Adaptive Generalized Predictive Control Subject to Input Constraints

Andrzej Królikowski*
Department of Control, Robotics and Computer Science
Poznań University of Technology
ul.Piotrowo 3a
60-965 Poznań, POLAND

Abstract

Generalized predictive control (GPC) problem of ARIMAX/ARMAX system in the presence of input constraints and parametric uncertainty is considered. An adaptive controller is implemented in an indirect way, and the considered constraints imposed on the control signal are of the rate, amplitude and energy type. A simulation comparative study of the adaptive control system behavior is given with respect to the design parameters and constraints. Additionally, two one-step controllers are compared by means of simulations.

1 Introduction

Predictive control seems to be one of the most popular topics in academic research and process control engineering mainly because of its simplicity and succesful industrial applications.

Because input constraints are ubiquitous in control engineering applications, the way of handling them in control system design is an important question. However, this does not often happen in the design of control algorithms reported in the literature. Disregarding constraints or imposing them on the control signal in a heuristic way can cause performance deterioration or even instability, especially in adaptive control with unstable systems. Taking constraints into account in the design stage leads inherently to a solution of constrained optimisation problem. It is well known that quadratic programming (QP) techniques can be applied to solve different kinds of predictive control problems under constraints.

In this paper, the generalized predictive control (GPC) is considered which is perhaps one of the most succesful representative amongst predictive control proposals. The application of the QP to solve the GPC is widely used, see for example the comments given in [5] and [7]. The constrained GPC has also been discussed in [3], [2] where the QP problem is transformed into the so called *Linear Complementarity Problem* which in turn is solved using Lemke's algorithm. This reduces the amount of computation compared with the QP. As an alternative to QP, a method based on a modification of Lawson's weighted least squares algorithm was proposed in [5] Another approach to solve the constrained optimization problem involves *Linear Matrix Inequalities* [7]. In [6], an interesting approach based on the dynamic programming was proposed to solve the constrained model predictive control. The desaturating approach for adaptive receding-horizon predictive control in the case of simultaneous amplitude and rate constraints is presented in [8].

*akrol@omega.ar-kari.put.poznan.pl

In recent years some research has been done on the predictive control of stochastic systems which contain parametric uncertainty. Here, as in all adaptive systems, it is necessary to combine both facets of the adaptive controller, the identification and the control algorithm, in order to obtain a proper interplay between them resulting in a robust performance of the adaptive controller.

In this paper, the constrained adaptive GPC (AGPC) for discrete-time stochastic system of ARIMAX/ARMAX structure with unknown but constant parameters is considered. For the indirect adaptive controller considered here, the controller parameters are tuned on the base of system parameter estimates along with the *Certainty Equivalence Principle*, and the rate, amplitude and energy constraints are assumed to be imposed on the control input. Few concepts about how the adaptive GPC can be realized are discussed in [9] and [10].

It is well known that the form of GPC makes the analytical examination of closed-loop stability and performance properties (including steady-state error) difficult. The objective of this paper is to present a simulation-based comparison of these properties with respect to control design parameters and constraints. To this end, stable, unstable as well as non-minimum phase second-order systems are taken for the simulation study.

2 Standard GPC

First, the standard unconstrained GPC problem of ARIMAX/ARMAX system will be shortly characterized. An ARMAX model is described by

$$A(q^{-1})y_t = q^{-1}B(q^{-1})u_t + C(q^{-1})e_t \tag{1}$$

where A, B, and C are polynomials in the backward shift operator q^{-1}, i.e.

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{na}q^{-na}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{nb}q^{-nb}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{nc}q^{-nc}$$

and y_t is the output , u_t is the control input and $\{e_t\}$ is assumed to be a sequence of independent variables with zero mean and variance σ_e^2.
In some applications it is more preferable to use an ARIMAX model given by

$$A(q^{-1})y_t = q^{-1}B(q^{-1})u_t + \frac{C(q^{-1})}{\Delta}e_t \tag{2}$$

where $\Delta = 1 - q^{-1}$.
The GPC cost function is of the form

$$J(N_y, N_u, q_u) = E[\sum_{i=1}^{N_y}(y_{t+i} - r_{t+i})^2 + q_u \sum_{i=1}^{N_u} u_{t+i-1}^{\star 2}] \tag{3}$$

where the weight $q_u \geq 0$ and the horizons N_y, N_u are basic design parameters of GPC. The object $u_t^\star$ is u_t for positional control based on an ARMAX model or Δu_t for incremental control when an ARIMAX model is assumed.

