CONTROL OF FLEXIBLE STRUCTURES USING MODELS WITH DEAD TIME

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Abstract

A new method for modeling and control of noncollocated flexible structures is proposed. The first step is modeling the controlled structure by a reduced order model including a delay between actuator excitation and the noncollocated sensor measurement, caused by the finite wave propagation velocity. A fixed order control scheme for compensating the response delay is then used. In addition, a new design methodology for the controller gain matrices, such that the residual dynamics is suppressed and the spillover effects are reduced, is suggested.

1. Introduction

Control of flexible structures is an important and one of the most complex problems in control theory. Low and frequently not well modeled damping, a large number of closely spaced resonances within control bandwidth, and high performance requirements often make this problem extremely difficult. Model based control methods such as pole placement or optimal control result in controllers having a dimension at least equal to that of the model. The infinite spectrum nature of the entire model makes it impossible to use such methods with finite dimension controller. Furthermore, the higher order modes are usually not well modeled and, as a rule, using them for controller design is not worth while. Thus, model reduction is necessary and there is a great variety of available methods for it. Some are specific for flexible structures, such as modal truncation, while others are general, e. g. L_2 optimization (Hyland and Bernstein, 1985) or balanced realization (Moore, 1981).

Although the design is based on a reduced model, the actual plant is still infinite dimensional, or at least of very high order so only some of the modes are controlled. Interaction between controlled and uncontrolled modes, known as spillover, degrades the performance and in certain cases may cause instability, as is shown in (Balas, 1978). Some approaches for alleviation of spillover effects were suggested (Longman, 1979; Sesak et al., 1979), but the problem still is a challenge. If sensors and actuators are collocated, then it is well known, that such structure has a positive real transfer function, that is stabilizable by any strictly positive real controller (Benhabib et al., 1981), regardless of spillover. However, in many, if not most, applications the output of interest and the applied input are spatially apart and collocated control means not having the physically meaningful feedback. Therefore in this paper we concentrate on noncollocated control systems as more general and potentially more effective.

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The phase shift in noncollocated system always exceeds 180 degrees above some finite frequency, which is inversely proportional to the distance between sensor and actuator. If this frequency lies within or straight behind the control bandwidth, then we have a stability and performance degradation. Conventional truncated modal models preserve the natural frequencies but have no mechanism to match the phase. This makes them adequate for collocated control, but not for control of noncollocated systems. In this paper we propose the new method for modeling of noncollocated flexible structures. The idea based on the observation that the separation between actuator and sensor creates input-output delay which is the time required for the waves to travel through this distance. This phenomenon was discussed in (Cannon and Schmitz, 1984; Spector and Flashner, 1987, 1990). Thus, the natural broadening of modal modeling to noncollocated systems is including the dead time into the reduced model. We derive an optimal L_2 order reduction method to model with dead time. Two possible cases, depending on the existence of rigid body modes, are analyzed and solved.

One of the advantages of modeling the structure as having dead time is the ability to use control strategies developed especially for such systems. These controllers are known as dead time compensators. Various schemes were suggested (Furukawa and Shimemura, 1983; Manitius and Olbrot, 1979; Watanabe and Ito, 1981), all of them are based on prediction of future values of plant output or state, and can be presented by a common scheme. Traditionally these schemes are associated with process control and to the best of our knowledge have never been applied to flexible structures.

In this work we use an observer-predictor control scheme consisting of an observer which reconstructs the state vector of the controlled subsystem, predictor which forecasts the future value of the state and a feedback gain. As was mentioned above, a finite dimension control of infinite dimensional flexible structure suffers from a spillover. A new method for calculating controller gains, which intended to suppress the residual dynamics is presented. The approach does not increase controller order or number of controller devices.

The paper is organized as follows. In the next section a motivating example is introduced. Section 3 deals with optimal order reduction with dead time. In section 4 the observer-predictor control scheme is described and the new gain calculation algorithm for suppression of residual subsystem dynamics is presented. The proposed method is demonstrated by an example in section 5. The results are summarized in section 6.

2. Motivating example

In order to justify our claim that a model with dead time is suitable for describing noncollocated flexible structures, we consider, as an example, the free-free uniform rod subjected to torque moment M(t) at one end which is shown in fig. 1.

