PI Controller Tuning via Multiobjective Optimization

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Abstract:
This paper presents a new method for tuning PI and PID controllers for models commonly used in process control. The proposed method is based on the satisfaction of more than one control design objectives. The design objectives are the satisfaction of certain phase and gain margin, the maximization of the resonant frequency and the minimization of a weighted integral of squa-red error. In solving the multiobjective optimization problem obtained, a simplified goal attain-ment formulation is proposed. The usefulness of the proposed method is demonstrated through simulation examples and a comparison with well known tuning formulas is also provided.

1 Introduction

A great number of PID controller tuning methods is now available to the designer of process control systems. Most of them are based on the satisfaction of single design objectives, such as the decay ratio, phase and gain margins, resonant peak and frequency, overshoot and certain error integral criteria (Astrom and Hagglund, 1995). However, these methods have several shortcomings that stem from the fact that all degrees of freedom , namely the controller adjustable parameters, are consumed in order to satisfy a single objective.

It is now widely recognized that the solutions of numerous design problems, in various branches of engineering, are incomplete because they fail to take into account all the important characteristics of the particular problem (Brayton *et al*., 1981). Controller design problems are among them. The problem that the controller designer faces is the simultaneous satisfaction of several criteria that are posed either on the time or on the frequency domain.

Zakian and Al-Naib (1973) were the first to suggest that control design problems should be posed as the satisfaction of more than one design objectives while Kreisselmeier and Steinhauser (1979) were the first to translate a control design problem into a set of performance objectives.

Harris and Mellichamp (1985) proposed a method for tuning PID controllers based on a combined performance index. This performance index is the weighted sum of common controller design objecti-

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ves posed in the frequency domain. Repetitive solution of complex nonlinear equation is essential for the calculation of performance index for given controller parameters. Abbas and Sawyer (1996) based on the technique known as the ε-constraint method for solving multiobjective optimization problems, proposed a new method for tuning controllers of given structure (such as PID controllers). The design objectives used were posed in the time domain and simulation of the systems dynamic response, under closed loop conditions, is used for the evaluation of each design objective.

The purpose of this study is to propose a new methodology for tuning PI and PID controllers that is based on a compromise between certain design objectives posed in the frequency as well as in the time domain. The study is concentrated on models commonly used to describe chemical processes such as first order plus delay time and integrator plus delay time models. Approximate equations for the calculation of the design objectives are summarized and a proper and simplified formulation of the goal attainment method for solving multiobjective optimization problems is proposed. It is shown that using these simplifications the computational effort is greatly reduced. In order to demonstrate the effectiveness of the proposed method and also to provide a comparison with well known tuning formulas, three simulation examples are given. In these examples the steps of the proposed method are further clarified and general guidelines for the application of the method are given.

2 Multiobjective Optimization and the Goal Attainment Method

In what follows a brief introduction of the multiobjective optimization theory is given. The interested reader is referenced to Clark and Westerberg, (1983), Grauer *et al.*, (1984) and Steuer (1986) for a detailed discussion of the subject.

Suppose that the controller parameters are denoted by the vector $\mathbf{c}=(c_1,\ldots,c_n)^T$, $\mathbf{c}\in R^n$, where R^n is called the input space. In the case where we have m control design objectives, they are expressed in vector form as

$$\mathbf{f}(\mathbf{c}) = \left(f_1(\mathbf{c}),\ldots,f_m(\mathbf{c})\right)^T \in R^m \tag{1}$$

where R^m is called the output space and $\mathbf{f}(\mathbf{c})$ is a point in the output space. It is desired to minimize each $f_i(\mathbf{c})$, $i\in M=\{1,2,\ .\ .\ .\ ,\ m\}$, but some of the f_i will conflict with each other. In other words, a change in $\mathbf{c}$, from $\mathbf{c}^k$ to $\mathbf{c}^k+\Delta\mathbf{c}$ increases some of the f_i but at the same time decreases some others.