The goal of the GPC is the output y_t to follow some reference signal r_t taking into account the control effort. This can be expressed in the following cost function

$$J(N_y, N_u, q_u) = (G\bar{u} + f - r)^T(G\bar{u} + f - r) + q_u\bar{u}^T\bar{u} \tag{4}$$

where the matrix G is composed of the impulse response coefficients, $\{g_i\}$, of the system model $\frac{B}{A\Delta}$ in the ARIMAX case

$$G = \begin{bmatrix} g_0 & 0 & \dots & 0 \\ g_1 & g_0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{N_y-1} & g_{N_y-2} & \cdots & g_{N_y-N_u} \end{bmatrix}$$

and

$$f = (f_{t+1}, \cdots, f_{t+N_y})^T$$

$$r = (r_{t+1}, \cdots, r_{t+N_y})^T$$

where the free response $f_{t+i} = \hat{y}_{t+i/t}$ with $\hat{y}_{t+i/t}$ being a part of the prediction $\hat{y}_{t+i}$, $i = 1, ..., N_y$, assuming that $\Delta u_{t+i} = 0, i = 0, ..., N_y - 1$. The unconstrained optimal control is then [1],[2],[4]

$$\bar{u}^* = (G^T G + q_u I)^{-1} G^T (r - f) \tag{5}$$

where

$$\bar{u}^* = (\Delta u_t^*, \cdots, \Delta u_{t+N_u-1}^*)^T \tag{6}$$

The first element of the sequence (6), i.e. Δu_t^*, is applied to the system. At the next time instant $(t + 1)$, the optimization procedure starts again with the current data. In the non-adaptive case, this means that in control law (5) only the vectors f, r should be updated.

The derivation of GPC controller for an ARMAX system needs some modifications to the two polynomial partitions (identities) [1]. In the first identity associated with the prediction partition, the polynomial A must be used, not $A\Delta$ as in the ARIMAX case. In the second identity concerning the control variables separation, the polynomials $G_i = \sum_{j=0}^{i-1} g_j q^{-j}$ appearing in the ARIMAX derivation must be replaced by $G_i \Delta$ for the ARMAX case. Thus, in the GPC for an ARMAX system, the coefficients g_i are the impulse response parameters of the transfer function $\frac{B}{A}$.

3 GPC subject to input constraints

Now, the GPC in the presence of rate, amplitude and energy constraints will be examined. As already mentioned, QP techniques are computationally demanding. However, as pointed out by Tsang and Clarke [4], for reducing the computation load the separate treating of amplitude and rate constraints can be advantageous.

First, the main results of [4] are given for the case of rate and amplitude constraints taking as a base the unconstrained GPC solution (5) to an ARIMAX model.

3.1 GPC under rate constraint

The rate of the control input is constrained in amplitude

$$| \Delta u_t | \leq \beta \tag{7}$$

In the case of ARIMAX model, using the Lagrange multipliers method as proposed in [4], the constrained optimal control can be found from (in the case when only one future control saturates at $+\beta$ or $-\beta$)

$$\bar{u}^c = \bar{u}^* + (G^T G + q_u I)^{-1} \lambda_j e_j \tag{8}$$

where $e_j = (0, 0, \cdots, 1, 0, \cdots, 0)^T$ where the entry 1 is at the $j - th$ place. To find the Lagrange multiplier the following equation has to be solved, say for $+\beta$

$$\beta = \Delta u^*_{t+j-1} + g_{jj}\lambda_j \tag{9}$$

wrt the Lagrange multiplier λ_j for $j = 2, \cdots N_u$, and g_{jj} is entry (j, j) of the matrix $(G^T G + q_u I)^{-1}$. Now, the optimal constrained Δu^c_t can be found putting λ_j back to (8).
The above procedure can be extended to the case when more than one future controls saturates, i.e. for $N_u > 2$. However, as pointed out in [4] the unsolved problem is on which limits the optimum lies. A reasonable solution, however heuristic, is based on the assumption that the constrained optimal control lies on the constraint which is violated by the solution to the unconstrained optimum. In particular, this reasonning is valid for $N_u = 2$.

3.2 GPC under amplitude constraint

The amplitude constraint imposed on the control input is given as follows

$$\mid u_t \mid \leq \alpha \tag{10}$$

First, it can be noted that the calculation of optimal amplitude-constrained control for an ARIMAX is more complex than for an ARMAX model. The opposite statement can be made in the case of rate constraint.
Again, the use of Lagrange multiplier method for solving the amplitude-constrained GPC for ARIMAX model is possible, however in general case the involved computations make this approach not beneficial anymore. The important case $N_u = 2$ makes an exception. Consider this case following the idea of [4]. When the future control is feasible then

$$u^c_t = sat[u^*_t; \alpha] \tag{11}$$

where $u^*_t = u_{t-1} + \Delta u^*_t$ is the control signal obtained by the standard algorithm and sat denotes the well-known saturation function with saturation limit α.
When the future control signal is infeasible, say $u_t = \alpha$ then