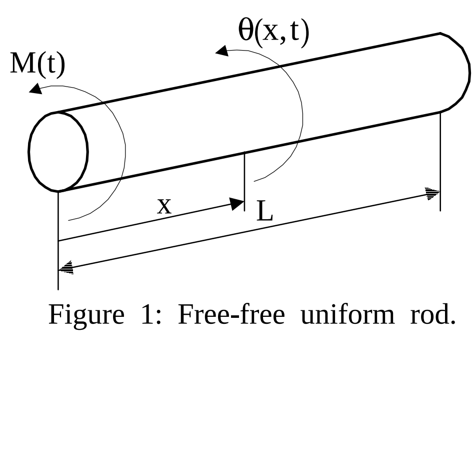

Figure 1: Free-free uniform rod.

The system is governed by the wave equation

$$\frac{\partial^2\theta(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\theta(x,t)}{\partial t^2} \tag{1}$$

with the boundary conditions

$$GI_p\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0}=-M(t),\qquad \left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=L}=0 \tag{2}$$

where I_p denotes the polar moment of inertia, G is the shear elasticity modulus, ρ is the material density and $c=\sqrt{G/\rho}$ is the wave propagation velocity.

Laplace transform with respect to time converts the partial differential equation (1) into an ordinary one. Consequently a transfer function from the applied moment to the torsion angle as a function of the spatial coordinate x (which can be interpreted as a sensor location) can be calculated, and is found to be

$$\frac{\theta(x,s)}{M(s)}=\frac{c}{GI_p}\cdot\frac{1}{s}\cdot\frac{e^{-\frac{2L}{c}(1-\beta)s}+1}{1-e^{-\frac{2L}{c}s}}\cdot e^{-\frac{L\beta}{c}s} \tag{3}$$

where $\beta=x/L$ denotes a nondimensional measurement coordinate. The expression $L\beta/c$ in the last term has units of time and represents the dead time in the response. To see this, consider the realization of (3) with causal blocks as is shown in fig. 2. The bold line is the fastest transmission from input to output and it passes through this delay term only.

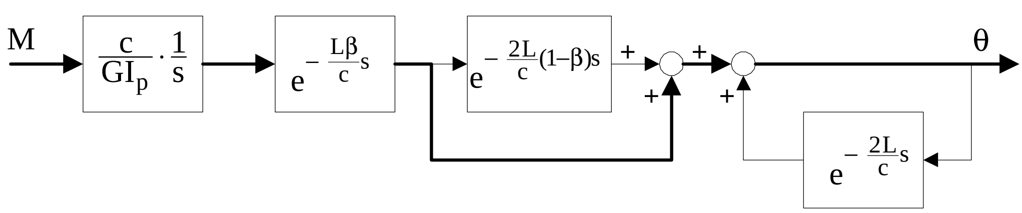

Figure 2: Transfer function (3) realization with causal blocks.

If the cross section or material properties of the rod are spatially dependent, the system is no longer modeled by the wave equation and there is no exact value of the dead time as in this example. In beams, which are governed by fourth order partial differential equations, the response has a dispersive nature, i. e. the propagation velocity, and hence the delay, are frequency dependent (Meirovitch, 1967). However, the phenomenon of delay between input and output exists in these cases, as well as in other kinds of flexible structures, and models containing dead time are good approximation in such cases also.

3. Optimal L_2 order reduction with dead time

The minimization criterion for L_2 optimal approximation of high order model by a low order one with dead time is of the form

$$J = \left\| G(s) - G_r(s) e^{-hs} \right\|_2^2 \tag{4}$$

There are two possible cases, depending on the existence of rigid body modes. Recall, that the rigid body mode has zero natural frequency, which corresponds to a double integrator. Thus, if the system does not have rigid body modes, then, with small amount of structural damping, which always presents in flexible structures, it is asymptotically stable and an existing method (Halevi, 1996) is applicable. On the other hand, a rigid body mode makes the system unstable and a different derivation is required. A key point is that the rigid body mode is an inherent property, which must be retained in the reduced model without any change. Based on this requirement, we write the cost (4) in a slightly different form

$$J = \left\| \left(G_s(s) - G_{rs}(s) e^{-hs} \right) \frac{1}{s^2} \right\|_2^2 \tag{5}$$

