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Nonlinear State Estimation for Rigid Body Motion with Low-Pass Sensors *

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Abstract

In this paper we consider the state estimation problem for the nonlinear kinematic equations of a rigid body observed under low pass sensors. On the way to solve that problem, the convergence of a state estimator for a generic stable time-varying linear system is shown. The problem is motivated from a walking robot application where inclinometers and gyros are the sensors used. We show that a non local high gain observer exists for the nonlinear rigid body kinematic equations and that it under a small angle assumption is possible to use one inclinometer only to estimate two angles.

Keywords: nonlinear state estimation, rigid body motion, linear time-varying systems, exponential observers, inclinometers

1 Introduction

In this paper we consider the state estimation problem for the nonlinear kinematic equations of a rigid body observed under low pass sensors. On the way to solve that problem, the convergence of a state estimator for a generic stable time-varying linear system is shown. Such a problem arises in many applications, for example, a legged mobile robot. For a walking robot, of paramount importance from a control perspective is a reliable estimate of pitch and roll. One kind of sensors that are often used for this kind of application are rate gyroscopes (gyros) measuring angular velocities. Even though gyros often have excellent bandwidth, the integrated signal providing angle measurements will never be reliable over an extended period of time. One type of sensors that provide an absolute angle reference are inclinometers. With prefiltering inclinometers can be modeled as low pass filter.

As is well known, the kinematics for rigid body are nonlinear. In this paper we consider first the state estimation problem where both pitch and roll are measured by inclinometers. We show that in this case, for an operating range that is reasonable for applications such as a walking robot, an observer with exponentially decaying error exists. We should emphasize here that this is not a local result. Then we consider the case where only one of the inclinometers is available. This is naturally more difficult and we show that for small angles there exists an exponential observer provided the system is "properly" excited. We should also point out that in the scenario we consider, the linearized version of the kinematics will be time-varying. Finally we illustrate the results with some simulations.

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2 Problem formulation

Consider the kinematics for a rotating rigid body which can be found in standard textbooks [Isidori(1995)] and are

$$\dot{R} = S(\omega)R \tag{1}$$

where $R \in SO(3)$ is the coordinate transformation relating a body fixed (B) frame to an inertial system (N) according to $x^B = Rx^N$. $S(\omega)$ is the scew-symmetric wedge matrix

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}$$

where ω_i is the components of the angular velocity vector expressed in the body fixed frame. A parameterization of $SO(3)$ suitable for this application is the yaw-pitch-roll (ψ, θ, ϕ) parameterization

$$R = \begin{bmatrix} \mathrm{c}\psi\mathrm{c}\theta & \mathrm{s}\psi\mathrm{c}\theta & -\mathrm{s}\theta \\ -\mathrm{s}\theta\mathrm{c}\theta + \mathrm{c}\psi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{c}\psi\mathrm{c}\phi + \mathrm{s}\psi\mathrm{s}\theta\mathrm{s}\phi & \mathrm{c}\theta\mathrm{s}\phi \\ \mathrm{s}\psi\mathrm{s}\phi + \mathrm{c}\psi\mathrm{s}\theta\mathrm{c}\phi & \mathrm{s}\psi\mathrm{s}\theta\mathrm{c}\phi - \mathrm{c}\psi\mathrm{s}\phi & \mathrm{c}\theta\mathrm{c}\phi \end{bmatrix}$$

defining local coordinates around $R = I$. This parameterization is suitable for many applications as the angles have an intuitive meaning and also, typical motions for mobile land robots are such that $|\theta|, |\phi| < \frac{\pi}{2}$ for which the parameterization is unique. In these coordinates the kinematics (1) are given by

$$\begin{aligned} \dot{\theta} &= \cos\phi\omega_2 - \sin\phi\omega_3 \\ \dot{\phi} &= \omega_1 + \sin\phi\tan\theta\omega_2 + \cos\phi\tan\theta\omega_3 \end{aligned}$$

where we only consider pitch (θ) and roll (ϕ).

