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Robust Missile Guidance Law against Highly Maneuvering Targets*

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Abstract

Simulation studies of future anti-missile defense scenarios clearly indicated that currently available guidance laws and estimation techniques are unable to guarantee a ”hit-to-kill” accuracy in the interception of the anticipated highly maneuvering targets.

In this paper the future interception scenarios of highly maneuvering anti-surface missiles are formulated as zero-sum pursuit-evasion games with imperfect information. The solution of the perfect information version of the game indicates that, if the actual target maneuver is known, a robust “hit-to-kill” homing accuracy can be guaranteed even with modest maneuverability and agility advantages. However, in a realistic environment with noise corrupted measurements the estimated target maneuver changes are observed with a delay, leading to a devastating affect on the guaranteed homing performance.

This paper describes the development of a new guidance law that explicitly takes into account the estimation delay and compensates for it. Applying this new guidance law leads to a significant reduction of the guaranteed miss distance and restores the robustness with respect to the actual target maneuver. The homing performance of the new guidance law was tested by a set of linearized Monte Carlo simulations, showing very promising results.

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1 Introduction

Motivated by the unexpected events of the Gulf War, the Faculty of Aerospace Engineering at the Technion has been involved since 1991 in the investigation of *future* anti-ballistic missile defense scenarios, where highly maneuvering tactical ballistic missiles (TBM) have to be intercepted [1- 13]. Previous experience with tactical interceptor missiles has established that if the interceptor has sufficient maneuverability advantage over a maneuvering target most guidance laws can guarantee small miss distances. The required maneuverability advantage depends on the guidance law. (It is about 4-5 for Proportional Navigation, 3-4 for Augmented Proportional Navigation and at least 2 for the four state guidance law [14], denoted in this paper as OGL.). In the last two guidance laws perfect knowledge of target maneuver, assumed to be constant, is needed. For some of the future interception scenarios, such as in Anti-Ballistic Missile Defense, ship defense or defense against maneuvering Cruise Missiles, this maneuverability advantage may not be achievable.

Recently developed anti-ballistic missile defense systems (ERINT and ARROW) demonstrated good homing accuracy against non maneuvering targets. The existing tactical ballistic missiles were not designed to maneuver, but due to their high reentry velocity they have a substantial maneuverability potential. Moreover, using this maneuverability potential requires only a modest technical effort. The same is true for future high speed anti-ship or cruise missiles. Against such threats the interceptor missiles will have only a marginal (if any) maneuverability advantage. Recent simulation studies of anticipated anti-missile defense scenarios clearly indicated that currently available guidance laws and estimation techniques are unable to guarantee an adequate homing accuracy for a kinetic ”hit-to-kill” in the interception of the expected highly maneuvering targets [2, 12, 13].

All known missile guidance laws used at the present were developed based on a linearized kinematical model and a linear quadratic optimal control concept where the information on the target maneuvers is obtained from an estimator [14]. As a consequence, the limited maneuver potential of the interceptor has not been explicitly taken into account in these guidance laws. Such an implementation has been based on the common practice to apply the *Certainty Equivalence Principle* which states that the estimation and control processes can be optimized separately, although this principle has been proven valid only for linear-quadratic problems without control constraints and with gaussian noise.

In recent Technion investigations the interception scenario of a highly maneuvering target has been formulated as a pursuit-evasion differential game with bounded controls [1, 3, 5, 7]. Assuming perfect information and using a linearized kinematical model, the guidance law obtained from the solution of such a game explicitly takes into account the available maneuvering potential of the interceptor as well as its dynamics and relies much less on target information than the guidance laws based on an optimal control concept. As a consequence, this guidance law guarantees an improved and robust homing accuracy against a highly maneuvering target, as long as the assumptions of linearization and perfect information are valid. The first missile guidance laws based on Differential Game Theory were developed, assuming perfect information and based on a linearized simplified model, several years ago [15-17] but have not yet been implemented. The implementation of such a guidance law in a realistic environment, where the perfect information assumption is not valid, has represented substantial difficulties and did not seem to be promising.

The objective of the current research activity, is to make the guidance law [17], based on perfect information Differential Game Theory, suitable for implementation in a realistic anti-missile defense scenario. A paper on the first step towards accounting for the phenomena of variable speed, variable maneuverability and nonlinear kinematics was presented recently [18].

This paper is focused on describing how to correct (or at least minimize) the inherent affects of the delay and the limited accuracy of the estimation process in a noise corrupted environment. The paper has the following structure. After the statement of the problem, the results of a deterministic (perfect information) game analysis are briefly summarized and a comparison with an optimal control guidance law is presented. It is followed by the implementation of differential game guidance laws in a noise corrupted environment, including the design of a suitable estimator and an analysis how the noisy measurements and the estimator affect the homing performance. In the next sections the development of a new guidance law compensating the estimation delay is presented and tested by linear Monte Carlo simulations.

2 Problem Statement

The interception scenario of a highly maneuvering target is formulated as a zero-sum pursuit-evasion game. The analysis of the scenario is based on the following set of assumptions:

(A-1) The designated target “T” of the anti-surface missile, protected by the defense system, is stationary.

(A-2) The engagement starts when the interceptor missile (the *pursuer*) "P" is launched against the anti-surface missile (the *evader*) "E".

(A-3) "P" has perfect information on "E", but "E" has information only on "T” and no state information on "P" .

(A-4) The interception must be completed within the "maximum effective range" of the defense system before "E" enters a prescribed "safety zone", defined with respect to "T”.

(A-5) The engagement between the two missiles takes place in a plane.

(A-6) Both missiles have constant velocities V_j and limited lateral accelerations. $|a_j| < (a_j)_{max}$ (j = E, P).

(A-7) The dynamics of both missiles are expressed by first-order transfer functions with the respective time constants τ_E and τ_P.

(A-8) The trajectories of both missile can be linearized along the initial line of sight.

(A-9) If the interception fails, "E" hits and destroys "T".

(A-10) The conditions of the engagement are such that (A-9) can be satisfied without restricting the motion of “E”.

In Fig. 1 a schematic view of the end-game geometry is shown. Note, that the respective velocity vectors of the missiles are generally not aligned with the reference line of sight. The aspect angles ϕ_p and ϕ_E are, however, small. Thus, the approximations $\cos(\phi_i) \approx 1$, $\sin(\phi_i) \approx (\phi_i)$, ($i$ = P, E), are uniformly valid and coherent with (A-8). Moreover, based on (A-6) and (A-8) the final time of the interception can be computed for any given initial conditions of the end-game

$$t_f = \arg\{X_f = X_0 - \int_{t_0}^{t_f}[V_e + V_p]\, dt = 0\} \tag{1}$$

allowing to define the time-to-go by

$$t_{go} = t_f - t \tag{2}$$

The equations of motion perpendicular to the initial line of sight and the respective initial conditions are written, based on assumptions (A-6) and (A-7), as follows:

$$y(t) \stackrel{\Delta}{=} y_E(t) - y_P(t) = x_1 \tag{3}$$

$$\dot{x}_1 = x_2 \;; \qquad x_1(0) = 0 \tag{4}$$

$$\dot{x}_2 = x_3 - x_4 \;; \qquad x_2(0) = V_E\, \phi_{E0} - V_P\, \phi_{P_0} \tag{5}$$

$$\dot{x}_3 = (a_E^c - x_3)/\tau_E \;; \qquad x_3(0) = 0 \tag{6}$$

$$\dot{x}_4 = (a_P^c - x_4)/\tau_P \;; \qquad x_4(0) = 0 \tag{7}$$

where a_E^c and a_P^c are the commanded lateral accelerations of "E" and "P" respectively:

$$a_E^c = (a_E)^{max}\, \mathbf{v} \;; \qquad |\mathbf{v}| \leq 1 \tag{8}$$

$$a_P^c = (a_p)^{max}\, \mathbf{u} \;; \qquad |\mathbf{u}| \leq 1 \tag{9}$$

The non zero initial conditions $V_E\, \phi_{E_0}$ and $V_P\, \phi_{P_0}$ represent the respective initial velocity components not aligned with the initial (reference) line of sight. By assumption (A-8) these components are small compared to the components along the line of sight.

