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An Outline for a Universal Logic System
A Logic System in Eight Truth Values

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A Universal Logic System establishes a truth value termed "neutral" between the contrary terms of truth and falsity. The Universal Logic System is composed of three primary logic sets: truth, falsity and neutrality together with three secondary sets: not-true, not-false and not-neutral. Furthermore, there are the Universal and Null logical sets. The Universal set is the union of all primary logical states. Distinction is made between the True set and the Universal set in the Universal logic system, unlike Boolean logic in which they are equated. This has fundamental implications as a many valued logic system. Traffic light states at a controlled intersection have been used as an illustration of Universal Logic.

I. Introduction

The Universal Logic System lays the foundation for an eight valued logic system. The term "Universal" refers to the fundamental reinterpretation of traditional logic values. Distinction is made between the True set and the Universal set in the Universal Logic system. The Universal set may be interpreted as certainty or uncertainty as is appropriate. The true set is defined as a unique primary element, unlike Boolean logic in which they are equated.

Following the tradition established by Boolean Algebra, logical states are designated as sets allowing logical states to be be analysed in Venn diagrams and defined in terms of union and intersection. The standard logic operations of conjunction, disjunction, implication and equivalence may be represented in truth tables of 8 x 8 matrices and, given the same preconditions, are fully consistent with that of Boolean Algebra. Each logical state is mutually consistent with other logic states and may be constructed either by complementing variables or by De-Morgan's rule.

II. Definitions for Truth Values

In Figure 1 the following unique set areas appear within the Venn diagram when the neutral truth value is introduced between the true and false labels.

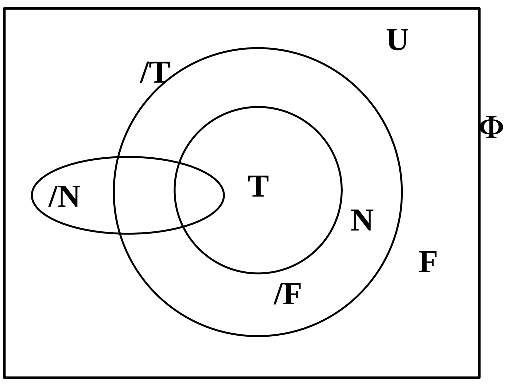


FIGURE 1 VENN DIAGRAM SHOWING ALL TRUTH VALUES

These labels are defined:
1. **T = True**
This is the truth label and consists of the area within the inner circle.
2. **F = False**
This is the false label and consists of the area beyond the outer circle.
3. **N = Neutral**
This is the neutral label and is the area between true and false.
4. **/N = Not-Neutral**
This is the complement of the neutral area and covers both true and false.
5. **/T = Not-True**
This is the complement of the true and consists of the areas that are false and neutral.
6. **/F = Not-False**
This is the complement of the false set and consists of the area that is both neutral and true.
7. **Φ = Null**
This is the empty set, **Φ,** and is the complement of the Universal set.
8. **U = Universal**
This denotes the entire logical set and consists of the union of the true, false and neutral areas.

Truth can be understood as the affirmation of the correctness of a given proposition. In propositional calculus, however, the True state can be defined in terms of other logical variables. A subsidiary definition of truth in terms of propositional calculus is provided by the Universal Logic System, where the True state is neither false nor neutral.

$\mathbf{T = / (F \cup N)}$

The False state is a primary logical variable being the negation or opposite of truth.

$\mathbf{F = \neg T}$

In the propositional calculus of a ternary logic falsity may be defined as being neither true nor neutral.

$\mathbf{F = / (T \cup N)}$ (T = True, N = Neutral)
$\mathbf{\quad = / (T + N)}$ (T, N disjoint)

The Neutral logic variable is a middle ground between the contrary terms of truth and falsity. A Neutral logic state is a primary or determinate logical state in that it is not a composite of other primary logic variables, however, it may be defined in terms of other logical variables.

Neutrality is initially defined as the state of being neither true nor false.

$\mathbf{N = / (T \cup F)}$
$\mathbf{\quad = / (T + F)}$ (Truth and Falsity are disjoint)

By De-Morgans rule Neutrality may be defined as being Not-True and Not-False.

