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Structural Properties of Inverse Linear Systems

by

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Abstract:

Inverse systems have long held an interest (Rosenbrock 1970; Rosenbrock & Van Der Weiden, 1977) for the design of linear systems. Rosenbrock (1970) first noted a form of polynomial system matrix realisation of the inverse of a square invertible transfer function matrix G(s) which was derivable directly from a polynomial realisation of the original transfer function matrix. The relationship between certain properties of the inverse system and the given system could then be easily described (Rosenbrock & Van Der Weiden, 1977). Thus, for example, it was established that the finite pole/zero structure of the inverse system was isomorphic to the finite zero/pole structure of the original system, with their finite decoupling zero structure being identical. The first part of the proposed paper establishes that such connection between the structure of any polynomial system matrix and that of a polynomial system matrix realisation of its inverse transfer function matrix is much more deeply rooted. Specifically it will be shown that the results extend in the generalised sense of being applicable not only to the finite frequency aspects but also to the infinite frequency structures. Thus for example the infinite decoupling zero structure of the inverse system is isomorphic to that of the original system, and the infinite transmission and system zeros of the inverse are identical to the transfer function and system poles at infinity of the original system, and vice versa.

Subsequently Kailath (1980) indicated that the same form of equivalence which relates two polynomial realisations of a given G(s) is induced between the derived

polynomial realisations of the inverse of G(s). Naturally, since only the finite frequency behaviour was of concern, the equivalence particularly considered was Rosenbrock's notion of strict system equivalence as formulated by Fuhrmann (1977). It will be established that this relationship extends to the notion of full system equivalence (f.s.e.) (Hayton et al., 1990) which is the basic underlying notion of equivalence for the generalised study of linear systems (Pugh et al., 1992). A further interesting connection was also noted by Kailath (1980) which concerns a form of polynomial system matrix originally suggested by Morf (1975). This system matrix in one sense can be viewed as the inverse of an extended or *generalised* transfer function matrix of G(s), but in another sense it is simply another realisation of the inverse of G(s). The type of equivalence induced on this form of system matrix by (f.s.e.) will also be determined.

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