How the membership function shape characterizes the dynamical behaviors of a fuzzy system

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1.Introduction

Fuzzy algebra represents a promising approach for the control of dynamic systems; in particular the fuzzy approach appears to be a very suitable tool in some cases, involving nonlinear or hill-known plants, that cannot be satisfactorily solved by using traditional control schemes.

Despite of the recent interest for the fuzzy logic strategy and of the large number of applications, that have induced a lot of research efforts, nowadays very little results are available about the theoretical aspects of plants containing fuzzy sub-systems. This lack of knowledge represents a drawback both for the design of a fuzzy controller (this topic will not be addressed in this paper; some results have been reported by the authors in [1]) and for the analysis. In this paper the latter problem will be taken into account in order to gain some insight about the influence of a set of parameters of the fuzzy sub-system on the full-system behaviour.

A fuzzy controller is built by using a set of fuzzy rules [2], that express in a linguistic form the control policy to be applied to the plant in order to obtain the desired system performances, in a closed loop scheme, as it is shown in Fig. 1

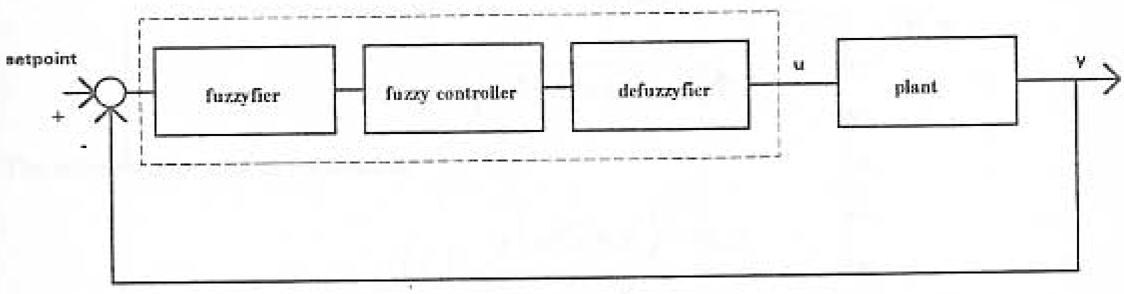

Fig. 1 - General scheme of a fuzzy control system

The two blocks named fuzzyfier and defuzzyfier have been introduced in the control scheme in order to allow the interfacing of the fuzzy world of the controller with the real one.
Each rule contained in the controller represents a fuzzy application from a set of measured variables to the space of the control variable. This application is defined by clustering the domain of each variable with a certain number of fuzzy sets, generally partially overlapped, labelled with linguistic expressions. A typical example of such a rule is reported in Fig.2.

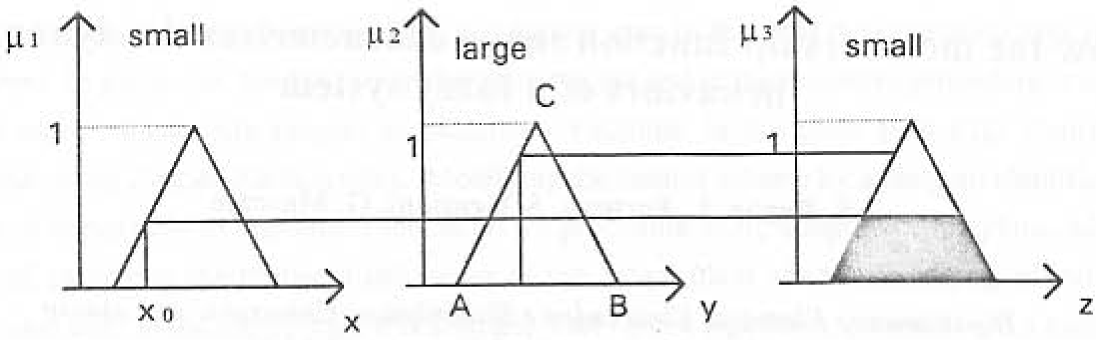

Fig. 2 - Example of fuzzy inference for the rule: "IF x IS small AND y IS large THEN z IS small"

The reported example refers to a fuzzy rule defined on the x and y variables that computes the z variable values. It possible to observe in the reported Fig. 2, that each fuzzy set is defined through a membership function $\mu(x)$ that associates to a given input $\mathbf{X}_0$ value a corresponding real value $\mu_1(x_0)$ belonging to the interval [0,1]. This value represents the evidence that the considered $\mathbf{X}_0$ value belongs to a given fuzzy set.

