The structure at infinity of linear delay systems and the row-by-row decoupling problem

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Abstract

The row-by-row decoupling problem is studied for linear delay systems. The structural approach is used to design a decoupling precompensator. The realization of the given precompensator by static state feedback is studied. Using various structural and geometric tools, a detailed description of the feedback is given, in particular, derivative of the delayed new reference can be needed in the realization of the precompensator.

1 Introduction

The structure at infinity or the Smith-McMillan form at infinity are well known tools for the characterization of the solvability of some control problems such as model matching, disturbance decoupling, row-by-row decoupling. For linear finite dimensional systems see (Silverman and Kitapçi, 1983) for instance. For linear infinite dimensional systems and in the particular case of bounded operators, the structure at infinity was introduced by Hautus (1975), described in several equivalent ways and used to solve some control problems in (Malabre and Rabah, 1990). The particular case of delay systems was studied in (Malabre and Rabah, 1993). However the structure at infinity defined there is too weak to insure a good solution for control problems: indeed the potential precompensators may be anticipative as it was pointed out in (Sename *et al*, 1995). In (Rabah and Malabre, 1996) we introduced the concept of strong structure at infinity which is more convenient for infinite dimensional systems (and for delay systems as a particular case). This structure is only well defined for some classes of systems. The positive result is that if this structure at infinity is well available then all potential solutions of control problems are non-anticipative and may be realized by static state feedback. Here we use, as in (Rabah and Malabre, 1998) for the disturbance rejection, the weak structure at infinity of the system in order to design a precompensator achieving decoupling, then this precompensator is decomposed in two parts: a strong proper precompensator which may be realized by static state feedback and a weak proper precompensator which can be realized by generalized static state feedback, feedback which contains the derivative of the new control. The results given here are in a general form at least for systems with commensurate delays. If the new control is not smooth enough, then the decoupling problem cannot be solvable by generalized static state

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feedback.
The paper is organized as follows. In Section 2 we describe the delay system considered in the paper and the problem of decoupling. In Section 3 we give basic notions and recall classical results concerning linear systems without delays, then we recall some notions and results for systems with delays in Section 4. In Section 5 we solve the row-by-row decoupling problem for delay systems in a general framework.

2 System description and problem formulation

2.1 System description

We consider the linear time-invariant systems with delays described by:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-1) + B_0 u(t) \\ y(t) = C_0 x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathcal{X} \approx \mathbb{R}^n$ is the state, $u(t) \in \mathcal{U} \approx \mathbb{R}^m$ is the control input, $y(t) \in \mathcal{Y} \approx \mathbb{R}^m$ is the output to be controlled. In order to simplify the notation and some computations, we limit ourself to systems with single delay in the states. All results and considerations given here remain valid for systems with several commensurate delays in the state, the control and the output.

The transfer function matrix of the system (1) is

$$T(s, e^{-s}) = C_0(sI - A_0 - A_1 e^{-s})^{-1} B_0$$

which may be expanded as follows

$$T(s, e^{-s}) \quad = \quad \sum_{j=0}^{\infty} T_j(s) e^{-js}, \tag{2}$$

where

$$T_j(s) = C_0(sI - A_0)^{-1} \left[A_1(sI - A_0)^{-1}\right]^j B_0.$$

Each matrix $T_j(s)$ may be decomposed using the following constant matrices introduced by Kirillova and Churakova and compared with other tools in (Tsoi, 1978):

$$\begin{array}{rcl} Q_i(j) & = & A_0 Q_{i-1}(j) + A_1 Q_{i-1}(j-1), \\ Q_0(0) & = & I, \; Q_i(j) \;=\; 0, \; i < 0 \text{ or } j < 0. \end{array} \tag{3}$$

We have

$$T_j(s) = \sum_{i=0}^{\infty} C_0 Q_i(j) B_0 s^{-(i+1)}.$$

Another expression which will be used in this paper is the following one

$$T(s, e^{-s}) = \sum_{i=0}^{\infty} \left(\sum_{j=0}^{i} C_0 Q_i(j) B_0 e^{-js} \right) s^{-(i+1)}. \tag{4}$$

These expressions may be obtained by a simple calculation using the relations (3), see (Sename *et al*, 1995; Tsoi, 1978).