$$\Delta u_t = \Delta u^*_t - \frac{\sigma_1}{\sigma_1 + \sigma_2}[\alpha' - (\Delta u^*_t + \Delta u^*_{t+1})] \tag{12}$$

where σ_1, σ_2 are the sums of the first and second rows of the $2x2$ matrix $(G^T G + q_u I)^{-1}$, respectively and $\alpha' = \alpha - u_{t-1}$. Note that $\alpha' = \Delta u_t + \Delta u_{t+1}$. The applied control signal follows then from

$$u^c_t = sat[u_{t-1} + \Delta u_t; \alpha] \tag{13}$$

where Δu_t is given by (12).

3.3 GPC under energy constraint

Consider the GPC for an ARMAX model with the cost function (4) for $q_u = 0$

$$J(N_y, N_u) = (G\bar{u} + f - r)^T (G\bar{u} + f - r) \tag{14}$$

under the energy constraint of input signal

$$\bar{u}^T \bar{u} \leq \gamma^2 \tag{15}$$

For the minimization of (14) under the constraint (15), the Kuhn-Tucker conditions yield

$$G^T G\bar{u} + G^T(f - r) + \lambda\bar{u} = 0 \tag{16}$$

$$\lambda(\bar{u}^T\bar{u} - \gamma^2) = 0 \tag{17}$$

$$\lambda \geq 0 \tag{18}$$

The optimal constrained control is then given by

$$\bar{u}^c = (G^T G + \lambda I)^{-1} G^T (r - f) \tag{19}$$

where the multiplier λ can be calculated from

$$(r - f)^T G(G^T G + \lambda I)^{-1T}(G^T G + \lambda I)^{-1} G^T (r - f) = \gamma^2 \tag{20}$$

Summarizing, when the constraint (15) is fulfilled the applied optimal control is the unconstrained optimal control $\bar{u}^*$ calculated for $q_u = 0$. Otherwise, the applied constrained control is given by (19) where the multiplier λ must be recalculated at each time step t whenever the constraint (15) is violated. It is worth to notice that the inversion of $(G^T G + \lambda I)$ can be calculated in a recursive way along with the increasing control horizon N_u [10].
The above constrained minimization problem can also be solved iteratively by the method of Carroll [12] using an unconstrained minimization technique for the modified cost function

$$J_m(N_y, N_u) = (G\bar{u} + f - r)^T(G\bar{u} + f - r) + r_k(\gamma^2 - \bar{u}^T\bar{u})^{-1} \tag{21}$$

where for monotonically decreasing r_k, so that $r_k \to 0$, the succsessive minimizations of (23) yield the constrained minimum.

4 One-step control

As a particular, however important case, consider GPC for an ARIMAX model with one-step control horizon ($N_u = 1$). From (5) one obtains

$$\Delta u_t = (g^T g + q_u)^{-1} g^T (r - f) \tag{22}$$

where g is the leading column in G. Thus, an explicit form for Δu_t can be obtained

$$\Delta u_t = \frac{\sum_{i=0}^{N_y-1} \epsilon_{t+i+1} g_i}{\sum_{i=0}^{N_y-1} g_i^2 + q_u} \tag{23}$$

where $\epsilon_{t+i} = r_{t+i} - f_{t+i}$, $i = 1, \cdots, N_y$ are the future reference deviations over the prediction horizon N_y. The coefficients g_i can be calculated recursively from

$$g_i = b_i - \sum_{k=1}^{i} a_k^* g_{i-k} \tag{24}$$

with $g_0 = b_0$, $b_i = 0$ for $i > nb$ and $a_j^* = a_j - a_{j-1}$ for $j = 1, \cdots, na + 1$ with $a_{na+1} = 0$ and $a_0 = 1$. It can be seen that the minimal value of N_y should be taken as $N_y = na + 1$.

Taking the rate constraint into consideration the constrained control law is

$$\Delta^c u_t = sat[\Delta u_t; \beta] \tag{25}$$

so, that $u_t^c = \Delta^c u_t + u_{t-1}$ becomes the next input to the system. It is worth to notice that controller (23) can be written in a convolution-type form

$$\Delta u_t = \sum_{i=1}^{N_y} p_i \epsilon_{t+i} \tag{26}$$

where parameters p_i are (complicated) functions of parameters in polynomials A, B and design parameters N_y, q_u. In this way, the numbers p_i can be treated as controller parameters.

