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Modified Internal Model Control for unstable systems

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Abstract

In the present paper, we propose a modified Internal Model Control systems that is implemented for unstable plants. This modification is simple and natural. Some characteristics of modified Internal Model Control such as stability, robust stability and so on are clarified .

Notations

R	field of real numbers
$\mathcal{C}$	field of complex numbers
$R(s)$	the space of all real-rational transfer function

1 Introduction

In the present paper, we examine Internal Model Control for linear time-invariant continuous time systems. Basic structure of Intenal Model Control is first proposed by Horowits. Garcia and Morari developed Internal Model Control (Garcia and Morari, 1982, 1985,b). Morari and Zafiriou summarized Internal Model Control over a wide range of control problem as a book (Morari and Zfiriou, 1989). Morari and Zafiriou discussed not only the basic characteristics of Internal Model Control but also systematic design method of Internal Model Control such as H_2 design, H_∞ design, and so on (Morari and Zfiriou, 1989). Watanabe and Yamada proposed a design method of Internal Model Control with low-sensitivity and robust stability characteristics (Watanabe and Yamada, 1993). Shu, Watanabe and Yamada gave a simple design methof of robust Internal Model Control with uncertain time-delay.

From (Garcia and Morari, 1982, 1985,b; Morari and Zfiriou, 1989), Internal Model Control has much advantage to design control systems. The stability of Internal Model Control is only depend on that of the controller and the nominal plant. In addition, even if the Internal Model Control system has control input saturation, stability of Intenal Model Conrol is only depend on that of the controller and the plant, too (Morari and Zfiriou, 1989; Morari, 1993). Morari proposed anti-windup scheme based on Internal Model Control using this characteristics of Internal Model Control (Morari and Zfiriou, 1989; Morari, 1993). Doyle et al. pointed out the problem, that Internal Model Control subject to control input saturation, does not have robust servo characeristics and showed an example(Doyle, Smith and Enns, 1987). Yamada proposed the simple degin method of the Internal Model Servo Control Systems with control input saturation baded on the idea of Internal Perturbed Model Control (Yamada, 1998).

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One more advantage of Internal Model Control is easy to shape sensitivity function and complementary sensitivity function. Yamada et al. proposed a design method of Internal Model Control using stable filtered inverse systems (Yamada and Watanabe, 1996) and gave simple design procedure by using filter parameter.

In this way, Internal Model Control has much advantage, but for unstable plants, Internal Model Control can not be applied. It is desirable to find a design method that has similar advantage to Internal Model Control for stable plants.

In the present paper, we propose a modified Internal Model Control systems that is implemented for unstable plants. This modification is simple and natural.

2 Internal Model Control

In this section, we shortly introduce the characteristics of Internal Model Control systems and describe a purpose of this paper.

Fig. 1 shows the Internal Model Control structure (Morari and Zfiriou, 1989). Here $G(s) \in$

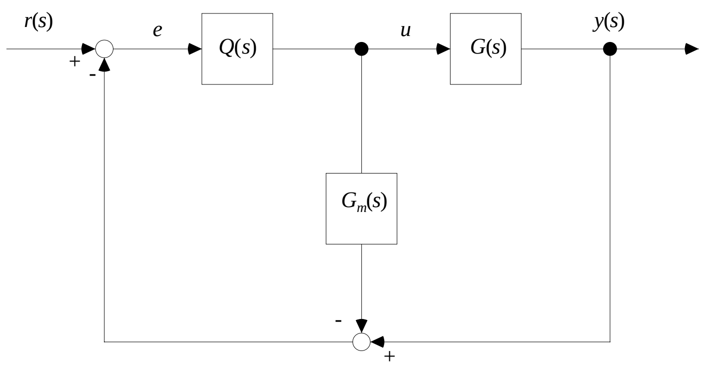

Figure 1: Block diagram of Internal Model Control

$R(s)$ is the single-input/single-outputplant, $G_m(s) \in R(s)$ is the nominal plant, $Q(s) \in R(s)$ is the controller, r is the reference input, y is the output and u is the contorol input.

The characteristics of Internal Model Control is summarized as follows. Stability of Fig. 1 is summarized as following theorem.

Theorem 1 *$G_m(s)$ is assumed to equal to $G(s)$. Internal Model Control system in Fig. 1 is internally stable if and only if both the plant $G(s)$ and the controller $Q(s)$ are stable(Morari and Zfiriou, 1989).*

In many cases, the error between the plant $G(s)$ and the nominal plant $G_m(s)$ exists. When $G(s)$ is denoted by

$$G(s) = G_m(s)(1 + \Delta(s)). \tag{1}$$

Here $\Delta(s)$ is arbitrary unceratinty included in Ω defined as follows.

