Proceedings of the 7th Mediterranean Conference on Control and Automation (MED99) Haifa, Israel - June 28-30, 1999

FLIGHT CONTROL DESIGN FOR A MISSILE. AN APPROXIMATE FEEDBACK LINEARIZATION APPROACH

Antonios Tsourdos* Anna L.Blumel†

Brian A.White‡

Department of Aerospace, Power and Sensors
Royal Military College of Science
Cranfield University
Shrivenham,SN6 8LA
England,UK

Abstract

Input-output approximate linearisation of a non-linear sixth order system has been studied. A method for controlling the non-linear system that is i/o linearisable is examined that retains the order and the relative degree of the system in the linearisation process, hence producing a linearised system with no internal or zero dynamics. Desired tracking performance for lateral accelerations and roll rate of the missile is achieved by using a non-linear control law that has been derived by selecting the lateral velocities and roll rate as the linearisation outputs. Simulation results are shown that exercise the final design and show that the linearisation and controller design are satisfactory.

1 Introduction

One of the main steps in designing a control system for a given physical plant is to derive a meaningful model of the plant, i.e. a model that captures the key dynamics of the plant in the operational range of interest. Models of physical systems come from various forms, depending on the modelling approach and assumptions. Some forms, however, lend themselves more easily to controller design.

A technique for transforming original system models into equivalent models of a simpler form is the so called Jacobian linearization or linearization about an equilibrium point.In this case it can be said that the linearization may not be a good approximation to the system for arbitrary configurations. Since the system is linearized about a single point, trajectory tracking can only be guaranteed in a sufficiently small ball of states about that point. There are several methods for circumventing this problem ; one of the most common is gain scheduling (Shamma *et al.*, 1990). To use gain scheduling, tracking controllers are designed for many different equilibrium points and gains are chosen based on the equilibrium points to which the system is nearest.

*Email: A.Tsourdos@rmcs.cranfield.ac.uk
†Email: A.L.Blumel@rmcs.cranfield.ac.uk
‡Email: B.A.White@rmcs.cranfield.ac.uk

An alternative technique is the nonlinear dynamic inversion or feedback linearization. Feedback linearization (FLN) deals with techniques for transforming original system models into equivalent models of a simpler form. FLN can be used as a nonlinear design methodology.

The use of input-output linearization techniques to produce a linear system with specified uncertainty has been examined. The main idea of i/o linearization is to algebraically transform a non-linear system into a linear form using state feedback (Isidori *et al.*, 1981; Hunt and Su , 1981; Su , 1982). There are few examples in the literature of the practical application of feedback linearisation (Hahn *et al.*, 1994), (Henson and Seborg , 1990). Applications to aerospace systems are also rare (Bezick *et al.*, 1995).

This paper looks at the application of feedback linearisation of a missile model that is described by look-up tables that define non-linear characteristics of the aerodynamics. One of the main problems with applying feedback linearisation techniques is that the process produces a system with the same relative degree as the original system, but usually with an order that is less. Indeed, the linearised system order is the same as the relative degree unless pre-compensators are used to artificially change the order and relative degree. This process results in zero or internal dynamics, which are modes that are effectively rendered unobservable by the linearisation process. If the system is non-minimum phase, then the zero dynamics are unstable. The analogy with linear systems is that a zero-pole system is linearised into an all-pole system by selecting the pole-zero excess as the order of the approximating system. In order to produce linearised systems that have no internal dynamics, techniques which preserve the dynamic order of the system are needed.

Several approaches are possible to the avoidance of internal or zero dynamics. One approach is to neglect terms in input derivatives until the required system order is reached (Hauser *et al.*, 1992). Another is to pre-compensate the system to increase the system relative degree artificially, and thus having some limited authority over the stability of the internal dynamics (Slotine and Li , 1991). Designing systems with unstable zero dynamics can also be achieved (Lu *et al.*, 1997) provided the input to the system remains bounded under feedback. A fourth way is to choose an output which has the required relative degree, and which is related to the required control output in some manner. The approach used in this paper is a combination of the first two (Tsourdos *et al.*, 1998): to select an output that relates to the variable that is to be controlled, but which gives a greater relative degree, and to neglect small terms that allow the final relative degree to be achieved.

