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Active Suspension Control of Ground Vehicle Heave and Pitch Motions

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Abstract

Ride quality depends on a combination of vertical displacement (heave) and angular displacement (pitch). Road irregularities are the main factor affecting ride comfort. Suspension elements between the road wheels and the vehicle body generate vertical forces which excite both heave and pitch motions. An active controller design based on time-scale separation and an "input decoupling transformation" is given. It is shown to give better performance that conventional passive suspension control.

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1 Introduction

The study of ride quality evaluates the passenger's response to road/terrain irregularities with the objective of improving comfort and road isolation while maintaining wheel/ground contact. Ride problems mainly arise from vehicle vibrations, which may be induced by variety of sources including external factors, such as roadway roughness or aerodynamics forces, or they may be internally generated forces produced by vehicle subsystems, such as the engine, powertrain, or the suspension mechanisms. Usually the surface irregularity acts as a major source that excites the vibration of the vehicle. Passenger comfort in a road vehicle depends on a combination of vertical motion (heave) and angular motion (pitch). Suspension elements between the road wheels and the vehicle body generate vertical forces which excite both heave and pitch motions. For some vehicle types the pitch response is very important while for other types the pitch is less important than heave.

Suspension system designs are mostly based on ride analysis. Vehicle suspensions using various types of springs, dampers, and linkages with tailored flexibility in various directions have been developed over the last century since the beginning of the automobile age. The simplest, and most common types of suspensions are passive in the sense that no external sources of energy are required. Passive suspensions do not allow independent control of pitch and heave.

With the development of modern control theory and the extraordinary development of inexpensive and reliable electronic devices it has become clear that increased performance is theoretically possible for suspensions which can sense aspects of vehicle motion and produce forces or motions through actuators in ways impossible for conventional passive suspensions. Karnopp (1987) presented a passive and active control of road heave and pitch motions for approximately uncoupled motions. Wong's work (1978, Ch. 7), considered a two-degrees-of Freedom Vehicle Model to study the heave and pitch motions neglecting the vehicle suspension dynamics.

It is the purpose of this paper to present an electronic control scheme to improve suspension performance in heave and pitch control as compared with simple passive suspensions. In general, the heave and pitch motions are coupled and an impulse at the front or rear wheels excites both motions. This means that pitch controllers and heave controllers cannot be independently designed. Therefore, our model includes the full vehicle suspension dynamics considering both heave and pitch motions. Yet, our controller design is based on time-scale separation and an "input decoupling transformation" that allows streamlined design yet gives performance better than over-simplified decoupled techniques.

It is difficult to improve body motion at frequencies above the wheel frequency w_0 without using complex controllers or mechanical modifications to the vehicle. A backstepping control approach is described in (Lin and Kanellakopoulos, 1997). Mechanical modifications involve adding extra damping masses on the unsprung mass. In our controller we improve performance above the wheel frequency by simply rolling off the damping coefficient above w_0. This cannot be accomplished using passive damping, but is not difficult to achieve using active damping control.

The pitch and heave control scheme proposed in this paper is similar to the stability augmentation system (SAS) used in aircraft control as shown in (Stevens and Lewis, 1992), which is simply a feedback control designed to increase the relative damping of a particular mode of motion of the system. This increase is achieved by augmenting one or more of the coefficients of the equation of motion by actuating the control signals in response to motion feedback variables. Heave and pitch natural frequencies are determined by the vehicle suspension dynamics and moment of inertia; and their damping is determined by the rate-dependent dynamics and moments. Automatic control SAS gives heave and pitch modes suitable damping and natural frequencies to enhance ride quality in such a manner that the effects of road excitation, or other disturbances upon the vehicle's motion are reduced significantly.

2 Four-degrees of Freedom Vehicle Model for Pitch and Bounce

The mass of the vehicle body is usually referred to as the sprung mass, whereas the mass of the running gear together with the associated components is referred to as the unsprung mass.

To study the vibrational characteristics of the vehicle, equations of motion based on Newton's second law for each mass have to be formulated (Alleyne *et* al., 1992; Karnopp, 1987; Wong, 1978). Natural frequencies and amplitude ratios can be determined by considering the principal modes (normal modes) of vibration of the system (Wong, 1978). When the excitation of the system is known, the response can be determined by solving the equations of motion.

The up and down linear motion (heave) and the angular motion (pitch) of the vehicle body and the motion of the wheels can be studied using the four-degrees-of-freedom (4-DOF) model shown in Fig. 1.