The sensors at hand are the rate gyro measuring the angular velocities ω and the inclinometers measuring θ and ϕ for which we use a first order model

$$\dot{y}_1 = \tau_1(\theta - y_1) \tag{2}$$

$$\dot{y}_2 = \tau_2(\phi - y_2) \tag{3}$$

where $\tau_i = 1/T_i$ is the inverse time constant of the inclinometer. Introducing $x_1 = [\theta, \phi]^T$ and $x_2 = [y_1, y_2]^T$ the system can be written as

$$\begin{aligned} \dot{x}_1 &= m(x_1)\omega \\ \dot{x}_2 &= \tau x_1 - \tau x_2 \\ y &= Cx \end{aligned} \tag{4}$$

where

$$m(x_1) = \begin{bmatrix} 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \end{bmatrix}$$

and

$$\tau = \mathrm{diag}(\tau_1, \tau_2)$$

and

$$C = [0 \quad I]$$

In this paper we consider the problem of reconstructing the pitch and roll from sensor measurements. Through out the paper, we assume that the measurements for ω are accurate enough, therefore they will be considered as known time-varying inputs to the pitch and roll equations.

3 Observers with two inclinometers

In this section we construct an observer for the case where both pitch and roll are measured. Using a high gain approach we show how to design an exponential observer that covers the range up to $\frac{\pi}{6}$ for the absolute value of the pitch angle. This is of course not a local result.

It will prove necessary to introduce bounds on the pitch angle θ. For our walking robot that is under development now [Wallentin *et al.*(1998)Wallentin, Jansson, and Andersson], we expect our control algorithms would keep the pitch in the range $-\frac{\pi}{6} < \theta < \frac{\pi}{6}$. The sensor time constants $\frac{1}{\tau_i}$ are the results of prefiltering and can be chosen as design parameters. We make the following assumption.

Assumption 3.1

$$\begin{array}{rcll} |\theta(t)| & \leq & \theta_b, & \theta_b = \frac{\pi}{6} - \frac{\delta}{3} \\ |\omega_i(t)| & \leq & \omega_i^m, & i = 2,3 \\ \tau_i & \leq & 2, & i = 2,3 \end{array} \tag{5}$$

where δ can be any small positive constant.

Theorem 3.1 *Consider a Luenberger type observer for (4)*

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, \omega) + L(y - C\hat{x}) \\ \hat{x}(0) &= \begin{bmatrix} 0 \\ y(0) \end{bmatrix} \end{aligned} \tag{6}$$

and let Assumption 3.1 be valid. Then, for each $\delta > 0$ there is an L such that the observer (6) is an observer with exponential error decay and the error tends to zero for all initial $|\theta(0)| \leq \theta_b$.

Proof
Let $L = [L_1^T \; L_2^T]^T$. Consider the error dynamics

$$\begin{aligned} \dot{\tilde{x}}_1 &= \tilde{m}(x_1, \hat{x}_1, \omega) - L_1 \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= \tau \tilde{x}_1 - (\tau + L_2)\tilde{x}_2 \end{aligned} \tag{7}$$

where

$$\tilde{x}_1 = \begin{bmatrix} \tilde{\theta} \\ \tilde{\phi} \end{bmatrix}$$

$$\tilde{x}_2 = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix}$$

are the errors and where the nonlinear terms are given by

$$\begin{aligned} \tilde{m}(x_1, \hat{x}_1, \omega) &= \begin{bmatrix} \cos\hat{\phi} - \cos\phi \\ \sin\hat{\phi}\tan\hat{\theta} - \sin\phi\tan\theta \end{bmatrix} \omega_2 \\ &+ \begin{bmatrix} -\sin\hat{\phi} + \sin\phi \\ \cos\hat{\phi}\tan\hat{\theta} - \cos\phi\tan\theta \end{bmatrix} \omega_3 \end{aligned}$$

The error dynamics (7) can be written

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + \begin{bmatrix} \tilde{m}(x_1(t), \hat{x}(t), \omega(t)) \\ 0 \end{bmatrix}$$