Definition 1 *$\Delta(s)$ is called a element of Ω if*

$$|\Delta(j\omega)| < |W(j\omega)| \ \forall\omega \in R, \tag{2}$$

and $\Delta(s)$ does not change the number of unstable poles of $G(s)$, that is, the nunber of unstable poles of $G(s)$ is assumed to be equal to that of $G_m(s)$. Here $W(s) \in R(s)$ is stable real-rational transfer function.

The robust stability condition of Internal Model Control System in Fig. 1 is summarized as follows.

Theorem 2 *The necessary and sufficient condition that Internal Model Control systems in Fig. 1 with arbitrary $\Delta(s)$ included in Ω, is*

$$|Q(j\omega)G_m(j\omega)W(j\omega)| \leq 1, \quad \omega \in R. \tag{3}$$

The transfer function from the reference input r to the error $e = r - y$ is given by

$$\begin{aligned} e &= r - y \\ &= (1 - G_m(s)Q(s))\, r. \end{aligned} \tag{4}$$

Here, the transfer function e/r denotes $S(s)$

$$S(s) = 1 - G_m(s)Q(s) \tag{5}$$

and called the sensitivity function. If $Q(s)$ is set to makes

- the case that $G_m(s)$ has no zeros in the closed right half plane

$$G_m(s)Q(s) = \frac{1}{(1 + s\tau)^{\alpha}}, \tag{6}$$

- the case that $G_m(s)$ has some zeros in the closed right half plane

$$G_m(s)Q(s) = \frac{G_{mi}(s)}{(1 + s\tau)^{\alpha}}, \tag{7}$$

then Fig. 1 has the robust servo characteristics for step reference input where $G_i(s)$ is the inner function of $G_m(s)$ satisfying

$$G_{mi}(-s)G_{mi}(s) = 1 \;\; G_{mi}(0) = 1.$$

That is the error $e = r - y$ tend to zero even if uncertainty between $G(s)$ and $G_m(s)$ or disturbances exist.

In this way, Fig. 1 has much advantage to design control systems:

1. Stability of Internal Model Control system is only depend on the stability of the nominal plant $G_m(s)$ and the controller $Q(s)$
2. Robust stability condition is easyily adjustable by using τ in (7). That is, for large $\Delta(s)$, if we set τ smaller then the robust stability condition is tend to be satisfied.
3. It is easy for shaping sensitivity function using τ.

But from Theorem 1, Internal Model Control can not be applied to unstable plants.

In the present paper, we modify Internal Model Control systems to be able to apply to unstable plants without loss of advantages of characteristics of Internal Model Control systems.

3 Basic idea of Modification of Internal Model Control

In this section, we describe the basic idea of modification of Internal Model Control for unstable plants. Modification of Internal Model Control is considered from the parametrization of the stabilizing controller based on Internal Model Control structure for unstable plants. In following section, $G(s)$ is assumed to be unstable and to satisfy parity interlacing property condition (Vidyasagar, 1985). This assumption implies that there exists stable stabilizing controller for $G(s)$.

For unstable plants, Morari and Zafiriou considered that Internal Model Control structure is used for the design of the feedback controller $C(s)$ in Fig. 2 . That is, Morari and Zafiriou considered to establish the condition of the controller $Q(s)$ that $C(s)$ has a structure given by

$$C(s) = \frac{Q(s)}{1 - Q(s)G_m(s)} \tag{8}$$

and $C(s)$ stabilize the plant $G(s)$. From (Morari and Zfiriou, 1989), Theorem 3 was obtained.

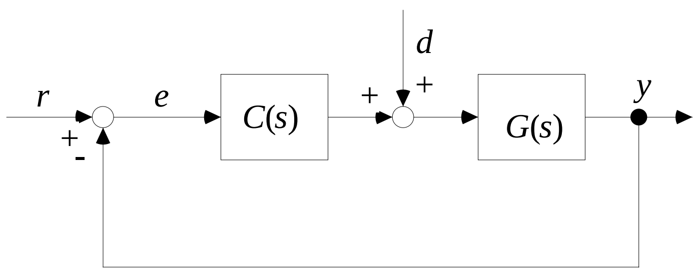

Figure 2: Closed loop system

Theorem 3 *Let us assume that $G(s)$ is equal to $G_m(s)$. The control system in Fig. 2 with the controller $C(s) = Q(s)/(1 - G_m(s)Q(s))$ is stable if and only if following expressions hold.*