Usually the design of a fuzzy controller is dealt without considering the influence of the membership function shape on the system dynamic characteristics and a triangular shape is chosen for its simplicity. In such a way, in order to fix each membership function, only three parameters are required (points A, B, C in Fig. 2) and therefore the design step is simplified. In the example reported in the follow, the fuzzy rules are fixed, while the membership function shape is varied.

In this paper the influence of the membership function shape on the steady-state behaviour of the closed loop plant is analyzed; moreover due to the consideration that a fuzzy controller is a nonlinear system, the behaviour of the whole system is studied in order to characterize nonlinear phenomena, including the possible presence of chaotic effects.

2.Parameterized Membership Function

In order to obtain a general characterization of the dependence of some properties of the plant on the shape of the membership function, a parametric representation via four independent parameters is used. The family of the membership functions has been obtained by considering adequate portions of gaussian function, as reported in Fig. 3.

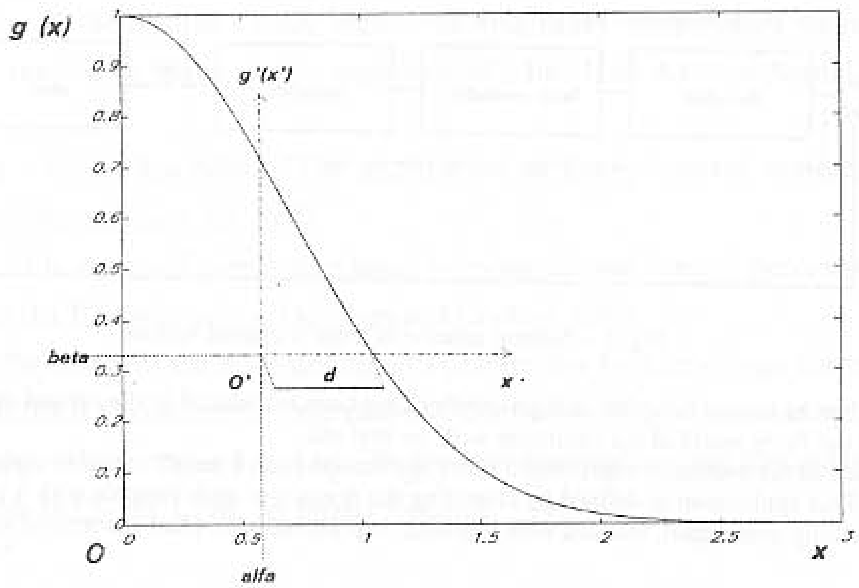

Fig. 3 Parameterization of the membership function

The membership functions are obtained by flipping around the $\mu^{'}\left(x^{'}\right)$ axis the portion of the gaussian curve contained in the quadrant defined by the axes traced with line-dot lines.

Referring to the symbols adopted in Fig. 3 we have

$$g(x) = e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^{2}} \qquad (1$$

For sake of simplicity, in this work the σ parameter is held constant.

Each particular membership function is obtained from the choice of a couple of values of ***alfa*** and ***beta***. A normalization is moreover required in order to obtain normal membership functions of the desired width. Generally if ***alfa*** and ***beta*** are almost zero, a quasi-gaussian shape is obtained, on the other hand if the ***beta*** value is almost equal to g(***alfa***) a straight line whose slope depends on the value of the first derivative of g(***alfa***) is found.

If L is the half-width of the desired membership function and y is the argument of the membership function to be computed, then from Fig. 3 we have:

$$x^{'} = y\frac{d}{L} \qquad (2$$

Moreover if the normality condition is imposed to the membership function, the following relation must occur:

$$\frac{g(alfa) - beta}{k} = 1 \qquad (3$$

therefore the k gain can be easily computed.