In searching for the best controller design, it is desirable to move in directions $\Delta\mathbf{c}$, in the input space , that satisfy the following (in the output space)

$$\Delta f_i = f_i(\mathbf{c}+\Delta\mathbf{c}) - f_i(\mathbf{c}) \le 0, \quad i\in M \tag{2}$$

i.e., the vector $\Delta\mathbf{f}$ defined on the output space, should have only non-positive components. The *Pareto critical point*, is any point in the input space where no $\Delta\mathbf{c}$ exists for which equation (2) to hold. The set of the Pareto critical points is called the *Pareto optimal* set. The image of a Pareto critical point is called a *noninferior point* and the set of all noninferior points is called the *noninferior solution set*. The set of points $\mathbf{q}\in R^m$ such that there exists $\mathbf{c}\in R^n$ where $\mathbf{q}=\mathbf{f}(\mathbf{c})$ is called the *attainable set*, denoted by Q, i.e.

$$Q = \left\{\mathbf{q}\in R^m \,\middle|\, \exists\mathbf{c}\in R^n:\ \mathbf{q}=\mathbf{f}(\mathbf{c})\right\} \tag{3}$$

The boundary of Q, denoted by ∂Q, is of great interest in the context of multiobjective optimization. The definitions given above are illustrated in Figure 1 for the case where there are only two objectives. Any point in the interior of the attainable set has a neighbor for which the values of both f_1 and f_2 are reduced. Points on the boundary, between points A and B do not have this property. A movement between the points A and B and on the boundary requires a trade-off: decreasing the one objective in the expense of increasing the other.

The *utopia point* is defined as the point in the input space with coordinates given by the solution of the following, scalar optimization problems

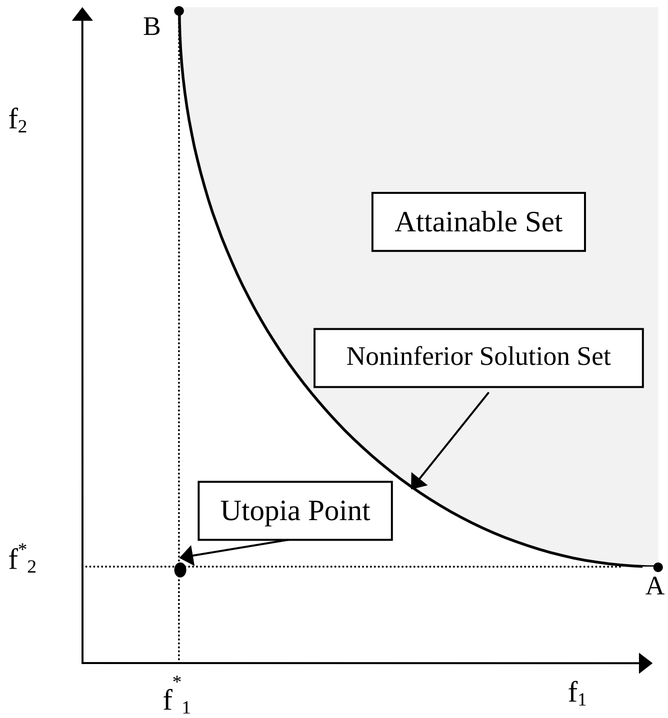

Figure 1. Graphical representation of the main definitions in MOP.

$$\min_{\mathbf{c}} \; f_i \,, \quad i \in M \tag{4}$$

Solution of a multiobjective optimization problems can be achieved in two ways. First, we can combine all objectives into a single one; a weighted sum

$$F(\mathbf{c}) = \sum_{i \in M} w_i f_i(\mathbf{c}) \tag{5}$$

where w_i>0 gives the relative importance of minimizing $f_i(\mathbf{c})$. Furthermore, each w_i must take into account the relative magnitudes or scales associated with $f_i(\mathbf{c})$, which may be specified in different units. This formulation gives rise to an unconstrained minimization problem. A similar approach is based on the solution of the following minimax problem

$$\min_{\mathbf{c}} \max_{i} \{ w_i f_i(\mathbf{c}) \} \tag{6}$$

where the weights are chosen as above. Second, we can generate the noninferior solution set and then find the best compromise solution by placing relative importance on the different objectives.

