Self-Tuning PID Controller Using d- Model Identification*

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Abstract

This contribution presents an application of a self-tuning digital PID controller for process control modelled by d- models. The process is identified by the regression (ARX) models using the recursive least square method (RLSM) with LD decomposition and applied directional forgetting. Controller synthesis is designed on the basis of a modified Ziegler-Nichols criterion for digital PID control loops. The ultimate (critical) proportional gain and period of oscillations have been derived for the second - order d- model. Control results using digital PID controller on the basic d- models and z - models are compared.

1 Introduction

The digital process computer as used as the control unit of the control loop operates only in discrete time sequences $t_k = kT_0$ (k = 0, 1, 2,...), where T_0 is the sampling period. In the case of continual technological process control we then consider continual control object and discrete controller. For this control loop to function well an interface between these differently operating dynamic systems is vital. Sampler and holder in combination with analogue - digital and digital - analogue converters are used for this interface. The sampler samples the continuous signal in k - multiples of sampling periods to produce an output signal as an impulse sequence. The height of impulses is equal to the value of the input signal over the sampling period. For technological process control the zero - order holder is used almost exclusively to hold the impulse constant over the entire sampling period. We must therefore use a suitable mathematical description to express the dynamic behaviour of the thus discretized members of the control loop. One such of description is an expression using the Z - transformation.

If $G(s)$ is the transfer function of a continual dynamic system, then the following expression for the discrete transfer function with the zero - order holder is valid

$$G(z) = \frac{z-1}{z} Z\left\{ L^{-1} \frac{G(s)}{s} \right\} \tag{1}$$

This step transfer function (1) is a rational polynomial function with variable z. The disadvantage of

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this transformation is that the Z - transformation parameters do not converge with the decrease of the sampling period to the Laplace - transformation continuous parameters from which they were derived. The simple model structure, identification from using measurable data, good suitability for the synthesis of the discrete control loop and for the description and expression of different types of the stochastic processes including disturbance modelling are advantages of the Z - transformation (1).

One of the disadvantages of the step transfer functions is its behaviour when sampling periods are decreased. The sampling period should be kept to minimum so that it does not result in a loss of information. On the other hand very small sampling periods yield the very small numbers from the transfer function numerator. The poles transfer function approaches unstable domain as the sampling period decreases.

2 Delta models and their identification

These disadvantages can be avoided by introducing a more suitable discrete model. For this purpose the d - model is the most suitable, where parameter d converges with decreased sampling period T_0 to a continuous operator s

$$\lim_{T_0 \to 0} d = s \tag{2}$$

Middleton and Goodwin (1990) published one of these approaches to the design of these new discrete models, and it was generalized by Mukhopadhyay et *al*. (1992). These models are widely known as delta models.

It is possible to prove (Mukhopadhyay *et al*., 1992), that equality

$$d = \frac{z-1}{a T_0 z + (1-a) T_0} \tag{3}$$

holds for interval $0 \le a \le 1$. By substituting a in equation (3) we obtain an infinite number of new d- models. There are the widely know and used d - models in practice

for $a = 0$ $\quad d = \dfrac{z-1}{T_0}$ $\quad$ forward d - model (4)

for $a = 1$ $\quad d = \dfrac{z-1}{zT_0}$ $\quad$ backward d - model (5)

for $a = 0.5$ $\quad d = \dfrac{2}{T_0}\dfrac{z-1}{z+1}$ $\quad$ Tustin d - model (6)

This paper will only concerned with the forward d - model (4).

The d - models will be used for process modelling for adaptive control based on the self - tuning controller (STC). The main idea of an STC is based of a recursive identification procedure and a selected control synthesis. For this reason it is necessary to apply suitable recursive identification algorithm to this model. For parameter estimation of the d - model, the recursive least squares method (RLSM) with LD decomposition and with directional forgetting is applied (Kulhavý, 1987).

A useful model to apply this method of identification is the regression (ARX) model which is often expressed in its compact form

$$y(k) = Q^T(k) f(k-1) + n(k) \tag{7}$$

where

$$Q^{T}(k)=\left[a_1, a_2, \ldots, a_{na}, b_1, b_2, \ldots, b_{nb}\right] \tag{8}$$

is the vector of the parameter estimates and

$$f^{T}(k-1)=[-y(k-1),-y(k-2),\ldots,-y(k-n),u(k-1),u(k-2),\ldots,u(k-n)] \tag{9}$$

is the regression vector ($y(k)$ is the process output variable, $u(k)$ is the controller output variable). The non-measurable random component $n(k)$ is assumed to have zero mean value $E[n(k)] = 0$ and constant covariance (dispersion) $R = E[n^2(k)]$.

We use for the PID controller digital synthesis *d*- second order model with transfer function

$$G(d)=\frac{y(d)}{u(d)}=\frac{b_1 d+b_2}{d^2+a_1 d+a_2} \tag{10}$$

which can be rearranged in to the form

$$d^2 y(d)=-a_1 d y(d)-a_2 y(d)+b_1 d u(d)+b_2 u(d) \tag{11}$$

By substituting *d* from relation (4) and multiplying z^{-2} we obtain the equation

$$\frac{1-2z^{-1}+z^{-2}}{T_0^2}Y(z)=-a_1\frac{z^{-1}-z^{-2}}{T_0}Y(z)-a_2 z^{-2}Y(z)+b_1\frac{z^{-1}-z^{-2}}{T_0}U(z)+b_2 z^{-2}U(z) \tag{12}$$

where $Y(z)$ and $U(z)$ are Z - transforms process output $y(k)$ and controller output $u(k)$ variable.

