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Actuator Design for Aircraft Robustness Versus Category II PIO

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Abstract

In this paper we deal with the analysis of Pilot in the Loop Oscillations (PIO) of Category II (with rate and position limiting), a phenomenon usually due to a misadaptation between the pilot and the aircraft response during some tasks in which tight closed loop control of the aircraft is required from the pilot, with the aircraft not responding to pilot commands as expected by the pilot himself. We propose an approach, based on robust stability analysis, which assumes that PIO are characterized by a limit cycle behaviour. In this approach the nonlinear elements are substituted by fictitious linear parameters, which can be considered time-invariant or time-varying; in this way we obtain two criteria for robustness versus Category II PIO. If, using the proposed criteria, the aircraft under consideration is shown to be Category II PIO prone, since limit cycles occurrence is due to a bad design of the nonlinear actuators, we propose an algorithm which, taking into account the trade-off between realization costs and performances, provides the guidelines for the design of actuators which should guarantee robustness versus Category II PIO. Finally, to demonstrate the use of the new proposed method, we apply our technique to a case study, namely the X-15 aircraft Landing Flare PIO (Matranga, 1961).

1 Introduction

Pilot In the loop Oscillations (PIO) is a well known and sadly famous phenomenon in the field of Handling Qualities of aircraft, which has been encountered and studied well before the advent of active control technology and Fly-by-Wire flight control systems (FCS). Its origin is a misadaptation between the pilot and the aircraft during some task in which tight closed loop control of the aircraft is required from the pilot, with the aircraft not responding to pilot commands as expected by the pilot himself. This situation can trigger a pilot action capable of driving the aircraft out of pilot control, which in some cases can only be recovered by the pilot releasing the column and exiting from the control loop.

The introduction of Fly By Wire FCS has in a sense exacerbated the problem of PIO, since the multitude of FCS modes, which can be easily designed and included into the new Digital FCS, can very easily disorientate the pilot in the interpretation of the aircraft response to his actions. Indeed three elements are considered in PIO analysis: the pilot, the aircraft dynamics and the trigger, an event which can introduce the misadaptation (McRuer *et al.*, 1997). Examples of the trigger are a FCS mode change, an unexpected nonlinear behaviour in the aircraft response, or a variation in the pilot control behaviour, such as an increase of

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the pilot gain. This situation has forced the U.S. military authorities to write down since 1982 explicit Flying Qualities Requirements for PIO in their Military Standard Specification Documents (Mil., 1980). An even greater emphasis is given to PIO detection criteria in the new issues of this Standard (Mil., 1990, 1994).

Because of the highly destructive potential of the PIO phenomenon a great effort has been spent in the last years in many research programs both in USA and Europe (Klyde *et al.*, 1995; Anderson and Page, 1995; McRuer *et al.*, 1997; Duda, 1997b) to study PIO, in order to derive methods which will be able to predict the tendency of aircraft to develop PIO.

PIO phenomena are commonly divided into three categories: *Category I* (the closed loop pilot vehicle system has a linear behaviour), *Category II* (the closed loop pilot vehicle system has a partial nonlinear behaviour), *Category III* (the closed loop pilot vehicle system has a highly nonlinear behaviour).

In this paper we focus on Category II PIO, which are mainly characterised by the saturation of position or rate limited elements. This kind of nonlinearity is unavoidably present in every aircraft, because of physical constraints of elements such as stick/column deflections, actuators position and rate limiters, limiters in the controller software and so on. In particular, rate limited actuators can expose the pilot to a sudden change of the dynamics of the augmented aircraft (flying qualities cliff) and have been indicated as the concurring cause to various high dramatic PIO incidents/accidents in the last years, both in high performance fighter aircraft, such as the YF22 (Dornheim, 1992) and the JAS-39 (Kullberg and Elcrona, 1995), and in transport aircraft, such as the C-17 (Iloputaife, 1997) and the B777 (Dornheim and Huges, 1995). The resulting PIO has the form of a limit cycle of the nonlinear system; thus limit cycle analysis is a sensible way to analyse the aircraft in order to predict this kind of PIO.

Two methods for the analysis of Category II PIO are currently available: the classical *Describing Function* (DF) method (see (Klyde *et al.*, 1995)), and the more recent *Open Loop Onset Point* (OLOP) method (see (Duda, 1997a)), which derives from the first. We have recently proposed (Scala, 1998; Amato *et al.*, 1999) a novel method to predict Category II PIO which has been shown to be equivalent to the DF method when there is only one nonlinear element in the loop; however the advantage of our method is that it does not suffer from the computational limitations of the DF approach and can be easily extended to the multi-nonlinear elements case.

This new detection criterion is based on a methodology for robustness analysis of dynamic properties of linear systems subject to time-invariant parameters (Verde, 1992). The methodology has been applied in the past with a good success to perform sensitivity analysis of flying qualities with respect to uncertainties of physical parameters of the augmented aircraft (Cavallo *et al.*, 1992a, 1990).

On the other hand we shall show that the methodology which tests robust stability versus time-invariant parameters (and therefore also DF based methods) may result optimistic in detecting PIO proneness of aircrafts; therefore a conservative approach which assumes time variation of the parameters is proposed; such method makes use of the Quadratic Stability approach (Barmish, 1983). By the use of both methods a complete analysis of the nonlinear system can be performed.

One of the goal of the paper is to provide a criterion, based on the above-mentioned analysis methods, to establish wether a given aircraft is free from PIO of Category II.

When the proposed test shows that the aircraft is PIO prone, since limit cycles occurrence is due to a bad design of the nonlinear actuators, we propose an algorithm which, taking into account the trade-off between realization costs and performances, provides the guidelines for the design of actuators which should render the aircraft robust versus Category II PIO.

Finally, to demonstrate the use of the new method, we shall apply our technique to a case study, namely the X-15 aircraft Landing Flare PIO (Matranga, 1961).

2 Problem Description

The formal definition of PIO given in U.S. Military Specifications (Mil., 1980) is:

There shall be no tendency for pilot-induced oscillations, that is, sustained or uncontrollable oscillations resulting from the efforts of the pilot to control the airplane.

As defined in the MIL specs, PIO is an umbrella under which the same phenomenon (closed loop pilot vehicle oscillation) can show with very different behaviours, mainly depending on the underlying cause of the PIO occurrence. In the following we give a classification of PIO (McRuer *et al.*, 1997) which takes into account some possible different behaviours of the closed loop pilot vehicle system during the PIO. In the given classification three different behaviours are recognised, leading to three PIO categories:

- **PIO Category I.** The closed loop pilot vehicle system has a linear behaviour.
- **PIO Category II.** The closed loop pilot vehicle system has a nonlinear behaviour, mainly characterised by the saturation of position or rate limited elements
- **PIO Category III.** The closed loop pilot vehicle system has a highly non linear behaviour, with no further peculiar characteristic.