The aim of this paper is to track the missile lateral acceleration demand in both the pitch and yaw plane as well as the roll rate in the roll plane, using the missile aileron, rudder and elevator; hence yielding a system with 3 inputs and 3 controlled outputs. The tracking and non-linear controllers are designed by defining lateral velocity as an output as it produces a higher relative degree than directly controlling lateral acceleration, which has a relative degree of zero. Lateral velocity is directly related to the lateral acceleration, as in steady state a constant incidence angle is associated with a constant lateral acceleration. The basic system is fifth order, with an integrator in front of the roll channel yielding a sixth order system.

2 HORTON Missile model

Data in the form of parametric relationship have been supplied by BADL and are described in (White , 1998). The lateral motion (Tsourdos *et al.*, 1998) is derived from the model defined in (Horton , 1992), while the roll model is derived from graphical relationships relating the moments generated by airelon, rudder and elevator action of the cruciform fin configuration.

These relationships are used to generated a 'parametric model' that is used for simulation and analysis. From this model a polynomial model was produced (White , 1998) to match the parametric model as close as possible in a least squares sense. This polynomial model is in form of polynomial relationships that are used for control synthesis. The description of the HORTON model was obtained from data supplied by Marta-BAe and detailed in (Horton , 1992).

The system in polar coordinates (with $z = \sqrt{(v^2 + w^2)}$ and $\lambda = arctan\frac{v}{w}$) is described as:

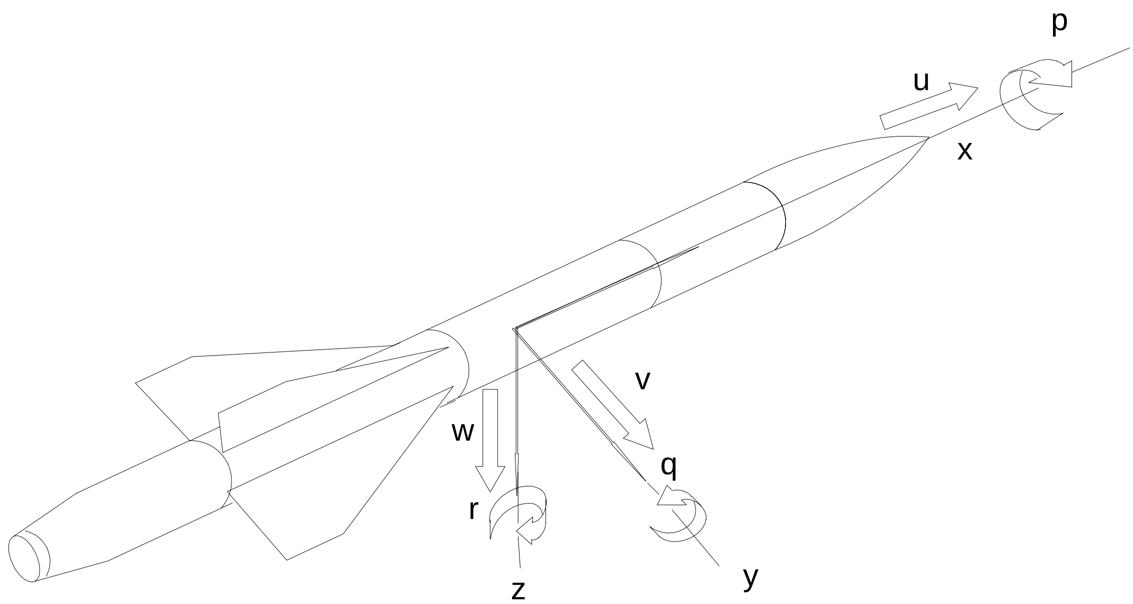

Figure 1: Airframe axes

$$\begin{aligned}
\dot{r} &= \frac{1}{2} I_z^{-1} \rho V_o S d(\frac{1}{2} d C_{n_r} r + C_{n_z} z + V_o C_{n_\xi} \xi + V_o C_{n_\zeta} \zeta) \\
\dot{q} &= \frac{1}{2} I_y^{-1} \rho V_o S d(\frac{1}{2} d C_{m_q} q + C_{m_z} z + V_o C_{m_\xi} \xi + V_o C_{m_\eta} \eta) \\
\dot{p} &= \frac{1}{2} I_x^{-1} \rho V_o S d(d C_{l_p} p + V_o C_{l_\zeta} \zeta + V_o C_{l_\eta} \eta + V_o C_{l_\xi} \xi) \\
\dot{z} &= \frac{1}{2} m^{-1} \rho V_o S (C_{yz_z} z + V_o C_{yz_\eta} \eta + V_o C_{yz_\zeta} \zeta) + u(\cos(\lambda) q - \sin(\lambda) r) \\
\dot{\lambda} &= \frac{1}{2} m^{-1} \rho V_o S (V_o C_{\lambda_\eta} \eta + V_o C_{\lambda_\zeta} \zeta) + u z^{-1} (\cos(\lambda) q - \sin(\lambda) r)
\end{aligned} \tag{1}$$

