FIRING SEQUENCES ESTIMATION FOR TIMED PETRI NETS

Dimitri Lefebvre
LAMPS - ENIBelfort / GEII - IUT Belfort Montbéliard
Espace Bartholdi - BP 525 - Belfort Technopôle - 90016 Belfort Cedex, France
E-mail:dimitri.lefebvre@enibe.fr

Abstract: This work deals with the firing sequences estimation for transitions - timed Petri nets by measurement of the places marking. Firing durations are unknown, but supposed not to be null. In fact, the Petri net marking is measured, on line, with a sampling period Δt small enough such that each transition is fired, at the most, one time during Δt. The estimation problem has exact and approximated solutions that are described. Sufficient conditions are given on the accuracy of the marking measurement, such that the estimation of the firing sequences is an exact one. If the estimation provides several solutions, the Petri net is completed in order to give a unique solution.

Keywords: Timed Petri nets, manufacturing systems, estimation, firing sequences, Moore Penrose inverse.

1. Introduction

Manufacturing systems often considered as discrete events systems and thus are described with discrete time models (Cassandras 1993, Cao *et al* 1990). Among the existing models, the transitions - timed Petri nets (TPN) (David *et al* 1992, Ramchandani 1973), are well adapted to represent assembly, disassembly, and manufacturing workshops composed of buffers and machines. With TPN models, buffers are represented by places, and machines are represented by transitions. The places marking stands for the buffer contents or for resources allocation, the transitions firing sequences represent the routings, and, for each transition, the minimal firing duration corresponds to the machine operating time.

This work is concerned with the firing sequences estimation for TPN by observation of the marking. Minimal firing durations are supposed to be unknown, but transitions with instantaneous firing are avoided. Conflictual situations as resources allocation are also considered. The problem is different from Petri nets state estimation (Borne *et al* 1990, Giua 1997, Kailath 1980), where transitions firing are observed, and marking is estimated. The proposed estimation is useful to provide the firing frequencies without studying the marking invariance properties (David *et al* 1992). Applications of our results are identification of deterministic or stochastic TPN (David *et al* 1992), and faults diagnosis (Isermann 1984, Knapp *et al* 1992, Wang *et al* 1993, Zeng *et al* 1991).

The section two is about PN, and TPN. Our notations and hypothesis are also presented. In the section three, the estimation of the firing sequences is obtained from the places marking measurement. The problem is solved with linear systems inversion (Gantmacher 1966, Rotella *et al* 1995). The estimation is obtained with the help of the Moore Penrose inverse (Ben-Israel *et al* 1974, Campbell *et al* 1979, Rotella *et al* 1995), of the incidence matrix and with a binary classifier function. When the set of solutions contains several elements, the TPN is completed such that a unique solution does fit the problem. A sufficient condition is also given on the marking measurement accuracy, such that the estimation of the firing sequences is the exact one. The last section is an illustration of the previous results.

2. Timed Petri nets

A PN with n places and p transitions is defined as < P, T, Pre, Post, M_0 > where $P=\{P_i\}_{i=1,\ldots,n}$ is a not empty finite set of places, $T=\{T_j\}_{j=1,\ldots,p}$ is a not empty finite set of transitions, such that $P \cap T = \emptyset$. IN is defined as the set of integer numbers and IR as the set of real numbers. Pre: $P \times T \rightarrow IN$ is the pre-incidence application (Pre (P_i, T_j) is the weight of the bond from place P_i to transition T_j), Post: $P \times T \rightarrow IN$ is the post-incidence application (Post (P_i, T_j) is the weight of the bond from transition T_j to place P_i). Let us also define $M(t) = (m_i(t))_{i=1,\ldots,n} \in IN^n$ as the marking vector at time t and $M_0 \in IN^n$ as the initial marking vector. The PN incidence matrix W is defined as $W = (\text{Post}(P_i, T_j) - \text{Pre}(P_i, T_j))_{i=1,\ldots,n,\ j=1,\ldots,p} \in IN^{n \times p}$. A firing sequence *S* is defined as an ordered series of transitions that are successively fired from marking M to marking M':

