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Optimal Combination of Identification and Control for Bounded-Noise ARX Systems

Andrzej Królikowski*
Department of Control, Robotics and Computer Science,
Technical University of Poznań
ul.Piotrowo 3a
60-965 Poznań, POLAND

Abstract

An optimal combination of sequential identification and control for linear bounded-input bounded-noise discrete-time ARX system is considered. Various configurations of identyfying/controlling sequences are investigated in order to find an optimal trade-off. A second-order tracking control system is simulated.

1 Introduction

The sequential identification and control can be considered as an alternative approach to the simultaneous identification and control (adaptive control, self-tuning). The reasons to use this approach could be problems with convergence of recursive parameter estimator in adaptive closed-loop or poor performance of adaptive control, see [1] for more details. This problem has also been considered in [2] for the case of stochastic noise systems. In this paper, an optimal combination of identification and control for bounded noise ARX systems is considered. The input applied in both identification and control periods is also assumed to be bounded in amplitude. Two identification algorithms [3],[4] are taken as a base for a design of identifying sequences, and the tracking is taken as a control objective. An optimal trade-off between identification and control periods has to be found in order to minimize the cost function.
The physical plant is assumed to be represented by a model of the ARX type

$$A(q^{-1})y_t = q^{-1}B(q^{-1})u_t + \eta_t \tag{1}$$

where A, B, are polynomials in the backward shift operator q^{-1}, i.e.

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{na} q^{-na}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{nb} q^{-nb}$$

y_t is the output, and u_t is the control input bounded in amplitude

$$| u_t | \leq \alpha \tag{2}$$

Different models can be used for η_t but as a result, the disturbance η_t is considered to be bounded with known upperbound δ_t

$$| \eta_t | \leq \delta_t \tag{3}$$

*akrol@omega.ar-kari.put.poznan.pl

2 Identification Algorithms

For parameter identification of bounded-noise model (1), the methods EW-RLS [3] or FHMV [4] are used.
The model (1) can be represented in the regressor form

$$y_t = \theta^T \varphi_t + \eta_t \tag{4}$$

where $\theta^T = (a_1, \cdots, a_{na}, b_0, \cdots, b_{nb})$ and

$$\varphi_t^T = (-y_{t-1}, \cdots, -y_{t-na}, u_{t-1}, \cdots, u_{t-nb-1}) \tag{5}$$

2.1 The EW-RLS algorithm

The exponentially weighted recursive least squares (EW-RLS) identification algorithm derived in [3] for model (4) has a form

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{d_t P_{t-1}\varphi_t}{\gamma_t}(\mid \epsilon_t \mid - \mid \delta_t \mid) sgn(\epsilon_t) \tag{6}$$

$$P_t = \frac{1}{\lambda}[P_{t-1} - \frac{d_t P_{t-1}\varphi_t\varphi_t^T P_{t-1}}{\gamma_t}(1 - \mid \frac{\delta_t}{\epsilon_t} \mid)] \tag{7}$$

where $0 < \lambda \leq 1$, $\gamma_t = \varphi_t^T P_{t-1}\varphi_t$, $\epsilon_t = y_t - \hat{\theta}_{t-1}^T \varphi_t$ and

$$d_t = \begin{cases} 0 & \text{if } \gamma_t = 0 \text{ or } \mid \epsilon_t \mid < \mid \delta_t \mid \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