3 Approximate Feedback Linearisation

The state–space form of the non-linear system of the home missile can now be written in a compact parametric format, as:

$$
\begin{aligned}
\dot{z} &= a_1 z + a_2 z^2 + a_3 r\sin(\lambda) - a_3 q\sin(\lambda) \\
&\quad + (a_4 z + a_5)(\sin(\lambda)\zeta + \cos(\lambda)\eta) \\
\dot{r} &= b_1 z^3 \sin(\lambda) + b_2 z^2 \sin(\lambda) + b_3 z\sin(\lambda) + b_4 z r + b_5 r \\
&\quad + (b_6 z + b_7)\zeta - (b_9 + b_8 z + b_1 0 z^2)\xi \\
\dot{q} &= -b_1 z^3 \cos(\lambda) - b_2 z^2 \cos(\lambda) - b_3 z\cos(\lambda) + b_4 z q + b_5 q \\
&\quad - (b_6 z + b_7)\eta + (b_9 + b_8 z + b_1 0 z^2)\xi \\
\dot{p} &= c_1 p + c_2 \xi + (c_3 + c_4 z)(\zeta + \eta) \\
\dot{\lambda} &= -a_3 z^{-1}(q\sin(\lambda) + r\cos(\lambda)) \\
&\quad + z^{-1}(a_4 z + a_5)(\sin(\lambda)\eta - \cos(\lambda)\zeta)
\end{aligned}
\tag{2}
$$

or in matrix form:

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
y &= h = \begin{bmatrix} v \\ w \\ p \end{bmatrix} = \begin{bmatrix} z\sin(\lambda) \\ z\cos(\lambda) \\ p \end{bmatrix}
\end{aligned}
\tag{3}
$$

This equation is now in standard form and input-output linearisation techniques can be applied to it. In order to retain the system order with no zero dynamics, an approximate input–output linearisation technique is applied to the missile model. It is based on an approximation method involving the modification of the function g presented in (Hauser *et al.*, 1992) and (Tsourdos *et al.*, 1998).

Using this approximation technique, terms are discarded in order to retain an approximate system with an equivalent order and relative degree. In other words the g vector field is modified. This is achieved by neglecting the term $\psi_{1,2}(x,u)$ shown in following equation as it will not affect the stability of the closed loop dynamics.

Let $\xi_1 = \phi_1 = h_1(x)$. Then:

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2 + \psi_1(x,\zeta) \\
\dot{\xi}_2 &= \alpha_1 + \beta_1 \zeta + \beta_2 \xi = \nu_1(x,\zeta,\xi)
\end{aligned}
\tag{4}
$$

where:

$$
\begin{aligned}
\alpha_1(x) &= (a_1\cos(\lambda) + 2a_2 z\cos(\lambda))(a_1 z + a_2 z^2 + a_3 r\sin(\lambda) - a_3 q\sin(\lambda)) \\
&\quad + (a_1 z\cos(\lambda) + a_2 z^2\cos(\lambda))(-a_3 z^{-1}(q\sin(\lambda) + r\cos(\lambda))) \\
&\quad + a3(b_1 z^3\sin(\lambda) + b_2 z^2\sin(\lambda) + b_3 z\sin(\lambda) + b_4 z r + b_5 r) \\
\beta_1(x) &= a_3(b_6 z + b_7) - (b_9 + b_8 z + b_{10} z^2) \\
\beta_2(x) &= a_3(b_9 + b_8 z + b_{10} z^2)
\end{aligned}
\tag{5}
$$