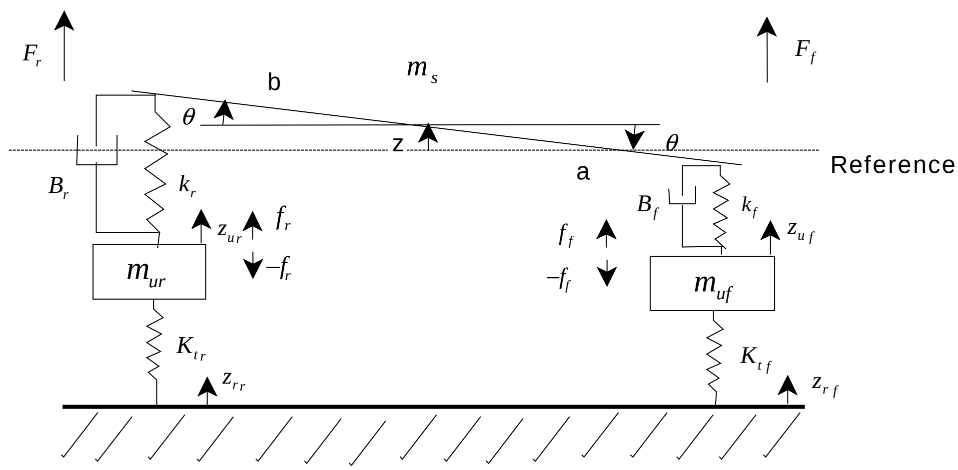


Fig. 1: Heave and Pitch/Terrain Mechanical Subsystem

For simplicity, in this model roll motion is neglected, and the pitch angles are assumed to be small. The mass of the body is m_s and its centroidal moment of inertia is J_y. Unsprung masses on the front and rear wheels are denoted by m_{uf} and m_{ur}, respectively. z_{rf} and z_{rr}, represent the road excitation on the front and rear wheels.

From Fig. 1, it can be seen that the displacements of the sprung masses are given by

Front wheel

$$z_{sf} = z - a \cdot \sin\theta \approx z - a \cdot \theta. \tag{1}$$

Rear wheel

$$z_{sr} = z + b \cdot \sin\theta \approx z + b \cdot \theta. \tag{2}$$

Equivalent forces in both wheels are given by

Front wheel

$$F_f = -k_f(z_{sf} - z_{uf}) - B_f(\dot{z}_{sf} - \dot{z}_{uf}) = -k_f(z - a\theta - z_{uf}) - B_f(\dot{z} - a\dot{\theta} - \dot{z}_{uf}) + Fr_f + f_f. \tag{3}$$

Rear wheel

$$F_r = -k_r(z_{sr} - z_{ur}) - B_r(\dot{z}_{sr} - \dot{z}_{ur}) = -k_r(z + b\theta - z_{ur}) - B_r(z + b\dot{\theta} - \dot{z}_{ur}) - Fr_r + f_r, \tag{4}$$

where Fr_f and Fr_r represent the front and rear wheel tire-road friction.

By applying Newton's second law and using the static equilibrium position as the origin for both the linear displacement of the center if gravity z and angular displacement of the vehicle body θ, the equations of motion for the system can be formulated.

The equation of motion for heave (Force balance in z direction) is

$$\ddot{z} = \frac{-(k_f + k_r)z - (B_f + B_r)\dot{z} + (ak_f - bk_r)\theta + (aB_f - bB_r)\dot{\theta} + k_f z_{uf} + k_r z_{ur} + B_f \dot{z}_{uf} + B_r \dot{z}_{ur}}{m_s} - \frac{Fr_f + Fr_r}{m_s} - g + $$
$$+ \frac{f_f + f_r}{m_s}. \tag{5}$$

The equation of motion for pitch (Moment of balance) is

$$J_y \ddot{\theta} = -F_f a \cos\theta + F_r b \cos\theta \approx -F_f a + F_r b$$
$$= ak_f (z - a\theta - z_{uf}) + aB_f (\dot{z} - a\dot{\theta} - \dot{z}_{uf}) - bk_r (z + b\theta - z_{ur}) - bB_r (\dot{z} + b\dot{\theta} - \dot{z}_{ur}) + aFr_f - bFr_r - af_f + bf_r.$$

Using $J_y = m_s r_y^2$,where r_y is the radius of gyration

$$\ddot{\theta} = \frac{1}{m_s r_y{}^2}\left\{-(a^2 k_f + b^2 k_r)\theta - (a^2 B_f + b^2 B_r)\dot{\theta} + (ak_f - bk_r)z + (aB_f - bB_r)\dot{z} + \right.$$
$$\left. -(ak_f z_{uf} - bk_r z_{ur}) - (aB_f \dot{z}_{uf} - bB_r \dot{z}_{ur}) + aFr_f - bFr_r - af_f + bf_r \right\}. \tag{6}$$

Using (1) and (2), equations (5) and (6) can be rewritten as

$$\ddot{z} = \frac{1}{m_s}\left\{-(B_f + B_r)\dot{z} + (aB_f - bB_r)\dot{\theta} - k_f (z_{sf} - z_{uf}) - k_r (z_{sf} - z_{ur}) + B_f \dot{z}_{uf} + B_r \dot{z}_{ur} - Fr_f - Fr_r + \right.$$
$$\left. - m_s g + f_f + f_r \right\}. \tag{7}$$

$$\ddot{\theta} = \frac{1}{m_s r_y{}^2}\left\{-(a^2 B_f + b^2 B_r)\dot{\theta} + (aB_f - bB_r)\dot{z} + ak_f (z_{sf} - z_{uf}) - bk_r (z_{sr} - z_{ur}) + \right.$$
$$\left. -(aB_f \dot{z}_{uf} - bB_r \dot{z}_{ur}) + aFr_f - bFr_r - af_f + bf_r \right\}. \tag{8}$$

By applying Newton's second law again on the front and rear wheel unsprung masses, the equations of motion can also be formulated.