$\mathbf{N = / T \cap / F}$

A proposition which is contingent or uncertain may be true or may be false and hence obtains the Not-Neutral logic status. The Not-Neutral logic state is a secondary logical variable in that it is a composite of the true and false states. In propositional calculus Not-Neutral set is defined as: $\mathbf{/ N = T \cup F}$

The Not-False set is the complement or contradictory of the false state. The Not False state includes the sets True and Neutral. It is therefore a secondary or potential logical variable in that it is composite of truth and neutrality. In propositional calculus, the Not-False set is defined as being either True or Neutral: $\mathbf{/ F = T \cup N}$ **or** $\mathbf{/ F = T + N}$ (as T and N are disjoint)

The Not-True state is the complement or contradictory of the true state. The Not-True state includes the False and Neutral sets. A proposition which is impossible to be true is regarded as being not-true. Non-truth is a secondary logical variable in that it consists of the union of the False and Neutral primary logical variables. In propositional calculus the Not-True set is defined as being False or Neutral

$\mathbf{/\,T = F \cup N}$

$\mathbf{/\,T = F + N}$

(Falsity and Neutrality are disjoint)

The null set, Φ, is defined as being neither True nor False nor Neutral. By De-Morgans law another definition of the null state may be derived.

$\mathbf{\Phi = /\,(T \cup F \cup N)}$

$\mathbf{\quad = /T \cap /F \cap /N = /U}$

The null set may be defined as being Not-True and Not-False and Not-Neutral. The empty logical state also denotes non-possibility or logical impossibility. For instance, it is impossible for the true and false sets to intersect as the two sets are disjoint. In propositional calculus this result has been formalised in the Principle of Non-Contradiction, that is: $\mathbf{T \cap F = \Phi}$

The Universal set represents the union of all members of the logical set, including propositions which are uncertain or indeterminate. In terms of probability the Universal set represents 100% certainty and is understood as occupying total logical possibility. As such the Universal class may be equated to "1", **U = 1**. The Universal set is a ternary logic variable, in that it is a composite of three primary logic variables: true, false and neutral. In propositional calculus it is denoted as being true or false or neutral: $\mathbf{U = T \cup F \cup N}$

The Universal set is the complement of the empty set meaning that it is a proposition which is not impossible. In propositional calculus it is expressed: $\mathbf{U = /\,\Phi}$

As the Universal set occupies total logical possibility the union of all primary truth variables must always equal one and in this sense it is understood as being complete certainty. However, the Universal set is interpreted both as certainty when equated with a single truth value and uncertainty when it is not. If the truth value of a given proposition is equated with the null set then the proposition is impossible. If the truth value of a given proposition is equated with the universal set then the proposition is certain. If the truth value of a given proposition is equated to neither the null nor the universal set then the proposition is uncertain.

III. Truth Tables for the Universal Logic System

A. Logical Conjunction

Logical conjunction is often referred to as intersection or simply as "and". When Boolean algebra is considered in two random state variables there are only 4 possible entries each variable may be assigned one of 2 states (T, F). The Boolean conditions may be stated as T= U = /F = /N = 1 and F = Φ = /T = N= 0.

q	p	$p \cap q$
F	F	F
T	F	F
F	T	F
T	T	T

TABLE 1. TRUTH TABLE SHOWING CONJUNCTION OF P AND Q IN BOOLEAN ALGEBRA.

It may be seen that the truth table for logical conjunction in 2 variables is a limited case of and is equivalent to the Universal Logic System case if falsity, F, is equal to Φ, and truth, T, is equal to 1 or the Universal state. F = Φ = 0 and T = U = 1 may be written for binary logic.

p	**q**	**p ∩q**
Φ	Φ	Φ
Φ	U	Φ
U	Φ	Φ
U	U	U

TABLE 2. TRUTH TABLE SHOWING P AND Q IN THE UNIVERSAL LOGIC SYSTEM

In the eight valued logic of the Universal system the truth table for logical conjunction may be written in a matrix form. For two random state variables p, q:

p \ **q**	**T**	**F**	**N**	**/N**	**/F**	**/T**	**Φ**	**U**
T	T	Φ	Φ	T	T	Φ	Φ	T
F	Φ	F	Φ	F	Φ	F	Φ	F
N	Φ	Φ	N	Φ	N	N	Φ	N
/N	T	F	Φ	/N	T	F	Φ	/N
/F	T	Φ	N	T	/F	N	Φ	/F
/T	Φ	F	N	F	N	/T	Φ	/T
Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ
U	T	F	N	/N	/F	/T	Φ	U

