ALGORITHMIC INTERNAL MODEL CONTROL OF UNSTABLE SYSTEMS

R. BERBER[1] and C. BROSILOW[2]

Abstract

An internally stable Algorithmic Internal Model Control (AIMC) strategy that uses linear or nonlinear model state feedback is proposed for unstable systems. The closed loop responses are those that would be obtained from a two degree of freedom IMC control system, if it were internally stable. Results of several simulations demonstrate the validity of the approach.

1. Introduction

The control of inherently stable processes have spurred very significant advances within the framework of internal model type control structures [6, 7, 8, 10]. Such control structures incorporate an online or "internal" model of the controlled process to directly infer how disturbances have influenced the controlled variable and take appropriate control action to counter the disturbances. A major practical advantage of such control systems is that they readily accommodate constraints on both the control efforts and controlled variables [6]. Control effort constraints are even more important in the control of inherently unstable processes.

As shown by Morari and Zafiriou [10], internal model control (IMC) can be used to design two degree of freedom controllers for unstable processes, but can not be used for implementation due to the internal instability of the IMC structure. This internal instability arises from two sources: (1) In an unstable process, the effect of disturbances, which enter through the process, on the controlled variable grows without bound, and (2) Differences between the model and process states grow without bound. To avoid internal stability problems, Morari and Zafiriou [10] recommend implementing the IMC control system as a standard two degree of freedom feedback control system, with the controller and set point filter designed to yield the same transfer functions as the IMC system. Unfortunately, the controller in such implementations can itself be unstable unless the IMC filter time constant is increased substantially beyond that required for robust performance. Algorithmic Internal Model Control (AIMC) described in this work provides an internally stable method for implementing two degree of freedom IMC for unstable systems that does not require increasing the filter time constant beyond that required for robust performance. The implementation method also readily accommodates both linear and nonlinear processes and control effort saturation.

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To explore the properties of stabilizing AIMC we first study two linear processes (first and second order). For the first order plus dead time process, a block diagram description of AIMC is obtained and its equivalence to a two degree of freedom IMC system is shown. The suggested AIMC strategy is then extended to nonlinear systems, and it is shown that it applies equally well to rather general nonlinear processes. Simulation examples are provided for both linear and nonlinear cases.

2. Algorithmic Internal Model Control

It is relatively easy to rearrange the structure of an internal model control system to estimate process disturbances, which are bounded, rather than, as is usual in IMC, the effects of the disturbances, which are unbounded, and thus eliminate one source of instability in the IMC structure. However, a more profound restructuring is needed to insure that the mismatch between the model and process states does not grow without bound. To accomplish the required restructuring, we define AIMC in terms of a sequence of tasks for a sampled system because the task description seems simpler and more natural. The tasks define the structure just as block diagrams define the structure of IMC. The AIMC tasks are:

(1) At each sampling time specify a desired behavior between the set point and the controlled variable. This behavior is that of a dynamical system driven by the set point and estimated disturbance. The dynamical system is generally composed of the IMC filter cascaded with the portion of the process model that the IMC controller does not invert. The initial state of the dynamical system is reset at each sampling time to the state of the model obtained in task (3) below.

(2) Estimate the disturbance entering the process, and, if there is a dead time between the control effort and the output, extrapolate the disturbance forward in time by the amount of the dead time.

(3) Predict the process output one dead time into the future based on current measurements, current and past controls, and the extrapolated disturbance. This step also provides projections of the model states that are used to compute the desired trajectory in task (1).

(4) Compute the control effort that forces the model to track the desired trajectory. For both linear and non linear models, the required control effort is found by equating the rth derivative of the minimum phase portion of the model output to the rth derivative of the desired dynamical system, where r is the relative order of the model.

(5) Go to the next sampling instant (or increase integration time in a simulation) and return to task (1) above.

The above strategy is shown in Figure 1 as a computing flow diagram for clarity.

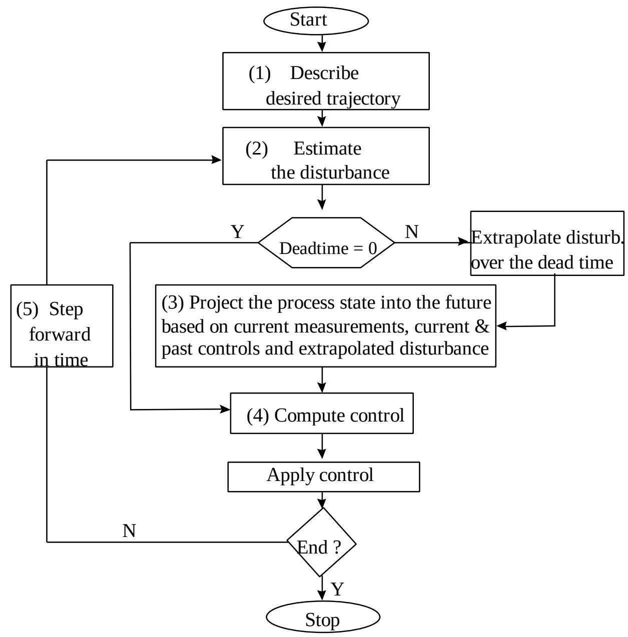

Figure 1. General flow diagram for stabilizing AIMC

Methods for accomplishing the tasks in AIMC and for designing the controller and estimator blocks in IMC depend on performance specifications, the process and disturbance models and the accuracy of the models. Process models to be used in the proposed approach should have unique trajectories for unique inputs. Further, the process outputs and states should depend continuously on the process inputs at each instant of time (i.e. infinitesimal changes in input functions should not cause jumps in the process outputs or states).

The filter in task 1 above allows the response speed of the dynamical system to be adjusted by manipulating the filter time constant, thereby providing a means for tuning the control system to achieve robustness in the case of modeling errors. If the model is a perfect representation of the process and exhibits minimum phase behavior, then the response of the process will be the same as the response of the dynamic system described by this filter equation. Most commonly, the filter will be chosen as a simple linear system with the same order as the relative order of the lag portion of the process model. A linear dynamic system can be used without loss of performance even when the process model is nonlinear provided that the model has no unstable transmission zeros [3].

The aim of task 2 is to obtain the disturbance estimate in such a way that the output of model, driven by the estimated disturbance, matches the filtered process output. Since the process output is often corrupted by noise, it is usually necessary to filter the measured output to prevent the noise from causing excessive actuator action. For linear and nonlinear systems we accomplish disturbance estimation as shown in Figure 2. In this figure, making the signal w zero allows one to get $\tilde{d}$, a filtered estimate of the disturbance (d) entering to the process. We used a Newton type algorithm for constructing the disturbance estimate in such a way that the filtered process output and the model output as shown in Figure 2 match exactly at each sampling instant making the signal w zero.