This set of equations (4)-(7) can be written in a compact form as a linear, time dependent, vector differential equation

$$\dot{X} = A(t)\, X + B(t)\, \mathbf{u} + C(t)\, \mathbf{v} \tag{10}$$

with the state vector

$$X = (x_1, x_2, x_3, x_4)^T \tag{11}$$

The natural cost function of the game is the miss distance

$$J = |D^T X(t)| = |x_1(t_f)| \tag{12}$$

where

$$D^T = (1, 0, 0, 0) \tag{13}$$

The problem involves two non-dimensional parameters of physical significance: the pursuer/evader maximum maneuverability ratio μ

$$\mu \triangleq (a_P)^{max} \,/\, (a_E)^{max} \tag{14}$$

and the ratio of evader/pursuer time constants ε

$$\varepsilon \triangleq \tau_E / \tau_P \tag{15}$$

The vector differential equation (10) can be reduced to a scalar one by using the *terminal projection* transformation

$$\boldsymbol{Z}(t) = D^T\, \boldsymbol{\Phi}(t_f, t)\, X(t) \tag{16}$$

where $\boldsymbol{\Phi}(t_f, t)$ is the well known transition matrix of the original homogeneous system. For the sake of generality non-dimensional variables are defined. The independent variable is normalized time-to-go

$$\Theta \triangleq (t_f - t) \,/\, \tau_P \,; \qquad \Theta(0) = t_f / \tau_P = \Theta_0 \tag{17}$$

and the non-dimensional state variable is the normalized *zero-effort miss distance*.

$$Z(\Theta) \triangleq \mathbf{Z}(t)/\tau_P^2 (a_E)^{max} = \{ x_1 + x_2 (\tau_P \Theta) + x_3\tau_E^2 (e^{-\Theta/\varepsilon} + \Theta/\varepsilon - 1) - x_4\tau_P^2 (e^{-\Theta} + \Theta - 1) \}/\tau_P^2 (a_E)^{max} \tag{18}$$

with the following normalized initial conditions

$$Z(\Theta_0) \triangleq Z_0 = (V_E\, \phi_{E_0} - V_P\, \phi_{P_0})\, \Theta_0 / \tau_P^2 (a_E)^{max} \tag{19}$$

This definition imbeds the assumption that all the original state variables (x_1 , x_2, as well as the lateral accelerations x_3 and x_4) are known to both players. Using the non-dimensional variables the normalized game dynamics becomes

$$\frac{dZ}{d\Theta} = \mu\, (e^{-\Theta} + \Theta - 1)\, \mathbf{u} - \varepsilon\, (e^{-\Theta/\varepsilon} + \Theta/\varepsilon - 1)\, \mathbf{v}\,; \qquad Z(\Theta_0) = Z_0 \tag{20}$$

The objective of the defense system is to minimize the non-dimensional pay-off function, which is the normalized *miss distance,*

$$J = |Z_f| \tag{21}$$

while the designer of the anti-surface missile wants to maximize it.

3 Deterministic Game Analysis

3.1 Perfect Information Game Solution

The perfect information version of the game is the worst case from the point of view of the defense. (It assumes that the actually “blind” anti-surface missile can observe the interceptor.) The solution of this game [17], is characterized by the decomposition of the (Θ, Z) game space into two regions of different strategies (called $\mathcal{D}_0$ and $\mathcal{D}_1$) as it is shown in Fig. 2. These regions are separated by a pair of optimal trajectories $\pm Z^*(\Theta)$ intersecting (tangentially) at the point ($Z = 0$, $\Theta = \Theta_s$), where $\Theta_s = \Theta_s(\mu, \varepsilon)$ is the non vanishing solution of the equation

$$\varepsilon\, (e^{-\Theta/\varepsilon} + \Theta/\varepsilon - 1) = \mu\, (e^{-\Theta} + \Theta - 1) \tag{22}$$

In $\mathcal{D}_1$ the optimal strategies are of the "bang-bang" type

$$\mathbf{u}^*(\Theta,Z) = \mathbf{v}^*(\Theta, Z) = \text{sign}\,\{Z\}\,; \quad Z \neq 0 \tag{23}$$

and the *value* of the game is a unique function of the initial conditions.

The boundary trajectories $\pm Z^*(\Theta)$ are obtained by integrating (20), starting at the point $(Z = 0, \Theta = \Theta_s)$ using (23). They enclose the region $\mathcal{D}_0$. Every trajectory starting in this region must go through the point $(Z = 0, \Theta = \Theta_s)$. Therefore in $\mathcal{D}_0$ the optimal strategies are arbitrary and the *value* of the game is constant (J_0^*). When a trajectory that starts in $\mathcal{D}_0$ reaches the point $(Z = 0, \Theta = \Theta_s)$, the evader must decide and select the direction of its maximal maneuver (either to the right or to the left) and the pursuer has to follow it. Therefore the optimal evasive maneuver that guarantees J_0^* is a maximal maneuver in a fixed direction for the duration of at least Θ_s.

Actually the entire segment $0 < \Theta \leq \Theta_s$ of the Θ axis is a *dispersal line* dominated by the evader ("E"). At any point along this segment the sign of the optimal maneuver can be randomly selected by the evader and the pursuer has to follow this choice. The boundary trajectories $\pm Z^*(\Theta)$ and the *dispersal line* belong to $\mathcal{D}_1$.

It can be easily shown that (22) has a non zero solution only if the product $\mu\varepsilon$, which has the physical interpretation of the pursuer/evader *agility ratio*, satisfies the inequality

$$\mu\varepsilon < 1 \tag{24}$$

Otherwise, the only solution of (22) is $\Theta = \Theta_s = 0$ and the *value* of the game (the guaranteed normalized miss distance) in $\mathcal{D}_0$ is also zero. If (24) is satisfied, then $\Theta_s > 0$ and the constant *value* of the game (the guaranteed normalized miss distance) in $\mathcal{D}_0$ is given by

$$J_0^* = \mu (1-\varepsilon) (e^{-\Theta_s} + \Theta_s - 1) - (\mu - 1) \Theta_s^2 / 2 \triangleq M_s(\mu, \varepsilon) \tag{25}$$

The values of Θ_s and M_s, both functions of μ and ε, are depicted in Figs. 3 and 4. These results are of great practical importance, because for the majority of cases the initial conditions of the engagement are in $\mathcal{D}_0$. Since in $\mathcal{D}_0$ the optimal strategies are arbitrary, the guidance law of the interceptor can be a linear one, or even a "bang-bang" type strategy as (23).

Summarizing this perfect information game solution, denoted as DGL/1, there are two cases to be distinguished. In the first case, where inequality (24) is satisfied, the guaranteed normalized miss distance M_s is not zero. Note, that this is a *saddle-point value* guaranteed for both players if they play optimally.

In the particular case of $\varepsilon = 0$, i.e. ideal evader dynamics, the information on the evader's maneuver (even if it is available) cannot be incorporated in the guidance law of the interceptor. This can be directly observed by taking the limits of (18) and (20) as $\varepsilon \to 0$. This case, denoted DGL/0, was first solved in [15] and discussed further in [16].

In the second case the inequality (24) is not satisfied, i.e. $\mu\varepsilon \geq 1$. In this case for all of the initial conditions of practical interest a *point capture* interception of the evader (zero miss distance) is guaranteed against any feasible maneuver. This is a robust “hit-to-kill” accuracy (a most desirable feature for anti-missile defense) based on the assumption of perfect knowledge of the target maneuver.

3.2 Comparison to OGL

It is interesting to compare the perfect information game solution to the performance of the four state *optimal guidance law* (OGL) outlined in [14], which explicitly incorporates the effect of an assumed constant target acceleration. Though OGL has been conceived to guarantee zero miss distance against constant target maneuvers, it is very sensitive to maneuver changes near to the final time of the interception. Against a deterministic *optimal evasion* the normalized *miss distance* obtained by OGL becomes quite large. This can be seen from the simulation results depicted in Fig. 5 which show, for different values of τ_E, the miss distance obtained by a "bang-bang" type acceleration command of the evader as the function of the time-to-go of the change in command.

Fig. 6 displays the simulation results of interceptions against the same type of maneuver using the guidance law of [15] denoted DGL/0, which does not include any information on target acceleration. From these two figures one can conclude that as long as $\varepsilon < 1$, the maximum miss distance of DGL/0 is smaller than the one of OGL. Moreover, DGL/0 has a uniform guaranteed upper limit for all values of ε. The performance of DGL/1 is, of course, even better. This comparison clearly demonstrates the advantage of *robust* guidance laws based on a pursuit-evasion game solution over an optimal control synthesis.

4 Guidance Law Implementation

As mentioned earlier, the robust "hit-to-kill" performance of DGL/1 is based on the assumption of perfect information, i.e. accurate knowledge of all the state variables including the actual lateral acceleration of the target. Unfortunately, this state variable cannot be directly measured. It has to be estimated based on the generally noise corrupted measurements of the available state variables. In this section the design of such an estimator (Kalman Filter) is outlined. The output of this estimator is used in the sequel in the implementation of DGL/1 and the homing performance in a noise-corrupted environment is evaluated by a large set of Monte Carlo simulations.

4.1 Estimator Design

The design is based on the dynamic model of (4) - (7). The available measurements are x_1, a product of the accurately measured range and a noise corrupted boresight angle, and x_4 the interceptor's own lateral acceleration (also measured with some error). The measurement noises are assumed to be zero mean, white and gaussian.