2.2 Problem formulation

Let be given the system (1). Find a precompensator $K(s, e^{-s})$ such that

$$T(s, e^{-s})K(s, e^{-s}) = \text{diag}\,\{h_1(s, e^{-s}), \ldots, h_m(s, e^{-s})\}$$

and which may be realized by generalized static state feedback of the form

$$u = F(e^{-s})x + G(s, e^{-s})v,$$

without anticipation. That is, the closed loop system, say $T_{F,G}(s, e^{-s})$ is such that:

$$T_{F,G}(s, e^{-s}) = \text{diag}\,\{h_1(s, e^{-s}), \ldots, h_m(s, e^{-s})\}.$$

This means that $F(e^{-s})$ and $G(s, e^{-s})$ may be decomposed as

$$\begin{aligned} F(e^{-s}) &= F_0 + F_1 e^{-s} + F_2 e^{-2s} + \cdots, \\ G(s, e^{-s}) &= G_0 + G_1(s)e^{-s} + G_2(s)e^{-2s} + \cdots, \end{aligned}$$

with (possible) polynomial matrices $G_i(s), i \geq 1$. This assumption allows to give a more general solution for a very large class of delay systems. If the problem is solvable we say that the *row-by-row decoupling problem* is solvable. The corresponding precompensator $K(s, e^{-s})$ is called *realizable* or *causal*.

3 Finite dimensional systems

The basic notion used is this paper is the notion of properness. Let us recall in this section the case of a classical linear system given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{5}$$

where $x(t) \in \mathcal{X} \approx \mathbb{R}^n$ is the state, $u(t) \in \mathcal{U} \approx \mathbb{R}^m$ is the control input, $y(t) \in \mathcal{Y} \approx \mathbb{R}^m$ is the output to be controlled. The transfer function matrix of the system is

$$T(s) = C(sI - A)^{-1}B.$$

The matrix $T(s)$ is rational and strictly proper, the properness being defined by the following.

Definition 3.1 *A complex valued function $f(s)$ is called proper if $\lim f(s)$ is finite when $|s| \to \infty$. It is called strictly proper if this limit is 0. A matrix $B(s)$ is biproper if it is proper and its inverse is also proper.*

As for linear systems in finite dimensional spaces one considers in fact only rational functions, properness means that the degree of the numerator is less than or equal to the degree of the denominator and strictly properness means that the equality cannot hold. A fundamental result is the existence of a canonical form at infinity (Smith-McMillan form at infinity) for strictly proper matrices (but also for general rational matrices).

Theorem 3.2 *There exist (non unique) biproper matrices $B_1(s)$ and $B_2(s)$ such that*

$$B_1(s)T(s)B_2(s) = \begin{bmatrix} \Delta(s) & 0 \\ 0 & 0 \end{bmatrix},$$

where $\Delta(s) = \operatorname{diag}\{s^{-n_1}, \ldots, s^{-n_r}\}$. The integers n_i are called the order of the zeros at infinity and the list of integers $\{n_1, \ldots, n_r\}$ is the structure at infinity *and is noted by $\Sigma_\infty(C, A, B)$ or $\Sigma_\infty T(s)$.*

The structure at infinity allows to describe the behavior of the system at time $t = 0$.

Another important tool which is useful to characterize several properties of linear systems is the maximal (A, B)-invariant subspace contained in $\operatorname{Ker} C$, see (Wonham, 1985). It will be noted by $\mathcal{V}_*(\operatorname{Ker} C, A, B)$. We shall also use the alternative expression of this subspace given by Hautus:

$$\mathcal{V}_*(\operatorname{Ker} C, A, B) = \{x \in \operatorname{Ker} C : x = (sI - A)\xi(s) - B\omega(s)\},$$

with strictly proper ξ and ω such that $\xi(s) \in \operatorname{Ker} C$ for $|s| > s_0$. The following result is well known and established by several authors. Let $\mathcal{B}$ denote the image of B.

