Robust continuous-time tracking and regulation for multirate sampled-data systems

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The problem of the asymptotic tracking and disturbance rejection of a linear multivariable system subject to unmeasurable disturbances was studied by many authors — see, e.g., [1,2,3]), and the references therein —. In most of these contributions it is required the compensator to maintain stability, asymptotic tracking and output regulation in spite of small – or, possibly, large — independent perturbations of all the elements of matrices describing the system. Recently, the problem of the asymptotic tracking and output regulation under uncertainties or perturbations of "physical" parameters affecting the description of the system was solved [4,5,6]. It was shown that robust solutions may exist even when no solution exists for wholly independent variations of the entries of matrices describing the system, i.e. when the Davison condition [1] is not satisfied.

If the problem of the asymptotic tracking is faced for a continuous-time plant making use of a multirate digital control system, the undesiderable ripple which may arise between sampling instants may become unacceptable if the sampling rate are small, and should be avoided. It is known that this can be robustly obtained if a continuous-time internal model of reference signals is included in the forward path of the feedback control system.

Here a method for obtaining such a continuous-time internal model of reference signals and disturbance functions is proposed follwing the same approach as in [4,5,6], for the case when the only uncertainties about the description of the plant concern the values of some "physical" parameters. Such a method allows the control requirements to be robustly satisfied, at least in a neighbourhood of the nominal "physical" parameters of the plant to be controlled, and, in particular, a continuous-time null steady-state error response to be guaranteed for all the values of the physical parameters in such a neighbourhood of the nominal ones.

Consider the linear time-invariant plant P described by

$$\dot{x}(t) = A(\beta)x(t) + B(\beta)u(t) + \sum_{i=1}^{\mu} M_i(\beta)d_i(t),$$

$$y(t) = C(\beta)x(t) + \sum_{i=1}^{\mu} N_i(\beta)d_i(t),$$

where $t \in \mathbb{R}$ is time, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the control input, $d_i(t) \in \mathbb{R}^{m_i}, i = 1,2,...,\mu$, are the unmeasurable and unknown disturbance inputs, $y(t) \in \mathbb{R}^q$ is the output to be controlled — which is assumed to be measurable — and $A(\beta)$, $B(\beta)$, $C(\beta)$, $M_i(\beta)$, $N_i(\beta)$, $i = 1, 2, ...\mu$, are matrices with real entries depending on a vector β of parameters, which are subject to variations and/or uncertain, $\beta \in \Omega \subseteq \mathbb{R}^h$, and play the role of the "physical" parameters of the plant. The nominal value β_0 of β is assumed to be an interior point of the set Ω which is assumed to be bounded. It is assumed that each of the first $\overline{q}$ components $y_1(t), ..., y_{\overline{q}}(t)$ of $y(t)$ must track the corresponding component of the reference vector $r(t) \in \mathbb{R}^{\overline{q}}$, $\overline{q} \leq q$. Therefore, the error signal $e(t) \in \mathbb{R}^q$ for P is defined by

$$e(t) := Vr(t) - y(t),$$

where

$$V := [I \ 0]^T.$$

It is also assumed that the classes $\mathcal{R}$ of reference signals $r(\cdot)$ to be asymptotically tracked and $\mathcal{D}_i$ of disturbance functions $d_i(\cdot), i = 1, 2, ..., \mu$, to be asymptotically rejected are defined as follows:

$$\begin{aligned} \mathcal{R} &= \mathcal{V}(\overline{q}, \alpha_1, k_1) \oplus \mathcal{V}(\overline{q}, \alpha_2, k_2) \oplus \cdots \oplus \mathcal{V}(\overline{q}, \alpha_{\overline{\mu}}, k_{\overline{\mu}}), \\ \mathcal{D}_i &= \mathcal{V}(m_i, \alpha_i, h_i), \quad i = 1, 2, ..., \mu, \end{aligned}$$

for some $\overline{\mu} \in \mathbb{Z}, 0 \leq \overline{\mu} \leq \mu$, $\alpha_i \in \mathbb{C}, i = 1, 2, ..., \mu$, $k_i \in \mathbb{Z}^+, i = 1, 2, ..., \overline{\mu}$, $h_i \in \mathbb{Z}^+, i = 1, 2, ..., \mu$, with

$$\mathcal{V}(\ell, \alpha, k) = \left\{ v(\cdot) : v(t) = \sum_{j=1}^{k} \frac{t^{j-1}}{(j-1)!}(\delta_j e^{\alpha t} + \delta_j^* e^{\alpha^* t}), \forall t \geq 0, \delta_j \in \mathbb{C}^\ell \right\},$$

where $\alpha \in \mathbb{C}, \ell \in \mathbb{Z}^+$, and * means complex conjugate.

