Margins and Bandwidth Limitations of NMP SISO Feedback Systems

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Abstract

Equations and graphs in order to evaluate the limitations and tradeoff between extreme cross over frequencies and gain and phase margins of an important class of open loop non minimum-phase transfer functions, as a function of the right half plane zeros or poles, are given.

1 Introduction

It is well known that the benefit of feedback for non-minimum-phase (NMP) plants is limited. This NMP phenomenon appears when the plant has right-half-plane (RHP) zeros, pure delay or if the open loop includes sampling. Classic examples includes flight control (elevation to aft δ_e control and elevation to throttle command as measured close to the aircraft center of gravity), and the inverted pendulum. Notable examples include: Sidi (1976) and Horowitz and Sidi (1978) who presented an optimal robust synthesis technique to design a feedback controller for an uncertain NMP plant to achieve a given closed loop performance. Their synthesis technique provides the designer with insight into the trade-off between closed loop performance and bandwidth, and also defines an implicit criterion for determining whether a solution exists. Sidi (1980) developed a criterion to estimate the maximum bandwidth of a sampled plant for given gain and phase margin. He assumed open loop transmission of the ideal Bode characteristics form and used asymptotic approximations. Horowitz and Liau (1984) extended this technique to stable plants with several RHP zeros. They showed how to achieve a large loop transmission in several frequency ranges, although there will always be some frequency ranges which are determined by the RHP zeros, in which the loop transmission must be less than $0dB$. This known fact was proven by Francis and Zames (1984) and by Freudenberg and Looze (1985) who showed that for NMP plants, a small sensitivity in one frequency range forces a large sensitivity in the complementary range. Freudenberg and Looze (1985, 1987) developed several constraints on the closed loop sensitivity of NMP and/or unstable plants in the form of weighted integrals of the sensitivity in log scale on all frequencies or on a frequency range where the open loop is much less than 1. Middleton (1991) used their results to provide a bandwidth limitation on NMP

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and/or unstable plants. Sidi (1997) used a different approach based on the Bode relationships to obtain graphical results on Bandwidth limitations for plants containing a single RHP zero or one unstable pole.

The purpose of this paper is to develop sensitivity limitations and design trade-offs for NMP plants (including unstable plants) using the notion of Phase Margin (PM), Gain Margin (GM) and cross-over frequency. The results of Sidi (1997) are extended for plants containing multiple RHP zeros and/or unstable poles. These limitations are developed now for systems whose type is at least 1. Type 0 stable systems can, using low enough gain, have any GM, thus type 0 systems will not be discussed.

In Fig. 1 the classical definitions of GM, M_H, PM, ϕ, cross over frequency, ω_ϕ, and GM frequency ω_M, are defined.

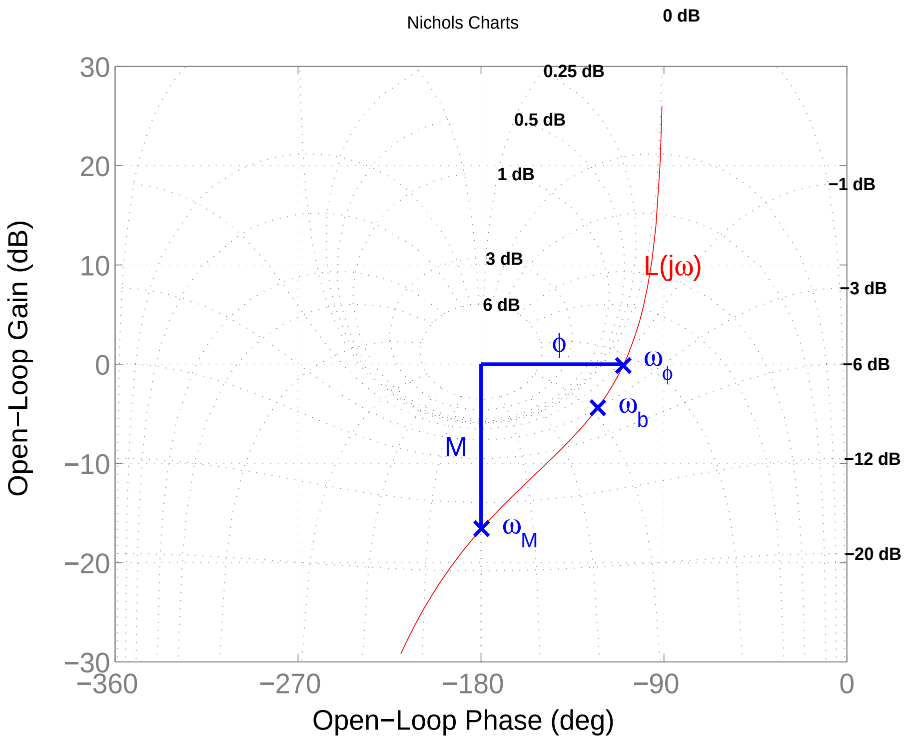

Figure 1: The classical definitions of GM, M, PM, ϕ, GM frequency, ω_M, and cross-over frequency, ω_ϕ

2 GM and PM Limitations for Stable NMP Plants

2.1 Stable plants with a single RHP zero

Let $L(s) = L_M(s)A(s)$ denote an open loop transfer function where $L_M(s)$ is minimum-phase and $A(s)$ is an all-pass transfer function. Based on the celebrated Bode relations which relate the amplitude and phase of minimum-phase transfer functions, and the practical criterion that the bandwidth of the controller should be bounded, we make the following assumption.

Assumption 2.1 *In the vicinity of the cross over frequency, ω_ϕ, and at least up to the GM* *frequency, ω_M, the minimum-phase transfer function, $L_M(s)$, can be approximated by*

$$L_M(s) \approx \frac{k}{s^{2\alpha}} \tag{1}$$

Note that assumption 2.1 is satisfied to a very good approximation in realistic problems and is justified by numerical examples in subsection 2.3 (see also Sidi (1997)).