Moreover, since the anti-surface missile is "blind" with respect to the interceptor, it must maneuver randomly. It is assumed that the evader's maneuver is a stochastic process in the form of a Random Telegraph signal characterized by a single parameter λ. (This assumption has been frequently used in missile guidance analysis.). For the estimator design such a process is represented by white noise going through a first-order *shaping filter* [19] with a time constant of $1/(2\lambda)$, because both have the same autocorrelation function. This representation makes the lateral aceleration command of the evader as an additional state variable of the estimator. The state variables and equations of motion are

$$x_1 = y_E(t) - y_p(t) \qquad \dot{x}_1 = x_2$$

$$x_2 = \dot{y}_E(t) - \dot{y}_p(t) \qquad \dot{x}_2 = x_3 - x_4$$

$$x_3 = \ddot{y}_E \qquad \dot{x}_3 = \frac{x_5 - x_3}{\tau_E}$$

$$x_4 = \ddot{y}_p \qquad \dot{x}_4 = \frac{a_p^c - x_4}{\tau_p}$$

$$x_5 = a_E^c \qquad \dot{x}_5 = (v_w - x_5)\cdot 2\lambda$$

(26)

which can be written in a matrix form as

$$\begin{aligned} \dot{X} &= AX + Bu + Gv_W \\ Y &= C_Y X + W \end{aligned} \qquad X = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{pmatrix}^T$$

(27)

where

$$B = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{\tau_p} & 0 \end{bmatrix}^T$$

(29)

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 2\lambda \end{bmatrix}^T \tag{30}$$

$$C_Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(31)

$$W = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T$$

(32)

In this notation v_w is the process noise, while w_1 and w_2 are the zero mean white gaussian measurement noises with the standard deviations σ_1 and σ_2 of x_1 and x_4 respectively. Since the system is observable an estimator design is possible. The well known continuos time Kalman Filter algorithm is

$$\dot{\hat{X}} = A\hat{X} + Bu + K(Y - C_Y\hat{X}) \qquad \hat{X}(t_0) = \hat{X}_0$$
$$\dot{P} = AP + PA^T - PC_Y^T R^{-1} C_Y P + GQG^T \qquad P(t_0) = P_0$$
$$K = PC_Y^T R^{-1} \tag{33}$$

where R is the covariance matrix of the measurement noise, X is a vector of the estimated states, P is the covariance matrix of the estimated state and K is the estimator gain matrix.

4.2 Monte Carlo Simulation Results

In simulating the implementation of the guidance laws DGL/0 and DGL/1, derived from perfect information differential game solutions, sets of 125 Monte Carlo runs with different noise samples were used. The parameters and the initial conditions of the simulations are summarized in Table 1.

parameters		states	estimated states	covariance matrix
$\lambda = 1.5$ 1/sec	$\tau_E = 0.2$ sec	$X_0 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	$\hat{X}_0 = \begin{bmatrix}0\\0\\0\\0\\a_E^{max}\end{bmatrix}$	$P_0 = \begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&(a_E^{max})^2\end{bmatrix}$
$t_f = 3$ sec	$\tau_P = 0.2$ sec			
$\sigma_1 = 1$ mrad	$a_E^{max} = 21g$			
$\sigma_2 = 0.1g$	$\mu = 2.25$			

Table 1: Parameters and initial conditions

The maneuvers of the TBM being of a randomly switched “bang-bang” type, the *worst* maneuver (the one that creates the largest miss distance), had to be identified. The accumulated miss distance distributions obtained by the *worst* maneuvers are shown in Figs. 7 and 8 for DGL/0 and DGL/1 respectively.

These results demonstrate that the direct implementation of DGL/1 using a Kalman Filter type estimator, which is a common practice in the missile guidance community (based on the unjustified application of the Certainty Equivalence Principle), fails to provide a satisfactory homing performance. The robust “hit-to-kill” accuracy is lost and the resulting miss distances may not be acceptable. Although DGL/0 also has a degraded homing performance due to the noise corrupted measurements, it seems to be less affected, because this guidance law doesn’t rely on the evader’s lateral acceleration.

The attempt to estimate the state variables for the implementation of a perfect information guidance law, based on noisy measurements creates two types of errors. Due to the measurement noise the estimated state, in our case the normalized *zero-effort miss distance* defined by (18), is never the actual one. The Kalman Filter has the role to minimize the root mean square of the estimation error. If the actual state variables are constant or vary rather slowly the estimation error is rather small. However, if there is a sudden change in one of the variables, the estimation error of this variable (and may be also others) becomes large and it may take a substantial time until the estimated state converges to its new value. This phenomenon is clearly seen in Fig. 9.

4.3 Affect of pure estimation delay

It is of common experience that even if the accuracy and the convergence of a position estimate is satisfactory, the estimated acceleration is less precise and it convergence much more slowly. Since this dynamic effect seems to be dominant, in this subsection it is assumed that the estimation of the variables x_1, x_2 and x_4 is ideal and the estimation process of x_3 (the evader's lateral acceleration) is approximated by a perfect information outcome delayed by the amount of Δt_{est} [10]. There is a lower bound for the value of Δt_{est} which can be found based on generic arguments, independent of the form of the estimator [20].

As a consequence of the estimation delay the maneuvering evader can generate a non zero miss distance even if $\mu\varepsilon \leq 1$, because DGL/1 is an optimal guidance law only for the perfect information case. For each value of Δt_{est} there exists an optimal evader maneuver that maximizes the miss distance. This maneuver consists of an optimally timed direction change ("switch") between maximal lateral acceleration command from one direction to the opposite side. In Fig. 10 the maximum normalized miss distance generated by the evader is plotted as the function of the normalized delay ($\Delta\Theta_{est} = \Delta t_{est}/\tau_P$) for a case with $\mu = 2$ and $\varepsilon = (0.25, 0.5, 1.0)$. The performance degradation due to estimation delay, very similar to the one observed in the Monte Carlo simulations, can be clearly observed. One can see that if the estimation delay is too long, it is better not to incorporate the target maneuver in the guidance law at all, i.e. to use DGL/0, where $\varepsilon = 0$ is assumed.

5 Modified guidance law compensating estimation delay

The objective of this section is to develop a new guidance law modifying the original DGL/1 in order to minimize the affect of the estimation delay. The approach is based on the following idea. Since the problem to be solved is a pursuit-evasion game with bounded controls, the classical Certainty Equivalence Principle, - allowing to use the estimated state variables to replace the actual ones in the separately optimized control law, - is not valid. Nevertheless, the separate design of the estimator is allowed but the optimization of the control law has to be based on the probability density function of the estimated state variables [21]. In other words, in the derivation of the guidance law the limitations of the estimation process has to be explicitly taken into account [11]. As the first step in this direction the interception scenario of maneuvering anti-surface missiles is reformulated as an imperfect information zero-sum pursuit-evasion game, or equivalently an imperfect information minmax control problem for the interceptor missile.

5.1 New problem formulation

Defense scenarios against maneuverable anti-surface missiles have an imperfect and asymmetrical information structure. The evader is "blind" with respect to the interceptor (pursuer), see (A-3). The interceptor is assumed to have information on the relative separation and velocity based on a set of noisy measurements. The pursuer can measure its own acceleration, but must estimate the evader acceleration from the other measurements. As mentioned earlier, the evader is "blind" and must maneuver randomly, which can be considered as a bounded disturbance in the minmax control formulation. In both formulations the payoff is the *miss distance* (the distance of closest approach).

The required solution of this problem is composed of the following triplet: (i) the *worst* bounded disturbance (the optimal evader strategy), (ii) the interceptor's guidance law (the optimal pursuer strategy), and (iii) the *guaranteed miss distance*. The solution of the corresponding perfect information pursuit-evasion game, using linearized planar kinematics and first order dynamic models for the sake of simplicity, was already presented in section 3. It serves as a point of reference for the imperfect information game of interest.

The estimation process of the evader's lateral acceleration is approximated by a delayed but correct outcome, as done already in the previous section, where its devastating effect on the guaranteed homing performance was also shown. These results are obvious, because DGL/1 is the optimal pursuer strategy only for the perfect information game without estimation delay ($\Delta t_{est} = 0$). For $\Delta t_{est} \neq 0$ the optimal pursuer strategy has to be different. The objective of this section is to find a new guidance law, that takes into consideration the imperfect information of the pursuer as suggested in [11], and provides an improved homing performance compared to both DGL/1 and DGL/0.

This problem is suitable to be formulated as a minmax control problem of the pursuer. Given the dynamic system (4)-(7) and a set of initial conditions, minimize the cost function $J = |x_1(t_f)|$, subject to the following set of available measurements

$$h_i(t) = x_i(t)\ ; \qquad i = 1, 2, 4\ ; \qquad\qquad h_3(t) = x_3(t - \Delta t_{est}) \tag{34}$$

In this formulation the minimizing control is **u**(t), while **v**(t) is considered as a bounded disturbance, both subject to the constraints (8) and (9). Based on this formulation an attempt is made in the sequel to obtain an improved guidance law which makes optimal use of the estimated lateral acceleration of the evader by taking into account the assumed delay of the estimation process.

