An MRAC Output Feedback Controller for Robot Manipulators

Howard Schwartz[†]

Department of Systems and Computer Engineering
Carleton University
1125 Colonel By Drive
Ottawa, Ontario K1S 5B6
Canada

Abstract

An adaptive controller is proposed, for the tracking control of robotic manipulators that does not require the measurement of joint velocities. The controller belongs to the class of model-reference adaptive controllers. An observer is used to generate an estimate of the joint velocities and an observer-based identifier with projection is used to update the parameter vector estimate. Simulation results are given to show the effectiveness of the control algorithm.

1. Introduction

The application of adaptive control to robot manipulators has been an active area of research for the last two decades. The problem of tracking control using state feedback, where both position and velocity are accessible, has been examined extensively (see for example (Slotine and Li, 1987), (Li and Slotine, 1989), and (Ortega and Spong, 1989)). In (Slotine and Li, 1987), an adaptive controller and a parameter adaptation rule are derived based on Lyapunov-like arguments, which guarantee the global asymptotic convergence of the tracking error. In (Li and Slotine, 1989) an indirect adaptive controller is derived which uses the torque estimation error to update the estimate of the parameter vector. Again, global asymptotic stability of the tracking error is shown. Several direct and indirect adaptive controllers are reviewed in (Ortega and Spong, 1989). However, although joint position measurement can be done very accurately, velocity measurements are noise-prone. This motivated attempts to design an observer-based controller. Several papers have been published with variations of this idea. In (Canudas De Wit and Fixot, 1992), an adaptive controller is proposed which utilizes a sliding observer to estimate the joint velocities. However, the discontinuities of the driving terms in the adaptation and observer differential equations induce high frequency signals that can create numerical stability problems. In (Lee and Khalil, 1997), an adaptive controller is presented which is based on a high-gain observer and an identifier with parameter projection. However, the control design requires estimating several parameters including the maximum torque possible.

In this paper, we review a recently proposed output feedback controller for robot manipulators which is based on the work of (Jankovic, 1996), and then present an new output feedback adaptive controller that belongs to the class of model reference adaptive controllers. A virtual control signal is computed and used to generate a linearizing control signal that uses an estimate of the joint velocities generated by an observer.

2. Review of a recently proposed controller

In this section we formulate the control problem and present the results of a recently proposed output feedback controller by (Hajjir and Schwartz, 1999).

The equation of motion for an n-link rigid robot is given by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \qquad (1)$$

† Email: schwartz@sce.carleton.ca

where $q \in R^n$ is the joint position vector, $u \in R^n$ is the input torque. $D(q) \in R^{n \times n}$ is the symmetric and uniformly positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in R^n$ is the Coriolis and centrifugal loading vector, and $g(q) \in R^n$ is the gravitational loading vector. An important property of equation (1) is that its left-hand side of can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y_1(q,\dot{q},\ddot{q})\phi \tag{2}$$

where $\phi \in R^p$ is the parameter vector and $Y_1(q,\dot{q},\ddot{q})$ is the regressor. Thus the robot equation of motion can be linearly parameterized.

The nonlinear dynamics (1) can be rendered linear by the following nonlinear control signal:

$$u = D(q)v + C(q,\dot{q})\dot{q} + g(q) \tag{3}$$

where $v \in R^n$ is the virtual control signal. Under the action of (2), equation (1) yields

$$\ddot{q} = v \tag{4}$$

We may now specify the virtual control by

$$v = \ddot{q}_d - K_D\dot{e} - K_P e \tag{5}$$

where K_D, K_P are positive definite diagonal matrices, and $e = q - q_d$ is the tracking error. Substituting (4) in (3), one gets the error equation

$$\ddot{e} + K_D\dot{e} + K_P e = 0 \tag{6}$$

which is asymptotically stable by proper choice of K_D, K_P.

