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ℓ^1 Optimal Control with Nonlinear Full State Feedback*

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Abstract

This paper considers ℓ^1 optimal control problems with full state feedback. In contrast to $\mathcal{H}^\infty$ optimal control, previous work has shown that linear ℓ^1 optimal controllers can be dynamic and of arbitrarily high order. However, this paper shows that continuous memoryless *nonlinear* state feedback performs as well as dynamic linear state feedback. The derivation, which is non-constructive, relies on concepts from viability theory.

Keywords: Disturbance rejection; nonlinear compensation; ℓ^1-optimal control; state feedback; viability theory.

1 Introduction

This paper investigates the structure of ℓ^1 optimal control problems [4] with full state feedback. The recent paper [5] has shown that even in the case of full state feedback, optimal and near-optimal linear controllers can be dynamic and of arbitrarily high order. This is in contrast to $\mathcal{H}^\infty$ near-optimal control (cf., [6] and references therein) for which full state feedback controllers can be static. This property ultimately reveals an underlying separation structure for $\mathcal{H}^\infty$ optimal control with output feedback. In light of the results of [5] (see also [7]), it seems unlikely that such a separation property holds for linear ℓ^1 optimal controllers.

In this paper, we follow on the work of [5] and consider the utility of *nonlinear* state feedback. We show that continuous nonlinear static state feedback performs as well as dynamic linear state feedback. Thus, the admission of nonlinear feedback removes the necessity of controller dynamics.

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The derivation, which is non-constructive, relies on concepts from viability theory [1][2][3]. The main idea is to show that disturbance rejection with a known bounded disturbance set is equivalent to restricting the plant state to evolve in a particular bounded region. In the terminology of viability theory, this bounded region is viable for the closed-loop dynamics with disturbances. Viability theory gives conditions for the existence of state feedback leading to viable trajectories. This feedback is then scaled to assure the desired performance for all disturbances.

2 Static Nonlinear State Feedback

We consider the following disturbance rejection problem. The plant dynamics are given by

$$x(k+1) = Ax(k) + B_1 d(k) + B_2 u(k),$$

$$z(k) = Cx(k) + D_{11} d(k) + D_{12} u(k),$$

where the vector signals z, d, and u denote regulated outputs, exogenous inputs, and control inputs, respectively.

Let the state have dimension n and control inputs have dimension m. The two full state feedback controller configurations under consideration are the following:

1. Linear dynamic feedback (K_{dy}):

$$w(k+1) = A_K w(k) + B_K x(k),$$

$$u(k) = C_K w(k) + D_K x(k).$$

2. Static nonlinear feedback (K_{st}):

$$u(k) = g(x(k)),$$

where $g : \mathcal{R}^n \to \mathcal{R}^m$ is continuous and $g(0) = 0$.

Definition 2.1 *A controller K_{dy} or K_{st} is said to be internally stabilizing with a performance of ρ if (1) the unforced dynamics ($d = 0$) are globally exponentially stable and (2) the forced dynamics with zero initial conditions satisfy $\|d \mapsto z\| < \rho$.*

See [4] for further discussion and motivation of such performance objectives.

Our main result is the following.

Theorem 2.1 *There exists an internally stabilizing linear dynamic controller, K_{dy}, with a performance of ρ only if there exists an internally stabilizing continuous static nonlinear feedback, K_{st}, with a performance of ρ.*

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