On the Solution Space of Discrete Time AR Representations.

by

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ABSTRACT

Consider a system of linear homogeneous difference and algebraic equations described in matrix form by :

$$A(\sigma)\,\beta(n)=0 \tag{1.1}$$

where σ is the shift operator i.e. $\sigma^i\beta(n)=\beta(n+i)$, $A(\sigma)=A_0+A_1\sigma+\cdots+A_k\sigma^k \in \mathbb{R}[\sigma]^{p\times m}$ with $\mathrm{rank}_{\mathbb{R}}A(\sigma)=r$ and $\beta(n) : \mathbb{N}\longrightarrow\mathbb{R}^m$. Following the terminology of Willems (1986) we call the set of equation (1.1) an *AR representation* (*AutoRegressive representation*) of $\mathcal{B}$ (*behaviour*), where $\mathcal{B}$ is the solution set of equations (1.1).

In system theory we need sometimes descriptions of dynamical systems where there is no distinction between inputs and outputs i.e. interconnection of systems. In such cases the model (1.1) is very useful (Blomberg 1983, Willems 1986,1991, Kuijper 1992). However as noted by Willems (1991) the natural definition of the behaviour of the system (1.1) remains an open problem. In this paper we give a solution of the above problem by investigating the set of solutions the system (1.1). More specifically in section 2 and 3 we investigate the solution sets of the system (1.1) which are due to the finite and infinite zero structure of $A(\sigma)$. Our assumption that $A(\sigma)\in\mathbb{R}[\sigma]^{p\times m}$ and $\mathrm{rank}_{\mathbb{R}}A(\sigma)=r$ i.e. $A(\sigma)$ is a

singular polynomial matrix, (where not necessary p=m=r), provides the polynomial matrix $A(\sigma)$ with extra structural invariants i.e. left and/or right kernels. As we show in section 4 the role of the right kernel of $A(\sigma)$ is to provide the system (1.1) with extra independent solutions while the left kernel (Section 5) gives rise to certain constraints between the initial conditions $\beta(0),\beta(1),..,\beta(k-1)$ which must be satisfied so that the system (1.1) has a solution. Finally in Section 6 we present the vector space of solutions of (1.1) and its relation with the structural invariants of the polynomial matrix $A(\sigma)$. A quite interesting result is that the solution vector space (behavior $\hat{B}$) of (1.1) is composed of equivalence classes and its dimension is equal to $f=\hat{n}+\hat{q}+\hat{\ell}$ i.e. is equal to the total number of zeros at $\mathbb{C}\cup\{\infty\}$ and the sum of the right minimal indices (order accounted for) in contrast to the regular AR representations i.e. $A(\sigma)$ is invertible, where each element of the solution vector space of (1.1) is a specific vector valued function and $f=\hat{n}+\hat{q}$ i.e. is equal to the total number of zeros at $\mathbb{C}\cup\{\infty\}$. The meaning of the algebraic structure of a polynomial matrix in relation to the solution vector spaces of singular AR representations has thus been elucidated.

Keyword : autoregressive representation, discrete linear homogeneous systems, behaviour, structural invariants.

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