2.2 The FHMV algorithm

The minimal volume recursive algorithm of Fogel-Huang (FHMV) [4] is given by

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{q_t}{\delta_t^2} Z_t \varphi_t \epsilon_t \tag{9}$$

$$Z_t = P_{t-1} - \frac{q_t P_{t-1}\varphi_t\varphi_t^T P_{t-1}}{\delta_t^2 + q_t\gamma_t} \tag{10}$$

where $P_t = z_t Z_t$, and

$$z_t = 1 + q_t - \frac{q_t}{\delta_t^2 + q_t\gamma_t}\epsilon_t^2$$

The way of computing the coefficients q_t is given as follows [4]

$$q_t = \begin{cases} 0 & \text{if } \alpha_2^2 - 4\alpha_1\alpha_3 < 0 \\ & \text{or } -\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3} \leq 0 \\ \frac{-\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}}{2\alpha_1} & \text{otherwise} \end{cases}$$

where $\alpha_1 = (n-1)R_t^2, \alpha_2 = R_t(2n - 1 - R_t^2 + \epsilon_t^2), \alpha_3 = n(1 - \epsilon_t^2) - R_t, R_t = \varphi_t^T P_{t-1}\varphi_t$ and $n = n_a + n_b + 1$.

2.3 Design of identifying sequences

The goal of the input identifying sequence $\{u_t^I\}$ is twofold: to speed up the achievement of identification accuracy given by some measure, i.e. to obtain a given accuracy in the shortest possibly time, or to achieve, in a given identification time, the highest possibly accuracy determined by some measure.

2.3.1 Input sequence design for EW-RLS

The identyfying input sequence is proposed to cause the steepest descent of $detP_t$ which is proportional to the volume of ellipsoid associated with parameter estimates error. From (7), it follows that

$$\max_{|u_{t-1}|\leq\alpha} \frac{detP_{t-1}}{detP_t} \equiv \max_{|u_{t-1}|\leq\alpha} \gamma_t \tag{11}$$

Taking into consideration

$$\gamma_t = s_{t-1}u_{t-1}^2 + t_{t-1}u_{t-1} + w_{t-1}$$

the maximization (11) yields

$$u_{t-1}^I = \alpha \; sgn \; t_{t-1} \tag{12}$$

where

$$\begin{gathered} t_{t-1} = p_{t-1}^{na+1,1}y_{t-1} + p_{t-1}^{na+1,2}y_{t-2} + \cdots + \\ p_{t-1}^{na+1,na}y_{t-1} + p_{t-1}^{1,na+1}y_{t-1} + p_{t-1}^{2,na+1}y_{t-2} + \cdots + \\ p_{t-1}^{na,na+1}y_{t-na+1}p_{t-1}^{na+2,na+1}u_{t-1} + \cdots + \\ p_{t-1}^{na+nb,na+1}u_{t-na+1} + p_{t-1}^{na+1,na+2}u_{t-1} + \cdots + \\ +p_{t-1}^{na+1,na+nb}u_{t-na+1} \end{gathered}$$

where $P_{t-1} = \{p_{t-1}^{k,l}\}$ for $k, l = 1, \cdots, na + nb$.

2.3.2 Input sequence design for FHMV

From Z_t and P_t (10), it follows that

$$P_t = z_t(I - \frac{q_t P_{t-1}\varphi_t\varphi_t^T}{1 + q_t\gamma_t})P_{t-1} \tag{13}$$

to give

$$\frac{detP_t}{detP_{t-1}} = \frac{z_t^{na+nb}}{1 + q_t\gamma_t} = \frac{1}{1 + q_t\gamma_t}(1 + q_t - \frac{q_t\epsilon_t^2}{\delta_t^2 + q_t\gamma_t})^{na+nb} \tag{14}$$

So, again the identyfying input has to be chosen as

$$\max_{|u_{t-1}|\leq\alpha} \frac{detP_{t-1}}{detP_t} \equiv \min_{|u_{t-1}|\leq\alpha} \frac{detP_t}{detP_{t-1}} \equiv \max_{|u_{t-1}|\leq\alpha} \gamma_t \tag{15}$$

yielding the similar result as (12).