For comparison, consider the ARIMAX model and the following one-step cost function

$$J_1(q_u) = E[(y_{t+1} - r_{t+1})^2 + q_u \Delta u_t^2 \mid y_t, y_{t-1}, \cdots, u_{t-1}, u_{t-2}] \tag{27}$$

under the rate constraint (7). Let Δu_t^* denote the unconstrained optimal control signal, i.e. the optimal control signal in the absence of any saturation limits. This control signal is given by

$$\Delta u_t^* = -\frac{b_0 H}{b_0 FB + q_u C} y_t + \frac{b_0 C}{b_0 FB + q_u C} r_{t+1} \tag{28}$$

If Δu_t^* is not feasible then the implemented control signal Δu_t differs from it by $u_t^+ = \Delta u_t^* - \Delta u_t$ which denotes the unimplemented portion of optimal contol signal. This correction takes into account the fact that the past values of cost function J_1 may not be zero due to saturation.

The control law including the saturation correction is as follows [11]

$$\Delta u_t = -\frac{b_0 H}{b_0 FB + q_u C} y_t - \frac{(b_0^2 + q_u)(1 - C)}{b_0 FB + q_u C} u_t^+ + \frac{b_0 C}{b_0 FB + q_u C} r_{t+1} \tag{29}$$

where H, F follow from the identity

$$C = \Delta AF + q^{-1} H \tag{30}$$

where the monic polynomial F is of degree 0. This means that the actually applied control signal is determined by

$$\Delta u_t^c = sat[\Delta u_t; \beta] \tag{31}$$

The control laws corresponding to (23), (28) in the case of ARMAX with the cost function

$$J_1 = E[(y_{t+1} - r_{t+1})^2 + q_u u_t^2 \mid y_t, y_{t-1}, \cdots, u_{t-1}, u_{t-2}] \tag{32}$$

can be easily derived.

5 Adaptive control

The adaptive controller which is proposed here is a *certainty equivalence* or *indirect* controller. To estimate the unknown system parameters $\underline{\theta} = (a_1, ..., a_{na}, b_0, ..., b_{nb}, c_1, ..., c_{nc})^T$ the *recursive extended least-squares*(RELS) algorithm is applied. Next, the current system parameter estimates $\hat{\underline{\theta}}_t$ are used for tuning of the GPC controller. Thus, the obtained adaptive GPC (AGPC) controller generates the current control signal. The procedure is repeated while the new output sample is available. From (5) it follows that actually applied unconstrained control signal is given by

$$u_t^* = u_{t-1}^* + q_1^T (r - f) \tag{33}$$

where q_1^T denotes the first row of matrix $(G^TG+q_uI)^{-1}G^T$. This means that in the unconstrained adaptive controller only q_1 and predictions f have to be recalculated at each discrete time instant t for current values of parameter estimates $\hat{\underline{\theta}}_t$ according to the standard derivation of the GPC algorithm. Implementing the constrained adaptive controller along with the derivations of Sections 3.1, 3.2 and 3.3, additional computations have to be performed at each discrete time instant t for current estimates $\hat{\underline{\theta}}_t$.

At present, there are no rigorous theoretical results regarding the stability of adaptive finite horizon predictive control. The lack of such results is more evident in adaptive control systems under amplitude or rate-constrained input. In this case, it is generally not possible to assure the closed-loop stability for unstable noisy systems. However, in some cases, e.g. for noise-free or bounded-noise e_t unstable systems, the close-loop stability can be obtained by an adaptive controller with constrained output. Then some stability-instability boundary can be evaluated in terms of initial conditions and initial parameter estimates wrt constraints. In simulations given below some runs are presented in order to evaluate the close-loop stability for given design parameters and initial parameter estimates wrt constraints imposed on the input.

6 Simulations

The following examples are simulated:

1. the second-order stable ARIMAX/ARMAX system with actual values of parameters $a_1 = -1.8, a_2 = 0.9, b_0 = 1.0, b_1 = 0.5$
2. the second-order unstable ARIMAX/ARMAX system with actual values of parameters $a_1 = 1.8, a_2 = -0.9, b_0 = 1.0, b_1 = 0.5$
3. the second-order non-minimum phase ARIMAX/ARMAX system with actual values of parameters $a_1 = -1.5, a_2 = 0.7, b_0 = -1.0, b_1 = 2.0$

The polynomial C is taken as $C = 1$ and the noise variance σ_e^2 was set at 0.1. System parameters were identified using the standard RELS method.

For the considered second-order ARIMAX example, the coefficients g_i in the GPC controller (23) for $N_y = 3$ are $g_0 = b_0, g_1 = b_1 - b_0(a_1 - 1), g_2 = -(a_1 - 1)[b_1 - b_0(a_1 - 1)] - b_0(a_2 - a_1)$.