The control given by (2) and (4) requires exact knowledge of the parameters of the robot, and that the joint velocity be available. In practice, the measured parameters are only approximate and may vary with time (e.g. when the load changes). Thus (2) and (4) are modified to

$$u = \hat{D}(q)(-Kx + \ddot{q}_d) + \hat{C}(q,\dot{q}) + \hat{g}(q) \tag{6}$$

Where, $x = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T = \begin{bmatrix} e^T & \dot{e}^T \end{bmatrix}^T$, and $K = \begin{bmatrix} K_P & K_D \end{bmatrix}$. The terms $\hat{D}(q)$, $\hat{C}(q,\dot{q})$ and $\hat{g}(q)$ are estimates of $D(q)$, $C(q,\dot{q})$ and $g(q)$, that satisfy linear parameterization, that is,

$$\hat{D}(q)\ddot{q} + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q) = Y_1(q,\dot{q},\ddot{q})\hat{\phi} \tag{7}$$

In a recent work (Hajjir and Schwartz, 1999), it is noted that

$$C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q})\theta \tag{8}$$

This follows from the linear parameterization property. The vector θ is obtained from ϕ by retaining only those parameters that specify $C(q,\dot{q})$ and $g(q)$, and $Y(q,\dot{q})$ is obtained from $Y_1(q,\dot{q},\ddot{q})$ by retaining the corresponding columns. If, in addition we assume that the joint velocity vector is not available to the controller then an observer has to be used to estimate it. The observer of (Hajjir and Schwartz, 1999) is given by

$$\dot{\hat{x}} = (A - \Gamma LC)\hat{x} + \Gamma LCx + B(\hat{D}^{-1}u - \hat{D}^{-1}\overline{Y}\hat{\theta} - \ddot{q}_d) \tag{9}$$

where $\hat{x} = \begin{bmatrix} \hat{x}_1^T & \hat{x}_2^T \end{bmatrix}^T = \begin{bmatrix} \hat{e}^T & \hat{\dot{e}}^T \end{bmatrix}^T$ is the observed error vector, and

$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ I_n \end{bmatrix},\ C = \begin{bmatrix} I_n & 0 \end{bmatrix},\ \Gamma = \begin{bmatrix} GI_n & 0 \\ 0 & G^2 I_n \end{bmatrix} \tag{10}$$

The positive scalar G is the observer gain constant. The matrix $\overline{Y} = Y(q_d + \overline{x}_1, \dot{q}_d + \overline{x}_2)$, where $\overline{x} = \begin{bmatrix} \overline{x}_1 & \overline{x}_2 \end{bmatrix} = Sat_{\Omega_x}(\hat{x})$ is the saturated error on a convex set Ω_x. The saturation function is applied component-wise and is defined as follows

$$Sat(x_i) = \begin{cases} -a_i & if \quad x_i < -a_i \\ x_i & if \quad |x_i| \le a_i \\ a_i & if \quad x_i > a_i \end{cases}$$

The positive scalars a_i define the convex set (hypercube) Ω_x.

The control law (Hajjir and Schwartz, 1999) is a modification of (6) to

$$u = \hat{D}(q)(-K\bar{x} + \ddot{q}_d) + \hat{C}(\bar{q}, \dot{\bar{q}}) + \hat{g}(\bar{q}) \tag{11}$$

where $\bar{q} = q_d + \bar{x}_1$ and $\dot{\bar{q}} = \dot{q}_d + \bar{x}_2$.

Similar to (8), we can write

$$\hat{C}(\bar{q}, \dot{\bar{q}})\dot{\bar{q}} + \hat{g}(\bar{q}) = Y(\bar{q}, \dot{\bar{q}})\hat{\theta} \tag{12}$$

Applying (11) to (1), we get the following error equation

$$\dot{x} = Ax + B(E\ddot{q}_d + D^{-1}\hat{D}(-K\bar{x}) + D^{-1}(\bar{Y}\hat{\theta} - Y\theta)) \tag{13}$$

Where,

$$E = D^{-1}\hat{D} - I$$

Substituting (11) in (9), the observer equation reduces to

$$\dot{\hat{x}} = A\hat{x} + \Gamma LC(x - \hat{x}) + B(-K\bar{x}) \tag{14}$$

An identifier (parameter estimator) is given by

$$\dot{\eta} = c_0(\bar{x}_2 - \eta) - \hat{D}^{-1}\bar{Y}\hat{\theta} + \hat{D}^{-1}u - \ddot{q}_d \equiv c_0(\bar{x}_2 - \eta) - K\bar{x} \tag{15a}$$