3 Tracking Control Problem

The tracking controller has a form [5]

$$u_t = \hat{\theta}_t^{*T} \varphi_t^{'} \tag{16}$$

where the controller parameters $\hat{\theta}_t^*$ are estimates of θ^* defined below and

$$\varphi_t^{'T} = (-y_t, \cdots, -y_{t-na+1}, -u_{t-1}, \cdots, -u_{t-nb}, r_{t+1}) \tag{17}$$

where r_t is a known reference signal to be followed by the output y_t. Here it holds

$$y_{t+1} = \alpha(q^{-1})y_t + \beta(q^{-1})u_t + \nu_t \tag{18}$$

where $\alpha(q^{-1}) = G(q^{-1}), \beta(q^{-1}) = F(q^{-1})B(q^{-1})$, $\mid \nu_t \mid \leq \nu_0$ while $F(q^{-1}), G(q^{-1})$ can be determined from diofantine equation

$$F(q^{-1})A(q^{-1}) + q^{-1}G(q^{-1}) = 1 \tag{19}$$

with $deg\alpha = na - 1, deg\beta = nb$. Here

$$\theta^* = (\theta_y^{*T}, \theta_u^{*T}, \theta_r^*)^T \tag{20}$$

where

$$\theta_y^{*T} = (\alpha_0^{'}, \cdots, \alpha_{na-1}^{'})$$

$$\theta_u^{*T} = (\beta_1^{'}, \cdots, \beta_{nb}^{'}), \quad \theta_r^* = \frac{1}{\beta_0}$$

with

$$\alpha_i^{'} = \frac{\alpha_i}{\beta_0}, \quad i = 1, \cdots, na - 1$$

$$\beta_j^{'} = \frac{\beta_j}{\beta_0}, \quad j = 1, \cdots, nb, \quad \beta_0^{'} = \beta_0 = b_0$$

Equation (18) can be written as

$$u_t = \theta^{*T} \varphi_t - \bar{\nu}_t \tag{21}$$

where

$$\varphi_t^T = (-y_t, \cdots, -y_{t-na+1}, -u_{t-1}, \cdots, -u_{t-nb}, y_{t+1}) \tag{22}$$

and $\mid \bar{\nu}_t \mid \leq \frac{\nu_0}{b_0}$. Looking at (16),(21) and (22) one can see the tracking and disturbance rejecting mechanism of the controller (16). The bounded control applied to the system is then

$$u_t^C = sat(u_t; \alpha) \tag{23}$$

4 Combination of Identification and Control

The combination of identification and control consists of consecutive identification in open-loop configuration, when the bounded identifying sequence $\{u_t^I\}$ is applied to the system, and the control in closed-loop configuration, when the control sequence $\{u_t^C\}$is applied. Identification and control have to be achieved within the time interval $[0, T]$ where T is given. The identification of system parameters lasts during the period $[0, T_I]$. So, the identification period T_I and the amplitude α of an identyfying signal are the only two experiment parameters considered. A controller obtained from the parameter estimates is used during the control time T_C, so $T = T_I + T_C$. The impact of identification accuracy measured by $detP_t$ on the control performance measured by the sum of tracking errors can be analyzed in order to find an optimal trade-off. To this end, various configurations of sequential combination of EW-RLS and FHMV identification together with tracking control are simulated.

5 Simulations

Consider the model (4) with numerical parameter values $a_1 = -1.8, a_2 = 0.9, b_0 = 1.0, b_1 = 0.5$, where η_t was taken as a truncated zero mean normal variable with variance $\sigma_\eta^2 = 0.2$, and $\delta_t = \delta = 0.4$. System parameters were identified using EW-RLS and FHMV algorithms with $\hat{\theta}_0 = 0$, $P_0 = 100I$, and $\lambda = 0.95$.
The identyfying input is taken as in (12). For the above example, we have for t_{t-1}

$$t_{t-1} = p_{t-1}^{3,1} y_{t-1} + p_{t-1}^{3,2} y_{t-2} +$$

$$p_{t-1}^{1,3} y_{t-1} + p_{t-1}^{2,3} y_{t-2} + p_{t-1}^{4,3} u_{t-2} + p_{t-1}^{3,4} u_{t-2}$$