Hence the output y_1 possesses a relative degree r_1 of 2.
Similarly, for the pitch plane, let $\xi_3 = \phi_3 = h_2(x)$. Then:

$$\begin{aligned} \dot{\xi}_3 &= \xi_4 + \psi_2(x, \eta, \xi) \\ \dot{\xi}_4 &= \alpha_2 + \beta_3 u_2 + \beta_4 u_3 = \nu_2(x, \eta, \xi) \end{aligned} \tag{6}$$

where:

$$\begin{aligned} \alpha_2(x) &= (a_1 sin(\lambda) + 2a_2 z sin(\lambda))(a_1 z + a_2 z^2 + a_3 r sin(\lambda) - a_3 q sin(\lambda)) \\ &\quad -(a_1 z sin(\lambda) + a_2 z^2 sin(\lambda))(-a_3 z^{-1}(q sin(\lambda) + r cos(\lambda))) \\ &\quad -a3(-b_1 z^3 cos(\lambda) - b_2 z^2 cos(\lambda) - b_3 z cos(\lambda) + b_4 z q + b_5 q) \\ \beta_3(x) &= a_3(b_6 z + b_7) - (b_9 + b_8 z + b_{10} z^2) \\ \beta_4(x) &= a_3(b_9 + b_8 z + b_{10} z^2) \end{aligned} \tag{7}$$

The output y_2 also possesses a relative degree r_2 of 2.
Finally, for the roll plane, for the linearisation process i.e. the design of the non-linear controller we take as output the roll rate p, but place an integrator in front of the roll channel to equalise the channel orders. Let $\xi_5 = \phi_5 = h_3(x)$, where $h_3(x)$ the roll angle. Then:

$$\begin{aligned} \dot{\xi}_5 &= \xi_6 \\ \dot{\xi}_6 &= \alpha_3 + \beta_5 \zeta + \beta_6 \eta + \beta_7 \xi = \nu_3(x, \zeta, \eta, \xi) \end{aligned} \tag{8}$$

where:

$$\begin{aligned} \alpha_3(x) &= c_1 \\ \beta_5(x) &= c_3 + c_4 z \\ \beta_6(x) &= c_3 + c_4 z \\ \beta_7(x) &= c_2 \end{aligned} \tag{9}$$

Hence the output y_3 possesses a relative degree r_3 of 2. The total relative degree of the system (Slotine and Li , 1991) is equal with the sum of the r_1, r_2, and r_3 is now 6, and has the same order as the original system and hence there are no internal dynamics. Since the total relative degree is equal with the order of the system, fully linearisation of the non-linear system can now be achieved.

The effect of neglecting the term $\psi_{1,2}$ in previous equation is to eliminate a non-linear zero in the system within the model description, and which is not taken into account in the non-linear control design. It had be shown in (White , 1998), this will not affect the performance of the control design in a significant manner as the zero can be approximated by:

$$\begin{aligned} z &\approx -\frac{(a_4 z + a_5)}{(2a_3 b_6 z + a_3 b_7)} \\ \frac{a_4}{a_3 b_6} &> 0 \\ \frac{a_5}{a_3 b_7} &> 0 \end{aligned} \tag{10}$$

When the velocity is defined as an output of the system then the relative degree is equal to the order of the system. In that case there is no zero dynamics involved into the design. If we don't neglect any term then the linearization will take place by solving the 2^{nd} derivative of the output for the highest derivative of the input $\dot{\psi}$. A pre-compensator will cancel the inherent zero in the i/o equation. Because the side-slip force(which for all missiles is negative), that zero will lie on the right hand s-plane, hence any pole cancellation must therefore be unstable. An approximation to this controller that does not include the cancellation pole can be used by neglecting the $\dot{\psi}$ term (the side-slip force acting on the control surfaces). This will tend to destabilise the system as an unstable zero will exist in the closed-loop system that is not taken into the analysis. This will produce a less stable solution than the cancellation feedback solution. But provided the side-slip force is not too great this will not affect the performance.

Equations (4), (6) and (8) represent a direct relationship between the outputs h_i and the inputs u_i. The required static state feedback for decoupled closed loop input/output behaviour is given by (Kravaris *et al.*, 1990) as:

$$u = E^{-1}\left\{\nu - \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}\right\} \tag{11}$$

where E is the characteristic (Kravaris *et al.*, 1990) or decoupling (Slotine and Li , 1991) matrix of the system, and is given by:

$$E = \begin{bmatrix} \beta_1 & 0 & \beta_2 \\ 0 & \beta_3 & \beta_4 \\ \beta_5 & \beta_6 & \beta_7 \end{bmatrix} \tag{12}$$

which is nonsingular. The determinant of the decoupling matrix is:

$$p(z) = \det(E) = p_0 + p_1 z + p_2 z^2 + p_3 z^3 + p_4 z^4 + p_5 z^5$$

There is not a value of interest for z which could make $p(z)$ (i.e. the determinant of the decoupling matrix) equal to zero.

