# APPLICATION OF $H_\infty$ METHODS TO THE CONTROL OF FLEXIBLE STRUCTURES

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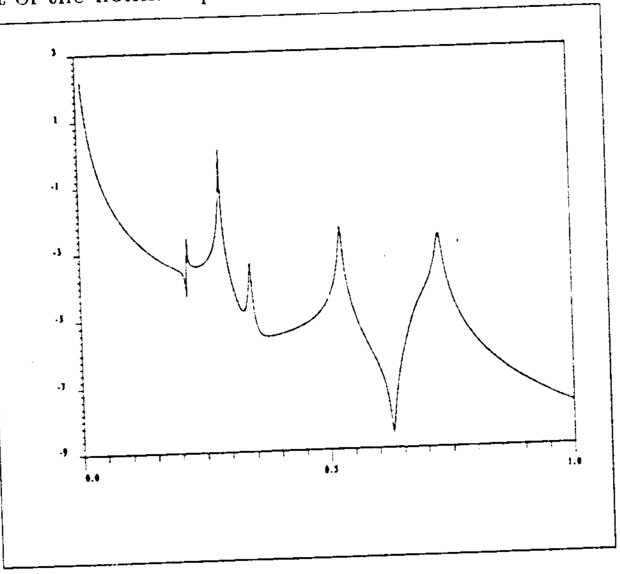
## 1 Introduction

In this paper we consider an $H_\infty$ controller design methodology for flexible systems. We illustrate our approach with a complete treatment of a SISO example drawn from an industrial application. The proposed method avoids undesirable (lightly damped) pole-zero cancellations and yields a controller of the same order as that of the plant, using an alternative formulation to the mixed-sensitivity formulation of the standard problem. This new formulation poses particular difficulties in that its construction and its solution require manipulation of improper rational transfer matrices in state-space form.

## 2 Problem presentation

We consider the problem of controlling an opto-mechanical device used in a line of sight stabilization system [1, 2, 3] The frequency response of the nominal plant is measured and identified using a least square method. The following figure shows the gain plot of the nominal plant.



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## 3 Problem solution

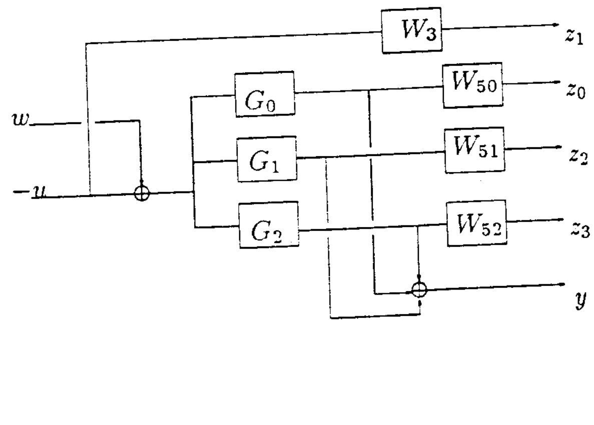
### 3.1 $H_\infty$ formulation

Robustness constraints on the control of flexible system are taken into account by a specific augmented plant which avoids lightly damped pole cancellations. Moreover, the synthesis ensures stability margin and high frequency robustness against spill-over effects by an adequate choice of the function penalizing the complementary sensitivity function.

**Preventing pole-zero cancellation:** In order to prevent lightly damped mode cancellations we penalize the function $G(I+GK)^{-1}$. In addition, $G$ is decomposed into its partial fraction expansion $G = (\sum G_i)$. Some of the $G_i$'s represent lightly damped modes, and it is possible to introduce the functions $G_i(I+GK)^{-1}$ in the $H_\infty$ criterion. This formulation allows to act independently on each mode, by penalizing the corresponding $G_i(I+GK)^{-1}$ function with a constant weighting function. (thus without increasing the order).

**Stability robustness:** It is obtained by penalizing the the complementary sensitivity function $GK(I+GK)^{-1}$ with $W_3(s)$. Choosing $W_3(s)$ as constant in the low frequency domain is equivalent to set the quality factor $Q$ of the closed loop system by ensuring the controlled open loop to be outside the corresponding M-circle. Choosing $W_3$ with a derivative action in the high frequency domain ensures robustness against spill-over effects. Finally, $W_3(s)$ is chosen as $Q^{-1}(\sum a_i s^i)$.