1. *$Q(s)$ is stable.*
2. *$G_m(s)Q(s)$ is stable.*
3. *$G_m(s)(1 - G_m(s)Q(s))$ is stable.*

When $G(s)$ is factorized by

$$G(s) = G_{un}(s)G_s(s), \tag{9}$$

where $G_s(s)$ is stable proper rational function and $G_{un}(s)$ is bi-proper antistable and minimum phase function. From Theorem 3, $Q(s)$ must hold following structure

$$Q(s) = \frac{\bar{Q}(s)}{G_{un}(s)}, \tag{10}$$

here $\bar{Q}(s)$ is stable . By using above equation, Fig. 2 is redrawn by Fig. 3 to have similar structure to Fig. 1 .

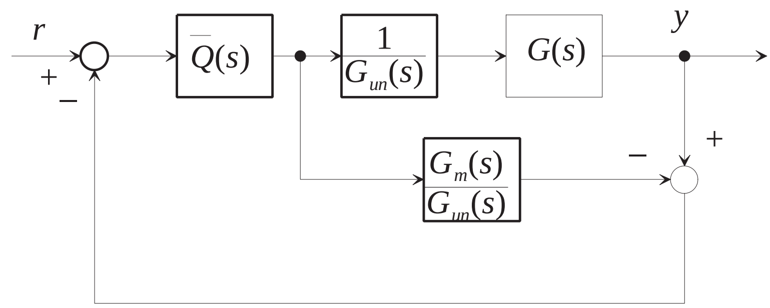

Figure 3:

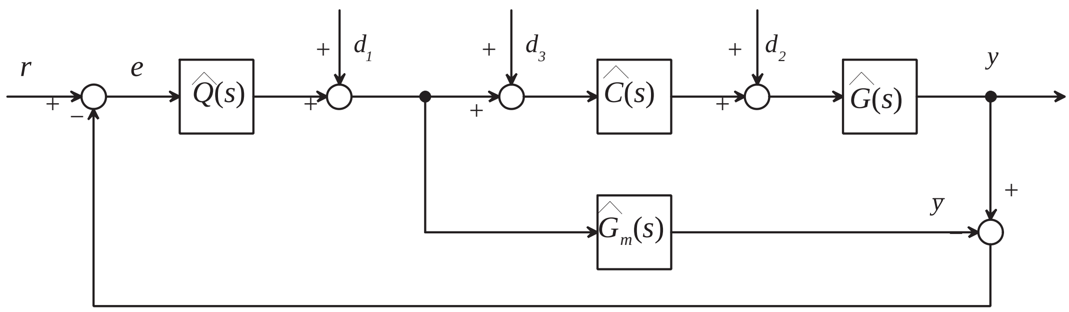

Figure 4: Basic structure of modified Internal Model Control

Since $\frac{1}{G_{un}(s)}G(s)$, $\bar{Q}(s)$ and $\frac{1}{G_{un}(s)}G_m(s)$ are stable, for unstable plants, Fig. 3 indicate natural expansion of Internal Model Control structure Fig. 1 . Therefore we propose Fig. 4 as a basic structure of modified Internal Model Control systems for unstable plants. Next, the characteristics of modified Internal Model Control system in Fig. 4 are considered. The stability of modified Internal Model Control system in Fig. 4 is summarized as Theorem 4.

Theorem 4 *Assume that $G_m(s)$ is equal to $G(s)C(s)$. The system in Fig. 4 is stable if and only if following expressions hold.*

1. *$\hat{G}(s)$ is stable.*
2. *$\hat{G}_m(s)$ is stable.*
3. *$\hat{C}(s)$ is stable.*
4. *$\hat{Q}(s)$ is stable.*