Theorem 3.3 *The following propositions are equivalent:*

1. *There exists a biproper precompensator $K(s)$ such that*

$$T(s)K(s) = \operatorname{diag}\{h_1(s), \ldots, h_m(s)\}$$

2. *There exist a feedback law $u = Fx + Gv$, such that*

$$C(sI - A - BF)^{-1}BG = \operatorname{diag}\{h_1(s), \ldots, h_m(s)\}$$

3. *The global structure at infinity is equal to the union of the row's structures at infinity:*

$$\Sigma_\infty(C, A, B) = \begin{bmatrix} \Sigma_\infty(c_1, A, B) \\ \vdots \\ \Sigma_\infty(c_m, A, B) \end{bmatrix},$$

where Σ_∞ denotes the canonical form at infinity for the given system, $c_i,\ i = 1, \ldots, p$ being the rows of the matrix C

4. $\operatorname{Im} B = \sum_{i=1}^{m} \operatorname{Im} B \cap \mathcal{V}_*(\mathfrak{C}_i, A, B)$, *where $\mathfrak{C}_i = \bigcap_{j \neq i}^{m} \operatorname{Ker} c_j$,*

5. *The so-called Falb-Wolovich matrix*

$$D = \begin{bmatrix} c_1 A^{n_1 - 1} B \\ \vdots \\ c_m A^{n_m - 1} B \end{bmatrix},$$

is invertible. The integers $n_i,\ i = 1, \ldots, m$ are the order of the zero at infinity of each row subsystem: $c_i A^{n_i - 1} B \neq 0$ and $c_i A^j B = 0$ for $j < n_i - 1$.

The relation between the precompensator $K(s)$ and the feedback law (F, G) is given by

$$K(s) = \left(I - F(sI - A)^{-1}B\right)^{-1} G. \tag{6}$$

PROOF: For the proofs see for example (Falb and Wolovich, 1967; Wonham, 1985; Commault *et al.*, 1986) and references given there. We need later the proof of the equivallence of the statements 1) and 4). Let us give a direct proof of this equivallence.

If the system is decouplable by precompensator (statement 1)), there exists a biproper precompensator $K(s)$ such that

$$C(sI - A)^{-1}BK(s) = \operatorname{diag}\{h_1(s), \dots, h_m(s)\}.$$

The matrix $K(s)$ may be written as $K(s) = V + W(s)$, where V is a non singular matrix and $W(s)$ is strictly proper. Let v_i and $\omega_i(s)$ be the i-th columns of the matrices V and $W(s)$ respectively. Then $\{v_i, i = 1, \dots, m\}$ forms a basis in $\mathbb{R}^m$. If we take

$$\xi_i(s) = (sI - A)^{-1}B(v_i + \omega_i(s)),$$

then $\xi_i(s) \in \mathfrak{C}_i$. On the other hand $\xi_i(s)$ and $\omega_i(s)$ are strictly proper. Hence, for all $i = 1, \dots, m$ one has

$$Bv_i \in \mathcal{V}_*(\mathfrak{C}_i, A, B)$$

As $\{v_i, i = 1, \dots, m\}$ forms a basis of $\mathcal{U}$ and B is assumed to be monic, then $\{Bv_i, i = 1, \dots, m\}$ is a basis of $\operatorname{Im} B$. Hence (8) holds.

Conversely assume that condition 4) is satisfied. Then for $\{v_i, i = 1, \dots, m\}$ linearly independent, one has

$$Bv_i = (sI - A)\xi_i(s) - B\omega_i(s)$$

with $\xi_i(s), \omega_i(s)$ strictly proper and $\xi_i(s) \in \mathfrak{C}_i$, i.e. $C\xi_i(s) = c_i\xi_i(s)$. For $V = [\, v_1 \quad \dots \quad v_m \,]$ and $W = [\, \omega_1(s) \quad \dots \quad \omega_m(s) \,]$, if we define $K(s) = V + W(s)$, then $K(s)$ is biproper and

$$C(sI - A)^{-1}BK(s) = \operatorname{diag}[c_1\xi_1(s), \dots, c_m\xi_m(s)].$$

Hence the system is row-by-row decoupled by the precompensator $K(s)$ and we have $h_i(s) = c_i\xi_i(s)$. ∎

4 Structural notions for delay systems

The transfer function matrix of a delay system is not rational. Moreover, it is not analytical at infinity. The notions of properness must be precised.

Definition 4.1 *A complex valued function $f(s)$, analytical for $\Re\mathrm{e}(s) \geq s_0$, is called weak proper if $\lim f(s)$ is finite when $s \in \mathbb{R}$ tends to ∞. It is called strictly weak proper if this limit is 0. A matrix $B(s)$ is weak biproper if it is weak proper and its inverse is also weak proper. Weak proper is replaced by strong proper if the same occurs when $s \in \mathbb{C}$ and $\Re\mathrm{e}(s) \to \infty$.*

It is obvious that strong properness implies weak properness. If the function is analytical at infinity both notions coincide, because the limits at infinity are the same.
The strong properness was used in (Hautus, 1975) and (Malabre and Rabah, 1990) in the description of the structure at infinity for infinite dimensional systems. In (Malabre and Rabah, 1993; Sename *et al*, 1995; Rabah and Malabre, 1998) the weak notion was used in order to define the structure at infinity of delay systems and to solve some control problems.