The stochastic discrete model for d - parameter estimates is then in the form

$$y_d(k)=-a_1 y_d(k-1)-a_2 y_d(k-2)+b_1 u_d(k-1)+b_2 u_d(k-2)+n(k) \tag{13}$$

where

$$\begin{aligned}
y_d(k)&=\frac{y(k)-2y(k-1)+y(k-2)}{T_0^2}\\
y_d(k-1)&=\frac{y(k-1)-y(k-2)}{T_0}\\
y_d(k-2)&=y(k-2)\\
u_d(k-1)&=\frac{u(k-1)-u(k-2)}{T_0}\\
u_d(k-2)&=u(k-2)
\end{aligned} \tag{14}$$

From equation (13) and (14) it is obvious that the parameter estimates have the same form as (8)

$$Q^{T}(k)=\left[a_1,\quad a_2,\quad b_1,\quad b_2\right] \tag{15}$$

and the regression vector is

$$\boldsymbol{f}^{T}(k-1)=\left[-\frac{y(k-1)-y(k-2)}{T_0},\quad -y(k-2),\quad \frac{u(k-1)-u(k-2)}{T_0},\quad u(k-2)\right] \tag{16}$$

3 PID controller synthesis

3.1 Calculating the critical parameters for the second order system

The PID controller setting, developed by Ziegler and Nichols (1942) more than half a century ago, is still widely used in industry. In this well-known approach the parameters of the controller are calculated from the ultimate (critical) gain K_{Pc} and the ultimate period of oscillations T_c of the closed loop system. These critical parameters are obtained from the experimental setting of the proportional controller in feedback. The gain of the controller is increased until oscillations with the constant amplitude are obtained. The analytic determination of the ultimate parameters K_{Pc} and T_c for the continuous transfer function differs from the discrete one. In the first case the poles of the characteristic polynomial of the closed loop system must lie on the imaginary axis and on the left-hand of the s - plane. The computation of K_{Pc} and T_c is made by substituting $s = jw_c$ in the characteristic polynomial. This solution is the same as the experimental setting of the proportional controller by the Ziegler-Nichols method. The Ziegler -Nichols formula gives for setting the PID controller these relations

$$K_P = 0.6K_{Pc}; \qquad T_I = 0.5T_c; \qquad T_D = 0.125T_c \tag{17}$$

where K_P is the proportional gain, T_I a T_D integral and derivative time constants.

The disadvantage of this experimental approach is that the system can become unstable and it can be a very demanding process to determine the stability boundary for a system with large time constants. The modified method for digital PID controllers setting does not have these disadvantages. The synthesis of the digital PID controller for the second order system was proposed by Bobál (1995) and the generalized approach for the n - order model is introduced in Bobál *et al*. (1997).

3.1.1 Calculating critical proportional gain

This method for setting digital PID controllers for d - model control will be now derived. Let the process be described by the single input single output (SISO) system d- model in the form of the discrete equation

$$y_d(k) = -a_1 y_d(k-1) - a_2 y_d(k-2) + b_1 u_d(k-1) + b_2 u_d(k-2) \tag{18}$$

is controlled by the proportional controller

$$u_d(k) = K_P\left[w_d(k) - y_d(k)\right] \tag{19}$$

Substituting of the equation (19) into equation (18) we obtain the closed control loop equation

$$y_d(k) + (a_1 + b_1K_P)y_d(k-1) + (a_2 + b_2K_P)y_d(k-2) = b_1K_P w_d(k-1) + b_2K_P w_d(k-2) \tag{20}$$

where

$$\begin{aligned} w_d(k-1) &= \frac{w(k-1) - w(k-2)}{T_0} \\ w_d(k-2) &= w(k-2) \end{aligned} \tag{21}$$

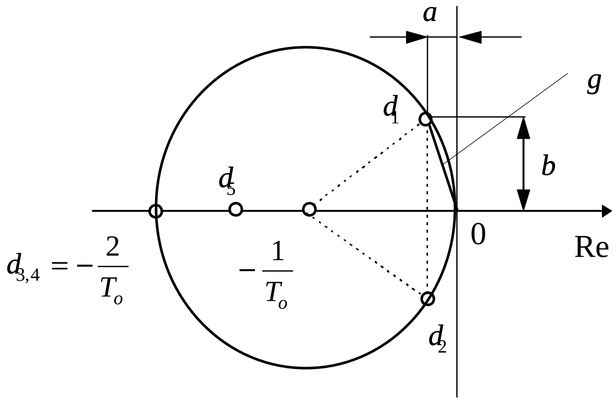


Figure 1. Stability region of the delta modification

By arrangement we then obtain the transfer function of the closed control loop

$$G_w(d) = \frac{y(d)}{w(d)} = \frac{b_1 K_P\, d + b_2 K_P}{d^2 + (a_1 + b_1 K_P)\, d + (a_2 + b_2 K_P)} \tag{22}$$

and by substituting

$$a_1 + b_1 K_P = b; \qquad a_2 + b_2 K_P = c \tag{23}$$

we obtain the transfer function in the form

$$G_w(d) = \frac{b_1 K_P\, d + b_2 K_P}{d^2 + b\, d + c} \tag{24}$$

The denominator of the transfer function (24) is the characteristic polynomial

$$D(d) = d^2 + b\, d + c \tag{25}$$

whose poles determine the behaviour of the closed control loop. At critical gain the poles of the characteristic polynomial (25) must lie on the stability boundary which is given by circle with its centre at point $-\frac{1}{T_0}$ and one of these points lies at the beginning of the d - plane (see Fig. 1).