If we denote the transition matrix associated with A by

$$\Phi(t) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{8}$$

and recall that

$$\tilde{x}_2(0) = 0. \tag{9}$$

then by integrating (7) and using (8) and (9) we can write

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}(t) = \begin{bmatrix} \Phi_{11}(t)\tilde{x}_1(0) \\ \Phi_{21}(t)\tilde{x}_1(0) \end{bmatrix} + \int_0^t \begin{bmatrix} \Phi_{11}(t-s)\tilde{m} \\ \Phi_{21}(t-s)\tilde{m} \end{bmatrix} ds \tag{10}$$

It is now straightforward to show that if we take

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 0 \\ 0 & -c_2 \\ -\tau_1 - l_1 & 0 \\ 0 & \tau_2 - l_2 \end{bmatrix}$$

and choose $l_i = 3l$ and $c_i = -2l^2/\tau$, for some $l > 0$ then

$$\Phi_{11} = (2e^{-lt} - e^{-2lt})I$$

$$\Phi_{21} = \frac{1}{l}(e^{-lt} - e^{-2lt})\mathrm{diag}(\tau_1, \tau_2)$$

which are bounded by

$$||\Phi_{11}(t)|| \le 2e^{-lt}$$
$$||\Phi_{21}(t)|| \le \max(\tau_1, \tau_2)e^{-lt}, \quad l > 1$$

where the norm used is the maximum norm.

To establish bounds on $\tilde{m}$, we use the mean value theorem stating that, for C^1-functions, $g(x+a) - g(x) = g'(x + \alpha a)a$ where $\alpha \in [0,1]$. For $\tilde{m}$ this gives

$$||\tilde{m}|| = (|\omega_2^m| + |\omega_3^m|)\max\Big(\tan(\theta + \alpha\tilde{\theta}), \frac{1}{\cos^2(\theta + \alpha\tilde{\theta})}\Big)||\tilde{x}_1(t)||$$

for some $\alpha \in [0,1]$. Now, assume that

$$|\tilde{\theta}(t)| \le \frac{\pi}{2} - \frac{2\delta}{3}. \tag{11}$$

It then holds that $|\theta + \alpha\tilde{\theta}| < \frac{\pi}{2} - \delta$ and thus

$$\begin{aligned} ||\tilde{m}|| &\le (|\omega_2^m + \omega_3^m|)\max\Big(\tan(\tfrac{\pi}{2} - \delta), \tfrac{1}{\cos^2(\pi/2-\delta)}\Big)||\tilde{x}_1(t)|| \\ &= K||\tilde{x}_1(t)|| \end{aligned}$$

where $e(t) = ||\,[\,\tilde{x}_1^T \quad \tilde{x}_2^T\,]\,||^T$. Using these bounds we get from the error equations (7) that

$$e(t) \le Me^{-lt}e(0) + \int_0^t Me^{-l(t-s)}Ke(s)ds$$

where $M = \max(\tau_1, \tau_2, 2)$ and $e(t) = ||\,[\,\tilde{x}_1^T \quad \tilde{x}_2^T\,]^T\,||$. Defining $p(t) = e^{lt}e(t)$ it holds that

$$p(t) \le Me(0) + \int_0^t Me(s)ds$$

and with Gronwall-Bellmans lemma we obtain

$$p(t) \le Me(0)e^{MKt}$$

We now have an exponential bound on the errors

$$e(t) \le Me(0)e^{-(l-MK)t}. \tag{12}$$

and for $l > MK$, the errors asymptotically tend to zero. To prove that the region $|\theta| < \frac{\pi}{6} - \frac{\delta}{6}, \quad |\tilde{\theta}| < \frac{\pi}{3} - \frac{2\delta}{3}$ is an invariant set for the observer (6) we note that our choice of initial guesses and the θ-bound gives $|e(0)| \le \frac{\pi}{6} - \frac{\delta}{3}$, which in consequence guarantees that $|\hat{\theta}| \le \frac{\pi}{3} - \frac{2\delta}{3} + \frac{\pi}{6} - \frac{\delta}{3} = \frac{\pi}{2} - \delta$ and thus (11) holds.