The diagram below represents the corresponding augmented plant.



In our application $G_0$ contains the rigid body mode of the system ($1/s^2$ factor) while $G_1$ and $G_2$ represent respectively the first two flexible modes of the model. The $H_\infty$ problem is the following:

Find internally stabilizing $K$ such that:

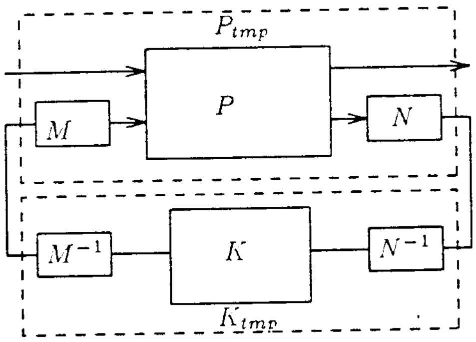
$$\left\| \begin{array}{c} W_3 GK(I+GK)^{-1} \\ W_{50} G_0 (I+GK)^{-1} \\ W_{51} G_1 (I+GK)^{-1} \\ W_{52} G_2 (I+GK)^{-1} \end{array} \right\|_\infty < 1$$

### 3.2 Numerical solution

To constrain the controller to contain an integration term, the synthesis model is multiplied by $1/s$ i.e the synthesis is performed with the pre-integrated system. This leads to a $G_0$ plant containing a factor $1/s^3$. The augmented plant is then:

$$\left[ \begin{array}{cc} 0 & -W_3 \\ W_{50} G_0 & -W_{50} G_0 \\ W_{51} G_1 & -W_{51} G_1 \\ W_{52} G_2 & -W_{52} G_2 \\ G_0 + G_1 + G_2 & -G_0 - G_1 - G_2 \end{array} \right]$$

With this formulation several difficulties arise which do not appear in the usual mixed sensitivity problem. $G$ is strictly proper ($D_{21} = 0$), with imaginary-axis poles and $W_3$ non proper (polynomial). To overcome these difficulties, we introduce pre- and post- compensators to "normalize" the problem [4, 5].

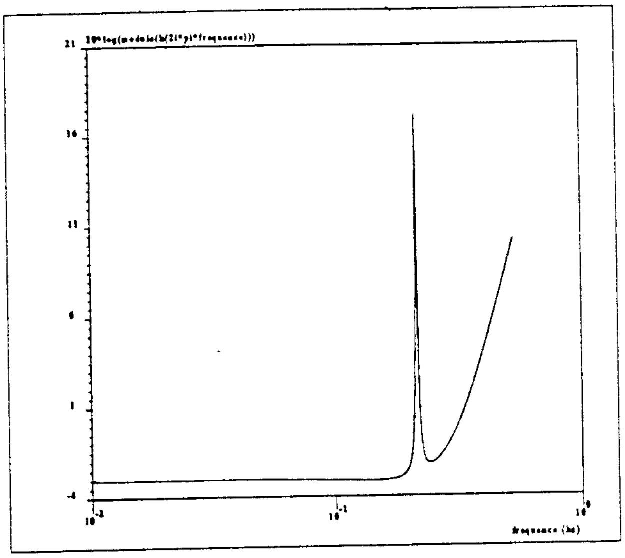



In the previous figure, $P$ does not satisfy the usual assumptions necessary to apply the Riccati based solution of the standard problem. However $P_{tmp}$ does (with proper choice of M and N), and the final controller $K$ is obtained from $K_{tmp}$ by $K = N K_{tmp} M$.