Front Wheel

$$m_{uf}\ddot{z}_{uf} = B_f (\dot{z}_{sf} - \dot{z}_{uf}) + k_f (z_{sf} - z_{uf}) - m_{uf} g + Fr_f - f_f - K_{tf}(z_{uf} - z_{rf})$$
$$= B_f (\dot{z} - a\dot{\theta} - \dot{z}_{uf}) + (k_f + K_{tf})(z_{sf} - z_{uf}) - K_{tf} z + K_{tf} a\theta - m_{uf} g + Fr_f - f_f + K_{tf} z_{rf}. \tag{9}$$

Rear Wheel

$$m_{ur}\ddot{z}_{ur} = B_r (\dot{z}_{sr} - \dot{z}_{ur}) + k_r (z_{sr} - z_{ur}) - m_{ur} g + Fr_r - f_r + K_{tr}(z_{ur} - z_{rr})$$
$$= B_r (\dot{z} + b\dot{\theta} - \dot{z}_{ur}) + (k_r + K_{tr})(z_{sr} - z_{ur}) - K_{tr} z - K_{tr} b\theta - m_{ur} g + Fr_r - f_r + K_{tr} z_{rr}. \tag{10}$$

Define the state as

$x_1 = z,$ *ride height (heave).*

$x_2 = \dot{z},$ *payload velocity.*

$x_3 = \theta,$ *pitch.*

$x_4 = \dot{\theta},$ *pitch velocity.*

$x_5 = z_{sf} - z_{uf},$ *front wheel suspension travel.*

$x_6 = \dot{z}_{uf},$ *front wheel unsprung mass velocity.*

$x_7 = z_{sr} - z_{ur},$ *rear wheel suspension travel.*

$x_8 = \dot{z}_{ur},$ *rear wheel unsprung mass velocity.*

The state-space equations for the mechanical subsystem are then given by

$$\begin{aligned}
\dot{x}_1 &= x_2\,,\\
\dot{x}_2 &= -\frac{B_f + B_r}{m_s}x_2 + \frac{aB_f - bB_r}{m_s}x_4 - \frac{k_f}{m_s}x_5 + \frac{B_f}{m_s}x_6 - \frac{k_r}{m_s}x_7 + \frac{B_r}{m_s}x_8 - \frac{Fr_f + Fr_r}{m_s} - g + \frac{f_f + f_r}{m_s}\,,\\
\dot{x}_3 &= x_4\,,\\
\dot{x}_4 &= \frac{aB_f - bB_r}{m_s r_y^{\,2}}x_2 - \frac{a^2B_f + b^2B_r}{m_s r_y^{\,2}}x_4 + \frac{ak_f}{m_s r_y^{\,2}}x_5 - \frac{aB_f}{m_s r_y^{\,2}}x_6 - \frac{bk_r}{m_s r_y^{\,2}}x_7 + \frac{bB_r}{m_s r_y^{\,2}}x_8 + \frac{aFr_f - bFr_r}{m_s r_y^{\,2}} +\\
&\quad + \frac{-af_f + bf_r}{m_s r_y^2}\,,\\
\dot{x}_5 &= x_2 - ax_4 - x_6\,,\\
\dot{x}_6 &= -\frac{K_{t\,f}}{m_{u\,f}}x_1 + \frac{B_f}{m_{u\,f}}x_2 + \frac{aK_{t\,f}}{m_{u\,f}}x_3 - \frac{aB_f}{m_{u\,f}}x_4 + \frac{k_f + K_{t\,f}}{m_{u\,f}}x_5 - \frac{B_f}{m_{u\,f}}x_6 + \frac{Fr_f}{m_{u\,f}} - g + \frac{K_{t\,f}}{m_{u\,f}}z_{r\,f} - \frac{f_f}{m_{u\,f}}\,,\\
\dot{x}_7 &= x_2 + bx_4 - x_8\,,\\
\dot{x}_8 &= -\frac{K_{t\,r}}{m_{u\,r}}x_1 + \frac{B_r}{m_{u\,r}}x_2 - \frac{bK_{t\,r}}{m_{u\,b}}x_3 + \frac{bB_r}{m_{u\,r}}x_4 + \frac{k_r + K_{t\,r}}{m_{u\,r}}x_7 - \frac{B_r}{m_{u\,r}}x_8 + \frac{Fr_r}{m_{u\,r}} - g + \frac{K_{t\,r}}{m_{u\,r}}z_{r\,r} - \frac{f_r}{m_{u\,r}}\,.
\end{aligned} \tag{11}$$

In matrix form

$$\begin{aligned}
\dot{x} &= Ax + B_d d + Bf\,,\\
y &= Cx,
\end{aligned} \tag{12}$$

with $f(t)$ a force input $f(t) = \begin{bmatrix} f_f(t) \\ f_r(t) \end{bmatrix}$ from the front and rear suspensions, and $d(t)$ the disturbance $d = \begin{bmatrix} Fr_f & Fr_r & g & z_{f\,r} & z_{r\,r} \end{bmatrix}^T$ where $Fr_f(t), Fr_r(t)$ are the frictions in the front and rear wheels, g is the acceleration due to gravity, and $z_{r\,f}(t), z_{r\,r}(t)$ are the terrain height disturbances in the front and rear wheels. The output $y(t)$ can be selected for specific performance analysis objectives.

The vehicle parameters selected for this study are:
Sprung mass $m_s = 1500$ Kg .
Unsprung masses $m_{u\,f} = m_{u\,r} = 59$ Kg.
Radius of gyration $r_y = 1.2$ m.
Distance between front axle and center of gravity $a = 1.4$ m.
Distance between rear and center of gravity $b = 1.7$ m.
Front spring stiffness $k_f = 35000$ N/m.
Rear spring stiffness $k_f = 38000$ N/m.
Front wheel damping $B_f = 1000$ N/m/s.
Rear wheel damping $B_r = 1100$ N/m/s.
Tire spring constants $K_{t\,f} = K_{t\,r} = 190000$ N/m.