TABLE 3. TRUTH TABLE SHOWING P AND Q IN THE UNIVERSAL LOGIC SYSTEM, P ∩ Q

B. Logical Disjunction

The operation of logical disjunction is often referred to as "union" or simply "or". As the word "or" is ambiguous in everyday English usage the union symbol "∪" denotes inclusive disjunction. Thus, "p ∪ q" is interpreted as meaning p or q or both. Inclusive disjunction has 64 entries in the eight valued logic system but only 4 entries in Boolean algebra. Consider the truth table for the union of two state variables p, q in Boolean algebra.

p	**q**	**p ∪q**
F	F	F
F	T	T
T	F	T
T	T	T

TABLE 4. TRUTH TABLE SHOWING UNION OF VARIABLES P AND Q IN BOOLEAN ALGEBRA

It can now be seen that the truth table for logical disjunction is a limited case of the Universal Logic System. When F = Φ = 0 and T = U = 1 the truth table for Universal Logic System reduces to the standard 4 valued case and is equivalent to Boolean disjunction.

p	**q**	**p ∪q**
Φ	Φ	Φ
Φ	U	U
U	Φ	U
U	U	U

TABLE 5. TRUTH TABLE SHOWING UNION P OR Q IN THE UNIVERSAL LOGIC SYSTEM

Disjunction of two logical variables can be seen to take the maximum truth value or if the variables are disjoint to be the addition of the two variables. Union in the Universal Logic System may be expressed for two variables p, q:

p \ q	T	F	N	/N	/F	/T	Φ	U
T	T	/N	/F	/N	/F	U	T	U
F	/N	F	/T	/N	U	/T	F	U
N	/F	/T	N	U	/F	/T	N	U
/N	/N	/N	U	/N	U	U	/N	U
/F	/F	U	/F	U	/F	U	/F	U
/T	U	/T	/T	U	U	/T	/T	U
Φ	T	F	N	/N	/F	/T	Φ	U
U	U	U	U	U	U	U	U	U

TABLE 6. TRUTH TABLE SHOWING UNION P ∪ Q IN THE UNIVERSAL LOGIC SYSTEM

C. Implication

If a premise p implies a conclusion q it is denoted as p→q. Implication is generally understood as meaning "if p then q". In the case of Boolean algebra the truth table for material implication can be demonstrated for the premise, p, and the conclusion, q. Let the truth value, T = U = 1 and false value, F = Φ = 0.

p	q	p → q
0	0	1
0	1	1
1	0	0
1	1	1

TABLE 7. TRUTH TABLE FOR IMPLICATION IN BOOLEAN ALGEBRA

The truth table for implication is derived by examining whether p is a subset or equal to q, that is whether or not p ⊆ q. If p is a logical subset of q then p implies q is true otherwise it is false. This appears to be the broadest definition of implication of all cases. The truth table for material implication in Boolean algebra is identical to the formulas: p →q = /(p ∩ /q) = /p ∪ q

However, the Boolean formula for material implication applies only in this limited case and not in this expanded form of logic. In the Universal Logic System p implies q means that p is a logical subset or equal to q. In symbolic notation, if p ⊆ q then p → q is true, if p is not a subset a false value is given, denoted p ⊄ q then p → q is false. The truth table for implication in the Universal Logic System may now be constructed.

p \ q	T	F	N	/N	/F	/T	Φ	U
T	T	F	F	T	T	F	F	T
F	F	T	F	T	F	T	F	T
N	F	F	T	F	T	T	F	T
/N	F	F	F	T	F	F	F	T
/F	F	F	F	F	T	F	F	T
/T	F	F	F	F	F	T	F	T
Φ	T	T	T	T	T	T	T	T
U	F	F	F	F	F	F	F	T

TABLE 8. IMPLICATION TRUTH TABLE IN THE UNIVERSAL LOGIC SYSTEM, P→Q

D. Material Equivalence

The bi-conditional p implies q and q implies p is known as material equivalence and is denoted p ≡ q or as p↔q. This operation usually has the meaning that p is true if and only if q is true. The bi-conditional p ≡ q may be represented in the Boolean formula: (/p ∩ /q) ∪ (p ∩ q)