$$M(S \rightarrow M' . \tag{1}$$

Such a sequence is represented by its characteristic vector $S = (s_j)_{j=1,\ldots,p} \in IN^p$ where s_j stands for the number of T_j firing. Equation (1) is equivalent to:

$$M' = M + W.S. \tag{2}$$

A TPN with n places and p transitions is defined as < PN, D_{min} > where PN is a Petri net, and $D_{min} = (d_{min\,j})_{j=1,\ldots,p} \in IR^{+\,p}$ a vector of positive non null transition minimal firing durations. The firing of transition T_j starts when T_j is enabled (there are enough parts in each upstream place), and ends after a positive non null duration equal to $d_{min\ j.}$ During the firing of T_j parts are reserved. Only non reserved parts are considered for enabling conditions. When two transitions T_j and $T_{j'}$ have a common upstream place, the TPN presents a structural conflict. The conflict becomes an effective one there are not enough parts in the common place to enable both. The conflict is solved according the TPN definition. Part in the common place are reserved for the firing of the transition with the smallest minimal firing duration.

Considering a sampling period Δt, $X_k = (x^k_j)_{j=1,\ldots,p} \in IN^p$ is defined as the characteristic vector of the firing sequence that occurs during [(k-1).Δt k.Δt[. Thus, considering $\Delta M_k = M(k.\Delta t) - M((k-1).\Delta t) \in IN^n$ as the variation of the marking during [(k-1).Δt k.Δt[, equation (2) results in:

$$\Delta M_k = W.X_k . \tag{3}$$

Let us assume that the sampling period Δt is small enough such that each transition of the PN could be fired, at the most, one time during [(k-1).Δt k.Δt[. With this restriction, $X_k \in \{0, 1\}^p$ with $x^k_j = 1$ if the transition T_j is fired during [(k-1).Δt k.Δt[, and $x^k_j = 0$ if not. Let us consider $B = \{B_j\}_{j=1,\ldots,p}$ with $B_j = (b^j_s)_{s=1,\ldots,p} \in IR^p$ such that $b^j_s = 0$ if $j \neq s$, and $b^j_j = 1$. B is the canonical basis of IR^p, and B_j represents the firing of transition T_j. Using the previous assumption, X_k belongs to a set composed of exactly 2^p different firing sequences, and can be written as in equation (4):

$$X_k = \sum_{j=1}^{p} x^k_j \, B_j \tag{4}$$

3. Estimation of the firing sequences

Let us define $\hat{M}_k = (\hat{m}^k_i)_{i=1,\ldots,n} \in IR^n$ as the approximated value of M(t) measured at each time $t = k.\Delta t$, $\Delta\hat{M}_k = \hat{M}_k - \hat{M}_{k-1} \in IR^n$ as the variation of the measured marking during [(k-1).Δt k.Δt[, and $\hat{X}_k = (\hat{x}^k_j)_{j=1,\ldots,p} \in IR^p$ as the corresponding approximated value of $X_{k.}$ Let us emphases the fact that $\hat{M}_k$ and $\Delta\hat{M}_k$ could contain integer values or real values according to the measurement sensor. But, in both cases the proposed approximation method results in vector $\hat{X}_k$ that contains real values. Equation (3) results in:

$$\Delta\hat{M}_k = W.\hat{X}_k . \tag{5}$$

Let us call r the rank of matrix $W \in IR^{n \times p}$. Let us also define h_k as the rank of matrix $(W, \Delta\hat{M}_k) \in IR^{n \times (p+1)}$, where matrix $(W, \Delta\hat{M}_k)$ stands for the aggregation of matrix W and vector $\Delta\hat{M}_k$.

To estimate the firing sequence characteristic vector, equation (5) has to be solved. This equation is considered as a set of n linear relations with a unknown vector $\hat{X}_k$ of dimension p. This equation may have one, several or no exact solution according to the values of n, p, r, and h_k (Gantmacher 1966, Rotella *et al* 1995). Let us define E_k as the set of solutions for equation (5) at time $t = k.\Delta t$, $(u_i)_{i=1\ldots p}$ as the column vectors of matrix W, and $Vect(W) = Vect\{u_1,\ldots u_p\}$ as the vector space defined by the vectors u_1 to u_p.