In this work, solution of the multiobjective optimization problem is achieved using the goal attainment method. In this method, noninferior solution vectors are found by solving the following problem (Gembicki and Haimes, 1975)

$$\begin{gathered} \min_{z,\,\mathbf{c}} \; z \\ \text{subject to} \\ f_i(\mathbf{c}) - w_i z \le f_i^* \end{gathered} \tag{7}$$

where w_i>0 are the weighting coefficients and z is an unrestricted scalar variable. The values of f_i^* can be thought as the desired levels of the performance index $f_i(\mathbf{c})$ (goals). The product zw_i is the degree of under- or overattainment of the goal f_i^*.

In the formulation given by equation (7) the designer expresses his intuitive knowledge of the problem through the selection of the weighting coefficients w_i. Heuristics are usually used in the selection of appropriate weights. However, at the beginning of a design, it is impossible to specify, in the ad hoc manner described above, the relative importance of each objective function. Hence, the design process is based on the adaptively revised assessment of the relative importance of each one of the various objectives and interaction between the designer and the optimization process is considered essential.

In order to avoid these problems, the method of Nye and Tits (1986) is used in this study. According to this method normalized objectives are defined by the transformation

$$\tilde{f}_i(\mathbf{c}) = \frac{f_i(\mathbf{c}) - f_i^G}{f_i^B - f_i^G} \tag{8}$$

where f_i^G and f_i^B are two chosen levels of satisfaction for the i objective function called the good and the bad level, respectively. They correspond to the higher and lower amount of designers satisfaction associated with each $f_i(\mathbf{c})$. Nye and Tits (1986) used the following rule of uniform satisfaction/ dissatisfaction for choosing the good and bad values: all objectives should provide to the designer the same level of satisfaction or dissatisfaction when they achieve their good and bad values, respectively. Thus, the following, modified goal attainment formulation is used in this study

$$\begin{gathered} \min_{z,\,\mathbf{c}} \; z \\ \text{subject to} \\ \tilde{f}_i(\mathbf{c}) - w_i z \le 0 \end{gathered} \tag{9}$$

In this formulation the worst level of designers dissatisfaction, over all design objectives, is minimized and the weighting coefficients w_i can now be set equal to one, since the levels of satisfaction/dissatisfaction are uniform. However, the weighting coefficients are included in the proposed formulation in order to be able to ensure that hard constraints, such as stability requirements, never become active.

3 Process and Controller Model

Most of the chemical processes exhibit monotone or essentially monotone step response and can be divided into two broad classes. The first class corresponds to self-regulating or stable processes that are usually described by the following first order plus delay time (FOPDT) model

$$G_P(s) = \frac{K_P e^{-ds}}{\tau s + 1} \tag{10}$$

where K_P is the process gain, τ the time constant and d the delay time. The second class corresponds to systems that show no self-regulation (unstable systems) that can be described by the integrator plus delay time (IPDT) model

$$G_P(s) = \frac{K_P e^{-ds}}{s} \tag{11}$$

The IPDT model can be seen as a limit of the FOPDT model with large values of the time constant.

In this study it is assumed that the controller used is the well known and extensively used PI controller having the following model

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right) \tag{12}$$

In tuning the PI controller the designer tries to satisfy several objectives considering the performance as well as the robustness of the closed loop system. A great number of tuning methods is now available for achieving single objectives such as decay ratio, phase and gain margins, resonant peak and frequency, overshoot and certain error integral criteria. However, these tuning methods are criticized for being case dependent. This means that when the same tuning method is applied to models of the same structure, but with different parameters, the closed loop response obtained may vary considerably.