This classification allows to categorise PIO detection criteria according to their potentiality to reveal the PIO tendency for the various categories.

In this paper we focus on Category II PIO; in this case, as said, PIO are mainly limit cycles due to the presence in the control loop of the nonlinear actuators. Since a limit cycle implies Lyapunov instability of the system, in the context of this paper the words "PIO", limit cycle and "instability" will be used with the same meaning.

Two methods for the analysis of Category II PIO are currently available: the classical *Describing Function* (DF) method (see (Klyde *et al.*, 1995)) and the more recent *Open Loop Onset Point* (OLOP) method (see (Duda, 1997a)).

The DF method is an "old" but honoured analytical tool for general nonlinear systems, which is capable to reveal PIO as limit cycles of the nonlinear system. Two drawbacks exist for this method. First, the graphical nature of the classical procedure limits the extension of its applicability. Second, the numerical approach, which has been recently proposed to make full use of the computing power of modern computers, requires an a priori estimate of possible limit cycles, because it is based on the solution of a nonlinear equation. Moreover a basic assumption to simplify the analysis is that the nonlinear elements are independent each other, i.e. their describing functions are those obtained in the case of a single nonlinearity.

In this paper we provide two different criteria for PIO analysis. Both criteria are based on the Robust Stability Analysis of a suitable linear system, obtained by substituting, in the original nonlinear system, the nonlinear element with an uncertain linear parameter. The first criterion, in which the uncertain parameter is assumed *time-invariant*, has been shown (see (Scala, 1998; Amato *et al.*, 1999)) to be equivalent, in the prediction of Category II PIO, to the DF analysis method (when we deal with a system containing a single nonlinear element). However, as shown in the above-referenced papers, the robust stability analysis is easier to perform, can be easily extended to deal with the multi-nonlinear elements case and can give more comprehensive results.

On the other hand, since the input-output gain associated with the saturation is time-varying, the methodology which tests robust stability versus time-invariant parameters (and therefore also DF based methods) may result optimistic in detecting PIO proneness of a given aircraft; therefore a second criterion which assumes *time variation* of the parameter is proposed; such method (which may give conservative results) makes use of the Quadratic Stability approach (Barmish, 1983). By the use of both methods a complete analysis of the nonlinear system can be performed.

The main objective of this paper is to show that, in the *analysis* context, methods based on robust stability can be used to establish PIO proneness of a given aircraft, while in a *synthesis* context they can be used to design suitable actuators which should not lead to aircraft instability.

Let us refer to the block diagram in Figure 1, where a classical closed loop scheme for the study of Category II PIO occurrence is considered. Here it is assumed that the pilot is engaged in some tracking task, for instance a pitch attitude control task. A roll-axis control task can be also analysed with no modifications to the method.

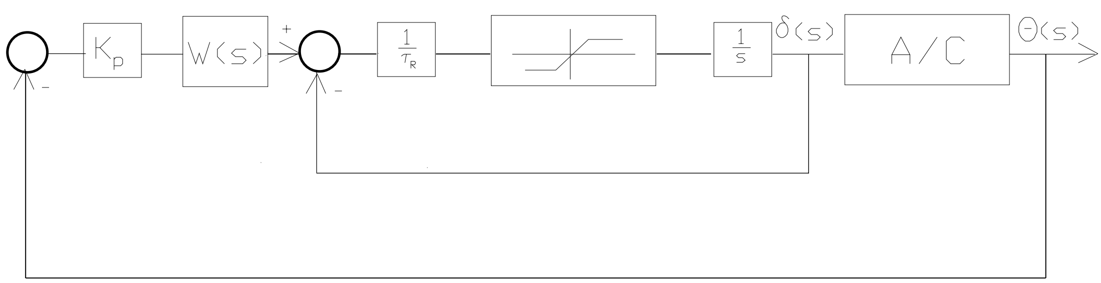

Figure 1: Closed loop scheme for the study of Category II PIO

The main blocks in Figure 1 are: the pilot transfer function, given by the series of the gain K_p and the normalized filter $W(s)$[1]; the nonlinear actuator, represented by the inner closed loop as a first order system of linear bandwidth $1/\tau_R$ with rate limiting provided by the saturation nonlinearity (normalised to be simmetric and with unitary slope) which precedes the position integrator; and the aircraft dynamics transfer function $\theta(s)/\delta(s)$ from the control surface position to the variable controlled by the pilot.

It is worth noting that in this paper, for the sake of simplicity, we consider a scheme with one nonlinear element; as said, the approach proposed in the paper can be easily extended to deal with the multi-nonlinearity case.

Concerning the notation related to the normalized nonlinearity, refer to Figure 2. With y_{max} we denote the maximum output amplitude; note that the nonlinearity output is dimensionally an angular rate and will be also denoted by $\dot{\delta}_{max}$. Obviously the linear treshold in input is given by $u_T = y_{max}$; finally u_{max} denotes the maximum input amplitude.

Now consider the scheme depicted in Figure 3, where the nonlinear element has been replaced by the linear gain L. It is clear that, when the actuator is not saturated, $L = 1$ (recall that the

[1] In this paper the problem of how to model the pilot behaviour, which also is of fundamental importance to gain more insights in the PIO generetion phenomena, is not treated; the interested reader is referred to (McRuer *et al.*, 1997).

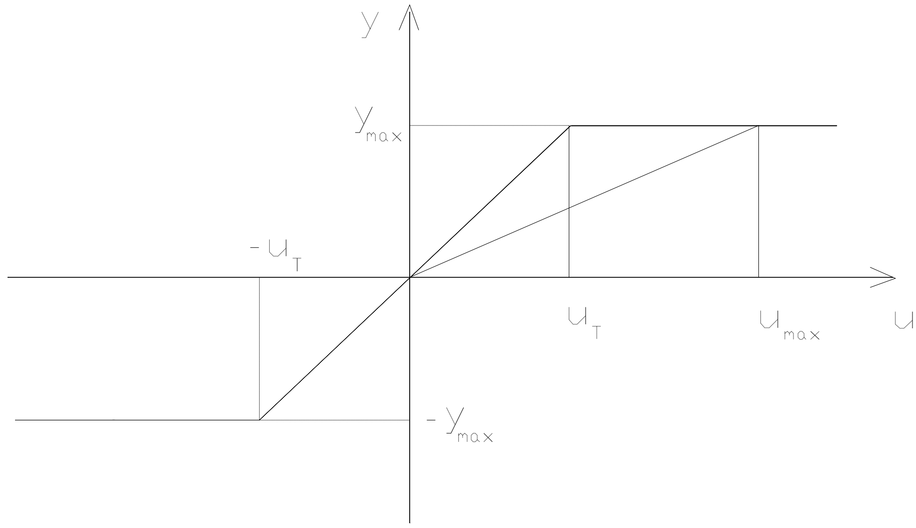

Figure 2: The normalized nonlinear element

nonlinearity has been normalized to have unitary slope); in the same way if we have an estimate of u_{max}, that is the maximum input entering the nonlinear element, the minimum value attained by L is $L_{min} := y_{max}/u_{max}$. Therefore we can conclude that $L \in [L_{min}, 1]$.

