CONTINUOUS–TIME DEADBEAT OBSERVATION PROBLEM

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ABSTRACT

Two observers are shown to estimate the states of a linear dynamic system deadbeatly in continuous time.

The first observer uses a finite number of delayed input/output measurements to reconstruct the state vector with zero error being provided with a process history over its largest time-delay. Moreover, the largest time-delay value puts a natural limit to the observer memory. The second observer comprises a conventional Luenberger observer and a finite-memory deadbeat observer to achieve zero estimation error at finite time.

INTRODUCTION

It is well known that deadbeat performance can be achieved in discrete-time systems by placing all roots of the characteristic polynomial of an observer or controller at the origin. Therefore, any pole-placement design technique provides the desired transient response property. In contrast, continuous-time deadbeat problem does not naturally arise from pole placement and has drown serious research attention only recently. As it can be seen, the continuous-time deadbeat controllers and observers for the plants described by ordinary linear differential equations belong to another, as the plants, class of dynamic systems. Presence of time delays in the control law or observer structure becomes inevitable to drive control or observation error to zero deadbeatly. Hence, continuous-time deadbeat structures are governed rather by differential-difference than ordinary differential equations and crave a special consideration.

Being a new research area, continuous-time deadbeat problem is treated only in a few papers. In [1,2], finite-memory deadbeat observation problem has been solved by a direct state-space approach r lated to deterministic least squares. Unfortuna enough, the resulting observer works in the open l mode only. In [3], a general solution for the de beat tracking and stabilization problems is obtai via finite Laplace transform , a technique commo differential-difference equations theory.

In the present paper, we develop a deadbeat o server for multivariable continuous systems to be u in a feedback control structure. The paper is co posed as follows. Firstly, we describe the Fini Memory Deadbeat Observer (FMDO) using a d ferent, than it is in [2], approach. Then, a co bination of a conventional Luenberger observer a FMDO is exploited to achieve desired performa in an Infinite-Memory Deadbeat Observer (IMD It is shown that the IMDO estimate can be used f feedback control since the time-delay part of the o server is intoduced in a way which does not chan the closed-loop stability properties.

1. FINITE–MEMORY OBSERVER

Let us consider the nonhomogeneous equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y &= Cx(t) \end{aligned} \qquad (1$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ the control vector, and $y(t) \in R^l$ is the observati vector. A, B, C are real matrices of appropriate d mensions.

Theorem 1 : Provided that the matrix

$$\mathcal{W}_k = \sum_{i=0}^{k} \exp(-A^T \tau_i) C^T C \exp(-A\tau_i)$$

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is positive definite, then the observer

$$\hat{x}_k(t) = \mathcal{W}_k^{-1} \sum_{i=0}^{k} \exp(-A^T \tau_i) C^T (y(t - \tau_i) + \int_{t-\tau_i}^{t} C \exp(A(t - \delta - \tau_i)) Bu(\delta) d\delta) \tag{1.2}$$

has the following properties (i) finite memory, limited by the largest time-delay τ_k; (ii) dead-beat performance, i.e $e(t) = x(t) - \hat{x}(t) = 0$; $t > \tau_k$ for any initial function $\phi_0 = y(t), t = [-\tau_k, 0]$.

Proof: Omitted.

Positive definiteness of the $\mathcal{W}_k$ matrix guarantees [1]that the system (1.1) (which is a time-delay system) is spectrally observable. In the case $\det(\mathcal{W}_k) = 0$, the observer (1.2) becomes a least-squares estimator without the deadbeat property, since it reconstructs the state vector only partly.

It is worth noting that whereas the only design parameter for a deadbeat observer is the sampling period, the finite-memory observer (1.2) has its parameters fully defined by a set of time delays $\tau_i, i = 0, \ldots, k$ which might be chosen by some relevant procedure considering for example observer's robustness against model uncertainties, optimality etc.

As Theorem 1 shows, there were no limitation on the choice of τ_i but positive definiteness of $\mathcal{W}_k$, and, hence, they may be taken arbitrarily as commensurate or noncommensurate positive reals. This becomes a merit of the method in comparison with the finite Laplace transform design procedure [3] which tackles only commensurate time delays.