On the other hand, the one-step controller Δu_t (29) has the following explicit form

$$\Delta u_t = -\frac{b_0}{b_0^2 + q_u}(h_0 y_t + h_1 y_{t-1} + h_2 y_{t-2} + b_1 \Delta u_{t-1} - r_{t+1})$$

where $h_0 = 1 - a_1, h_1 = a_1 - a_2, h_2 = a_2$.
Simulation runs were performed for a square wave as a reference signal given by

$$r_{25N+t+5} = 5(-1)^N \quad t = 0, 1, ..., 24 \quad N = 0, 1, ...$$

and for horizons $N_u = 2, N_y = 3$ with the weight $q_u = 0.1$ if not given otherwise.

6.1 Adaptive GPC under rate constraint

Fig.1 shows the control behavior for example 1 (ARIMAX) with $\beta = 1.5$. For β lower than 0.5 the performance is poor. Example 2 (ARIMAX) is simulated in Fig.2 for $\beta = 90$, and for $\beta < 75$ the system destabilizes. Obviously, in this case as well as in all other examples related with unstable systems, for a given set of design parameters N_u, N_y, q_u, the unconstrained closed-loop system is stable for known actual values of parameters. Simulations of example 3 (ARMAX) are shown in Figs.3,4 for $\beta = 0.5$, $\beta = 1.5$, respectively. In this example the horizons $N_u = 3, N_y = 5$ are needed to stabilize the system.

6.2 Adaptive GPC under amplitude constraint

The results for example 1 (ARMAX) are shown in Figs.5,6 with $\alpha = 0.6$ and $\alpha = 2.2$, respectively. Example 2 (ARMAX) is simulated in Fig.7 for $\alpha = 35$. For $\alpha < 27$ the system gets unstable. Here again, for a given set of design parameters N_u, N_y, q_u the closed-loop system is stable for actual values of parameters. Simulation of example 3 (ARMAX) is shown in Fig.8 for $\alpha = 4$ and $N_u = 3, N_y = 5$. The tracking deteriorates for lower values of constraint α, and for $\alpha < 2$ the system gets unstable. The similar behavior can also be observed for an ARIMAX model not presented here.

6.3 Adaptive GPC under energy constraint

The control behavior for example 1 (ARMAX), is shown in Fig.9 with $\gamma = 3$ where the plot of of λ is also given. For $\gamma < 1$, the performance essentially deteriorates. Example 2 (ARMAX) is simulated in Fig.10 for $\gamma = 45$. For $\gamma < 31$ the system falls into instability. Simulation of example 3 (ARMAX) is given in Fig.11 for $\gamma = 3$.

6.4 Adaptive one-step controllers

Fig.12 shows the one-step control behavior (25) for example 1 (ARIMAX) with $\beta = 18$ and $N_u = 1, N_y = 3$ while the one-step controller (31) is simulated in Fig.13 for the same example with $N_u = 1, N_y = 1$. The tracking performance in Fig.13 is much better than in Fig.12. This is because the one-step controller (31) takes the saturation correction into account, and moreover the matrix G in (23) is not of full dimension $3x3$ but it is truncated to a $3x1$ matrix ($N_u = 1$).

7 Conclusions

The GPC problem is presented in the case of rate, amplitude and energy-constrained input for one and multi-stage cost functions. An indirect adaptive version of GPC with parametric uncertainty is also discussed. A second-order example is simulated as an indirect adaptive control system for different configurations of design parameters. As expected, the simulation results show an essential influence of design parameters and constraints on the stability and control performance. This is very crucial point in the analysis of the closed-loop stability, especially of unstable open-loop systems. In general, these systems can not be stabilized using adaptation under input constraints. However in practice, by a proper choice of design parameters, the close-loop stability of unstable systems can be achieved when the initial parameter estimates are close to actual values of parameters and the noise level is low. In this case, given the assumed initial conditions some stability boundaries wrt constraints can be established through simulations. This is for example, illustrated in Figs.2,7,10 where the stable response is obtained for large values of constraint β, α, γ, i.e. for practically unconstrained control.
In all cases, the tracking performance is poor at the initial phase of control and gets better later on along with the identification of system parameters, however, in this regard the non-minimumphase systems show the worst performance.

References

[1] Bitmead, R.R., Gevers, M. and Wertz, V. (1990): *Adaptive Optimal Control*, Prentice Hall International.

[2] Camacho, E.F. and Bordons, C. (1995): *Model Predictive Control in the process industry*, Springer.

[3] Camacho, E.F. (1993): *"Constrained generalised predictive control,"* IEEE Trans. Automat. Contr., **38**, No.2, pp.327-332.

[4] Tsang, T.T.C. and Clarke, D.W. (1988): *Generalised predictive control with input constraints*, IEE Proc., Pt.D, **135**, No.6, pp.451-460.

[5] Rossiter, J.A. and Kouvaritakis, B. (1993): *"Constrained stable generalised predictive control,"* IEE Proc., Pt.D, **140**, No.4, pp.243-254.

