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STOPPING OF ALGORITHMS AND FAULTS DETECTION IN KALMAN FILTER APPLICATION

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Abstract

An approach to the generation of stopping rules in parametric identification problems is proposed on the basis of the computation of a statistic of the difference between two successive estimates. This statistic is also used for fault detection in the Kalman filter. The developed decision rules are applied to a linear system identification problem. Experimental results are presented to demonstrate the performance of the proposed algorithms.

1. Introduction

The identification of dynamic objects described by difference equations of a known order, but with unknown coefficients, entails the estimation of these coefficients, i.e., it is a parametric problem. It is always attended by the problem of determining the stopping time of the computations or what essentially amounts to the verification of sufficiency of the number of observations when a prescribed accuracy is attained on the part of the computed estimates.

The results of the general mathematical theory of optimal stopping rules [1-3], a latter-day branch of probability theory, have not enjoyed any appreciable application in the generation of stopping rules in parametric identification problems. This situation is attributable to the complexity of adapting various statistical tests of a general nature to real applied problems and algorithms.

The application of the rules proposed in [4,5] for stopping of the identification process runs into several difficulties, one of which is the need to specify an admissible error ellipsoid or an admissible measure of this ellipsoid.

In this article we propose a stopping rule that is free of these shortcomings; it is based on the comparison of a statistic of the difference between two successive estimates with a predetermined confidence limit of the chi-square distribution. The indicated statistic is also used for fault detection in the Kalman filter.

2. Generation of stopping rules

We introduce the following stopping rule in application to multidimensional parametric identification problems:

$$r_i^2 = (\hat{\theta}_i - \hat{\theta}_{i-1})^T D^{-1}_{\Delta\theta_i} (\hat{\theta}_i - \hat{\theta}_{i-1}) \le \varepsilon, \qquad (1)$$

where $D_{\Delta\theta_i}$ is the covariance matrix of the discrepancy between two successive estimates $\hat{\theta}_i$ and $\hat{\theta}_{i-1}$, and ε is a predetermined small number.

We assume that the well-developed theory of Kalman filtering is used to estimate the parameters from a sequence of observations with Gaussian tolerances of the measurement errors and system noise. In this situation the Kalman filter yields an estimate with expected value equal to the estimated quantity and a Gaussian distribution function. The discrepancy $\hat{\theta}_i - \hat{\theta}_{i-1}$ then has a normal distribution as well, since it is a linear combination of two Gaussian random variables [6]. With these considerations in mind we know that the statistic r^2 has a χ^2 distribution with n degrees of freedom (n is the number of dimensions of the vector θ), and the threshold values of r^2 can be found by determining the tabulated values of the χ^2 distribution for a given level of significance.

It is evident from relation (1) that the smaller the value of r^2 , the greather will be the consistency of the estimates. Usually in the testing of consistency in such cases the lower limit of the confidence interval must be equal to zero, and the upper limit is determined by the level of significance α_1.

To test the consistency of the estimates, we adopt the level of significance α_1, which corresponds to the confidence coefficient $\beta_1=1-\alpha_1$. We specify the threshold χ_β^2 in terms of this probability, using the distribution of the investigated statistic r^2:

$$P\left\{\chi^2 < \chi^2_{\beta_1}\right\} = \beta_1, 0 < \beta_1 < 1.$$

We stop the estimation process when $r_i^2<\chi_{\beta_1}^2$, since further observations yield insignificant improvement of the identified model and are deemed impractical in this event. If the quadratic form r^2 is larger than or equal to the specified threshold $\chi_{\beta_1}^2$,estimation should be continued.

This stopping rule can be used to make a timely decision to stop the estimation process in the identification of dynamic systems, and it does not require large computational expenditures.

