A Separation Principle for Dissipative Control Affine System

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Keywords : nonlinear dissipative systems, observers, separation principle, feedback stabilization.

In this paper, we consider a multi input nonlinear system of the form

$$(\Sigma) \qquad \begin{cases} \dot{x} = f(x) + \sum_{i=1}^{m} u_i(t) g_i(x) \\ y = h(x) \end{cases}$$

with $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}^m$. (Σ) is supposed to be dissipative, that is to say there exists a C^∞ function $V : \mathbb{R}^n \to \mathbb{R}^+$ verifying the following properties :

$$(P.1) \qquad \lim_{\|x\| \to +\infty} V(x) = +\infty \,;\, V(0) = 0.$$

$$(P.2) \qquad L_f(V)(x) \leq 0, \ \forall x \in \mathbb{R}^n$$

In the case where $L_f(V) = 0$ the system is said to be conservative.

The aim is to show that if this class of systems are stabilizable by some feedback control $u(x)$ then it can be performed by some feedback $u(\hat{x})$ where $\hat{x}$ is an estimate of x coming from an observer.

For some particular classes of nonlinear systems, this problem has been treated in [TS], [HA, BU].

Here we present an extension of the bilinear dissipative separation principle proposed in [GA, KU].

First we recall the stabilization result for (Σ).

This result has been already proved for conservative nonlinear systems in [GA, BO], and for bilinear dissipative systems in [GA, KU]. The result is the following :

Theorem 1 :

Assume that (Σ) satisfies :

a : there exists a real valued function V with $V(0) = 0$ satisfying $(P.1)$ and $(P.2)$

b : $dV(x) = 0 \iff x = 0$

c : $f(0) = 0$

d : the vector space E_x spanned by $\{f(x), ad_f^k(g_i)(x), k \in \mathbb{N}, i = 1, \cdots, m\}$ is n-dimensional, for any x such that $L_f(V)(x) = 0, L_{g_i}(V)(x) = 0$ and $x \neq 0$.

Then the feedback $u(x) = \begin{pmatrix} u_1(x) \\ \vdots \\ u_m(x) \end{pmatrix}$ where $u_i(x) = -r_i(x)L_{g_i}(V)(x)$ stabilizes (Σ), and that for any C^∞ function $r_i(x)$, $r_i(x) > 0\ \forall x \in \mathbb{R}^n \setminus \{0\}$.

The proof of this theorem uses the same steps as in [GA,BO].

Before stating the main theorem, we need the following assumptions :

$(P.3)$ There exists an observer $\mathcal{O}$ for (Σ) :

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}) + \sum_{i=1}^m u_i g(\hat{x}) + K(t)(h(\hat{x}) - y) \\ \dot{K}(t) = P(K(t), u(t), y(t), \hat{x}(t)) \quad \text{with } \hat{x} \in \mathbb{R}^n, \end{cases}$$

$\mathcal{O}$ converges exponentially for any input $u \in L^\infty(\mathbb{R}), \|u\|_{L^\infty} \leq r$ and the gain $K(t)$ is uniformly bounded (Note that the boundedness of the gain $K(t)$ is always satisfied in the litterature).

$(P.4)$ The system (Σ) satisfies the conditions of theorem 1 and the $r_i(x)$'s are chosen such that $|u_i(x)| \leq r$.

Consider the following system :

$$(G) \quad \begin{cases} \dot{x} = f(x) + \sum_{i=1}^m u_i(\hat{x})g_i(x) \text{ with } u_i(\hat{x}) = -r_i(\hat{x})L_{g_i}V(\hat{x}) & (1) \\ \dot{\hat{x}} = f(\hat{x}) + \sum_{i=1}^m u_i g_i(\hat{x}) + K(t)(h(\hat{x}) - h(x)) & (2) \\ \dot{K}(t) = P(K(t), u(t), y(t), \hat{x}(t)) & (3) \end{cases}$$

We obtain the main result :

Theorem 2 (separation principle) :

With the hypotheses (P.3) and (P.4), if, moreover :

(P.5) For any compact K of $\mathbb{R}^n$, there exists a compact $K'(K \subset K')$ such that for any trajectory of (G) which are issued from $K \times K \times \mathbb{R}^n$, its components $x(t)$ and $\hat{x}(t)$ lie in $K' \times K'$.

Then the set $\{0\} \times \{0\} \times \mathbb{R}^n$ is an attractor of any trajectory of (G) issued from $K \times K \times \mathbb{R}^n$. Moreover the subsystem formed by (1) and (2) is globally asymptotically stable.

In what follows, we give a sufficient condition for which the system (G) satisfies (P.5). For this, recall that a Lyapunov function for an autonomous system $\dot{x} = F(x)$ at the equilibrium point 0 is a C^1 function :

$W : \mathbb{R}^n \longrightarrow \mathbb{R}^+$ such that :

$i) W(x) = 0 \iff x = 0$

$ii) \lim_{x\to\infty} W(x) = +\infty$

$iii) \frac{\delta W(x)}{\delta x}(x).F(x) < 0,\ \forall x \neq 0.$

$(\frac{\delta W(x)}{\delta x}(x)$ is a one-line vector and $F(x)$ is a one-column vector.)

Remarks :

1) The above function $V(x)$ is not, in general, a Lyapunov function.

2) By J.L. Massera's theorem [MA], any C^1 globally asymptotically stable system admits a Lyapunov function.

Using the same notations as above, and denoting by (Σ_C) the closed loop system $\dot{x} = f(x)+\sum_{i=1}^{m} u_i(x)g_i(x)$ with $u_i(x) = -r_i(x)L_{g_i}(V(x))$ we have :

Proposition 3 :

The condition (P.5) of theorem 2 holds if one of the following assumption is satisfied :

$(\mathcal{A}_1)$: The above function $V(x)$ verifies : $\|\frac{\partial V}{\partial x}\| \leq \lambda_1(1+|V(x)|), \ \forall x \in \mathbb{R}^n, \ \lambda_1 > 0$

$(\mathcal{A}_2)$:
i) There exists a Lyapunov function $W(x)$ for Σ_C which satisfies : $\|\frac{\partial W}{\partial x}\| \leq \lambda_2(1+|W(x)|), \ \forall x \in \mathbb{R}^n, \ \lambda_2 > 0$
ii) The output function $h(x)$ is globally Lipschitzian.

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