

# Modelling and identification of a high temperature short time pasteurization process including delays

Carlos F. Alastruey (1), Manuel De la Sen (2)  
and Mario García-Sanz (1)

(1) Department of Automatic Control and Computing Systems,  
Public University of Navarra,  
31006 Pamplona, Spain  
E-mail: karlos@upna.es

(2) Department of Electricity and Electronics,  
University of the Basque Country  
48080 Bilbao, Spain

## Abstract

In this paper, an improved mathematical model for a High Temperature Short Time (HTST) pasteurization plant is proposed. The main differences from previous models are that the four interconnected blocks of the heat exchanger model are assumed to be of third order; therefore fundamental physical properties of the plant are not neglected. In addition, time delays at the output of the heat exchangers are considered in order to take into account the fact by which the temperature sensor is not physically within the heat exchanger. Following the proposed model, a parameter identification procedure is suggested, by using stable filtering for the input and output signals.

## 1. Introduction and problem statement

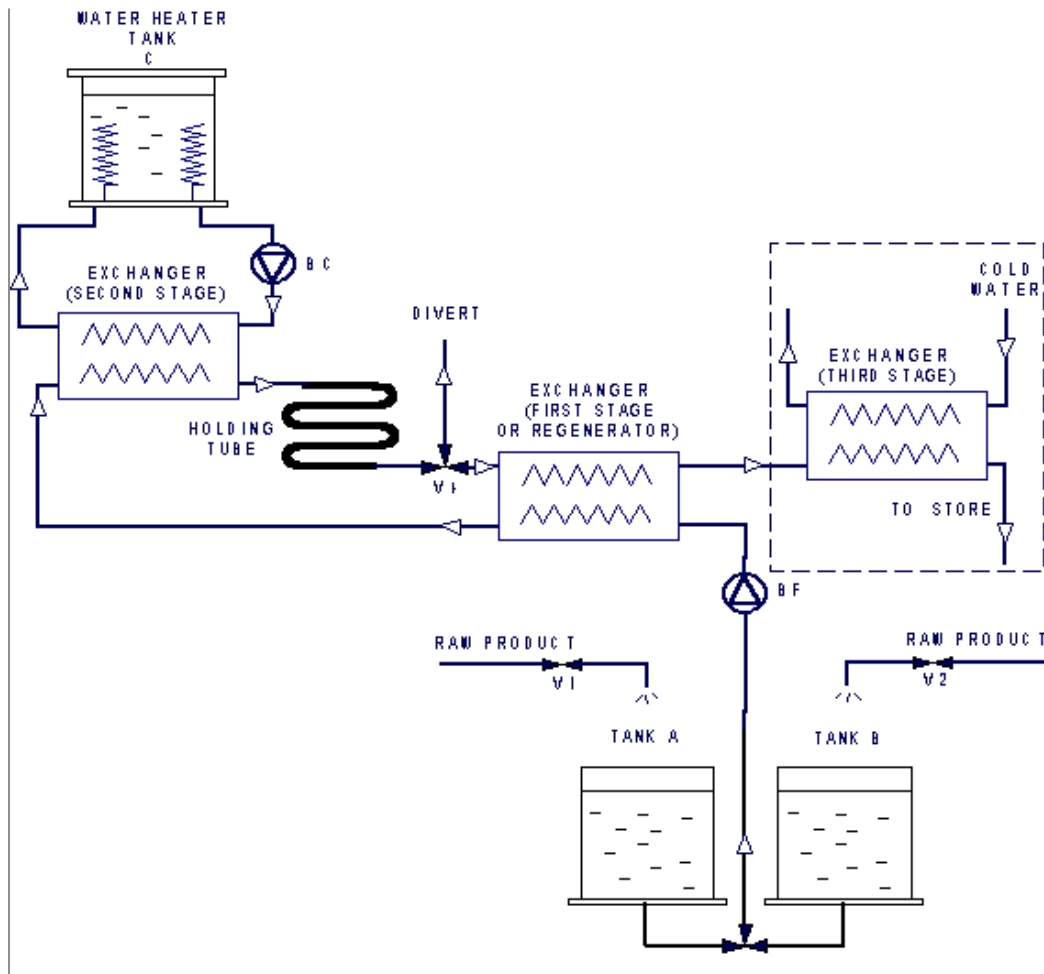
Pasteurization is a well known procedure in food industry, particularly when processing liquid products such as milk and fruit juices. Mainly, pasteurization implies that a food product is exposed to some temperature profile during a predetermined period of time, in order to reduce the proportion of microorganisms. Among many pasteurization methods, the High Temperature Short Time (HTST) method is usually accepted as the industry standard.

