Motion Planning for a Class of Active Robotic Systems

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Abstract

Parallel manipulator systems for use in exploratory and active robotic tasks are examined, with emphasis on the related motion planning issues. Motion planning is done in the 3-dimensional configuration-space of a planar parallel manipulator system and is based on paths that optimize a curvature-squared cost functional and, as a consequence, optimize both kinematic and dynamic characteristics of the system. This can be formulated as an optimal control problem for a left-invariant system, which falls in the framework of the classical problem of elastica. The results for the problem of elastica are used in selecting configuration-space paths between prespecified end-points of the manipulator that either avoid kinematic singularities or cross them in a desired way. Our approach to planning is not limited to parallel manipulators, but can be adapted to other robotic systems where planning in the configuration space may be of interest.

Introduction

Exploratory and active tasks occur when a robotic system relocates its mechanical or sensory subsystems or alters their characteristics in order to collect information about its environment, about the tasks it is required to perform and, possibly, about the robotic system itself. Actions of this type can also facilitate the subsequent processing and understanding by the system of the sensory information that was obtained, since both spatially and temporally novel information is added in a controlled way (see (Aloimonos & Tsakiris [1991]) and references there).

Active robotic systems depend on a mechanism that carries the sensors, has the ability to translate or reorient them and is able to perform fine, accurate and, at the same time, fast motions. Our approach is to implement this sensor-carrying platform as a *parallel manipulator*. An example of such a system is the so-called "Stewart platform" where six legs with linear motors support a platform carrying the sensors. By varying the lengths of the legs, the platform translates and/or reorients itself. This platform can be attached at the end-effector of a serial robot and provide a light-weight, yet strong and compact system with full 6 degrees-of-freedom motion and with increased accuracy and resolution (Fichter [1986]).

Motion Planning

In the case of serial manipulators planning in "joint space" is considered computationally more efficient compared to "Cartesian space" planning because of the need, in the later case, to solve the inverse kinematics problem at run time. However, for parallel manipulators, planning in Cartesian space becomes feasible, since the inverse kinematics is easily and uniquely solvable. Since the orientation of the platform is also of significant interest in case the platform carries sensors, and since the sensory information interpretation subsystems typically use configuration space–based reasoning, we consider planning in the *configuration space* of the parallel manipulator (Latombe [1991]).

Our main problem is the specification of a trajectory for the parallel manipulator system that has optimal shape characteristics and meets prespecified boundary conditions. Such a trajectory is a curve in the configuration space, which is usually a subgroup of the Special Euclidean group.

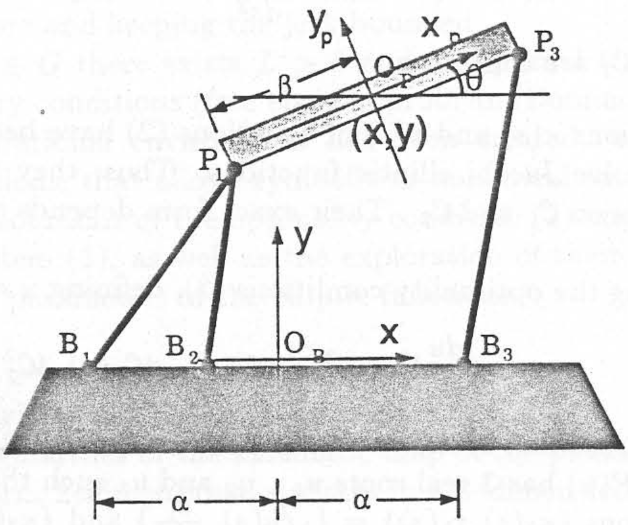

Fig. 1

In the case where the configuration space is a chart of a 3–dimensional manifold, as in a *planar* parallel manipulator (fig. 1), we use the Frenet-Serret apparatus to express the path of the parallel manipulator in configuration space as an arc–length parametrized curve of a left–invariant system on the Lie group G $= SE(3)$, of the form:

$$\frac{d\chi}{ds}(s) = \chi(s)\begin{pmatrix} 0 & -\kappa(s) & 0 & 1 \\ \kappa(s) & 0 & -\tau(s) & 0 \\ 0 & \tau(s) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \chi(s)\mathcal{A}_4 + \kappa(s)\chi(s)\mathcal{A}_3 + \tau(s)\chi(s)\mathcal{A}_1 \quad , \ (1)$$

where $\chi \in G$, $\mathcal{A}_i \in \mathcal{L}(\mathrm{G})$, the Lie algebra of G and κ is the curvature and τ the torsion of the path.