Three possibilities for pole placement of the characteristic polynomial on the circle may occur so that closed control loop is on the stability boundary:

a) The characteristic polynomial (25) includes a pair of complex conjugate poles

$$d_{1,2} = a \pm jb; \qquad a^2 + b^2 = g^2 \tag{26}$$

Then the characteristic polynomial (25) with poles (26) expressed as a product of root factors has the form

$$D(d) = (d - a - jb)(d - a + jb) = d^2 - 2a\, d + a^2 + b^2 \tag{27}$$

Since it is valid according to the Euklide’s leg theorem (see Figure 1)

$$g^2 = a\, \frac{2}{T_0} \tag{28}$$

then with regard to the second expression (26), expressions (23) and substitution $K_P = K_{Pc}$ we obtain the polynomial equation in the form

$$d^2 + \left(a_1 + K_{Pc} b_1\right) d + a_2 + K_{Pc} b_2 = d^2 - 2a\, d - a\, \frac{2}{T_0} \tag{29}$$

By comparison the same powers of d in the equation (29) two equations are then obtained in the form

$$a_1 + K_{Pc} b_1 = -2a\,; \qquad a_2 + K_{Pc} b_2 = -a\, \frac{2}{T_0} \tag{30}$$

Equations (30) give relations to calculate the ultimate gain and real part of complex conjugate poles

$$K_{Pc} = \frac{a_1 - a_2 T_0}{b_2 T_0 - b_1}\,;\; a = -\frac{a_1 + K_{Pc} b_1}{2} \tag{31}$$

b) The characteristic polynomial (25) includes double real poles $d_{3,4} = -\frac{2}{T_0}$ or $d_{3,4} = 0$ (imaginary component b = 0). The control loop is on the stability boundary only where $d_{3,4} = -\frac{2}{T_0}$, because the pole $d_{3,4} = 0$ will not put the control loop into oscillation, so that the characteristic polynomial (25) expressed as a product of root factors has the form

$$D(d) = \left(d + \frac{2}{T_0}\right)^2 = d^2 + \frac{4}{T_0}\, d + \frac{4}{T_0^2} \tag{32}$$

and we obtain the next polynomial equation

$$d^2 + \left(a_1 + K_{Pc} b_1\right) d + a_2 + K_{Pc} b_2 = d^2 + \frac{4}{T_0}\, d + \frac{4}{T_0^2} \tag{33}$$

By comparing the same powers of d in equation (33) two equations are then obtained in the form

$$a_1 + K_{Pc} b_1 = \frac{4}{T_0}\,; \qquad a_2 + K_{Pc} b_2 = \frac{4}{T_0^2} \tag{34}$$

By solving equations (34) we obtain equation (31) to calculate critical gain once more.

c) The characteristic polynomial (25) includes one real pole $d_4 = -\frac{2}{T_0}$ a and a second real pole $d_5 = l$ ($|\,l| < \frac{2}{T_0}$). The characteristic polynomial (25) expressed as a product of root factors has the form

$$D(d) = \left(d + \frac{2}{T_0}\right)\left(d - l\right) = d^2 + \left(\frac{2}{T_0} - l\right) d - \frac{2\,l}{T_0} \tag{35}$$

and we obtain the next polynomial equation

$$d^2 + \left(a_1 + K_{Pc} b_1\right) d + a_2 + K_{Pc} b_2 = d^2 + \left(\frac{2}{T_0} - l\right) d - \frac{2}{T_0} \tag{36}$$

By comparing the same powers of d in equation (36) two equations are then obtained in the form

$$a_1 + K_{Pc} b_1 = \frac{2}{T_0} - l; \qquad a_2 + K_{Pc} b_2 = -\frac{2l}{T_0} \tag{37}$$

By solving equations (37) we obtain the expression

$$K_{Pc} = \frac{4 - 2T_0 a_1 + T_0^2 a_2}{2T_0 b_1 - T_0^2 b_2} \tag{38}$$

to calculate critical gain. The critical gain K_{Pc} is then computed either from equation (31) or from equation (38) according to fulfilment of the condition (see equations (23) and (25))

$$d = b^2 - 4c \leq 0 \tag{39}$$

It is evident that when fulfilling condition (39), the discriminate of the characteristic polynomial (25) is negative or zero, therefore it has a pair of complex conjugate or double real poles (case a) or b)). If condition (39) is not fulfilled the discriminate of the characteristic polynomial (25) is positive and therefore it has two real different poles (case c)).