Proof: Transfer function from inputs to outputs are given by

$$\begin{bmatrix} u_1(s) \\ u_2(s) \\ y(s) \\ \bar{y}(s) \\ e(s) \end{bmatrix}$$

$$= \begin{bmatrix} \hat{Q}(s) & 0 & -\hat{Q}(s)\hat{G}(s) & -\hat{Q}(s)\hat{G}(s)\hat{C}(s) \\ \hat{C}(s)\hat{Q}(s) & \hat{C}(s) & -\hat{C}(s)\hat{Q}(s)\hat{G}(s) & \hat{C}(s)(1-\hat{Q}(s)\hat{G}_m(s)) \\ \hat{G}(s)\hat{C}(s)\hat{Q}(s) & \hat{Q}(s)\hat{C}(s) & \hat{G}(s)(1-\hat{Q}(s)\hat{G}_m(s)) & \hat{G}(s)\hat{C}(s)(1-\hat{Q}(s)\hat{G}_m(s)) \\ \hat{G}_m(s)\hat{Q}(s) & \hat{G}_m(s) & -\hat{G}_m(s)\hat{Q}(s)\hat{G}(s) & -\hat{G}_m(s)\hat{Q}(s)\hat{G}(s)\hat{C}(s) \\ 1 & 0 & -\hat{G}(s) & -\hat{G}(s)\hat{C}(s) \end{bmatrix} \begin{bmatrix} r \\ d_1 \\ d_2 \\ d_3 \end{bmatrix}. \tag{11}$$

From the definition of internal stability (Doyle, Francis and Tannenbaum, 1992), all of transfer function in above equation must be stable for the internally stability of the system in Fig. 3 . Therefore the proof of this theorem is obviously obtained. ■

From the above consideration, we propose a design method of modified Internal Model Control below. Basic consept is summarized below.

1. Since $\hat{G}(s)$ in Fig. 4 must be stable from Theorem 4, $\hat{G}(s)$ is considered as a system that is stabilized by using local feedback loop like Fig. 5 , here $K(s)$ is a stable stbilizing controller of $G(s)$. That is,

$$\hat{G}(s) = \frac{G(s)}{1+K(s)G(s)} \tag{12}$$

is asymptotically stable. From the assumption that $G(s)$ satisfies parity interlacing property condition, such $K(s)$ exists without fail. Fig. 5 shows the structure of $\hat{G}(s)$ given by (12).

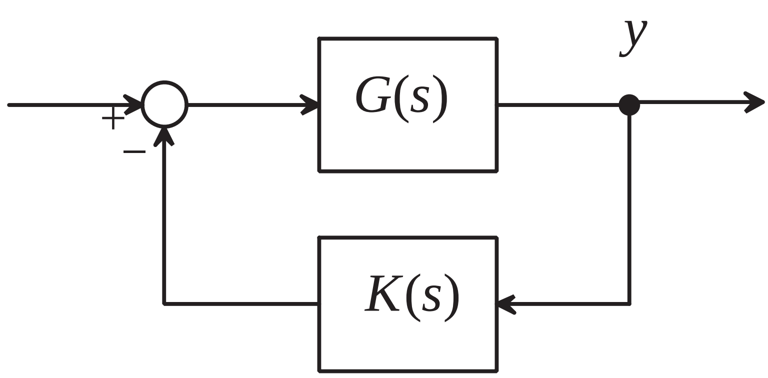

Figure 5: Structure of $\hat{G}(s)$

2. Let $\hat{C}(s)$ be

$$\hat{C}(s) = \frac{1+K(s)G_m(s)}{G_{un}(s)}. \tag{13}$$

From the assumption that $K(s)$ is asymptotically stable, the unstable poles of $1+K(s)G(s)$ is identical to that of $G(s)$. Therefore $\hat{C}(s)$ is asymptotically stable. In addition, since both $1+K(s)G(s)$ and $G_{un}(s)$ are bi-proper, $\hat{C}(s)$ is bi-proper, too. Thus $\hat{C}(s)$ is stable and bi-proper system.

3. Let $\hat{G}_m(s)$ satisfy

$$\hat{G}_m(s) = \hat{C}(s)\hat{G}(s). \tag{14}$$

From (9), (12) and (13), $\hat{G}_m(s)$ is rewritten by

$$\begin{aligned}\hat{G}_m(s) &= \frac{G_m(s)}{G_{un}(s)} \\ &= G_s(s). \end{aligned}\tag{15}$$

$\hat{G}_m(s)$ is stable because $G_s(s)$ is stable.