3. Fault detection in the Kalman filter

The chi-square test discussed above can also be used to troubleshoot the Kalman filter. The statistic r^2 must be compared with the confidence limit of the χ^2 distribution determined from the expression

$$P\left\{\chi^2 < \chi^2_{\beta 2}\right\} = \beta_2, 0 < \beta_2 < 1.$$

and a decision must be made on the basis of the rule

$r_i^2 \le \chi_{\beta_2}^2$, the Kalman filter is operating normally;
$r_i^2 > \chi_{\beta_2}^2$, faults are present.

Consequently, by comaring the above-defined statistic r_i^2 with the confidence limits obtained for the corresponding χ^2 distribution it is possible to solve two problems at once: to determine the stopping time of the identification process and to detect faults in the Kalman filter in due time.

Figure 1 shows a graph of the probability density function of the χ^2 distribution with n=4 degrees of freedom and the computed confidence limits for $\beta_1=1-\alpha_1=0.2$ and $\beta_2=1-\alpha_2=0.95$ (α_2 is the level of significance), where the numerals indicate: 1) the zone of stopping of observations; 2) the zone of estimation; 3) the fault-detection zone.

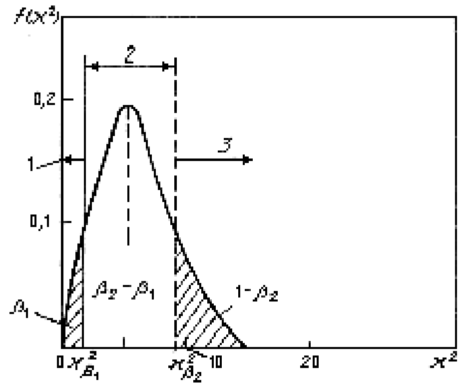

Fig.1.The domain of possible Kalman filter employment

It is evident from Fig. 1 that the domain of potential aplication of the Kalman filter is partitioned into three zones. Estimation is assumed to continue as the value of the statistic r_i^2 is determined between the confidence limits $\chi_{\beta_1}^{\ 2}$ and $\chi_{\beta_2}^{\ 2}$. If $r_i^2<\chi_{\beta_1}^{\ 2}$, the estimation process should be stopped, since further observations are assumed to yield insignificant improvement of the accuracy of estimation . If $r_i^2>\chi_{\beta_2}^{\ 2}$, faults are present in the Kalman filter, inducing large deviations of the estimates. In this case a decision is made as to the need for, and the character of, corrective actions in the estimation process.

The expressions for the areas shown in Fig. 1 are written in the form

$$\beta_1 = \int_0^{\chi^2_{\beta_1}} f(\chi^2)d\chi^2; \beta_2 = \int_0^{\chi^2_{\beta_2}} f(\chi^2)d\chi^2; \beta_2 - \beta_1 = \int_{\chi^2_{\beta_1}}^{\chi^2_{\beta_2}} f(\chi^2)d\chi^2 .$$

In the solution of a number of applied problems the threshold $\chi_{\beta_2}^{\ 2}$ (the upper confidence limit) can be replaced by $\chi^2_{(1+\beta_1)/2}$. It is then no longer necessary to use two levels of significance. However, this substitution is feasible only for isolated special cases, when the required probability of a correct decision as to normal operation of the Kalman filter is not too high, owing to the small value of β_1 .

Allowing for the fact that the stated problems of generating stopping rules and troubleshooting the Kalman filter are separate problems and can require the use of different levels of significance , two levels of significance α_1 and α_2 are recommended for the general treatment of the proposed approach.

4.Computation of the covariance matrix of the discrepancy between two successive estimates

Let us consider a linear system of the form

$$y_i = x_i^T \theta; i = \overline{1,n}, \qquad (2)$$

where $x_i^T = \left[1, p_i, p_i^2, ..., p_i^m\right]$ is the vector of the input variables ; $\theta^T = \left[a_0, a_1, a_2, ..., a_m\right]$ is the vector of the unknown parameters (the parameters being estimated).

The output signal of the object y_i is recorded by a measuring instrument

$$z_i = y_i + h_i,$$

where h_i is the measurement error with a zero mean and the variance σ^2.