The wheel frequency for this system is given approximately by
$w_0 \approx \sqrt{K_t / m_u} = 56.75$ rad/s or $f_0 = 9.03$ Hz.

3 Active Control System

The proposed closed-loop system is given in Fig. 2. It uses some ideas from aircraft control (Stevens and Lewis, 1992). It consists of inner loops to reject the terrain disturbances, outer loops to stabilize heave and pitch, and an input-decoupling transformation to blend these two control actions.

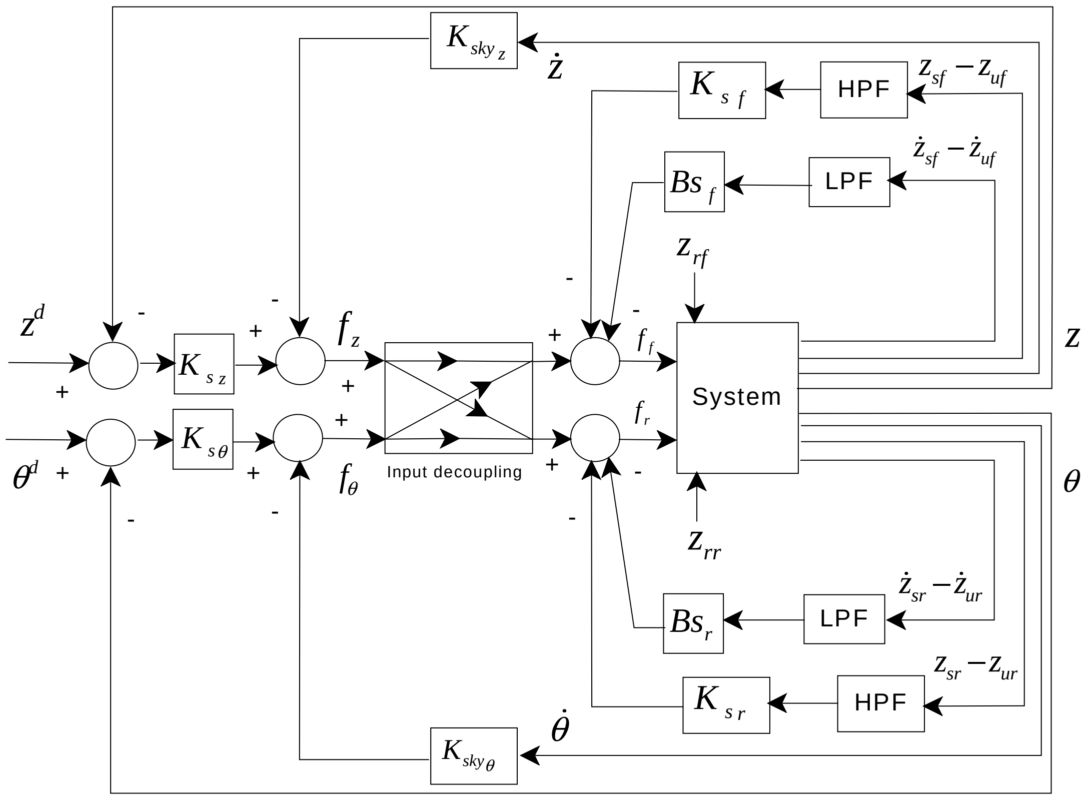


Fig. 2 Generation of Target Strut Force

3.1 Input Decoupling Transformation

According to (11), one has only two control inputs f_f and f_r, yet wishes to control four variables, namely pitch, heave, and front and rear body motions due to road disturbances. One can confront this problem by using inner control loops to affect road disturbance rejection, and outer loops to control pitch and heave. The relation between signals in the inner and outer loops is provided by noting that f_f and f_r are inputs to the equations for $\dot{x}_6$ and $\dot{x}_8$ respectively, while the sum of these two affects heave motion and a weighted difference affects pitch.

Pitch and heave equivalent forces can be decoupled into front and rear suspension forces by the following relations

$$f_z = f_f + f_r \equiv \textit{heave control}, \tag{13}$$

$$f_\theta = -af_f + bf_r \equiv \textit{pitch control}. \tag{14}$$

The equivalent forces can be expressed in matrix form as

$$\begin{bmatrix} f_z \\ f_\theta \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -a & b \end{bmatrix} \begin{bmatrix} f_f \\ f_r \end{bmatrix}. \tag{15}$$

The inverse transforms (i.e. front and rear suspension forces from heave and pitch equivalent forces) are given by

$$\begin{bmatrix} f_f \\ f_r \end{bmatrix} = \frac{1}{a+b} \begin{bmatrix} b & -1 \\ a & 1 \end{bmatrix} \begin{bmatrix} f_z \\ f_\theta \end{bmatrix}. \tag{16}$$

This decoupling scheme is known as a Butterfly input-decoupling transformation (IDT) and it is shown in Fig. 3 and used in Fig. 2 to blend the inner and outer control loops.