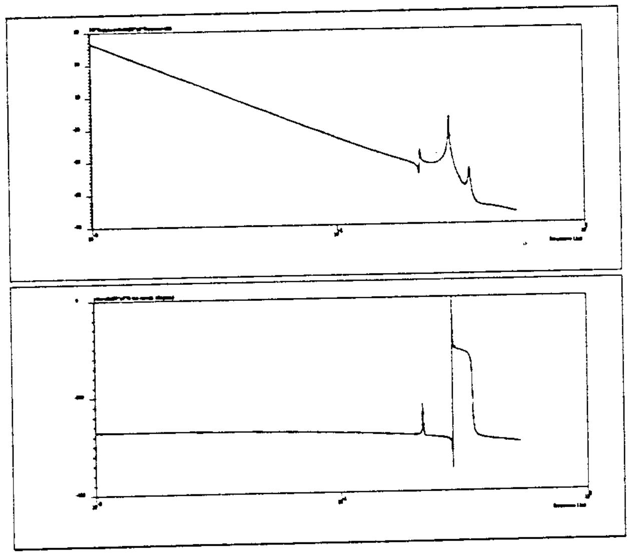
The minimal realization of the augmented plant is obtained using the Ψlab's primitives [1] which allow the manipulation of improper systems.

Below is the gain plot of $W_3$:

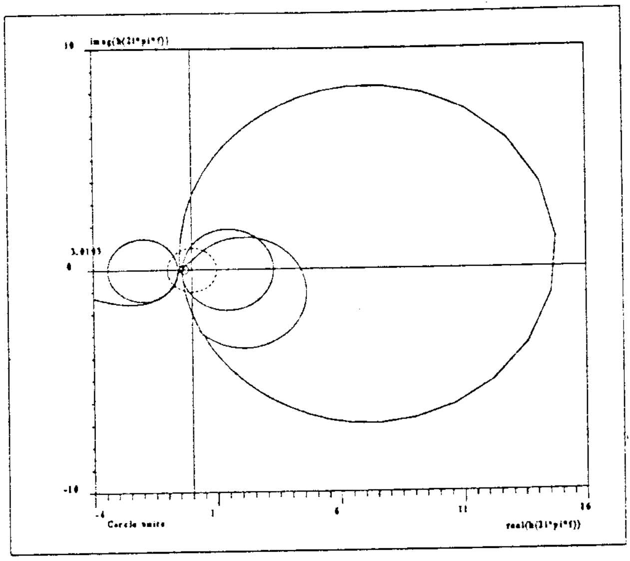
[1] Ψlab is a scientific software package for control and signal processing developed at INRIA [6].



The synthesis model (including the factor $1/s$) is the following:

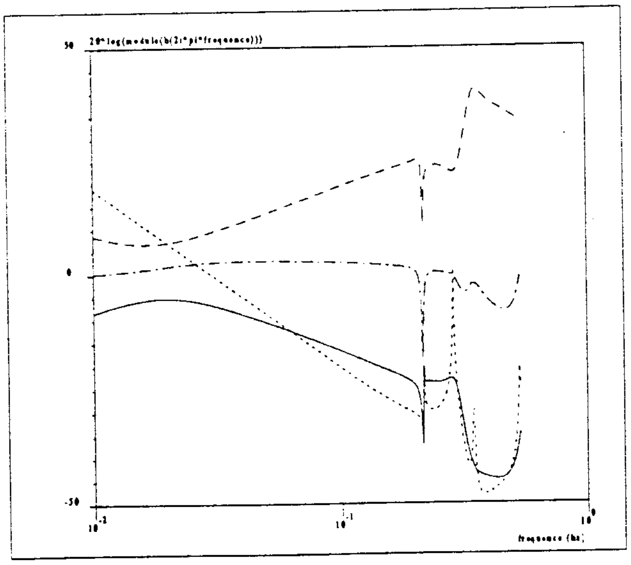


The $W_5$'s weighting constants and the derivative action of $W_3(s)$ are chosen so that the $H_\infty$ algorithm converges with $\gamma = 1$. The controlled open-loop is plotted in the Nyquist plane in order to analyze the stability of the system:



Clearly, the cancellations have been avoided, leading to a positive control of lightly damped modes. In addition, the open loop Nyquist plot is outside the specified M-circle ensuring the desired stability margin.

The following diagram shows the nominal plant, the function $G/(1+KG)$, the controller and the complementary sensitivity function $KG/(1+KG)$:



## References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