Given the precondition that T = U = 1 and F = **Φ** = 0, the truth table generated by the Universal Logic System is equivalent to that of Boolean algebra. However, it is found that the formula for material equivalence applies only in the limited case of Boolean Algebra.

p	**q**	**p ≡ q**
0	0	1
0	1	0
1	0	0
1	1	1

TABLE 9. TRUTH TABLE FOR MATERIAL EQUIVALENCE IN BOOLEAN ALGEBRA

If material equivalence, denoted p↔q, is interpreted in its broadest sense it means that the two statements p and q imply each other. It is interpreted to be the same formula as that of (p→q) ∩ (q→p). Material equivalence acts simply as equivalence of logical status between two propositions. The truth table for Material implication in the Universal Logic System is shown:

p \ **q**	**T**	**F**	**N**	**/N**	**/F**	**/T**	**Φ**	**U**
T	T	Φ	Φ	Φ	Φ	Φ	Φ	Φ
F	Φ	T	Φ	Φ	Φ	Φ	Φ	Φ
N	Φ	Φ	T	Φ	Φ	Φ	Φ	Φ
/N	Φ	Φ	Φ	T	Φ	Φ	Φ	Φ
/F	Φ	Φ	Φ	Φ	T	Φ	Φ	Φ
/T	Φ	Φ	Φ	Φ	Φ	T	Φ	Φ
Φ	Φ	Φ	Φ	Φ	Φ	Φ	T	Φ
U	Φ	Φ	Φ	Φ	Φ	Φ	Φ	T

TABLE 10. TRUTH TABLE FOR MATERIAL IMPLICATION IN THE UNIVERSAL LOGIC SYSTEM**, P↔Q**

This description of material equivalence gains a true entry, T, only when logical truth values are equivalent and a null, **Φ**, entry if not. In this case the truth table generated by the Universal Logic System strongly resembles that of Boolean algebra. The null entry is generated by the conjunction of disjoint variables true and false.

E. The Negation and Complement operation

Negation is the contrary or opposite of the given variable. It is often generated by the negative prefixes such as “un” or “dis” or ”im”. For example, the term “true” has its opposite in the term “untrue”. Negation and the Complement operation are distinguished in the 3 valued case of the Universal Logic System, in contradistinction to the Boolean case, where they are found to be equivalent.

p	**¬p**
T	F
F	T
N	/N
/T	/F
/F	/T
/N	N
Φ	U
U	**Φ**

TABLE 11. TRUTH TABLE FOR NEGATION IN THE UNIVERSAL LOGIC SYSTEM

The Complement operation provides the complement of a given variable. In the most precise sense of the word the complement is simply the "not" operation. The complement is generated in English by placing the "not" prefix before the term. For example, the term "true" has its complement in the term "not-true". Consider the truth table for Complement operation for a proposition p.

p	**/p**
T	/T
F	/F
N	/N
/N	N
/F	F
/T	T
Φ	U
U	Φ

TABLE 12. TRUTH TABLE FOR COMPLEMENT OPERATION IN THE UNIVERSAL LOGIC SYSTEM

IV. De-Morgan's Laws in the Universal Logic System

De-Morgan's theorem gives an equivalence between /(p ∩ q) and /p ∪ /q. A truth table for /p ∪ /q (not p or not q) should be identical to the one listed above if this is the case.

p \ **q**	**T**	**F**	**/T**	**/F**	**N**	**/N**	**Φ**	**U**
T	/T	U	U	/T	U	/T	U	/T
F	U	/F	/F	U	U	/F	U	/F
/T	U	/F	T	/N	/N	/F	U	T
/F	/T	U	/N	F	/N	/T	U	F
N	U	U	/N	/N	/N	U	U	/N
/N	/T	/F	/F	/T	U	N	U	N
Φ	U	U	U	U	U	U	U	U
U	/T	/F	T	F	/N	N	U	Φ

TABLE 13. TRUTH TABLE SHOWING NON-P AND Q IN UNIVERSAL LOGIC, /(P∩Q)

q \ **p**	**T**	**F**	**/T**	**/F**	**N**	**/N**	**Φ**	**U**
T	/T	U	U	/T	U	/T	U	/T
F	U	/F	/F	U	U	/F	U	/F
/T	U	/F	T	/N	/N	/F	U	T
/F	/T	U	/N	F	/N	/T	U	F
N	U	U	/N	/N	/N	U	U	/N
/N	/T	/F	/F	/T	U	N	U	N
Φ	U	U	U	U	U	U	U	U
U	/T	/F	T	F	/N	N	U	Φ