Under assumption 2.1, if $L(s)$ includes a single RHP zero at a, then

$$\arg L(j\omega) = -\alpha\pi - 2\tan^{-1}(\omega/a); \quad \omega_\phi \leq \omega \leq \omega_M, \tag{2}$$

which yields

$$\omega_{a\phi} \stackrel{\text{def}}{=} \frac{\omega_\phi}{a} = \tan\frac{(1-\alpha)\pi-\phi}{2} \tag{3}$$

$$\omega_{aM} \stackrel{\text{def}}{=} \frac{\omega_M}{a} = \tan\frac{(1-\alpha)\pi}{2}. \tag{4}$$

From equations (3,4) and assumption 2.1, the high frequency GM is

$$M_H = \left[\frac{\omega_M}{\omega_\phi}\right]^{2\alpha} = \left[\frac{\tan\frac{\alpha\pi+\phi}{2}}{\tan\frac{\alpha\pi}{2}}\right]^{2\alpha}. \tag{5}$$

Also from equations (3,4)

$$\frac{\omega_{aM}}{\omega_{a\phi}}\omega_{a\phi} = \frac{\omega_M}{\omega_\phi}\omega_{a\phi} = \frac{\omega_{a\phi}+\tan(\phi/2)}{1-\omega_{a\phi}\tan(\phi/2)} \tag{6}$$

Equation (5) for M_H and the PM, ϕ, as a function of the slope, α, can be solved numerically, its solution is presented graphically in Fig. 2.

The results in Fig. 2 are a good practical estimation to the relation between its parameters. For a quantitative discussion see subsection 2.3

One reasonable approximate to the results in Fig. 2 is the hyperbolic expression

$$\frac{\omega_\phi}{a} = \frac{k_1(\phi)}{M_H + k_2(\phi)}. \tag{7}$$

A good approximation for k_1 and k_2 in the range $\phi = 30^\circ - 45^\circ$ and $M_H = 4 - 12dB$ (M_H in dB units and ϕ in degree units) is

$$\frac{\omega_\phi}{a} = \frac{0.02\phi + 1.6}{M_H - 0.026\phi - 0.24} \tag{8}$$

<u>Example 1:</u> For the following two plants, use loop shaping to achieve maximum ω_ϕ for PM, 45°, and GM, $10dB$.

$$P_1(s) = \frac{1}{s}\frac{3-s}{8+s}; \quad P_2(s) = \frac{1}{s^2}\frac{3-s}{8+s}.$$

<u>Solution:</u> The shaped open loops for P_1 and P_2, respectively, are

$$L_1(s) = \frac{2.5(3-s)(s+0.5)}{s(s+8)(s+0.1)}$$

$$L_2(s) = \frac{2.5(3-s)(s+0.4)}{s^2(s+8)}$$

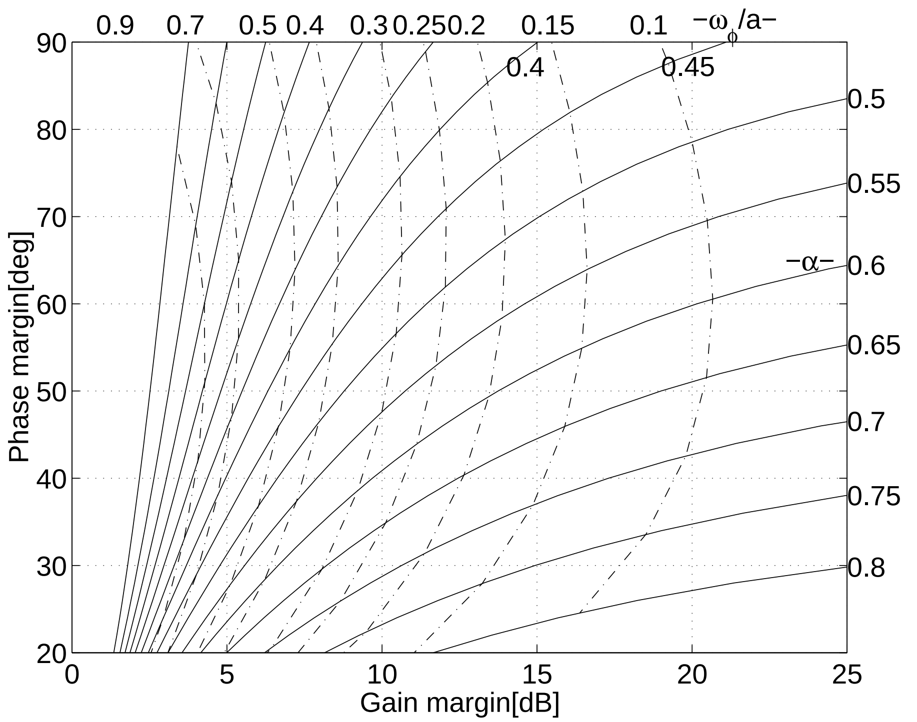

Figure 2: Typical plots of PM versus GM, also shown constant contours of ω_ϕ/a and α

for which $\omega_\phi = 1$ which is about 10% more than the estimated value in Fig. 2 and equation (8). The reason is that assumption 2.1 was violated by slightly increasing the phase of $L_M(j\omega)$ in frequencies larger than ω_ϕ. Nichols and Bode plots of $L_1(s)$ and $L_2(s)$ are given in Fig. 3 and Fig. 4, respectively. Note that adding more integrators to the open loop without violating the phase margin will keep ω_ϕ almost unchanged (but it increases the benefit of feedback below the cut-off frequency, ω_ϕ).

2.2 Extension to several RHP zeros

NMP plants with only real RHP zeros: A reasonable estimation for the relation between GM, PM and cross over frequency can be achieved by replacing the RHP zeros by an equivalent single RHP zero whose phase is the first order approximation of the original RHP zeros. A simple formula is derived as follows: Let the RHP zeros be located at $z_1, \cdots, z_n$, the zero z which replaces them is found from the following low frequency first order approximation

$$\arg \frac{1 - s/z_1}{1 + s/z_1} \cdots \frac{1 - s/z_n}{1 + s/z_n} \approx \arg \frac{1 - s/z}{1 + s/z}.$$

The argument due to all RHP zeros and all LHP poles at some ω is

$$\arg[A(s)]_{s=j\omega} = m\pi - 2 \sum_{i=1}^{i=m} \arctan \frac{\omega}{z_i}.$$

But since the smallest RHP zero, let say z_1, is the important one, and the additional RHP zeros can only aggravate the stability problem at lower frequencies, then at the frequency range $\omega < z_i$