$$\dot{\hat{\theta}} = \text{Proj}\,(-c_1\bar{Y}^T\hat{D}^{-1}(\bar{x}_2 - \eta)) \tag{15b}$$

where c_0 and c_1 are positive constants. The projection operator in (15b) is applied component-wise, and is given by (Lee and Khalil, 1996)

$$[\text{Proj}(\phi)]_i = \begin{cases} & if \;\; a_i < \hat{\theta}_i < b_i \;\; or \\ \phi_i & if \;\; \hat{\theta}_i \ge b_i \;\; and \;\; \phi_i \le 0 \;\; or \\ & if \;\; \hat{\theta}_i \le a_i \;\; and \;\; \phi_i \ge 0 \\ (1 + \dfrac{b_i - \hat{\theta}_i}{\delta})\phi_i & if \;\; \hat{\theta}_i \ge b_i \;\; and \;\; \phi_i \ge 0 \\ (1 + \dfrac{\hat{\theta}_i - a_i}{\delta})\phi_i & if \;\; \hat{\theta} \le a_i \;\; and \;\; \phi_i \le 0 \end{cases} \tag{16}$$

where it is assumed that the actual parameter vector θ belongs to the set Θ defined by

$$\Theta = \{\theta \mid a_i - \delta \le \theta_i \le b_i + \delta\}$$

This completes the specification of the controller of (Hajjir and Schwartz, 1999). A simulation study of this controller is presented in section 4. The following section presented the new controller of this paper, and its performance will be compared to (Hajjir and Schwartz, 1999) by simulation.

3. The MRAC controller

In this paper, we assume that the desired trajectory $q_d(t)$ is the output of a second order reference model,

$$q_d = W_m(s)r \tag{17}$$

where $W_m(s)$ is the reference model transfer function, and r is the reference input. We design v as follows (Narendra and Annaswamy, 1989)

$$v = d_0 q + d_1^T \omega_1 + d_2^T \omega_2 + kr \tag{18}$$

where

$$\dot{\omega}_1 = \Lambda\omega_1 + lv \tag{19a}$$

$$\dot{\omega}_2 = \Lambda\omega_2 + lq \tag{19b}$$

The vectors $\omega_1, \omega_2 \in R^n$, $d_0, k \in R$, and the diagonal matrices $d_1, d_2, l, \Lambda \in R^{n\times n}$. We will assume that $\Lambda = -\lambda I_n$, $\lambda > 0$, so that $(sI-\Lambda)^{-1} = \dfrac{1}{\lambda(s)} I_n$, where $\lambda(s) = s + \lambda$. From equations (6), we have

$$\omega_1(s) = (sI - \Lambda)^{-1} l\, v(s) \tag{20a}$$

and $$\omega_2(s) = (sI - \Lambda)^{-1} l\, q(s) \tag{20b}$$

Define $$\frac{d_1(s)}{\lambda(s)} I_n = d_1^T (sI - \Lambda)^{-1} l \tag{21a}$$

and $$\frac{d_2(s)}{\lambda(s)} I_n = d_2^T (sI - \Lambda)^{-1} l \tag{21b}$$

Using (21a) and (21b) in (18), we obtain

$$v = d_0 q + \frac{d_1(s)}{\lambda(s)} v + \frac{d_2(s)}{\lambda(s)} q + kr \tag{22}$$

Rearranging, and using (4),

$$(1 - \frac{d_1(s)}{\lambda(s)})\ddot{q} = d_0 q + \frac{d_2(s)}{\lambda(s)} q + kr$$

Therefore,

$$q = \frac{k\lambda(s) r}{s^2(\lambda(s) - d_1(s)) - d_0\lambda(s) - d_2(s)} = W(s)r \tag{23}$$

Now we can select d_0, d_1, d_2, l, k such that

$$W(s) = W_m(s)$$

The above development was carried out assuming perfect knowledge of the robot model parameters, and that the joint velocities were available to the controller. In the case where only an estimate of the parameter vector is available, the control (2) is modified to

$$u = \hat{D}(q)v + \hat{C}(q,\hat{\dot{q}})\hat{\dot{q}} + \hat{g}(q) \tag{24}$$