where $G_s(s) = s^2 G(s)$ and $G_{rs}(s) = s^2 G_r(s)$ denote the multiplicatively stable parts of the full and reduced models respectively. The special structure of models of flexible mechanical systems guarantees the properness of $G_s(s)$, thus it has a state space realization (A, B, C, D). In the derivation we adopt the idea of Wilson and Mishra (1979), who derive an optimal approximation for the system response to input of the form $u(t) = t^k / k!$. In our case the problem is interpreted as an optimal order reduction one with ramp input and reduced model with dead time.
The properness of $G_r(s)$ implies that the realization of the stable rational part $G_{rs}(s)$ is given in the following way

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t) \tag{6}$$

$$y_r(t) = C_r x_r(t) + D_r u(t) + E_r \dot{u}(t) \tag{7}$$

Using Parseval's theorem, criterion (5) is transformed to time domain

$$J = trace\left\{ \int_0^\infty \left(R(t) - R_r(t-h) \right)^T \left(R(t) - R_r(t-h) \right) dt \right\} \tag{8}$$

where $R(t)$ and $R_r(t)$ are ramp responses of $G_s(s)$ and $G_{rs}(s)$ respectively and $R_r(t-h)$ is the delayed version of $R_r(t)$. The responses in (8) are given by

$$R(t) = Ce^{At} A^{-2} B + \left(D - CA^{-1} B \right) t - CA^{-2} B \tag{9}$$

$$R_r(t-h) = 1(t-h)\cdot\left(C_r e^{A_r(t-h)} A_r^{-2} B_r + \left(D_r - C_r A_r^{-1} B_r\right)(t-h) + \left(E_r - C_r A_r^{-2} B_r\right)\right) \quad (10)$$

Here $1(t-h)$ denotes Heavyside function applied at $t = h$.

The criterion (8) is finite only if the two responses (9), (10) are equal at steady state. This requirement is satisfied by the following choice of D_r and E_r

$$D_r = D - CA^{-1}B + C_r A_r^{-1} B_r \quad (11)$$

$$E_r = C_r A_r^{-2} B_r - CA^{-2}B + \left(D - CA^{-1}B\right)h \quad (12)$$

Substituting of (11) and (12) into (8) we obtain

$$\begin{aligned} J = trace\left\{\int_0^h R(t)^T R(t) dt\right\} + \\ trace\left\{\int_h^\infty \left(Ce^{At}A^{-2}B - C_r e^{A_r(t-h)} A_r^{-2} B_r\right)^T \left(Ce^{At}A^{-2}B - C_r e^{A_r(t-h)} A_r^{-2} B_r\right) dt\right\} \end{aligned} \quad (13)$$

Assume that h is fixwd, the first term does not include parameters of the reduced model and, therefore, does not affect the minimization process. By changing the integration variable to $\tau = t - h$ in the second term, the equivalent optimization criterion is

$$J' = trace\left\{\int_0^\infty \left(Ce^{A\tau}e^{Ah}A^{-2}B - C_r e^{A_r\tau} A_r^{-2} B_r\right)^T \left(Ce^{A\tau}e^{Ah}A^{-2}B - C_r e^{A_r\tau} A_r^{-2} B_r\right) d\tau\right\} \quad (14)$$

Introducing the definitions $\overline{B} = e^{Ah}A^{-2}B$ and $\overline{B}_r = A_r^{-2}B_r$, the criterion (14) is identical to standard L_2 order reduction (Hyland and Bernstein, 1985) with $\overline{B}_r$ and $\overline{B}$ instead of B_r and B respectively.

To conclude, the optimal solution for the rational part of reduced model for any given value of the dead time, is given by

$$G_r(s) = \frac{1}{s^2}\left(C_r\left(sI - A_r\right)^{-1} B_r + D_r + E_r s\right) \quad (15)$$

where D_r and E_r are given in the (11)-(12), $B_r = A_r^2 \overline{B}_r$ and $\left(A_r, \overline{B}_r, C_r\right)$ is obtained from standard L_2 order reduction of $\left(A, e^{Ah}A^{-2}B, C\right)$. An optimal value of the dead time, which minimizes the cost (4), can be found iteratively by cost evaluation for different values of h. It should be pointed out, that suboptimal solution is possible by use of truncated realization method (Moore, 1981), which is easier to compute.