[6] Soufian, M., Sandoz, D.J. and Soufian, M. (1997): *"Dynamic programming approach for constrained model predictive control,"* Preprints 4th IFAC Conf. on Syst.Structure and Contr., (Bucharest), Oct.23-25, pp.232-237.

[7] Kothare, M.V., Balakrishnan, V. and Morari, M. (1996): "Robust constrained model predictive control using linear matrix inequalities,",*Automatica*, **32**, No.10, pp.1361-1379.

[8] De Nicolao, G., Scattolini, R. and Sala, G. (1996): "An adaptive predictive regulator with input saturations," *Automatica*, **32**, No.4, pp.597-601.

[9] Landau, I.D., Lozano, R. and M'Saad, M. (1998): *Adaptive Control*, Springer.

[10] Niederliński, A., Mościński, J. and Ogonowski, Z. (1995): *Adaptive Control*, WNT, Warsaw, (in Polish).

[11] Segall, N.L., Macgregor, J.F. and Wright, J.D. (1991): "One-step optimal saturation correction," *Automatica*, **27**, No.1, pp.135-139.

[12] Bazaraa, M.S. and Shetty, C.M. (1979): *Nonlinear Programming. Theory and Algorithms*, John Wiley and Sons.

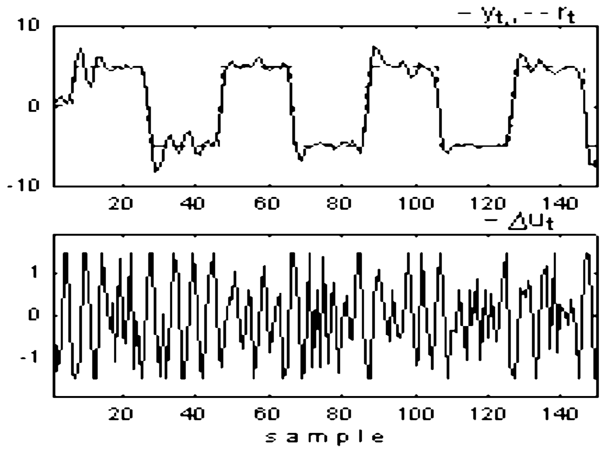

Figure 1: AGPC under rate constraint

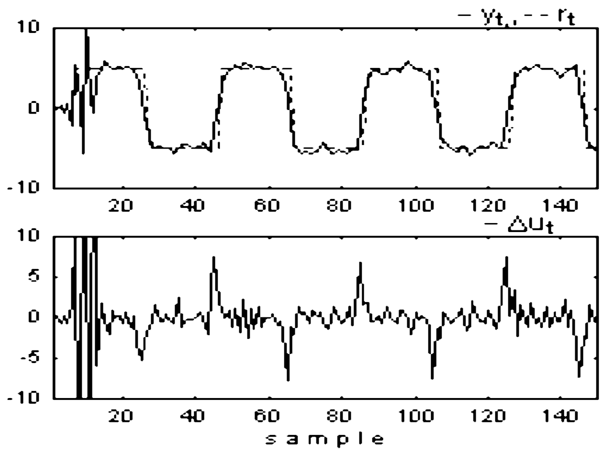

Figure 2: AGPC under rate constraint

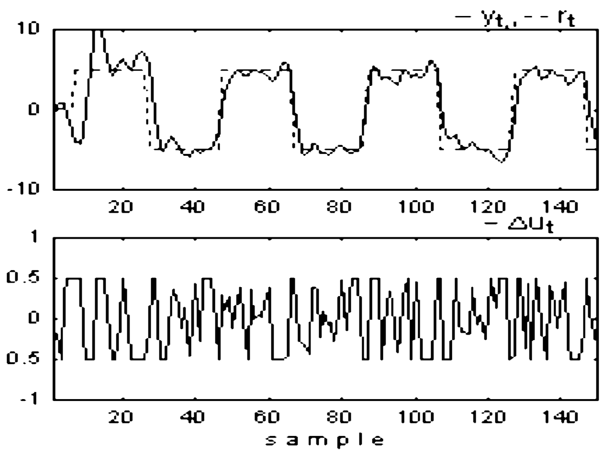

Figure 3: AGPC under rate constraint

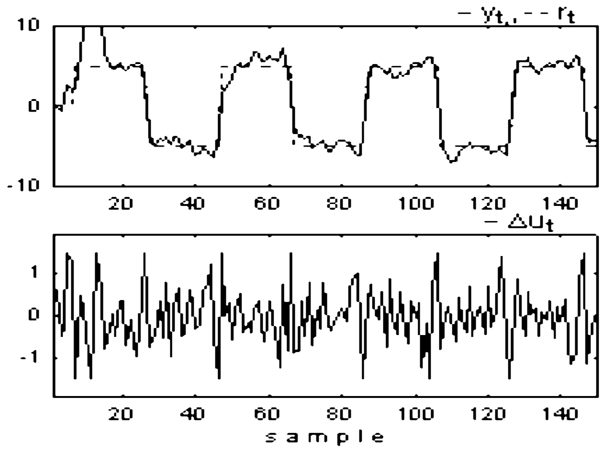

Figure 4: AGPC under rate constraint

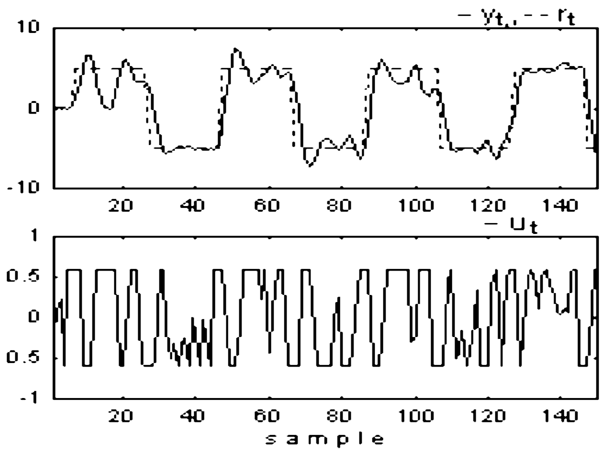

Figure 5: AGPC under amplitude constraint