The plant PCT23, manufactured by Armfield (UK), is a laboratory version of a real industrial pasteurization process (see Figure 1, Ibarrola et al). It consists of a bench-mounted process unit to which is connected a dedicated control console. An interface card DT2811-PGH is used for monitoring and controlling the process through a computer.

During the pasteurization process, liquid is pumped at a preset flow rate from one of two storage tanks to an indirect plate heat exchanger. This heat exchanger increments the temperature of the liquid to a certain value. The pasteurization process requires the liquid stream to be maintained at such a temperature for some time. This purpose is achieved through the use of a so-called holding tube followed by a temperature-activated diverter valve. The holding tube can be regarded as an effective distance/velocity time lag. After that, the liquid is refrigerated by firstly exchanging heat with incoming liquid, and then by using externally supplied cooling water. Thus, the plate heat exchanger is composed of three interconnected elements: 1- feed preheat / regeneration, 2- heating, and 3- cooling. The three heat exchangers are in fact sharing the same

physical block, but for the sake of simplicity it is reasonable to assume that they do not have any coupling effect among them.

Ibarrola et al (1998) proposed a structure of four interconnected first order blocks for every heat exchanger stage of the same HTST plant. However, in this paper we propose a model consisting on four interconnected third order blocks with an output delay for each one of the three heat exchanger stages. Such a corrected model can take into account essential physical effects, i. e., relaxed or non relaxed oscillations that are related to vibrating systems and heating processes (Krabs, 1992). Obviously, those oscillatory effects cannot be represented by an association of first-order blocks, because such an association cannot involve any complex pole.

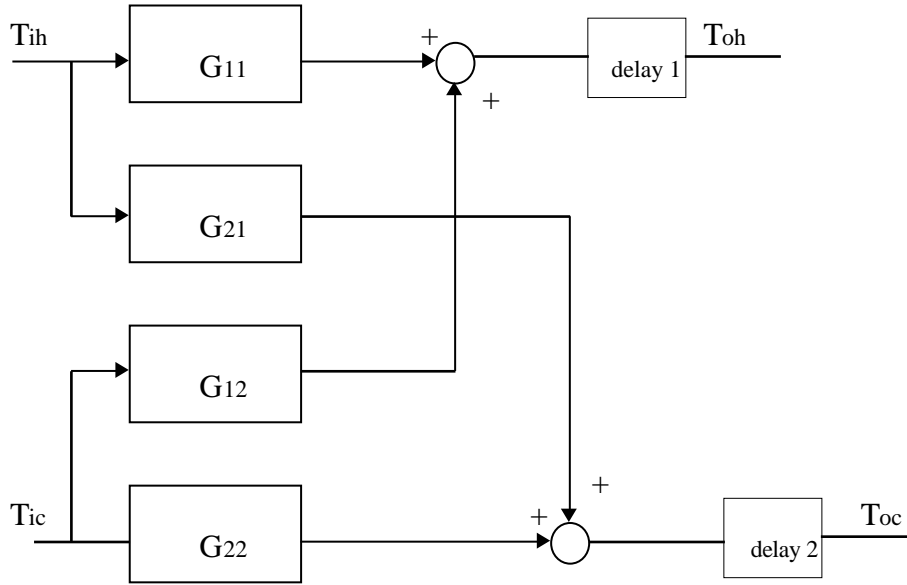


**Figure 1** HTST Plant diagram

In addition to this, for the identification of the parameters of the model, inputs with relatively high frequencies will be used. It is known that parameter identification with low-varying inputs in certain cases can converge to a limit that could be not necessarily the correct one. At least, frequencies of input test signals for identification purposes are related to the sampling frequency, especially when delays are involved (Chen and Loparo, 1993).

## 2. The heat exchanger second stage model

The heat exchanger second stage has proved to be the most important part of the plant (Ibarrola et al, 1998). For that reason, we will focus our modelling on such a second stage. The MIMO structure for the second stage is shown in Figure 2.



**Figure 2** Block diagram for the second stage model of the heat exchanger

This very structure is also suitable for the first stage or regenerator, but with different parameter values due to the different size of the heat exchanger stages. The same remains valid for the third stage.  $T_{ih}$  and  $T_{oh}$  in Figure 2 are the input and output temperatures of the hot water which is utilized to heat the liquid to be pasteurized.  $T_{ic}$  and  $T_{oc}$ , on the other hand, represent the input and output temperatures of the liquid to be pasteurized, which, as a result of traversing this second stage, becomes heated. Different third order transfer functions are postulated for every block in Figure 2. In addition, two point time delays must be included at the two outputs (not at the output of each block). Such an output time delay is introduced by the temperature sensor, which is located physically outside the heat exchanger, and its reading process becomes delayed some seconds as a result (Franklin et al, 1991). In fact, those delays are flow-dependent. Let us suppose a linear dependence of those time delays with respect to the flows  $F_h$ ,  $F_c$ , which are, respectively, the fluid flow of the hot water, and the fluid flow of the (cold) liquid to be pasteurized.