We are interested in specifying a family of paths between two given manipulator configurations parametrized by the length L of each path and by a set of parameters determining its shape, but with the requirement that each path in this family will be as "non–curvy" as possible. This is very similar to the classical problem of elastica, where a flexible thin rod of known length is allowed to deform, while its ends remain fixed (Bryant & Griffiths [1986]; Langer & Singer [1984]). The equilibrium configurations to the elastica problem are known to correspond to a constrained variational problem, which minimizes a cost integral involving the squared curvature of the rod. In our case we look for curves of the left–invariant system on G that minimize $\frac{1}{2}\int_0^L \kappa^2(s)ds$.

This variational problem is solved by using a suitable generalization of the Maximum Principle and the theory of Hamiltonian systems (Arnold [1978]).

Proposition : (Griffiths [1983]; Jurdjevic [1990]) The curvature and torsion corresponding to the regular extremals of the variational problem of minimizing $\frac{1}{2}\int_0^L \kappa^2(s)ds$ on the system (1) satisfy the optimality conditions:

$$\kappa^2\tau = C_1 \quad \text{and} \quad \frac{d^2\kappa}{ds^2} + \kappa^3 - 2C_2\kappa - 2\tau^2\kappa = 0\,, \tag{2}$$

where C_1 and C_2 are constants.

The solutions $\kappa(s)$ and $\tau(s)$ of equations (2) have been derived and, in the generic case, they involve Jacobi elliptic functions. Thus, they are periodic functions, whose period depends on C_1 and C_2 . Their exact form depends on the boundary conditions of the problem.

Combining the optimality conditions (2), defining $u = \kappa^2$ and integrating, we get:

$$(\frac{du}{ds})^2 + u^3 - 4C_2u^2 - 4C_3u + 4C_1^2 = 0\,. \tag{3}$$

When the discriminant of the cubic $P(u) = u^3 - 4C_2u^2 - 4C_3u + 4C_1^2$ is negative and when $C_2 < 0$, then $P(u)$ has 3 real roots u_1 , u_2 and u_3 such that $-u_1 \le 0 \le u_2 \le u_3$ and (3) has the solutions $\{\kappa_1(s), \tau_1(s)\} = \{\sqrt{u(s)}, \frac{C_1}{u(s)}\}$ and $\{\kappa_2(s), \tau_2(s)\} = \{-\sqrt{u(s)}, \frac{C_1}{u(s)}\}$, where $u(s) = u_3[1 - \frac{k^2}{w^2}\,\mathrm{sn}^2(\frac{\sqrt{u_3+u_1}}{2}s\,,\,k)]$, $w^2 \overset{\text{def}}{=} \frac{u_3}{u_3+u_1}$, $k^2 \overset{\text{def}}{=} \frac{u_3-u_2}{u_3+u_1}$ and sn is one of the Jacobi elliptic functions (for a more detailed discussion see (Tsakiris & Krishnaprasad [1993])).

In the case $u_1 = 0$ (i.e. $w = 1$) we get the planar "orbitlike" elasticae (fig. 2). In the case $u_2 = 0$ (i.e. $w = k$) we get the planar "wavelike" elasticae (fig. 3,4). In the case $u_2 = u_3$ (i.e. $k = 0$) we get the spatial circular helices (fig. 5).