Let us recall the following results using the weak properness and introduced in (Malabre and Rabah, 1993; Rabah and Malabre, 1996).

Theorem 4.2 *There exist (non unique) weak biproper matrices $B_1(s,e^{-s})$ and $B_2(s,e^{-s})$ such that*

$$B_1(s,e^{-s})T(s)B_2(s,e^{-s}) = \begin{bmatrix} \Delta_0(s) & 0 & \cdots & 0 & 0 \\ 0 & \Delta_1(s)e^{-s} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \Delta_k(s)e^{-ks} & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

where $\Delta_i(s) = \mathrm{diag}\{s^{-n_{i,1}}, \ldots, s^{-n_{i,j_i}}\}$ and $n_{i,j} \leq n_{i,j+1},\ i = 1, \ldots, k$. The list of integer

$$\{n_{i,j},\ i = 1, \ldots, k; j = j_1, \ldots, j_i\}$$

is called the weak structure at infinity *of the system $T(s,e^{-s})$ and is noted $\Sigma_\infty^w T(s,e^{-s})$.*

Some additional assumptions may insure that the weak structure at infinity also gives a strong structure at infinity (the biproper matrices B_i are strongly biproper).

5 The row-by-row decoupling problem for delay systems

Our purpose is to give for a linear time delay system a more general solution for the row-by-row decoupling problem.
The given problem was studied by several authors (Rekasius and Milzareck, 1977; Tzafestas and Paraskevopoulos, 1973; Sename *et al*, 1995) but only partial solutions were given. In (Rabah and Malabre, 1997) an abstract geometric approach is developed using Hautus' definition of (A,B)-invariant subspaces, however it is difficult to compute the used subspace and the result given there is limited to the strong definition of properness. Our approach developped first in (Rabah and Malabre, 1998) for the disturbance rejection problem is extended here to the row-by-row decoupling problem. The weak structure at infinity given in the previous section allows to give the following general formulation and solution for this control problem by generalized static state feedback.

Theorem 5.1 *The following propositions are equivalent:*

1. *The weak structure at infinity verifies:*

$$\Sigma_\infty^w(C_0, A_0, A_1, B_0) = \begin{bmatrix} \Sigma_\infty^w(c_1, A_0, A_1, B_0) \\ \vdots \\ \Sigma_\infty^w(c_p, A_0, A_1, B_0) \end{bmatrix},$$

   where c_i's are the rows of the matrix C_0.

2. *The matrix*

$$D_0 = \begin{bmatrix} c_1 Q_{n_1-1}(k_1)B_0 \\ \vdots \\ c_m Q_{n_m-1}(k_m)B_0 \end{bmatrix},$$

   is invertible, where for each row i the integers n_i and k_i are such that: $c_i Q_{n_i-1}(k_i)B_0 \neq 0$ and $c_i Q_j B_0 = 0$ for $j < n_i - 1$ and $j < k_i$.

3. *The row-by-row decoupling problem for the delay system (1) is solvable by a weak biproper precompensator :*

$$T(s,e^{-s})K(s,e^{-s}) = \text{diag } \left\{h_1(s,e^{-s}),\ldots,h_m(s,e^{-s})\right\}.$$

4. *The decoupling problem is solvable by generalized static state feedback*

$$u = F(e^{-s})x + G(s,e^{-s})v,$$

where

$$\begin{array}{rcl} F(e^{-s}) & = & F_0 + F_1 e^{-s} + \cdots, \\ G(s,e^{-s}) & = & G_0 + G_1(s)e^{-s} + \cdots, \end{array}$$

with (possible) polynomial matrices $G_i(s),\ i \geq 1,\ G_0 = D_0^{-1}$ *and constant matrices* $F_i,\ i \in \mathbb{N}$. *The relation between the precompensator* $K(s,e^{-s})$ *and the feedback law*

$$u = F(e^{-s})x + G(s,e^{-s})v$$

is given by

$$K(s,e^{-s}) = \left(I - F(e^{-s})(sI - A(e^{-s}))^{-1}B_0\right)^{-1} G(s,e^{-s}),$$

where $A(e^{-s}) = A_0 + A_1 e^{-s}$.

