DYNAMIC STABILIZATION OF NONLINEAR STOCHASTIC SYSTEMS

Patrick FLORCHINGER *
URA CNRS No 399, Département de Mathématiques,
UFR MIM, Université de Metz,
Ile du Saulcy,
F 57045 METZ Cedex, France.
Phone (33) 87 63 39 95, Fax (33) 87 38 03 83
email florchin@ilm.loria.fr

IEEE Mediteranean Symposium on new Directions in Control Theory and Applications

Key Words Nonlinear control stochastic systems, Stochastic stability, Feedback laws, Dynamic control.

The purpose of this work is to study the stabilization of nonlinear stochastic systems by means of a dynamic controller.

Denote by $(\Omega, \mathcal{F}, P)$ an usual probability space and by w a standard $\mathbb{R}^m$-valued Wiener process defined on this space.

Consider the multi-inputs stochastic differential system in $\mathbb{R}^n$,

$$x_t = x_0 + \int_0^t f(x_s, u)\, ds + \int_0^t g(x_s) \circ dw_s \tag{1}$$

where

1. x_0 is given in $\mathbb{R}^n$.
2. u is an $\mathbb{R}^p$-valued control law.

*also : INRIA Lorraine, Projet Congé, CESCOM, Technopole de Metz 2000, 4 Rue Marconi, F57070 METZ, France.

3. f and g are Lipschitz functionals mapping $\mathbb{R}^n \times \mathbb{R}^p$ (respectively $\mathbb{R}^n$) into $\mathbb{R}^n$ (respectively $\mathbb{R}^m$) such that
   - $f(0,0) = 0$ and $g(0) = 0$,
   - there exists a non-negative constant K such that for any $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^p$,

$$|f(x,u)| + |g(x)| \leq K(1 + |x| + |u|).$$

The stochastic differential system (1) is said to be locally feedback stabilizable in probability at the origin if there exist a neighbourhood D of the origin in $\mathbb{R}^n$ and a functional ϕ mapping D into $\mathbb{R}^p$ such that

1. $\phi(0) = 0$.
2. For every $x \in D$, the solution of the closed-loop system

$$x_t = x + \int_0^t f(x_s, \phi(x_s))\, ds + \int_0^t g(x_s)\, o dw_s \qquad (2)$$

   is uniquely defined.
3. The equilibrium solution $x_t \equiv 0$ of the closed-loop system (2) is asymptotically stable in probability.

The concept of dynamic stabilization for deterministic nonlinear control systems has been introduced by Sontag and Sussmann [1].

Then, one can extend this concept to control stochastic differential system as follows,

The stochastic differential system (1) is said to be locally dynamic asymptotically stabilizable in probability at the origin if the stochastic differential system

$$\begin{pmatrix} x_t \\ z_t \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \end{pmatrix} + \int_0^t \begin{pmatrix} f(x_s, z_s) \\ v \end{pmatrix} ds + \int_0^t \begin{pmatrix} g(x_s) \\ 0 \end{pmatrix} o dw_s \qquad (3)$$

is locally feedback stabilizable in probability at the origin.

The stochastic differential system (1) is said to satisfy a Dynamic stochastic Lyapunov condition at the origin if there exists a functional V mapping $\mathbb{R}^n \times \mathbb{R}^p$ into $\mathbb{R}$ such that V is continuous and positive definite on a neighbourhood D of the origin in $\mathbb{R}^n \times \mathbb{R}^p$, is smooth on $D \setminus \{0\}$ and such that for any $(x,z) \in D \setminus \{0\}$ such that $\frac{\partial V}{\partial z}(x,z) = 0$ it follows that $LV(x,z) < 0$ where L denotes the infinitesimal generator of the stochastic differential system (1).

Then, one can prove the following theorem which extends theorem 3 from Tsinias [2] to the dynamic stabilization of stochastic systems.

Theorem 1) If the stochastic differential system (1) satifies a Dynamic stochastic Lyapunov condition at the origin, then it is locally asymptotically stabilizable in probability at the origin.
2) If the stochastic differential system (1) satisfies a Dynamic stochastic Lyapunov condition at the origin and the equation $\frac{\partial V}{\partial z}(x,z) = 0$ has a solution $z = \phi(x)$, $\phi(0) = 0$ which is continuous on a neighbourhood S of the origin in $\mathbb{R}^n$ and smooth on $S \setminus \{0\}$, then the stochastic differential system (1) is locally feedback stabilizable in probability at the origin by means of the control law $u = \phi(x)$.
3) The stochastic differential system (1) is locally feedback stabilizable in probability at the origin by means of a locally smooth feedback law $u = \phi(x)$ with $\phi(0) = 0$, if and only if (1) satisfies a Dynamic stochastic Lyapunov condition at the origin with V smooth in a neighbourhood of the origin and $\det \frac{\partial^2 V}{\partial z^2}(0,0) \neq 0$.
In this case, the stochastic differential system (1) is locally dynamic asymptotically stabilizable in probability at the origin by means of a continuous feedback law $v = r(x,z)$ which is smooth for any $(x,z) \neq 0$ in a neighbourhood of the origin in $\mathbb{R}^n \times \mathbb{R}^p$.

References

[1] E.D.Sontag, H.J.Sussmann, Remarks on continuous feedback. *Proceedings of the 19th IEEE Conference on Decision and Control*, Albuquerque, NM (1980) 916–921.

[2] J.Tsinias, Sufficient Lyapunov–like conditions for stabilization. *Mathematics of Control Signals and Systems* 2 (1989) 343–357.