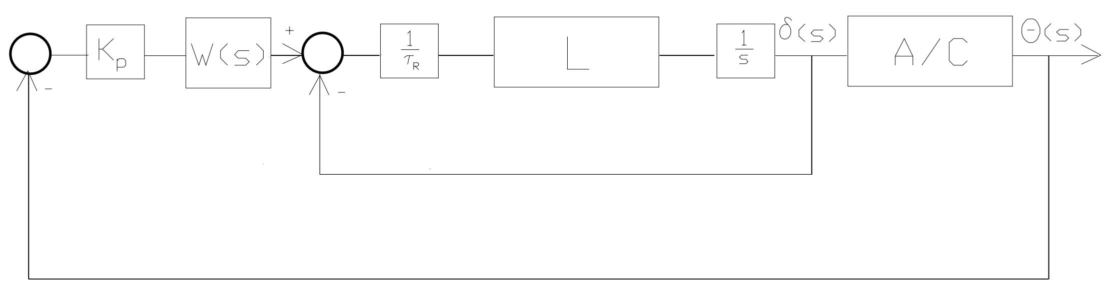

Figure 3: Replacement of the nonlinear element with the linear gain L

As said, in the next sections two different stability analysis will be performed using the linear scheme in Figure 3. Stability will be tested versus the parameter L which, representing the gain of the nonlinear element, plays a fundamental rôle in the possible development of Category II PIO. Another parameter which is considered in the stability analysis is the pilot gain K_p; indeed it is clear that delicate and full attention manoeuvres, like tracking, aerial refuelling, etc., require an high pilot gain which can trigger the PIO occurrence.

The first analysis assumes that the gain L is time-invariant; the goal is to determine the region in the $K_p - L$ plane for which the closed loop dynamic matrix associated to the scheme in Figure 1 is Hurwitz; we shall refer to this analysis as the "optimistic" or the "weak" one, because, as we shall see, it does not guarantee stability of the nonlinear system in Figure 1. Conversely, the second analysis, referred as the "strong" one, assumes that the parameter L

is a time-varying function with co-domain in $[L_{min}, 1]$; this, at the price of some conservatism, guarantees stability of the nonlinear scheme in Figure 1.

Obviously by the use of both methods a complete analysis of the nonlinear system can be performed. In particular, if the stability regions computed according to the two procedures are close to each other, one analysis validates the other one.

3 A PIO Detection Procedure Based on Robust Stability Analysis of a LTI System Subject to Uncertain Time-Invariant Parameters

We assume that the aircraft under consideration is Category I PIO free; this means that for $L = 1$ (actuator working in the linear range) a full excursion of the admissible pilot gain, say $[0, K_{p_{max}}]$, is allowed without destabilizing the aircraft.

Let $A_{CL}(K_p, L)$ be the closed loop system matrix associated to the scheme in Figure 3; note that the dependence on both parameters K_p and L is affine. Now let us denote, in the $K_p - L$ plane, by $\mathcal{R}_{oper}$ and by $\mathcal{R}_H$ the operating region and the Hurwitz stability region defined as follows:

$$\mathcal{R}_{oper} := [L_{min}, 1] \times [0, K_{p_{max}}] \tag{1}$$
$$\mathcal{R}_H := \{(L, K_p) \,:\, A_{CL}(L, K_p)\,\text{is Hurwitz}\} \tag{2}$$

Condition 1 *A condition for robustness versus Category II PIO (weak version)*
The aircraft depicted in Figure 1 is Category II PIO free if

$$\mathcal{R}_H \supseteq \mathcal{R}_{oper} \tag{3}$$

■

We call (3) "weak condition" because it does not guarantee stability of the original nonlinear system depicted in Figure 1. This is due to the fact that, in the definition of the stability region $\mathcal{R}_H$, we consider the gain L as an uncertain time-invariant parameter (indeed we just check Hurwitzness of $A_{CL}(K_p, L)$), while such parameter actually varies in time since, during saturation, it equals the ratio between y_{max} and the instantaneous input to the nonlinearity (see also Section 4). Robust stability versus time-invariant parameters does not imply stability versus time-varying parameters (Amato *et al.*, 1992). Therefore the PIO robustness condition (3) must be validated through a conservative analysis which takes into account the variation of the parameter L; this is done in Section 4.

Condition (3) is equivalent to require that the Boundary of the Hurwitz stability region $\mathcal{R}_H$ does not intersect the boundary of the operating region $\mathcal{R}_{oper}$. Based on the above observation, a Category II PIO free aircraft must *necessarely* exhibit an Hurwitz stability region like the one depicted in Figure 4.

The situation reported in Figure 5 is associated to an aircraft which is PIO prone, because for some admissible values of K_p and L the system becomes unstable. It is interesting to note that the point where the boundary of $\mathcal{R}_H$ crosses the boundary of the operating envelope corresponds to a pair of pure imaginary poles of the closed loop system whose frequency coincides with the limit cycle frequency obtained by a Describing Function analysis of the system (see (Scala, 1998)).