The linearised closed loop system is now given by:

$$\ddot{y}_i = \nu_i \tag{13}$$

Where ν is the new linearised system input. Now choose the new control input to be:

$$\nu = \ddot{y}_d - k_1\dot{e} - k_2 e \tag{14}$$

where $e \equiv y - y_d$. The close–loop system is thus characterised by:

$$\ddot{e} + k_1\dot{e} + k_2 e = 0 \tag{15}$$

where k_1 and k_2 are chosen such that all roots of $s^2 + k_1 s + k_2 = 0$ are in the open left–half plane, which ensures $\lim_{t\to\infty} e(t) = 0$.

It can be said that now the tracking control problem for the non-linear system described by equation (3) has been solved using the control law in equation (11) and (14). Indeed, since equation (15) has the same order as the non-linear system, there is no part of the system dynamics which is rendered "unobservable" in the approximate input–output linearisation. Since there are no zero dynamics in the linearised system, the stability of the linearised system can be guaranteed and the tracking problem has been solved (Slotine and Li , 1991), (Hauser *et al.*, 1992).

4 Trajectory controller design

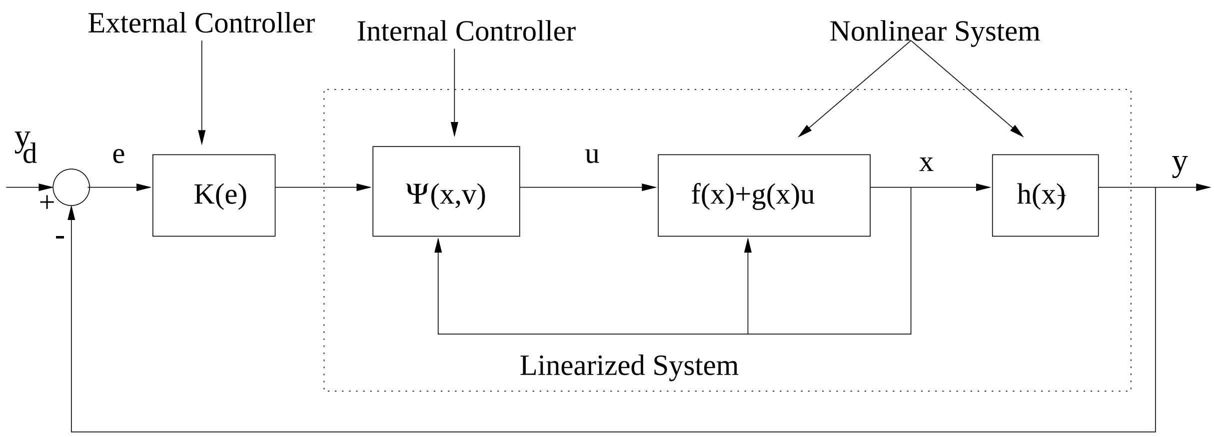

Figure 2: Trajectory control design

Figure 2 shows the non-linear controller structure. A fast linear actuator with natural frequency of 250 rad / sec has been included in the non-linear system. The desired acceleration a_d is achieved by using the non-linear equation $a_d = f(v)$. Therefore the trajectory controller performs by defined a desired acceleration as a function of the lateral velocity demand. The error dynamics are constructed using the a_d signal and the feedback of the actual states - velocity, rate and acceleration.

The error coefficients in (15) are chosen to satisfy a Hurwitz polynomial. For the second order error equation in the acceleration channel, $k_1 = 2\zeta w_n$ and $k_2 = w_n^2$, where $w_n = 60(rad/sec)$ and $\zeta = 0.7$ while for the roll channel $w_n = 80(rad/sec)$. This speed of response is significantly faster than the open loop response and so should exercise the dynamics of the non-linear missile.

The results for diffrent demand in acceleration is shown in Figure 3 and Figure 5. As expected for a non-linear system, the relationship between lateral velocity and lateral acceleration is non-linear. The results also show that the actuator does not significantly affect the design. The non-linear approach is also shown to be reasonably accurate, as the predicted and actual performances are very close.