5.2 Minmax Certainty Equivalence

The classical Certainty Equivalence Principle mentioned earlier, was formulated for stochastic control problems, where the disturbances are modeled as a stochastic process and the payoff is the expected value of a deterministic cost function. This principle was proven to be valid for problems with linear unconstrained dynamics, a quadratic cost and gaussian white noise (LQG). Recently a different certainty equivalence theorem, applied to minmax control problems where the disturbances are bounded and the payoff is a maximum, i.e. independent of the disturbances statistics, was proven [22]. It is called the Minmax Certainty Equivalence Principle (MCEP).

The set of hypotheses needed for applying the principle are the following:

(i) The observation process has to be consistent, non anticipative and of perfect recall;

(ii) The perfect information game associated with the problem admits an optimal strategy (**u***, **v***) and a smooth value function;

(iii) The solution of an *auxiliary problem*, that serves to determine the *worst* possible disturbance compatible with the actual measurement sequence, is **unique**.

If all these conditions are met, than the *worst* possible state can be computed and used in the perfect information feedback strategy of the pursuer (minimizer) instead of the unavailable actual *true* state.

In the attempt of applying MCEP to the above outlined minmax control problem the following observation can be made.

(I) The observation process of the problem satisfies the first hypothesis.

(ii) The value function of the perfect information game (see section 3) is only piecewise smooth. Along the boundary trajectories $\pm Z^*(\Theta)$ and the *dispersal line*$\{Z = 0 \cap \Theta_s \geq \Theta \geq 0\}$ the gradient of the value function is discontinuous.

The violation of the second hypothesis is not an essential problem. It is merely necessary for solving the *auxiliary problem* using the Hamilton-Jacobi equation. In our case, due to the structure of the problem the *worst* possible state can be computed directly, as it is shown in the sequel.

For the present problem the true state, the normalized *zero-effort miss distance*, can be written based on (18) as

$$Z(t) = Z^0(t) + \Delta Z_E(t) \tag{35}$$

where

$$Z^0(t) = x_1(t) + x_2(t)\, t_{go} - \Delta Z_P(t) \tag{36}$$

and

$$\Delta Z_P(t) = \tau_P^2 (e^{-\theta} + \theta - 1)\, x_4(t) = \tau_P^2\, \psi\, (t_{go}/\tau_P)\, x_4(t) \tag{37}$$

$$\Delta Z_E(t) = \tau_E^2 (e^{-\theta/\varepsilon} + \theta/\varepsilon - 1)\, x_3(t) = \tau_E^2\, \psi\, (t_{go}/\tau_E)\, x_3(t) \tag{38}$$

Because of the estimation delay one observes, instead of the actual value of $\Delta Z_E(t)$,

$$\{\Delta Z_E(t)\}_{est} = \tau_E^2\, \psi\, (t_{go}/\tau_E)\, x_3(t - \Delta t_{est}) \tag{39}$$

and the observed *zero-effort miss distance* is

$$Z_{est}(t) = Z^0(t) + \tau_E^2\, \psi\, (t_{go}/\tau_E)\, x_3(t - \Delta t_{est}) \tag{40}$$

Given the delayed measurement $x_3(t - \Delta t_{est})$ the uncertain value of $\Delta Z_E(t)$ is bounded (see Fig. 11) by

$$[\Delta Z_E(t)]_{min} \le \Delta Z_E(t) \le [\Delta Z_E(t)]_{max} \tag{41}$$

where

$$[\Delta Z_E(t)]_{min} = \tau_E^2\, \psi\, (t_{go}/\tau_E)\left[x_3(t-\Delta t_{est})e^{-\frac{\Delta t_{est}}{\tau_E}} - a_E^{max}(1-e^{-\frac{\Delta t_{est}}{\tau_E}})\right] \tag{42}$$

$$[\Delta Z_E(t)]_{max} = \tau_E^2\, \psi\, (t_{go}/\tau_E)\left[x_3(t-\Delta t_{est})e^{-\frac{\Delta t_{est}}{\tau_E}} + a_E^{max}(1-e^{-\frac{\Delta t_{est}}{\tau_E}})\right] \tag{43}$$

As a consequence, two candidates of the *worst* possible state can be computed and the truly *worst* possible state is determined by

$$[Z\ (t)]_{worst} = \arg\max\ \{|[Z\ (t)]_{min}|,\ |[Z\ (t)]_{max}|\} \tag{44}$$

where

$$[Z\ (t)]_{min} = Z^0(t) + [\Delta Z_E(t)]_{min} \tag{45}$$

$$[Z\ (t)]_{max} = Z^0(t) + [\Delta Z_E(t)]_{max} \tag{46}$$

Since the application of MCEP suggests to use u* = sign{$[Z\ (t)]_{worst}$}, there is a difficulty if $|[\Delta Z_E(t)]_{min}| = |[\Delta Z_E(t)]_{max}|$. In this case the *worst* possible state is not unique and as a consequence, the MCEP cannot be applied.

5.3 Guidance Law Synthesis Based on the Reachable Set Concept

This difficulty was circumvented by using a different approach, namely the concept of *reachable sets*. Such an approach was presented in [23, 24], dealing with pursuit evasion games with delayed information. It suggests to create, based on the available information such as $Z^0(t)$ and $\{\Delta Z_E(t)\}_{est}$ at every point of the time "t" the *reachable set* of the evader and to aim to the center of the convex hall of this *reachable set*.

As long as the *reachable set* remains in $\mathcal{D}_0$ the cost is not effected by the disturbance and it remains J_0*. If the *reachable set* stays in $\mathcal{D}_1$, the *worst* possible disturbance is unique (**v*** = sign{Z}). The true state remains in $\mathcal{D}_1$ and if the pursuer uses the optimal strategy (**u*** = sign{Z_0}), the cost is equal to the value function of the perfect information game.

Based on the system dynamics, $\Delta Z_E(t)$ is bounded

$$[\Delta Z_E(t)]_{min} \le \Delta Z_E(t) \le [\Delta Z_E(t)]_{max} \tag{47}$$

In the present problem the *reachable set* of Z(t) is the segment (see Fig. 11).

$$[Z(t)]_{min} \leq Z(t) \leq [Z(t)]_{max} \qquad (48)$$

The approach suggests [22, 23] that the optimal pursuer strategy in the game with delayed information should be

$$\mathbf{u}^* = \text{sign}\,\{Z(t)_{av}\} \qquad \{Z(t)_{av}\} \neq 0 \qquad (49)$$

where

$$Z(t)_{av} = \{\,[Z(t)]_{max} + [Z(t)]_{min}\,\}/2 \qquad (50)$$

If $Z(t)_{av} = 0$, as for example on the *dispersal line*, the *reachable set* approach suggests $\mathbf{u}^* = 0$. It turns out that this optimal pursuer strategy is identical with the application of MCEP outlined earlier, if the *worst* possible state is unique, because

$$\text{sign}\,\{[Z\,(t)]_{worst}\} = \text{sign}\,\{Z(t)_{av}\} \qquad (51)$$

The control strategy of the pursuer, as defined above, is adopted as a new interceptor guidance law, called in the sequel as DGL/C.

5.4 Deterministic Results

The homing performance of this guidance law was tested in a large set of simulations and yielded very encouraging results. This is illustrated by a numerical example and the corresponding Figs. 12-13. This example describes a short end-game ($\Theta_0 = 7.0$) with a non zero initial condition and the set of parameters ($\mu = 2.25$, $\varepsilon = 1.0$) that guarantees in the perfect information game DGL/1 achieves a zero miss distance. The engagement starts with a non zero initial condition in $\mathcal{D}_0$ $\{Z(\Theta_0)=2.8.\}$ and the evader (assumed to have perfect information) starts with a full acceleration command $\mathbf{v}^* = \text{sign}\{Z(\Theta_0)\} = +1$.

In Fig. 12 (with a fixed estimation delay $\Delta\Theta_{est} = 1.0$) one can directly observe the robust nature of DGL/C with respect to the timing of the evader's switch in the maneuver direction in contrast to the great sensitivity of DGL/1 to the same parameter. This is similar to the robust behavior of DGL/0, which does not use the information on the acceleration of the evader, but the *guaranteed* normalized miss distance is much smaller. In Fig. 13 the *guaranteed* normalized miss distances of DGL/C are compared, for different values of the normalized estimation delay, to those of DGL/0 and DGL/1. From this figure the superior performance of DGL/C is very clear. The much smaller *miss distances* and the robust performance are the consequence of taking the imperfect nature of the available information into explicit consideration.

5.5 Comparison by Monte Carlo Simulations

The homing performance of the new guidance law DGL/C was also tested in the environment of noise corrupted measurements described in section 4. The guidance law was implemented with the same estimator described in 4.2. As in the other cases an ensemble of 125 Monte Carlo runs with different noise samples were tested.

In the attempt to identify the *worst* case it was found that, similarly to DGL/0, there is a *guaranteed* maximum miss distance for all feasible maneuver sequences of the evader. The equivalent pure estimation delay which provides the best compensation of the affects of the noise corrupted measurements was identified to be $\Delta\theta_{est}=0.5$ and used in the computations of DGL/C.

