# On the Generation of the Most Promising Control Structure for Large Dimensional Systems

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**Abstract:**
An algorithmic method for variable pairing selection of large dimensional systems is proposed. The proposed method relies on the main properties of the RGA and RIA matrices and the concepts of interaction, integrity and stability. It is shown that the minimization of the overall interaction in multi-input, multi-output large scale systems under several stability and structural constraints can be formulated either as a Mixed Integer Nonlinear Programming problem or as a Mixed Integer Linear Programming problem when the RGA or the RIA matrices, respectively, are used as interaction measures. The proposed method can be readily applied to systems with arbitrarily large dimensions, providing a simple and quick solution to the problem of the generation of feasible and promising control structures.

## 1 Introduction

Multivariable controllers have several advantages over single loop controllers for multivariable plants. However, in process control applications, decentralized control systems are far more common than any multivariable controller. This is due to several characteristics of the decentralized controller, such as flexibility in operation, failure tolerance and simplified design and tuning, that are particularly desirable in process control applications; see Chang and Yu (1991), Campo and Morari (1994), for a detailed analysis of this issue. In designing decentralized control systems, the first problem that has to be solved is the problem of control structure selection or input-output variable pairing problem, i.e. which of the available manipulated variables is to be used in order to control each of the controlled variables.

It is now recognized that the selection of the control structure can dramatically affect control performance (Campo and Morari, 1994). The number of alternative control structures can be enormous, especially in plant-wide control problems, and the exhaustive comparison of their perfor-

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mance is not feasible. To this end, much effort has been expended towards the development of efficient, low effort algorithms for the quick ranking of the alternative control structures. This ranking is mainly based on the evaluation of the loop interaction associated with each control structure as well as on its disturbance attenuation properties using open-loop screening tools (Narraway *et al.,* 1991; Hovd and Skogestad, 1992; McAvoy and Ye, 1994; Yi and Luyben, 1995; Banerjee and Arkun, 1995).

The relative gain array (RGA) was first proposed in (Bristol, 1966) as a steady-state interaction measure. The acceptance of the RGA as an efficient interaction measure is unique in both industry and academia. A number of interesting properties of the RGA have been established and its dynamic (frequency) definition has also been proposed (for an overview, see McAvoy, 1983a; Sinskey, 1988). Furthermore, a number of fundamental closed-loop system properties (such as stability and integrity) was expressed in terms of the open-loop RGA (McAvoy, 1983a, 1983b; Grosdidier *et al.*, 1985; Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996). Thus, its theoretical basis is now better understood and combined with the simplicity of its calculation make RGA far the most useful and comprehensive screening tool for control structure selection.

Recently, based on the RGA matrix, a new interaction measure called the relative interaction array, has been proposed in (Zhu and Jutan, 1993, 1996a, 1996b; Zhu 1996a, 1996b). In these papers it has been shown that the RIA has several advantages over RGA, particularly when the overall interaction in a multivariable system is assessed. Furthermore, alternative conditions for interaction, stability and integrity in terms of the RIA elements have been proposed

The aim of the present paper is to propose a systematic framework for the generation of the most promising control structure for large dimensional systems. A new algorithm, based on the main properties of the RGA and RIA matrices and the concepts of interaction, integrity and stability, is proposed. It is shown that the minimization of the overall interaction in multi-input, multi-output systems under several stability and structural constraints can be formulated as Mixed Integer Nonlinear Programming (MINLP) problem in terms of the RGA matrix. Furthermore, it is shown that, when the RIA based overall interaction measure is used, the problem can formulated as a Mixed Integer Linear Programming problem (MILP). In order to demonstrate the usefulness of the proposed algorithm as a rigorous and systematic solution to the problem of automatic generation of promising control structures, two large scale industrial problems, namely the hydrodealkylation of toluene (HAD) process and the Tennessee Eastman problem, are considered in the paper.

