Title: A polynomial solution to the scalar 4-block general distance problem

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Abstract:

A 4-block general distance problem is solved for the scalar case with the system represented in polynomial form. The problem arises from the H-infinity mixed sensitivity with measurement noise (MS+MN) problem. Thus the formulation used for the 4-block problem is chosen to retain some of the structural properties of the 4-block problem originating from a feedback problem such as the MS+MN.

The solution procedure employs a reduction technique resulting in a (one block) weighted Nehari problem. For this reduction the equations of Kwakernaak(87) (see also Boekhoudt(88) for proofs) are used. These equations are preferred to those of Doyle(84) because they involve a scalar spectral factorisation problem. Thus two spectral factorisations are required.

The weighted Nehari problem must then be solved. Normally to solve this problem a further spectral factorisation is required. If however only equalising solutions are sought, the structure of the problem may be exploited so that no extra factorisation is required. The resulting equation requires the solution of a linear homogeneous overdetermined set of equations.

The solution procedure therefore requires solving two spectral factorisations and one linear polynomial equation per (gamma-) iteration. A bisection method may be used for the method of iteration to obtain the optimal cost.

The proof for these results relies on ideas from Kwakernaak(85). Useful insights are also provided by the theory of Fuhrmann(91).

An advantage of the proposed solution to the SISO 4-block problem is that there are simplifications compared with the solution procedure of Kwakernaak(87) or of Doyle(84), resulting in savings of one spectral factorisation. Also all the polynomial equations used in the solution are scalar with the most demanding one being the spectral factorisation, for whose solution numerically reliable algorithms exist. An alternative polynomial procedure that could be used here is the J-spectral factorisation approach (Kwakernaak 90). Although this is more general than the proposed method and theoretically very attractive, it requires the solution of J-spectral factorisation equations. These are inevitably multivariable (2x2 in this case) and the numerical solution is not without problems at the present stage of development.

References

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