The accumulated miss distance distribution obtained for the *guaranteed* maximum miss distance is compared in Figs. 14 with the *worst* results obtained for DGL/1, as it was shown on Fig. 8. It can been clearly seen that the modification introduced into DGL/C leads not only to a restored robustness but also to a substantially improved homing performance compared to DGL/1. Nevertheless, it has to be admitted that even with this compensation a “hit-to-kill” homing accuracy could not be achieved.

6 Conclusions

In this paper the development of a new guidance law, that explicitly takes into account the inherent delay of the estimation process in a noise corrupted environment and compensates for it, is described. By modeling the affect of noisy measurements and a typical linear estimator by a pure delay in the observation of the target maneuver, a compensation scheme was determined. Applying this new compensated guidance law leads to a significant reduction of the guaranteed miss distance. Moreover, it restores the robustness of the original guidance law, derived by using perfect information Differential Game Theory, with respect to the form of actual (bounded) target maneuver. The homing performance of the new guidance law was tested by a set of linearized Monte Carlo simulations, demonstrating a substantial improvement in the homing performance.

There is no doubt that the recently completed phase succeeded to demonstrate an impressive potential for an improved and robust homing performance that can be achieved by making use of the missile guidance concept based on Differential Game Theory.

This has been, however, only the first step towards the development of an improved robust guidance law that can guarantee satisfactory homing performance against highly maneuvering anti-surface missiles expected in the future. The proposed compensation is based on a rather rough approximation of the noise and estimator affects by a pure delay. Most probably a more accurate model may lead to even better results. Moreover, the estimator used in the implementation should be optimized in order to reduce the equivalent estimation delay.

In any case there is a need to validate the new guidance law in a wide range of scenario parameters and in a realistic nonlinear simulation environment. The compensation for affects of a realistic anti-missile defense scenario with variable speed and maneuverability, as well as of the three dimensional nonlinear geometry, have to be also incorporated in the new guidance law and tested.