The control input is taken as in (23) where for the considered example we have

$$u_t = \frac{1}{b_0}(a_1 y_t + a_2 y_{t-1} - b_1 u_{t-1} + r_{t+1})$$

and

$$\alpha_0^{'} = \frac{-a_1}{b_0}, \quad \alpha_1^{'} = \frac{-a_2}{b_0}, \quad \beta_1^{'} = \frac{b_1}{b_0}, \quad \theta_r^* = \frac{1}{b_0}$$

The reference signal was taken as $r_t = 0$ in the identification period, i.e. for $t = 1, \cdots, T_I$, while in the control period $r_t = 4$, i.e. for $t = T_I + 1, \cdots, T$.
The optimal combination of consecutive identification and control periods is considered regarding the cost $J = \frac{1}{T}\sum_{t=1}^{T} e_t^2$, where the tracking error $e_t = r_t - y_t$, and $T = 50$. For optimization, the sample cost $\bar{J}$ averaged over 30 realizations was taken into consideration. The obtained results concerning optimal identification periods T_I^{opt} are shown in Figs.1,2,3 for the EW-RLS method with $\alpha = 4, 6, 8$, where $T_I^{opt} = 5, 6, 9$, respectively. The corresponding plots for the FHMV method are given in Figs.4,5,6, where $T_I^{opt} = 10, 10, 9$, respectively. Plot of signals u_t^I, u_t^C, y_t is shown in Fig.7 for $\alpha = 4$ and $T_I^{opt} = 5$ when the EW-RLS method is used. In Fig.8, the corresponding plot is shown for $T_I^{opt} = 10$ when the FHMV method is used. The decreasing rate of $\bar{J}_t$ corresponding to Figs.1,2,3 with optimal T_I^{opt} is given in Fig.9 for the EW-RLS method. The corresponding plots of $\bar{J}_t$ for the FHMV method are shown in Fig.10. Additionally, the plots of estimates for EW-RLS and FHMV methods are presented in Figs.11,12, respectively for the identification period $T_I = 10$.

6 Conclusions

An optimal combination of identification and control problem is formulated and corresponding algorithms are proposed for an ARX model with bounded disturbance. A second-order example is simulated showing that the EW-RLS identification algorithm converges faster than the FHMV algorithm. The obtained results show an optimal interplay between T_I and T_C regarding the cost $\bar{J}$ for a given α, where the identification period T_I^{opt} is shorter for the EW-RLS method than for the FHMV method. It can also be seen that T_I^{opt} depends on the imposed constraint α. An interesting, however to be expected, fact is that when the constraint α is enough large, i.e. when the control signal does not saturate, then the obtained costs $\bar{J}$ for both methods are almost equal with the same T_I^{opt}, see the case $\alpha = 8, T_I^{opt} = 9$.

References

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[5] Annaswamy, A.M. and Karason S.P. (1995): "Discrete time adaptive control in the presence of input constraints," *Automatica*, **31**, No.10, pp.1421-1431.