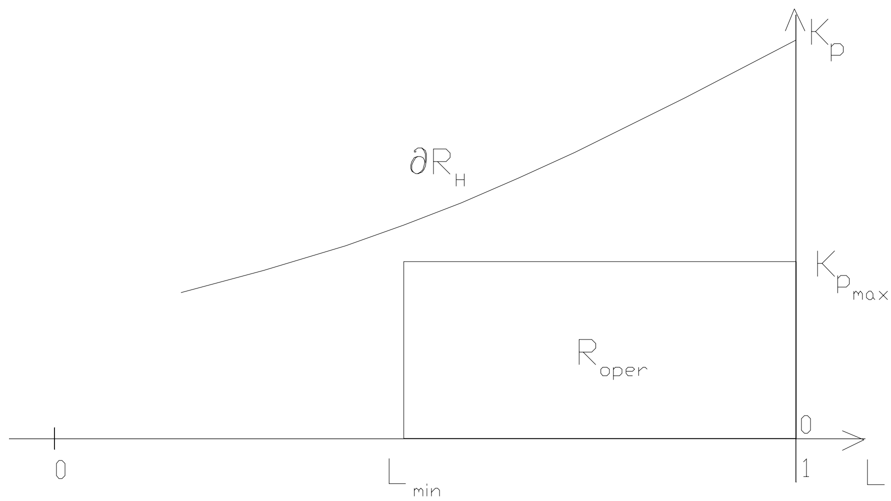

Figure 4: The Hurwitz stability region for an aircraft satisfying the condition for robustness versus Category II PIO (weak version)

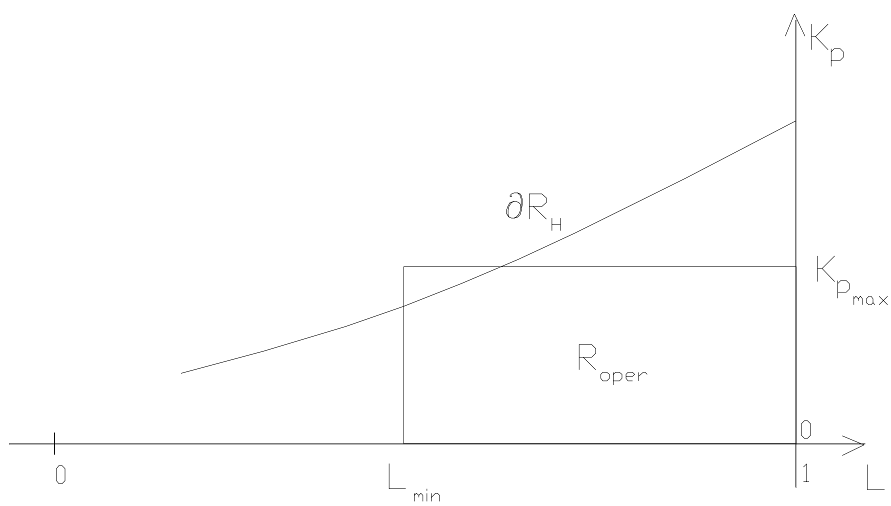

Figure 5: The Hurwitz stability region for a Category II PIO prone aircraft

To compute the region $\mathcal{R}_H$ we present an algorithm, ROBAN, which performs the robust stability analysis of a LTI system subject to parametric time-invariant uncertainties. This algorithm is based on a polynomial approach (see (Verde, 1992)), i.e. on the availability of the characteristic polynomial of the LTI system and is presented in the general case of systems depending on p uncertain parameters.

3.1 The algorithm ROBAN

Let n be the order of the dynamic system and p the number of uncertain parameters affecting the behaviour of the system. Let $\pi = [\pi_1, \ldots, \pi_p]^T \in \Pi \subset \mathcal{R}^p$ be the vector of uncertain parameters, ranging in the hyper-rectangle Π, $\pi_0 \in \Pi$ be the vector of nominal values of the uncertain parameters and

$$H^w(\pi_0, \rho) := \{\pi : \|\pi - \pi_0\|_\infty^w < \rho\}$$

be an hyper-rectangular neighbourhood of π_0 with radius ρ where

$$\|x\|_\infty^w = \max_i w_i\,|x_i| \qquad w_i > 0.$$

It is always possible to select a weighting vector w_0 such that $H^{w_0}(\pi_0, 1) = \Pi$, i.e. to assume that the uncertain parameters range in a hyper-rectangle of unitary radius.

Now let $a(\,\cdot\,) : \Pi \to \mathcal{R}^n$ be a continuous vector function of π, and $L : a(\Pi) \to P^n$ the linear operator mapping $a(\Pi)$ into P^n, the set of monic polynomials of degree n. Define the compound operator L_a as $L \circ a$, and let the function

$$p(s, a) = s^n + a_1 s^{n-1} + a_n\,,\, s \in \mathcal{C}\,.$$

The complete behaviour of the uncertain dynamic system is described by the following family of monic polynomials

$$L_a(\Pi) = \{p(\,\cdot\,, a(\pi))\ |\pi \in \Pi\}\ .$$

Definition 1 Let $\tilde{\Pi} \subseteq \Pi$; the family of polynomials $L_a(\tilde{\Pi})$ is said to be Hurwitz if the roots of $p(\,\cdot\,, a(\pi))$ are in open left half of the complex plane for all $\pi \in \tilde{\Pi}$.

Definition 2 The stability region $\mathcal{S}$ in the parameter space Π, is the set composed of all $\pi \in \Pi$ such that $L_a(\mathcal{S})$ is Hurwitz.

With respect to the above definitions, we can pose the following problem, which is solved by ROBAN.

Problem 1 Find the stability region $\mathcal{S}$ in the parameter space Π.

In (Verde, 1992) a test condition, say Test1, on the family of polynomials $L_a(\Pi_T)$, which, when satisfied, assures that the tested hyper-rectangle Π_T has no intersections with the boundary of the stability region, $\partial\mathcal{S}$, i.e. Π_T is completely contained in or completely external to $\mathcal{S}$, is given. Viceversa if Test1 is not satisfied, then Π_T intersects $\partial\mathcal{S}$. An algorithm which performs Test1 is provided in (Verde, 1992).

Based on such algorithm, the following procedure, implemented at Centro Italiano Ricerche Aerospaziali (Italian Aerospace Research Center) in the software ROBAN gives a solution to Problem 1, i.e. computes the boundary of the stability region $\partial\mathcal{S}$ up to a desired resolution. Any dependence of $L_a(\,\cdot\,)$ on the parameter π is covered by ROBAN.

Procedure 1

Step 0 Say Π the hyper-rectangle in which the uncertain parameters range and assume it as the initial hyper-rectangle for which stability has to be tested.

Step 1 Initialise three empty lists of hyper-rectangles, L_1 to L_3, and set $i1 = 0$.

Step 2 Put Π in the list L_1, and set $i1 = 1$.

Step 3 Run Test1 for the hyper-rectangle Π_{i1} of L_1 with the algorithm of (Verde, 1992).

Step 4 **If** Test1 is satisfied **then** put Π_{i1} in L_3

else if the radius of Π_{i1} is greater than a predefined threshold put the 2^p hyper-rectangles obtained by dividing Π_{i1} in equal parts along its sides into L_2 **end**.