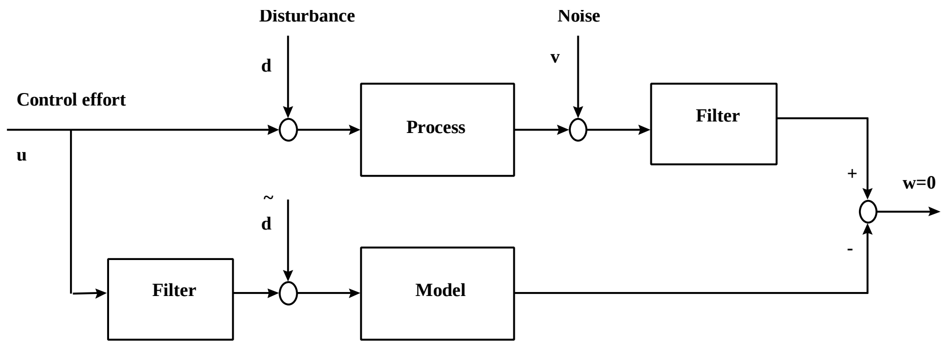

Figure 2. Disturbance estimation

Prediction of process states one dead time into the future (task 3) simply involves applying past controls and extrapolated disturbances to the model. (If the model is nonlinear, then the dead time is the least time for a change in the control effort to appear in the output.) This prediction step effectively ‘removes' the dead time from the control calculations in the same way that the Smith Predictor ‘removes’ the dead time from the control system.

The degree of complexity in computing the controls to force the process model to track a desired trajectory in task 4 depends mainly on the number of controls and output variables, and the order of the process model. For this task, we use the linearizing model state feedback control approach suggested in a previous paper by the authors (Berber & Brosilow 1998 [1]). This approach applies to both linear and nonlinear processes. One advantage of the model state feedback (MSF) implementation is that it overcomes the potentially poor behavior of the standard IMC structure due to control effort saturation. In an IMC system with a perfect model, control effort saturation is ignored leading to sluggish responses, overshoots, or pseudo non minimum phase behavior. In MSF, the control effort is calculated at each sampling instant based on the current model state and a desired response starting from that same state.

While the AIMC tasks are most naturally applied to sampled processes, there is no conceptual difficulty in passing to the limit of very small sampling times and thereby obtaining a continuous version of AIMC. For the simple linear systems we provide the analytic calculations of the AIMC tasks in continuous time and also provide the block diagram implementation for comparative purposes, whereas for higher order linear systems and nonlinear systems we give only numerical calculation procedures.

We applied the AIMC strategy to examples of several linear and nonlinear systems, with and without parametric uncertainty. In all cases, the model was assumed to be a perfect representation of the process. Simulations were done in MATLAB-SIMULINK® environment via numerical computation. Results demonstrate that the AIMC strategy allows one to achieve performance which is otherwise unachievable using classical single or two degree of freedom feedback control systems.

3. Application to First Order Plus Dead Time Processes

Here we provide the application of the suggested stabilizing AIMC algorithm to a first order plus dead time process, and show that this is equivalent to the two degree of freedom IMC system. The argument follows that in Brosilow and Cheng [2].

In the Laplace domain, a first order plus dead time process is given by

$$p(s) = \frac{K}{ts+1} e^{-sq} \tag{1}$$

where K is the process gain, τ is the process time constant (the process is unstable if τ < 0) and θ is the process dead time.

For a clearer understanding of the control strategy, let us now consider the analytic calculation of the control effort for such simple processes. Since the disturbance enters through the process, the following equation holds;

$$y = p(s)\left[u(s) + d(s)\right] \tag{2}$$

where y is the controlled process output

For the first task in AIMC, we choose the following first order filter as the lag portion of the dynamic system to be followed.

$$f(s) = \frac{1}{es+1} \tag{3}$$

where ε is the filter time constant, or tuning parameter for the speed of response.

From Figure 2 the disturbance estimate is given by:

$$e^{-sq}\tilde{d}(s) = p^{-1}(s)\, f(s, e_d)\, y(s) - f(s, e_d)\, u(s) \tag{4}$$

In the above, the filter used for disturbance estimation is chosen to be the same form as the controller filter, but the filter time constant e_d is, in general, smaller than that of the controller filter whose purpose is to accommodate modeling errors, and not just excessive noise amplification.

A prediction of the disturbance for θ time units into the future is also needed. The simplest possible prediction is to assume that the disturbance remains constant at the currently estimated value. That is

$$d(t-q+s) = d(t-q) \quad \text{for } 0 \le \sigma \le \theta \tag{5}$$

Prediction of the process state at t + θ also involves projecting the effect of past controls. The output and the state of a first order system are the same, so estimation of the current state is simply the output or some suitable filtered version thereof. Prediction of the state one dead time ahead requires projection of the current state, past controls, and the projected disturbances. For the continuous representation,

$$y(t+q) = e^{-q/t} y(t) + K\int_t^{t+q} e^{-(t+q-s)/t} u(s-q)\,ds + K(1-e^{-q/t})\,d(t-q) \tag{6}$$

The integral on the right is the convolution integral evaluated over a period θ, starting from time t. (Note that when σ = t the integrand is $e^{-q/t}u(t-q)$, while when σ = t + θ the integrand is u(t) so that only past controls are needed for the projection.) Because the integral is evaluated over a finite period, it remains bounded even when τ is negative.

The control u(t) is chosen to make the process model follow a desired linear system as it moves toward the set point. The model given by (1) can be expressed in the time domain as

$$t\,\frac{d}{dt}\,y(t+q)+y(t+q)=K(u(t)+d(t)) \qquad (7)$$

The desired output response, $y_d(t)$, to a set point change and the disturbance is:

$$e\,\frac{d}{dt}\,y_d(t+q)+y_d(t+q)=y_{sp} \qquad (8)$$

Since the desired response always starts from the projected model state,(i.e. $y_d(t+q)=y(t+q)$),then $y(t+q)$ from (7) will track $y_d(t+q)$ from (8) if we choose u(t) to make the derivatives of $y(t+q)$ and $y_d(t+q)$ equal to each other. From (7) and (8) this requires:

$$(K(u(t)+d(t))-y(t+q))/t=(y_{sp}-y_d(t+q))/e \qquad (9)$$

Setting $y_d(t+q)=y(t+q)$, projecting *d(t)* from (5) as *d(t -q)* and solving for *u(t)* gives:

$$u(t)=-d(t-q)+((t/e)y_{sp}+(1-t/e)y(t+q))/K$$

(10)

Figure 3 gives the block diagram for the continuous version of the AIMC algorithm for a first order plus dead time process.. This figure comes from equation, (1), (4), (5) and the transforms of equations (6) and (10).