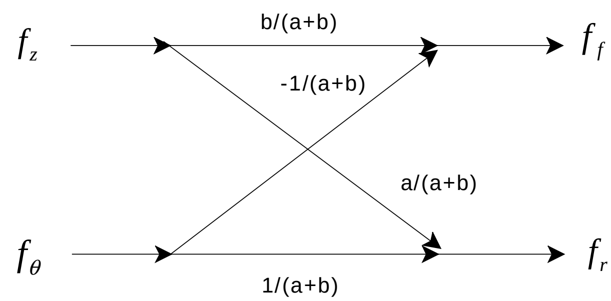


Fig. 3. Butterfly IDT

3.2 Inner Loop Design

It is not possible to decrease body motions at the wheel frequency w_0 using control inputs applied only between the sprung and unsprung masses, as in all standard strut geometries. This is due to the fact that using such control inputs, the system has a pole-zero cancellation at w_0 arising from an uncontrollable mode.

It is not difficult to improve performance below the wheel frequency w_0. This may be accomplished by rolling off the spring constant at low frequencies (LF) or using skyhook damping.

It is difficult to improve body motion at frequencies above the wheel frequency w_0 without using complex controllers or mechanical modifications to the vehicle. A backstepping control approach is described in (Lin and Kanellakopoulos, 1997). Mechanical modifications involve adding extra damping masses on the unsprung mass. In our controller we improve performance above the wheel frequency by simply rolling off the damping coefficient above w_0. This cannot be accomplished using passive damping, but is not difficult to achieve using active damping control.

To roll off the damping constants at high frequency one may introduce low pass filters (LPF) in the damping loops by defining

$$\begin{aligned} x_9 &= \frac{w_{bf}}{s+w_{bf}}(\dot{z}_{sf} - \dot{z}_{uf}) = \frac{w_{bf}}{s+w_{bf}}(\dot{z} - a\dot{\theta} - \dot{z}_{uf}) = \frac{w_{bf}}{s+w_{bf}}(x_2 - ax_4 - x_5), \\ x_{10} &= \frac{w_{br}}{s+w_{br}}(\dot{z}_{sr} - \dot{z}_{ur}) = \frac{w_{br}}{s+w_{br}}(\dot{z} + b\dot{\theta} - \dot{z}_{ur}) = \frac{w_{br}}{s+w_{br}}(x_2 + bx_4 - x_7), \end{aligned} \tag{17}$$

where w_{bf}, w_{br} are the roll-off frequencies for front and rear wheels damping, respectively. This can be realized by adding the state equations

$$\begin{aligned} \dot{x}_9 &= -w_{bf}x_9 + w_{bf}(x_2 - ax_4 - x_5), \\ \dot{x}_{10} &= -w_{br}x_{10} + w_{br}(x_2 + bx_4 - x_7). \end{aligned} \tag{18}$$

To roll off the spring constants at low frequency one may introduce a high pass filter (HPF) by defining

$$x_{5W} = \frac{s}{s+w_{kf}}(z_{sf} - z_{uf}) = \frac{s}{s+w_{kf}} x_5,$$
$$x_{7W} = \frac{s}{s+w_{kr}}(z_{sr} - z_{ur}) = \frac{s}{s+w_{kr}} x_7, \tag{19}$$

where w_{bf}, w_{br} are the roll-off frequencies for the spring constants in the front and rear wheels, respectively. This HPF is a washout circuit like those used in aircraft control. To realize the HPF as a state system one may write

$$x_{5W} = \frac{s}{s+w_{kf}} x_5 = \left[1 - \frac{w_{kf}}{s+w_{kf}}\right] x_5 = x_5 - w_{kf}\bar{x}_5,$$
$$x_{7W} = \frac{s}{s+w_{kf}} x_7 = \left[1 - \frac{w_{kr}}{s+w_{kr}}\right] x_7 = x_7 - w_{kr}\bar{x}_7, \tag{20}$$

and define additional states $x_{11} = \bar{x}_5$ and $x_{12} = \bar{x}_7$ so that

$$\begin{aligned}
\dot{x}_{11} &= -w_{kf}x_{11} + x_5, \\
x_{5W} &= -w_{kf}x_{11} + x_5, \\
\dot{x}_{12} &= -w_{kr}x_{12} + x_7, \\
x_{7W} &= -w_{kr}x_{12} + x_7.
\end{aligned} \tag{21}$$

This feedback scheme is shown in Fig. 2. Note that this scheme cannot be implemented using passive feedback, since the spring constant and damping coefficient are being dynamically filtered.

The desired damping and spring constants are provided through the external feedback loops

$$f = \begin{bmatrix} f_f \\ f_r \end{bmatrix} = \begin{bmatrix} B_{sf} & K_{sf} & 0 & 0 \\ 0 & 0 & B_{sr} & K_{sr} \end{bmatrix} \begin{bmatrix} x_9 \\ x_{5W} \\ x_{10} \\ x_{7W} \end{bmatrix}. \tag{22}$$

The variables selected to study ride quality were heave and pitch accelerations due to front wheel road excitation.

We selected the same values for the damping and spring constants as in the original passive damping system, namely $B_{sf} = 1000$ N/m/sec, $B_{sr} = 1100$ N/m/sec, $K_{sf} = 35000$ N/m, $K_{sr} = 38000$ N/m. The damping roll-off frequencies w_{bf}, w_{br} and the spring constant roll-off frequencies w_{kf}, w_{kr} were selected equal to the wheel frequency w_0. Bode plots on Fig. 4(a) and 4(b) show improvements due to the effects of rolling off B_{sf}, B_{sr} at high frequencies, and rolling off K_{sf}, K_{sr} at low frequencies. It can be seen that the wheel frequency is an invariant.