5 Conclusions

There are three ways to increase the relative degree of a non-linear system. These are either to propose a new output that is an approximation of the desired one, to neglect sufficiently small terms during the differentiation process or finally to design a pre-compensator for the system. This paper presented the trajectory control design of non-linear missile model that is a combination of the first two. It results in a linear equivalent system with no internal or zero dynamics, and with a design of a trajectory control which gives small tracking errors for both the lateral velocity and lateral acceleration. The design has involved increasing the speed of response of the system sufficiently for a linear approximation for the system to be inadequate for design purposes, and the response for both small and large demands has been shown to be invariant. Other techniques are now being researched that involve a quasi-linear approach, or involve pre-compensation to look at techniques that can be applied to the lateral acceleration directly. This involves dealing with a non-minimum phase system that yields unstable zero dynamics with direct linearisation methods.

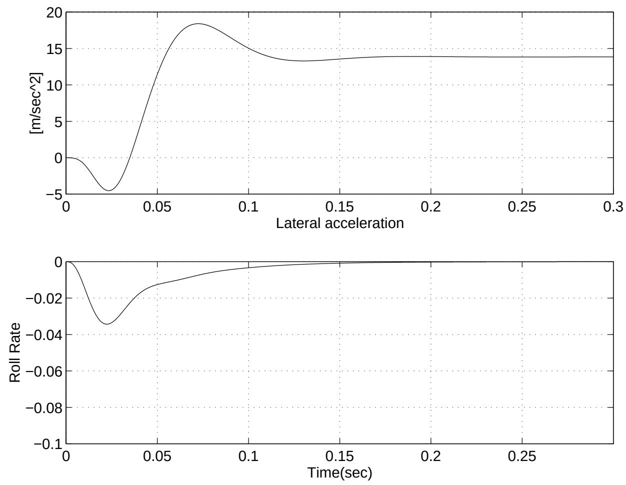

Figure 3: Lateral Accelerations and Roll Rate for $a_d = 14$ and $p_d = 0$

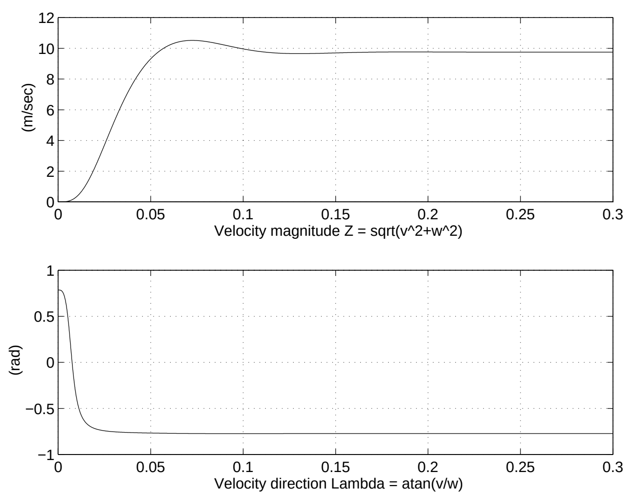

Figure 4: Magnitude and direction of velocity for $a_d = 14$ and $p_d = 0$

References

Kravaris, C., M. Soroush (1990). "Syntesis of Multivariable Nonlinear Controllers by Input–Output Linearization," *AIChE Journal*,**36**, no. 2, pp. 249–264.

Hahn, H., A. Piepenbrink, and K. D. Leimbach (1994). "Input–Output Linearization Control of an Electro Servo–hydraulic Actuator," *Proceedings of the 1994 IEEE Conference on Control Applications*, vol. 2, pp. 995–1000.

Hauser, J., S. Sastry, and P. Kokotovic (1992). "Nonlinear Control Via Appriximate Input–Output Linearization: The Ball and Beam Example," *IEEE Transactions on Automatic Control*,**37**, no. 3, pp. 392–398.

