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A HIGH GAIN OBSERVER FOR ROBUST STATE FEEDBACK CONTROLLER

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Abstract:
A High gain observer is introduced in this paper to reconstruct unmeasurable system states for nonlinear feedback control strategies. The analysis of the observer shows promising properties. The validity its performance shown through numerical example in the presence of uncertainty in the system.

Key Words: High gain observer, output feedback control, sliding mode control, robot manipulators.

1. Introduction

There exists a vast literature on the design of robust state feedback controllers for uncertain dynamic systems when the uncertainty is matched. Typical techniques are high gain feedback control, Lyapunov Min-Max control, and Variable Structure System (VSS) with Sliding Mode Control (SMC), Marino (1985), Corless (1993) and Utkin (1992). These control strategies assume that states of the system are available for the output feedback which not the cases in practice. Also it is important that a robust observer design is necessary in order to implement such designs to the mechanical systems, such as robot manipulators, by utilizing only output feedback rather than full state feedback, Spong and Vidyasagar (1989).

In the case of output feedback SMC there are two type design procedures are available; a compensator type and an observer type. The results of compensator type nonlinear feedback controllers are limited to relative degree one and minimum phase systems, Heck *et al.*, (1995). The available nonlinear observers for nonlinear feedback controllers are well reviewed in, Walcott *et al*., (1989) and Misawa and Hedrick (1989). Another type of nonlinear observer is the discontinuous observer, Walcott and Zak (1988) where is restricted by the Positive Real Lemma, Slotine and Li (1991), and can be used when velocity is available for particular mechanical systems. In the case the only position is measurable this type of observer cannot be applicable in the electromechanical systems. Note that all methods have their own positive aspects, either as extensions of linear techniques, or as novel nonlinear techniques.

Here, Variable Structure System (VSS) theory is utilized in the design of output feedback regulators for systems subject to bounded uncertainties and disturbances. A robust high-gain observer is developed to estimate the state variables for design VSS with Sliding

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Mode Control (SMC). The validity and performance of the observer is discussed and illustrated with an example: a robotic manipulator.

2. System Description

Consider an uncertain dynamical system is given as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + f(x,u,t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $u(t)$ and $y(t)$ are respectively scalar system input and output. Here $\dot{x}(t)$ is the derivative of the state vector of $x=[x_1, x_2, \ldots x_n]^T$. The system matrixes are in the following forms;

$$A = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The unknown function f: $\hat{A}^n \times \hat{A}_+ \to \hat{A}^n$, which represents the uncertainties and nonlinearities in the systems. It is assumed that the parameter uncertainties and disturbances are matching, occurring only on the control channels,

$$f(x,u,t) = Bx(x,t) \tag{2}$$

where x: $\hat{A}^n \times \hat{A}_+ \to \hat{A}$ is unknown, but bounded, so that

$$\|x(x,t)\| \leq r \qquad \forall x \in \hat{A}^n, \, t \geq 0 \tag{3}$$

where r is the positive scalar function and $\| \, . \, \|$ denotes the euclidean norm. Furthermore the system is defined in (1) satisfies the following assumptions.

A1 Let $x_0(\hat{x},t)$ be known nominal models of $x(x,t)$ and satisfy following conditions

$$x_0(\hat{x},t) - x(x,t) = \Delta x(e,t) \qquad \forall t \geq 0$$

$$\|\Delta x(e_i,t)\| \leq L_F \|\hat{x}_i - x_i\| \qquad i = 1,\ldots,n$$

where L_f is the positive Lipschitz constant and $e = \hat{x} - x$.

A2 The initial value of the system state $x(0)$ is bounded as constant.

A3 The whole system is *BIBO*.

The equation in (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + B[u(t) + x(x,t)] \tag{4}$$

where $x(x,t)$ represents the lumped uncertainties.

3. The properties of VSS with Sliding Mode and High Gain System

Variable Structure System utilizes a high-speed switching control law to drive the plant state trajectories onto a specified and designer-chosen surface in the state space, called switching surface or sliding mode, and then maintain the plant state trajectories on this surface. The controller generated via VSS with sliding mode is nonlinear and known as sliding mode control Utkin (1992). Sliding mode control is somewhat related to the Lyapunov control. The outstanding feature of these controllers is their excellent robustness and invariance properties. The fundamental features of VSS with sliding mode and the Lyapunov control and their applications are given in Utkin (1992) and Zinober (1990). Moreover, Lyaponov control and sliding mode control, which are commonly used for the systems have matching uncertainty, generate nonlinear state feedback laws.