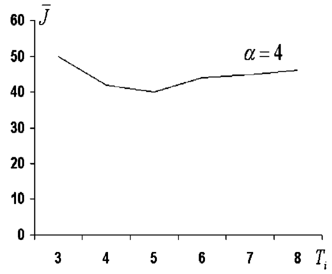

Figure 1: Optimization results, $T_I^{opt} = 5$

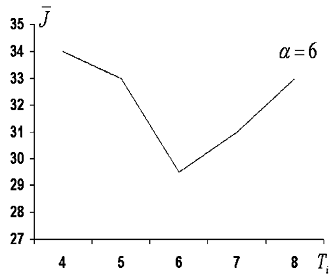

Figure 2: Optimization results, $T_I^{opt} = 6$

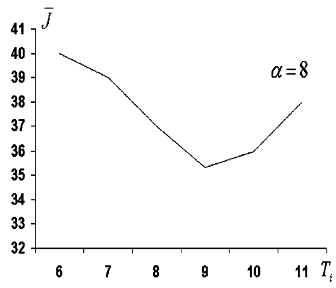

Figure 3: Optimization results, $T_I^{opt} = 9$

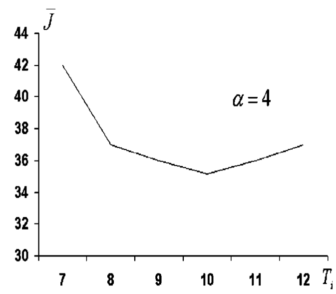

Figure 4: Optimization results, $T_I^{opt} = 10$

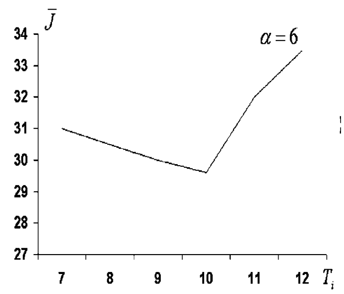

Figure 5: Optimization results, $T_I^{opt} = 10$

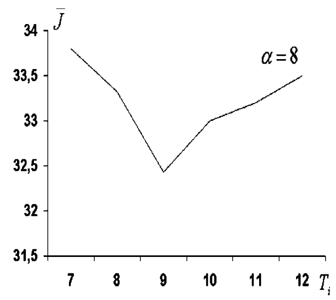

Figure 6: Optimization results, $T_I^{opt} = 9$

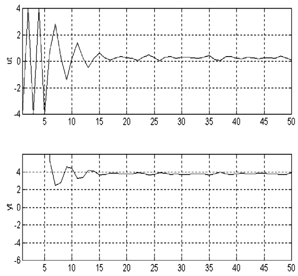

Figure 7: Identification/control: $\alpha = 4, T_I^{opt} = 5$

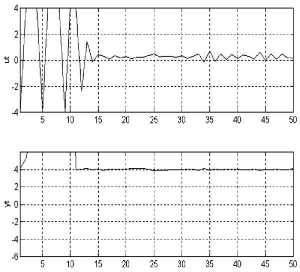

Figure 8: Identification/control: $\alpha = 4, T_I^{opt} = 10$

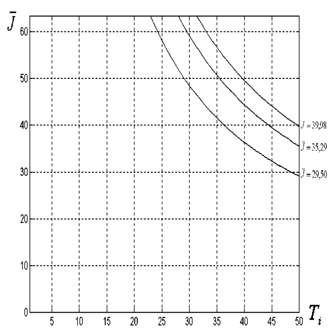

Figure 9: Sample cost $\bar{J}$

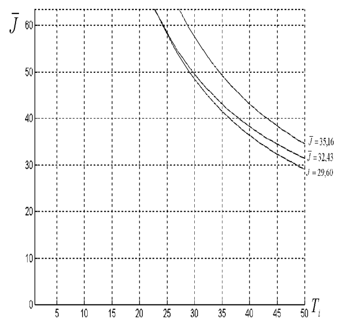

Figure 10: Sample cost $\bar{J}$

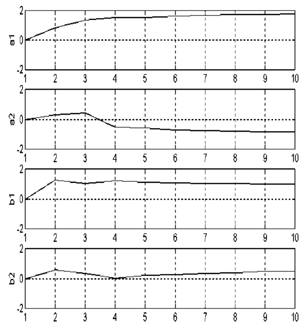

Figure 11: Estimates for EW-RLS method

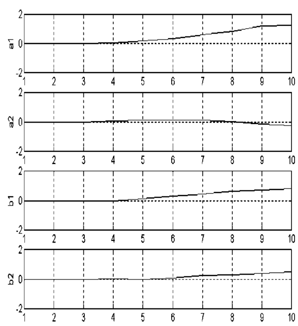

Figure 12: Estimates for FHMV method