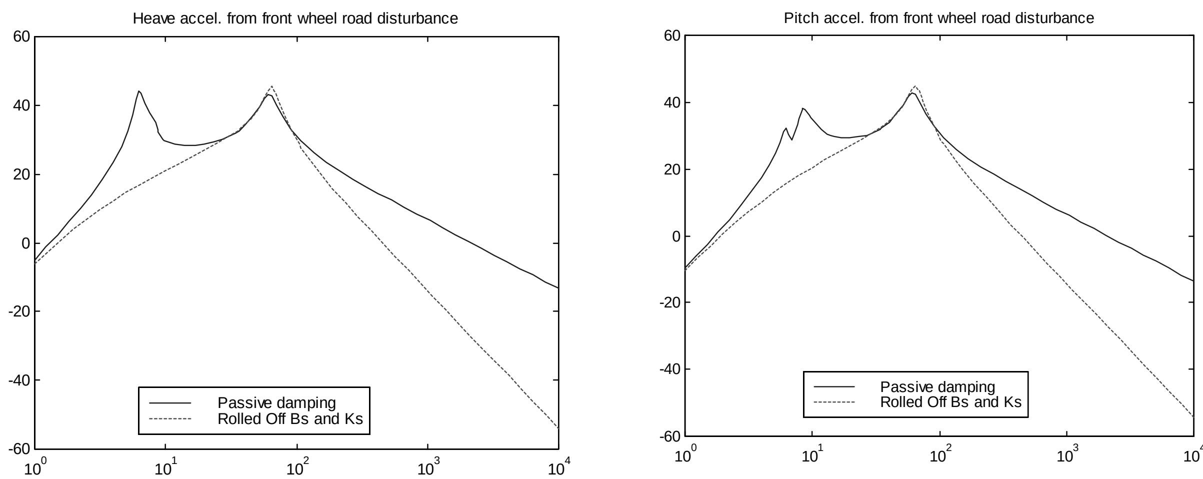
Rolling off the spring constants at frequencies below the wheel frequency w_0 reduces disturbance effects at the heave and pitch vibration modes, but it may be undesirable since the soft spring may hit the suspension travel limits. Soft spring constants require motion limiting logic (or hard springs at the limits) as well as ride height control to center the suspension travel excursions.

3.3 Outer Loop Design

We use "skyhook damping" to generate the heave and pitch equivalent forces. Skyhook damping (Alleyne *et* al., 1992; Karnopp, 1983; Karnopp *et* al., 1991) describes feedback of the "absolute" heave and pitch body velocities rather than the velocity of the body relative to the wheels.

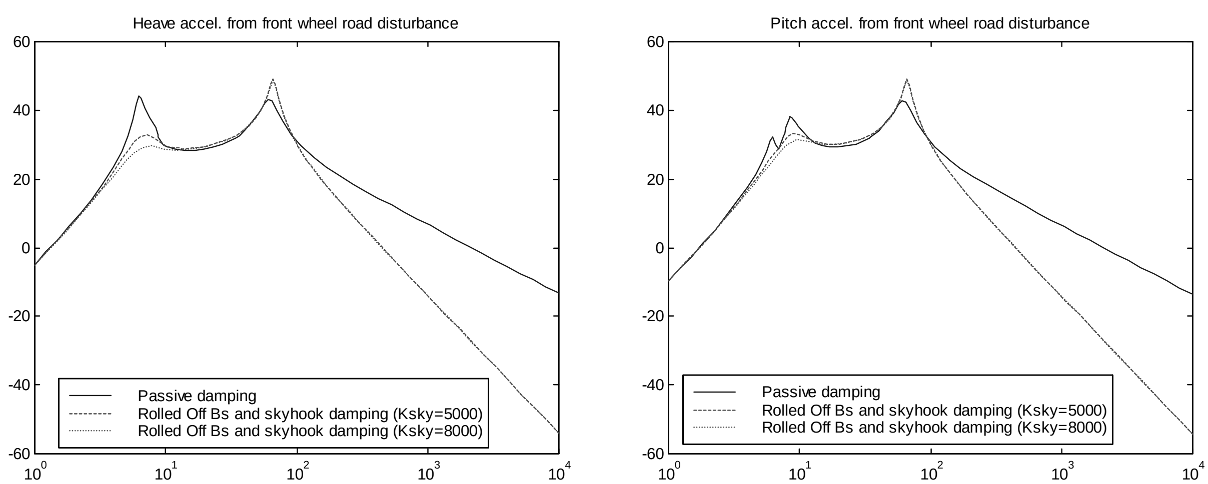
3.4 Overall Performance

Bode plots in Fig. 6(a) and 6(b) show the HF improvements due to the effects of rolling off $B_{s\,f}$, $B_{s\,r}$ at high frequencies, and the LF improvements due to LF roll off of spring constant and skyhook damping. As expected, no improvements are achieved at the wheel frequency. The heave and pitch modes were found to occurred at frequencies of $w = 6$ rad/s and $w = 8$ rad/s, respectively.