The d value can be found observing that if x is equal to ***alfa+d*** then the gaussian function value is ***beta:***

$$g(alfa + d) = beta \qquad (4$$

$$e^{-\frac{1}{2}\left(\frac{alfa+d}{\sigma}\right)^{2}} = beta \qquad (5$$

$$d = \sigma\sqrt{-2\ln(beta)} - alfa \qquad (6$$

The membership value is obtained as:

$$g^{'}\left(x^{'}\right) = \frac{g\left(alfa + x^{'}\right) - beta}{k} \qquad (7$$

In the above expressions it is supposed that:

$$0 < alfa$$

$$0 < beta < g(alfa)$$

3.A Case Study

In the following a study regarding the influence of the introduced parameters of the fuzzy controller applied to a nonlinear controlled plant is described.

The plant to be controlled is represented by the following differential equation [3]

$$\frac{dy}{dt} = -y + 0.5y^2 + u. \quad (8$$

The fuzzy controller is built up with four control rules:

if error=ep and rate=rp then output=on;

if error=en and rate=rn then output=oz;

if error=en and rate=rp then output=oz;

if error=en and rate=rn then output=op;

and the associated fuzzy sets, in the particular case of triangular membership function shape, i.e. ***beta*** $\longrightarrow$ *g(**alfa**)*, are reported in Fig. 4.

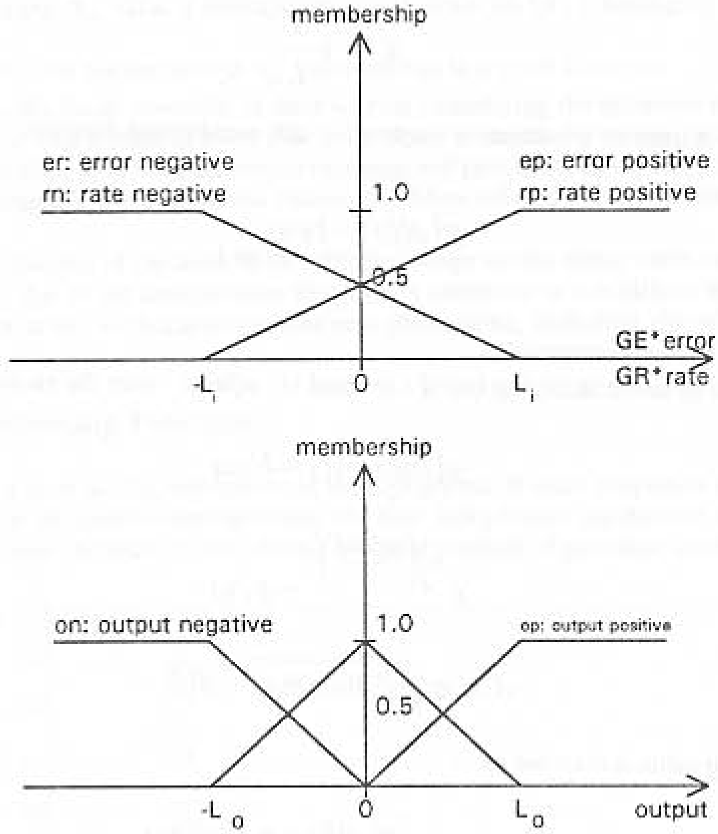

Fig. 4 Membership functions for the inputs ant the output of the fuzzy controller

The GE and GR parameters appearing in Fig. 4 represent two gains that multiply the error and its rate respectively. In the proposed application we have fixed GE=20 GR=8.75.

The control policy is obtained from the output of the fuzzy controller, by using the following well known defuzzyfication approach:

$$d_{nl} = \frac{\sum\left[(value\_of\_member) * (membership\_of\_member)\right]}{\sum(memberships)}$$

In the previous expression the term ***value_of_member*** corresponds to the value of the universe of discourse of the output variable that is associated with the unitary degree of membership, in the considered case $-L_0$ and L_0.

Let us consider the ***alfa-beta*** plane named "shape-plane", the analysis has been performed by varying ***alfa*** in the interval $[0, 2\sigma]$ and ***beta*** in the related interval $[0, g(alfa)[$.