To estimate the parameters of system (2) an algorithm takes into account the errors of the input variables is presented in [7]:

$$\begin{aligned}
\hat{\theta}_i &= \hat{\theta}_{i-1} + K_i (y_i - x_i^T \hat{\theta}_{i-1}), \\
K_i &= \frac{P_{i-1} x_i}{\sigma_i^2 + \hat{\theta}_{i-1}^T D_{xi} \hat{\theta}_{i-1} + x_i^T P_{i-1} x_i}, \qquad (3) \\
P_i &= P_{i-1} - \frac{P_{i-1} x_i x_i^T P_{i-1}}{\sigma_i^2 + \hat{\theta}_{i-1}^T D_{xi} \hat{\theta}_{i-1} + x_i^T P_{i-1} x_i}.
\end{aligned}$$

where K_i is the gain of the filter being examined ; P_i is the covariance matrix of the errors of the estimates; D_{xi} is the covariance matrix for input standard signals errors.

We investigate the problem of determining the covariance matrix of the discrepancy between successive estimates. The estimates $\hat{\theta}_{i-1}$ and $\hat{\theta}_i$ are determined from sets of successive measurements $\left\{z_1, z_2, ..., z_{i-1}\right\} = Z_1^{i-1}$ and $\left\{z_1, z_2, ..., z_{i-1}, z_i\right\} = Z_2^i$, respectively:

$$\hat{\theta}_{i-1} = E\left\{\theta_{i-1} / Z_1^{i-1}\right\}; \hat{\theta}_i = E\left\{\theta_i / Z_2^i\right\}.$$

These estimates of the parameters are correlated, since common data are used . The correlation between them is also attributable to shared initial conditions and shared system noise.

We consider the magnitude of the discrepancy between two successive estimates:

$$\beta_i^* = \hat{\theta}_i - \hat{\theta}_{i-1}.$$

Since the Kalman filter (3) is linear, the investigated estimates are unbiased , i.e., $E\left\{\beta_i^*\right\}=0.$

The covariance for β_i^* is written in the form

$$E\left\{\beta_i^*\beta_i^{*T}\right\}=P_1+P_2-P_{21}^T-P_{12}^T=D_{\Delta\theta i}, \quad (4)$$

where P_1 and P_2 are the covariance matrices of the errors of two successive estimates $\hat{\theta}_i$ and $\hat{\theta}_{i-1}$ respectively, and P_{12} and P_{21} are the cross covariance matrices between the errors of mentioned estimates, $P_{12}=P_{21}^T$ [8].

Consequently, to compute the covariance of the difference between two successive estimates of the Kalman filter, it is necessary to obtain the cross terms of the covariance of the error of the extended state vector

$$P_{ex}=\begin{pmatrix} P_1 & P_{12} \\ P_{12}^T & P_2 \end{pmatrix}.$$

Algorithms for determining the extended covariance P_{ex} are described in detail in [8,9].

Once P_1,P_2, and P_{12} have been found, Eq.(4) is used to determine the covariance matrix $D_{\Delta\theta_i}$,and a comparison of the statistic r_i^2 computed from this matrix with the confidence limits obtained of the corresponding χ^2 distribution leads to decisions whether to stop the estimation process and whether the synthesized Kalman filter is operating normally.

The covariance matrix $D_{\Delta\theta_i}$calculation is usually a very diffucult problem, because it requires the determination of the cross covariance between errors of two considered estimates $\hat{\theta}_i$ and $\hat{\theta}_{i-1}$.

Investigated estimates of Kalman filter $\hat{\theta}_i$ and $\hat{\theta}_{i-1}$ are evaluated based on the same sistem state model, their initial conditions are equal, the initial covariances of errors of mentioned estimates and the initial cross covariance between them are also equal. Then the covariance matrix $D_{\Delta\theta_i}$may be written in the form

$$D_{\Delta\theta_i}=P_2-P_1=P_{i-1}-P_i.$$

5.Experimental results

For example we shall investigate the problem of calibration of measuring devices.