## 2 RGA and RIA Based Overall Interaction Measures

In Bristol (1966), it has been suggested that in order to evaluate the interaction in a process, each open loop steady-state gain should be evaluated first with all other loops open and then reevaluated with all other loops closed and all other controlled variables held at their set points. The ratio of these gains is defined as the relative gain of the controlled variable $y_i$ to a manipulated variable $u_j$ and is given by the following expression

$$\lambda_{ij}(0) = \frac{\left( \partial y_i \big/ \partial u_j \right)_{OL}}{\left( \partial y_i \big/ \partial u_j \right)_{CL}} \tag{2.1}$$

In the sequel the subscript (0) is dropped for clarity and steady state is assumed unless otherwise stated. For the $2\times 2$ plant, depicted in Figure 1, the open-loop gain is simply given by

$$\left( \partial y_1 \big/ \partial u_1 \right)_{OL} = g_{11} \tag{2.2}$$

while, when the second loop is closed, the corresponding closed-loop gain is

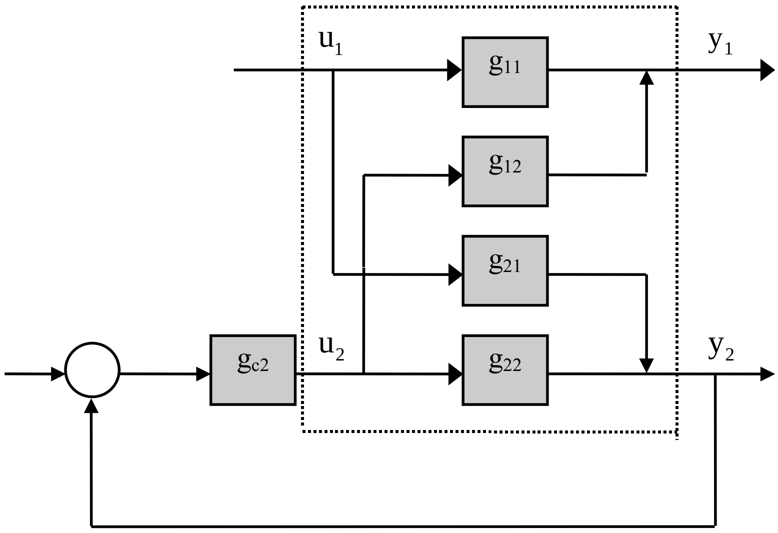


Figure 1. Block diagram for $2 \times 2$ systems with the loop 2 closed

$$\left( \partial y_1 / \partial u_1 \right)_{CL} = g_{11} - \frac{g_{C2} g_{12} g_{21}}{1 + g_{C2} g_{22}} \tag{2.3}$$

Under the assumption that the controller is infinite at the steady state (integral action is included), we take

$$\left( \partial y_1 / \partial u_1 \right)_{CL} = g_{11} - \frac{g_{12} g_{21}}{g_{22}} \tag{2.4}$$

and

$$\lambda_{11} = \frac{g_{11} g_{22}}{g_{11} g_{22} - g_{12} g_{21}} \tag{2.5}$$

Originally, the RGA matrix was defined as a steady-state interaction measure (see Bristol, 1966). For a number of practical problems the steady-state definition is sufficient. However, in order to take into consideration dynamics, it is necessary to use frequency dependent definitions of the RGA matrix. Let **G**(s) denote the transfer function matrix of an $n \times n$ plant. The dynamic RGA matrix is defined as (Mcavoy, 1983a, 1983b; Grosdidier *et al.*, 1985; Witcher and McAvoy, 1977)

$$\mathbf{RGA}(s) = \mathbf{G}(s) \times \left( \mathbf{G}^{-1}(s) \right)^T \tag{2.6}$$

where the symbol $\times$, denotes element by element (Hadamard) multiplication. It should be noted that the assumption of perfect control at all frequencies is implied in this definition. This, in general, is not a valid assumption and many authors discard the use of the dynamic RGA (see for example Manousiouthakis *et al.,* 1986; Grosdidier and Morari, 1986). In what follows, the main properties of the RGA matrix as well as a number of important known results are summarized.

1. The sum of the elements of each row and each column of the RGA is unity.
2. The RGA is invariant under input and output scaling.
3. Any permutation of rows and columns in the transfer function matrix **G**(s) results in the same permutation in the RGA.
4. For closed-loop stable systems with diagonal pairing, a decentralized control system where the individual loops are stable, possesses integrity against ith loop failure only if (Zhu and Jutan 1996b)

$$\lambda_{ii} > 0 \ , \ \forall i = 1,2,\ldots,n \tag{2.7}$$

5. If $\lambda_{ii} < 0$ then, for any diagonal compensator **C**(s) with integral action in all elements and strictly proper loop transfer function, the closed-loop system has one of the following properties (Grosdidier *et al.,* 1985):
   a) the overall closed-loop system is unstable
   b) the ith loop is unstable by itself
   c) The closed-loop system is unstable if the ith loop is opened