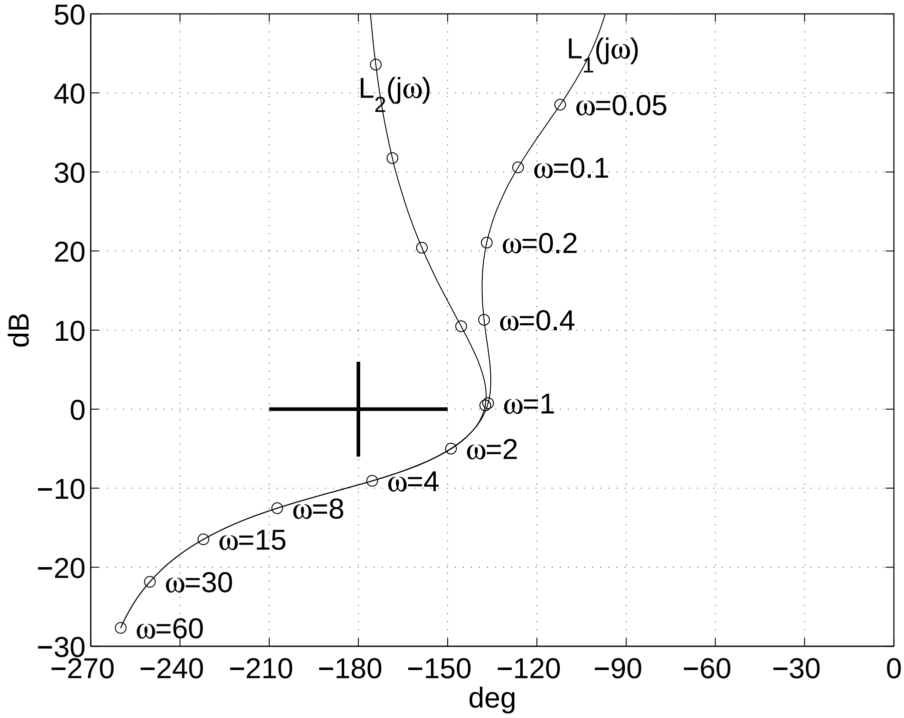

Figure 3: Nichols of $L_1(s)$ and $L_2(s)$

it holds

$$\arctan\frac{\omega}{z_i} \approx \frac{\omega}{z_i}$$

therefore

$$\arg[A(s)]_{s=j\omega} \approx m\pi - 2\sum_{i=1}^{i=m}\arctan\frac{\omega}{z_i} = m\pi - 2\frac{\omega}{z_i},$$

hince

$$\frac{1}{z} \approx \frac{1}{z_1} + \cdots + \frac{1}{z_n}. \tag{9}$$

The reason for choosing this approximation is that the frequency range in which all the parameters involved is in a range where the linear approximation of $\tan\omega/z_i \approx \omega/z_i$ is applicable. This result can be exemplified with the following open-loop transfer functions:

$$L_1(s) = \frac{-2.35(s-2)(s+0.4)}{s(s+0.1)(s+8)}, \quad L_2(s) = \frac{3.8(s-3)(s-6)(s+0.35)}{s(s+0.1)(s+8)(s+15)}$$

where the equivalent NMP zero of the RHP zeros of L_2 is at 2 (by approximation of equation (9)). Both transfer functions are shown in Fig. 5 for which the GM is $10.2dB$ and the PM is $45°$. From Fig. 2, $\omega_\phi/a = 0.3$ thus by the approximation $\omega_\phi = 2 \times 0.3 = 0.6$ is expected. By loop shaping of $L_1(j\omega)$, $\omega_\phi = 0.65$ and of $L_2(j\omega)$, $\omega_\phi = 0.67$ where obtained, thus equation (9) is a very good approximation.

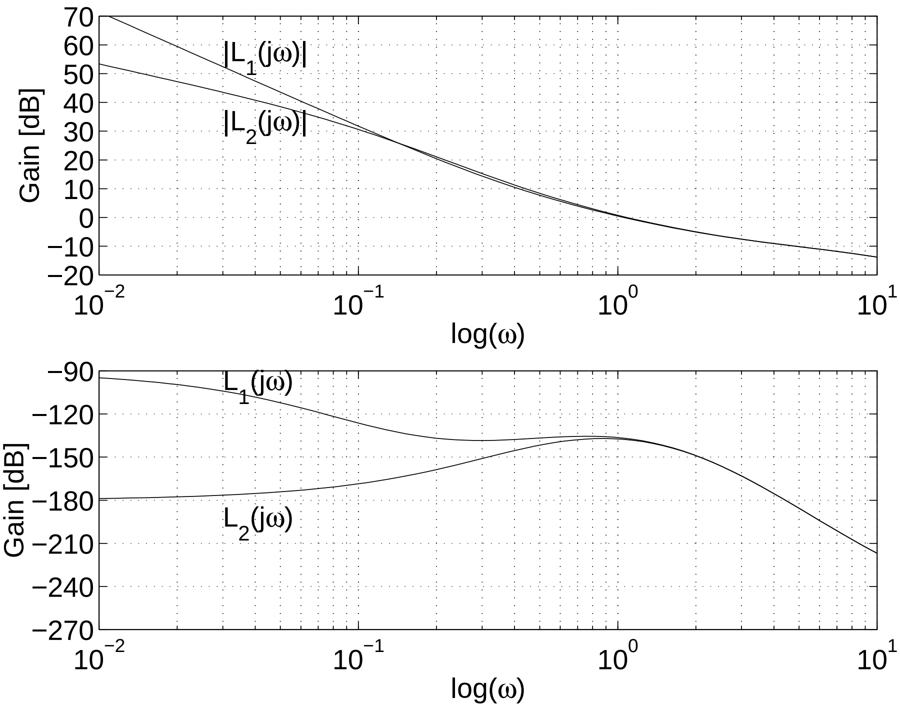

Figure 4: Bode of $L_1(s)$ and $L_2(s)$

NMP plants with highly under-damped RHP zeros: Under assumption 2.1, $L(s)$ has a simple property, as the damping factor tends to zero its Nichols plot converges to the simple 3 straight lines structure of Fig. 6. Hence, the relation between the GM, M_H, PM, ϕ, ω_M and cross-over frequency, ω_ϕ, for a highly under-damped RHP complex zero at a converges, as $\xi \to 0$, to

$$\begin{aligned} \phi &= \pi(1-\alpha) \\ \omega_M &= a \\ \log \frac{\omega_\phi}{a} &= \frac{M_H/20}{2\alpha}. \end{aligned}$$