Taking Laplace transform of (1.2) yields

$$\hat{x}_k(s) = \mathcal{W}_k^{-1} \sum_{i=0}^{k} \exp(-A^T \tau_i) C^T C \exp(-s\tau_i) \times (x(s) + (sI - A)^{-1} Bu(s)) - (sI - A)^{-1} Bu(s)$$

which hints to the conclusion that the applicability of FMDO to feedback control is questionable. Indeed, had $\hat{x}_k(s)$ been fed back as $u(s) = G\hat{x}_k(s)$, would all the time dealys τ_i appear in the characteristic polynom of the closed loop system. Of course, it does not necessarily mean poor stability, but rather makes the task of procuring reasonable transient behavior more complicated.

2. INFINITE-MEMORY OBSERVER

Continuous-time deadbeat phenomenon is not bound only to finite-memory structures, but can be also accomplished in infinite-memory structures.

<u>*Theorem 2 :*</u> Provided that the matrix $(A - KC)$ is Hurwitz and the matrix

$$\mathcal{W}_k = \sum_{i=0}^{k} \exp((KC - A)^T \tau_i) C^T C \exp((KC - A)\tau_i)$$

is positive definite, then the observer

$$\begin{aligned} \hat{x}(t) &= \bar{x}(t) + e_d(t) \\ \dot{\bar{x}}(t) &= A\bar{x}(t) + Bu(t) + K(y(t) - C\bar{x}(t)) \\ e_d(t) &= \mathcal{W}_k^{-1} \sum_{i=0}^{k} \exp(-(A - KC)^T \tau_i) C^T \times \\ &\quad (y(t - \tau_i) - C\bar{x}(t - \tau_i)) \end{aligned} \tag{2.1}$$

is a deadbeat observer in the sense that the estimatio error $e(t) = x(t) - \hat{x}(t)$ is zero for all $t \geq \tau_k$.

Proof: Omitted.

Strictly speaking, it is not necessary to impos the stability limitation on the matrix $(A - KC)$ sinc the FMDO is as well able to reconstruct the state vec tor of an unstable system. However, from a practica standpoint, the presence of unbounded signals in a observer seems to be irrational.

As it follows from (1.2), the FMDO structur cannot be directly used for feedback control as fo closed-loop stability is not guaranteed. The followin theorem shows that the IMDO structure overcome this difficulty.

<u>*Theorem 3 :*</u> Suppose the feedback control law

$$u(t) = G\hat{x}(t) \tag{2.2}$$

is used to stabilize the plant (1.1). Then, the closed loop system (1.1), (2.1), (2.2) is asymptotically stab iff $(A - KC)$ and $(A + BG)$ are Hurwitz matrices.

Proof: Omitted.

The observer-based control system (2.2),(2.1), comparison with the conventional feedback controll $u(t) = G\bar{x}(t)$, retains closed-loop stability and simu taniously improves transient response of the contr loop attaining the plant state vector perfectly follov the model

$$\dot{x}(t) = (A + BG)x(t)$$

after the deadbeat settling time τ_k has been expire In this sense, the deadbeat observer compensates a ditional transient effects caused by the Luenberg observer. This can be also illustrated by evaluati the closed-loop transfer function.

<u>*Theorem 4*</u> : With the control law

$$u(t) = G\hat{x}(t) + r(t)$$

used for the system (1.1), (2.1), the closed-loop tran fer function is given by

$$y(s) = C(sI - A - BG)^{-1} Br(s)$$

Proof: Omitted.

Thus, observer's own dynamics is cancelled in the closed-loop transfer function and does not affect controller performance, provided transient response in IMDO settled deadbeatly.

4. CONCLUSIONS

A deadbeat observation problem is solved for continuous linear multivariable systems. Two deadbeat observers possessing respectively finite and infinite process memory are introduced.

The memory limit imposed intrinsically by the finite-memory observer structure might be helpful in preventing it from possible divergence and makes observer residuals more sensitive to recent measurements.

The infinite memory deadbeat observer is shown to improve the controller performance in a feedback control system by effectively cancelling the control error transient terms caused by observer dynamics.

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