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Variable Structure Control with Varying Bounds of Robot Manipulators *

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Abstract

A controller for robot manipulators is proposed in this paper. The design is based on a Lyapunov approach with V.S.C. and sliding mode technique. Fundamental properties of the robot, as well as some engineering considerations are taken into account during the design procedure. Chattering is tackled by indexing the magnitude of the bound to the tracking error. Simulations reveal a great reduction of chattering while maintaining the controller performance.

1 Introduction

Increasing demand for high performance robots has lead to the development of various advance control techniques. Two general controller approaches may be considered for robot manipulators, the model-based approach and the non-model-based approach. The model based controllers consider some of the system structures in their designs. In contrast non-model-based controllers do not take account of the system properties. The latter group of controllers has been largely used in industry because of the ease of implementation and relatively good performances. However the increasing demand for high speed response and accuracy leads to more and more interest in the model-based controllers. The idea behind model-based controllers is to use the dynamic equations of the system as feedforward terms of the control algorithm, transforming the non-linear system into a new linear system incorporating the robot and its controller. This linearization is only effective if the exact dynamic equations are available, which in practice is not the case as uncertainties on the parameters as well as on the model equations exist. To deal with these imperfections, a robust algorithm can be used. Variable Structure Control in conjunction with sliding mode is an efficient technique, that can render the system insensitive to parameter variations as well as to disturbances as demonstrated by (Drazenovic, 1969; Kaynak *et al.*, 1985 or Utkin, 1978).

The first application of V.S.C. with sliding mode to robotics is due to (Young, 1978), and the number of reported experiments since has kept increasing with (Ritcher *et al.*, 1982; Slotine, 1984; Kai S. Yeung *et al.*, 1988 and Chee-Fai Yung *et al.*, 1994) to only cite those ones.

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2 Proposed V.S.C. algorithm

Accurate trajectory following in robotics is an important problem, and many control algorithms using V.S.C. and sliding mode have been proposed by (Slotine *et al.*, 1983; Bailey *et al.*, 1987; Wijesoma *et al.*, 1990; Yi-Feng Chen *et al.*, 1990; Gao *et al.*, 1990; Gorez, 1997 and Yu Tang, 1998). The limitation with the use of sliding mode is the high frequency switching, commonly known as chattering. Chattering is unacceptable in robotics as it may excite unmodeled high frequency modes, which could damage the robot manipulator. The general approach to over come the problem is to replace the non-linear switching function by a smooth one as in (Slotine *et al.*, 1983 and Ambrosino *et al.*, 1984). This method however seriously alters the performance of the controller, because of the high degree of smoothness needed to completely overcome chattering.

The proposed V.S.C. algorithm is based on a general Lyapunov approach, physical constraints are used to establish the convergence to zero of the errors. The problem of chattering is overcome by making the magnitude of the discontinuous element a function of the errors.

The design procedure starts with the selection of n linear sliding surfaces S, where n is the number of joints of the robot manipulator.

$$S_i = (\dot{\theta}_i - \dot{\theta}_{di}) + \lambda(\theta_i - \theta_{di}), i = 1, ..., n \tag{1}$$

where θ_i and $\dot{\theta}_i$ are the angular position and velocity of the ith joint, and θ_{di} and $\dot{\theta}_{di}$ the are demand angular position and velocity for the ith joint. By the defining the position error $(\theta - \theta_d)=\widetilde{\theta}$ and $S = [S_1...S_n]^T$.

$$S = \dot{\widetilde{\theta}} + \lambda\widetilde{\theta} \tag{2}$$

and

$$\dot{S} = \ddot{\widetilde{\theta}} + \lambda\, \dot{\widetilde{\theta}} \tag{3}$$

Now define

$$e = (-\,\dot{\theta}_d + \lambda\, \widetilde{\theta}) \tag{4}$$

$$\dot{e} = (-\,\ddot{\theta}_d + \lambda\, \dot{\widetilde{\theta}}) \tag{5}$$

The inverse dynamics of the robot are given as

$$Ta = M(\theta)\,\ddot{\theta} + C(\theta, \dot{\theta})\,\dot{\theta} + G(\theta) \tag{6}$$

$$M(\theta)\,\ddot{\theta} = Ta - C(\theta, \dot{\theta}) - G(\theta) \tag{7}$$

where $M(\theta)$ is the inertia matrix, which is symmetric positive definite and $\ddot{\theta}$ is the joint acceleration. $C(\theta, \dot{\theta})\ \dot{\theta}$ represents the Coriolis and centripetal effects with $\dot{\theta}$ the joint velocity and the vector $C(\theta, \dot{\theta})$ is defined such that it verifies the skew-symmetric properties: $x^T \left[\dot{M}(\theta) - 2C(\theta, \dot{\theta})\right] x = 0$, for all $x \in R^n$. The gravitational torque is given by $G(\theta)$.