- When $r = h_k$, $\Delta\hat{M}_k \in Vect\{u_1,\ldots u_p\}$. The equation (5) has at least one exact solution, and E_k is defined as the set of exact solutions for (5). In this case the system is said to be compatible (Rotella *et al* 1995).
- When $r < h_k$, $\Delta\hat{M}_k \notin Vect\{u_1,\ldots u_p\}$. The equation (5) has no exact solution. But (5) has one or several approximated solutions. The vector $\hat{X}_k$ is called an approximated solution for equation (5) if it minimises the difference $\| \Delta\hat{M}_k - W.\hat{X}_k \|$ where $\| . \|$ stands for the Euclidean norm (Rotella *et al* 1995). In this case, E_k is defined as the set of approximated solutions for (5), and the system is said to be not compatible (Rotella *et al* 1995).

For all the cases, the solutions (the exact ones or the approximated ones) could be obtained with the same result. The set of solutions for (5) can be expressed with the Moore Penrose inverse of matrix W (Ben-Israel *et al* 1974, Campbell *et al* 1979, Rotella *et al* 1995). The Moore Penrose inverse of $W \in IR^{n \times p}$ is the unique matrix $W^+ \in IR^{p \times n}$, that verifies the properties $W.W^+.W=W$, $W^+.W.W^+= W^+$, $(W.W^+)^T=W.W^+$, and $(W^+.W)^T = W^+.W$.

Theorem 1: *For system (5), the set of solutions (exact or approximated) is given by:*

$$E_k = \left\{ \hat{X}_k \;\middle/\; \hat{X}_k = W^+.\Delta\hat{M}_k + (I_p - W^+W)z \right\}, \tag{6}$$

where I_p stands for the identity matrix of $IR^{p \times p}$, and z stands for any vector of IR^p. The solution $\hat{X}^0_k$ corresponding to z=0 is such that:

$$\left\| \hat{X}_k{}^0 \right\| = \inf_{\hat{X}_k \in E_k} \left\| \hat{X}_k \right\| . \tag{7}$$

The Moore Penrose inverse of matrix W can be worked out with a help of maximal rank factorisation of W.

Theorem 2: *Let be $W \in IR^{n \times p}$ of rank r. There exists two matrices $W_l \in IR^{n \times r}$ of full column rank r, and $W_r \in IR^{r \times p}$ of full row rank r, such that $W= W_l . W_r$. In this case the Moore Penrose inverse of matrix W is given by:*

$$W^+ = W_r{}^T (W_l{}^T.W.W_r{}^T)^{-1} W_l{}^T . \tag{8}$$

Theorem 3: *The dimension of the set E_k is given by p-r. When p = r the system (5) has a unique solution (exact or approximated).*

For the sake of brevity, the proof of theorems 1, 2 and 3 is omitted (Campbell *et al* 1979, Rotella *et al* 1995). From a numerical point of view, the maximal rank factorisation of matrix W is not easy to obtain. A more efficiency method consists to apply the Greville constructive algorithm (Ben-Israel *et al* 1974, Golub *et al* 1986, Rotella *et al* 1995).

Let us remind that the components of vector $\hat{X}_k$ are real values. To deal with firing sequences, and according to our assumptions the vector $\hat{X}_k$ has to be transformed into a vector $\overline{X}_k$ = $(\overline{x}^k{}_j)_{j=1,\dots,p} \in \{0, 1\}^p$. Let us define $\overline{x}^k{}_j$ as $\overline{x}^k{}_j = \text{sign}(\hat{x}^k{}_j - ½)$. The function sign is a binary classifier such that $\overline{x}^k{}_j = 1$ if $\hat{x}^k{}_j \geq ½$, and $\overline{x}^k{}_j = 0$ if $\hat{x}^k{}_j < ½$. We call the vector $\overline{X}_k$ as the estimated value of X_k.

Two problems may occur regarding the approximation $\hat{X}_k$.

- E_k could contain several solutions. In this case, some complementary information must be added to the PN in order to choice the good one. This difficulty will be studied in paragraph 3.1.
- E_k may also contain no exact solution. In this case, a sufficient condition is given on the marking measurement accuracy, such that $\overline{X}_k = X_k$. The approximation error is studied in section 3.2.