In what follows, a number of simplified expressions for the calculation of phase and gain margins, resonant frequency and error integral indices are going to be summarized. These expressions are valid for processes that are adequately described by FOPDT and IPDT models and controlled using PI controllers.

FOPDT. Ho *et al*. (1995) have shown that when a PI controller is used in order to control a process that is described by FOPDT model, then, the phase and gain margins achieved can be approximated by the following equations

$$A_m = \frac{\pi}{4K_P K_C}\theta\left(1+\sqrt{1+\frac{4\theta}{\pi}\left(1-\frac{\tau}{\tau_I}\right)}\right) \tag{13}$$

$$\Phi_m = \frac{\pi}{2}+\frac{\pi}{4K_P K_C}\left(1-\frac{\tau}{\tau_I}\right)-K_P K_C\theta \tag{14}$$

where $\theta=d/\tau$, A_m is the gain margin and Φ_m is the phase margin. The resonant peak, M_P, is defined as

$$M_P = \max_{\omega}\left|\frac{G_C G_P}{1+G_C G_P}\right| \tag{15}$$

and the frequency at which this maximum occurs is called resonant frequency (ω_r). The closest point to the (-1,0) point on a Nyquist plot occurs at a frequency near to frequency ω_G the gain crossover frequency, defined as the smallest frequency for which the following condition holds

$$\left|G_C(j\omega_G)G_P(j\omega_G)\right|=1 \tag{16}$$

Often ω_G is a good approximation to ω_r (Rohrs *et al*., 1993) and can be calculated by the following approximation (Ho *et al*., 1995)

$$d\omega_G = \frac{\Phi_M+\left(\frac{\pi}{2}\right)\left(A_M-1\right)}{A_M^2-1} \tag{17}$$

Nishikawa *et al*. (1984) have shown that for the case of FOPDT models quite acceptable closed loop response is obtained when the PI controller tuning is based on the minimization of the following weighted integral of squared error (WISE)

$$J_{WISE}(\lambda)=\int_0^{\infty}\left(e(t)e^{\lambda t}\right)^2 dt \tag{18}$$

where t is the time and e(t) the error (deviation) between the desired and the real response of the process. The parameter λ is chosen to satisfy

$$\lambda=\frac{\gamma}{P_u} \tag{19}$$

where P_u is the ultimate period and the parameter γ is assigned skillfully so as to get the desired damping of the closed loop response. The Laplace transform of the weighted error can in general be written as the ratio of two polynomials in s, i.e.

$$L\left[e(t)e^{\lambda t}\right]=\frac{N(s-\lambda)}{D(s-\lambda)} \tag{20}$$

where

$$D(s-\lambda)=a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n \tag{21}$$

and

$$N(s-\lambda)N(-s+\lambda)=b_0 s^{2n-2}+b_1 s^{2n-4}+\cdots+b_{n-2}s^2+b_{n-1} \tag{22}$$

Then, J_{WISE} is obtained by the ratio of two determinants as follows (Newton *et al*., 1957)

$$J_{WISE} = (-1)^{n-1} \frac{1}{2a_0} \frac{B_n}{H_n} \tag{23}$$

where H_n is the Hurwitz determinant of D(s-λ) and B_n is obtained from H_n by replacing the first row by $(b_0,b_1,...,b_{n-1})$.