3.1.2 Calculating the critical period of oscillations

According to the properties the d - transformation (see Middleton and Goodwin, 1990) it is obvious that d - transforms with polynomials which can be expressed in the denominator in the form

$$D(d) = d^2 + \frac{2(1-\cos wT_0)}{T_0} d + \frac{2(1-\cos wT_0)}{T_0^2} \tag{40}$$

correspond to time functions $\cos wt, \sin wt$ which are the harmonic undamped oscillations on the stability boundary. By comparing polynomials (25) and (40) it is obvious that

$$b = \frac{2(1-\cos wT_0)}{T_0}; \qquad c = \frac{2(1-\cos wT_0)}{T_0^2} \tag{41}$$

From equations (41) it is evident that $b = cT_0$. We can derive the relation to calculate of the critical frequency from the first or second equation (41). Substituting (23) into (41) we obtain expressions

$$w_c = \frac{1}{T_0} \arccos\left(\frac{2 - a_1 T_0 - b_1 K_{Pc} T_0}{2} \right); \qquad w_c = \frac{1}{T_0} \arccos\left(\frac{2 - a_2 T_0^2 - b_2 K_{Pc} T_0^2}{2} \right) \tag{42}$$

and for the critical period of oscillations T_c holds

$$T_c = \frac{2p}{w_c} \tag{43}$$

Flow diagram of the controller is presented in Figure 2.

3.2 Digital PID controllers

Let the continuous PID controller be described by a transfer function in the form

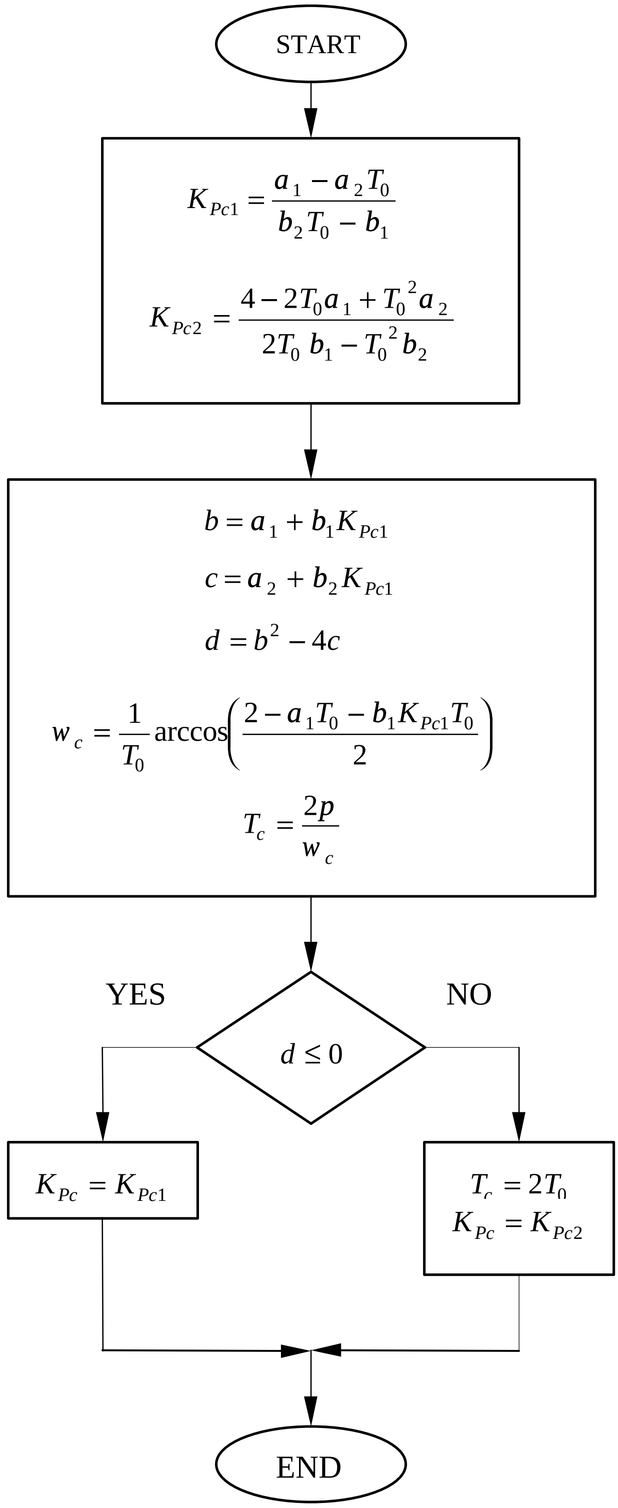


Figure 2. Flow diagram of the PID controller

$$G_C(s)=\frac{U(s)}{E(s)}=K_p\left(1+\frac{1}{T_I s}+T_D s\right) \tag{44}$$

where $Y(s)$ and $E(s) = W(s) - Y(s)$ are the Laplace transforms of the process output and error, $W(s)$ is the Laplace transform of the reference signal.

To get a digital version of the PID controller, we should discretize the integral and derivative component of equation (46). For discretizing the integral component we usually employ the forward rectangular method (FRM), backward rectangular method (BRM) or trapezoidal method (TRM). The derivative component is mostly replaced by the 1st order difference (two-point difference). For practical use the recurrent control algorithms which compute the actual value of the controller output $u(k)$ from the previous value $u(k-1)$ and from compensation increment seem to be suitable

$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1) \tag{45}$$

where controller parameters are

$$q_0,q_1,q_2=f\left(K_P,T_I,T_D,T_0\right) \tag{46}$$

It is subsequently possible to derive further variants of digital PID controllers, for example the PID controller with a filter constant in the D-part (Isermann, 1989). The PID controller designed by Takahshi *et al*. (1971) has been modified because the amplitude changes of the controller output variable $u(k)$ are further reduced if the reference variable $w(k)$ is only present in the integration form. The change of the process output variable $y(k)$ on the reference value is then mainly controlled through the integral component

$$u(k)=K_P\left\{-y(k)+y(k-1)+\frac{T_0}{T_I}\left[w(k)-y(k)\right]+\frac{T_D}{T_0}\left[2y(k-1)-y(k)-y(k-2)\right]\right\}+u(k-1) \tag{47}$$

where

$$K_P=0.6K_{Pc}\left(1-\frac{T_0}{T_c}\right); \qquad T_I=\frac{K_PT_c}{1.2K_{Pc}}; \qquad T_D=\frac{3K_{Pc}T_c}{40K_P} \tag{48}$$

The digital PID controller (47) and (48) has been verified in simulation and in laboratory conditions operating in real time.