3.1. Petri net complement

When E_k contains several elements, the marking vector $\hat{M}_k$ does not contain enough information to work out an unique estimation of the firing sequences. In this case, one solution is to increase the dimension of the marking vector, by adding some new places to the PN such that the rank of matrix W will increase to p. The places must be located such that the rank of the incidence matrix increases.

Theorem 4: *When equation (5) has several solutions, the approximation vector $\hat{X}_k$ could be obtained by the addition of p - r complementary places located such that the rank of matrix W increases to p.*

Proof (constructive): The equation (5) has a unique solution if and only if r = p. When r<p, the addition of well chosen complementary rows to the matrix W increases the rank of W. Let us assume that $W \in IR^{n \times p}$ is of rank r<p and let us define w_i as the i^{th} row of matrix W. There exists $w_{n+1}{}^T \in IR^p$ such that $\text{rank}(w_1{}^T, \dots, w_n{}^T, w_{n+1}{}^T) = r+1$. Repeating the same operation p-r times, the rank of augmented matrix W increases to p. Each row defines the location of a new place in the Petri net. Thus the approximation of the vector X_k requires the addition of p-r places.

3.2. Estimation error

From a theoretic point of view the non-compatible case does not occur, because the equation (3) is always compatible. The marking vector results from the PN evolution, and considering any firing sequence X_k, the resulting vector ΔM_k belongs to Vect(W). Moreover, if equation (6)

provides several solutions, the TPN is completed according the result given by theorem 4, such that only a unique solution does fit the approximation problem. Thus X_k is unique.
From a numerical point of view, the compatibility of equation (5) is not warranted. Only an approximation $\Delta\hat{M}_k$ of ΔM_k is measured, and this approximation is not exact. In this case, $r<h_k$, and equation (5) has one or several approximated solutions according to the value of n, p and r. The vector $\Delta\hat{M}_k$ includes a measurement error e_k and for this reason may be out of Vect(W):

$$\Delta M_k = \Delta\hat{M}_k + e_k \, , \tag{9}$$

Equation (3) results in:

$$X_k = W^+.\Delta\hat{M}_k + W^+.e_k \tag{10}$$

where $\hat{X}_k = W^+.\Delta\hat{M}_k$ and $\varepsilon_k = W^+.e_k$ represents the influence of the measurement error on the approximation $\hat{X}_k$ of X_k.

Theorem 5: *The approximation* $\hat{X}_k$ *of* X_k *includes an error* ε_k *such that:*

$$\|\varepsilon_k\| \le \sqrt{\sigma}.\|e_k\| \, , \tag{11}$$

where σ *stands for the spectral radius of matrix* $(W^+)^T.W^+$ *(*σ *is the maximal module of the eigenvalues of matrix* $(W^+)^T.W^+$*).*

Proof: Let us mention that the Euclidean norm is a multiplicative norm, thus:

$$\|\varepsilon_k\| = \left\|W^+ e_k\right\| \le \left\|W^+\right\|.\left\|e_k\right\| . \tag{12}$$

The Euclidean norm of matrix W^+ results from the vectorial Euclidean norm:

$$\left\|W^+\right\| = \max_{\|x\|=1}\left\{\left\|W^+ x\right\| \right\}. \tag{13}$$

with $\|x\| = \sqrt{x^T x}$ and $\left\|W^+ x\right\| = \sqrt{x^T (W^+)^T W^+ x}$.