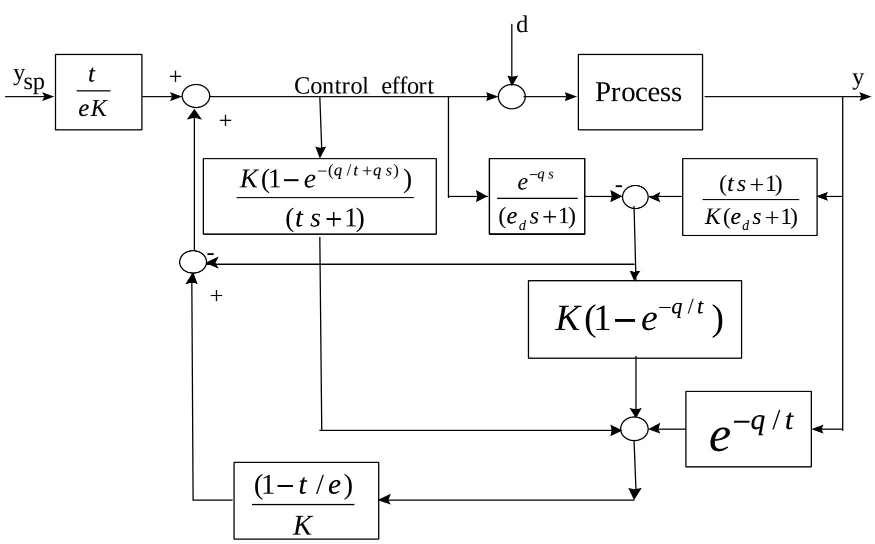

Figure 3. AIMC block diagram for FOPDT process

By (a lot of) block diagram algebra, one can show that the transfer functions between the output, y(s), and the disturbance, d(s), and set point, r(s) for figure 3 are the same as for figure 4 below. However, figure 3 is internally stable even when the process is unstable (i.e. $\tau < 0$), while the diagram of figure 4 is internally unstable. To see why this is so, we note that the transform of the integral in (6), as derived in Appendix A, is:

$$\mathcal{L}\left\{K\int_{t}^{t+q} e^{-(t+q-s)/t}u(s-q)ds\right\} = \frac{K(1-e^{-q/t}e^{-qs})}{ts+1}u(s) \tag{11}$$

The right hand side of this equation implies that integral in (6) can be evaluated as

$$K\int_{t}^{t+q} e^{-(t+q-s)/t}u(s-q)ds = x(t) - e^{-q/t}x(t-q) \tag{12}$$

where x(t) is given by the solution of

$$t\frac{d}{dt}x(t) + x(t) = Ku(t) \tag{13}$$

Equation (12) is a valid method of computation only for stable x(t). If x(t) is unstable, the right hand side of the equation (12) is the difference between two numbers growing without bound and infinite precision is required to get a finite result. This difficulty is the reason why the two degree of freedom IMC control system given in Figure 4 is computationally equivalent to the AIMC structure only for stable systems.

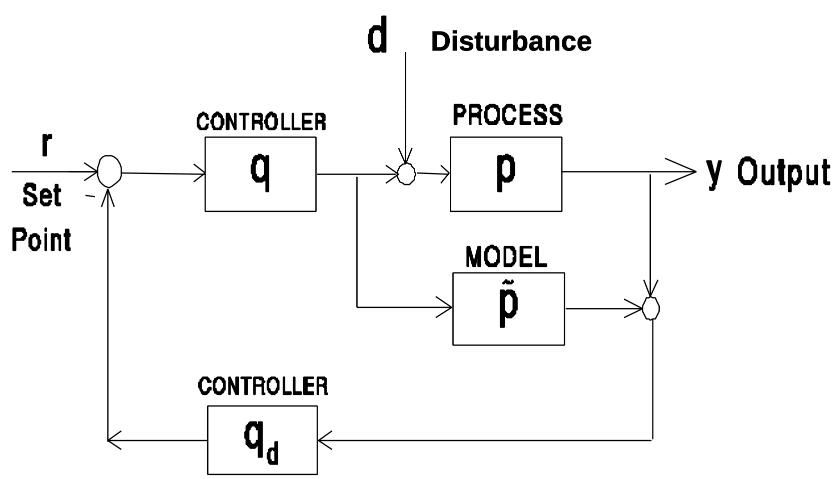

Figure 4a. Two degree of freedom IMC control system equivalent to Figure 3 for Stable Processes

In figure 4a, q(s) and q_d(s) are:

$$q(s) = \frac{ts+1}{(es+1)K} \tag{14}$$

$$q_d(s) = a + \frac{(1-a)(ts+1)}{(e_d s+1)} \tag{15}$$

where $\qquad a \equiv (1 - e/t)e^{-q/t}$

An alternate, internally stable, implementation of figure 4a for unstable processes suggested by Morari and Zafiriou (1989) is the simple feedback system of figure 4b.

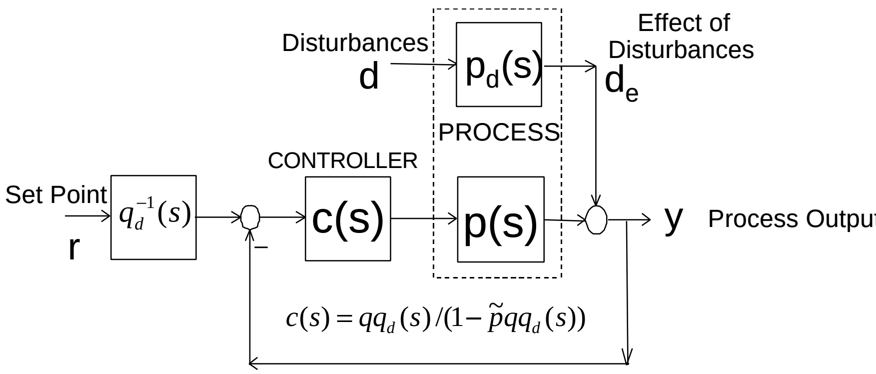

Figure 4b Alternate Feedback Configuration of Figure 4a

The controller, c(s), in figure 4b must be implemented as a single transfer function (or a single set of differential equations) where any dead times are replaced by finite order (Pade) approximations in order for the control system to be internally stable for an unstable process. Further, the controller c(s) must be stable. At a minimum, this means that the right half plane zeros at $s=-1/\tau$ ($\tau < 0$) in the numerator and denominator of c(s) must be made to cancel exactly. Further, the denominator of c(s) must not have any additional right half plane zeros. Unfortunately, for small values of the filter time constants ε, and ε_d, the denominator of c(s) does have additional right half plane zeros, and the control system given by figure 4b is unstable even for perfect models, whereas the control system given by figure 3 will be stable.

From figure 4a, the response of the system in figure 3 to disturbances and set point changes for a perfect model is:

$$\begin{aligned} y(s) &= pq(s)y_{sp}(s) + (1-\tilde{p}qq_d(s))p(s)d(s) \\ &= \frac{y_{sp}(s)}{es+1} + \left(1 - (\frac{e^{-sq}}{es+1})(a - \frac{(1-a)(ts+1)}{(e_d s+1)})\right)\frac{Ke^{-sq}d(s)}{ts+1} \end{aligned} \tag{16}$$

By the definition of a in (15) the term in brackets in (16) has a zero at $s = -1/\tau$ for any values of ε and ε_d.

