elements some nonlinear functions of u and y and X is the state vector. If we assume in this stage that the <u>whole state vector is measurable</u> then,

PROPOSITION 2.1 :

*The control law* $D=(x+s-s_0)^2$ asymptotically stabilizes the system (2.3) to the equilibrium point ( $\tilde{x}=0$, $s=0$).

## 3.The observer

The observer we will use is the one proposed by Hammouri-DeLeon ([4]), concerning systems of the form (2.4). It is well-known that for nonlinear systems the property of observability is input-dependent. But since our system of culture fermentation is observable for any input applied, by nature, we can use this observer for every admissible input. Then we have:

PROPOSITION 3.1:

*The system*

$$\dot{z} = A(u,y)z + B(u,y) - S^{-1}C^{T}(Cz-y) \tag{3.1}$$

$$\dot{S} = -\theta S - A^{T}(u,y)S - SA(u,y) + C^{T}C$$

*is an observer for system (2.1) in the sense that*

$$\|z(t) - X(t)\| \leq Me^{-\theta t}$$

*for $\theta$ large enough.*

The proof can be found in [4].

## 4.Observation and Control

The final and most important part is, to prove that we can implement the control law of proposition (2.1) using the output of the observer instead of the actual X. For this we have to prove the

THEOREM 4.1:

*If the system (2.3) is stabilized around $x = x_r$ by the control law $u = D = (x + s - s_0)^2$, then the composite system:*

$$\dot{z} = A(u,y)z + B(u,y) - S^{-1}C^{T}C\epsilon \tag{4.1}$$

$$\dot{\epsilon} = (A(u,y) - S^{-1}C^{T}C\epsilon$$

$$\dot{S} = -\theta S - A^{T}(u,y)S - SA(u,y) + C^{T}C$$

*is globally asymptotically stable for $\theta$ large enough.*

The proof is based on the following three lemmas:

LEMMA 4.2: *The matrix $S(t)$ is bounded for $\theta$ large enough.*

LEMMA 4.3: *For $S(t)$, $\lim_{t \to +\infty} S(t) = S_\infty$, where $S_\infty$ is the unique solution of: $0 = -\theta S_\infty - A^T(u,y)S - SA(u,y) + C^T C$*

LEMMA 4.4: *The composite system (4.1) is locally asymptotically stable at the point $(0,0,S_\infty)$ by the feedback law $D=(x+s-s_0)^2$.*


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