The vector $\hat{\dot{q}}$ is an estimate of the joint velocity vector. It is generated by an observer that will be specified later. The terms $\hat{D}(q)$, $\hat{C}(q,\hat{\dot{q}})$ and $\hat{g}(q)$ are estimates of $D(q)$, $C(q,\dot{q})$ and $g(q)$, respectively such that the linear parameterization property is preserved, i.e.

$$\hat{D}(q)v + \hat{C}(q,\hat{\dot{q}})\hat{\dot{q}} + \hat{g}(q) = Y(q,\hat{\dot{q}},v)\hat{\theta} \tag{25}$$

Applying the control (24) to the robot model (1), we obtain

$$\begin{aligned}\ddot{q} &= D^{-1}(q)[\hat{D}(q)v + \hat{C}(q,\hat{\dot{q}})\hat{\dot{q}} + \hat{g}(q) - C(q,\dot{q})\dot{q} - g(q)] \\ &= D^{-1}(q)[D(q)v + Y(q,\hat{\dot{q}},v)\hat{\theta} - Y(q,\dot{q},v)\theta] \\ &= v + D^{-1}(q)[Y(q,\hat{\dot{q}},v)\hat{\theta} - Y(q,\dot{q},v)\theta] \\ &= v + \eta\end{aligned} \tag{26}$$

where

$$\eta = D^{-1}(q)[Y(q,\hat{\dot{q}},v)\hat{\theta} - Y(q,\dot{q},v)\theta] \tag{27}$$

Note that the vector η results from parameter estimation error and velocity estimation error.

Using (26) in (22), and manipulating, we arrive at

$$q = W(s)r + W_\eta(s)\eta \tag{28}$$

where

$$W_\eta(s) = \frac{\lambda(s) - d_1(s)}{s^2(\lambda(s) - d_1(s)) - d_0\lambda(s) - d_2(s)} = \frac{s + a_2}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{29}$$

Since d_0, d_1, d_2, l, k can be selected such that $W(s) = W_m(s)$, we can express the tracking error as

$$e = q - q_d = W_\eta(s)\eta \tag{30}$$

where we have used (4) in equation (15).

In order to estimate the joint velocity vector $\dot{q}$, we use the following observer

$$\frac{d}{dt}\begin{bmatrix}\hat{q}\\ \hat{\dot{q}}\end{bmatrix} = \begin{bmatrix}-L_1 & I_n\\ -L_2 & 0\end{bmatrix}\begin{bmatrix}\hat{q}\\ \hat{\dot{q}}\end{bmatrix} + \begin{bmatrix}L_1\\ L_2\end{bmatrix}q + \begin{bmatrix}0\\ I_n\end{bmatrix}v \tag{31}$$

It can be easily shown that the observation error equation is given by

$$\frac{d}{dt}\begin{bmatrix}\tilde{q}\\ \tilde{\dot{q}}\end{bmatrix} = \begin{bmatrix}-L_1 & I_n\\ -L_2 & 0\end{bmatrix}\begin{bmatrix}\tilde{q}\\ \tilde{\dot{q}}\end{bmatrix} - \begin{bmatrix}0\\ I_n\end{bmatrix}\eta \tag{32}$$

or $$\dot{\tilde{x}} = A_0\tilde{x} + B_0\eta \tag{33}$$

where $\tilde{x} = \begin{bmatrix}\tilde{q}^T & \dot{\tilde{q}}^T\end{bmatrix}^T$, $A_0 = \begin{bmatrix} -L_1 & I_n \\ -L_2 & 0 \end{bmatrix}$ and $B_0 = -\begin{bmatrix}0 & I_n\end{bmatrix}^T$.

Finally, to update the parameter vector estimate, we define the virtual error as

$$e_v = v - \ddot{q}_d \tag{34}$$

This is the error between the virtual control and desired joint acceleration. Using (26) and (30), we have

$$e_v = \ddot{q} - \eta - \ddot{q}_d = (s^2 W_\eta(s) - 1)\eta \equiv W_\phi(s)\eta \tag{35}$$

where $$W_\phi(s) = \frac{d_0\lambda(s) + d_2(s)}{s^2(\lambda(s) - d_1(s)) - d_0\lambda(s) - d_2(s)} \tag{36}$$

Now we select the parameter update law as

$$\dot{\hat{\theta}} = \mathrm{Proj}(\zeta(t)e_v) \tag{37}$$

where $$\zeta(t) = W_\phi(s)[Y^T(q, \hat{\dot{q}}, v)] \tag{38}$$

The projection operator in (37) is used to ensure that $\hat{\theta}$ remains bounded. This completes the specification of the algorithm.

