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THE PARTIAL NON-INTERACTING PROBLEM : GEOMETRIC AND STRUCTURAL SOLUTIONS

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The aim of this paper is to propose a new contribution in the domain of the famous Decoupling Problem for Linear Time-Invariant systems, directly inspired from [D.L.M.83] and [M.M.92] . We introduce here the so-called k^{th} - order Partial Non Interacting Problem (PNIP(k)):

k^{th} - order Partial Non-Interacting Problem : Let $k \in \mathbb{N}$ be given, and let us consider a m-inputs, p-outputs (C,A,B) system , with n-dimensional state space $\mathcal{X}$ and described by : $\dot{x}(t)=Ax(t)+Bu(t)$, $y(t)=Cx(t)$. Does there exist a regular static state feedback law : $u = Fx + Gv$ (with G invertible), such that the k first Markov parameters of the closed loop system, $C(A+BF)^{\mu-1}BG$, have all their non-diagonal elements equal to zero ? Note that some diagonal elements of the corresponding matrices may be zero ; moreover we do not require any full row rank property for the considered system.

This problem is interesting for several reasons :

* it is a weakened version of the famous Decoupling Problem, which corresponds to "infinite order" Partial Non Interaction : indeed, our solution brings back to the classical well-known results for that particular case. When Exact (regular) Decoupling is not solvable, our procedure gives more information on this pathology (typically we are able to know from which step the inherent couplings of the system cannot be cancelled).

* a similar problem has been introduced in the early 80's [E.S.80], related to Partial Model Matching, and its geometric and structural solutions considered in [M.M.92], with an interesting application in the field of systems with delays [M.R.92] : the present results will be a starting point for the study of the existence of non-anticipatory solutions for the decoupling problem of linear systems with delays ; indeed, as in [M.R.92], we can consider, for these systems, the non interacting problem with fixed (finite) horizon k.

We first extend the result initially stated by [F.W.67]. Let n'_i denote the order of the zero at infinity of the i^{th} row-subsystem (c_i,A,B) , i.e. $n'_i =: \mathrm{Inf}\ \{j \text{ such that } c_jA^{j-1}B\neq 0\}$ and let D_k denote the matrix formed by the rows $c_iA^{n'_i-1}B$, but only for $n'_i \leq k$. Our first result is :

Theorem 1 : Let k and (C,A,B) be given. The PNIP(k) has a solution if and only if D_k is epic.

Our next result is a structural "translation" of Theorem 1. Let $\{n_1, n_2, \ldots, n_r\}$ be the global structure at infinity of (C,A,B), where r denotes the rank of (C,A,B). This structure can be described through various ways, including the so-called Smith Mc Millan form at infinity or various geometric supports (see [C.D.82], [D.L.M.83],...).

Theorem 2 : Let k and (C,A,B) be given. The PNIP(k) has a solution if and only if the set of all the n_i which are smaller than or equal to k is equal to the set of all the n'_i which are smaller than or equal to k.

Our third result is an extension of [M.W.71] and gives a geometric equivalent for the previous Theorems. Let us consider the following Invariant Subspace Algorithms :

$$\begin{cases} \mathcal{T}_i^0 = \mathcal{X} \\ \mathcal{T}_i^{\mu+1} = \bigcap_{j\neq i} \mathrm{Ker}\ c_j \cap A^{-1}\ (\mathrm{Im}B + \mathcal{T}_i^{\mu}) \end{cases}$$

Theorem 3 : Let k and (C,A,B) be given. The PNIP(k) has a solution if and only if :

$$\mathrm{Im}B = \sum_{i=1}^{p} \mathrm{Im}B \cap \mathcal{T}_i^{\mu}, \text{ for all } \mu \le k.$$

As an important consequence, we derive the following :

Corollary : The Exact Decoupling Problem is equivalent to the PNIP(k) for k = sup $\{n'_i\}$.

Our future extensions will concern the study of linear systems with delay, but also the solution of the similar problems with internal stability requirement, which is of course demanded for applications. Our presentation here is limited to the row-by-row non interaction (or decoupling), but it is quite direct to extend Theorems 2 and 3 to the block case : indeed, most of our proofs are directly inspired from [D.L.M.83], which concerns the block case.

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