2.3 Discussion of the applicability of Assumption 2.1

It is possible that the relations (summarized in Fig. 2) between GM, PM, and ω_ϕ/a are violated if assumption 2.1 is violated. In order to check the results for transfer functions that deviate strongly from assumption 2.1, we performed the following comparison: Eight open-loop transfer functions which on the Nichols chart, at frequencies higher than $\omega_{a\phi}$, were shaped such that (i) the unity feedback closed loop contours are $0, 1, 3, 6dB$ in Fig. 7a for $\omega > \omega_\phi$; and (ii) the same transfer functions having all the same GM, $6dB$, with PM of $60, 54.2, 45.3, 34$ deg in Fig. 7b.

The results obtained are summarized in the tables below. The difference in ω_ϕ between the one obtained based on assumption 2.1 (given in Fig. 2) and the results given in the tables is less than 16%. This error is a result of large deviations from assumption 2.1, which means that α is not kept constant, moreover $\alpha \to 0$ at frequencies not much larger than ω_ϕ, which is not a practical realization.

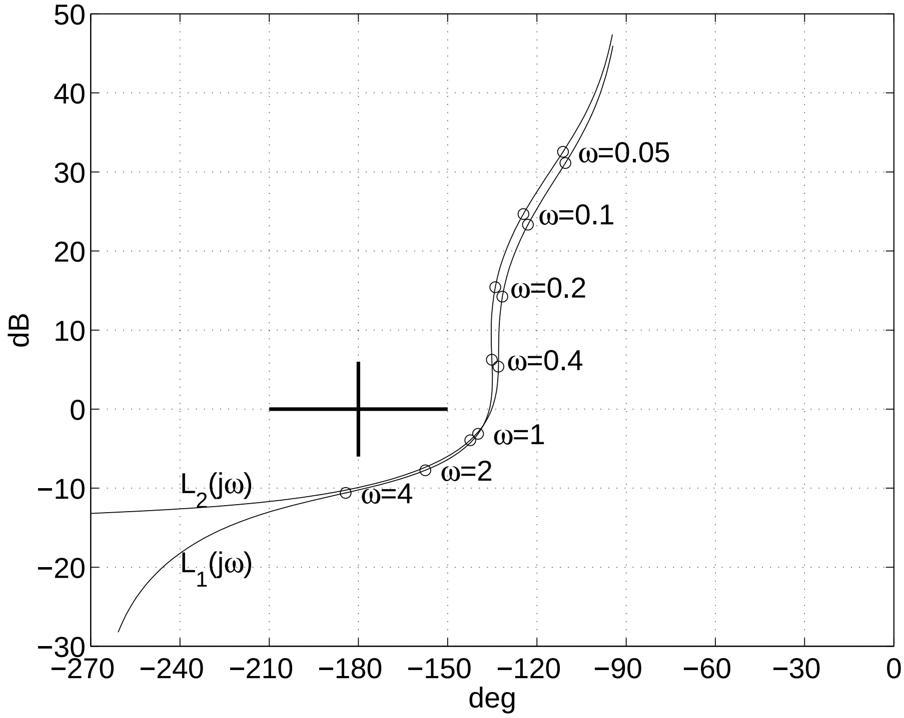

Figure 5: Nichols graphs for two TFs containing one and two NMP zeros

Errors for approximated ω_ϕ of $L_1(s)$ to $L_4(s)$ of Fig. 7a				
	$L_1 = \frac{-0.5(s-3)(s+0.1)}{s(s+0.1)}$	$L_2 = \frac{-0.52(s-3)(s+0.28)}{s(s+0.1)}$	$L_3 = \frac{-0.575(s-3)(s+0.55)}{s(s+0.1)}$	$L_4 = \frac{-0.65(s-3)(s+0.95)}{s(s+0.1)}$
$\|T\|dB$	0	1	3	6
$GM[dB]$	6	5.68	4.81	3.74
$PM[deg]$	60	52.0	42.2	29.7
ω_ϕ	1.74	1.86	2.2	2.8
ω_ϕ (Fig. 2)	1.85	1.93	2.17	2.36
error ω_ϕ %	6.6	3.5	1.1	16

Errors for approximated ω_ϕ of $L_1(s)$ to $L_4(s)$ of Fig. 7b				
	$L_1 = \frac{-0.5(s-3)(s+0.1)}{s(s+0.1)}$	$L_2 = \frac{-0.5(s-3)(s+0.28)}{s(s+0.1)}$	$L_3 = \frac{-0.5(s-3)(s+0.55)}{s(s+0.1)}$	$L_4 = \frac{-0.5(s-3)(s+0.95)}{s(s+0.1)}$
$GM[dB]$	6	6.1	6	6
$PM[deg]$	60	54.2	45.3	34
ω_ϕ	1.74	1.73	1.89	1.97
ω_ϕ (Fig. 2)	1.85	1.81	1.69	1.42
error ω_ϕ %	6.6	4.7	6.0	27