4. Simulation

The proposed controller was simulated on a two-link revolute joint robot. For this robot, we have

$$D(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3\cos q_2 & \theta_2 + \theta_3\cos q_2 \\ \theta_2 + \theta_3\cos q_2 & \theta_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3\sin(q_2)\dot{q}_2 & -\theta_3\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ \theta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} \theta_4 g\cos q_1 + \theta_5 g\cos(q_1 + q_2) \\ \theta_5 g\cos(q_1 + q_2) \end{bmatrix}$$

where θ_1-θ_5 are dependent on the physical properties of the two links and are given by, $\theta_1 = m_1 l_{c1}^2 + I_1 + m_2 l_1^2$, $\theta_2 = I_2 + m_2 l_{c2}^2$, $\theta_3 = m_2 l_1 l_{c2}$, $\theta_4 = m_1 l_{c1} + m_2 l_1$, $\theta_5 = m_2 l_{c2}$, and $g = 9.81$ m/s^2.

The following physical parameters were assumed:

$$m_1 = 10, m_2 = 5, l_1 = 1, l_2 = 1, l_{c1} = l_1/2, l_{c2} = l_2/2, \; I_i = m_i l_i^2/12 .$$

With these values, the actual value of the parameter vector is

$$\theta = [8.33,\ 1.67,\ 2.5, 10,\ 2.5]^T$$

The initial estimate was set to 75% of the actual value. The reference trajectory for both joints was set to the step response of the second-order reference model with double pole at $p_i = -5$. Thus the reference model is given by

$$W_m(s) = \frac{25}{s^2 + 10s + 25}$$

The corresponding design parameters are

$$\lambda = 10,\ d_0 = -125,\ d_1 = -10,\ d_2 = 1000,\ k = 25,\ l = 1$$

The observer gain matrix was set to

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 200 I_2 \\ 10^4 I_2 \end{bmatrix}$$

Figure 1 shows the tracking error performance of the two joints. The maximum tracking error is 0.32 radian for the first joint and 0.5 for the second joint. The tracking errors quickly converge to zero after two seconds. The parameter estimate trajectory is shown in Figure 2.

We then compared the performance of the new MRAC controller with the controller of (Hajjir and Schwartz, 1999). The same desired trajectories and initial parameter estimate were used. The controller parameters were selected as follows

$$G = 10$$

$$L = \begin{bmatrix} 20 & 0 \\ 0 & 20 \\ 100 & 0 \\ 0 & 100 \end{bmatrix}, \qquad K = \begin{bmatrix} 900 & 0 & 60 & 0 \\ 0 & 900 & 0 & 60 \end{bmatrix},\ c_0 = 2,\ c_1 = 1$$

Figure 3 shows the resulting tracking error. The maximum tracking error is 0.05 radian for the first joint and 0.26 radian for the second joint. The tracking error for both joints tends to a very small neighborhood of the origin. Figures 4 show the parameter estimate trajectory obtained with this controller. Only three parameters can be updated; namely, θ_3, θ_4 and θ_5, which are the parameters of $C(q,\dot{q})$ and $g(q)$.

5. Conclusions

In this paper, we reviewed a recent output feedback controller proposed in (Hajjir and Schwartz, 1999) and then presented a new output feedback controller that belongs to the class of model reference adaptive controllers. An important advantage of the new controller is the ability to adapt all the robot parameters unlike the controller of (Hajjir and Schwartz, 1999) in which the estimate of the inertia matrix parameters is kept fixed.

Given any reference model, the new controller parameters can be directly computed that result in a closed loop system with a matching transfer function. It is shown that in the absence of exact knowledge of the robot parameters, and its actual joint velocity; the tracking error is the output of a linear filter whose input is a nonlinear function of the parameter estimate error, and the velocity estimate error (equation (28)). The effectiveness of the controller is demonstrated by simulation.