Step 5 Set $i1 = i1 + 1$.

Step 6 **If** exists Π_{i1} **then goto** Step 3
else if list L_2 is empty, **goto** Step 7
else clear list L_1; **set** $i1 = 1$; **copy** list L_2 into list L_1; **clear** list L_2; **goto** Step 3 **end**

Step 7 For each hyper-rectangle Π_i in L_3 check the stability of the polynomial $L_a(\pi_{ci})$, evaluated at the centre of Π_i. The stability region Π_S is approximated by the union of those Π_i for which $L_a(\pi_{ci})$ is stable.

4 A PIO Detection Procedure Based on Robust Stability Analysis Versus Time-Varying Parameters

First of all note that, for a given input function $u(\cdot)$ to the nonlinear element in Figure 1, the input-output instantaneous gain is defined as

$$L(t) := \frac{y(t)}{u(t)} \qquad L(t) \in [L_{min}, 1]\,. \tag{4}$$

Therefore the parameter L in the scheme of Figure 3 is actually time-varying. The analysis performed in Section 3 guarantees stability versus a time-invariant L because it checks the location of the eigenvalues of $A_{CL}(K_p, L)$ on the complex plane. However negativeness of the real part of the eigenvalues does not guarantee stability versus a time-varying parameter (Amato *et al.*, 1992); hence the approach based on robust stability analysis versus a time-invariant parameter may be "optimistic" in determining PIO proneness of an aircraft.

Based on the above consideration, in this section we propose a different approach which provides a stability test which also guarantees *certain* global asymptotic stability of the nonlinear system depicted in Figure 1. Obviously this approach will provide an useful tool to estimate how optimistic the approach of Section 3 is.

First note that the nonlinear system in Figure 1 is embedded into the linear system in Figure 3, provided we allow the function $L(\cdot)$ to be any member of the set

$$\{L(\cdot)\,:\,[0,+\infty) \to \mathbb{R}\,|\,L(t) \in [L_{min}, 1]\} \tag{5}$$

On the basis of the above considerations we can conclude the following:

Fact 1 *Asymptotic stability of the linear system in Figure 3 versus all time realizations of the gain $L(\cdot)$ belonging to the set of functions (5) implies global asymptotic stability of the nonlinear system in Figure 1.*

Next Section provides a test for stability analysis in presence of time-varying parameters.

4.1 A test for robust stability analysis of a system subject to uncertain time-varying parameters

Consider the uncertain system

$$\dot{x}(t) = A(\pi(t))x(t) \tag{6}$$

where $\pi(\cdot)\,:\,[0,\infty) \to \Pi \subset \mathbb{R}^p$ is a p-component vector of time-varying parameters, $A(\cdot)$ is assumed to be *affine* in its argument and, as usual, Π is a hyper-rectangle. We denote the vertices of Π by $\pi_{(i)}$, $i = 1, \ldots, 2^p$.

To study the stability properties of system (6) we need to use a Lyapunov function approach. In particular let $V(x) = x^T P x$ be a quadratic Lyapunov function, P being a positive definite matrix. Using properties of quadratic forms it is possible to prove the following well known result.

Fact 2 *System (6) is asymptotically stable for all parameter realizations $\pi(\cdot)$ if*

$$A^T(\pi)P + PA(\pi) < 0 \qquad \forall \, \pi \in \Pi \tag{7}$$

The matrix inequality (7) is also known as the *Quadratic Stability* (QS) condition for system (6) (Barmish, 1983). Since the LHS in (7) depends affinely on the parameter vector π, according to (Boyd *et al.*, 1993), inequality (7) is equivalent to the following set of inequalities:

$$A^T(\pi_{(i)})P + PA(\pi_{(i)}) < 0 \qquad i = 1, \ldots, 2^p \tag{8}$$

Therefore quadratic stability of system (6) is equivalent to the solvability of the following feasibility problem

Problem 2 Find a positive definite matrix P such that

$$A^T(\pi_{(i)})P + PA(\pi_{(i)}) < 0 \qquad i = 1, \ldots, 2^p \tag{9}$$

■

Note that the constraints in Problem 2 are Linear Matrix Inequalities (LMIs, (Boyd *et al.*, 1993)); such problem can be solved with widely available software (see for example the Matlab LMI Toolbox).

4.2 A “strong” condition for Category II PIO free aircraft

Consider the zero-input closed loop system associated to the scheme in Figure 3

$$\dot{x}(t) = A_{CL}(K_p, L)x(t) \tag{10}$$

where $K_p \in [0, K_{p_{max}}]$ is *time-invariant* and $L \in [L_{min}, 1]$ is *time-varying.* Let us define the following stability region in the $K_p - L$ plane

$$\mathcal{R}_{QS} := \{(L, K_p) \; : \; \text{system (10) is Hurwitz with respect to } K_p \text{ and QS with respect to } L\} \tag{11}$$

Condition 2 *A condition for robustness versus Category II PIO (strong version)*
The aircraft depicted in Figure 1 is Category II PIO free if

$$\mathcal{R}_{QS} \supseteq \mathcal{R}_{oper} \tag{12}$$

■

By virtue of Facts 1-2 it is readily seen that Condition 2 guarantees global asymptotic stability of the nonlinear system depicted in Figure 1. Note however that Condition 2 is only a *sufficient*, i.e. conservative, condition for stability. For this reason we call it a “strong condition”.

Next we provide an algorithm to determine the region $\mathcal{R}_{QS}$ which takes into account the different nature of the two parameters (K_p time-invariant and L time-varying) involved in the analysis.

Procedure 2

Step 1 **Let** $\Delta K = 0.01$, $K_{min} = 0$, $K_{max} = \Delta K$;

Step 2 Try to solve the following feasibility problem

Problem 3 Find $P > 0$ such that

$$A_{CL}^T(L, K_p)P + PA_{CL}(L, K_p) < 0 \qquad L = L_{min}, 1 \qquad K_p = K_{min}, K_{max}$$

Step 3 **If** Problem (3) is not feasible **then** plot the box $[L_{min}, 1] \times [0, K_{min}]$ and **let** $L_{min} = Lmin + \Delta L$

else let $K_{min} = K_{max}$, $K_{max} = K_{min} + \Delta K$ **end**

If $L_{min} < 1$ **then goto** Step 2; **else STOP**.