TABLE 14. TRUTH TABLE OF NOT P OR NOT Q IN THE UNIVERSAL LOGIC SYSTEM /P ∪ /Q

The logical variables for each case are calculated from the union and intersection truth tables already given. The two truth tables can be seen to be identical, confirming De-Morgan's rule and the validity of the Universal logic. Logical non-disjunction of two terms p and q may be expressed in English usage as "it is neither p nor q". De-Morgan's theorem states that the expression, "it is neither p nor q", is equivalent to the statement, "it is not-p and not-q". Thus material equivalence may be expressed in propositional calculus as /(p ∪ q) ≡ /p ∩ /q. If this is the case, a truth table for the /p ∩ /q should be equivalent to the truth table of /(p ∪ q)

p \ q	T	F	/T	/F	N	/N	Φ	U
T	/T	N	Φ	F	F	N	/T	Φ
F	N	/F	T	Φ	T	N	/F	Φ
/T	Φ	T	T	Φ	T	Φ	T	Φ
/F	F	Φ	Φ	F	F	Φ	F	Φ
N	F	T	T	F	/N	Φ	/N	Φ
/N	N	N	Φ	Φ	Φ	N	N	Φ
Φ	/T	/F	T	F	/N	N	U	Φ
U	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ

TABLE 15. TRUTH TABLE OF NON-DISJUNCTION IN THE UNIVERSAL LOGIC SYSTEM, /(P∪Q)

p \ q	T	F	/T	/F	N	/N	Φ	U
T	/T	N	Φ	F	F	N	/T	Φ
F	N	/F	T	Φ	T	N	/F	Φ
/T	Φ	T	T	Φ	T	Φ	T	Φ
/F	F	Φ	Φ	F	F	Φ	F	Φ
N	F	T	T	F	/N	Φ	/N	Φ
/N	N	N	Φ	Φ	Φ	N	N	Φ
Φ	/T	/F	T	F	/N	N	U	Φ
U	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ

TABLE 16. TRUTH TABLE OF "NOT P AND NOT Q" IN THE UNIVERSAL LOGIC SYSTEM, /P ∩ /Q

The truth tables for the /p ∩ /q and /(p ∪ q) are shown to be equivalent as is expected.

V. The Generalised Boolean Algebra and the Universal Logic System

The generalised Boolean Algebra for dimensions, m≥2 bears a strong structural similarity to the Universal Logic and a comparison of the systems of logic should be made. The Universal Logic system appears to be the extended Boolean Algebra known as B(8). The structure of the extended Boolean Algebra is based upon that of the finite distributive lattice and it can be shown that the lattice structure of Boolean Algebra is identical to that of Universal Logic in the same dimension.

However, basic differences arise between the two systems of logic when interpretations of the truth values are considered. The truth tables for implication and material equivalence are quite different. It is in the interpretation of the truth and falsity as primary elements or atoms of the lattice in Universal Logic where significant differences between the two systems occur. The extended Boolean Algebra and Universal Logic is considered in the two dimensional case and give rise to a total of 4 truth values. It is a point of note that Universal Logic affixes the primary variables of the lattice to be truth, falsity and neutrality while the extended Boolean Algebra leaves the "atoms" of the lattice as unassigned mathematical variables. The case of B(4) and U(4) are considered in the truth tables below and Φ=0, T=1, F=2, U=3[9] are used for the purposed of comparison. Universal Logic in 4 truth values is denoted as U(4) and Boolean Algebra in 4 truth values is denoted as B(4).

B(4)

	+	Q 0	1	2	3
P	0	0	1	2	3
	1	1	1	3	3
	2	2	3	2	3
	3	3	3	3	3

U(4)

	$\cup$	Q 0	1	2	3
P	0	0	1	2	3
	1	1	1	3	3
	2	2	3	2	3
	3	3	3	3	3

	$\bullet$	Q 0	1	2	3
P	0	0	0	0	0
	1	0	1	0	1
	2	0	0	2	2
	3	0	1	2	3

	$\cap$	Q 0	1	2	3
P	0	0	0	0	0
	1	0	1	0	1
	2	0	0	2	2
	3	0	1	2	3

TABLE 17. TRUTH TABLES FOR B(4) AND U(4) LATTICES

Implication corresponds with the highest element in the lattice and the highest element in the lattice is interpreted as "is the case" for the extended Boolean Algebra[12]. A comparison of the truth table shows that implication and material equivalence are not equal in the two systems.