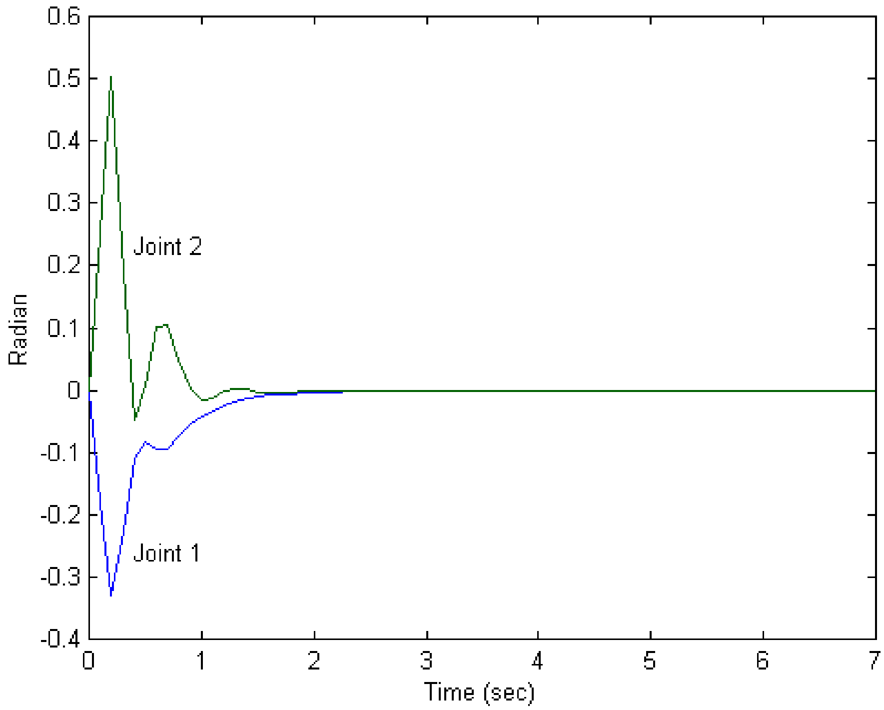

Figure 1. Tracking Error of the two joints with the new controller

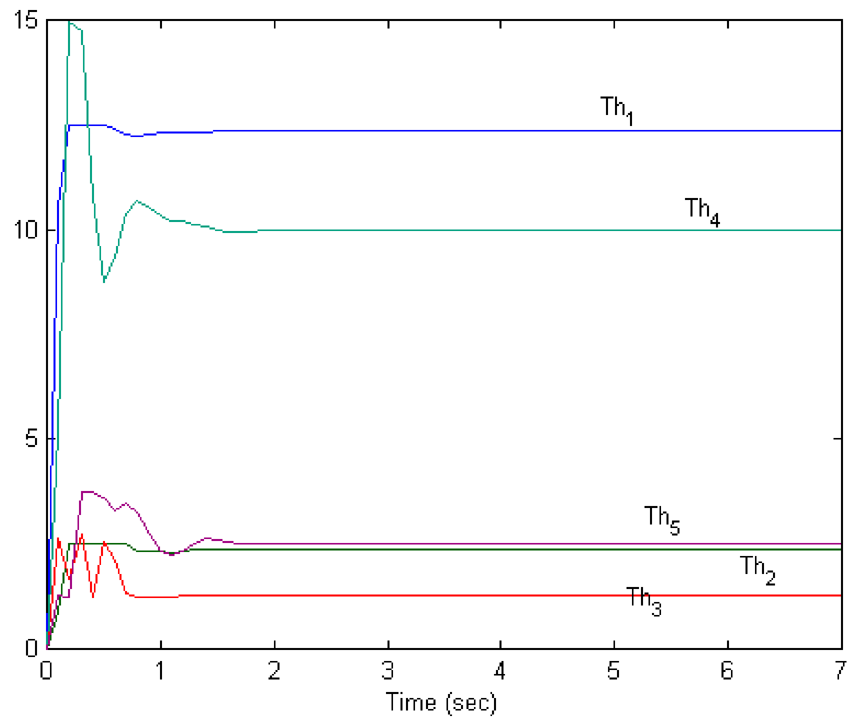

Figure 2. Parameter estimate trajectories

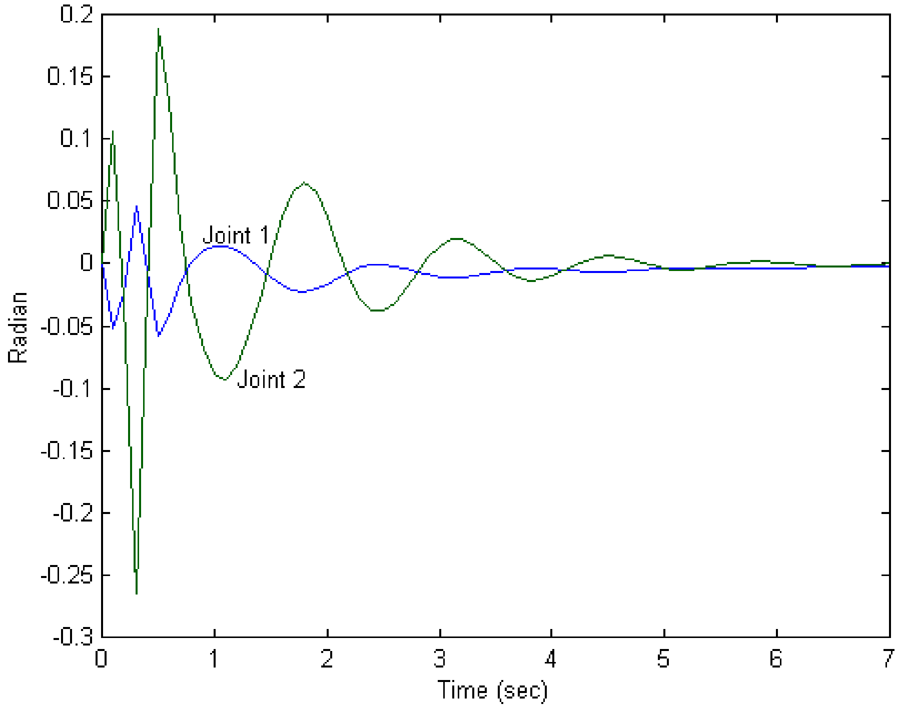