Example (continued)

We apply now the proposed method of model reduction to the example introduced in section 2. The measurement point was taken at the far end of the rod, that is $\beta = 1$. For comparison, we derive 8-th order models by both modal truncation and optimal order reduction with dead time. The minimum of the optimization cost (4) is achieved at the theoretical value of the dead time $L\beta/c$ (for the parameters used in the example). Fig. 3 shows frequency responses of a full and both reduced models. As can be seen from the figure, the main improvement, resulting from the inclusion of dead time in the reduced model, is the better phase matching in the residual, i.e. high frequency, region. Seemingly, the quality of the model in that region is of limited importance because of the small gain. However, large error in phase angle of residual modes, as in the case of modal truncation, can cause instability of the closed loop due to spillover. The better phase matching in the case of approximation with dead time reduces this possibility. While the poles are almost identical in the two reduced models, it is interesting to study the zeros patterns. There are no zeros and infinity number of poles in the full model. Model reduction unavoidably reduces the difference between the number of poles and zeros (this difference is revealed in the phase lag at high frequencies). In order to compensate for this reduction, RHP zeros, which add phase lag, appear in the modally truncated model. In the time domain this results in oscillations about the zero axis, imitating the dead time. Existence of RHP zeros in the model imposes severe limitations on the achievable performance. However, in this case these zeros appear artificially due to a certain modeling method. The dead time adds the necessary phase lag and the reduced order model avoids the RHP zeros problem.

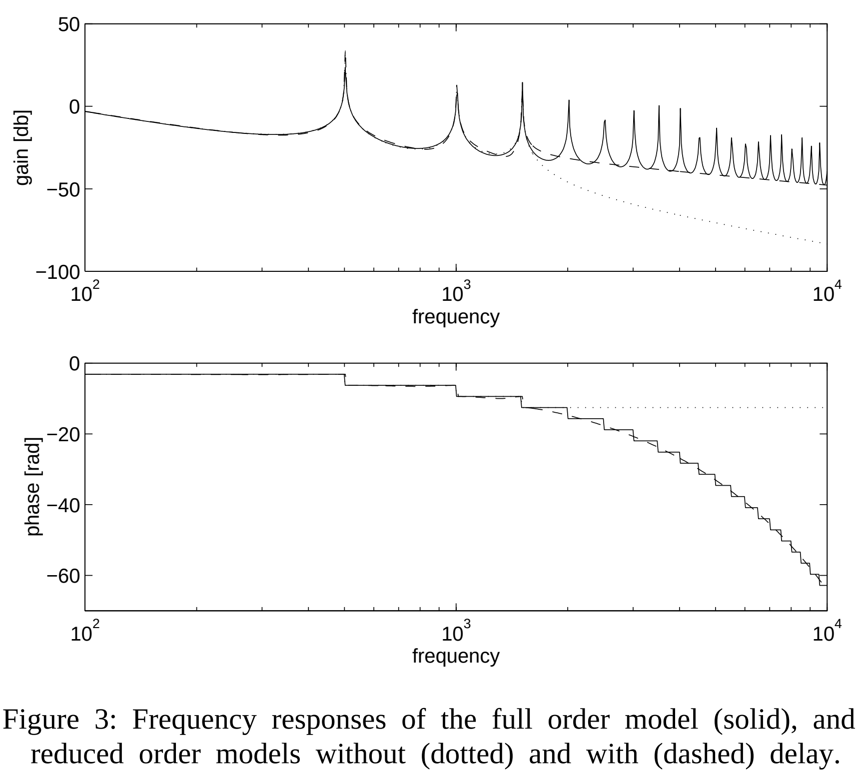

Figure 3: Frequency responses of the full order model (solid), and reduced order models without (dotted) and with (dashed) delay.

4. Control strategy

In this section a control law for noncollocated flexible structures is proposed. The idea is to apply an observer-predictor control scheme, derived for delayed systems, to reduced model with dead time, which was obtained in the previous section. The appropriate adjustments in the control gains calculation will be done for compensating of undesirable spillover effects.