4 Observers with one inclinometer

In this section we consider the problem of reconstructing both pitch and roll in the case where only one inclinometer is available. Without loss of generality, we assume the inclinometer for pitch is available. Solving this problem would be interesting if one of the inclinometers becomes unreliable due to malfunction, or as a result of impact in one direction.

In this paper we consider only the case of small angles. Then the nonlinear dynamics can be simplified as

$$\begin{array}{rcl} \dot\theta & = & -\omega_3(t)\phi + \omega_2 \\ \dot\phi & = & \omega_3(t)\theta + \omega_1 \\ \dot y & = & \tau\theta - \tau y \end{array} \tag{13}$$

which is linear but time varying.

We show now that under some rather mild conditions there exist exponential observers for the system.

Let us first rewrite (13) into the matrix form:

$$\begin{array}{rcl} \dot x & = & A(t)x + B\omega \\ y & = & Cx \end{array}$$

where $x = [\theta, \phi, y]^T$, $C = [0\ 0\ 1]$ and

$$A(t) = \begin{bmatrix} 0 & -\omega_3(t) & 0 \\ \omega_3(t) & 0 & 0 \\ \tau & 0 & -\tau \end{bmatrix} \tag{14}$$

Theorem 4.1 *Suppose $\omega_3(t)$ is such that the following observability condition is satisfied:*

$$\int_0^T \Phi^*(t+s,t)C^*C\Phi(t+s,t)ds \geq \epsilon I, \tag{15}$$

for some $\epsilon > 0$, $T > 0$ and any $t \geq 0$ where $\Phi(t,s)$ is the transition matrix of $A(t)$. Then there exists an exponential observer in the following form:

$$\dot{\hat x} = A(t)\hat x - L(t)(C\hat x - y(t)) \tag{16}$$

Remark: (15) is fulfilled for example by constant $\omega_3 \neq 0$ and for $\omega_3 = \sin(t)$.
Before we give a constructive proof for the theorem, we have to show a general result for linear time-varying systems, which, we believe, is new and interesting in its own right.

Proposition 4.1 *Consider*

$$\begin{array}{rcl} \dot x & = & A(t)x \\ y & = & Cx \end{array}$$

If there exists a $P(t)$, $0 < mI \leq P(t) \leq MI$ such that

$$A^*P + PA + \dot P \leq 0$$

and there exist $\epsilon > 0$ and $T > 0$ such that for any $t \geq 0$

$$\int_0^T \Phi^*(t+s,t)C^*C\Phi(t+s,t)ds \geq \epsilon I,$$

where $\Phi(t,s)$ is the transition matrix of $A(t)$, then

$$\dot{\hat x} = A(t)\hat x - P(t)^{-1}C^*(C\hat x - y(t))$$

is an observer with exponentially decaying error.

We will prove the proposition at the end of the section. Let us first use it to show Theorem 4.1.

Proof of Theorem 4.1:
For system (13), let

$$P(t) = I + \begin{bmatrix} \alpha\alpha^T & -\alpha \\ -\alpha^T & 0 \end{bmatrix} \tag{17}$$

where $\alpha(t)$ satisfies

$$\begin{aligned} \dot{\alpha} &= \begin{bmatrix} -\tau & -\omega_3(t) \\ \omega_3(t) & -\tau \end{bmatrix} \alpha + \begin{bmatrix} \tau \\ 0 \end{bmatrix} \\ \alpha(0) &= 0 \end{aligned} \tag{18}$$