Usually, the calibration of any measuring device is made by the help of the taken as etalon standard measuring instrument. But, each standard instrument also reproduces signals with some definite errors (even if they are very small). If these errors are not taken into account during calibration process, the final results will contain several errors.

A new algorithm taking into account the errors of the standard instrument is presented in [7] with following initial conditions:

1)Calibration characteristics of the measuring device (in the present case, differential pressure gage)is described by 2 order polynomial as follows:

$$y(p) = a_0 + a_1 p + a_2 p^2 , \qquad (5)$$

2)It is assumed, that the standard instrument used for calibration also have some small errors.

3) Measurements contain random Gaussian noises (h_i) with a zero mean

$$z_i = a_0 + a_1 p_i + a_2 p_i^2 + h_i .$$

The coefficients in the polynomial (4) are evaluated in [7] by the algorithm (3).

In the calculations the following data and initial conditions are taken:

$$\sigma_{pi} = 0{,}00026 \; ; \quad \sigma_i = 0{,}0026,$$

where σ_{pi} is the standard error deviation of the standard measuring instrument, $\sigma_i$is the standard error deviation of the differential pressure gage.

The range of changing sample pressures is $0 \le p_i \le 1600$ bar.

The covariance matrix for input standard signals errors [9]:

$$D_{xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (0.00026)^2 & 0 \\ 0 & 0 & 4p_i^2(0.00026)^2 \end{bmatrix}.$$

As initial conditions the following values are chosen

$$\theta_0^T = [0 \;\; 1 \;\; 2 \,] \; ; \qquad P_0 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

The coefficients a_0, a_1 and a_2 found by estimation via algorithm (3), are given in Fig.2, and their errors variances in Fig.3. The curves in Fig.2 are characterized behaviors of values $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$ from iteration number. As it can be seen from the curves after some iterations the deviations of the investigated values are very small.

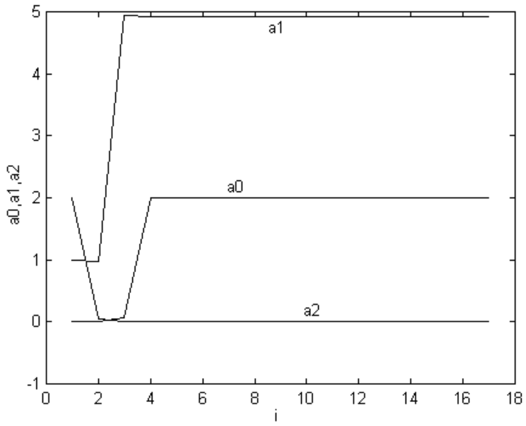

Fig.2. a_0, a_1, a_2 coefficients values behaviours

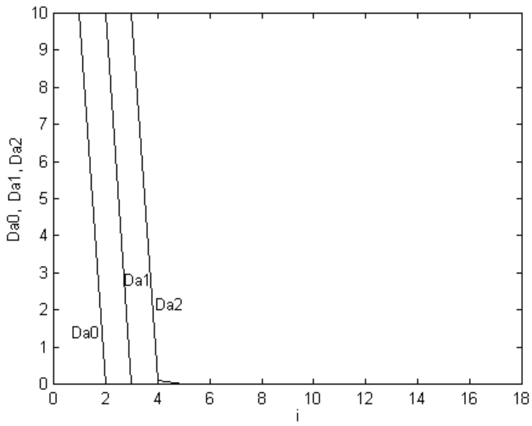

Fig.3. Variances of the errors value D_{a0}, D_{a1}, D_{a2}

Let us apply the developing in this article stopping rule and fault detection algorithms to the differential pressure gage calibration problem.

The experimental results are shown in Fig.4 and Fig.5, in which the threshold ($\chi_{\beta_1}^2$ and $\chi_{\beta_2}^2$) and r^2 statistic values are given respectively. We adopt the following confidence coefficients: a) for stopping rule problem β_1=0.01; b) for fault detection problem β_2=0.95. Their thresholds are: $\chi_{\beta_1}^2$=0,115 and $\chi_{\beta_2}^2$=7.8 respectively.