4 Simulation example and laboratory verification

As an example of verification by computer simulation, we show of a proportional second-order system with the transfer function

$$G(s)=\frac{0.2}{s^2+1.2s+0.2} \tag{49}$$

for the very small sampling period T_0 = 0.01. The output value of the controller was limited within the range from u_{min} = 0 to u_{max} = 1. The initial vector of parameter estimates was $\hat{Q}^T(0)=\left[0.1, 0.1, 0.2, 0.2\right]$. Figure 3 shows the control process with identification using z - model, Figure 4 shows the control process using identification d- model. By comparing of the both Figures it is obvious very favourable influence for the parameter estimates convergent in the case d- model and good influence of the controller output $u(k)$ course. The d- parameter estimates converge for the small sampling period to continuous parameters of the model (49), the controller output $u(k)$ converges to steady state. The z - parameter estimates $\hat{b}_1$ and $\hat{b}_2$ are very small, they converge slowly and the controller output $u(k)$ oscillates.

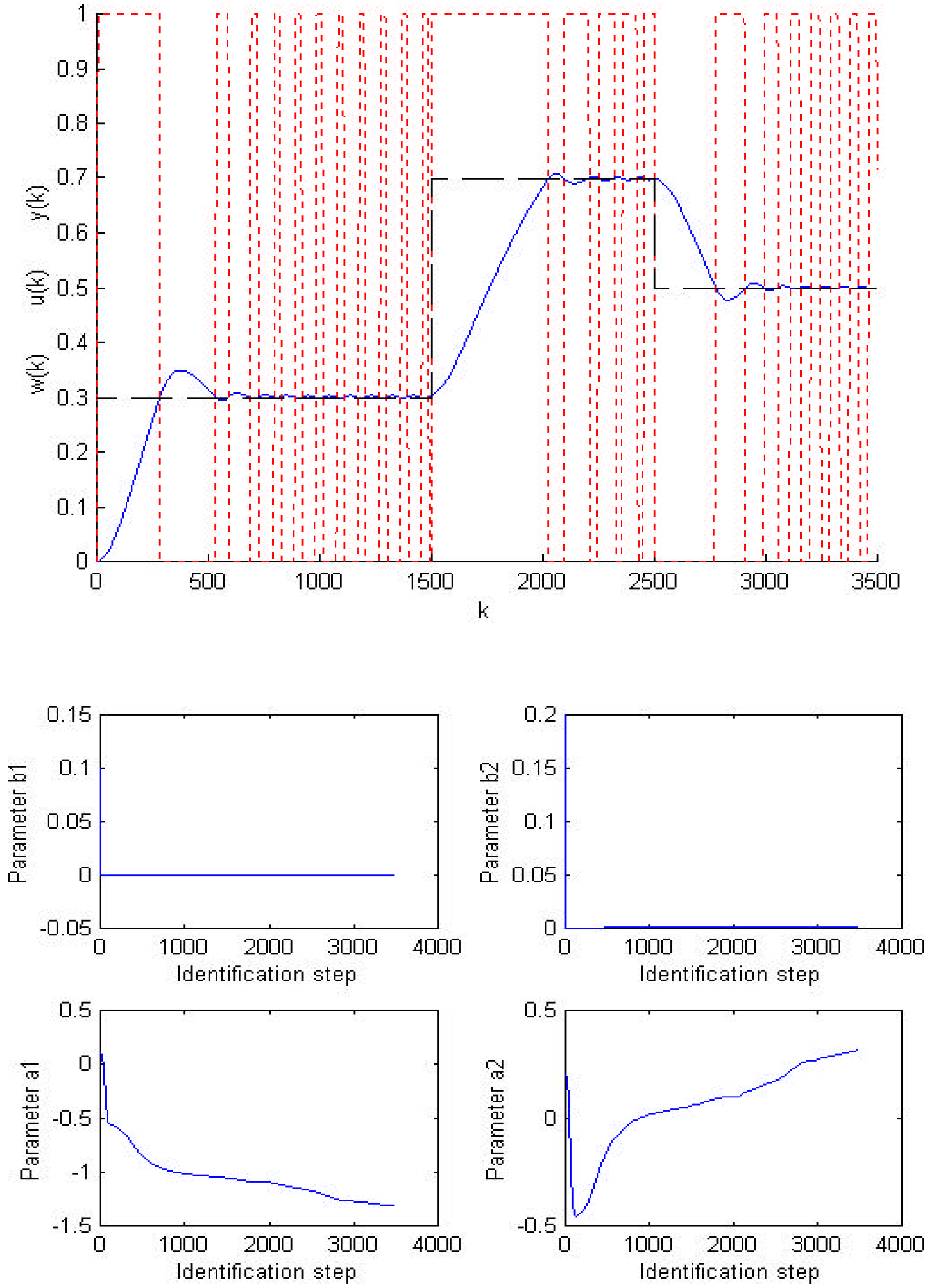


Figure 3. Simulation results: control of model (49) (z - model identification) and convergence of parameter estimates ($y(k)$ - solid, $u(k)$ - dotted, $w(k)$ - dashed)