4.1 Observer-predictor control scheme

We start with a realization of $G_r(s)e^{-hs}$

$$\dot{x}(t) = A_r x(t) + B_r u(t-h) \tag{16}$$

$$y_r(t) = C_r x(t) \tag{17}$$

(the delay in a model can be attributed to either the input or the output, where only the notation changes, and we use here input delay). A control law which stabilizes (16)-(17), is given as follows (Furukawa and Shimemura, 1983)

$$u(t) = -Fx(t+h) \tag{18}$$

where F is the state feedback gain, such that matrix $A_r - B_r F$ is stable. The future state $x(t+h)$ is found from the solution of the state equation

$$x(t+h) = e^{A_r h}x(t) + \int_t^{t+h} e^{A_r(t+h-\tau)}B_r u(\tau - h)d\tau = e^{A_r h}x(t) + \int_{t-h}^{t} e^{A_r(t-\tau)}B_r u(\tau)d\tau \tag{19}$$

This expression does not include any future information, thus can be realized. The second term can be implemented in the integral form as in (19) or by means of a dynamic model of the system, as shown in (Furukawa and Shimemura, 1983). The entire state vector $x(t)$ is not measurable and has to be reconstructed from the available input and output signals by means of an observer

$$\dot{\hat{x}}(t) = A_r\hat{x}(t) + B_r u(t-h) + L\big(y(t) - C_r\hat{x}(t)\big) \tag{20}$$

where L is the observer gain, such that matrix $A_r - LC_r$ is stable. The overall control law consists of three stages, as is shown in fig. 4: estimation of the delayed state vector, prediction of its future value and predicted state feedback. We refer to this scheme as observer-predictor control law.

As was discussed in the introduction, control systems, using finite dimensional models of flexible structures, undergo spillover effects, degrading performance and stability margins. With a given controller structure, the gains F and L are the only parameters which can be used to suppress undesired residual dynamics. The gains calculation for spillover alleviation is the subject of the next subsection.

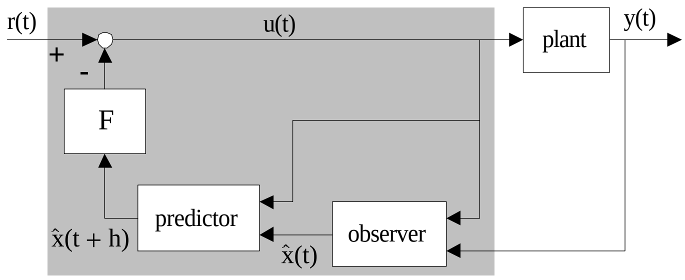

Figure 4: Observer-predictor control scheme.

4.2 Controller gains and spillover suppression

The approach for spillover suppression, which is developed here, applies to collocated as well as noncollocated designs, and is not directly related to the model with dead time. The starting point is as follows. Given the dynamic system

$$\dot{x} = Ax + Bu + w \tag{21}$$

$$y = Cx + v \tag{22}$$

which is driven by a deterministic control input u and by a white noise process w of intensity W>0. The measurement contains white noise v of intensity V>0, and the cross correlation between two noises is V_{cross}. The realization is such that A is block diagonal and we partition the system to controlled and suppressed subsystems

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} u + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{23}$$

$$y = \begin{bmatrix} C_c & C_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + v \tag{24}$$

with $x_c \in R^{n_c}$ and $x_s \in R^{n_s}$. It is assumed that the suppressed subsystem is well modeled but can not be controlled because of controller order limitations. In terms of the previous section the system (A, B, C) is a realization of $G_r(s)$ and $n_r = n_c + n_s$. That means that in the modeling process we choose n_r which is somewhat greater than the fixed controller order n_c.

The term $B_s u$ represents the input spillover and $C_s x_s$ is the output spillover term. In the next two subsections an estimator for x_c with suppressed influence of the output spillover is constructed (subsection 4.2.1) and state feedback gain matrix alleviating the input spillover is calculated (subsection 4.2.2).