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Figures

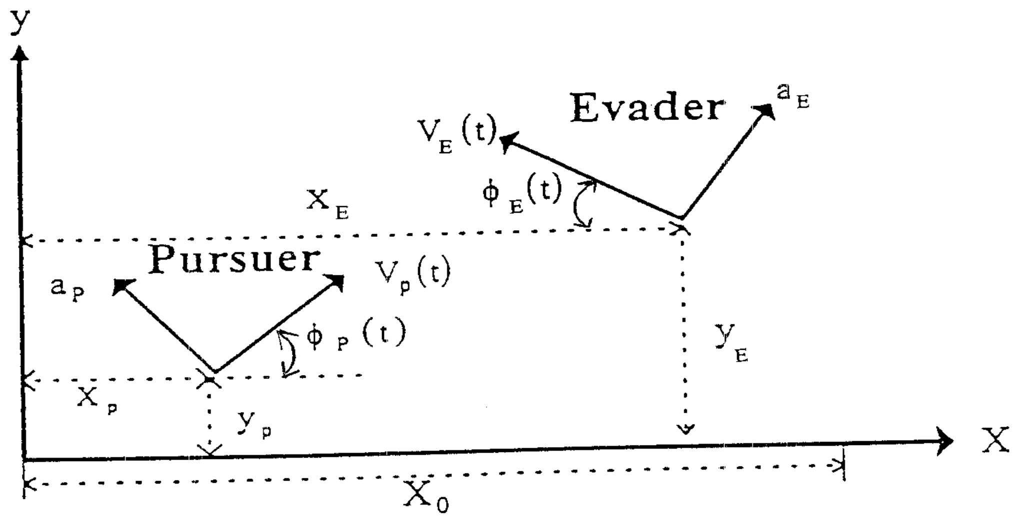

Fig.1 Interception geometry

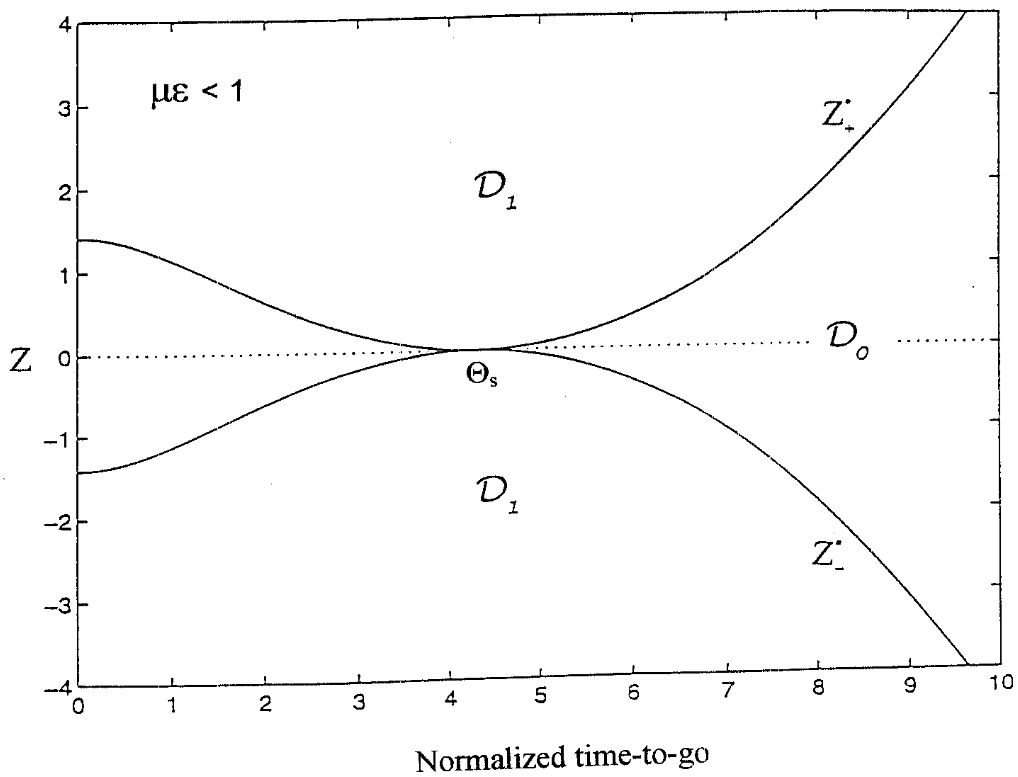

Fig. 2 Decomposition of the game space

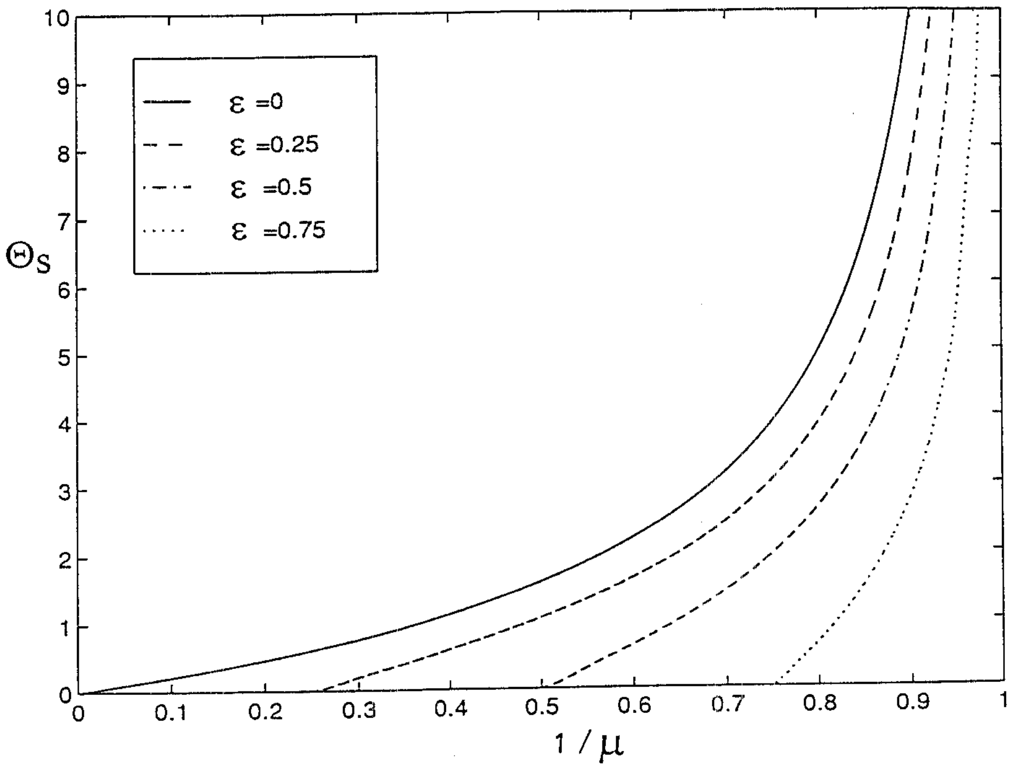

Fig. 3 Normalized critical time, Θ_s

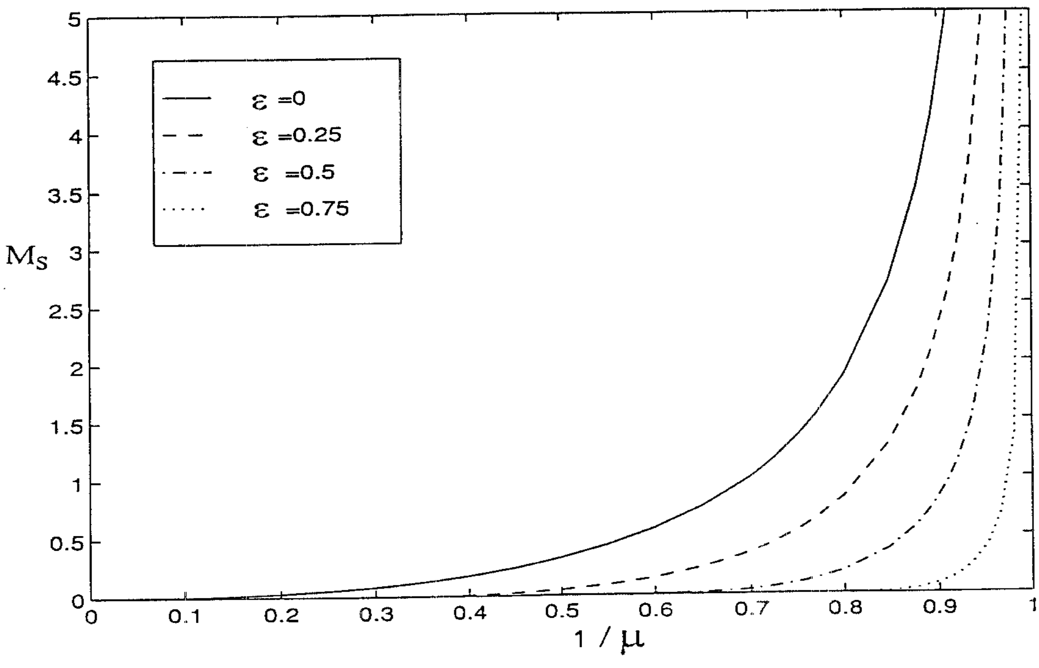

Fig. 4 Normalized guaranteed miss distance in $\mathcal{D}_0$, M_s