IPDT For the case of open loop unstable systems, that can be adequately described by IPDT model and are controlled using PI controllers, Kookos *et al*. (1997) have shown that phase and gain margins can be approximated by the equations

$$\Phi_M = \frac{\pi}{2}\left(\frac{\alpha-1}{\alpha}\right) - \frac{\alpha\beta}{2\pi} \tag{24}$$

$$A_M = \left(\frac{2\alpha}{\pi}\right)\frac{1}{4}\left\{\pi + (\pi^2 - 4\beta)^{1/2}\right\} \tag{25}$$

where the parameters α and β are given by

$$\alpha = \frac{\pi}{2K_P K_C d} \tag{26}$$

$$\beta = 4\frac{d}{\tau_I} \tag{27}$$

Kookos *et al*., have also shown that equation (16) holds true for the case of IPDT model. Furthermore, they have shown that the WISE method of Nishikawa *et al*., when applied to systems modeled as IPDT systems, gives quite acceptable PI controller tunings.

It is interesting to note that, in the cases where the model under study is given by

$$G'_P(s) = \frac{G_P(s)}{\tau' s + 1}, \ \tau >> \tau' \tag{28}$$

where the transfer function $G_P(s)$ is given by equations (10) or (11), then, the method presented in this section is also applicable by using a PID controller of the following form

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)(1 + \tau_D s) \tag{29}$$

and using $\tau_D = \tau'$.

4 PI Controller Tuning using the Goal Attainment Method

The PI controller parameters, K_C and τ_I , are adjusted so as to achieve the best compromise solution to a chosen, vector form, objective function. The particular control objectives are the satisfaction of certain phase and gain margins specifications, the maximization of the gain crossover frequency and the minimization of the WISE performance index. The good level for each objective function is chosen to correspond to the coordinates of the utopia point. The bad levels are chosen to represent uniform levels of dissatisfaction.

In the sequel, a number of simulation examples are given in order to clarify the method proposed above and also to provide a comparison of the proposed method with existing and well known tuning methods.

4.1 Example 1: Second order plant with delay

Edgar *et al*. (1981), Edgar and Hougen (1981), Harris and Mellichamp (1985), Seborg *et al*. (1989) and Abbas and Sawyer (1995) have extensively studied the following process model

$$G(s) = \frac{e^{-s}}{(10s+1)(5s+1)} \tag{30}$$

Using process reaction curve analysis the following approximate model can be obtained

$$G(s) = \frac{e^{-2.8s}}{12.9s + 1} \tag{31}$$

Based on the FOPDT approximation given by equation (31), a number of PI controller tunings have been proposed (the reader can consult the references given above).

In order to apply the proposed method, the good values of the controller design objectives are specified first. The values chosen are: 60° phase margin specification, 3.2 gain margin specification, 45 rad for the $d\omega_G$ product specification while the good value for the WISE is calculated through the solution of the scalar optimization problem

$$\min_{c_1, c_2} J_{WISE} \tag{32}$$

where the parameters c_1, c_2 over which the optimum is searched, are given by

$$c_1 = \frac{K_P K_C}{\theta}\ ,\ c_2 = \left(\frac{1}{1+\theta}\right)\left(\frac{\tau_I}{\tau}\right) \tag{33}$$

The controller parameters that satisfy each design objective are easily calculated through the simplified expressions given in the previous section. In this way, the utopia point is immediately found. Then, the bad values, i.e. values that provide uniform levels of dissatisfaction for each objective, have to be specified. To this end, extensive simulations were performed in order to examine the associated levels of dissatisfaction as we move away from the optimal or good values. It was found that a phase margin specification of 45°, a gain margin specification of 2.2, a $d\omega_G$ product specification of 30 rad and a 10% increase of the WISE performance index correspond to roughly uniform dissatisfaction levels.

Finally, the modified (with all weighting coefficients equal to one) goal attainment problem is solved and the controller parameters are obtained. For the process described by equation (31) the controller parameters are Kc=2.45 and τ_I=12.96. In Figures 2.a and 2.b, the closed loop response obtained is shown. For comparison, the close loop response obtained using the controller parameters proposed by Harris and Mellichamp (1985), is also included in Figure 2. As it is depicted, the proposed method yields grater overshoot and longer rise time when compared to the method of Harris and Mellichamp. However, the proposed method has the advantage that the input peak and input effort are smaller. Furthermore, the settling time achieved, when the proposed method is used, is considerably smaller than the one obtained with Harris and Mellichamp tunings.