Then a trivial calculation gives that

$$V(x,t) = x^T P(t) x \tag{19}$$

is a Lyapunov function for (13), or in other words,

$$\dot{P}(t) + A(t)P(t) + P(t)A^T(t) = Q \leq 0$$

Now we only need to show $P(t)$ is bounded below and above:

$$mI < P(t) < MI$$

To show this, note that (18) has a Lyapunov function $V_1(x,t) = ||x||^2$ with $\dot{V}_1 = -2\tau||x||^2$ and thus is exponentially stable. Therefore α is bounded and as a consequence, $P(t)$ is bounded above.
To show that $P(t)$ is bounded below we consider the eigenvalues. It is tedious but straightforward to find the minimal eigenvalue

$$\lambda_{min} = 1 + \frac{||\alpha||^2 - \sqrt{||\alpha||^4 + 4||\alpha||^2}}{2}$$

for which it holds that $\lambda_{min} > 0$ and for bounded α, $\lambda_{min} \geq \epsilon > 0$.
Therefore, the hypotheses of Proposition 4.1 are satisfied and if we take $L(t) = P(t)^{-1}C^*$, (16) in Theorem 4.1 is an exponential observer, provided the observability condition is also fulfilled.
Now we prove Proposition 4.1. Before we proceed with the proof, we need the following two lemmas [Brocket(1970)].

Lemma 4.1 *Let $K(\cdot) \in L_\infty(0,+\infty)$ be $n \times n$-matrix function. Consider the system*

$$\dot{z} = K(t)z. \tag{20}$$

Assume that

$$\int_{t_0}^{\infty} |z(t)|^2 \, dt \leq c^2 \, |z(t_0)|^2 \tag{21}$$

for any solution $z(\cdot)$ of (20) and any $t_0 \geq 0$ with the constant $c > 0$ being independent of $z(\cdot)$ and t_0.
Then and only then,

$$|z(t)| \leq b|z(t_0)|e^{-r(t-t_0)}$$

for some $b > 0$, $r > 0$.

Lemma 4.2 *Let $\phi(\cdot) \in L_2([t_0,+\infty) \to R^n)$ and $\theta > 0$. Denote*

$$\overline{\phi}(t) := \int_0^\theta \phi(t+s) \, ds \quad \forall t \geq t_0. \tag{22}$$

Then $\overline{\phi}(\cdot) \in L_2([t_0,+\infty) \to R^n)$ and

$$\left|\overline{\phi}(\cdot)\right|_2 \leq \theta|\phi(\cdot)|_2. \tag{23}$$

Proof: By using the Cauchy-Schwartz inequality, we get

$$|\overline{\phi}(t)| \leq \sqrt{\theta}\left(\int_t^{t+\theta} |\phi(s)|^2\, ds\right)^{1/2}.$$

Hence,

$$\int_{t_0}^{\infty} \left|\overline{\phi}(t)\right|^2\, dt \leq \theta \int_{t_0}^{\infty} dt \int_t^{t+\theta} |\phi(s)|^2\, ds = \theta \int_{t_0}^{\infty} |\phi(s)|^2\, ds \int_{s-\theta}^{s} dt = \theta^2 |\phi(\cdot)|_2^2.$$

Thus, $\overline{\phi}(\cdot) \in L_2$ and (23) is true.
Now we are ready to prove the proposition.
Proof: Denote

$$z = x(t) - \hat{x}.$$

Then

$$\dot{z} = Az - P^{-1}C^*Cz \tag{24}$$

and

$$\frac{d}{dt}(z^*Pz) = z^*\left(PA + A^*P + \dot{P}\right)z - 2z^*C^*Cz \leq -2|Cz|^2.$$