Figure 3. Tracking error with the controller of (Hajjir and Schwartz, 1999)

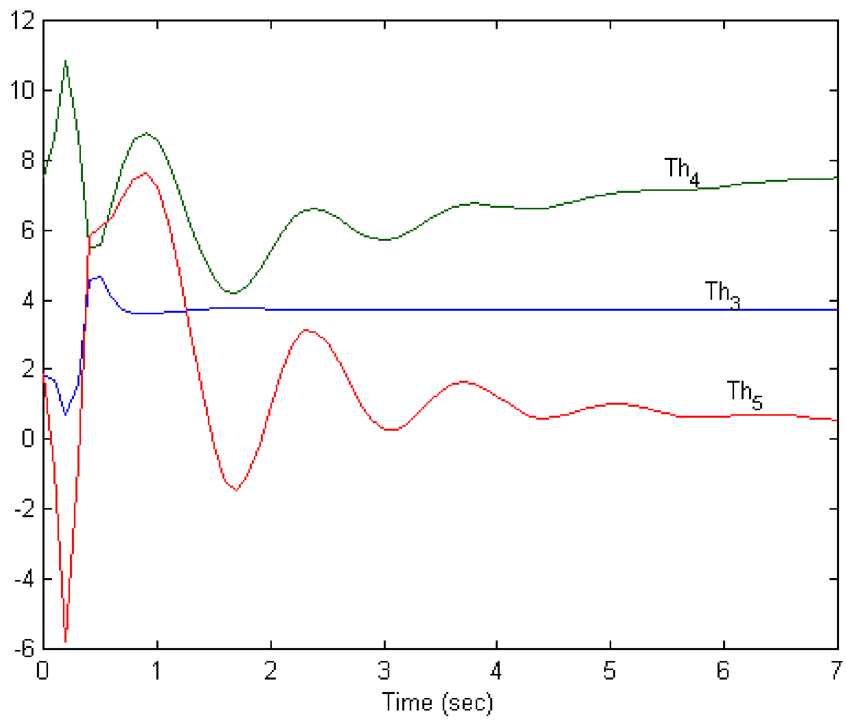

Figure 4. Parameter estimate trajectories for the controller of (Hajjir and Schwartz, 1999)

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