STABILIZATION OF BILINEAR SYSTEMS USING AN OBSERVER CONFIGURATION

R. CHABOUR * H. HAMMOURI **

* INRIA LORRAINE, Technopôle Metz 2 000, CESCOM, 4 rue Marconi 57070 METZ (FRANCE), TEL. 00 33 87 20 35 13, FAX. 00 33 87 76 39 77, e.mail. chabour@ilm.loria.fr.

** Université Claude Bernard Lyon I, LAGEP, Bât. 721, 43, Bld du 11/11/1918 69622 VILLEURBANNE CEDEX (FRANCE).

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INTRODUCTION

This paper is a contribution to the stabilizability problem in observer design of bilinear system

$$\begin{cases} \dot{x} = Ax + uBx \\ y = Cx \end{cases}$$

This problem depends on three factors: the existence of stabilizing feedback law u (x), the existence of an observer giving an estimation $\hat{x}$ of the state x, and the fact that the closed-loop system is asymptotically stable. The most general result of stabilizability of n dimensional bilinear systems follows from the Jurdjevic-Quinn theorem [6]. More recently, in [4], the authors give a class of planar bilinear systems which are not stabilizable by any continuous feedback at the origin and stabilizable thanks to an homogeneous feedback of degree zero. In [1-2] the authors construct a Kalman-like observer for bilinear systems and note that the class of bad inputs constitute the singularity of the problem (a bad input is an input which renders the system unobservable).

For n-dimensional systems observable for any input and for which the drift is dissipative, J. P. GAUTHIER and I. KUPKA [5] prove the stability in observer design by means of Jurdjevic-Quinn feedback law. Using Lyapunov techniques and homogeneous feedback laws, the authors prove in [3] the stability in observer configuration for planar systems with non dissipative drift in the class of bilinear systems observable for any input. In this paper, using a theorem recently proved by Rosier [7], we generalize the result given in [3] to the n-dimensional case.

Main theorem

Consider the following bilinear system

$$\sum\begin{cases}\dot{x} = Ax + uBx \\ y = Cx\end{cases} \qquad x \in \mathbb{R}^n,\ u \in \mathbb{R}$$

If

(i) Σ is observable for any input

(ii) Σ is stabilizable by an homogeneous feedback of degree zero

Then the system

$$\begin{cases}\dot{\hat{x}} = A\hat{x} + u(\hat{x})B\hat{x} - S^{-1}C^{T}(C\hat{x} - y) \\ \dot{e} = (A + u(\hat{x})B - S^{-1}C^{T}C)e\end{cases}$$

is globally asymptotically stable, where $S(t)$ is a solution of

$\dot{S} = -\theta S - A^{T}(u(\hat{x}))S - SA(u(\hat{x})) + C^{T}C$, with $S(0)$ symmetric positive, and $e(t) = \hat{x}(t) - x(t)$.

Proof

Let $u(x)$ be a stabilizing homogeneous feedback law of degree zero the closed-loop system

$$\dot{x} = Ax + u(x)Bx$$

is a continuous homogeneous vector field of degree 1.

According to [7], there exists an homogeneous Lyapunov function V such that:

1) $V(0) = 0 \quad V(x) > 0$ for all $x \neq 0$ and $V(x) \to +\infty \quad \|x\| \to +\infty$

2) $\langle Ax + u(x)Bx, \quad \nabla V(x)\rangle < 0 \qquad \forall x \neq 0$

3) the partial derivatives $\dfrac{\partial V}{\partial x_i}$ are also homogeneous.

Hence, we have $\left\|\dfrac{\partial V}{\partial x}\right\| \leq \alpha(1 + V(x))$ and the proof follows from theorem 1 [3].

Example 1 (see [5])

J. P. GAUTHIER and I. KUPKA prove the stability in observer design for a bilinear system

$$\mathrm{I}\begin{cases}\dot{x} = Ax + u(Bx + b)\\ y = Cx\end{cases}$$

such that

1) (I) is observable for any input

2) $\langle Ax, x\rangle \le 0$ for all x

3) Span $\{Ax, \mathrm{ad}^0 Ax(Bx+b), \ldots \mathrm{ad}^k Ax(Bx+b)\} = \mathbb{R}^n \quad \forall x \in W - \{0\}$

$$W = \{x / \langle x, Ax\rangle = 0\} \cap \{x / \langle x, Bx + b\rangle = 0\}$$

To derive the particular case b = 0 from the theorem given bellow, we have just to remark that $u(x) = \dfrac{-\langle x, Bx\rangle}{\langle x, x\rangle}$ if $x \neq 0$ is an homogeneous stabilizing feedback law of degree zero.

Example 2 (see [3])

$$\begin{cases}\dot{x}_1 = x_1 + 3x_2 + ux_1\\ \dot{x}_2 = -2x_2 + x_1 - ux_2\\ y = h_1x_1 + h_2x_2 \end{cases} \qquad h_1h_2 \neq 0$$

and $$u = \begin{cases}\dfrac{-6x_1^2 - 6x_1x_2 + 9x_2^2}{3x_2^2 + 2x_1^2} & \text{if } (x_1, x_2 \neq (0,0)\\ 0 & \text{otherwise}\end{cases}$$

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