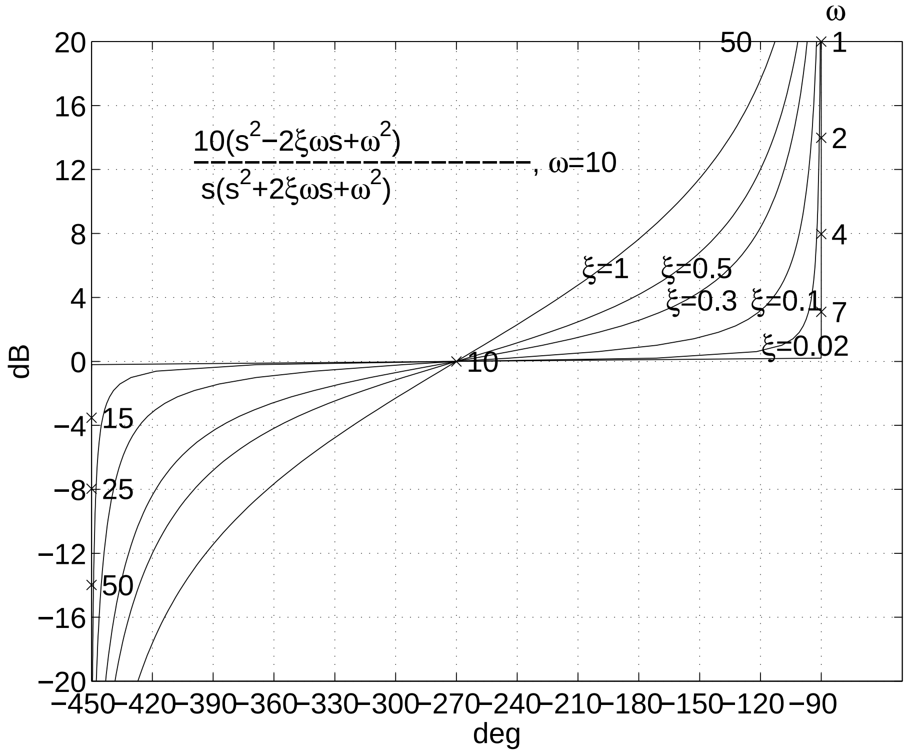

Figure 6: Nichols plot of $\frac{10}{s}\,\frac{s^2-2\xi s\omega+\omega^2}{s^2+2\xi s\omega+\omega^2}$ for several ξ values

3 Margins and ω_ϕ Relations in Unstable Plants

3.1 Statement and origin of the problem

The existence of unstable poles renders the transfer function to be NMP in the sense of Bode. The existence of unstable poles puts a lower bound on the achievable bandwidth. The phase of a single unstable pole located at a is $-\pi + \tan^{-1}\omega/a$ and the phase of an open loop with at least one RHP pole at $\omega = 0$ must be $-k\pi$, therefore it has GM due to crossing of its Nyquist plot by the -180° line above $0dB$ (denoted by M_L), thus there must exist ω_ϕ, but high frequency GM may not exist. As is well known, practical physical systems have at least 2 more poles than zeros, moreover in order to minimize sensor noise amplification, high frequencies must be attenuated. Hence the controller must have more poles than zeros, thus the phase of the open loop at high frequencies must approach at least -270°, therefore ω_M exists. The problem considered here will then be: Given low frequency GM, M_L, high frequency GM, M_H, PM, ϕ and the system's RHP poles; find the minimal ω_ϕ. A graphical representation of all the parameters involved, i.e., ϕ, M_L, M_H, ω_ϕ and ω_M is given in Fig. 8.

3.2 Unstable plants with a single RHP pole

The simplest unstable open loop transmission with a single RHP pole and finite GM will be used for our discussion, it is of the form (same form was used by Sidi 1997)

$$L(s) = \frac{k}{s/a - 1}\,\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{10}$$

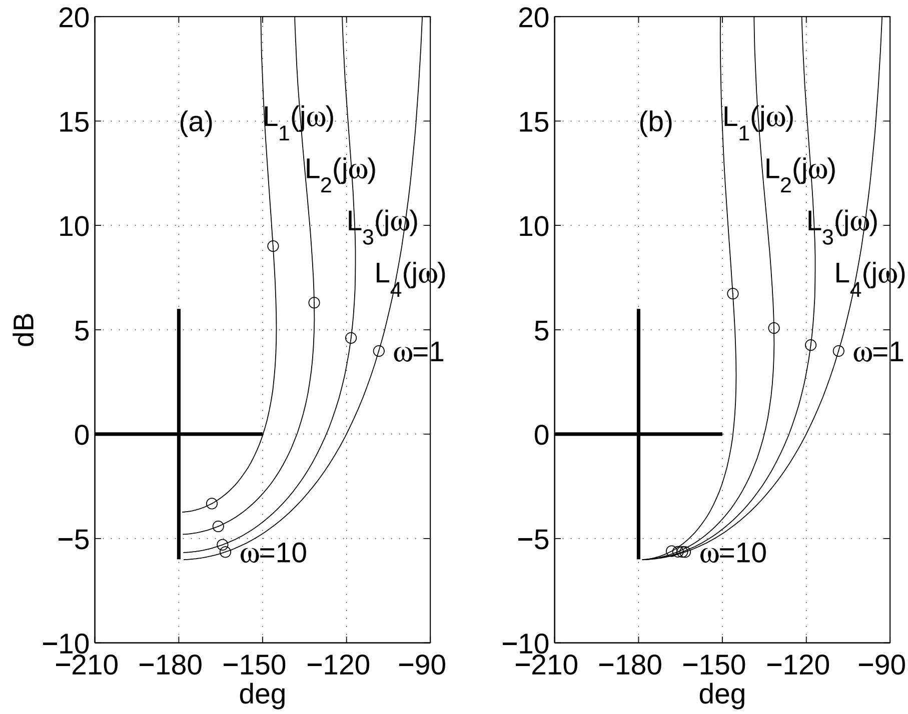

Figure 7: Nichols plots for NMP TFs $L(s)$ for which the closed loop $|L/(1+L)|$ is close to $0, 1, 3, 6dB$ (a), and same TFs as per (a) but gain change for GM $6dB$ (b)

Without loss of generality we can normalize the pole such that $a = 1$ (if $a \neq 1$ the following equations and results are true where s is replaced by s/a, ω by ω/a and ω_n by ω_n/a). Using standard complex numbers arithmetic (M_L in arithmetic units):

$$|L(j\omega)| = \frac{M_L \omega_n^2}{\sqrt{1+\omega^2}\sqrt{(\omega^2-\omega_n^2)^2+(2\xi\omega_n\omega)^2}} \tag{11}$$

$$\arg L(j\omega) = -\pi + \tan^{-1}\omega - \tan^{-1}\frac{2\xi\omega_n\omega}{\omega_n^2-\omega^2}. \tag{12}$$