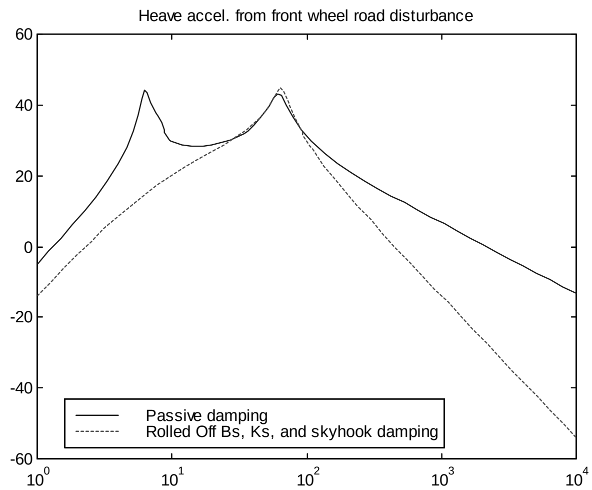
(a) (b)

Fig. 4: Effects of Active Damping. LF spring constant and HF roll-off.

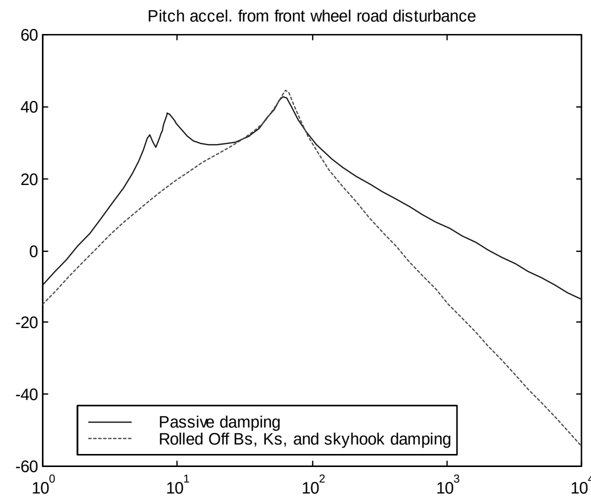


(a) (b)

Fig. 5: Damping roll off at HF and skyhook damping.



(a)



(b)

Fig. 6: LF spring constant, HF damping roll off and skyhook damping.

4 Simulations

Let the road disturbance on the front wheel $z_{r\,f}(t)$ be sinusoidal with wavelength λ_r (meters) and amplitude μ_r (meters). If the vehicle travels a speed v, then the road disturbance input is given by

$$z_{r\,f}(t) = \mu_r \sin\left(\frac{2\pi v}{\lambda_r} t\right), \tag{23}$$

where the road disturbance frequency is $w_r = \dfrac{2\pi v}{\lambda_r}$.

The road disturbance affecting the rear wheel will be a delayed version of the one in the front wheel. The delay for a constant velocity is given by $\tau = L/v$ where $L = a + b$ is the distance between front and rear axles of the vehicle.
Then,

$$z_{r\,r}(t) = \mu_r \sin\left(\frac{2\pi v}{\lambda_r}(t + L/v)\right). \tag{24}$$

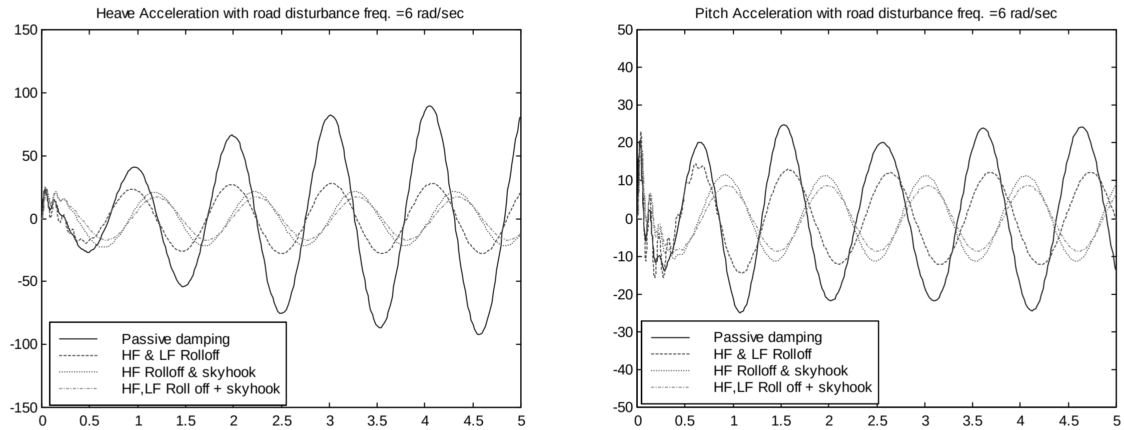
The simulation parameters selected were

- Simulation time: 5 seconds.
- Road Excitation: $\mu_r = 0.5$ meters, $\lambda = 3$ meters, $v = 22.22$ meters/sec.
- Road Excitation Frequency: $w_r = 6, 8, 57, 150$ rad/s

From Fig. 7 and 8, it can be seen that active damping improves heave and pitch accelerations at low frequencies. Note that a phase shift is introduced by rolling off the spring constant at low frequencies. Better performance is achieved by LF spring constant roll off. Skyhook damping improves the results even further.