The RGA matrix is used for input-output variable pairing according to the following rule of thumb: Input and output variables should be paired in such a way that, the corresponding diagonal RGA elements are as close to one as possible. This can be expressed as (Zhu and Jutan, 1993)

$$\min \sum \left|\lambda_{ii} - 1\right| \tag{2.8}$$

that is, the distance of the paired RGA elements and 1 is minimized over all possible pairings. However, the usefulness of this overall interaction measure is restricted due to the fact that the distance between RGA elements and 1 does not quantify the amount of interaction (Zhu 1996a, 1996b). Consider for example two different $2 \times 2$ systems having $\lambda_{11} = 0$ and $\lambda_{11} = 2$. Although they are totally different from the interaction point of view, they are indistinguishable according to relation (2.8). An improved interaction measure, proposed in (Zhu, 1996b) and called the *relative interaction array (RIA)*, is defined as follows

$$\phi_{ij} = \frac{1}{\lambda_{ij}} - 1 \tag{2.9}$$

where $\phi_{ij} = [\mathrm{RIA}]_{ij}$. Using the definition of RIA, equation (2.4) can be written as

$$\left( \partial y_1 \big/ \partial u_1 \right)_{CL} = g_{11}\left(1 + \phi_{11}\right) \tag{2.10}$$

Thus, as it is shown for the case of $2 \times 2$ systems, RIA represents the relative amount of interaction in the loop. A comparison between RGA and RIA as interaction measures is also given in (Zhu, 1996b).

It is interesting to note that, the RIA interaction measure is also related to some of the most well known interaction measures. For a $2 \times 2$ system, the *interaction quotient* or *Rijnsdorp interaction measure* $\kappa$ (Rijnsdrop, 1965; Kominek and Smith, 1979), is defined to be

$$\kappa = \frac{g_{12}g_{21}}{g_{11}g_{22}} \tag{2.11}$$

or

$$\kappa = 1 - \frac{1}{\lambda_{11}} \tag{2.12}$$

Thus, the negative of the RIA interaction measure is equal to the interaction quotient or Rijnsdorp interaction measure. It is also interesting that the Rijnsdrop's conclusions (Rijnsdrop, 1965) about the interaction quotient are similar to the conclusions of Zhu (1996b) about the RIA interaction measure.

The $\mu$ interaction measure is defined as the Structured Singular Value of the relative error matrix **E**(s), which is given by (Grosdidier and Morari, 1987)

$$\mathbf{E}(s) = \left(\mathbf{G}(s) - \tilde{\mathbf{G}}(s)\right)\tilde{\mathbf{G}}^{-1}(s) \tag{2.13}$$

where $\tilde{\mathbf{G}}(s) = \mathrm{diag}\{\mathbf{G}(s)\}$. For the case of $2 \times 2$ systems, the $\mu$ interaction measure and the RIA interaction measure are related through the following expression

$$\mu(\mathbf{E}) = \sqrt{\left|\phi_{11}\right|} \tag{2.14}$$

In (Grosdidier and Morari, 1987), it has been concluded that the square of the $\mu$ interaction measure is the most natural extension of the Rijnsdorp interaction measure to systems larger than $2 \times 2$.

Finally, in the case of $2 \times 2$ systems described by first order with dead time transfer function, it has been proven in Marino-Galarraga *et al.* (1987) that the ratio of the closed-loop ultimate frequency to the open-loop ultimate frequency (MIMO) is given by

$$\frac{(\omega_u)_{OL}}{(\omega_u)_{CL}} = 1 - \frac{1.5\tan^{-1}\left(\sqrt{\phi_{11}}\right)}{\pi} \tag{2.15}$$

while the ratio of the closed loop ultimate gain to the open-loop ultimate gain is given by

$$\frac{(K_u)_{CL}}{(K_u)_{OL}} = \sqrt{\lambda_{11}}\left(1 - \frac{\tan^{-1}\left(\sqrt{\phi_{11}}\right)}{\pi}\right) \tag{2.16}$$

Thus, a number of authors, using different starting points for interaction analysis, concluded that the interaction is more effectively analyzed in terms of the RIA interaction measure.

According to the definition of the RIA interaction measure, variables should be paired in such a way so that

a) all the RIA elements are close to 0,
b) all the RIA elements are greater than -1, and
c) RIA elements close to -1 are avoided.