The fundamental property of high gain systems is given in Kokotovic and Khalil (1986) is their relationship with singularly perturbed systems. Young *et. al.* (1977) have studied high-gain feedback systems and obtained results that the slow motions of high gain systems are the same as sliding motions in VSS.

Consider the following first order system.

$$\dot{x}_n(t) = f(x,t) + b(x,t)u(t) \tag{5}$$

where $y = x_1$ and u are respectively the scalar output and input and $b(x,t)$ is invertible. The state vector of $x=[x_1, x_2, \ldots x_n]^T$ and $\dot{x}_i(t) = x_{i+1}$ (i=1,2,…,n-1). The dynamic of $f(x,t)$ is not exactly known but estimated as $\hat{f}(x,t)$. The error between $\hat{f}(x,t)$ and $f(x,t)$ is defined as $\Delta f(e,t)$ and assumed that it satisfies the assumption **A1** for any finite interval [1 T].

$$\Delta f(e,t) \le L_f \tag{6}$$

where L_f is a positive constant. Now consider the model as

$$\dot{\hat{x}}(t) = \hat{f}(x,t) \tag{7}$$

where $\hat{x}$ is the model output. In order to have track $x(t) = \hat{x}(t)$, we define sliding surface and tracking error

$$s(t) = \hat{x}(t) - x(t). \tag{8}$$

For this tracking problem let we first select the *SMC* as

$$u = \frac{k}{b}\operatorname{sgn}(s(t)) \tag{9}$$

where k is the positive constant and sgn is the sign function. Subtract equation (5) from (7), we then have

$$\dot{s}(t) = \Delta f(e,t) - k\,\mathrm{sgn}(s(t)) \qquad (10)$$

The equations (8) and (10) are simply means that a sliding mode does exist on the discontinuity surface whenever the distances to this surface and the velocity of its change $\dot{s}$ are of opposite sign. The sliding condition is given as follows

$$\dot{s}(t)s < 0 \qquad (11)$$

Thus the sliding condition is satisfy and tracing is achieved despite the existing nonzero component $\Delta f(t,x)$ in finite time if the control gain k is chosen to be large enough as

$$k \geq L_f + h \qquad (12)$$

where h >0.

Now replacing the discontinuous control from the equation (10) with a continuous feedback control as

$$u = ks(t) \qquad (13)$$

with this controller the system is given in (10) can be rewritten as follows

$$m\frac{ds(t)}{dt} = m\Delta f(e,t) - b(x,t)s(t) \qquad (14)$$

where m =1/k. For the sufficiently small but non-zero value of m the sign of $s(t)$ is opposite to that of its derivative $ds(t)/dt$. This means that the sliding mode occurs for this system as well. It is clear that the sliding mode motion will not be exactly on the manifold $s(t) = 0$ as in the discontinuous control system, but in the vicinity of the switching surface.

For sufficiently high gain k this system is singularly perturbed system Kokotovic and Khalil (1986). With $m = 0$ equation becomes as $b(t).s(t) = 0$ has not a unique solution and does not allow one to separate slow and fast motions of high gain system. But we show that the sliding motion is occur for the equation in (14). As the case with the design of discontinuous control systems where the equivalent control method Utkin (1992) permits one to form the equations of the motion on the manifold of discontinuity surface. Therefore from the sliding mode condition in (11), the equivalent control has unique solution to the algebraic equation in (14) as

$$0 = \frac{\Delta f(e,t)}{k} - b(x,t)s(t) \qquad (15)$$

By choosing gain k in equation (13) to be large enough, we can now guarantee that the convergence condition is given in (11) is satisfied despite the existing nonzero component $\Delta f(e,t)$ and the initial conditions.

It is clear that this magnitude of k brings the system motion to the vicinity of the switching surface. Thus the motion of equivalent control system is identical to the motion of the slow and subsystems of the high gain feedback system for $k \to \infty$.

4. Design of the High gain Observer

In order to estimate unmeasurable state of (4) except x_1 the following form is only given to design an observer. Note this is not appropriate as an observer in its current form, since the last term e contains all state variables.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \hat{z}_0(x,t) - Le \\ y(t) &= Cx(t) \end{aligned} \tag{16}$$

where $L = diag[l_1, l_2, \ldots, l_n]$ and the system matrixes are same as in (4). The structure of the equation in (16) is based on the nominal system of (4) and the only freedom we have now is design of the gains l_i (i=1,2,...,n).

The error between the system (16) and (4) is;

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_{n-1} \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \Delta z\,(e,t) \end{bmatrix} - \begin{bmatrix} l_1 & 0 & \dots & \dots & 0 \\ 0 & l_2 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & l_{n-1} & 0 \\ 0 & \dots & \dots & \dots & l_n \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix} \tag{17}$$

It is clear that this error dynamic system is the asymptotic stable by choosing appropriate design parameter L.