PROOF: Suppose that the condition $1)$ is verified. The integers n_i and k_i for $i = 1,\ldots,m$ describe the weak structure at infinity of each row i. Then

$$T(s,e^{-s}) = \text{diag } \left\{s^{-n_1}e^{-k_1 s},\ldots,s^{-n_m}e^{-k_m s}\right\}(D_0 + W(s,e^{-s})) \tag{7}$$

and $W(s,e^{-s})$ is strictly weak proper. If D_0 is not invertible, then by elementary operations one can reduce some column of D_0 and then the global structure at infinity would not coincide with $\text{diag }\{s^{-n_1}e^{-k_1 s},\ldots,s^{-n_m}e^{-k_m s}\}$, which is not possible by hypothesis. This gives that $2)$ holds.

Suppose that $2)$ is verified. Then from the factorisation 7 (which is always true), one has $D_0 + W(s,e^{-s})$ weak biproper, because

$$\lim_{\mathbb{R}\ni s\to\infty}(D_0 + W(s,e^{-s}) = D_0.$$

Then $K(s,e^{-s}) = (D_0 + W(s,e^{-s}))^{-1}$ is also biproper and

$$T(s,e^{-s})K(s,e^{-s}) = \text{diag } \left\{s^{-n_1}e^{-k_1 s},\ldots,s^{-n_m}e^{-k_m s}\right\},$$

which means that $3)$ holds.

Assume now that the decoupling problem is solvable by a weak biproper compensator and let us show that there also exists a solution by generalized static state feedback. Let us first note that $3)$ implies $1)$ (and $2)$) This means that one can define from (7)

$$K(s,e^{-s}) = \left(D_0 + W(s,e^{-s})\right)^{-1},$$

and then

$$K^{-1}(s, e^{-s}) = D_0 + W_1(s, e^{-s}) + W_2(s, e^{-s}),$$

where $D_0 = \sum_{j=0}^{\infty} C_0 Q_{\nu-1}(\kappa + j) e^{-js} B_0$, where ν and κ denote for each row i the integers n_i and k_i respectively (as in the matrix of Falb-Wolovich). The matrix $W_1(s, e^{-s})$ is the strong proper part of $W(s, e^{-s})$:

$$W_1(s, e^{-s}) = \sum_{j=0}^{\infty} \sum_{i=1}^{\infty} C_0 Q_{i+\nu-1}(\kappa + j) B_0 s^{-i} e^{-js},$$

and $W_2(s, e^{-s}) = W(s, e^{-s}) - W_1(s, e^{-s})$. The matrix $W_2(s, e^{-s})$ contains generalized proper terms like se^{-s}. Then $D_0 + W_1(s, e^{-s})$ is strong biproper. Let us denote

$$K_1(s, e^{-s}) = (D_0 + W_1(s, e^{-s}))^{-1},$$

and $K_2(s, e^{-s}) = K(s, e^{-s}) - K_1(s, e^{-s})$. The matrix $K_2(s, e^{-s})$ contains generalized proper terms like se^{-s} as $W_2(s, e^{-s})$. The precompensator $K_1(s, e^{-s})$ is strongly biproper and may be realized by static state feedback, see (Rabah and Malabre, 1996), (Sename *et al*, 1995) where additional conditions are given in order to insure the strong biproperness of the precompensator and the feedback is explicetely designed:

$$\begin{array}{rcl} F^1(e^{-s}) & = & F_0 + F_1 e^{-s} + F_2 e^{-2s} + \cdots, \\ G^1(e^{-s}) & = & G_0^1 + G_1^1 e^{-s} + G_2^1 e^{-2s} + \cdots, \end{array}$$

where, for example, $G_0^1 = D_0^{-1}$ and

$$F_0 = -G_0^1 \begin{bmatrix} c_1 Q_{n_1}(k_1) \\ \vdots \\ c_m Q_{n_m}(k_m) \end{bmatrix}$$

the other matrices are computed as in (Rabah and Malabre, 1996) and (Sename *et al*, 1995). This gives

$$K_1(s, e^{-s}) = \left(I - F^1(e^{-s})(sI - A(e^{-s}))^{-1} B_0\right)^{-1} G^1(e^{-s}).$$