The time domain response for a step input in set point is an exponential rise which approaches a step as the time constant, ε, approaches zero. The time domain response to a step disturbance for $\varepsilon=\varepsilon_d=0$ is

$$\begin{aligned} y(t) &= 0 && \text{for} \quad 0 \le t \le \theta \\ &= (1-e^{-t/\tau})K && \qquad\quad \theta \le t \le 2\theta \\ &= 0 && \qquad\quad 2\theta < t \end{aligned} \tag{16a}$$

No controller without a priori knowledge of when the step disturbance will occur can give a better response than (16a). When ε and ε_d are greater than zero, the response is the same as that given by (16a) for $t \le 2\theta$, and is the sum of exponentials given below for $t \ge 2\theta$.

$$y(t) = \boldsymbol{a} e^{-(t-2q)/e_d} + \boldsymbol{b} e^{-(t-2q)/e} \quad ; \quad t > 2\theta \tag{16b}$$

where $\alpha \equiv (1\text{-}a)c$; $\beta \equiv (1\text{-}a)(1\text{-}c)+a\text{-}e^{-(\theta/\tau)}$; and $c \equiv (1\text{-}\varepsilon/\varepsilon_d)^{-1}$

Notice that in (16b), $y(2\theta)=1\text{-} e^{-(\theta/\tau)}$ so that y is continuous at $t=2\theta$.

Results of applying the strategy given by figure 3 to an unstable process where K=1, τ = -1, θ = .8 are presented in Figures 5 and 6 for disturbance rejection and set point tracking respectively. Figure 5 shows the output and control effort responses to a step disturbance of 1 introduced at time t= 0. Figure 6 shows the set point tracking capability of the control system. In these simulations, controller filter and disturbance estimation filter are chosen to be ε =1, ε_d = 1. These figures also compare the results of AIMC to those obtained from figure 4b where the controller c(s) is approximated by the PID controller computed as in Lee et al. [9](i.e $c(s) \cong -1.62(1+1/(8.27s)+.359s/(.018s+1))$. The differences between the AIMC controller and the PID controller are due to the fact that for ε =1, ε_d = 1, the PID controller is not a very good approximation to the controller, c(s), in figure 4b. Indeed, the controller c(s) becomes unstable when $\varepsilon = \varepsilon_d < .83$.

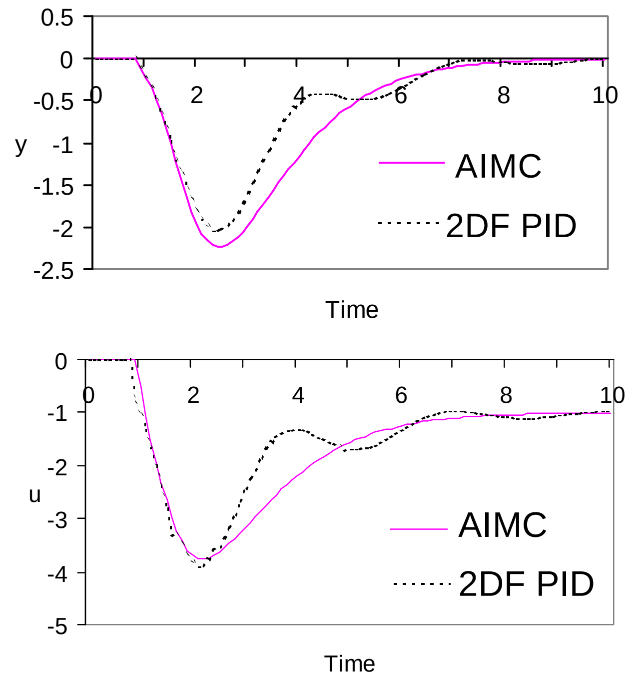

Figure 5. Regulatory response of first order linear system

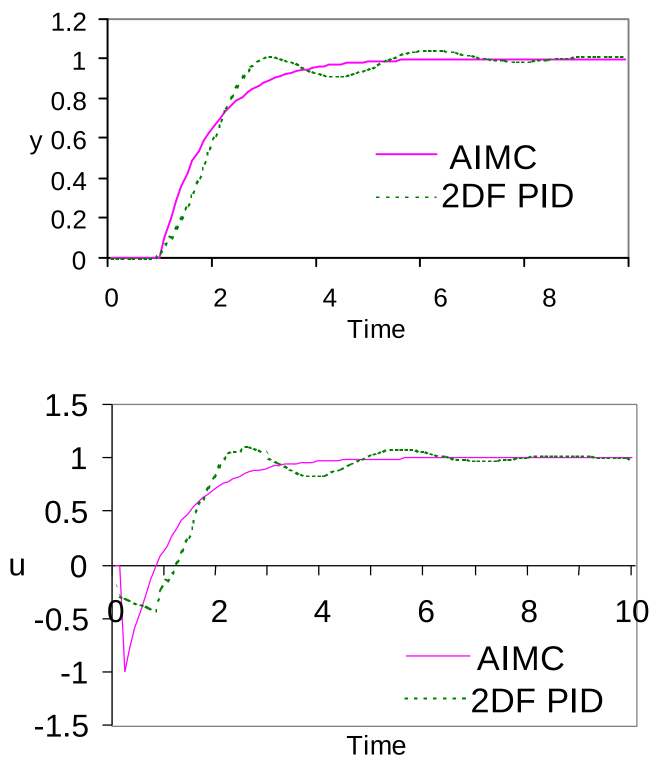

Figure 6. Servo response of first order linear system

4. Application of AIMC to a Second Order Linear System

The example process is:

$$p(s) = \frac{K}{(t_1 s + 1)(t_2 s + 1)} e^{-sq} \tag{17}$$

where $\tau_1 = 1$, $\tau_2 = -1.1$ and $\theta = 1$.

A perfect model was assumed in all simulations. The AIMC control algorithm of Section 2 was implemented according to the specified tasks as follows:

- The desired trajectory is set as a second order filter;

$$e^2 \ddot{y}_f + 2e\dot{y}_f + y_f = y_{sp} \tag{18}$$

- Disturbance is estimated as described in Figure 2.

- Process output is projected into the future by integrating the model equation (17) from the current time, t_k, to $t_k+\theta$ based on current measurement, current and past controls, and the extrapolated disturbances. In the time domain, and in presence of d introduced through the process, (17) becomes:

$$t_1 t_2 \ddot{y} + (t_1 + t_2)\dot{y} + y = K(u + d) \tag{19}$$

  In the course of integration, the effect of past controls on the projection was taken into account by shifting the past trajectory of the control effort forward in time by θ units. Since the derivative of the process output constitutes one of the states of this model, this projection implies that, not only the process output but also its first derivative was projected into the future.