So, for any two instants $t \geq t_0 \geq 0$,

$$z(t)^*P(t)z(t) \leq z(t_0)^*P(t_0)z(t_0) - 2\int_{t_0}^{t} |Cz(s)|^2\, ds,$$

Therefore

$$z(t)^*P(t)z(t) \leq z(t_0)^*P(t_0)z(t_0) \leq M|z(t_0)|^2,$$

$$2\int_{t_0}^{t} |Cz(s)|^2\, ds \leq M|z(t_0)|^2.$$

By assumption $a^*Pa \geq m|a|^2$. So we have

$$m|z(t)|^2 \leq z(t)^*P(t)z(t) \leq M|z(t_0)|^2,$$

$$|z(t)| \leq \sqrt{\frac{M}{m}}|z(t_0)|, \tag{25}$$

$$\int_{t_0}^{\infty} |Cz(s)|^2\, ds \leq \frac{M}{2}|z(t_0)|^2, \tag{26}$$

Solving the error equation we have

$$z(t+s) = \Phi(t+s,t)z(t) - \underbrace{\int_t^{t+s} \phi(t+s,\theta)\zeta(\theta)\, d\theta}_{\varphi_s(t)}. \tag{27}$$

where $\zeta(t) = P^{-1}C^*Cz(t)$. Let $t, s \geq 0, s \leq T$ where T is the constant from the assumption. Since the matrix $A(t)$ is Lyapunov stable, $\|\Phi(t+\tau,t)\| \leq \alpha < \infty$ for all $\tau \geq 0$. So, in (27),

$$|\varphi_s(t)| \leq \underbrace{\alpha \int_t^{t+T} |\zeta(\theta)|\, d\theta}_{\chi(t)} \tag{28}$$

where, by Lemma 4.2,

$$|\chi(\cdot)|_2 \leq \alpha T|\zeta(\cdot)|_2 \leq \alpha\frac{1}{m}\|C^*\|\sqrt{\frac{M}{2}}T\,|z(t_0)|. \tag{29}$$

Now we get

$$\int_0^T |C\Phi(t+s,t)z(t)|^2 \ ds = \int_0^T |C(z(t+s)+\varphi_s(t))|^2 \ ds$$
$$\leq 2\int_0^T |Cz(t+s)|^2 \ ds + 2\int_0^T |C\varphi_s(t)|^2 \ ds$$

Then,

$$\begin{aligned}\int_{t_0}^{\infty} dt \int_0^T |C\Phi(t+s,t)z(t)|^2 \ ds &\leq 2\int_{t_0}^{\infty} dt (\int_0^T |Cz(t+s)|^2 \ ds + \int_0^T |C\varphi_s(t)|^2 \ ds) \\ &\leq 2\int_{t_0}^{\infty} |Cz(r)|^2 dr \int_{max\{t_0, r-T\}}^{r} dt + 2\int_{t_0}^{\infty} T\|C\|^2 \chi(t) dt \\ &\leq TM(1+\frac{T^2}{m^2}\alpha^2\|C\|^4)|z(t_0)|^2.\end{aligned}$$

On the other hand,

$$\int_{t_0}^{\infty} dt \int_0^T |C\Phi(t+s,t)z(t)|^2 \ ds = \int_{t_0}^{\infty} \left(z(t)^* \left[\int_0^T \Phi^*(t+s,t)C^*\Phi(t+s,t)C \ ds\right] z(t)\right) \ dt.$$

Then the observability hypothesis yields

$$\int_{t_0}^{\infty} dt \int_0^T |C\Phi(t+s,t)z(t)|^2 \ ds \geq \varepsilon \int_{t_0}^{\infty} |z(t)|^2 \ dt.$$

Thus,

$$\int_{t_0}^{\infty} |z(t)|^2 \ dt \leq \frac{TM}{\varepsilon}\left(1+\frac{T^2}{m^2}\alpha^2\|C\|^4\right)|z(t_0)|^2.$$

This means that the system (24) satisfies the hypotheses of Lemma 4.1. In other words, the assumptions of Lemma 4.1 are fulfilled with respect to the matrix-function:

$$K(t) := A(t) - P(t)^{-1}C^*C. \tag{30}$$

Therefore, the error tends to zero exponentially.