The Lyapunov function used for the design : $V = \frac{1}{2}S^T.M.S$, represents the pseudo-kinetic energy of the system. Differentiating the function V with respect to time gives:

$$\dot{V} = S^T.M.\dot{S} + \frac{1}{2}S^T.\dot{M}.S \tag{8}$$

$$\dot{V} = S^T.M.\ddot{\theta} + S^T.M.(-\ddot{\theta} + \lambda\dot{\tilde{\theta}}) + \frac{1}{2}S^T.\dot{M}.S \tag{9}$$

Substituting equation (7) and equation (4) and its derivative equation (5) into equation (9), gives:

$$\dot{V} = S^T.\left[Ta - C(\theta,\dot{\theta})\dot{\theta} - G(\theta) - C(\theta,\dot{\theta})e + C(\theta,\dot{\theta})e\right] + S^T.M.\dot{e} + \frac{1}{2}S^T.\dot{M}.S \tag{10}$$

Using the skew-symmetry property, the expression is reduced to:

$$\dot{V} = S^T.\left[Ta - G(\theta) + C(\theta,\dot{\theta})e + M.\dot{e}\right] \tag{11}$$

Letting $Ta = \widehat{G}(\theta) - \widehat{C}(\theta,\dot{\theta})e - \widehat{M}.\dot{e} + Ts$, and substituting it into equation (11) gives

$$\dot{V} = S^T.\left[\widetilde{G}(\theta) - \widetilde{C}(\theta,\dot{\theta})e - \widetilde{M}.\dot{e} + Ts\right] \tag{12}$$

where $(\widehat{\cdot})$ is the parameter estimate and $(\widetilde{\cdot}) = (\widehat{\cdot}) - (\cdot)$, is the parameter error.

The value for Ts that will guarantee the negativeness of $\dot{V}$ is found by considering the physics of the system.

Assumption 1 For a robot manipulator, since the link lengths and joint displacements (linear or rotational) are bounded, the gravitational torque and its estimate are bounded, and so is the gravitational error $\widetilde{G}_i(\theta)$ for each link. i.e. : $\widetilde{G}_i(\theta) < Kg_i$.

Assumption 2 The $C(\theta,\dot{\theta})$ matrix relates the velocity/position to torque. This matrix depends linearly on the velocity. Again, for a robot manipulator with bounded joint displacements, the velocity of the joint is bounded as well (maximum velocity of the actuator). The matrix $C(\theta,\dot{\theta})$ is then bounded and so is its estimate (maximum velocity profile set in the trajectory generation control algorithm). So it can be said that the error $\widetilde{C}(\theta,\dot{\theta})$ is also bounded. i.e. : $\widetilde{C}_{ij}(\theta,\dot{\theta}) < Kc_{ij}$.

Assumption 3 The inertia matrix is a function of the position of the joint. Since each joint position is bounded, each term of the inertia matrix is bounded, and so are the estimated terms, the errors on the inertia matrix terms are then bounded. i.e. : $\widetilde{M}_{ij} < Km_{ij}$.

Assumption 4 The function $e = (-\dot{\theta}_d + \lambda\widetilde{\theta})$, represents a velocity demand function, which incorporates the position error. The velocity demand is bounded in the trajectory generation algorithm (safety limit). The position error is assumed to be and to remain bounded by taking Ke_i sufficiently large. i.e. : $e_i < Ke_i$.

Assumption 5 For $\dot{e} = (-\ddot{\theta}_d + \lambda\dot{\tilde{\theta}})$, a similar argument as for e is used. The demand acceleration is bounded in the trajectory generation algorithm (safety limit), and the velocity error is assumed to be and to remain bounded by taking $K\dot{e}_i$ sufficiently large. i.e. : $\dot{e} < K\dot{e}_i$.

The two last assumptions will to be smoothen later on.