Equation (11) for the cross over frequency ω_ϕ ($|L(j\omega_\phi) = 1|$) yields

$$M_L = \sqrt{1+\omega_\phi^2}\sqrt{(\omega_\phi^2-\omega_n^2)^2+(2\xi\omega_n\omega_\phi)^2}/\omega_n^2. \tag{13}$$

Note that from stability considerations $M_L > 1$, hence the following inequality must hold

$$\left((\omega_\phi^2-\omega_n^2)^2+(2\xi\omega_n\omega_\phi)^2\right)(1+\omega_\phi^2) > \omega_n^2 \tag{14}$$

Equation (12) for the high frequency GM ($\arg L(j\omega_M) = -\pi$) yields

$$-\pi = -\pi + \tan^{-1}\omega_M - \tan^{-1}\frac{2\xi\omega_n\omega_M}{\omega_n^2-\omega_M^2}$$

which gives

$$\omega_M = \sqrt{\omega_n^2 - 2\xi\omega_n}. \tag{15}$$

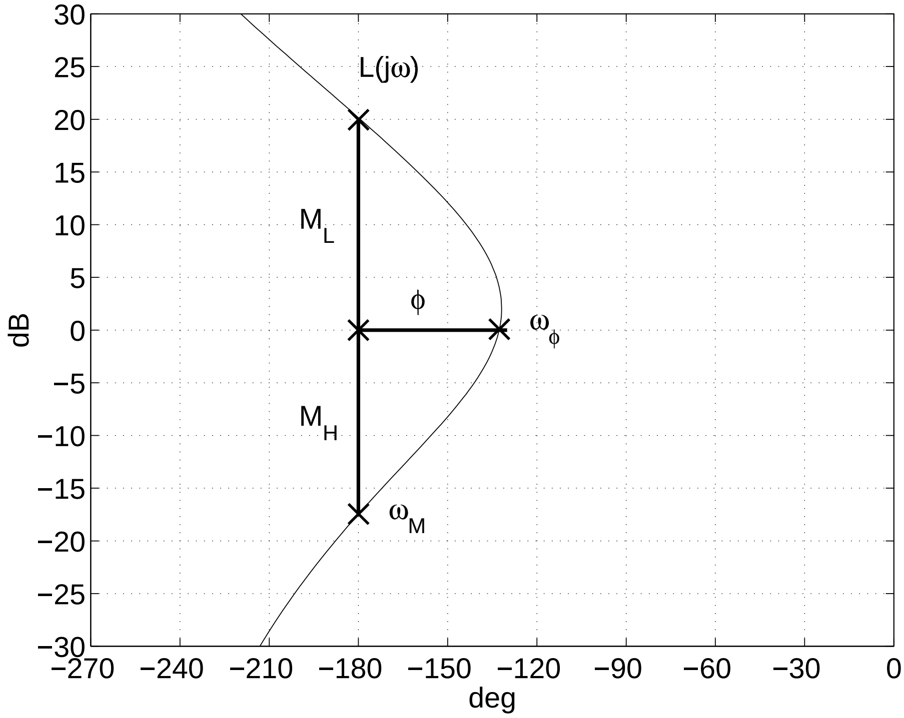

Figure 8: Definition of the margins M_H, M_L, ω_ϕ and ω_M

Substituting equation (15) in equation (11) gives (M_H in arithmetic units),

$$\frac{1}{M_H} = \frac{\omega_n M_L}{2\xi(1+\omega_n^2-2\xi\omega_n)}. \tag{16}$$

Note that from stability considerations $M_H > 1$, hence the following inequality must hold

$$\frac{\omega_n M_L}{2\xi(1+\omega_n^2-2\xi\omega_n)} < 1. \tag{17}$$

Equation (16) and equation (13) links between the GMs, M_L, M_H and the cross over frequency, ω_ϕ.

The next important relationship is related to the PM, ϕ. By equation (12) it satisfies

$$-\pi+\phi = -\pi+\tan^{-1}\omega_\phi - \tan^{-1}\frac{2\xi\omega_n\omega_\phi}{\omega_n^2-\omega_\phi^2}, \tag{18}$$

$$\frac{2\xi\omega_n\omega_\phi}{\omega_n^2-\omega_\phi^2} = \frac{\omega_\phi-\tan\phi}{1+\omega_\phi\tan\phi}. \tag{19}$$

By use of equations (13,16,19) we can get a set of graphical relationships between the PM, ϕ, GMs M_L, M_H and ω_ϕ for different ξ's. First we differentiate equation (12) in order to obtain the maximum argument of $L(j\omega)$ to be at $\omega = \omega_\phi$. The result is the solution of the following $4th$ order equation

$$(1-2\xi\omega_n)\,\omega_\phi^4 + \left(-2\omega_n^2+4\xi^2\omega_n^2-2\xi\omega_n^3-2\xi\omega_n\right)\omega_\phi^2 + \left(\omega_n^4-2\xi\omega_n^3\right) = 0.$$

Having found ω_ϕ as a function of ω_n we get the PM by equation (18), and M_L and M_H by equation (13) and equation (16), respectively. These relations are shown in Fig. 9 for $\xi =$

0.5 via ω_ϕ/a, ϕ, M_H and M_L as a function of ω_n/a. Fig. 10 shows the resulting open loop transmissions on the Nichols chart for $\omega_n/a = 3.65, 5.3, 8.3, 14.5, 33$ and 100 with respective PM of $30, 40, 50, 60, 70$ and 80deg.