5 Simulations

5.1 High gain state estimation for rigid body motion.

To illustrate the results in section 3, consider a rigid body (4) equipped with the sensors described in section 2. Let it be subject to the angular velocities $\omega = [1\sin(2\pi t)\ 0.7\sin(\pi t)\ 7\sin(6\pi t)]$. Let the time constants be given by $\tau_i = 1$ and take for instance $\delta = 0.15$ in Assumption 3.1. Then it holds (12) that $l > 690$ guarantees convergent estimates. Let the initial state be given by $[\theta, \phi, y_1, y_2](0) = [\pi/6 - 1.1\delta, \pi/8, 0, 0]$. As can be seen in figure 1, our choice of initial values and angular velocities give pitch angles within the prescribed bounds. From the figure it is also obvious that the inclinometer output is not suitable for control purposes.
Applying the proposed high gain observer, the errors converge to zero as expected. For this case the convergence is considerably faster than the bound (12) which also is shown in the figure.

5.2 State Estimation with one inclinometer

In section 4 it was shown that, for small angles it is possible to estimate pitch and roll using only an inclinometer for the pitch angle. To illustrate this, we consider a motion generated by $[\theta\ \phi\ y_1](0) = [\frac{\pi}{8}\ \frac{-\pi}{8}\ 0]$ and $\omega = [0\ 0\ \sin t]$. In figure 2, pitch, roll, estimates and errors are given. The observer is seen to converge.

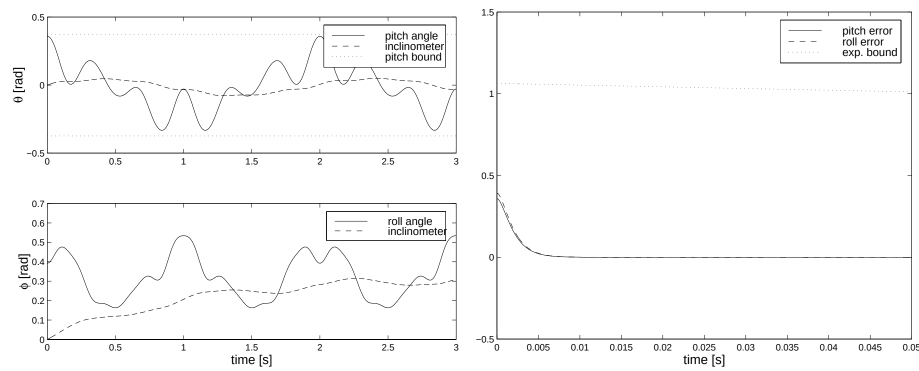

Figure 1: Estimation with two inclinometers.

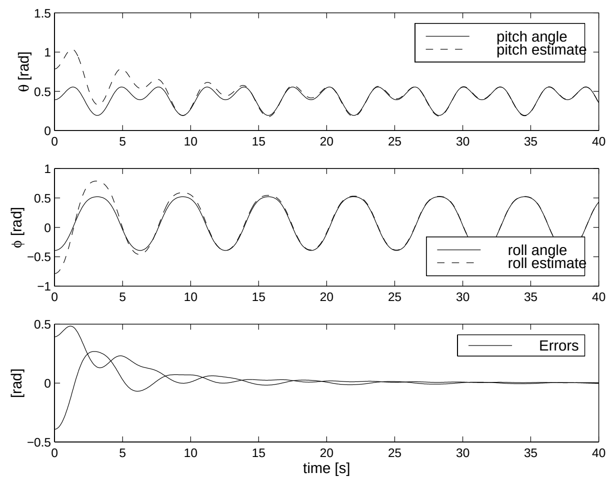

Figure 2: Estimation with one inclinometer

6 Summary

We have studies the state estimation problem for rigid bodies where the sensors used are inclinometers and gyros. The existence of an exponential observer of high gain type is shown. If one inclinometer is out of order or unreliable due to for example a sudden impact it is still possible to estimate pitch and roll given small angles and strong observability. Future work amounts to considering other observers such as extended Kalman filters and to build an experimental sensor platform and study the algorithms performance in the real world. Inclinometers are sensitive to translational accelerations so it is likely that a switching scheme based on acceleration triggering has to be considered. Finally the sensor system will be implemented on a walking robot.

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