4.2.1 State estimation and output spillover suppression

The optimization cost for the estimation problem is

$$J = \lim_{t\to\infty} E\left\{\left(x_c(t) - \hat{x}_c(t)\right)^T R\left(x_c(t) - \hat{x}_c(t)\right)\right\} \tag{25}$$

where R>0 is a weighting matrix. The controlled subsystem can be written as

$$\dot{x}_c = A_c x_c + B_c u + w_1 \tag{26}$$

$$y = C_c x_c + v_1 \tag{27}$$

where v_1 is the output of suppressed subsystem driven by white noise w_2 plus the white measurement noise

$$\dot{x}_s = A_s x_s + w_2 \tag{28}$$

$$v_1 = C_s x_s + v \tag{29}$$

We omit the deterministic input term in the suppressed subsystem as it will be suppressed by feedback gain calculation. Notice also, that the control signal u is a linear combination of the controlled state variables, thus its frequency content is mostly outside the bandwidth of the residual subsystem. Now we can treat the suppressed subsystem as a shaping filter and with such interpretation the measurement disturbance v_1 in controlled subsystem is a colored noise. Thus, representation (26)-(29) is equivalent to the problem of fixed order optimal filtering of colored noises, solved in (Halevi, 1990). The observer is given as

$$\dot{\hat{x}}_c = A_c \hat{x}_c + B_c u + L\left(y - C_c \hat{x}_c\right) \tag{30}$$

where an optimal gain L is given in terms of solution of modified Riccati and Lyapunov equations having the dimension of the full system and coupled by a projection matrix. Thus, the price for spillover suppression is increased off-line computational load, which is not particularly important. The dynamic model used for the on-line state estimation has the dimension of the controlled subsystem, i.e. the proposed method addresses the output spillover problem without increasing the controller order and without increasing the number of sensors.

4.2.2 State feedback gain and input spillover suppression

For the control part the quadratic cost is given as

$$J = \lim_{t\to\infty} E\left\{x(t)^T R_1 x(t) + u(t)^T R_2 u(t)\right\} \tag{31}$$

where $R_1 \geq 0$ and $R_2 > 0$ are weighting matrices. The control input is constrained to include only the controlled states

$$u(t) = -Fx_c(t) \tag{32}$$

It is required now to find the feedback gain F that minimizes the cost (31).
The solution to this problem is dual to solution in the previous subsection, and, as in the previous case, the suppression is achieved at the expense of increased off-line calculations, but the controller order does not change.

The approach described here is somewhat in the spirit of the results of Bernstein and Hyland (1984), which derive a fixed order optimal control law. The solution there results in two Riccati equations and two Lyapunov equations, all of them having the full model dimension and coupled by projection matrix. The controller is given by a single formula and in general does not decompose into an observer-state feedback pattern. We separate the problem to two independent parts. This separation is essential for the control scheme in fig. 4 because it includes a predictor between the observer and feedback blocks.

5. Example

To demonstrate the proposed method we consider a system consisting of ten masses $m_i = 1$ connected by springs $k_i = 1$ and dashpots $c_i = 0.01$ as shown in fig. 5. The force acting on the first mass constitutes the input, and the output to be controlled is the displacement of the last mass. Such system can represent the lumped model of a flexible shaft (with rotation rather than translation as in the fig. 5) with a rigid body mode. Measurements at both ends are available and the performance is described in the terms of rise time and overshoot. First, we apply the collocated rate feedback. with gain $c_u = 1$, corresponding to the maximum attainable damping in the flexible modes. With collocated rate loop closed, the system has one pole at the origin. We exploit it to obtain zero steady state error for step input.

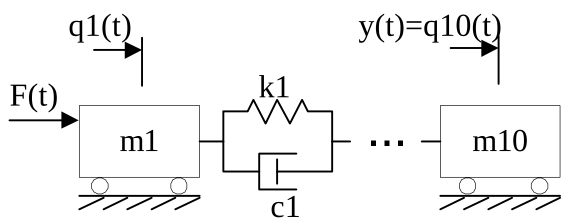

Figure 5: The open loop system in example.

With the collocated loop fixed, the next step is the design of an outer noncollocated control law. We compare four observer-based designs. The feedback gain is obtained from minimization of a quadratic cost criterion which includes the noncollocated output, its rate and the control input

$$J = \int_0^\infty \left(4y(t)^2 + \dot{y}(t)^2 + u(t)^2\right)dt \tag{33}$$

The process noise enters the system together with the deterministic input. Since the criterion and the noise structure do not depend on a specific model, different schemes are comparable. The candidate designs are:

a) A standard LQG controller based on reduced model of order 4 (2 modes) without dead time and without spillover suppression.
b) A controller having the structure shown in fig. 4, based on a 4th order reduced model with dead time, without spillover suppression.
c) An LQG controller of order 4 with one residual mode suppressed according to the proposed technique.
d) A 4th order controller as in fig. 4, with one mode suppressed.