- To calculate the control so that the model output tracks the filter output given by (18), the AIMC control law requires that the second derivative of the model output, from (19) be equal to the second derivative of the desired trajectory from (18). This gives:

$$u(t_k) = \frac{t_1 t_2}{Ke^2} y_{sp} - d + \frac{1}{K}\left(t_1 + t_2 - \frac{2t_1 t_2}{e}\right)\dot{y}(t_k + q) + \frac{1}{K}\left(1 - \frac{t_1 t_2}{e^2}\right) y(t_k + q) \tag{20}$$

In the derivation of the above equation, the first derivative of the filter output was made equal to the first derivative of the model output. All model states are projected one dead time into the future, and the control effort is calculated from this equation with output and its first derivative being evaluated at time $t_k+\theta$.

Results from the application of the stabilizing AIMC strategy to the unstable process, given by (17) are shown in Figures 7-9. Figures 7 and 8 depict the performance of the controller for set point tracking and disturbance rejection (under an introduced disturbance of d = 1 through the process at t=0). Figure 9 shows the transient behavior of the system under a set point change of r = 1 and a disturbance of d= - 0.3. All these simulations were done with $\varepsilon = \varepsilon_d = 1$. For this example, the controller c(s) is stable for $\varepsilon = \varepsilon_d > .36$. However, to get a PID controller which yields a good response requires $\varepsilon = \varepsilon_d > 3$.

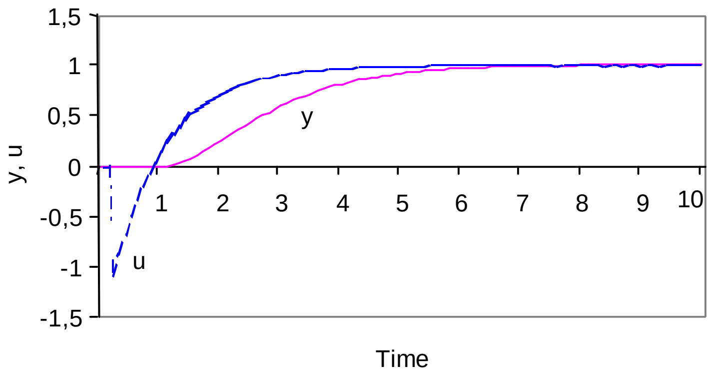

Figure 7. Second order linear system, Set Point Response

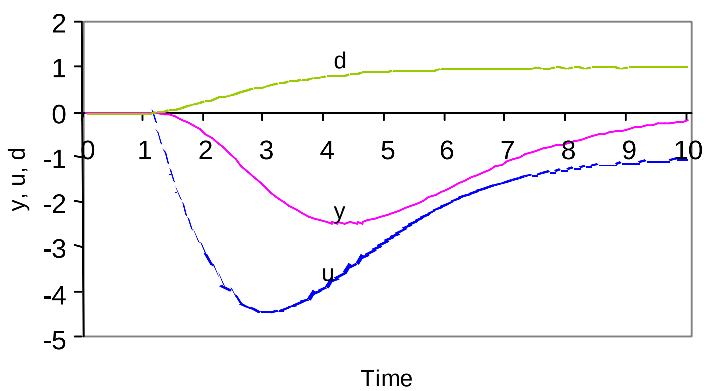

Figure 8. Second order linear system, response to a step disturbance

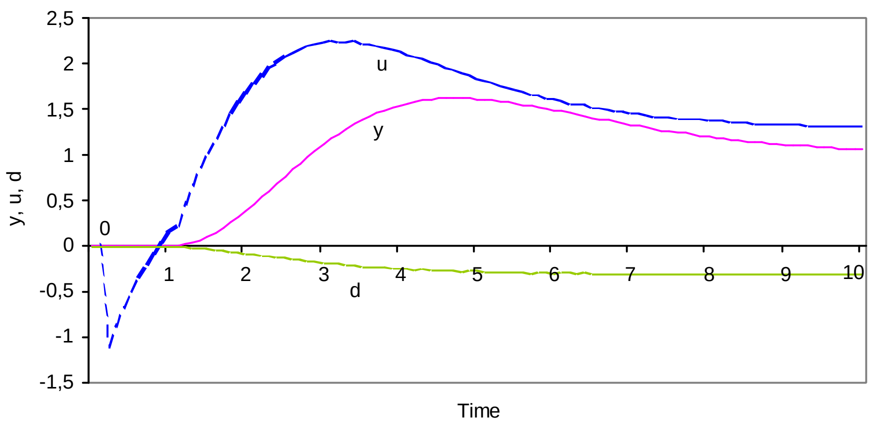

Figure 9. Second order system, under set point change and step disturbance

5. Application to Nonlinear Systems

To illustrate the applicability of the algorithm to nonlinear systems, we have chosen two examples one stable and the other unstable. The stable example was studied by Soroush and Kravaris [11]. It considers a CSTR in which the following parallel reactions take place:

$$A \xrightarrow{k_1} U_1 \xrightarrow{k_2} U_2 \; ; \quad A \xrightarrow{k_3} D$$

where U_1 and U_2 are undesired side products, and D is the desired product.

Feed to the reactor is free of any products. The control objective is to keep the reactor temperature at its set point at 400 °K. This is a stable operating point corresponding to high conversion. Manipulating variable is the heat input to the reactor (Q). Following standard assumptions and considering that the concentration of D does not affect the reactor temperature, mass balance for A and energy balance give the reactor model:

$$\dot{C}_A = -k_1 C_A^3 - k_2 C_A^{0.5} - k_d C_A + \frac{C_{Ai} - C_A}{t} \tag{21}$$

$$\dot{T} = \frac{-\Delta h_1 k_1 C_A^3 - \Delta h_2 k_2 C_A^{0.5} - \Delta h_d k_d C_A}{rc} + \frac{T_i - T}{t} + \frac{Q}{rcV} \tag{22}$$

The kinetic rate constants follow the Arrhenius relation, $k_i = k_{oi} \exp(-E_{ai}/RT)$. The operating and kinetic parameters are given in Table 1. The presence of steady state multiplicities makes the reactor start-up a true control problem, because if the necessary steady state heat input is applied to the reactor in open loop it reaches the lowest conversion operating point instead of the highest conversion set point.

Table 1. Parameters for the CSTR with parallel reactions

R	R=8.345 kJ/(kmol.K)
k_{01}	k_{01}= 2x10^3 m^6/(kmol^2.s)
k_{02}	k_{02}= 3.4x10^6 kmol^{05}/(m$^{-1.5}$.s)
k_{0d}	k_{0d}= 2.63x10^5 s^{-1}
E_{01}	E_{01}= 4.9x10^4 kJ/kmol
E_{01}	E_{01}= 6.5x10^4 kJ/kmol
E_{01}	E_{01}= 5.7x10^4 kJ/kmol
$-\Delta H_1$	$-\Delta H_1$=4.5x10^4 kJ/kmol
$-\Delta H_2$	$-\Delta H_2$=5.0x10^4 kJ/kmol
$-\Delta H_d$	$-\Delta H_d$=6.0x10^4 kJ/kmol
ρ	ρ=1000 kg/m^3
C	c=4.2 kJ/(kg.K)

The performance of the AIMC controller for start-up to reach the desired operating state is compared to two other nonlinear controllers, namely, model state feedback (MSF) as presented by Berber and Brosilow (Kluwer, 1998), and global linearizing control (GLC in Two Degree of Freedom Output Feedback form) of Soroush and Kravaris [11]. Tuning parameters for AIMC was $\varepsilon = \varepsilon_d = 50$ whereas for GLC $\beta_1 = \gamma_1 = -0.98$. Maximum control effort was set to 30. Results are shown in Figures 10a, b. The results show that all controllers perform similarly for set point tracking.