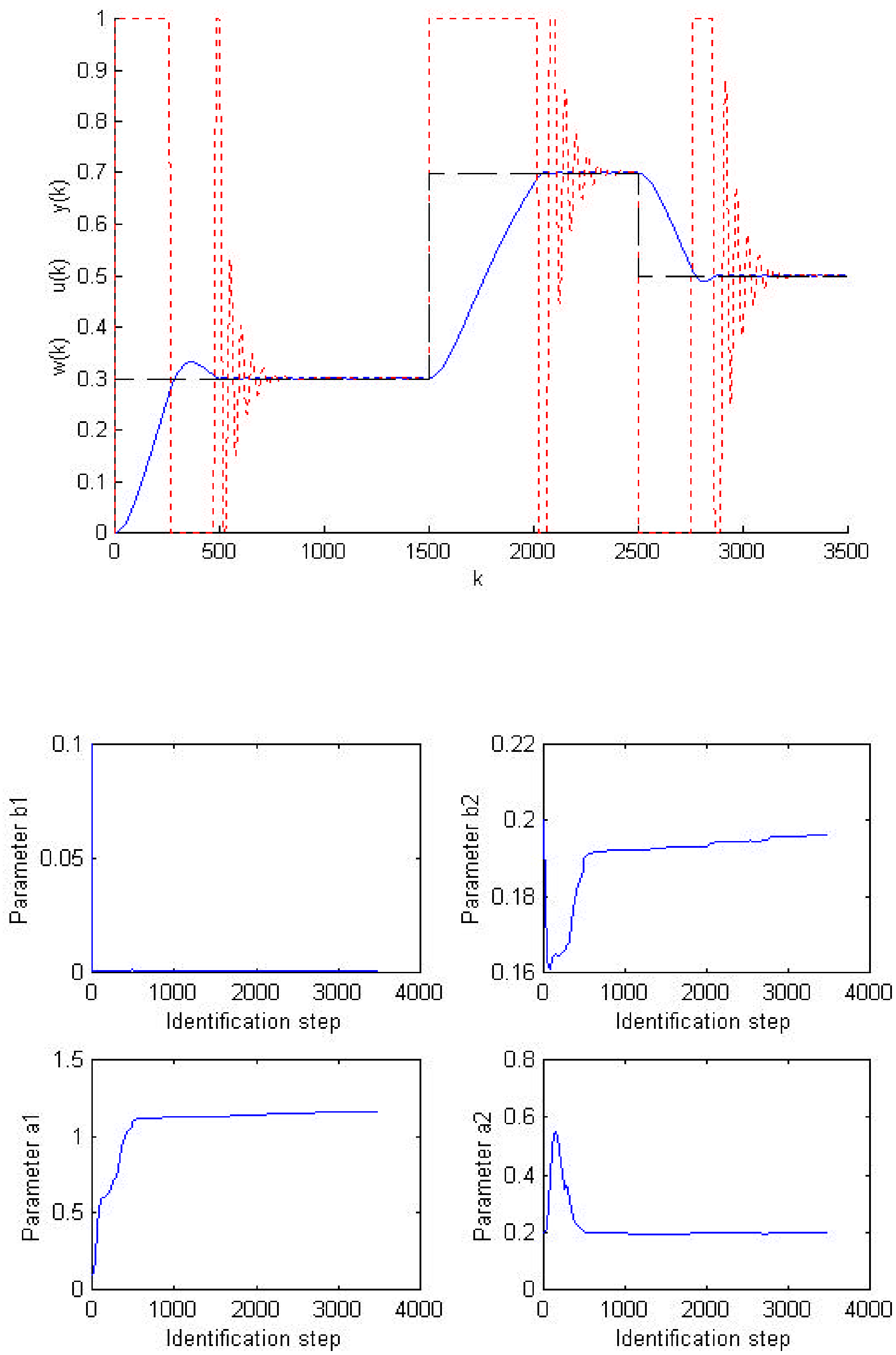
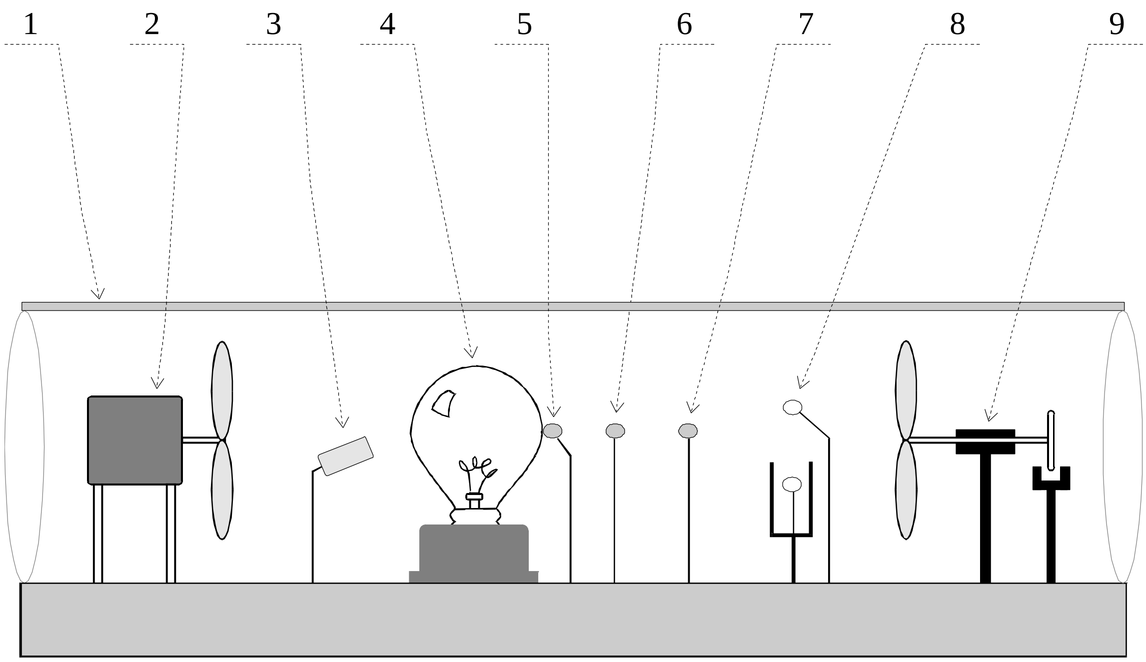


