# EXPONENTIAL MEAN SQUARE STABILITY OF PARTIALLY LINEAR STOCHASTIC SYSTEMS

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Denote by $(\Omega, \mathcal{F}, P)$ an usual probability space and by $w$ a standard $\mathbb{R}^m$-valued Wiener process defined on this space.

Consider the multi-inputs partially linear stochastic differential system in $\mathbb{R}^n \times \mathbb{R}^p$,

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$$\begin{cases} x_t = x_0 + \int_0^t (f(x_s) + G(x_s, \xi_s)\xi_s)ds + \int_0^t g(x_s)dw_s \\ \xi_t = \xi_0 + \int_0^t (A\xi_s + Bu)ds \end{cases} \tag{1}$$

where

1. $f$ and $g$ are $C^2$ functionals mapping $\mathbb{R}^n$ into $\mathbb{R}^n$ and $\mathbb{R}^{n\times p}$ respectively with bounded derivatives and such that $f(0) = g(0) = 0$.
2. $G$ is a functional mapping $\mathbb{R}^n \times \mathbb{R}^p$ into $\mathbb{R}^p$ such that there exists a non decreasing scalar function $\gamma(||\xi||) \geq 0$ bounded for all bounded $\xi$ such that
$$||G(x,\xi)|| \leq \gamma(||\xi||)||x|| \quad \forall (x,\xi) \in \mathbb{R}^n \times \mathbb{R}^p.$$
3. $A$ and $B$ are matrices in $\mathcal{M}_{p\times p}(\mathbb{R})$ and $\mathcal{M}_{p\times q}(\mathbb{R})$ respectively such that the pair $(A, B)$ is stabilizable.
4. $u$ is a $\mathbb{R}^q$-valued control.

Then, we can prove the following stabilizing result,

**Theorem** If the equilibrium solution $x_t \equiv 0$ of the stochastic differential equation
$$x_t = x_0 + \int_0^t f(x_s)ds + \int_0^t g(x_s)dw_s$$
is exponentially stable in mean square then, the equilibrium solution $(x_t, \xi_t) \equiv (0,0)$ of the control stochastic differential system (1) is exponentially stable in mean square for every linear feedback law $u$ defined by $u(\xi) = K\xi$ where $K$ is a matrice in $\mathcal{M}_{q\times p}(\mathbb{R})$ such that the matrice $A + BK$ is asymptotically stable (i.e. all the eigenvalues of the matrice $A + BK$ have negative real parts).

The proof of this result relies on the stochastic Lyapunov machinery developped by Khasminskii [1].
To conclude, note that a determinstic version of this result is due to Saberi, Kokotovic and Sussmann [2].

## References


[1] R.Z.Khasminskii, Stochastic stability of differential equations. Sijthoff & Noordhoff, Alphen aan den Rijn (1980).

[2] A.Saberi, P.V.Kokotovic, H.J.Sussmann, Global stabilization of partially linear composite systems. *SIAM Journal of Control and Optimization* 28 No 6 (1990) 1491–1503.