B(4)

	$\rightarrow$	Q 0	1	2	3
P	0	3	3	3	3
	1	0	3	0	3
	2	0	0	3	3
	3	0	0	0	3

U(4)

	$\rightarrow$	Q 0	1	2	3
P	0	1	1	1	1
	1	2	1	2	1
	2	2	2	1	1
	3	2	2	2	1

	$\equiv$	Q 0	1	2	3
P	0	3	0	0	0
	1	0	3	0	0
	2	0	0	3	0
	3	0	0	0	3

	$\equiv$	Q 0	1	2	3
P	0	1	0	0	0
	1	0	1	0	0
	2	0	0	1	0
	3	0	0	0	1

TABLE 18. TRUTH TABLES FOR B(4) AND U(4) LATTICES

However, the comparison between the two systems in this way cannot actually be made as truth is equivalent to the unit (4) and falsity the null set (0) in B-algebra, while there is no analogue for lattice elements (1) and (2). The conclusion is that Universal Logic is not equivalent to the lattices of extended Boolean Algebra for m≥2 dimensions.

VI. Logical Sets and Subsets

The 8 truth labels may be viewed as sets and subsets of one another as there exist 6 possible subset relations in the Universal Logic system:

(1) $\Phi \subseteq T \subseteq /F \subseteq U$
(2) $\Phi \subseteq T \subseteq /N \subseteq U$
(3) $\Phi \subseteq F \subseteq /T \subseteq U$
(4) $\Phi \subseteq F \subseteq /N \subseteq U$
(5) $\Phi \subseteq N \subseteq /T \subseteq U$
(6) $\Phi \subseteq N \subseteq /F \subseteq U$

These results may be interpreted in the following way:
The null set Φ is the proper subset of all other logical sets.
The true set T is the proper subset of the not-false, not-neutral and universal sets.
The false set F is the proper subset of the not-true, not-neutral and universal sets.
The neutral set is the proper subset of the not-true, not-false and universal sets.
The not-true set is a proper subset of the universal.
The not-false set is a proper subset of the universal.
The not-neutral set is a proper subset of the universal.
The universal set is the set of all other logical sets.

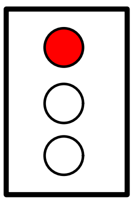
Note that the subset symbol $\subseteq$ means that the set may be equivalent to or a proper subset of another set. Similarly, all 8 logical states may be equated by the above listed subset relations as the case requires.

A necessary outcome of the new system produces some important results. One important axiom of Boolean algebra is that **1 ≠ 0**, and this axiom effectively retains consistency within the logic system thereby banning paradox. Thus, Boolean lattices are found to exist only in dimensions 1 and greater. The new interpretation of Universal logic allows a zero dimensional lattice element to exist with consistency maintained for all theorems. The number of logical values n per dimension is given by the formula, $n = 2^m$, where m is the dimension of the lattice. If we let m= 0 and n=1, all truth values are equivalent such that U= Φ. The zero dimensional lattice is of course the single point and therefore Universal Logic lattices exist in dimensions greater than or equal to zero.

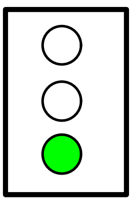
VII. A Many Valued Logic System

A traffic light at a controlled intersection may be employed as an illustration of Universal logic. If a proposition is defined: such that p is the statement, “Proceed through the controlled intersection”. The traffic light states can be used to confirm or deny this proposition.

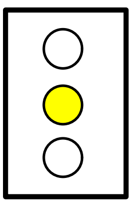
Standard Traffic Light States



The red traffic light may be regarded as the denial of the proposition, p. The proposition, p, is false (p = F). Thus the motorist is required to stop at the controlled intersection.

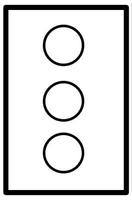


The green traffic light may be regarded as the affirmation of the proposition, p. The proposition, p, is true (p = T).Thus, the motorist is required to proceed through the controlled intersection.