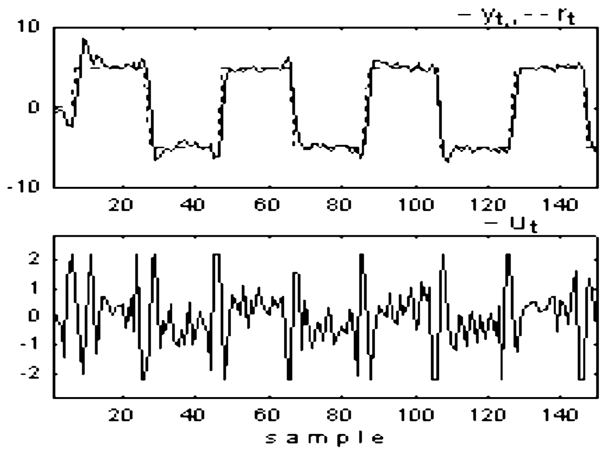

Figure 6: AGPC under amplitude constraint

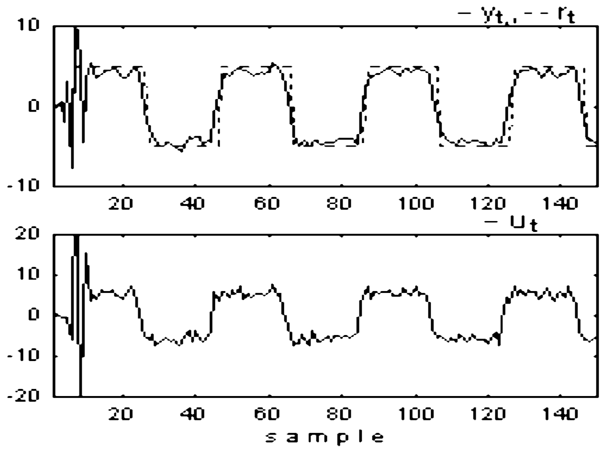

Figure 7: AGPC under amplitude constraint

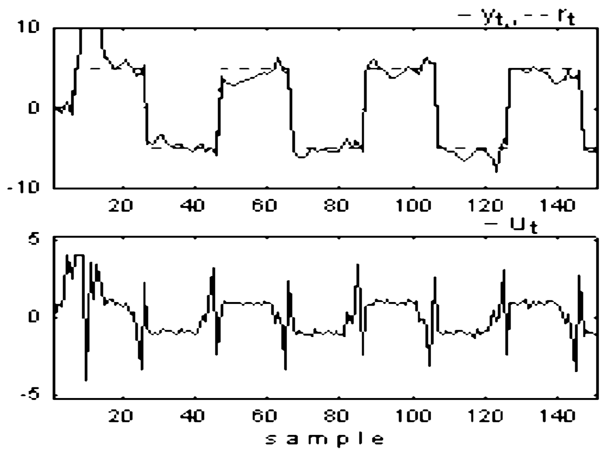

Figure 8: AGPC under amplitude constraint

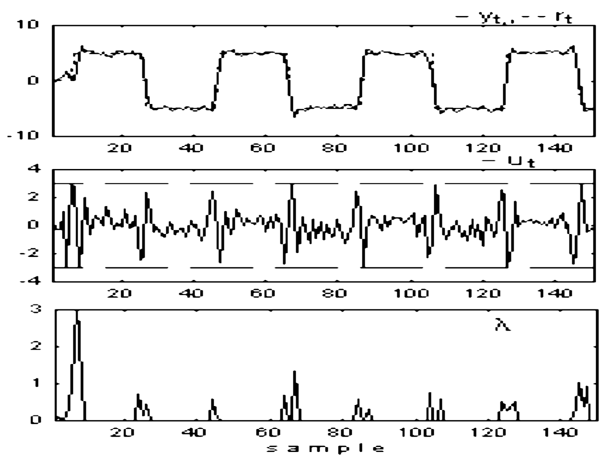

Figure 9: AGPC under energy constraint

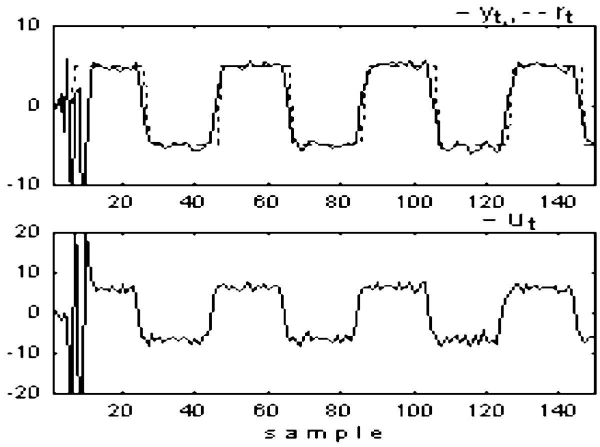

Figure 10: AGPC under energy constraint

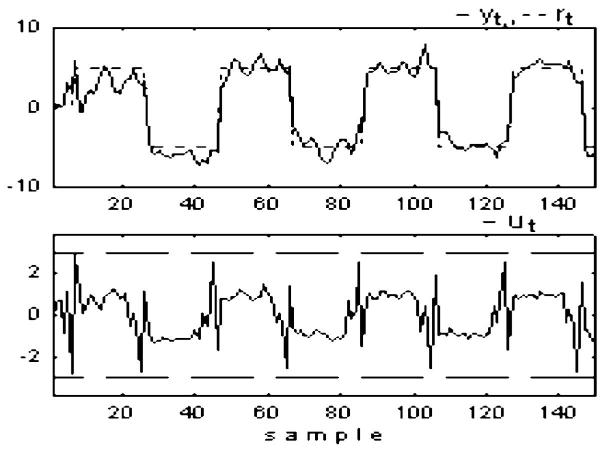

Figure 11: AGPC under energy constraint

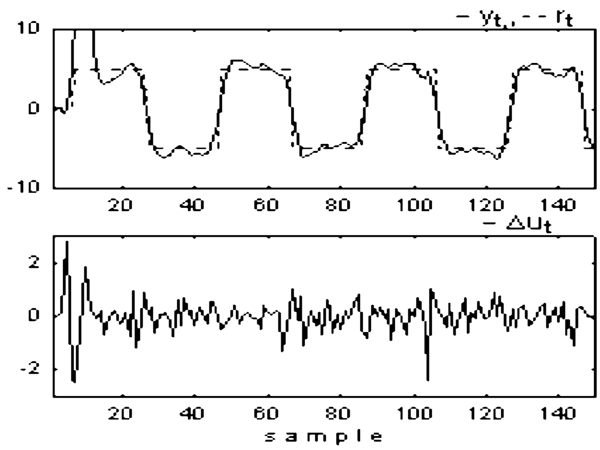

Figure 12: One step control under rate constraint

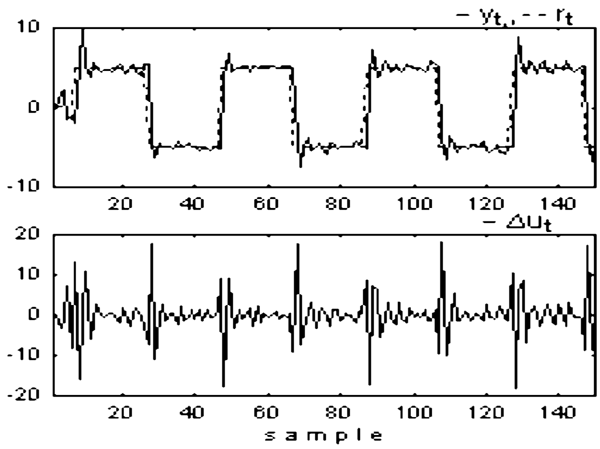

Figure 13: One step control under rate constraint