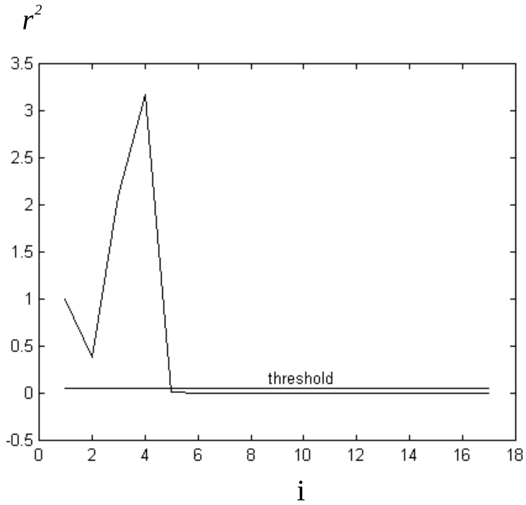

Fig.4. r^2 statistic values behaviours when Kalman filter operates normally

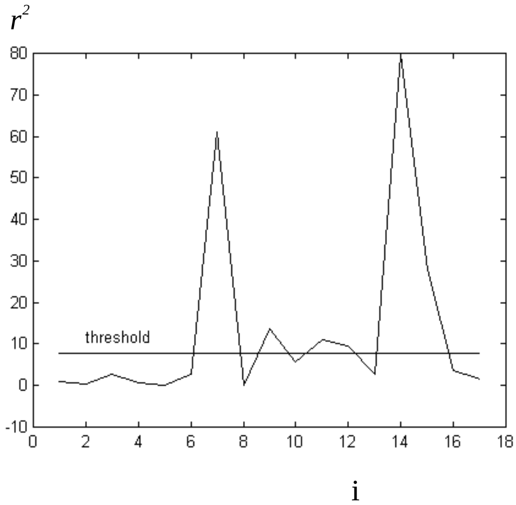

Fig.5. r^2 statistic values behaviours when fault occurs in the Kalman filter

It is evident from Fig. 4, in 5th calculation step r^2 statistic value is less than threshold value $\chi_{\beta_1}^2$. Therefore in this case it is recomended to stop the estimation process, since further observations yield insignificant improvement of the estimated coefficients a_0, a_1 and a_2 and are deemed impractical.

Experimental fault detection result is shown in Fig.5 (faults in measurement channel are simulated adding 1 to the measurement results in 6th and 14th estimation steps). It is evident from Fig.5, faults in the Kalman filter detected operatively with use presented statistic r^2.

6.Conclusions

We have thus proposed a new approach to the generation of stopping rules in parametric identification problems on the basis of the above-introduced statistic r^2 with a χ^2 distribution. The computed statistic r^2 is also used to detect faults in the Kalman filter.

The domain of possible utilization of the Kalman filter is partitioned into three zones according to our rule. When the value of r^2 lies between the confidence limits $\chi_{\beta_1}^2$ and $\chi_{\beta_2}^2$ (see Fig.1) of the corresponding χ^2 distribution, the decision is made to continue the estimation. When r^2 attains the confidence limits, the decision is made to stop the estimation process and, accordingly, the kind of corrective actions in the estimation process is decided.

The stopping rule developed here has the advantage that its application does not require the specification of an admissible error ellipsoid, whose construction represents an independent problem.

In some cases it may be necessary to process a large number of observations in order for the value of the statistic to reach the indicated “estimation stopping” threshold, resulting in large expenditures of computer time. On the other hand, the processing of a smaller number of observations lowers the accuracy of estimation. By the same token, if the “fault-detection” threshold is too high, the effects of faults in the Kalman filter tend to be smoothed out, and if the threshold is too low, the probability of false alarm increases. Consequently, the choice of these thresholds (the confidence limits of the χ^2 distribution) can have a decisive influence on the efficient utilization of computer time.

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