For the block  $G_{11}$  it is postulated the following transfer function

$$G_{11}(s) = \frac{K_{11}}{(s + \sigma_{11})(s + \sigma_{11} + j\omega_{11})(s + \sigma_{11} - j\omega_{11})} \quad (1)$$

which contains a real pole, and an additional couple of conjugated complex poles in order to take into account oscillatory modes and vibrating effects. It is also specified a CD gain  $K_{11}$ . As usual, “j” stands for the imaginary unit. Similar third order transfer functions are postulated for the remaining blocks in Figure 2. Time delay blocks 1 and 2 have, respectively, the transfer functions  $e^{-h_1 F_h s}$ ,  $e^{-h_2 F_c s}$  corresponding to pure time delays. Therefore, the number of parameters to be identified for the heat exchanger second stage is 18, which can be regarded as elements of a vector of parameters to be identified, namely:

$$= (K_{11}, K_{12}, K_{21}, K_{22}, \tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}, \tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}, \tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}, h_1, h_2) \quad (2)$$

In the next section a procedure will be outlined for the identification of the first 16 parameters of . A linear transformation of such a reduced parameter vector will be denoted in the sequel. The last two parameters can be regarded as input delays for the third stage of the heat exchanger, and can be identified according to a very suitable algorithm described by Chen and Loparo (1993).

### 3. Identification of the second stage

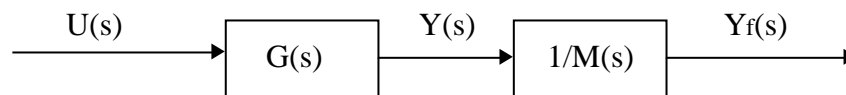
Assume that  $\hat{\theta}$  is the estimation of a linear transformation of the reduced vector of parameters, namely  $\theta$ . The following estimation procedure is proposed:

$$\begin{aligned} \dot{\hat{\theta}} &= P e = P [y_f - \hat{y}^T] = P [y_f - \hat{y}] \\ \dot{P} &= -P^T P \quad \dot{P}^{-1} = P^{-T} \quad \text{with } P(0) = P^T(0) > 0 \\ e &= y_f - \hat{y}^T = P^T \hat{\theta} - \hat{y}^T = -\tilde{y}^T \end{aligned} \quad (3)$$

where  $\hat{y}$  stands for the data regressor (a matrix relating the output  $y$  with the vector of parameters to be identified) (Ferretti et al, 1995), “e” stands for the error in the estimation of the output, and

$y_f = L^{-1} \frac{1}{M(s)} Y(s)$  being  $M(s)$  strictly stable and of n-th order. This filtering is included in order to ensure that the measured output is n-differentiable, a circumstance that is not guaranteed because in some measured outputs of the HTST pasteurization plant the first derivative was only piecewise (Ibarrola et al, 1998). Such a filtering can be realized in theory according to two different schemes:

#### Scheme 1



#### Scheme 2



Scheme 1 is preferred in practice because it produces a really differentiable signal. Scheme 2 is still useful for block diagram manipulation. Then, by using the differential operator  $D$ , the next chain of implications follows:

$$y(t) = G(D)u(t) \quad M(D)y_f(t) = G(D)u(t) = G(D)M(D)u_f(t) + v(t) \quad (4)$$

where  $v(t)$  is a vanishing time function associated with the filtering process. Then, assuming null initial conditions, Eq. (4) implies

$$y_f(t) = G(D)u_f(t) + v(t) = \frac{B(D)}{A(D)}u_f(t) + v(t) \quad (5)$$

Or alternatively, by decomposing  $A(D) = D^n + A_0(D)$ , and, without loss of generality, assuming  $A(D)$  and  $M(D)$  monic polynomials, and supposing  $v(t)$  negligible,

$$A(D)y_f(t) = B(D)u_f(t) \quad y_f^{(n)}(t) = -A_0(D)y_f(t) + B(D)u_f(t) = \quad^T \quad (6)$$

Observe that existence for  $y_f^{(n)}$ ,  $u_f^{(n)}$  is ensured if  $M = n$ . Then one has for the filtered signals

$$A(D)y_f(t) = B(D)u_f(t) \quad y_f^{(n)}(t) = -A_0(D)y_f(t) + B(D)u_f(t) \quad (7)$$