Taking $F(e^{-s}) = F^1(e^{-s})$ and

$$G^2(s, e^{-s}) = \left(I - F(e^{-s})(sI - A(e^{-s}))^{-1} B_0\right) K_2(s, e^{-s}),$$

one obtains

$$K_1(s, e^{-s}) + K_2(s, e^{-s}) = \left(I - F(e^{-s})(sI - A(e^{-s}))^{-1} B_0\right)^{-1} \left(G^1(e^{-s})) + G^2(s, e^{-s})\right).$$

Then

$$K(s, e^{-s}) = \left(I - F(e^{-s})(sI - A(e^{-s}))^{-1} B_0\right)^{-1} G(s, e^{-s}),$$

with $G(s, e^{-s}) = G^1(e^{-s}) + G^2(s, e^{-s})$. Hence $4)$ is satisfied.

If the decoupling problem is solvable by generalized static state feedback, then

$$T(s, e^{-s}) K(s, e^{-s}) = \operatorname{diag}\left\{h_1(s, e^{-s}), \ldots, h_m(s, e^{-s})\right\}.$$

with weak biproper $K(s, e^{-s})$ and $h_i(s, e^{-s}) \neq 0$ for each i. This gives, for each i:

$$c_i(sI - A(e^{-s})^{-1} B_0 K(s, e^{-s}) = \begin{bmatrix} 0 & \cdots & h_i(s, e^{-s}) & \cdots & 0 \end{bmatrix}$$

and then each row i of the system has the structure at infinity of

$$[0 \quad \cdots \quad h_i(s, e^{-s}) \quad \cdots \quad 0]$$

and this gives 1). This ends the proof. ∎

Corollary 5.2 *If in the theorem weak structure (or properness) is replaced by the strong one, then the feedback contains only static terms, no derivative of the reference is needed.*

PROOF: The assumptions of the corollary imply that the weak structure at infinity is also the strong structure at infinity (Rabah and Malabre, 1996), this gives $K(s, e^{-s}) = K^1(s, e^{-s})$, and then $G(s, e^{-s}) = G^1(e^{-s})$ and $F = F^1(e^{-s})$. The precompensator is realizable by static state feedback. No derivation of the delayed reference is needed. ∎

In Theorem 5.1, the "geometric" formulation was omitted (statement 4 in Theorem 3.3). The following proposition gives an analogous result. In order to formulate this result let us introduce a Hautus' (A, B)-invariant subspace for delay systems (see (Malabre and Rabah, 1993) for the introduction of this tool and application to disturbance decoupling for delay systems and (Rabah and Malabre, 1997) for the same statements in terms of strong properness and for the design of strong decoupling precompensator).

For

$$\mathfrak{C}_i = \bigcap_{j \neq i}^{m} \operatorname{Ker} c_j,$$

let $\mathcal{V}_\Sigma(\mathfrak{C}_i, A(e^{-s}), B_0),\ i = 1, \ldots, m$ be the subspaces

$$\mathcal{V}_*(\mathfrak{C}_i, A(e^{-s}), B_0) = \left\{ x \in \mathfrak{C}_i : x = (sI - A(e^{-s}))\xi(s) - B_0\omega(s) \right\},$$

with strictly weak proper ξ and ω such that $\xi(s) \in \mathfrak{C}_i$ for $s > s_0$.

Theorem 5.3 *The system (1) is decouplable iff*

$$\operatorname{Im} B_0 = \sum_{i=1}^{m} \operatorname{Im} B_0 \cap \mathcal{V}_\Sigma(\mathfrak{C}_i, A(e^{-s}), B_0), \tag{8}$$

with $\operatorname{Im} B_0 \cap \mathcal{V}_\Sigma(\mathfrak{C}_i, A(e^{-s}), B_0) \neq 0$.

PROOF: The proof is the same as the proof of the equivallence of statements 1) and 4) for the Theorem 3.3 (see Section 3). We need only to replace $\mathcal{V}_*$ by $\mathcal{V}_\Sigma$, the matrices A by $A(e^{-s})$ and B by B_0. ∎

Note that the statements and some details of the proofs must be reformulated if this system has delay in control and output

6 Conclusion

In order to solve in a general form and without prediction the row-by-row decoupling problem for delay systems we use the weak structure at infinity which is well defined for linear time delay systems. The general solution is of feedback type. However we need some smoothness of the new reference v. This is the counterpart of the generality. For practical use this means that if the reference is not smooth enough, we need in fact very high gain in approximation. The results given here may be also considered, with some modification, for more general delay systems: systems with distributed delays or of neutral type.

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