Two type of experiments are described; in the former the setpoint has been fixed to the value of $\text{setpoint}(t) = 3$ in order to emphasize the variation of some system parameter such as steady state error, settling time and overshoot. In Fig. 5 it is shown the plant output waveform against the ***beta*** parameter variation for a fixed ***alfa*** value ***alfa***=0.

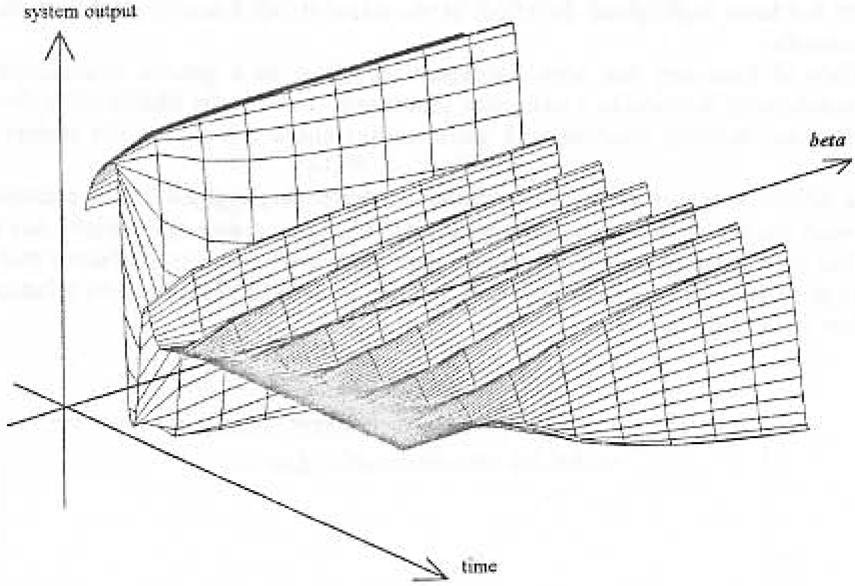

Fig. 5 System output for ***alfa***=0 and 0<***beta***<***1***.

It is easy to observe that while for ***beta***=0 the system steady state behavior has a fixed point with zero error, with increasing values of ***beta*** the qualitative behavior changes from an equilibrium point to a limit cycle; moreover overshoot and settling time increase too.

The latter experiment has been performed by setting $\text{setpoint}(t) = 3 + \frac{1}{10} sin\left(\frac{2\pi t}{10}\right)$; in this way, with an easy change of variables, it is possible to show that this is equivalent to increase the order of the corresponding system with zero-input by one (the time) [4].

Since the system has now three state variables it is possible to investigate about the presence of complex nonlinear phenomena in the system steady state response, as chaos. In Fig. 6 are shown the state space system trajectories in two different points of the "shape-plane".