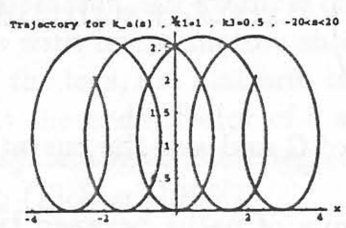

Fig. 2

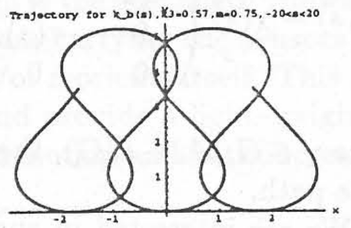

Fig. 3

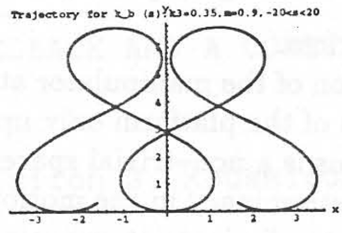

Fig. 4

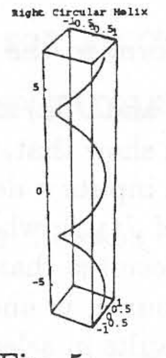

Fig. 5

Assuming that the path curvature and torsion are known, the system equations (1) can be integrated to derive the optimal configuration–space paths. In certain cases of interest (e.g. when the magnitude of the velocity is constant), those optimal paths also satisfy dynamic constraints by minimizing acceleration, optimizing power requirements for the system actuators and keeping the jerk bounded.

For any χ_0 , $\chi_1 \in G$ there exists $L > 0$ and an optimal trajectory $(\kappa(s)\ ,\ \chi(s))$ satisfying the boundary conditions (free elastica problem (Bonnard et al. [1982])).

A special computational environment has been created, using Mathematica on UNIX–based workstations, that allows symbolic or numerical computation and graphical display of various solutions of the optimality condition (2) and of the corresponding trajectories of the system (1), as well as the exploration of their dependence on initial conditions and on the parameters of the elliptic functions.

Kinematic Singularities

We study the singularities of the kinematic map of the planar parallel manipulator and show that they form 2–dimensional surfaces in a 3–dimensional configuration space (fig. 6).

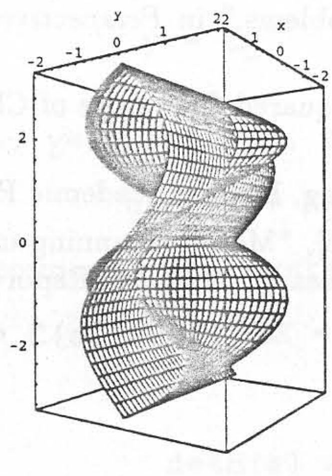

Fig. 6

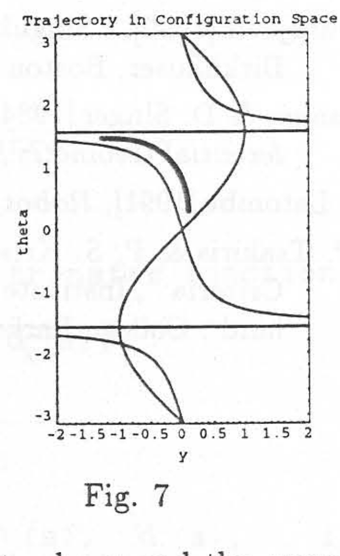

Fig. 7

Let the configuration matrix of the manipulator be χ and the corresponding leg lengths be σ. Then its velocity kinematics relate the spatial and angular velocities (ξ, ω)

of the platform to the rate of change $\dot{\sigma}$ of the leg lengths by: $\Sigma(\sigma)\dot{\sigma} = J(\chi)\begin{pmatrix}\xi \\ \omega\end{pmatrix}$, where $\Sigma(\sigma)$ and $J(\chi)$ are appropriate matrices.

We can show that, if we start the motion of the manipulator at a singular configuration χ_*, the inputs $\dot{\sigma}$ determine the motion of the platform only up to an element of the null space of $J(\chi_*)$; when χ_* is singular, this is a non–trivial space. The physical meaning of this result is that there exists an indeterminacy in the motion of the manipulator, which, of course, is undesirable. In order to eliminate it, we use our previous motion planning results in selecting paths between prespecified configurations that either avoid singularities or cross the singular surfaces in a desired way (fig. 7). The flexibility on the shape of the optimal paths achieved by the previous formulation becomes, in this case, very important.

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