Our aim is that use this structure to design a high gain observer such that the structure in (16) has to be change and the last term in the right side of this equation is include e_1 only. Consider the first equation of the system error dynamics in (17)

$$\dot{e}_1 = e_2 - l_1 e_1 . \tag{18}$$

In order to have e_1 converges to zero asymptotically, we define a sliding surface as $e_1 = x_2 - x_1$. The equation in (18) may be rewritten

$$\frac{de_1}{d t} = m e_2 - e_1 \tag{19}$$

where t = t/m, $m = 1/l_1$. This equation is a singularly perturbed system. Since for $m \to 0$ in this system with a stretched time scale e_1 and $\dot{e}_1$ have opposite sign. The condition for the existing of the sliding condition is given in (11) is satisfied despite the existing nonzero component e_2.

So that sliding motion will always occur for this system and using the property of equivalent control for the system in (19);

$$\begin{aligned} &\dot{e}_1 = 0 \\ &e_2 \rightarrow l_1 e_1 \end{aligned} \tag{20}$$

where $l_1 \rightarrow \infty$.

In the previous section it has been shown that for finite magnitude l_1 the error e_1 converges to vicinity of switching surface. Let define the distance to this vicinity as $\boldsymbol{d}_1$ and chose the gain l_1 according to this vicinity e_1 converges to $\boldsymbol{d}_1$ asymptotically. After the sliding motion for e_1 takes place equation (20) is substituted in second equation of (17) as

$$\begin{aligned} \dot{e}_2 &= e_3 - l_2 e_2 \\ &= e_3 - l_2 l_1 e_1 \end{aligned} \tag{21}$$

This equation can be consider as equation for e_1 and l_2 is chosen such that the sliding motion for e_2 will take place after sliding motion of e_1. This provides the second equation of the error system again becomes asymptotically stable with the condition; $l_2 << l_1$.

The manipulation is given for e_1 is carry on until e_{n-1} and the gains are chosen such that the following conditions satisfy for every equations of the error system as

$$\frac{l_i}{l_j} \rightarrow \infty \qquad i < j\,. \tag{22}$$

This means that the sliding motion for l_i takes place first and its differential equation will reduce to the algebraic equation. Then substitute the solution of this algebraic equation in to the error equation is defined for e_j and l_j chosen such that sliding motion for e_j will take place after that the sliding motion of e_i. The hierarchy is carry on until e_n with the condition given in (22) and sliding motion occurs for all the component of the error system e_i except for the last term e_n. The last equation of the error system in

$$\frac{de_n}{dt_n} = m_n \Delta f(e,t) - e_n \tag{23}$$

where $t_n = t_n/m_n$, $m_n = 1/l_n$. For the sufficently small value of $m_n \rightarrow 0$ $\dot{e}_n$ and e_n have opposite sign.

The existing condition of sliding mode is satisfied in the presence of function $\Delta z\,(e,t)$. In the case only e_1 is known the equation in (23) with the above procedure is became as

$$\frac{de_n}{dt_n} = m_n \Delta z\,(e,t) - l_{n-1} l_{n-2}, \ldots l_2 l_1 e_1 \tag{24}$$

Thus the robust high gain observer is designed as;

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \hat{z}_0(x,t) - Ke_1 \tag{25}$$

where $K = [k_1, k_2, \ldots, k_n]^T$ is the design parameter and

$$K = \begin{bmatrix} l_1 \\ l_2 l_1 \\ \vdots \\ l_{n-1} l_{n-2} \ldots l_2 l_1 \\ l_n l_{n-1} \ldots l_2 l_1 \end{bmatrix} \tag{26}$$

With this observer the system states $\hat{x}_i$ (i = 2,3,…,n) now can be estimated and converges to its real value x_i using only output variable x_1 and again providing that the error system obained between (25) and (4) is asymptotically stable in the presence of $\Delta z\,(e,t)$.