Using all the previous assumptions, the inverse dynamics error $(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e})$ can be bounded by some coefficients.

$$\left|\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right|_i < K_i \tag{13}$$

$$\left(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right)_i - K_i < 0 \tag{14}$$

The negative definiteness of $\dot{V}$ as well as the reaching condition for equation (12), can be guaranteed by taking $Ts_i = -K_i.sign(S_i)$.

If $S_i > 0$,

$$\left(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right)_i - K_i.sign(S_i) < 0 \tag{15}$$

$$S_i.\left(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right)_i - K_i.sign(S_i) < 0 \tag{16}$$

If $S_i < 0$,

$$\left(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right)_i - K_i.sign(S_i) > 0 \tag{17}$$

$$S_i.\left(\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e}\right)_i - K_i < 0 \tag{18}$$

So

$$\dot{V} = S^T.\left[\widetilde{G}(\theta) - \widetilde{C}(\theta, \dot{\theta})e - \widetilde{M}. \dot{e} - K.sign(S)\right] < 0 \tag{19}$$

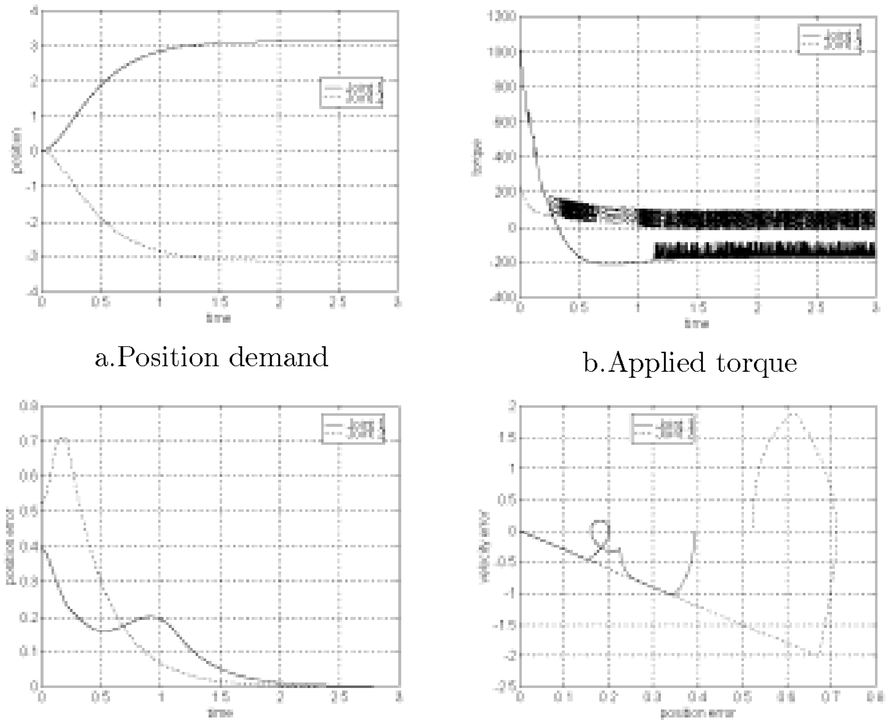
The controller has the form :

$$Ta = -\widehat{M}. \dot{e} - \widehat{C}(\theta, \dot{\theta})e + \widehat{G}(\theta) - K.sgn(S) \tag{20}$$

3 Evaluations

3.1 Simulations

The initial control law, equation (20) was simulated on a two degrees of freedom robot manipulator as described in (Slotine's book, 1986, p.229). The kinetic terms used to compute the control law presented an error on their values of 20%. The position demand is shown in figure 1.a, the robot manipulator starts with a position error of $\frac{\Pi}{8}$ for joint 1 and $\frac{\Pi}{6}$ for joint 2. The torque applied to the system (figure 1.b), exhibits a large amount of chattering which would be unacceptable in a real application. Figure 1.c and 1.d shown the global convergence of the errors to zero.