The matrix $(W^+)^T.W^+ \in IR^{n \times n}$ is a symmetric non negative matrix. There exists an orthogonal matrix $P \in IR^{n \times n}$ such that $(W^+)^T.W^+ = P^T.D.P$ where $D \in IR^{n \times n}$ is the diagonal matrix that contains the eigenvalues $\{d_i\}_{i=1,\ldots,n}$, of $(W^+)^T.W^+$. Thus:

$$\left\|W^+ x\right\| = \sqrt{x^T.P^T.D.P.x} = \sqrt{y^T.D.y} = \sqrt{\sum_{i=1}^{n} d_i y_i^2} \, ,$$

with $y = P.x = (y_i)_{i=1\ldots n}$. Let us notice that $\|x\| = 1 \Leftrightarrow x^T x = 1 \Leftrightarrow x^T P^T P x = 1 \Leftrightarrow y^T y = 1 \Leftrightarrow \|y\| = 1$. Calling σ as the maximal eigenvalue of $(W^+)^T.W^+$, we have:

$$\left\|W^+ x\right\| \le \sqrt{\sigma.\sum_{i=1}^{n} y_i^2} = \sqrt{\sigma}$$

and this maximal value is reached. Thus $\left\|W^{+}\right\| \le \sqrt{\sigma}$, and equation (11) holds.

The estimation of the measurement error obtained with the last result is composed of 2 terms: the term $\sqrt{\sigma}$ depends only from the structure of the PN, and the term $\|e_k\|$ depends only from the place marking measurement method. Because of the non linear sign function, it is not easy to work out the estimation error $\left\|X_k - \overline{X}_k\right\|$. But a result is proposed that gives a sufficient condition such that the estimation $\overline{X}_k$ of X_k has no error.

Theorem 6: *The estimation vector verifies* $\overline{X}_k = X_k$ *if the measurement error is such that:*

$$\left\|M_k - \hat{M}_k\right\| < \frac{1}{4.\sqrt{\sigma}}, \quad k \ge 0, \tag{14}$$

Proof: Let us assume that equation (14) holds. Thus:

$$\left\|M_k - \hat{M}_k\right\| + \left\|M_{k-1} - \hat{M}_{k-1}\right\| < \frac{1}{2.\sqrt{\sigma}}, \quad k \ge 1.$$

But $\left\|M_k - \hat{M}_k\right\| + \left\|M_{k-1} - \hat{M}_{k-1}\right\| \ge \left\|M_k - \hat{M}_k + \hat{M}_{k-1} - M_{k-1}\right\| = \left\|\Delta M_k - \Delta\hat{M}_k\right\| = \|e_k\|$.

Thus $\|e_k\|$ < 1/ (2.$\sqrt{\sigma}$), and according to theorem 5, for every j=1,…,p, we have:

$$\left|x_j^k - \hat{x}_j^k\right| \le \sqrt{\sum_{s=1}^{p}\left(x_s^k - \hat{x}_s^k\right)^2} < \frac{1}{2},$$

and $\hat{x}_j^k$ is bounded by:

$$x_j^k - \frac{1}{2} < \hat{x}_j^k < x_j^k + \frac{1}{2}. \tag{15}$$

According relation (15), and the definition of the estimation vector $\overline{X}_k$, if $x_j^k = 0$, $\hat{x}_j^k - 1/2 < 0$, and $\overline{x}_j^k = 0$. On the contrary, if $x_j^k = 1$, $\hat{x}_j^k - 1/2 > 0$, and $\overline{x}_j^k = 1$. Thus $\overline{X}_k = X_k$.

Let us notice that a sufficient condition to verify equation (14) is:

$$\left|m_i(k.\Delta t) - \hat{m}_i^k\right| < \frac{1}{4.\sqrt{n.\sigma}}, \quad k \ge 0, \quad i = 1,\dots,n, \tag{16}$$

where $\left|m_i(k.\Delta t) - \hat{m}_i^k\right|$ is the approximation marking error for place P_i at time t=k.Δt.

4. Example

Let us consider, as an example, the TPN in figure 1 (David, *et al* 1992) with 8 places and 4 transitions of unknown minimal firing durations d_1, d_2, d_3, and d_4 (not smaller than 1/6 second). The places P_5 to P_8 limit the number of simultaneous firing of transitions T_1 to T_4 to one. The system is working at maximal speed (i.e. parts are immediately reserved when a transition is enabled). This PN has a structural conflict because the place P_4 has two downstream transitions T_3 and T_4. The conflict becomes an effective one if there is a unique part in place P_4 and if the parts in P_7 and P_8 are both not reserved. In this case, the conflict is solved according the TPN definition. The part in place P_4 is reserved for the firing of T_4.Let us notice that the estimation procedure is not related to the conflict resolution rule, and does work for any other rule.