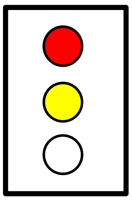


The amber traffic light is understood as a contingent statement: prepare to stop or proceed through the intersection if it is safe to do so. The proposition, p, is true or false (p = /N).Thus, the motorist is required to use discretion at the controlled intersection.

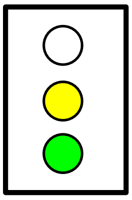
Aberrant Traffic Light States



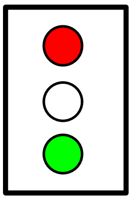
No traffic lights on mean that the traffic light imperatives do not apply. The traffic lights have no bearing on the truth or falsity of proposition and p is neither true nor false (that is neutral, $p = N$). Thus, the motorist is required to ignore traffic lights and apply the standing road rules at the intersection.



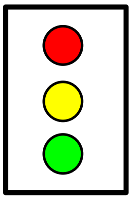
Both red and amber lights on mean that there is an imperative to stop and an instruction prepare to stop or proceed if it is safe. In propositional calculus, $p = F \cap /N = F$. Thus the imperative to stop overrides the contingent and the driver must stop at the controlled intersection.



Green and amber lights on mean that there is an imperative to go or prepare to stop. In propositional calculus, $p = T \cap /N = T$. Thus, the imperative to go overrides the contingent and the driver must proceed through the controlled intersection.



Green and red lights on mean that there is an imperative to go and to stop ! In propositional calculus, $p = T \cap F = \Phi$. The imperative to go and stop at once is impossible and cannot be executed. This is the condition for paradox.



Green, amber and red lights on mean that there is an imperative to go and to stop and to prepare to stop or to proceed if it is safe! In propositional calculus, $p = T \cap F \cap /N = \Phi$. Similarly, the imperative to go and stop and stop or go is impossible and cannot be executed simultaneously.

The traffic light as a demonstration of a many valued logic system reveals five of eight possible logical states in the Universal Logic system, that is, true, false, neutral, not-neutral and impossible logic states. The results of this analysis also concur with our natural interpretations of the traffic light states.

VIII. Conclusion

An eight valued logic system has been outlined that allows a broader range of propositions to be analysed. Uncertain or contingent propositions may be given the appropriate Universal or Not-Neutral logical states. Partially - certain variables such as Not-True and Not-False states may also be given a logical state without loss of precision. Impossible or paradoxical statements may be given the null or empty logical state. Irrelevant or meaningless propositions may be assigned as neutral. Finally, standard true and false logical values may also be assigned to relevant propositions.

A practical development of the Universal Logic System is the fact that logical variables are calculated by the standard set operations of union and intersection as set variables, unlike several of the many valued matrix logic systems which have been constructed by arbitrary rules. The truth tables of Universal Logic can be seen to be identical to those of the equivalent Boolean functions given the same preconditions. Essentially, the Universal Logic System offers an expansion of binary logic while retaining and building upon this system of logic.

As the Universal Logic develops pivotal elements of the interpretation of a many valued logic system, a new formula for validity is required. Typically, a tautological statement is valid and is equated to the unit, U, or 1. A well formed formula is called a tautology if its value is truth, t, for every system of values of its variables[3]. In Boolean Algebra a statement is tautological if it is equated to the unit, U, or 1. The identity formula, p ⊃ p, is a standard example of a tautology as it is true in all cases. Universal Logic upholds the definition of validity with the further criterion that truth is to be equated to the Universal state and is thus certain, that is, **T = U =1**. A further requirement for validity can be imposed, such that, no tautological statement which is certainly true can also be paradoxical or impossible. In Universal logic, impossibility is associated with the null set, Φ. Therefore, the fully developed criterion for validity is that *truth is certain and is not null*. The truth values may be written, **T = U ≠ Φ**. This formulation for validity allows the system of logic to maintain consistency within all its theorems without contradiction.

Universal Logic establishes a consistent logic system with a third intermediate truth value neutrality between true and false, and in combination with other meta-logic states forms a coherent schemata for an expanded logic system. Universal logic may be viewed as a finite distributive lattice, however, truth values in the first three dimensions are affixed rather than assigned to "variables" of the lattice. In this sense the Venn diagram is probably the best way of examining the new logic. An axiomitisation of the Universal Logic system is being undertaken.

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