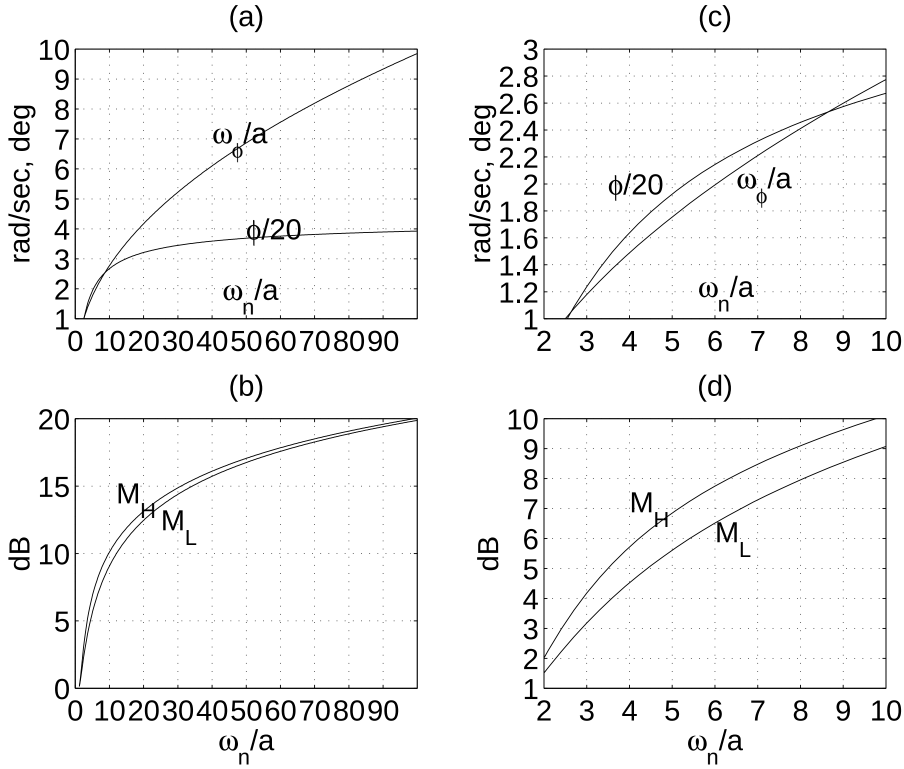

Figure 9: PM, ϕ and upper and lower GMs, M_H, M_L versus ω_n/a for $\xi = 0.5$

3.3 Application of the results in practical problems

In practical feedback control problem, we are interested in reducing ω_n/a as much as possible, in order to minimize the sensor's noise amplification at the plant input, see Horowitz (1963) and Horowitz and Sidi (1972). The results in Fig. 9 can be used to find what the constraints posed by the open loop RHP pole are. For instance, let us suppose that we need PM of $\phi = 40°$ then from Fig. 9, $\omega_n/a = 5.3$ is the smallest value that can be used, for which (from the same graph) $\omega_\phi/a = 1.8$, $M_L = 5.8dB$ and $M_H = 7.2dB$. These results are confirmed in the Nichols chart of Fig. 10 on which are shown some $L(j\omega)$ of equation (10).

Practical open-loop transfer functions will be more complicated than the assumed form here, i.e., will contain more poles and zeros than equation (10). However, optimal shaping of the open-loop transfer function will have the basic margin characteristics shown for the simplified structure of equation (10) given in Fig. 10, for which the results in Fig. 9 exactly hold.

3.4 Extension to several RHP poles

NMP plants with only real RHP poles: A reasonable estimation for the relation between GM, PM and cross over frequency can be achieved by replacing the RHP poles by a single equivalent RHP pole whose phase is the first order approximation of the original RHP poles in high

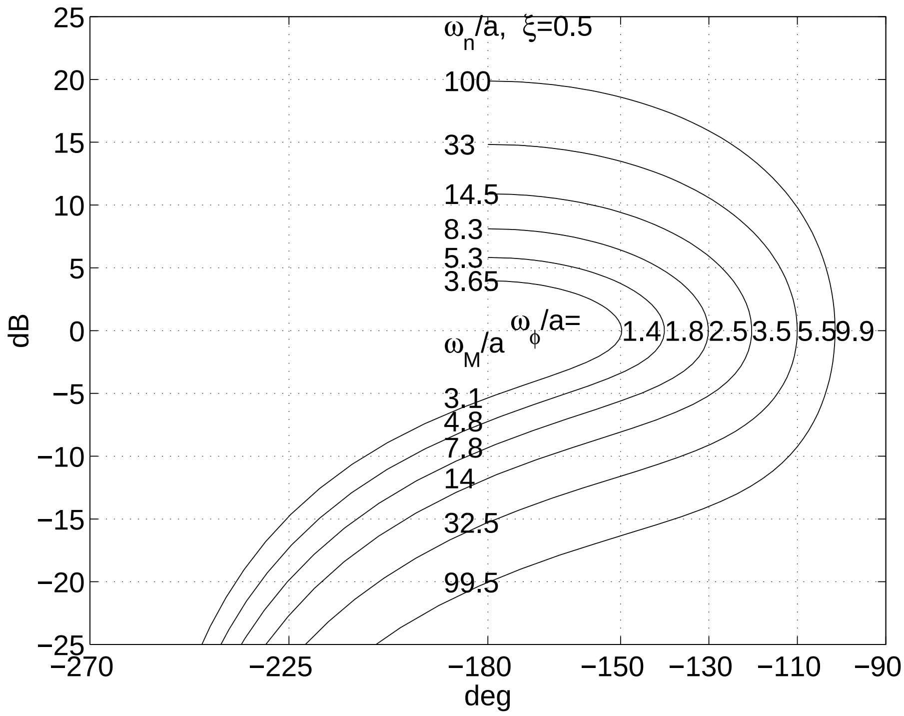

Figure 10: Several Nichols plot which confirm the relations of Fig. 9

frequencies. This is because high frequencies are the region that dominate the bandwidth equations. A simple formula is derived as follows: Let the RHP poles be located at $p_1, \cdots, p_n$. The argument of the all-pass due to all RHP poles and all LHP zeros at some ω is

$$\arg[A(s)]_{s=j\omega} = -m\pi + 2\sum_{i=1}^{i=m} ctan^{-1}\frac{p_i}{\omega}.$$

But since the largest RHP pole, let say p_1, is the important one, and the additional RHP poles can only aggravate the stability problem at higher frequencies, then at the frequency range $\omega > p_i$, it holds

$$ctan^{-1}\frac{p_i}{\omega} \approx \frac{p_i}{\omega}$$

therefore

$$\arg[A(s)]_{s=j\omega} \approx -m\pi + 2\sum_{i=1}^{i=m} ctan^{-1}\frac{p_i}{\omega} = -m\pi + 2\frac{p_i}{\omega},$$

and the pole p which replaces them is

$$p \approx p_1 + \cdots + p_n. \tag{20}$$

This approximation is exemplified with the following transfer functions

$$\begin{aligned} L_1(s) &= \frac{8}{(s-4)(s^2/21.2^2 + s/21.2 + 1)}, \\ L_2(s) &= \frac{16(s+2.2)(s+5)}{(s-1)(s-3)(s^2/20^2 + 1.6s/20 + 1)(s+15)}. \end{aligned}$$

In this case the equivalent unstable pole is by equation (20) at 4. Both transfer functions are shown in Fig. 11, whose PM are 40° and $M_H + M_L$ are approximately the same for both cases ($13.5dB$). For $\phi = 40°$ we find by Fig. 9 that $\omega_n/a = 1.85$, so that $\omega_\phi = 7.4$ which is a good approximation for the cross over frequency $\omega_\phi = 7.5$ of $L_2(s)$ and 7.8 of $L_1(s)$, as shown in Fig. 12.