4.2 Example 2: Process with large delay time

Control of systems with large delay time is notoriously difficult (Astrom and Hagglund, 1995). The transfer function given by

$$G(s) = \frac{e^{-3s}}{(s+1)^2(2s+1)} \tag{34}$$

has been studied by Yuwana and Seborg (1982), Jutan and Rodriguez (1984), Chen (1989), Lee *et al.* (1990) and Krishnaswamy and Pangalah (1996) as a severe test of their tuning and identification methods due to the large delay time involved. Using process reaction curve analysis the following approximate model is obtained

$$G(s) = \frac{e^{-4.5s}}{2.92s + 1} \tag{35}$$

The modified goal attainment problem is solved using the same good and bad values as in Example 1. The controller parameters obtained are K_C=0.424 and τ_I=3.537. For comparison, the refined Ziegler-Nichols tuning formula proposed by Astrom *et al.* (1992) and Astrom and Hagglund (1995) is also applied. In Figures 3.a. and 3.b. the closed loop responses are shown. The superiority of the proposed method over the method of Astrom *et al.* is clearly depicted. The reader is also encouraged to compare the responses given in Figure 3 with the ones given in the references above.

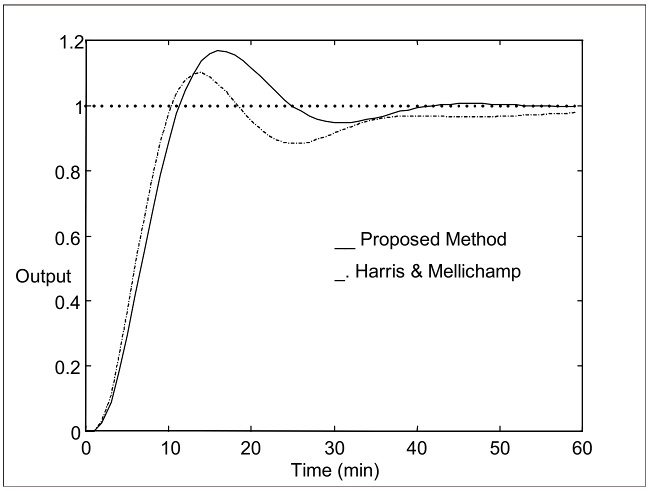

Figure 2.a Output response for Example 1.

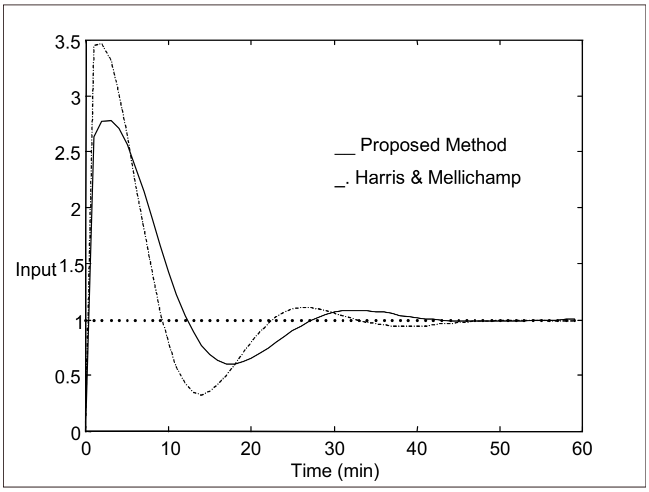

Figure 2.b. Input variation for Example 1.