The reduced models were obtained using the truncated balanced realization method applied to the system with the collocated rate loop. An integrator was treated as step input using similar techniques to those in section 3.
The step responses of the four control systems (a)-(d) are given in fig. 6. As can be seen from the fig. 6a, the standard observer-state feedback control based on the reduced order model results in unstable response due to residual modes. The introduction of the dead time into the model and the corresponding modification of the control scheme stabilizes response (fig. 6b). The same effect is reached with suppression of residual dynamics by technique of section 4 (fig. 6c). Suppression of residual mode in control with dead time smoothes the response (fig. 6d). Stability margins for controls (b)-(d) are given in table 1.

design	(b)	(c)	(d)
gain margin	1.75	1.47	2.
phase margin	60°	55°	63°

Table 1: Stability margins of designs (b)-(d).

It might be argued, that a comparison of the observer-predictor-state feedback controller with the standard observer-state feedback controller is not justified because of different order of controllers. Indeed, the predictor uses a dynamic model of the process and is of the same order as the observer. However, the overall order of controller cannot be considered as doubled, because the two components can be implemented in parallel. Thus the on-line computational load increase is not as considerable as might be conceived.

6. Conclusions

A new and comprehensive methodology of modeling and control of noncollocated flexible structures is proposed. The main idea in the modeling stage is based on including dead time in reduced models of flexible structures. The time delay arises from finite wave propagation velocity and it exists whenever the actuator and the sensor are noncollocated. After a model with dead time is obtained, control laws derived for such systems can be used. In this work we utilize the controller which consists of an observer for the delayed system state vector, a prediction of its future value and a state feedback based on the predicted value.

A new approach in the controller gains design, which is independent of modeling with time delay, is based on the observation that residual dynamics can be presented as a colored noise corrupting the measurement. As a result, suppression of output spillover is solved as a fixed order filtering problem of colored noises. Input spillover suppression is achieved by calculating the state feedback gain in a dual manner. The performance of proposed approach was demonstrated by means of an example.

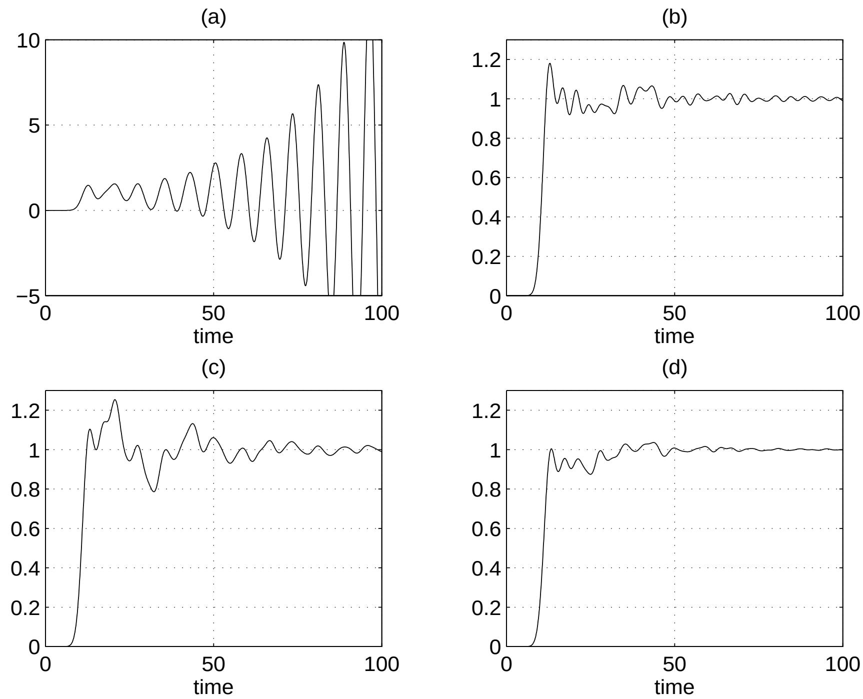

Figure 6: Step responses of noncollocated designs (a)-(d).

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