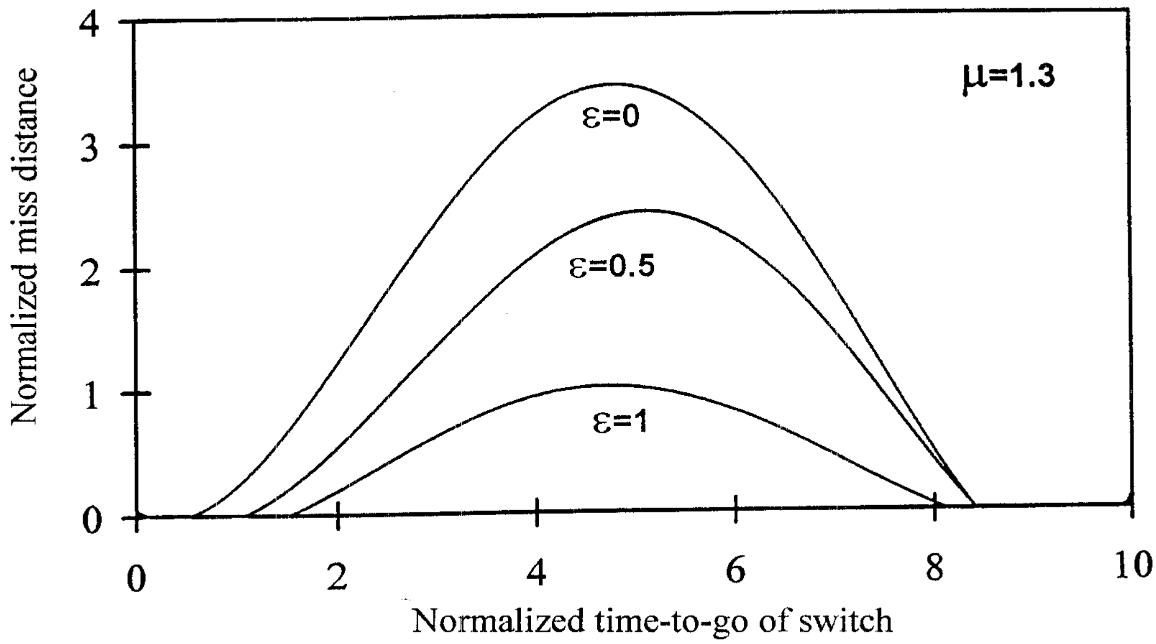

Fig.5 Homing performance of OGL

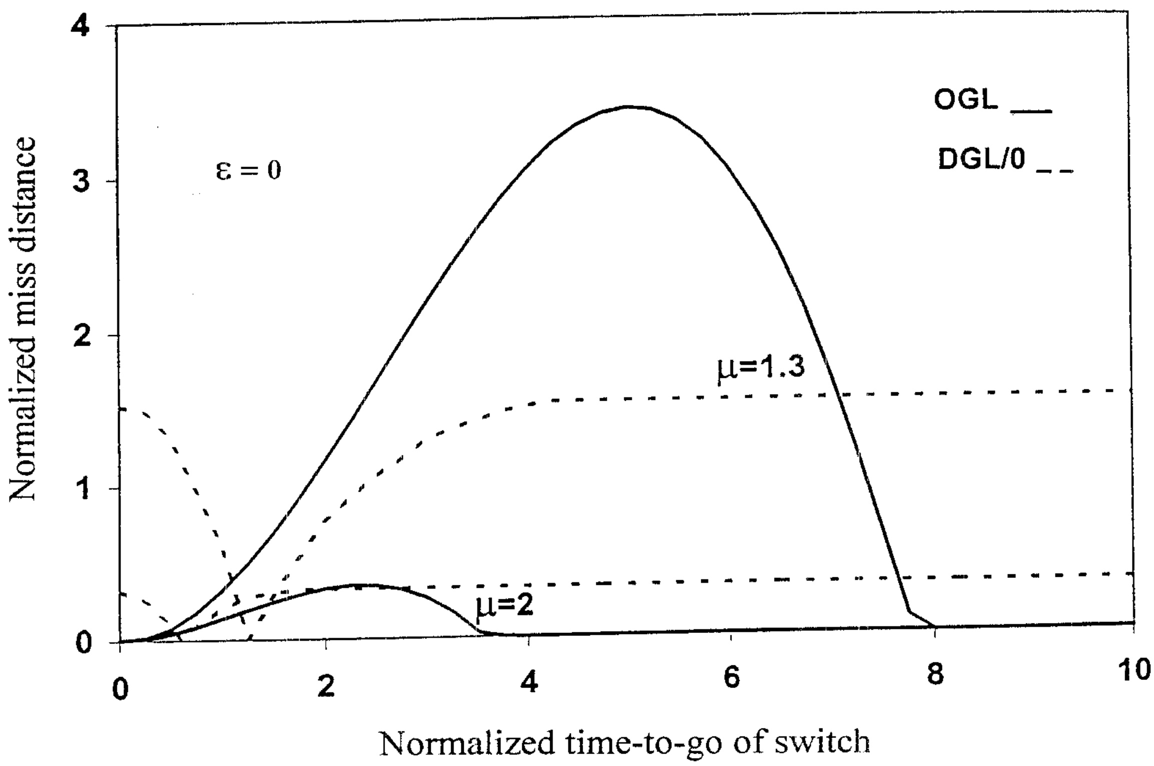

Fig.6 Homing performance of DGL/0

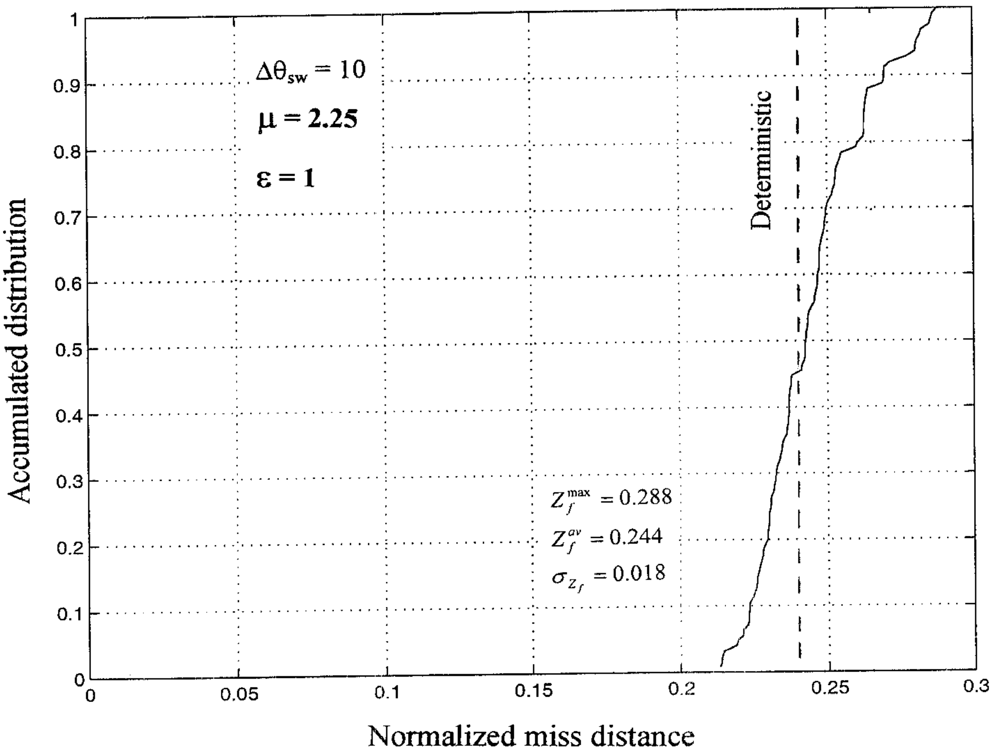

Fig. 7 Accumulated miss distance distribution DGL/0 (*worst* case)

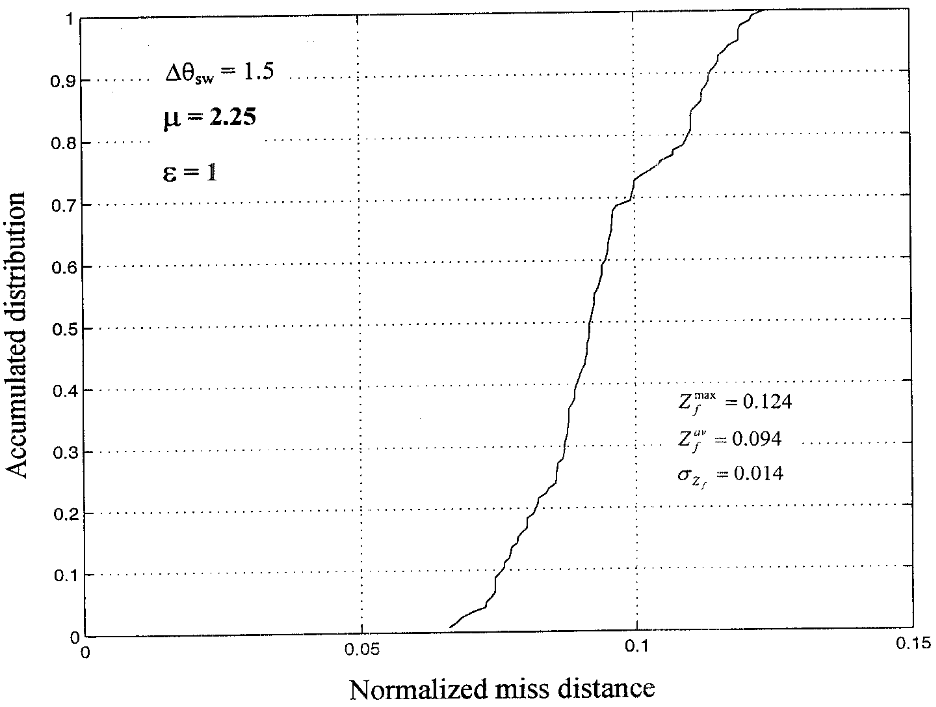

Fig. 8 Accumulated miss distance distribution DGL/1 (*worst* case)