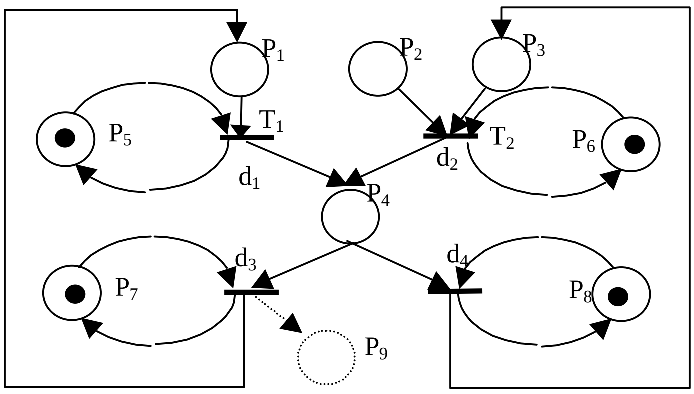

Figure 1: Example of TPN

The initial marking vector is given by M_0=(10 20 0 0 1 1 1 1)^T. There is no interest to consider the marking evolution of places P_5 to P_8 because the marking of these places is always equal to 1. Thus, only a reduced incidence matrix W is considered that defines the marking evolution of places P_1 to $P_{4.}$

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}_k = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}_{k-1} + \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}_k . \tag{17}$$

The matrix $W \in IR^{4 \times 4}$ is of rank 3. Thus E_k contains several solutions, and to estimate the vector X_k, the system (17) must be completed with another row that is independent from each other. Applying theorem 4, a possible solution is to add a new place P_5 to measure the flow coming from the transition T_3, such that the rank of W increases to 4 (this solution is represented with dashed point in figure 1). Equation (17) results in (18):

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{pmatrix}_k = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{pmatrix}_{k-1} + \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}_k \Rightarrow \hat{X}_k = \frac{1}{3} \begin{pmatrix} -2 & 0 & 1 & 1 & 3 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \\ -1 & -3 & 2 & -1 & 0 \end{pmatrix} \Delta\hat{M}_k \tag{18}$$

The resulting estimation equation is given by (18) and the marking evolution is given in figure 2. The estimation of the firing sequences is achieved by measurement of the marking vector with a sampling period Δt = 0.13 seconds. Let us notice that Δt < 1/6 second, that is to say Δt is small enough such that each transition of the PN could be fired at the most one time during [(k-1).Δt k.Δt[.

An uniformly distributed random error is considered. Thus, depending on the error, the vector $\Delta\hat{M}_k$ may belong or not in Vect(W), and the approximation $\hat{X}_k$ is either an exact solution, either an approximated one for equation (18). For transition T_4, the estimation $\overline{X}_k$, as the exact value of X_k are represented in figure 3, for a local measurement error of maximal value α = 0.05. Similar results are obtained for the transitions T_1 to T_3. One can notice that the estimation $\overline{X}_k$ corresponds exactly (with a delay) to the vector X_k.