Therefore, an alternative estimation procedure is given by

$$\begin{aligned} \dot{\hat{e}} &= P \dot{e} = P \left[ y_f - \hat{y}^T \right] = P \left[ y_f - \hat{y} \right] \\ \dot{P} &= -P^T P \quad \dot{P}^{-1} = \quad^T \quad \text{with } P(0) = P^T(0) > 0 \\ e(t) &= y_f^{(n)} - \hat{e}^T \end{aligned} \quad (8)$$

given that  $A = M = n$ ,  $B = m \leq n$ . On the other hand, for the non filtered signals the following identities hold:

$$\begin{aligned} A(D)y_f(t) &= B(D)u_f(t) \quad y(t) = M(D)y_f(t) = M(D)y_f(t) - A(D)y_f(t) + B(D)u_f(t) \\ &= [M(D) - A(D)]y_f(t) + B(D)u_f(t) \end{aligned} \quad (9)$$

Assuming  $M(D)$  and  $A(D)$  monic and  $M(D) = D^n + M_0(D)$ ,  $A(D) = D^n + A_0(D)$  then Eq. (9) leads to

$$y(t) = [M_0(D) - A_0(D)]y_f(t) + B(D)u_f(t) \quad (10)$$

Observe in Eq. (10) that  $M_0$  is known, but  $A_0$  is in principle unknown. Let us write

$$M_0(D) = \sum_{i=1}^n m_{0i} D^{n-i}; \quad A_0(D) = \sum_{i=1}^n a_{0i} D^{n-i} \quad (11)$$

Thus, the output can be expressed in the following manner

$$y(t) = \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix}^T = \begin{bmatrix} m_{0n} - a_{0n} & L & m_{01} - a_{01} & b_{0m} & L & b_{01} \end{bmatrix} \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix} \quad (12)$$

and the estimated output is given by

$$\hat{y}(t) = \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix}^T = \begin{bmatrix} m_{0n} - \hat{a}_{0n} & L & m_{01} - \hat{a}_{01} & \hat{b}_{0m} & L & \hat{b}_{01} \end{bmatrix} \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix} \quad (13)$$

In a similar way one can obtain relations for the filtered output and for the estimated filtered output:

$$y_f(t) = \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix}^T = \begin{bmatrix} -a_{0n} & L & -a_{01} & b_{0m} & L & b_{01} \end{bmatrix} \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix} \quad (14)$$

$$\hat{y}_f(t) = \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix}^T = \begin{bmatrix} -\hat{a}_{0n} & L & -\hat{a}_{01} & \hat{b}_{0m} & L & \hat{b}_{01} \end{bmatrix} \begin{bmatrix} y_f(t) \\ M \\ y_f^{(n-1)}(t) \\ u_f(t) \\ M \\ u_f^{(m-1)}(t) \end{bmatrix} \quad (15)$$

Therefore one gets

$$\hat{y}_{ij} = \frac{m_{0(n-j+1)} - \hat{a}_{0(n-j+1)}}{\hat{b}_j} = m_{0(n-j+1)} - \hat{a}_{0(n-j+1)} \quad n < j \leq n+m+1 \quad (16)$$

Note that an estimation of the vector of parameters as shown in Eq. (16) can be linearly transformed to the 16 first elements of  $\hat{y}$ , and then the estimation procedure would be completed.

## **Conclusions**

In this paper an improved mathematical model for a High Temperature Short Time (HTST) pasteurization plant has been proposed. Differences from previous models are that the four interconnected blocks of the heat exchanger model are assumed to be of third order; this fact allows taking into account vibrating and oscillatory effects in the heat exchanger. In addition, time delays at the output of the heat exchangers were considered. A parameter identification procedure has been also suggested, by using stable filtering for the relevant signals.

## **Acknowledgments**

The authors are very grateful to the Spanish DGICYT, by its support through a research project (ref. PB96-0257), and to the Spanish CICYT, by its support through a research grant No. TAP97-0471.

## **References**

- R. Chen and K.A. Loparo; "Identification of time delays in linear stochastic systems"; Int. J. Control, Vol. 57, No. 6, pp. 1273-1291. 1993.
- G. Ferretti, C. Maffezzoni and R. Scattolini; "The Recursive Estimation of Time Delay in Sampled-Data Control Systems"; published in "Techniques in Discrete-Time Stochastic Control Systems", edited by C.T. Leondes; Academic Press, 1995.
- G.F. Franklin, J.D. Powell and A. Emami-Naeini; "Feedback Control of Dynamic Systems"; Addison-Wesley Publishing Co.; Massachussets, USA; 1991.
- J.J. Ibarrola, J.C. Guillen, J.M. Sandoval and M. Garcia-Sanz; "Modelling of a high temperature short time pasteurization process"; Food Control, Vol. 9, No. 5, pp. 267-277. 1998.
- W. Krabs; "On Moment Theory and Controllability of One-Dimensional Vibrating Systems and Heating Processes"; Lecture Notes in Control and Information Sciences, No. 173; Springer-Verlag, 1992.