Based on the definition of RIA, Zhu proposed the following overall interaction measure

$$\min \sum |\phi_{ii}| \tag{2.17}$$

The main advantage in using the RIA interaction measure is that ambiguities observed when we use the RGA interaction measure are avoided.

# 3 Variable Pairing Selection Based on the RIA Interaction Measure

In this Section, the mathematical formulation of the variable pairing selection problem (also called control structure selection problem) based on overall interaction measures is presented.

The set of outputs will be denoted by the index ***OUT***={j} and the set of inputs will be denoted by the index ***IN***={i}. Let k be an alias of both i and j. The following integer variables are introduced

$$Y_{kj} = 0,1$$

If $Y_{kj}$ is equal to one, the manipulated variable k $(u_k)$ is used to control the controlled variable j $(y_j)$. Let $\mathbf{M} = [\mu_{ik}]$, $i \in \boldsymbol{OUT}$, $k \in \boldsymbol{IN}$, be the RGA matrix that corresponds to diagonal pairings of manipulated and controlled variables $(Y_{kk} = 1)$. Then, for any realization of the integer variables $Y_{kj}$, the i-jth element of the corresponding RGA matrix is given by

$$\lambda_{ij} = \sum_{k=1}^{n} \mu_{ik} Y_{kj} \tag{3.1}$$

Since only one manipulated variable is assigned to each controlled variable, the following conditions have to be satisfied

$$\sum_{k \in \boldsymbol{IN}} Y_{kj} - 1 = 0 \ , \ j \in \boldsymbol{OUT} \tag{3.2}$$

$$\sum_{j \in \boldsymbol{OUT}} Y_{kj} - 1 = 0 \ , \ k \in \boldsymbol{IN} \tag{3.3}$$

Now, let ***NEG*** be the index set defined as

$$\boldsymbol{NEG} = \{(i,j) : \lambda_{ji} \le 0 \text{ or } \phi_{ji} \le -1\} \tag{3.4}$$

In order to exclude these undesirable pairings, the corresponding binary variables are set equal to zero. That is

$$Y_{ij} = 0 \ , \ (i,j) \in \boldsymbol{NEG} \tag{3.5}$$

In order to find all possible solutions, an additional constraint, called integer cut constraint, should be added. Let $\boldsymbol{I}$ be the index set of $Y_{ij}$s that take value equal to one and $|\boldsymbol{I}|$ be the cardinality of $\boldsymbol{I}$. Then, the additional constraint has the following form

$$\sum_{(k,j)\in \boldsymbol{I}} Y_{kj} \leq |\boldsymbol{I}| - 1 \tag{3.6}$$

The mathematical formulation, which is the basis for the proposed method for quick control structure selection, called in the sequel problem P1, can be stated as follows:

**Problem P1:** Find

$$\min_{\boldsymbol{Y}} \sum_{k=1}^{n} \left| \frac{1}{\lambda_{kk}} - 1 \right| \tag{3.7}$$

subject to the constraints

$$\lambda_{ij} - \sum_{k=1}^{n} \mu_{ik} Y_{kj} = 0 \tag{3.8a}$$

$$\sum_{k\in \boldsymbol{IN}} Y_{kj} - 1 = 0 \ , \ j\in \boldsymbol{OUT} \tag{3.8b}$$

$$\sum_{j\in \boldsymbol{OUT}} Y_{kj} - 1 = 0 \ , \ k\in \boldsymbol{IN} \tag{3.8c}$$

$$Y_{kj} = 0 \ , \ (k,j)\in \boldsymbol{NEG} \tag{3.8d}$$

$$\sum_{(k,j)\in \boldsymbol{I}} Y_{kj} \leq |\boldsymbol{I}| - 1 \tag{3.8e}$$

The mathematical formulation of Problem P1 corresponds to a Mixed Integer Nonlinear Programming (MINLP) problem, since it contains both integer and continuous variables. The nonlinearity of the problem is only due to the nonlinear objective function.