Since $A_{CL}(\cdot, \cdot)$ is affine in its arguments, the four LMIs contained in Problem 3 guarantee asymptotic stability versus a time-varying parameter L as well as versus a time-varying parameter K_p in the interval $[K_{min}, K_{max}]$ (see Problem 2); since in this study K_p is assumed to be time-invariant, the constraints expressed by the above-mentioned LMIs could seem conservative. This is not true in practice, because the interval $[K_{min}, K_{max}]$ has been chosen very small and stability versus time-varying parameters with a small excursion is practically equivalent to stability versus time-invariant parameters.

An aircraft which exhibits a QS stability region like the one depicted in Figure 6 is *certainly* Category II PIO free.

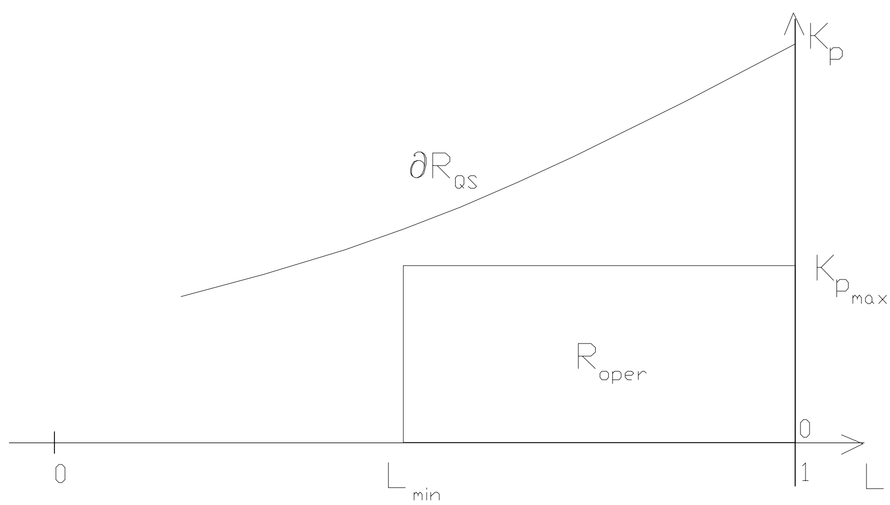

Figure 6: The Quadratic Stability region for an aircraft satisfying the condition for robustness versus Category II PIO (strong version)

The situation reported in Figure 7 is associated to an aircraft which *could be* (remember that the QS analysis is conservative) PIO prone.

In the next section we shall apply the methodologies of Sections 3 and 4 to design, if needed, an actuator guaranteeing robustness versus Category II PIO.

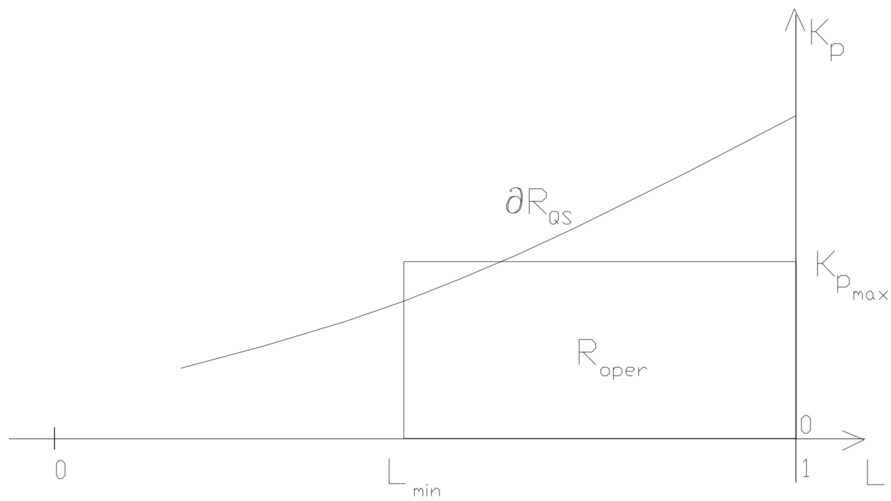

Figure 7: The Quadratic Stability region for a Category II PIO prone aircraft (strong version)

5 Design of Actuators Guaranteeing Robustness Versus Category II PIO

Let us come back to the definitions of the stability regions $\mathcal{R}_H$ and $\mathcal{R}_{QS}$ given in the previous sections; taking into account that the boundary of $\mathcal{R}_H$ is always above the boundary of $\mathcal{R}_{QS}$, because Quadratic Stability is more demanding than Hurwitz Stability, we have that three situations can happen:

i) $\mathcal{R}_{QS} \supseteq \mathcal{R}_{oper}$ (Figure 6); in this case the aircraft is Category II PIO free; the actuator is fine as it stands.

ii) $\mathcal{R}_H \supseteq \mathcal{R}_{oper}$ and $\mathcal{R}_{QS} \not\supseteq \mathcal{R}_{oper}$ (Figure 8). In this case we have that the aircraft satisfies the condition for *weak* PIO robustness but not the *strong* one. Nothing can be said in general and only simulations can certify the goodness of the actuator.

iii) $\mathcal{R}_H \not\supseteq \mathcal{R}_{oper}$ (Figure 5). In this case the aircraft is definitely Category II PIO prone; a redesign of the actuator is necessary.

In the case iii) we propose the following empirical procedure for the actuator redesign. The new actuator will have a larger linear range; this is simply obtained by modifying the value of the maximum output amplitude, say $y^*_{max} > y_{max}$.

Procedure 3

Step 1 Determine the intersection $(L^*, K_{p_{max}})$ between the boundary of the region $\mathcal{R}_{QS}$ and the operating region (see Figure 9).

Step 2 Choose L^*_{min} such that

$$L^*_{min} > L^* \,. \tag{13}$$

The relation between L^*_{min} and y^*_{max} is given by

$$L^*_{min} = \frac{y^*_{max}}{u_{max}} \tag{14}$$

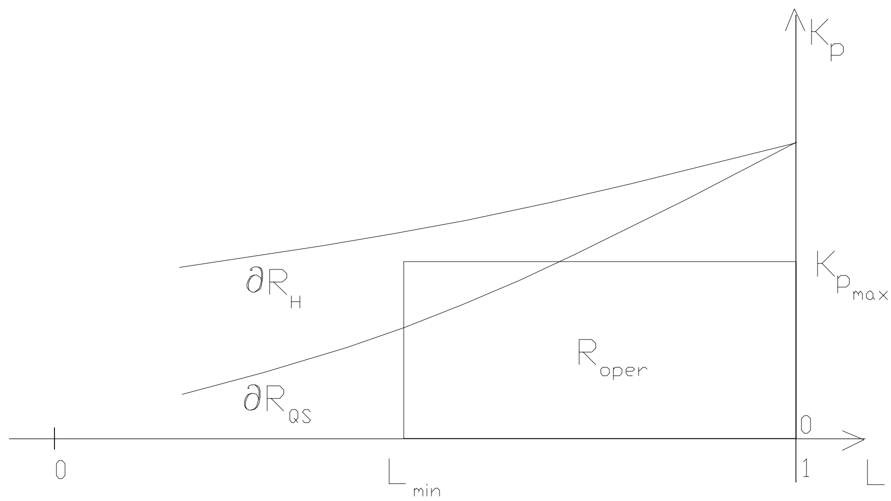

Figure 8: The Hurwitz Stability region contains the operating region but the Quadratic Stability region does not