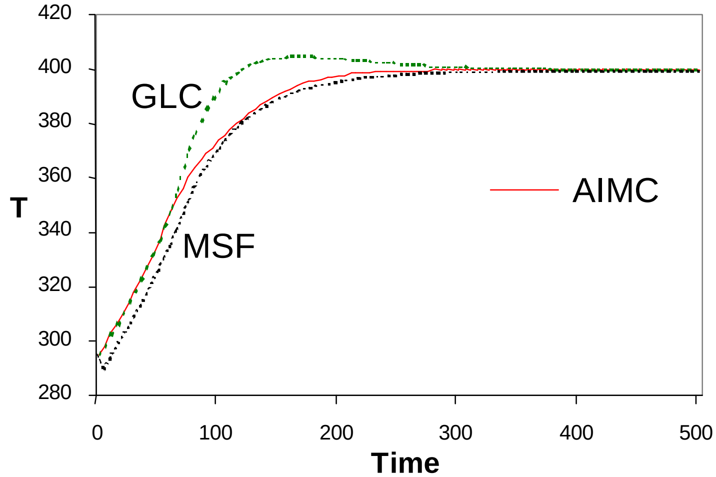

Figure 10a. Reactor start-up profile, comparison of three controllers

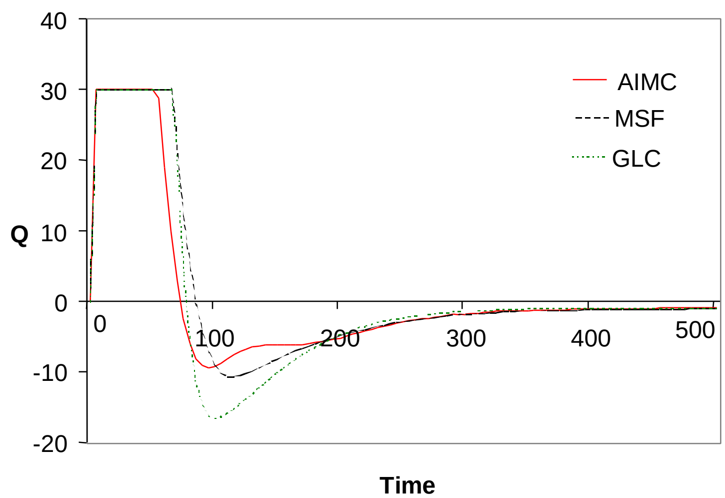

Figure 10b. Control effort in reactor start-up corresponding to Figure 10a

To test the AIMC and compare it to GLC for regulatory behavior, an unmeasured step disturbance of +20 °K was introduced to the feed temperature. Tuning parameters were the same as those in the set point tracking. The performance of the two controllers are indistinguishable when the tuning parameters in GLC were set as $\beta_1 = \gamma_1 = -0.99$, as shown in Figures 11a, b.

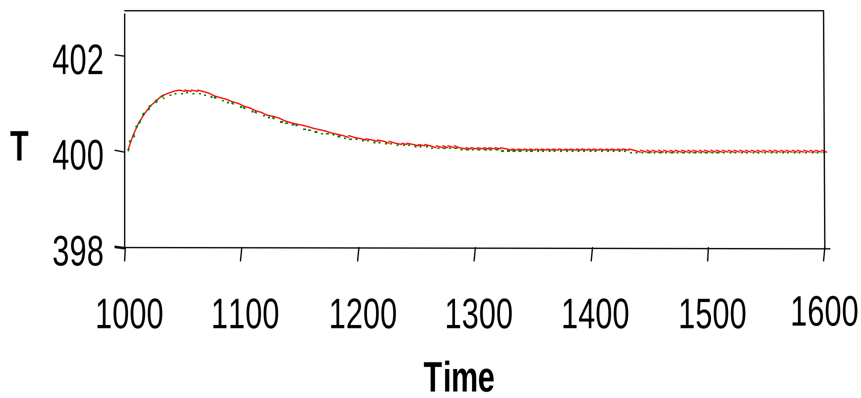

Figure 11a. Regulatory performance of AIMC and GLC to a step disturbance

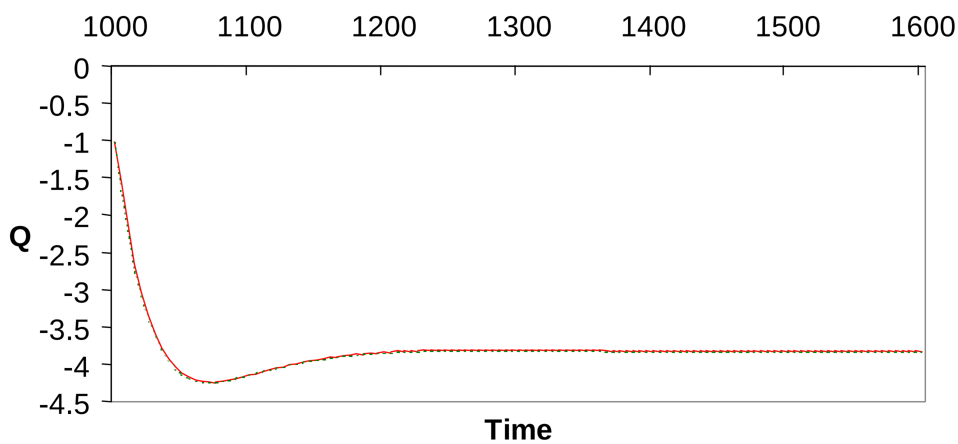

Figure 11b. Control effort in reactor start-up corresponding to Figure 11a.

The second nonlinear example was previously presented by Calvet and Arkun [4, 5] originating from the work of Uppal *et al*. [12]. This example considers a continuous stirred tank reactor in which a first order, exothermic, irreversible reaction is taking place. The dimensionless model equations are given by:

$$\dot{x}_1 = -x_1 + D_a(1-x_1)e^{\frac{x_x}{1+(x_2/n)}} - d_2 \tag{21a}$$

$$\dot{x}_2 = -x_2 + BD_a(1-x_1)e^{\frac{x_x}{1+(x_2/n)}} - b(x_2 - x_{20}) + bu + d_1 \tag{21b}$$

where: x_1 and x_2 are the dimensionless composition and temperature
D_a, B,ν, β and x_{20} are the standard dimensionless parameters
d_1 and d_2 are the dimensionless feed temperature and feed composition fluctuations [4].