Figure 4. Simulation results: control of model (49) (d - model identification) and convergence of parameter estimates ($y(k)$ - solid, $u(k)$ - dotted, $w(k)$ - dashed)



1 - Tunnel	**2 -** Ventilator	**3 -** Photoresistor
4 - Electric bulb	**5 -** Thermistor T1	**6 -** Thermistor T2
7 - Thermistor T3	**8 -** Thermoanemometer	**9 -** Flow indicator

Figure 5. Laboratory through - flow heater

This self-tuning controller PID (STC PID) has been verified for the control of the laboratory through-flow heater (see Figure 5). This laboratory equipment is multi input - multi output (MIMO) system. Input variables are the heat source (electric bulb) and the air flow source (ventilator). Output variables are the air temperature and its flow and bulb brightness. Figure 6 and 7 show the process control verification, where the flow indicator (position **9**) speed value is the process output and the speed ventilator (position **2**) value is the controller output. The sampling period T_0 = 0.5 s, the initial vector of parameter estimates was $\hat{Q}^T(0) = [0.2, 0.1, 0.1, 0.05]$. From Figures 6 and 7 it is obvious that the d- model identification improves the control quality.

5 Conclusions

The proposed STC PID algorithm using identification d- model is simple, sufficiently robust and suitable for control a large class of systems. The simulation results and verification in laboratory conditions operating in real time of the proposed controller confirmed theoretical assumptions. The advantages of this approach include in the numeric stability of the recursive identification algorithm, in good parameter estimates convergent and very good of the controller output course for the small sampling period values.