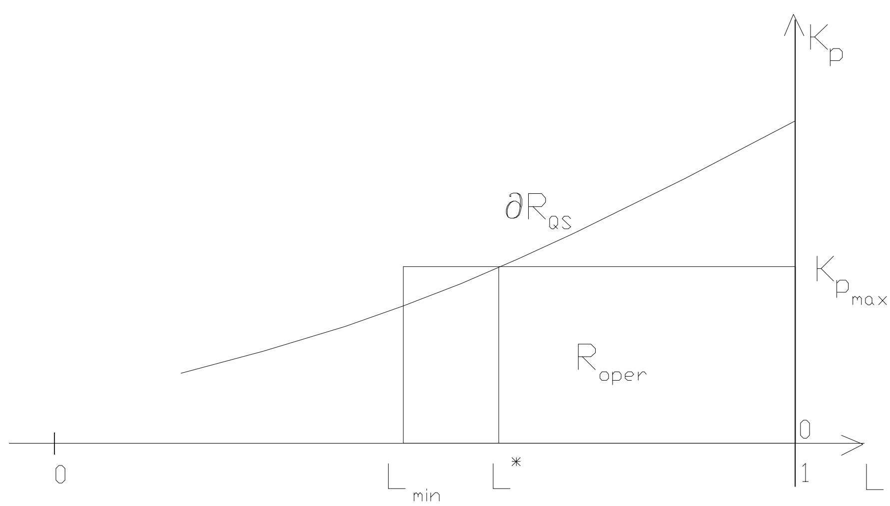

Figure 9: The intersection between the boundary of the QS region and the operating region

from which we obtain the design value of y^*_{max}

$$y^*_{max} = L^*_{min} u_{max} \,. \tag{15}$$

The new operating region $\mathcal{R}^*_{oper}$ depicted in Figure 10 does not intersect now the boundary of the QS region, and therefore the aircraft should become Category II PIO free.

■

Concerning the above procedure a couple of comments are in order.

a) Since the quadratic stability condition is a conservative condition for global asymptotic stability of the nonlinear system in Figure 1, it is possible that a value of the maximum output amplitude of the nonlinear element smaller than $L^*_{min} u_{max}$ will not destabilize the system. This is important because the actuator cost is strictly related to the linear range that it exhibits;

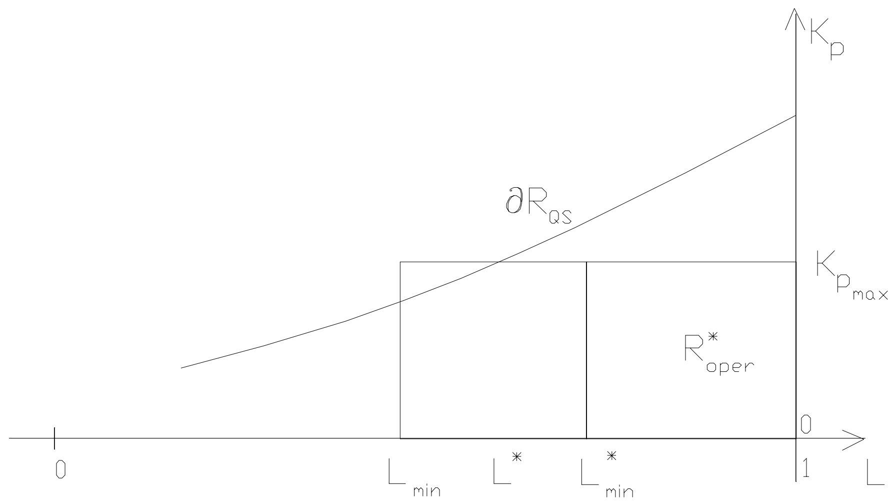

Figure 10: The new operating region

b) The value of u_{max} is computed numerically by performing several simulations on the nonlinear system in Figure 1 in presence of the original actuator. In the formula (14) it is assumed that u_{max} remains unchanged also in correspondence of the new actuator; this, in general, is not true. Therefore it could be necessary to choose a value of the output magnitude of the nonlinear element larger than the theoretical one $L^*_{min} u_{max}$.

In other words, after the actuator has been redesigned according to Procedure 3, a tuning action guided by simulations is needed to adjust the value of y^*_{max}, in order to perform a trade-off between Category II PIO robustness and actuator cost.

6 Category II PIO Analysis and Synthesis: A Case Study

We have chosen to demonstrate the capabilities of our proposed method on the same test case presented in (Klyde *et al.*, 1997), which is based on the X-15 PIO occurred on June 8, 1959 during a landing flare (Matranga, 1961).

The model of the aircraft with rate limited actuator is shown in Figure 11 (note that in this case the pilot filter $W(s) = 1$).

The numerical values of the elements in the block diagram are:

$$\begin{aligned} \frac{\theta(s)}{\delta(s)} &= \frac{3.476(s+0.0292)(s+0.883)}{(s^2+0.019s+0.01)(s^2+0.8418s+5.29)} \\ \tau_r &= 0.04\,sec \end{aligned} \tag{16}$$

For $L = 1$ (linear actuator) we have that for $K_p \in [0, 7.1]$ the system is asymptotically stable. Since this aircraft has to be considered Category I (linear) PIO free, it must satisfy the existing criteria for Category I PIO robustness; in particular the Gain/Phase Template - Average Phase Rate Criterion see (McRuer *et al.*, 1997) requires (in the worst case) a gain margin of 2.51 ($8db$) for the whole range of admissible pilot gains. Since for $K_p > 7.1$ the linear system becomes unstable, we obtain that $K_{p_{max}} = 7.1/2.51 = 2.82$, which is approximated, for the sake of safety, by $K_{p_{max}} = 3$. From numerical simulation we obtain that $u_{max} = 500deg^2$.