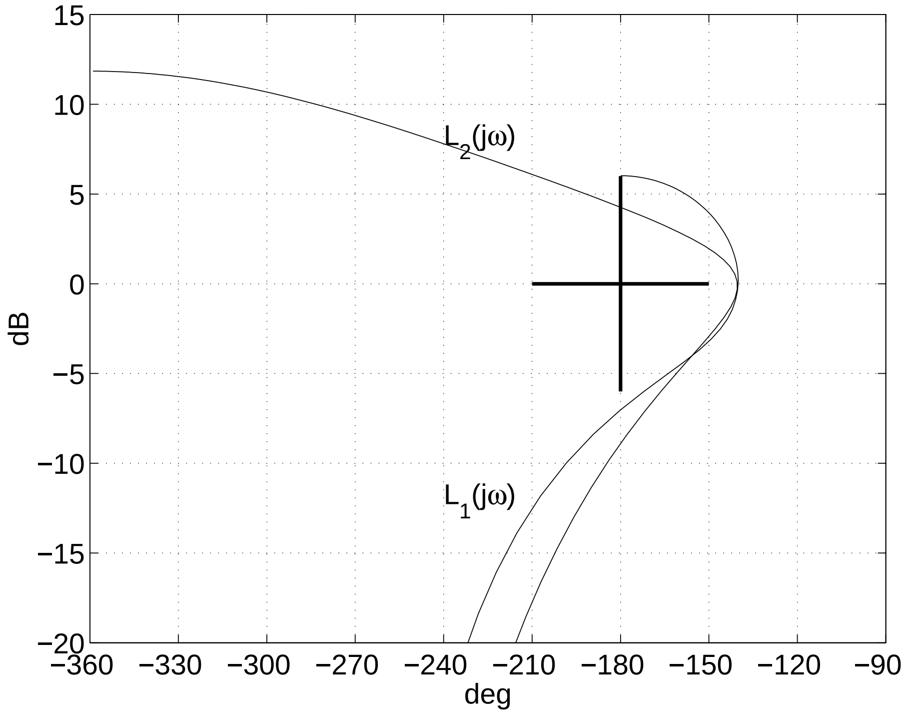

Figure 11: Nichols graphs of a TF containing two NMP poles and a TF containing an equivalent single RHP pole

<u>NMP plants with highly under-damped RHP poles:</u> Under assumption 2.1, $L(s)$ has a simple property, as the damping factor tends to zero, its Nichols plot converges to its minimum phase part for frequencies larger than the under-damped poles, thus can be treated as minimum phase transfer function in that range.

4 Conclusions

The existence of a RHP zero in a plant, limits the achievable bandwidth of the open loop transmission, thus limiting the benefits of feedback with respect to the closed loop sensitivity. This paper gives simple equations and graphs in order to find limitations and tradeoff between the cross over frequency and gain margin and phase margin as a function of the RHP zeros; where the slope of its Bode plot around the cross over frequency is given.

The existence of an unstable pole has an opposite effect, it puts a lower bound on the cross over frequency. Equations and graphs in order to find limitations and tradeoff between the cross over frequency and gain margin and phase margin as a function of the RHP poles, based on a simplified structure of its loop transmission around the cross over frequency are given.

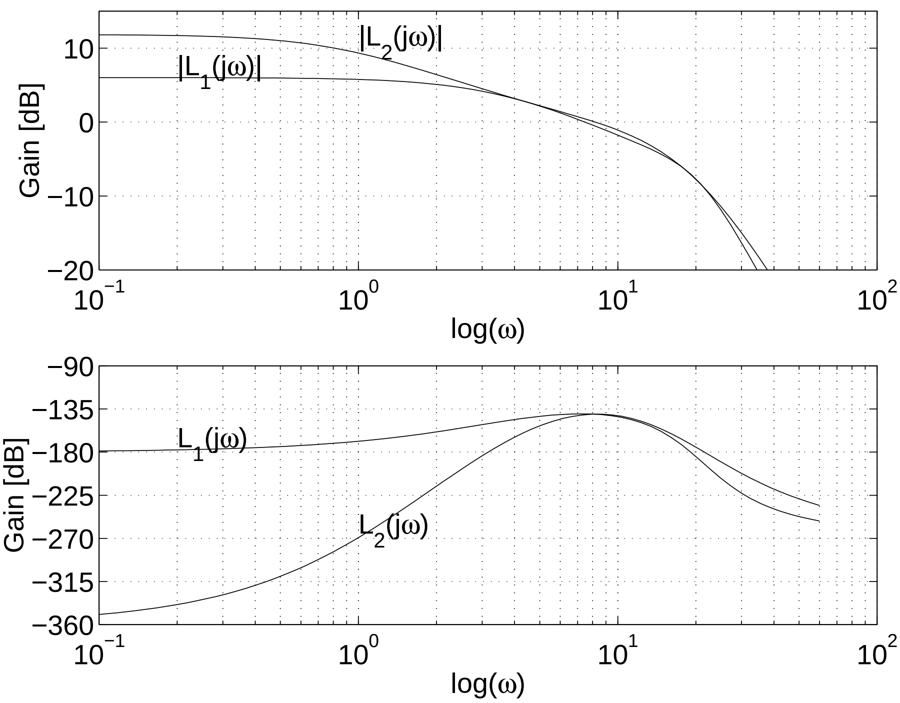

Figure 12: Bode plot of the TF in Fig. 11, a TF containing two NMP poles and a TF containing an equivalent single RHP pole

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