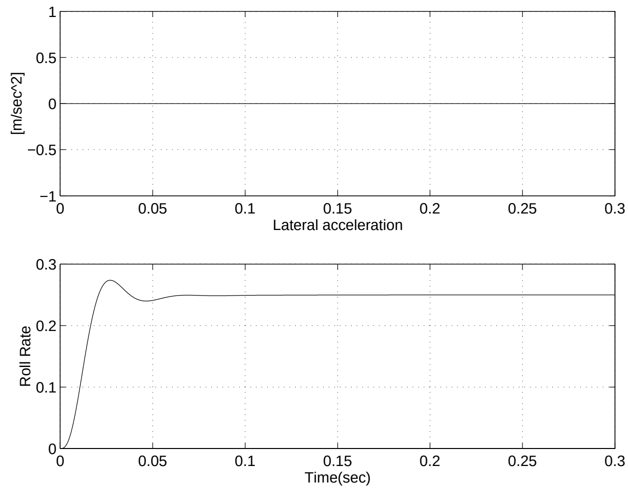

Figure 5: Lateral Accelerations and Roll Rate for $a_d = 0$ and $p_d = 0.25$

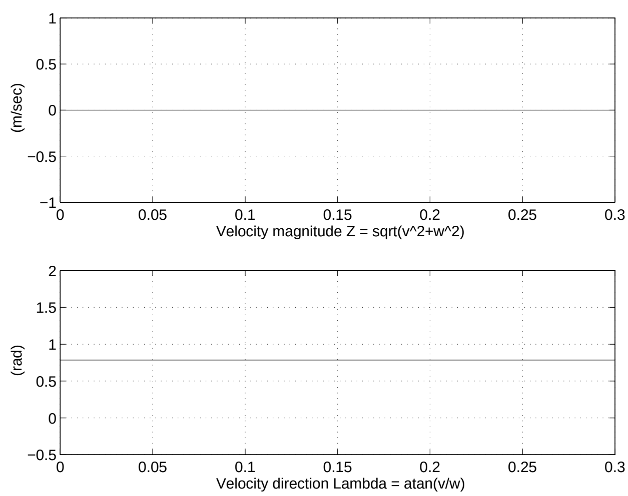

Figure 6: Magnitude and direction of velocity for $a_d = 0$ and $p_d = 0.25$

Horton, M. P. (1992). "A study of autopilots for the adaptive control of tactical guided missiles," *Master's thesis, University of Bath*

Isidori, A., A. Krener, and C. Monaco (1981). "Nonlinear decoupling via feedback: A differential-geometric approach," *IEEE Transactions on Automatic Control*,**26**, pp. 331–345.

Hunt, L.R. and R. Su (1981). "Linear equivalents of nonlinear time-varying systems," *Proc. Int. Symp. on Math. Thy. of Newt. and Sys.*, pp. 119–123.

Henson, M. and D. Seborg (1990). "Input–Output Linearization of General Nonlinear Processes," *AIChE Journal*,**36**, no. 11, pp. 1754–1757.

Bezick, S., I. Rusnak, and S. Gray (1995). "Guidance of a Homing Missile via Nonlinear Geometric Control Methods," *Journal of Guidance, Control and Dynamics*,**18**, no. 3, pp. 441–448.

Slotine, J.J and W. Li (1991). *Applied Nonlinear Control*, Prentice Hall.

Su, R. (1982). "On the linear equvalents of nonlinear systems," *Systems and Control Letters*, **2**, pp. 48–52.

Tsourdos, A., A.L. Blumel, and B.A. White (1998). "Autopilot design of a nonlinear missile," *Proc. IEE Control'98*, no. 3, pp. 889–894.

Tsourdos, A., A.L. Blumel, and B.A. White (1998). "Trajectory Control of a Nonlinear Homing Missile," *Proc. of the* 14^{th} *IFAC Symp. on Aut. Control in Aerospace*, pp. 118–124.

Tsourdos, A., A.L. Blumel, and B.A. White (1998). "Lateral Acceleration Control Design of a Nonlinear Homing Missile," *Proc. of the* 4^{th} *IFAC Nonlinear Control Systems Design Symposium*, pp. 708–713.

White, B.A. (1998). "Non-linear Auto-pilot of the HORTON and GENERIC Missile," *Tech.report, DAPS,RMCS,Cranfield Uni,UK.*

Lu, X.Y., S. Spurgeon, and I. Postlethwaite (1997). "Robust Variable Structure Control of a PVTOL Aircraft," *International Journal of Systems Science*,**28**, no. 6, pp. 547–558.

Shamma, J.S., and M.Athans (1990). "Analysis of nonlinear gain scheduled control systems," *IEEE Trans. on Automatic Control*,**35**, pp. 898–907.