Now, let $\xi_{ik}$ be the absolute value of the i-kth element of the RIA interaction matrix. Then, the formulation of Problem P1 can be rewritten to the form of the following Problem P2:

**Problem P2:** Find

$$\min_{\boldsymbol{Y}} \sum_{k=1}^{n} \phi_{kk} \tag{3.9}$$

subject to the constraints

$$\phi_{ij} - \sum_{k=1}^{n} \xi_{ik} Y_{kj} = 0 \tag{3.10a}$$

$$\sum_{k\in \boldsymbol{IN}} Y_{kj} - 1 = 0 \ , \ j\in \boldsymbol{OUT} \tag{3.10b}$$

$$\sum_{j\in \boldsymbol{OUT}} Y_{kj} - 1 = 0 \ , \ k\in \boldsymbol{IN} \tag{3.10c}$$

$$Y_{kj} = 0 \ , \ (k,j)\in \boldsymbol{NEG} \tag{3.10d}$$

$$\sum_{(k,j)\in \boldsymbol{I}} Y_{kj} \leq |\boldsymbol{I}| - 1 \tag{3.10e}$$

The mathematical formulation of Problem P2 corresponds to a Mixed Integer Linear Programming (MILP) problem, since it involves integer variables and linear objective function and constraints. Solution of the Problem P2 can be achieved with standard branch and bound techniques making the proposed formulation more applicable for systems with large dimensions.

For the cases where the number of inputs (m) exceed the number of outputs (n), then n out of m input variables have to be chosen in order to control the n outputs. This can easily be included in the proposed formulation using the following constraint

$$\sum_{k\in \boldsymbol{IN}} Y_{kj} - 1 \leq 0 \ , \ j\in \boldsymbol{OUT} \tag{3.11}$$

instead of constraint (3.2).

It is interesting to note that the proposed algorithm for variable pairing selection is not restricted to zero frequency. Its applicability can easily be extended to any frequency of practical interest. This is of great practical importance, since control structures with undesirable high frequency characteristics can effectively be eliminated.

# 4 Illustrative Examples

In this Section, three examples are presented in order to demonstrate the effectiveness of the proposed method for quick variable pairing selection. In the first example, it is shown that when the proposed formulation is applied to $4 \times 4$ systems (24 alternative control structures), the solution of the problem is straightforward. The proposed method is more useful when the dimension of the system is greater than five (more than 720 alternative control structures). To demonstrate this, in the second example the proposed method is applied to a nonsquare $5 \times 13$, large dimensional system. Finally, in the third example, the control structure of the Tennessee Eastman problem, consisting of a $7 \times 7$ system, is considered.

### 4.1 Example 1: Distillation Column

In this example the FS configuration of heat integrated distillation columns, studied in Chiang and Luyben (1988), is considered. It is a $4 \times 4$ system with the following steady-state gain matrix

$$\boldsymbol{G}_{4\times 4}(0) = \begin{bmatrix} 4.45 & -7.4 & 0 & 0.35 \\ 17.3 & -4.1 & 0 & 0.92 \\ 0.22 & -4.66 & 3.6 & 0.042 \\ 1.82 & -34.5 & 12.2 & -6.92 \end{bmatrix}$$

The corresponding RGA matrix is given by

$$\mathrm{RGA}_{4\times 4}(0) = \begin{bmatrix} 2.0979 & -0.9979 & 0 & -0.0999 \\ -1.0389 & 1.3315 & 0 & 0.7074 \\ 0.0409 & -0.5626 & 1.5137 & 0.0079 \\ 0.0999 & 1.2290 & -0.5137 & 0.3846 \end{bmatrix}$$

The index set ***NEG*** is then calculated as

$$\boldsymbol{NEG} = \{(2,1),(3,1),(4,1),(1,2),(3,2),(2,3),(3,4)\}$$

Since $Y_{2,1}$, $Y_{3,1}$ and $Y_{4,1}$ are equal to 0, $Y_{1,1}$ must be equal to 1, in order to satisfy equation (3.2) (for i=1). Furthermore, since $Y_{3,1}$, $Y_{3,2}$ and $Y_{3,4}$ are equal to 0, $Y_{3,3}$ must be equal to 1, in order to satisfy equation (3.3) (for j=3). Using equation (3.2) (for i=3), it follows that $Y_{1,3} = 0$ and $Y_{4,3} = 0$. In a similar way, we take $Y_{1,4} = 0$. Thus the **Y** matrix takes the following form

$$\boldsymbol{Y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \times & 0 & \times \\ 0 & 0 & 1 & 0 \\ 0 & \times & 0 & \times \end{bmatrix}$$

The problem is then simplified to the $2 \times 2$ problem of pairing outputs 2 and 4 with inputs 2 and 4 (2 alternatives instead of 24). The corresponding $2 \times 2$ RIA matrix is then calculated as

$$\mathrm{RIA} = \begin{bmatrix} -0.249 & 0.4146 \\ -0.186 & 1.600 \end{bmatrix}$$