5. Numarical Example: A Cylindric Manipulator

Here sliding mode control is designed for the *q -r* manipulator in Wolcott and Zak (1988) based on the high gain observer given in (25) where only position is measured. Reading from Wolcott and Zak (1988) the equation of Cylindric manipulator is given in the state space form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{10 + M x_3{}^2} & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{M} \end{bmatrix} \begin{bmatrix} u_1 + F_1 \\ u_2 + F_2 \end{bmatrix}$$

where x_1 is the angle of the rotation of the cylinder, x_3 is the radyal distance of the end of telescoping arm from the center of the rotation. The mass of the load is defined by M and the inputs are given in a torque, u_1 applied the hub in the direction of x_1, and a translational force, u_2 applied in the direction of x_3. The nonlinear fonctions have the following forms;

$$F_1 = -(2Mx_2x_3x_4 + 10\cos x_1(10 - Mx_3))$$
$$F_2 = M\,(x_2{}^2 x_3 - 10\sin x_1)$$

The bounds of the system state variables, the load and system nonlinearites respectively are; $|x_1| < p$ *rad*, $|x_2| < 2$ *rad/s*, $|x_3| < 2$ *m*, ,$|x_4| < 2$ *m/s* ,$|M| < 2$ *kg*, $\|F_1\|$ and $\|F_2\|$.

Here it is assumed that the position variables are mesurable, thus the system output is;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

a. Observer design

The observer for this system has MIMO form is designed as;

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0.0556u_1 \\ 0.5u_2 \end{bmatrix} + \begin{bmatrix} |n_1| \\ |n_2| \end{bmatrix} \right\} - \begin{bmatrix} l_1 & 0 \\ l_2 l_1 & 0 \\ 0 & l_3 \\ 0 & l_4 l_{31} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where n_1 and n_2 are

$$n_1 = -(2M\hat{x}_2\hat{x}_3\hat{x}_4 + 10\cos\hat{x}_1(10 - M\hat{x}_3))/(10 + M\hat{x}_3{}^2)$$
$$n_2 = M(\hat{x}_2{}^2\hat{x}_3 - 10\sin\hat{x}_1)/M$$

b. Sliding Mode controller design based on observer

The controller is designed such that the final position for desired outputs are x_{d1}=1.5 *rad* and x_{d3}=2 *m*. It is important that switching surfaces can not be designed, since it needs x_2 and x_4. The switching surface based on the estimated state for the desired behavior is $s = S\hat{x} + x_d$ where $S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and the observed state based controller is

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\hat{x}_2(10 + M\hat{x}_3{}^2) - 172\dfrac{s^T Sb_1}{\|s^T Sb_1\| + h_1} \\ -\hat{x}_4 M - 38\dfrac{s^T Sb_2}{\|s^T Sb_2\| + h_2} \end{bmatrix}$$

where η_1 and η_2 are positive constant to reduce the chattering. Here Sb_1 and Sb_2 are the first and second column of *SB* matrix respectively.

The simulation results are presented in Fig. 1 and Fig. 2 for the system with initial conditions are $x(0)=[0, 0, 1, 0]^T$. The actual and estimated angular velocity are depicted in Fig. 1 (a) and (b), respectively and shows that the observed state x_2 has a small error where it may be reduced by chose observed gain large enough. The actual radial velocity and its estimated are depicted in Fig. 1 (c) and (d). The control signal designed based on observed states are depicted in Fig. 2 (a) and (b). The switching surfaces are depicted in Fig. 2 (c) and (d).