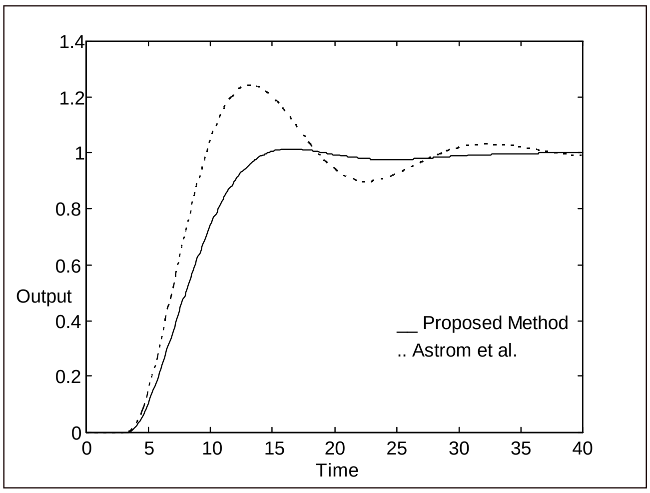

Figure 3.a. Output response for Example 2.

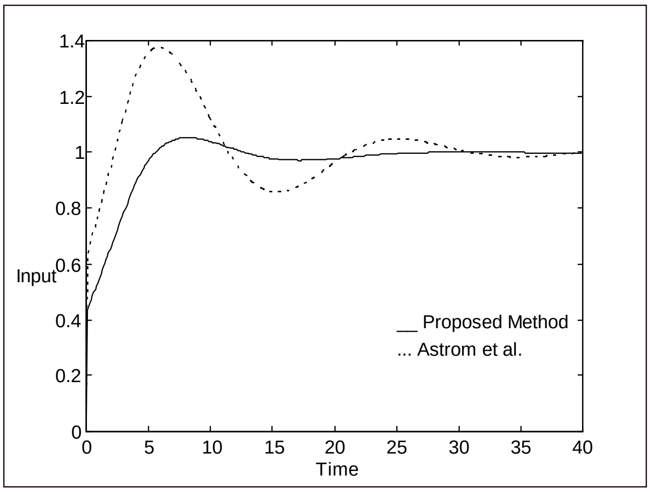

Figure 3.b. Input variation for Example 2.

4.3 Example 3: Integrator plus delay time process

Integrator plus dead time (IPDT) model was found to be a suitable model for a number of chemical processes. Chien and Fruehauf (1990), Tyreus and Luyben (1992), Friman and Waller (1994) and Luyben (1996) suggested that using the IPDT model for feedback controller tuning has several advantages. For SISO systems that contain two parameters, only one experiment is needed for the estimation of these parameters. For MIMO systems the parameters of the off-diagonal elements of the transfer function matrix can also be estimated during the relay experiment.

The process studied by Chien and Freuhauf (1990) and later by Tyreus and Luyben (1992) is considered. Parameter values are K_P=0.2 %/min and d=7.4 min. The good and bad values used in the modified, goal attainment formulation are the same as in the previous examples. The resulting controller parameters are K_C=0.3685 and τ_I=42.6 and correspond to α=2.88 and β=0.695. These values of the adjustable parameters α and β, are also valid with any IPDT process model, irrespectively of the particular numerical values of the model parameters.

In Figures 4.a and 4.b. the closed loop response to a set point step change is shown. 50% error in the estimation of the time delay is assumed in this simulation. It is observed that the proposed method gives quite acceptable results while the Chien and Fruehauf (CF) method yields an (marginally) unstable closed loop system. The Tyreus and Luyben (TL) method gives a fairly conservative closed loop response and the deviation between the actual and desired output remains significant even for times up to 20d. At the same time the variation of the input variable is close to the one obtained by the proposed method.

5 Conclusions

In this paper a new method for tuning PI or PID controllers for models commonly used in process control is presented. The method is based on the simultaneous satisfaction of more than one control objectives posed either in time or frequency domain. The problem is formulated as a multiobjective optimization problem and is solved using a modified goal attainment technique. The proposed method is compared with well known tuning methods through a number of simulation examples. The proposed method gives satisfactory results for models such as integrator plus delay time models and first order plus delay time models. Furthermore, the method is shown to give acceptable tunings even in the extreme cases where the delay time is the dominant feature of the system under study. Thus, the proposed method is applicable in a wide range of controller design problems commonly encountered in process control giving satisfactory results.