Finally, since 0.186+0.4146<0.249+1.600, the proposed variable pairing is

$$y_1 \leftrightarrow u_1 \;\;,\;\; y_4 \leftrightarrow u_2 \;\;,\;\; y_3 \leftrightarrow u_3 \;\text{ and }\; y_2 \leftrightarrow u_4$$

### 4.2 Example 2: HAD Process of Toluene

In this example, the hydrodealkylation (HAD) process of toluene, studied in Skogestad and Postlethwaite (1996), is considered. The plant has 5 controlled outputs and 13 candidate manipulations. As a result there are 1287 possible combinations with 5 inputs and 5 outputs. The steady-state gain matrix is given by

$$\boldsymbol{G}_{\mathrm{HDA}}^{\mathrm{T}}(0)=\begin{bmatrix} 0.7878 & 1.1489 & 2.6640 & -3.0928 & -0.0703 \\ 0.6055 & 0.8814 & -0.1079 & -2.3769 & -0.0540 \\ 1.4722 & -5.0025 & -1.3279 & 8.8609 & 0.1824 \\ -1.5477 & -0.1083 & -0.0872 & 0.7539 & -0.0551 \\ 2.5653 & 6.9433 & 2.2032 & -1.5170 & 8.7714 \\ 1.4459 & 7.6959 & -0.9927 & -8.1797 & -0.2565 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1097 & -0.7272 & -0.1991 & 1.2574 & 0.0217 \\ 0.3485 & -2.9909 & -0.8223 & 5.2178 & 0.0853 \\ -1.5899 & -0.9647 & -0.3648 & 1.1514 & -8.5365 \\ 0.0000 & 0.0002 & -0.5397 & -0.0001 & 0.0000 \\ -0.0323 & -0.1351 & 0.0164 & 0.1451 & 0.0041 \\ -0.0443 & -0.1859 & 0.0212 & 0.1951 & 0.0054 \end{bmatrix}$$

The corresponding RGA matrix has the form

$$\mathrm{RGA}_{\mathrm{HDA}}^{\mathrm{T}}(0)=\begin{bmatrix} 0.1275 & -0.0755 & 0.5907 & 0.1215 & 0.0034 \\ 0.0656 & -0.0523 & 0.0030 & 0.1294 & 0.0002 \\ 0.2780 & 0.0044 & 0.0463 & 0.4055 & -0.0060 \\ 0.3684 & -0.0081 & 0.0009 & 0.0383 & -0.0018 \\ -0.0599 & 0.9017 & 0.2079 & -0.1459 & 0.0443 \\ 0.1683 & 0.4042 & 0.1359 & 0.1376 & 0.0089 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0014 & -0.0017 & 0.0013 & 0.0099 & 0.0000 \\ 0.0129 & -0.0451 & 0.0230 & 0.1873 & -0.0005 \\ 0.0374 & -0.1277 & -0.0359 & 0.1163 & 0.9516 \\ 0.0000 & 0.0000 & 0.0268 & 0.0000 & 0.0000 \\ 0.0001 & 0.0001 & 0.0000 & 0.0001 & 0.0000 \\ 0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0000 \end{bmatrix}$$

Using the formulation P2, the five more promising control structures are calculated and are listed in Table 4.1.

### 4.3 Example 3: The Tennessee-Eastman Problem

In this final example, the selection of the control structure for the Tennessee Eastman problem, studied in McAvoy and Ye (1994), Banerjee and Arkun (1995), is considered. The controlled and manipulated variables as well as the steady-state gain matrix of the system are given in the same

| | INPUT-OUTPUT PAIRING | | | | | $\sum\lvert\phi_{ii}\rvert$ |
|---|---|---|---|---|---|---|
| S1 | 1-4 | 2-5 | 3-1 | 4-3 | 5-10 | 3.998 |
| S2 | 1-4 | 2-6 | 3-1 | 4-3 | 5-10 | 5.358 |
| S3 | 1-4 | 2-5 | 3-1 | 4-9 | 5-10 | 6.878 |
| S4 | 1-6 | 2-5 | 3-1 | 4-3 | 5-10 | 7.228 |
| S5 | 1-3 | 2-5 | 3-1 | 4-9 | 5-10 | 7.758 |

Table 4.1. Most promising control structures for the HAD process

references. The interested reader is also referenced to those papers for a detailed presentation of the problem.