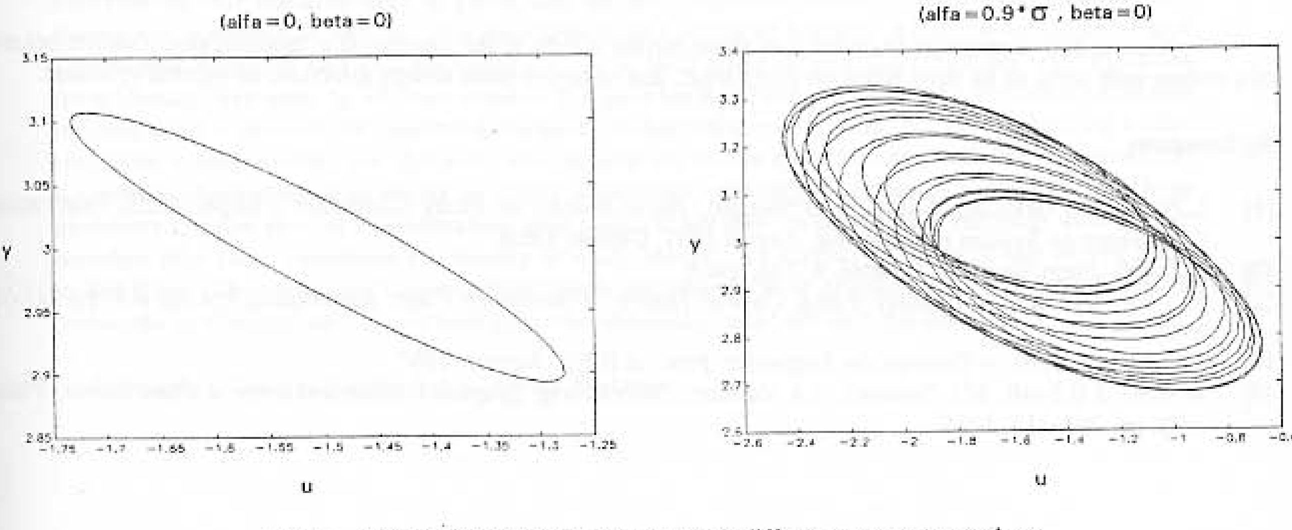

Fig. 6 System steady state trajectory with different parameters values.

While a limit cycle is present in the case *(alfa=0, beta=0)*; the system largest Lyapunov exponent has been computed [5] for the case *(alfa*= 0.9*σ, *beta*=0) resulting in a positive value; this allows us to confirm the presence of chaotic effects for some choice of the considered parameters.

4.Conclusions

In this paper has been highlighted the effect of the membership function shape on the behavior of a plant containing a fuzzy controller.

A large variety of behaviors has been discovered revealing, as a general characteristic, that more is the resemblance of the membership function to a triangular shaped one, more is the likelihood to find complex behaviors; on the other side when membership function with gaussian-like shape are chosen the system shows more regular characteristics.

In Fig. 7 the different regions in the "shape-plane" have been highlighted for the example with the sinusoidal setpoint; in the **k**–region the system present a limit cycle, in the **l**-region a quasi-periodicity has been observed and in the **m**-region chaos has been detected from the lyapunov exponents analysis. It is to observe that the three considered regions are not impinge to the ***alfa*** axis due to the fact that our analysis is performed on a lattice and must begin for values of ***beta*** different from zero.

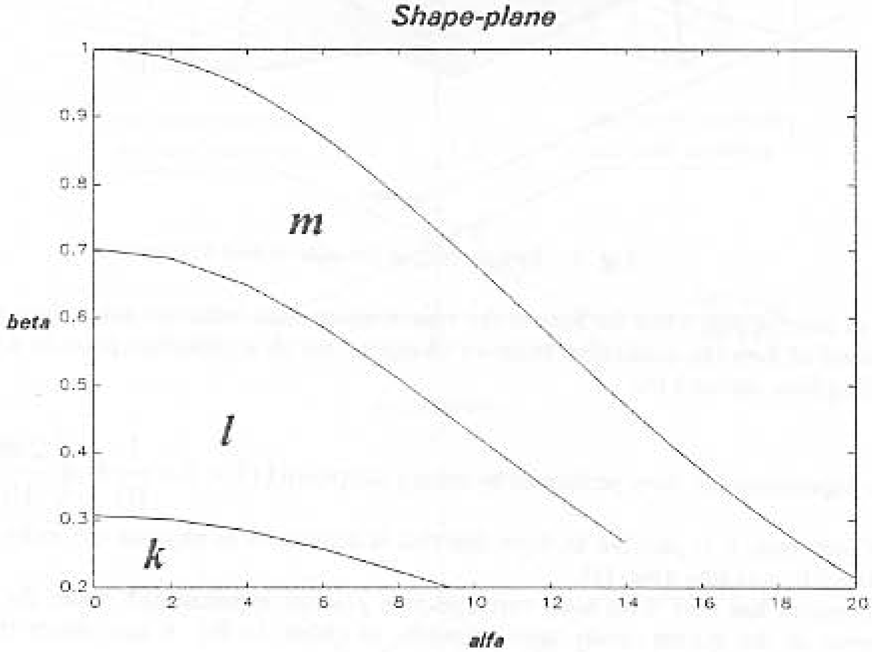

Fig. 7 The "Shape-plane" with the different system behavior regions for the third order system.

Work is still in progress to investigate about further effects of the membership function shape on the behavior of a system with some of its parts based on fuzzy logic, and to derive some design directives or general relations.

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