In this work, d_2 is assumed to be zero. A step disturbance was introduced by assigning a specified value to the term d_1. With the parameters $D_a = 0.072$, $B = 8$, $\beta = 0.3$, $\nu = 20$, and $x_{20} = 0$, the CSTR exhibits an ignition/extinction behavior. The control problem is to operate the reactor at the unstable operating condition $u^{op} = -0.20$, $x_1^{op} = 0.5$ and $x_2^{op} = 3.03$. The reactor is initially assumed to be operating at a stable steady state of $x_1 = 0.2$ and $x_2 = 1.33$.

Simulation results when the AIMC structure is implemented are given in Figures 12-14. The two plots in Figure 12 show the response of the dimensionless temperature to a step change in set point for two different values of the filter time constant. Tuning parameters used were ε=1 and ε=2. The transition of the process variable reflects a linear behavior, and is smooth allowing the reactor to be operated at an unstable operating point by the AIMC algorithm.

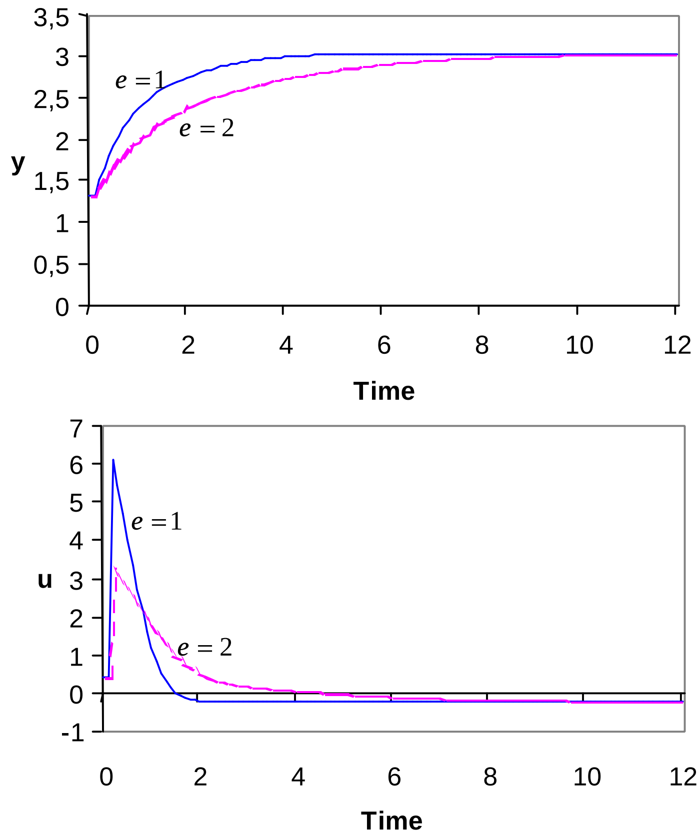

Figure 12. Nonlinear system under a step change; process variable and control effort

Figures 13a and 13b present the transient behavior of the process variable and the control effort when an unmeasured step disturbance of d_1 =.3 is introduced to the process. Figure 13c shows the estimated step disturbance that makes the model output equal to the process output if it were introduced to the model as an additive signal to the control effort similar to the linear case. It is to be noted that the estimated additive step disturbance of d_e=1 is equivalent to the specified unmeasured disturbance of d=.3 at the steady state. The disturbance estimation given in figure 13c also makes the model and process output match at each sampling time throughout the transient. It is seen that this disturbance is successfully compensated by the AIMC system and the reactor is kept at the unstable operating point using two different filter time constants. Figure 14 shows the behavior of the AIMC system for a set point change and an unmeasured disturbance of d_1= 0.3. The controller successfully tracks the set point under the influence of the unmeasured disturbance.

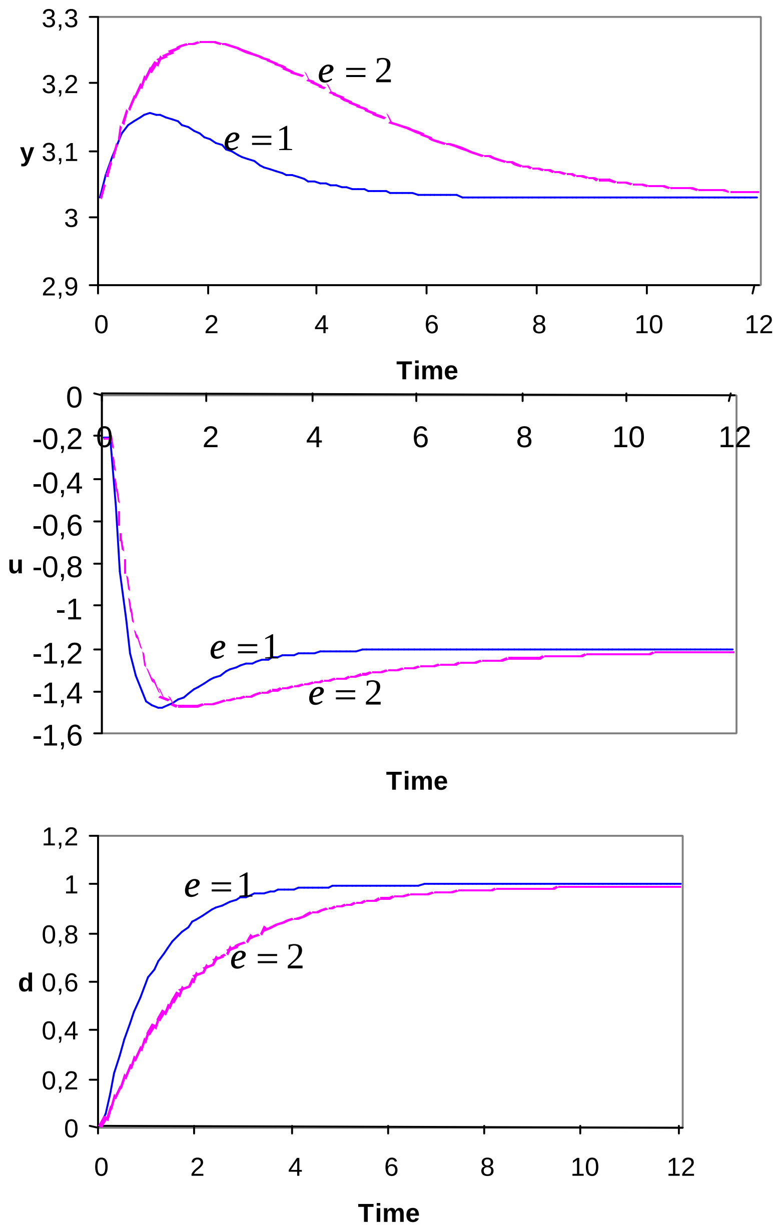

Figure 13. Nonlinear system under an unmeasured disturbance; process variable, control effort and disturbance estimate