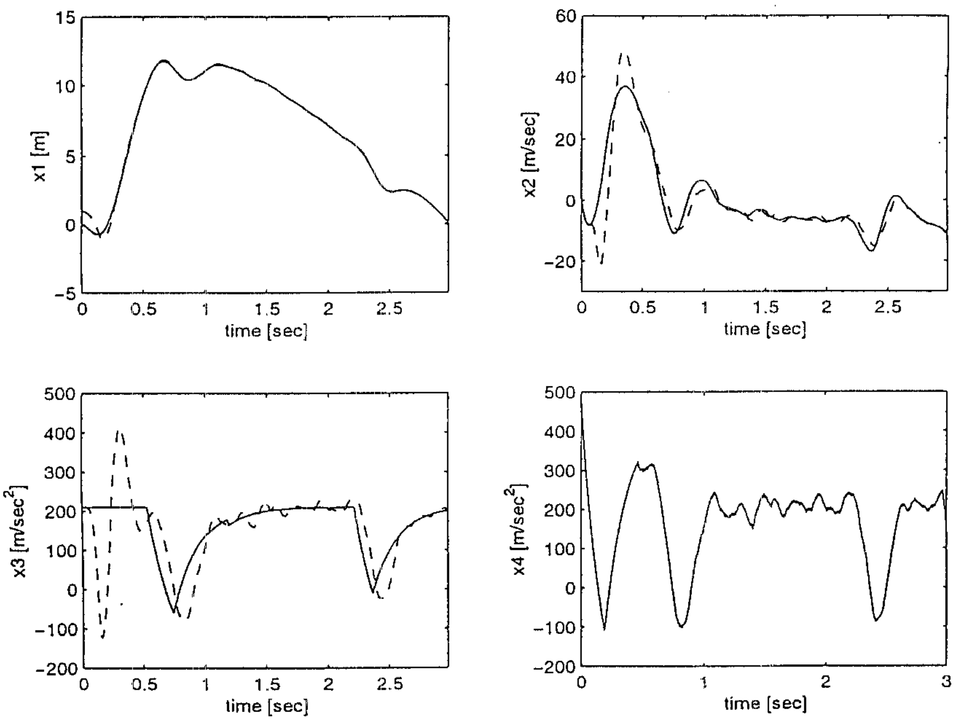

Fig. 9 Estimator performance

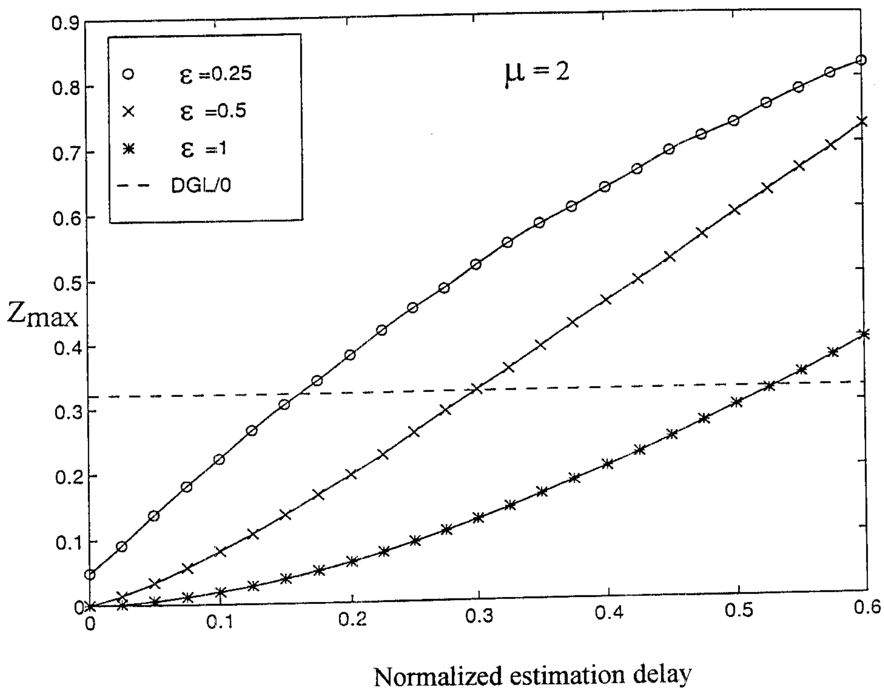

Fig. 10 Affect of pure estimation delay

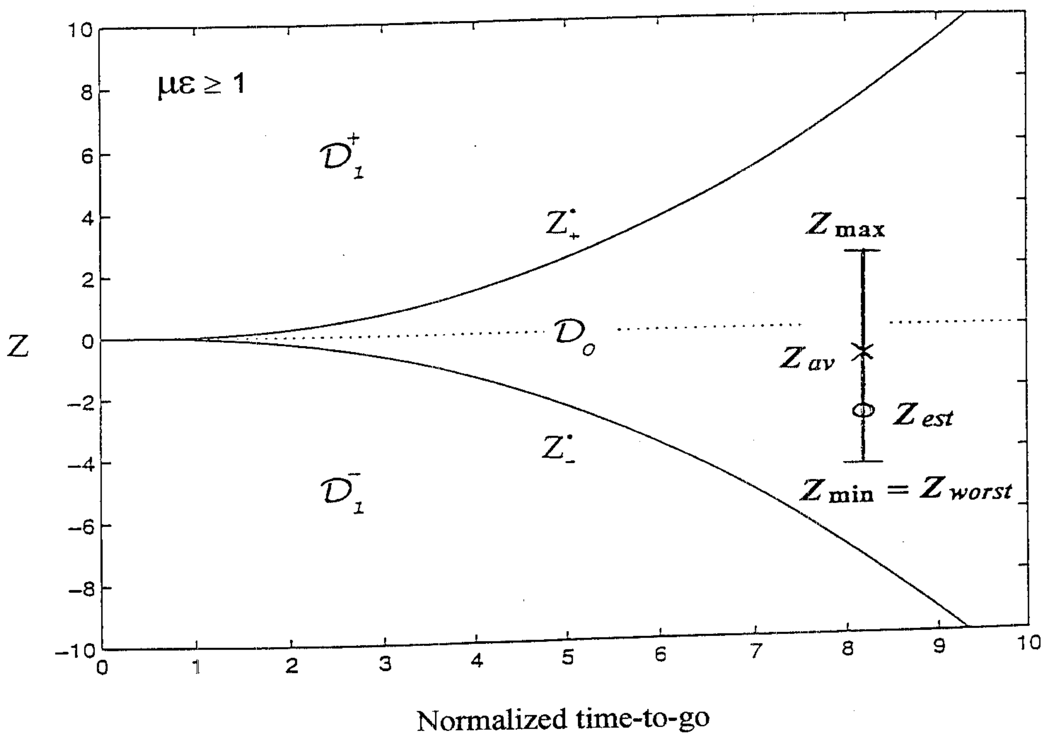

Fig. 11 The uncertainty bounds of Z

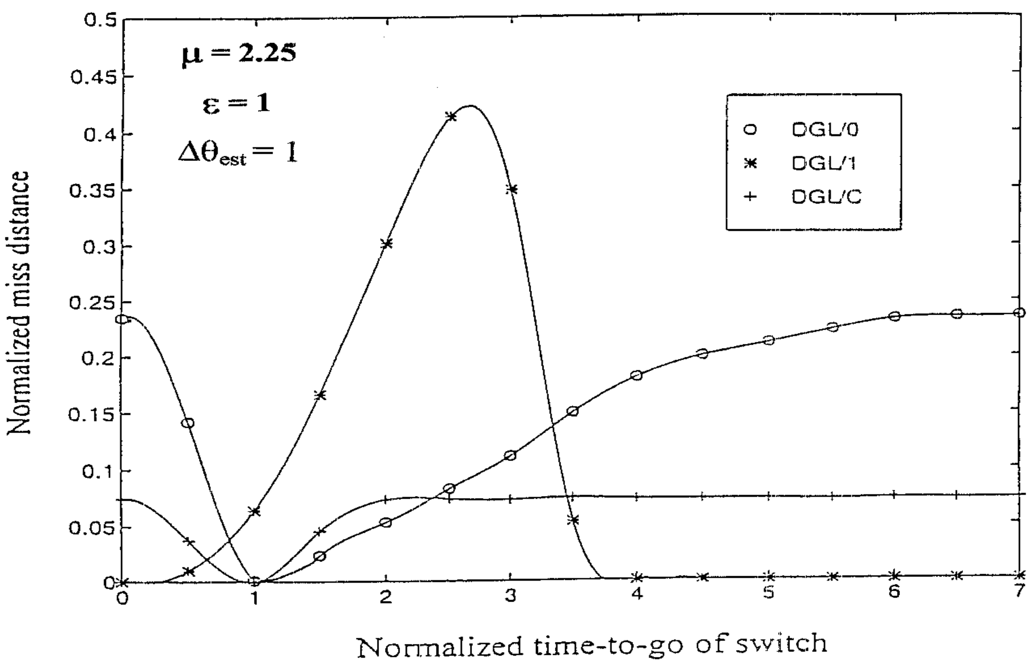

Fig. 12 Homing performance of DGL/C, constant delay

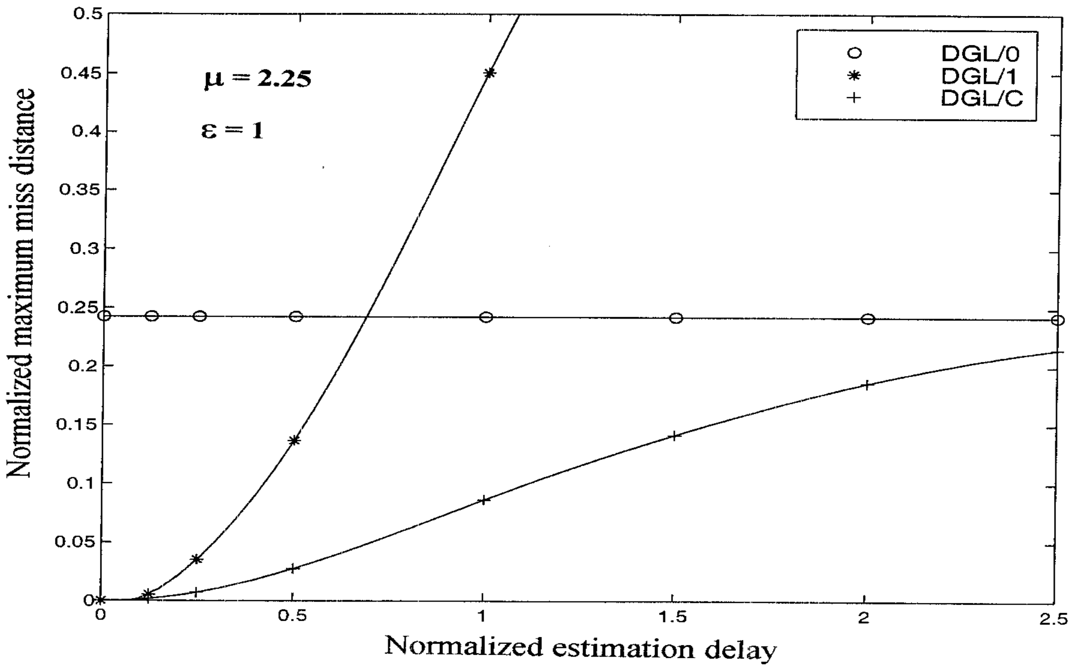

Fig. 13 Guaranteed normalized miss distance as a function of estimation delay

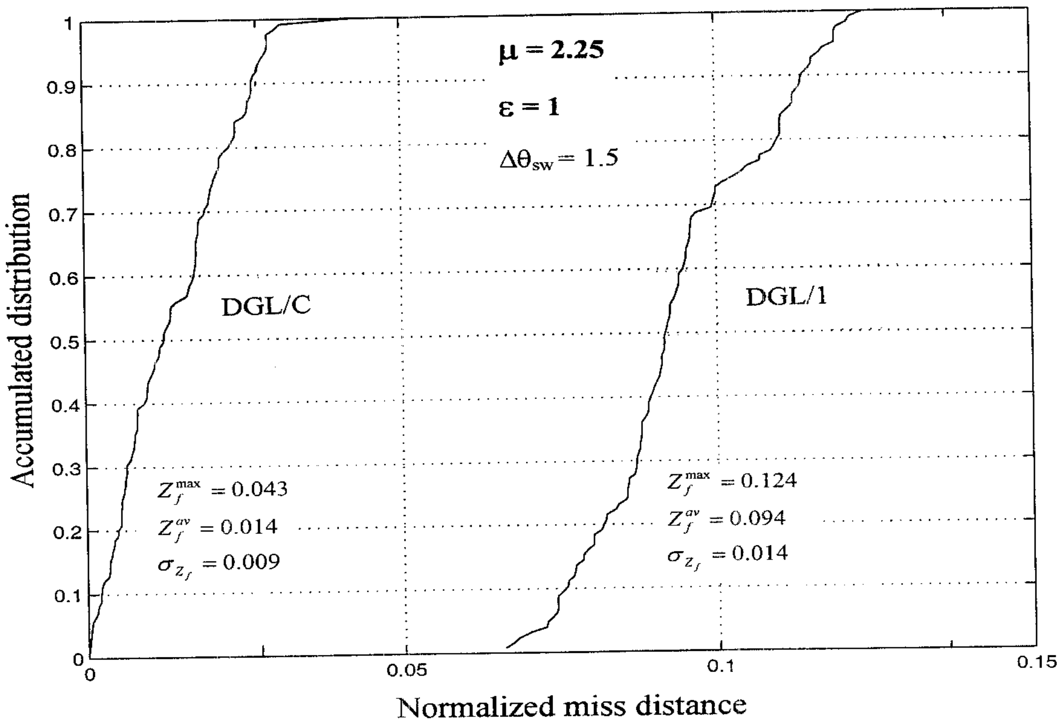

Fig. 14 Accumulated miss distance distribution DGL/C comparison to DGL/1