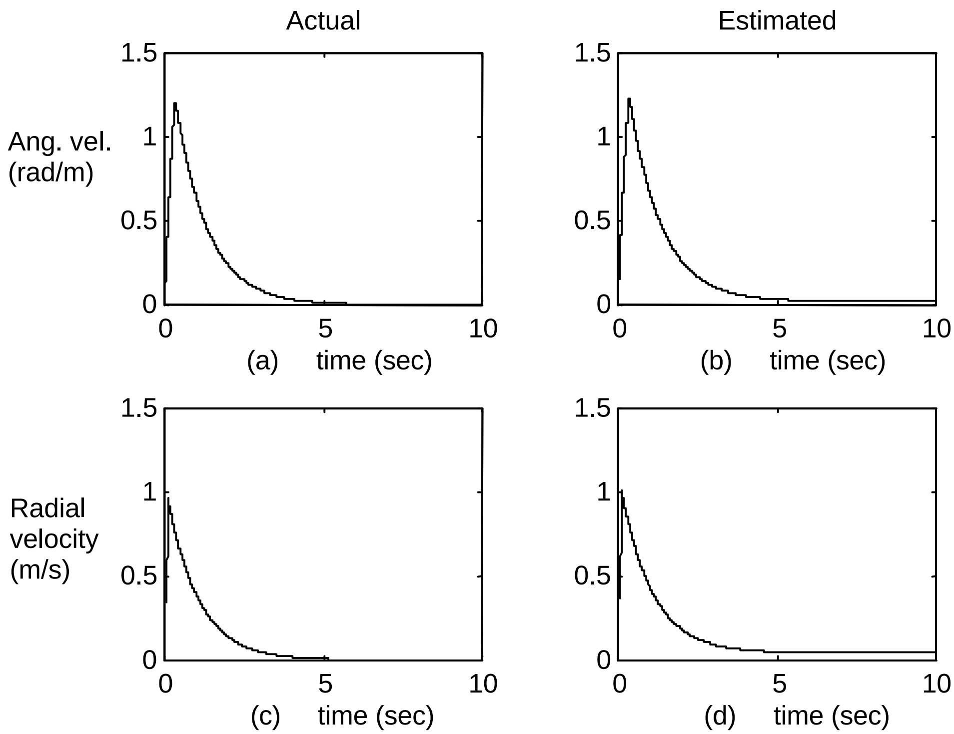

Fig. 1: The actual and estimated states of the close observer-controller feedback loop.

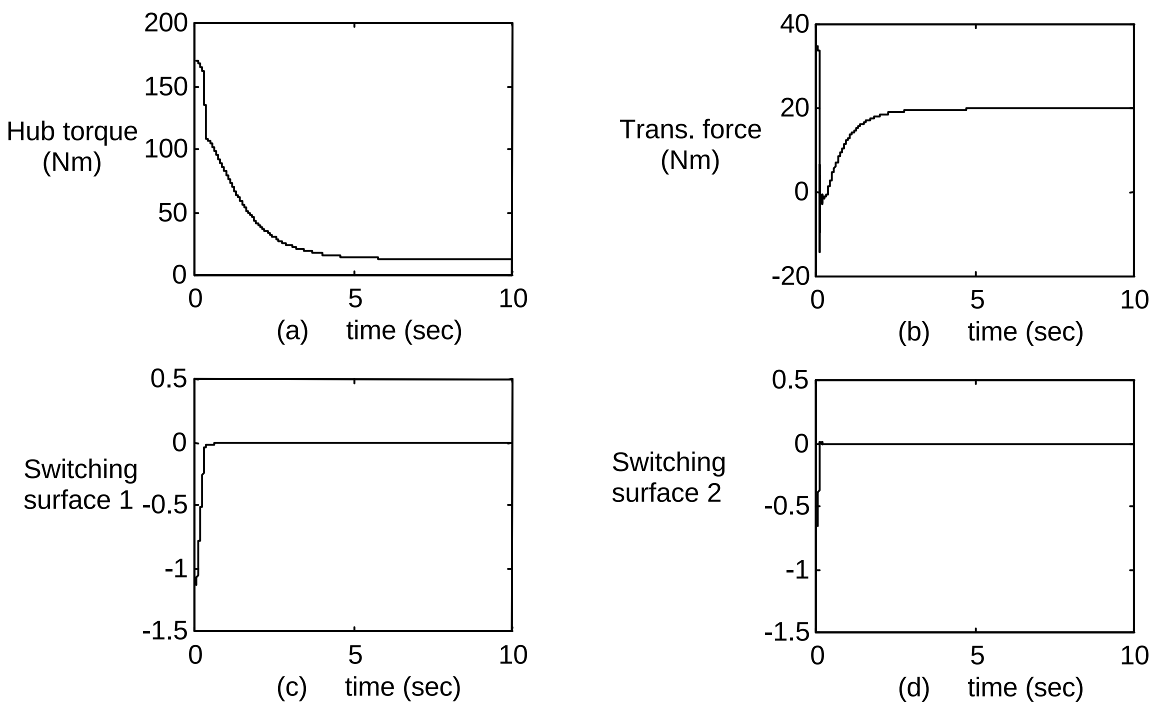

Fig. 2: The controllers and switching surfaces based on the estimated states.

The final values of the switching surfaces are occur on the vicinity of the ideal sliding motion as expected. These results are compare the results are given in Walcott and Zak (1988). Since their observer designed based on the positive realness lemma the velocity has to be measurable. However velocity measurement by means of tachometer is often contaminated by noise Ahmet (1995). This circumstance may reduce the dynamic performances of a robot, since, in the practice, the controller gain matrices are limited by noise present in the velocity measurement, De Wit *et al.*, (1992). Although the velocity can be obtained from the integration of the acceleration, however the acceleration measurement is very expensive Lewis, Ucar and Bishop (1997). From these circumstances he observer structure given in this paper may be used to constrain the states for the nonlinear control design techniques.

6. Conculution

In this paper a high gain observer is presented. The proposed observed shows excelent performance and properties. By employing this observer practical stability also can be garantied in the presence of the uncertainities. Simulation results illustrate that we can indeed achive an insensitive control loop.

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