a.Position demand b.Applied torque

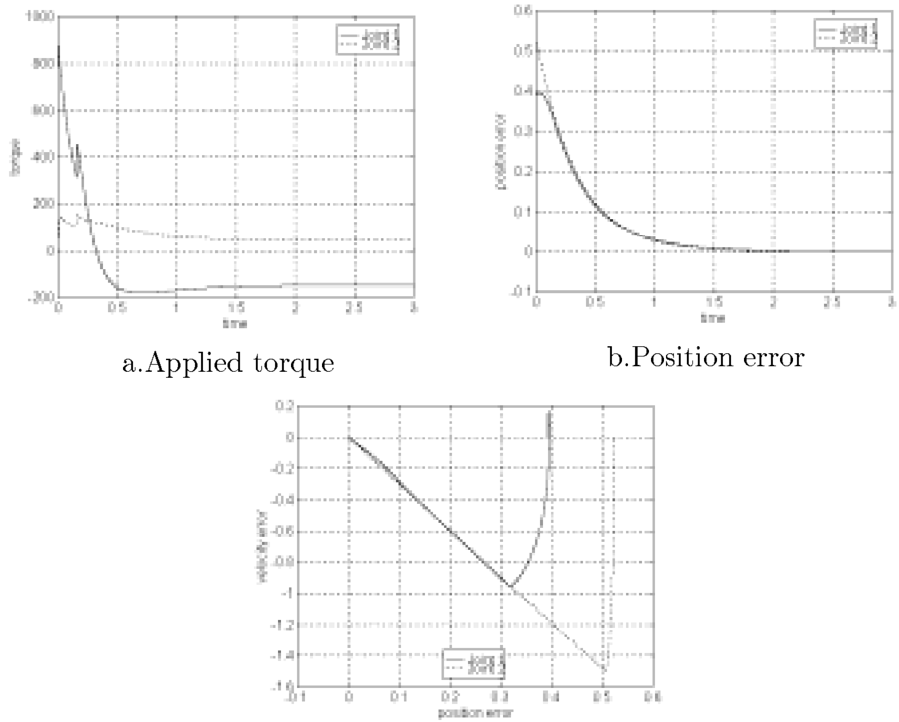
c.Error position d.State space tracking error

Figure 1

The problem with the initial design is the high degree of chattering, which is due to the large value of the switching bound. This large value is mainly due to assumptions 4 and 5, where the magnitude is chosen to be always greater than the position and velocity error. This over large magnitude can be drastically reduced by indexing it with the position and velocity error. In the following parts, the magnitude of the bounds are made directly proportional to the position and velocity error. Initial tests reveal that with this form of control, an offset occurs in the steady state. This phenomena is simply due to the fact that the magnitude of the switching term is not large enough to guarantee the sliding condition when the error is close to zero, this is because of the linearity between the magnitude of the switching terms of the errors. To overcome this problem, an integral term is added to the control law such that the new control law is now of the form :

$$Ta = -\widehat{M}.\,\dot{e} - \widehat{C}(\theta,\dot{\theta})e + \widehat{G}(\theta) - K(\dot{\tilde{\theta}},\tilde{\theta}).sgn(S) + K.\int_{0}^{T}\tilde{\theta}.dt \tag{21}$$

Simulation of the proposed control law, equation (21) on the same system (figure 2), reveals a great reduction of the chattering in the applied torque while guaranteeing trajectory tracking and zero steady state error.



a.Applied torque b.Position error

c.State space tracking error
Figure 2

4 Conclusion

In this paper, a new type of controller for robot manipulator has been proposed, equation (21). The design of this controller is based on a Lyapunov approach with sliding mode. During the derivation of the algorithm, the robot fundamental properties are taken into account and engineering considerations are used to establish bounds on the system uncertainties. The initial form of the controller is tested and the effectiveness of the approach is confirmed by simulations. To guarantee the negativeness of the Lyapunov function derivative and that sliding mode occurs on each sliding surfaces, the bounds have to conservatively selected. This imposed a large value on the magnitude of the switching terms and results in a high level of chattering on the applied torque, making the algorithm unsuitable for practical applications. By reconsidering some of the terms to be bounded, it appears that the constant high magnitude for the bounds can be reduced by indexing it with the position and velocity error. This results in significant reduction of the chattering magnitude, an offset however is generated. An integral term is then added to the control law to remove the steady state error. Simulations of the control laws reveal that chattering is greatly reduced if not completely eliminated whilst maintaining the performances of the control algorithm. The effectiveness of the scheme for the purpose of trajectory following is clearly demonstrated by simulations results and promising practical implementations are currently conducted (Nigrowsky *et al.*, May 1999; Nigrowsky *et al.*, June 1999).

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