Chaotic behaviours of a two revolute joint robot controlled with a PD algorithm

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Introduction

A two revolute joint robot is a four-dimensional dynamical system, which has many sources of non-linearities. Moreover, it can be programmed for executing repetitive tasks, which play the role of periodical excitations in this system. So, it is in the situation of producing complex dynamical behaviours such as : subharmonic, higher harmonic and fractional harmonic resonances, or synchronisation, and also quasi periodic and chaotic motions [Mir 87b] [Mir 90] [Mah 92].

This paper presents the results of simulation of a SCARA robot controlled with a PD algorithm, which can give rise to such complex behaviours. Three different numerical tools have enabled the analysis of global behaviours of this non-linear mechanical articulated system making repetitive and periodical tasks [Str 89] [Vak 90].

Modelisation

Using the Lagrangian formulation, the model is established in the state space in the following form [Cra 89] [Lop 88]:

$$
(\mathrm{I})\quad
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = \dfrac{1}{p_3(x_3)}\left[\begin{array}{l} K_3(x_3)\times\left(K_2(x_3)x_2^2 + N_2\left(2x_2x_4 + x_4^2\right)\right) \\ +N_2(\Gamma_1(t) - f_{v1}x_2) - K_2(x_3)(\Gamma_2(t) - f_{v2}x_4) \end{array}\right] \\
\dot{x}_3 = x_4 \\
\dot{x}_4 = \dfrac{1}{p_3(x_3)}\left[\begin{array}{l} -K_3(x_3)\times\left(K_1(x_3)x_2^2 + K_2(x_3)\left(2x_2x_4 + x_4^2\right)\right) \\ -K_2(x_3)(\Gamma_1(t) - f_{v1}x_2) + K_1(x_3)(\Gamma_2(t) - f_{v2}x_4) \end{array}\right]
\end{cases}
$$

where :

$$x_1 = \theta_1 \quad x_2 = \dot{\theta}_1 \quad x_3 = \theta_2 \quad x_4 = \dot{\theta}_2$$

$$
\begin{aligned}
K_1(x_3) &= (m_1 + m_2)d_1^2 + m_2 d_2^2 + 2m_1 m_2 d_1 d_2 \cos(x_3) \\
K_2(x_3) &= m_2 d_2^2 + m_2 d_1 d_2 \cos(x_3) \\
K_3(x_3) &= m_2 d_1 d_2 \sin(x_3) \\
N_2 &= m_2 d_2^2 \\
P_3(x_3) &= (m_1 + m_2)d_1^2 m_2 d_2^2 - (m_2 d_1 d_2)^2 \cos^2(x_3)
\end{aligned}
$$

and ,

m_i : mass of link i

d_i: length of link i

$\Gamma_i(t)$: torque applied to the joint i

f_{vi}: viscous friction coefficient of the joint i

PD algorithm

PD control law has the following basic form for the joint i [Dom 88] [Can 92]:

$$\Gamma_i(t) = K_{pi}(\theta_{di}(t) - \theta_i(t)) + K_{vi}(\dot{\theta}_{di}(t) - \dot{\theta}_i(t))$$

Global behaviours relative to the values of the parameters K_{pi} and K_{vi} are analysed. By making (K_{p2}, K_{v2}) dependent on (K_{p1}, K_{v1}), it is shown that the number of parameters can be reduced to only the pair (K_p, K_v)

Results in the phase space

The dynamical system is excited with periodical inputs $\theta_{di}(t)$ corresponding to a repetitive task in the joint space $(0, X_0, Y_0, Z_0)$. These excitations are :

$$\theta_{d1}(t) = A_1 \sin(\omega_1 t) \quad \theta_{d2}(t) = A_2 \sin\left(\frac{\omega_1}{n} t\right), n \in N$$

Poincaré section :

The Poincaré section consists in sampling the evolution of the variable $x_i(t)$ with a sampling period τ of the external excitation : $\tau = \dfrac{2\pi}{\omega_1}$

This section generates the associated map T :

$$\begin{cases} R^4 \rightarrow R^4 \\ X(n) \rightarrow X(n+1) = \varphi(X(n), K_p, K_v, \tau) \end{cases}$$

where $X = (x_1, x_2, x_3, x_4)$ and $\varphi(X(n), K_p, K_v, \tau)$ is the solution of the differential system (I) for the sampled times $t=(n+1)\tau$.

An order k cycle of T (corresponding to an order k subharmonic for (I)) is defined by :

$$X(n+k) = T^k(X(n)) = X(n) \quad , \quad X(n+r) \neq X(n)\ ,\ r < k$$

The Poincaré section of Fig 1 shows a chaotic behaviour (strange attractor) for a given choice of (Kp, Kv). $K_p=10$, $K_v=7$, $A_1=90°$, $A_2=45°$, $\omega_1=2\pi$, n=2.

Results in the parametric plane

For the same imposed task, regions of the parametric plane (Kp,Kv) corresponding to stable subharmonic and chaotic behaviors, are identified.

Two methods are used. The first one consists in an automatic scanning of the parameter plane, which identifies regions associated with a subharmonic of a given order. The second method is related to the numerical determination of the bifurcation curves (boundary of the above regions) by using their mathematical definition.

a- Scanning

For a given initialisation of x_i (i=1,...,4) the scanning of the (Kp, Kv) parameters plane shows that different subharmonic behaviours may occur. It is worth of note that this tool does not permit to take into account any perturbation.

Fig 2 represents the seven first attractive cycles subharmonics for the initialisation $x_1(0)=x_2(0)=x_3(0)=x_4(0)=0$, each order having an associated color.

b- Bifurcation curves

A bifurcation being related to a qualitative change of the system behaviour, a bifurcation curve, in the parametric plane, is a locus where such phenomena occur. The paper gives only a first set of results limited to low order k=1,...,7.

Information furnished by the bifurcation curves will enable to determine regions of multi-stability (simultaneous existence of several attractors of different orders) and to predict the evolution of the system if parameters are changed during the motion.

Conclusion

The study of behaviours in the phase space and in the parametric plane allows the determination of the different sets of parameters Kp and Kv values permitting the definition of an imposed repetitive task.

Keywords
Robotics, chaos, PD controller, Poincaré section, bifurcation.

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