In order to address the $9\times 9$ pairing problem, $3.63\times 10^5$ alternatives have to be considered. However, two of the manipulated variables are used for control of product mix and production rate control. Thus, two of the controlled variables are also dropped. These are the stripper and separator pressures. This reduces the number of alternative control structures to $5.04\times 10^3$, since the final system is a $7\times 7$ system. In McAvoy and Ye (1994), several qualitative quite complicated arguments are used in order to further reduce the dimension of the system. In the present paper, the full $7\times 7$ system is considered and the proposed algorithm is applied in order to generate the most promising $7\times 7$ control structure. The solution of the Mixed Integer Linear Programming problem P2 for the present case is given in Table 4.2.

It is interesting at this point to note that in McAvoy and Ye (1994), it has been concluded that the pairs

Reactor pressure $\leftrightarrow$ A feed flow s.p.
Stripper temperature $\leftrightarrow$ Stripper steam flow s.p.
Compressor power $\leftrightarrow$ Compressor recycle valve
Reactor temperature $\leftrightarrow$ Stirred speed

are common to all of the promising control structures. This is in accordance with the results taken from the proposed formulation of the problem. However, the pair Reactor temperature $\leftrightarrow$ Reactor cooling water temperature, which is included in some of the promising variable pairs proposed in McAvoy and Ye (1994), is not a valid pair for the $7\times 7$ control structure, since, as it can be easily checked using the proposed formulation, it corresponds to a negative element of the RGA matrix. Therefore, using the proposed formulation acceptable pairs are quickly identified in spite of the great number of alternatives.

| CONTROLLED | MANIPULATED | $\lambda_{ij}$ |
|---|---|---|
| Reactor feed flow | Purge s.p. | 0.623 |
| Reactor temperature | Stirred speed | 99.97 |
| Reactor pressure | A feed flow s.p. | 2.136 |
| Separator temperature | Condenser cooling water s.p. | 0.503 |
| Stripper temperature | Stripper steam flow s.p. | 0.910 |
| Recycle flow | Reactor cooling water s.p. | 186.7 |
| Compressor power | Compressor recycle valve | 0.749 |

Table 4.2. Optimal solution for the Tennessee Eastman variable pairing problem

## 5 Conclusions

A systematic framework for the generation of the most promising control structures for large dimensional systems has been proposed in this paper. The proposed algorithm is based on the main properties of the RGA and RIA matrices and the concepts of interaction, integrity and stability. It has been shown that the minimization of the overall interaction in multi-input, multi-output systems under several stability and structural constraints can be formulated as Mixed Integer Nonlinear Programming problem when the RGA matrix is used as the interaction measure. Moreover, it has been shown that, when the RIA based overall interaction measure is used, the problem can be reduced to a Mixed Integer Linear Programming problem. This last formulation can easily be applied and solved even for systems of arbitrarily large dimension, in order to automatically generate the most promising control structures. Thus, from the application point of view, the proposed algorithm offers a rigorous, systematic and simple solution to the problem of the automatic generation of feasible and promising control structures for large scale industrial problems such as the HAD process and the Tennessee Eastman problem.

## 6 Nomenclature

| | |
|---|---|
| $\mathbf{E}(s)$ | : Relative error matrix |
| $\mathbf{G}(s)$ | : Transfer function matrix of the plant |
| $g_{ij}(s)$ | : i-jth element of the **G**(s) matrix |
| $K_u$ | : Ultimate gain |
| $\boldsymbol{I}$ | : Index set of promising control structures |
| $\mathbf{Y}$ | : Permutation matrix |
| $Y_{ij}$ | : i-jth element of the **Y** matrix |
| $y_j$ | : jth output variable |
| $u_i$ | : ith input variable |
| $\kappa$ | : Interaction quotient or Rijnsdorp interaction measure |
| $\lambda_{ij}$ | : i-jth element of the RGA matrix |
| $\mu$ | : Structured Singular Value of the **E**(s) matrix |
| $\mu_{ij}$ | : i-jth element of the RGA matrix that corresponds to diagonal pairings |
| $\xi_{ij}$ | : Absolute value of the i-jth element of the RIA matrix that corresponds to diagonal pairings |
| $\phi_{ij}$ | : Absolute value of the i-jth element of the RIA matrix |
| $\omega_u$ | : Ultimate frequency |

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