[2] The actual value of u_{max} entering the nonlinear actuator is $20deg$, which is multiplied by $1/\tau_r = 25$; therefore we obtain that the signal entering the normalized nonlinearity has a maximum amplitude of about $500deg$.

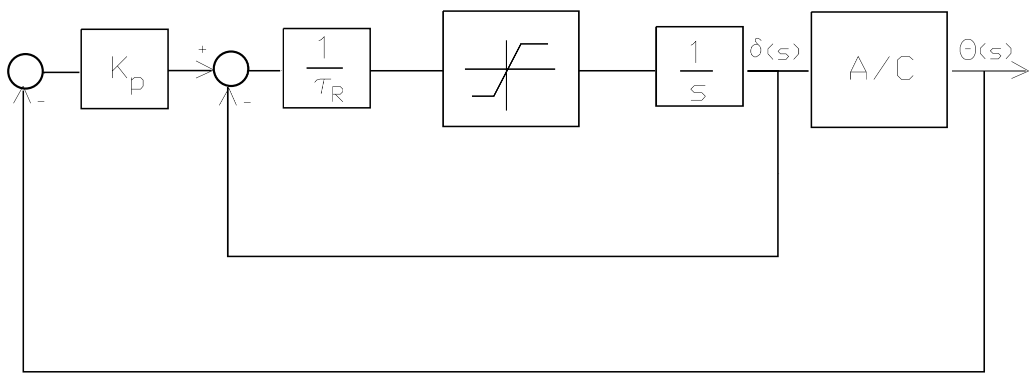

Figure 11: The X-15 Model

Moreover $y_{max} = \dot{\delta}_{max} = 15 deg/sec$, therefore $L_{min} = 0.03$. The boundary of the regions $\mathcal{R}_H$ (continuous line) and $\mathcal{R}_{QS}$ (dashdot line), computed according to the methodologies proposed in the previous sections, are depicted in Figure 12 together with the operating region $\mathcal{R}_{oper}$; this aircraft, as experience confirmed, is clearly Category II PIO prone.

We note that the boundary of $\mathcal{R}_{QS}$ intersects the operating region in correspondence of the point $(L^*, K_{p_{max}}) = (0.36, 3.0)$. According to Step 2 of Procedure 3 we set $L^*_{min} = 0.37$. Therefore we obtain

$$\dot{\delta}^*_{max} = y^*_{max} = 185 deg \tag{17}$$

Extensive numerical simulations shows that a value of $y^*_{max} = 170 deg$ is actually sufficient to guarantee global asymptotic stability of the nonlinear system in Figure 11. Note that, in this case, a very high quality actuator is required to render the aircraft robust versus Category II PIO's.

7 Concluding Remarks

In this paper we have presented a new method derived from robust stability analysis of linear systems for the analysis of Category II PIO. When the analysis shows PIO proneness of the aircraft, the proposed method provides also some guidelines for the design of actuators which should render the aircraft itself robust versus Category II PIO's.

A further investigation of the following issues is required:

- Pilot model(s) to be used for the analysis;
- Test of other structural properties to be used as PIO indicators in alternative to system stability;
- Application of the method to cases extracted from existing PIO databases involving multiple nonlinearities / multiaxis PIO.

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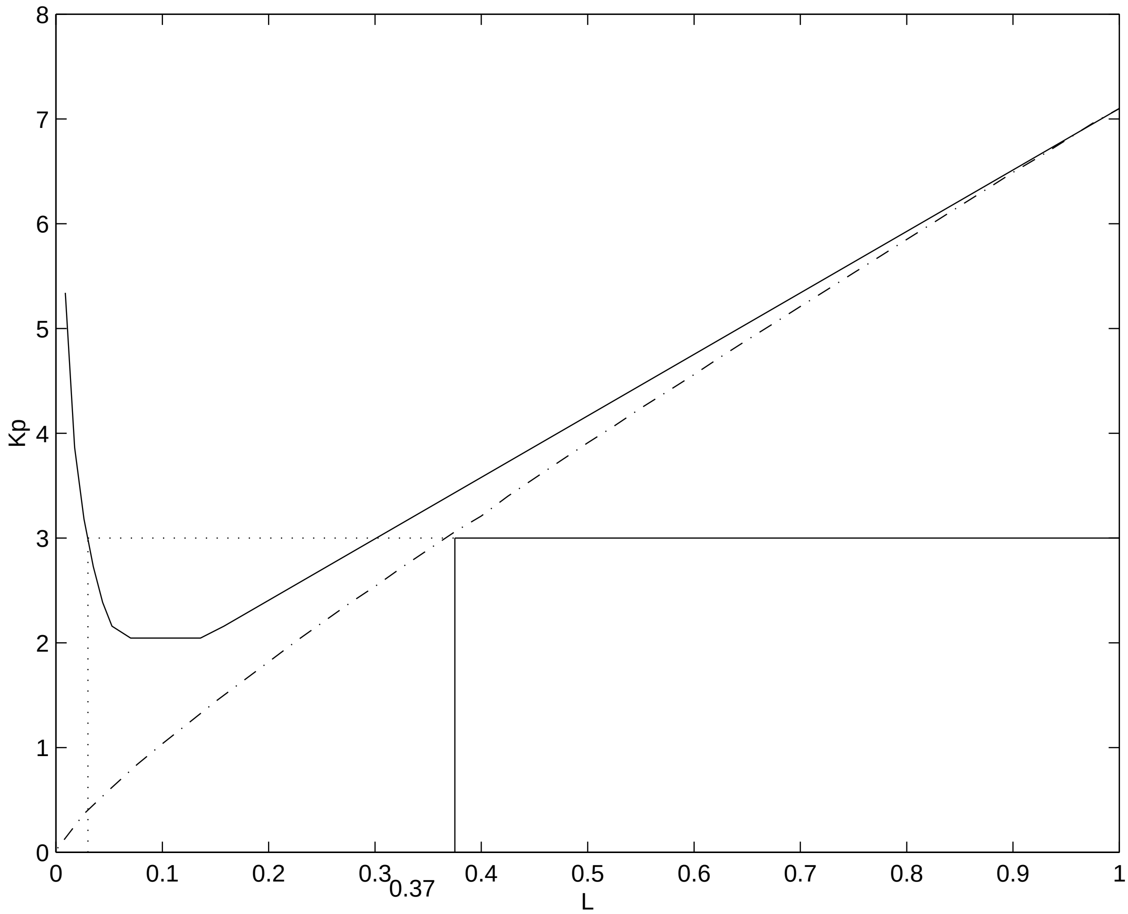

Figure 12: The boundary of Hurwitz and Quadratic Stability regions for the X-15 model

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