When for plant P a multirate digital control system is used, it seems reasonable to require for the error $e(t)$ not only its convergence to zero at sampling times but the stronger ripple-free requirement, i.e.

$$\lim_{t \to +\infty} e(t) = 0.$$

Such a requirement, as well as asymptotic stability, should be satisfied, in principle, not only at the nominal parameters of the plant P — i.e. at $\beta = \beta_0$ — but also for some variations and/or uncertainties of them: at least for all the values of β belonging to some neighbourhood $\Psi \subseteq \Omega$ of β_0. In order to satisfy the above requirements, the control scheme depicted in fig. 1 is proposed here, where K_D is a periodic discrete-time subcompensator, H and S are a zero-order multirate holder and a multirate sampler, respectively, and K_C is a time-invariant continuous-time subcompensator, such that the series connection S_C of K_C and P provides a continuous-time internal model of reference signals and disturbance functions.

The following problem is solved here.

Problem 1 *(<u>Robust continuous-time tracking and regulation problem</u>). Find, if any, the linear dynamic compensators K_D and K_C such that the following requirements are satisfied by the overall hybrid multirate control system Σ represented in Figure 1:*

(a) Σ is exponentially stable at the nominal parameters of the plant P, i.e. for $\beta = \beta_0$;

(b) relation () is satisfied for each disturbance function $d_i(\cdot) \in \mathcal{D}_i, i = 1, 2, ..., \mu$, and for each reference signal $r(\cdot) \in \mathcal{R}$, for $\beta = \beta_0$;

(c) properties (a) and (b) are preserved for all β in some neighbourhood $\Psi \subseteq \Omega$ of β_0 (or, possibly, in some "large" subset Ψ of Ω containing β_0 as an interior point).

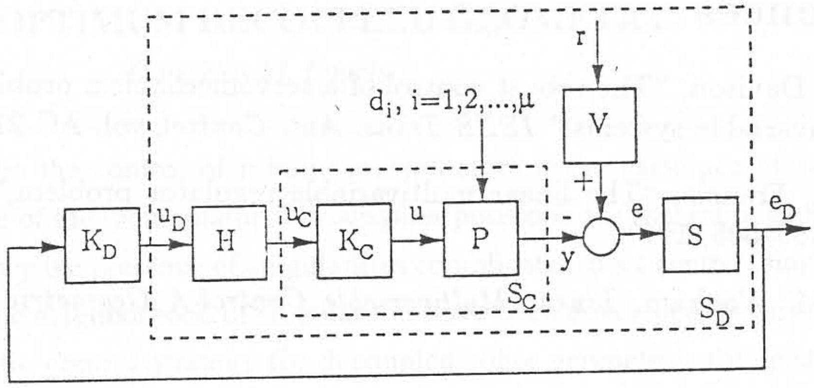

Figure 1: Structure of the hybrid multirate control system Σ.

Such a problem is solved under the following technical assumptions.

Assumption 1 *There exists a closed neighbourhood $\Psi_a \subseteq \Omega$ of β_0 such that all the entries of $A(\beta)$, $B(\beta)$, $C(\beta)$ are continuous functions of β in Ψ_a, and, in addition,*

$$\operatorname{rank}\begin{bmatrix} A(\beta_0)-\alpha_i I & B(\beta_0) \\ C(\beta_0) & 0 \end{bmatrix} = n+q \qquad (\text{i.e., full row}-\text{rank}), \quad i=1,2,\ldots,\mu.$$

Assumption 2 *None of the values $\jmath 2\pi i/\omega$, $i \neq 0$, $i \in \mathbb{Z}$, is an element of $\Gamma(\beta_0) := \sigma(A(\beta_0)) \cup \{\alpha_1, \alpha_1^*, \alpha_2, \alpha_2^*, \ldots, \alpha_\mu, \alpha_\mu^*\}$, where $\sigma(A(\beta_0))$ is the set of the eigenvalues of $A(\beta)$; and, for each element γ of $\Gamma(\beta_0)$, none of the values $\gamma + \jmath 2\pi i/\omega$, $i \neq 0$, $i \in \mathbb{Z}$, is an element of $\Gamma(\beta_0)$, where $\jmath$ is the imaginary unit, and ω is the least common multiple of the sampling periods and hold intervals.*

Theorem 1 *There exist a periodic discrete-time compensator K_D and a time-invariant continuous-time compensator K_C which constitute a solution of Problem 1, under Assumptions 1 and 2, if and only if the triplet $(A(\beta_0), B(\beta_0), C(\beta_0))$ is stabilizable and detectable.*

The solution of Problem 1 proposed by the sufficiency proof of Theorem 1 is based on the construction of a suitable internal model of reference signals and disturbance functions to be connected in series with the plant to be controlled, whose physical parameters are uncertain, as the design method proposed in [4,5,6] is.

Statements similar to Theorem 1 hold for the case when requirement (a) of Problem 1 is strengthened by prescribing the degree of exponential stability of Σ at $\beta = \beta_0$, or the dead-beat convergence of the free responses of Σ at $\beta = \beta_0$, and, correspondingly, requirement (c) is properly amended, as in [4]. Specifically, if (a) is strenghthened in the latter way, and stabilizability and detectability are substituted by reachability and observability in Theorem 1, it is easy to see that Theorem 1 still holds, in order to obtain a ripple-free dead-beat convergence to zero of both the error response and the state free response of the hybrid control system Σ at $\beta = \beta_0$, and to maintain a convergence to zero with an arbitrary degree of exponential decay provided that the parameter perturbations are constrained within a sufficiently small neighbourhood of β_0.

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