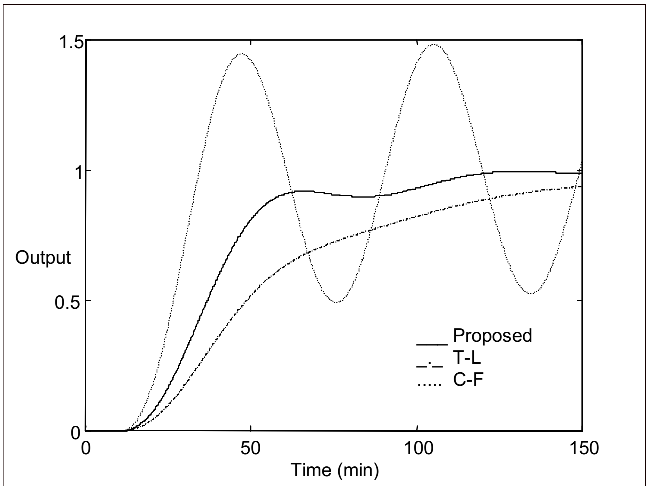

Figure 4.a. Output response for Example 3.

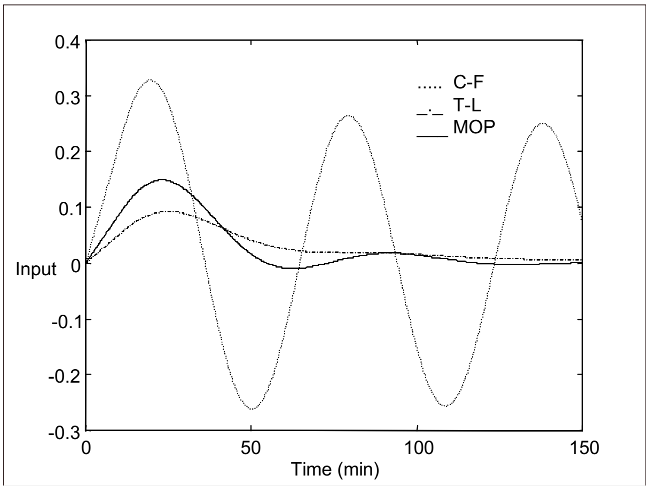

Figure 4.b. Input variation for Example 3.

6 Nomenclature

c = vector of the design parameters
D(s) = denominator polynomial of the Laplace transform of the weighted error
d = delay time (min)
e(t) = deviation between the desired and actual output of the system
f_i = I design objective
f^G = good level of design objectives
f^B = bad level of design objective
G_C = transfer function of the controller
G_P = transfer function of the process
H_n = Hurwitz determinant
J = performance index
K_C = controller gain (%/%)
K_P = process gain (%/min)
K_u = critical gain (%/%)
L = Laplace transformation operator

M_P = maximum of the complementary sensitivity function
N(s) = numerator polynomial of the Laplace transform of the error
P_u = ultimate period
Q = attainable set, defined in equation (3)
s = Laplace transform variable
t = time (min)
w_i = weighting coefficient
z = unrestricted variable used in the goal attainment formulation
A_M = gain margin
α = adjustable parameter given in equation (26)
β = adjustable parameter given in equation (27)
γ = adjustable parameter given in equation (19)
θ = dimensionless ratio
λ = time weight
τ_I = reset time (min)
τ_{CL} = closed loop time constant (min)
Φ_M = phase margin (rad)
ω = frequency (rad/min)
ω_G = gain crossover frequency (rad/min)
ω_r = resonant (rad/min)
ω_u = ultimate frequency (rad/min)

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