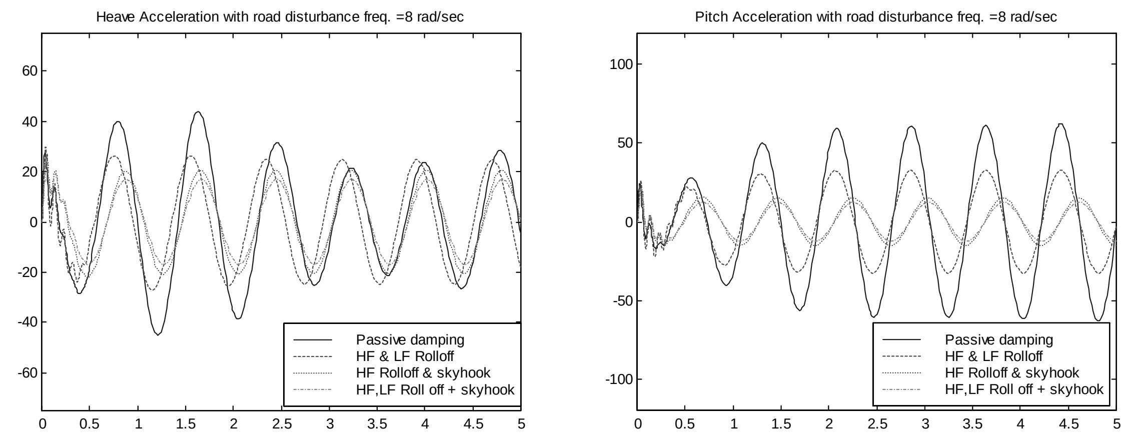
No significant improvements are achieved at the wheel frequency as depicted in Fig. 9. In fact HF damping roll off and skyhook damping make the performance worse at the wheel frequency. However, LF spring constant roll off introduces a slight improvement.

Fig. 10 shows that roll-off damping at high frequencies improves performance beyond the wheel frequency.



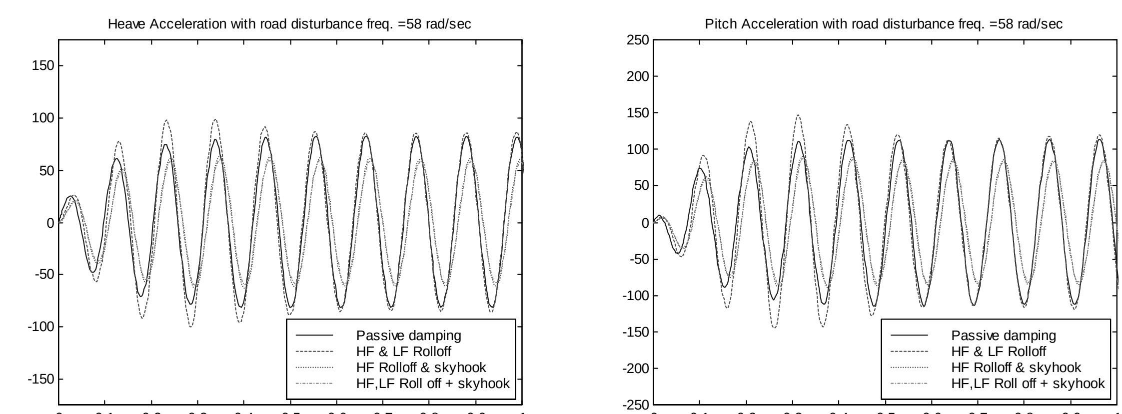
(a) (b)

Fig.7: Simulation results for $w_r = 6$ rad/s.



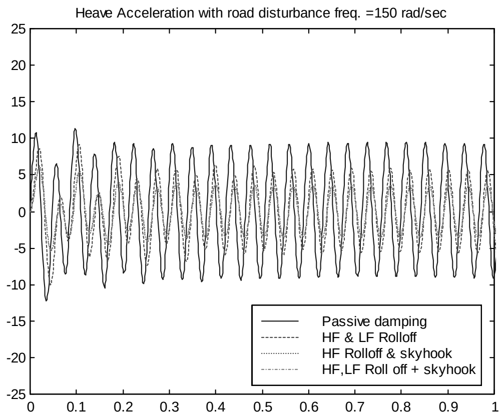
(a) (b)

Fig. 8: Simulation results for $w_r = 8$ rad/s.

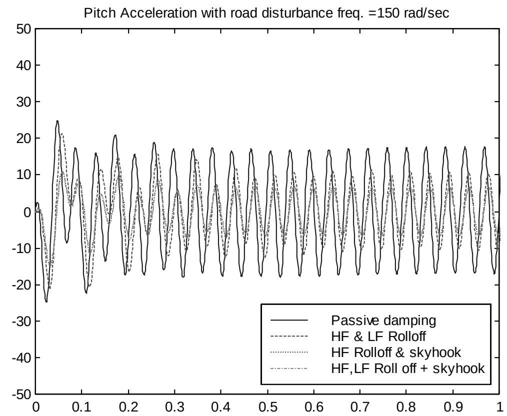


(a) (b)

Fig. 9: Simulation results for $w_r = 58$ rad/s.



(a)



(b)

Fig. 10: Simulation results for $w_r = 150$ rad/s.

5 Conclusions

In this paper, an active controller design based on time-scale separation and an "input decoupling transformation" has been proposed to reduce heave and pitch motion and improve ride quality of the vehicle. Improvement can be achieved below and above the wheel frequency by using the proposed active controller. It is not possible to decrease body motions at the wheel frequency w_0 using control inputs applied only between the sprung and unsprung masses, because the system has a pole-zero cancellation at w_0 arising from an uncontrollable mode.

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