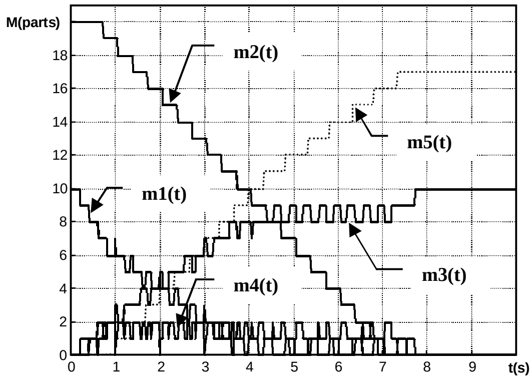

Figure 2: Marking evolution

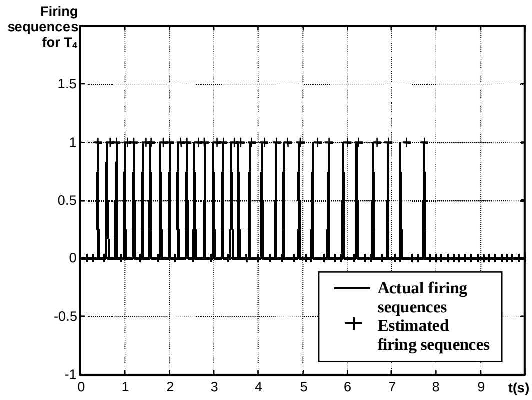

Figure 3: Estimation of X_k for T_4

The set of eigenvalues of $(W^+)^T.W^+$ is given by {0, 0.1529, 0.3820, 2.1805, 2.6180}. Thus σ=2.6180 and applying theorem 6, the maximal admissible measurement error such that $\overline{X}_k = X_k$ is given by equation (19):

$$\left\|M_k - \hat{M}_k\right\| < \frac{1}{4\sqrt{\sigma}} = \frac{1}{4\sqrt{2.6180}} = 0.15, \quad k \geq 0\,. \tag{19}$$

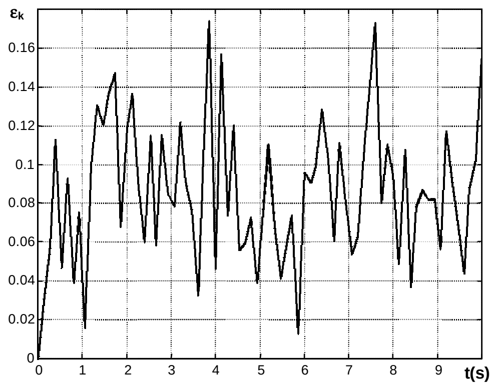

Figure 4: Error ε_k for α=0.05

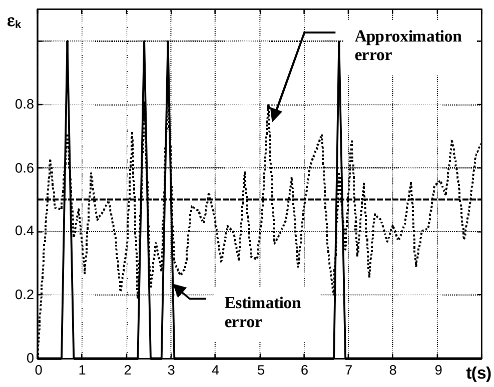

Figure 5: Error ε_k for and α=0.2

This condition is satisfied if each component of $\hat{M}_k$ verifies $\left|m_i^k - \hat{m}_i^k\right| < 0.051$. Considering, on one hand, the previous local measurement error of maximal value α = 0.05 for each place marking, the approximation error verifies ε_k < 0.5, as represented in figure 4, and $\overline{X}_k = X_k$. Considering, on the other hand, a local measurement error of maximal value α = 0.2 for each place marking, the accuracy of the measurement is not good enough, as represented in figure 5.

There exist several values of k such that $\varepsilon_k > 0.5$, that result in estimation $\overline{X}_k$ different from X_k. But there exist also values of k such that $\varepsilon_k > 0.5$, that result in $\overline{X}_k = X_k$. This illustrates the fact that the condition given in theorem 6 is a sufficient but not a necessary one.

It is also interesting to notice that decreasing the sampling period does not improve the firing sequences estimation. The sampling period must only verifies the hypotheses given in section two, that is to say, for this example, $\Delta t<1/6$ (smallest firing duration of transitions).

5. Conclusions

Using transitions - timed Petri nets for the modelling of manufacturing systems, an estimation of the firing sequences was proposed. This problem results in linear system inversion for which the Moore Penrose inverse of the incidence matrix has to be worked out. The set of solutions was described (theorems 1 to 3). When several solutions exist, the Petri net was completed with additional relations in order to provide a unique solution (theorem 4). A sufficient condition was given on the marking measurement accuracy, such that the estimation of the firing sequences is an exact one. (theorems 5 and 6). The proposed method is not limited to transitions – timed Petri nets. The same results hold in the cases of places – timed Petri nets, or stochastic – timed Petri nets.

Our further work is to apply the firing sequences estimation to determine firing occurrences, and firing frequencies. Applications like faults diagnosis and timed Petri nets identification will also be considered.

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