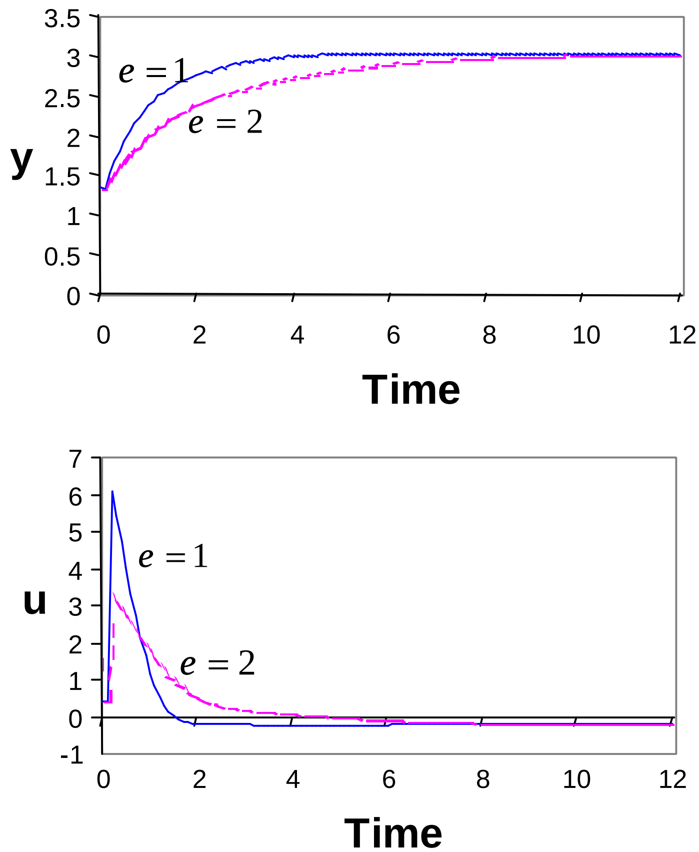

Figure 14. Nonlinear system under step change and disturbance; process variable and control effort

6. Conclusions

The problem of the control of unstable systems has been tackled, and a solution that is applicable to linear as well as nonlinear processes is provided. An algorithmic IMC algorithm, applicable to a very broad range of process models, is presented to stabilize the unstable processes. For a first order plus dead time process, AIMC is analytically equivalent to the two degree of freedom IMC. AIMC, however, is internally stable for unstable models whereas the classical implementation of two degree of freedom IMC is not. Simulations show that AIMC can be effectively used for both stable and unstable linear and nonlinear processes.

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Appendix A. Laplace Transform of the Effect of Past Controls on the Future Process Output

The influence of past controls on the predicted output $y(t+\theta)$, represented by y_m here, can be evaluated through the following convolution integral:

$$y_m \equiv K \int_{t}^{t+q} e^{-(t-q-s)/t} u(s-q) ds \tag{A1}$$

To obtain the transform of (A1), we first express y_m as the difference between two integrals, and then change variables so as to get each of the integrals in standard convolution form as follows:

$$y_m / K = \int_0^{t+q} e^{-(t-q-s)/t} u(s-q) ds \; - \; \int_0^{t} e^{-(t-q-s)/t} u(s-q) ds$$

$$= \int_q^{t+q} e^{-(t-q-s)/t} u(s-q) ds \; - \; \int_0^{t} e^{-(t-q-s)/t} u(s-q) ds \qquad \text{(A2)}$$

since $m(\sigma - \theta)$ is zero for $\sigma < \theta$.

Now let $\phi = \sigma - \theta$ in the first integral and factor $e^{-\theta/\tau}$ from the second integral. This gives

$$y_m / K = \int_0^{t} e^{-(t-f)/t} u(f) d f \; - \; e^{-\theta/\tau} \int_0^{t} e^{-(t-s)/t} u(s-q) ds \qquad \text{(A3)}$$

Transforming (A3) gives

$$\mathcal{L}\{y_m / K\} = \frac{u(s)}{ts+1} - \frac{(e^{-q/t})(e^{-sq} u(s))}{ts+1}$$

$$= \frac{1}{ts+1} u(s) - \frac{e^{-q/t}}{ts+1} e^{-qs} u(s)$$

Appendix B. The Analyical Equivalence of AIMC and Two Degree of Freedom IMC

The Laplace transforms of eqns. (10) and (6) in the text are, respectively,

$$u(s) = - e^{-s\theta} d(s) + \frac{\tau}{\varepsilon K} y_{SP}(s) + \frac{(1-\tau/\varepsilon)}{K} (e^{s\theta} y(s)) \qquad \text{(B1)}$$

$$[e^{s\theta} y(s)] = e^{-\theta/\tau} y(s) + \frac{K(1-e^{-\theta/\tau} e^{-s\theta})}{\tau s+1} u(s) + K(1-e^{-\theta/\tau}) e^{-s\theta} d(s) \qquad \text{(B2)}$$

Letting $e^{-\theta/\tau} \equiv b$ and rearranging terms gives

$$[e^{-s\theta} y(s)] = b\,[y(s) - \frac{Ke^{-s\theta}}{\tau s+1} u(s)] + \frac{K}{\tau s+1} u(s) + K(1-b) e^{-s\theta} d(s)$$

(B3)

The term in parentheses in (B3) is measured output less the effect of all past controls on the output as predicted by the model and as shown in Figure 4.
Let

$$d_e(s) \equiv y(s) - \frac{Ke^{-s\theta}}{\tau s+1} u(s) \tag{B4}$$

Substituting (B3) and (B4) into (B1) gives

$$u(s) = \frac{(1-\tau/\varepsilon)}{K} \left[bd_e(s) + \frac{K}{ts+1} u(s) + K(1\text{-}b)e^{-s\theta} d(s) \right] + \frac{\tau}{\varepsilon K} y_{SP}(s) - e^{-s\theta} d(s) \tag{B5}$$

Rearranging terms to solve for u(s) from (B5) gives

$$\left[\frac{\varepsilon s+1}{\tau s+1} \right] u(s) = -\, a\, d_e(s) + (a\text{-}1)Ke^{-s\theta} d(s) + y_{SP}(s)$$

(B5a)

where $a \equiv (1\text{-}\varepsilon / \tau)b$

From the manner in which $e^{-s\theta} d(s)$ is computed, c.f. eqn. (2) in the text,

$$e^{-s\theta} d(s) = \frac{\tau s+1}{(\varepsilon_d s+1)K} d_e(s)$$

(B6)

Substituting (B6) into (B5a) gives

$$u(s) = \frac{\tau s+1}{K(\varepsilon s+1)} \left[y_{SP}(s) - \left[a + (1\text{-}a) \frac{(ts+1)}{e_d s+1} \right] d_e(s) \right] \tag{B7}$$

Letting

$$q_d(s) \equiv a + (1\text{-}a) \frac{(\tau s+1)}{\varepsilon_d s+1}$$

and

$$q(s) \equiv \frac{\tau s+1}{K(\varepsilon s+1)}$$

gives exactly the relationships shown diagrammatically in figure 4a.