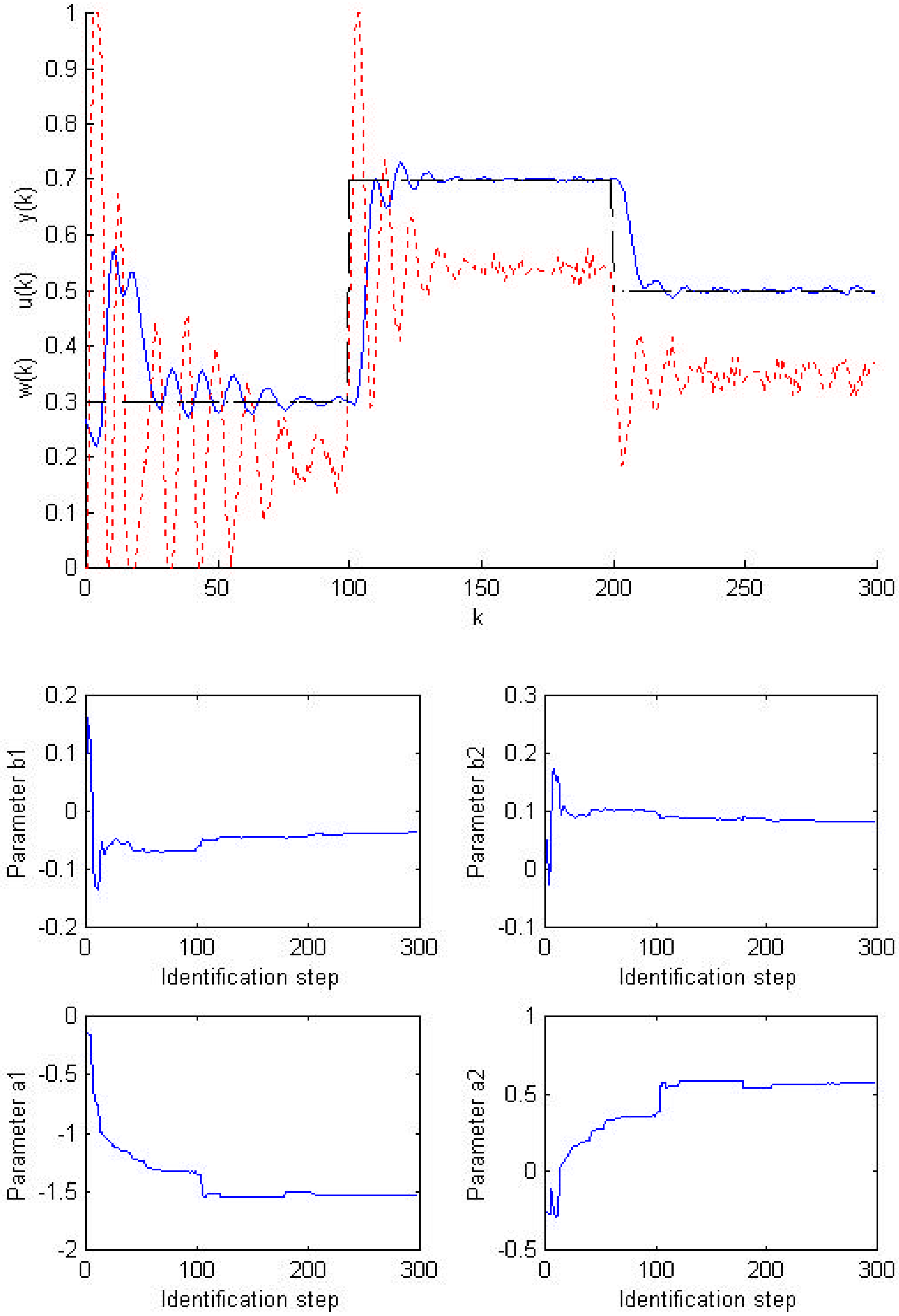


Figure 6. Air flow control of the laboratory through-flow heater (*z* - model identification) and convergence of parameter estimates (*y*(*k*) - solid, *u*(*k*) - dotted, *w*(*k*) - dashed)

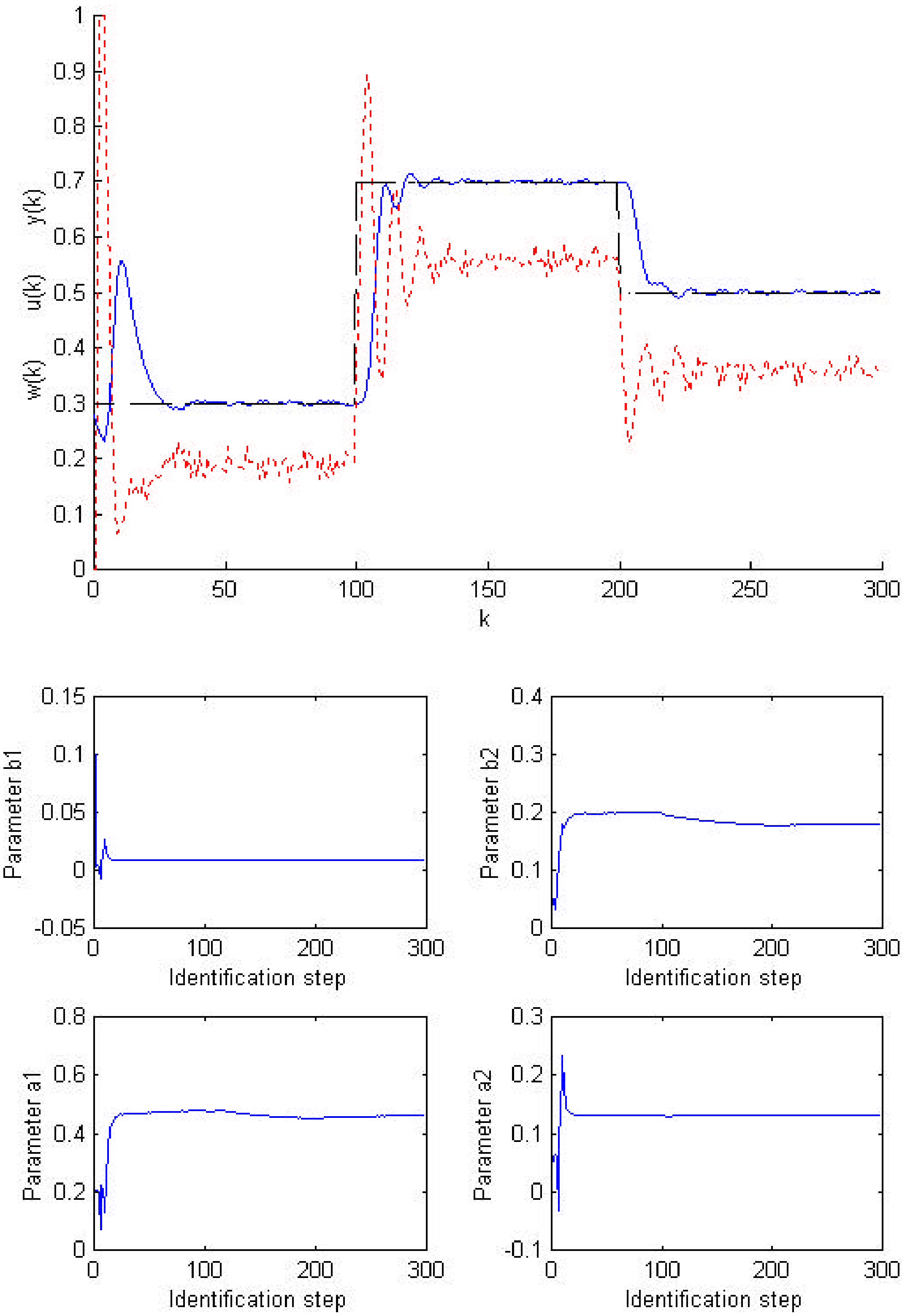


Figure 7. Air flow control of the laboratory through-flow heater (***d***- model identification) and convergence of parameter estimates ($y(k)$ - solid, $u(k)$ - dotted, $w(k)$ - dashed)

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