4. Above expressions are summarized in Fig. 6

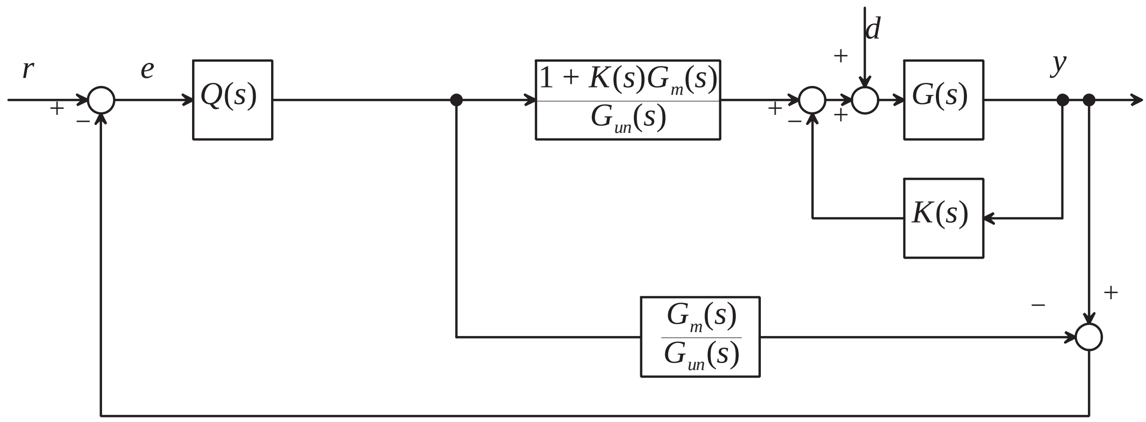

Figure 6: Modified Internal Model Control

From Theorem 4, Modified Internal Model Control in Fig. 6 is intenally stable. In this way, we can simplly modify Internal Model Control to be applicable for unstable plants.

4 Characteristics of Modified Internal Model Control

In this section, some characteristics of Modified Internal Model Control in Fig. 6 such as input-output property, robust stability and sensitivity characteristics, are presented.

Transfer function from the reference input r to the output y is given by

$$y(s) = \frac{\bar{Q}(s)(1+K(s)G_m(s))G_s(s)(1+\Delta(s))}{1+K(s)G_m(s)+(K(s)G_{un}(s)+Q(s))(1+\Delta(s))}r(s). \tag{16}$$

From above equation, transfer function from r to $r-y$ is obtained by

$$r(s)-y(s) = \frac{(1-Q(s)G_s(s))\,(1+K(s)G_m(s)+K(s)G_m(s)\Delta(s))}{1+K(s)G_m(s)+(K(s)G_{un}(s)+Q(s))(1+\Delta(s))}r(s). \tag{17}$$

Therefore if

$$(1-Q(s)G_s(s))|_{s=0} = 0, \tag{18}$$

Modified Internal Model Control has robust servo characteristics for step reference input. This characteristics is same to that of Intenal Model Control in Fig. 1 .

The robust stability condition of Modified Internal Model Control is summarized in following theorem.

Theorem 5 *Let us assume that Modified Internal Model Control is internally stable in the case of $\Delta(s) = 0$. Modified Internal Model Control in Fig. 6 is robustly stable for arbitraly uncertanty included in Ω if and only if*

$$\left\| \frac{(K(s)G_{un}(s) + Q(s))\, G_s(s)}{1 + K(s)G_m(s)} W(s) \right\|_\infty \leq 1 \tag{19}$$

holds.

Proof: If the Nyquist plot of the characteristics polynomial of Fig. 6 for all $\Delta(s)$ included in Ω encircles the origin the number of unstable poles of $G_m(s)$ in the counter-clockwise direction, then system in Fig. 6 is robustly stable. From (16), characteristic polynomial of Fig. 6 is equal to

$$\begin{aligned} & 1 + K(s)G_m(s) + (K(s)G_{un}(s) + Q(s))(1 + \Delta(s)) \\ = \; & (1 + K(s)G_m(s)) \left(1 + \frac{(K(s) + Q(s))G_m(s)\Delta(s)}{1 + K(s)G_m(s)} \right). \end{aligned}$$

From the assumption that $K(s)$ stabilize $G_m(s)$ and $K(s)$ is asymptotically stable, the Nyquist plot of $1 + K(s)G_m(s)$ encircle the origin the number of unstable poles of $G_m(s)$ times in the counter-clockwise direction. Therefore the necessary and sufficient condition that the system in Fig. 6 is robustly stable for arbitrary uncertainty in Ω is equivalent to the condition that the Nyquist plot of

$$\left(1 + \frac{(K(s)G_{un}(s) + Q(s))G_s(s)\Delta(s)}{1 + K(s)G_m(s)} \right)$$

does not encircle the origin any times.

The remaining problem is to prove the necessary and sufficient condition that the Nyquist plot of

$$\left(1 + \frac{(K(s)G_{un}(s) + Q(s))G_s(s)\Delta(s)}{1 + K(s)G_m(s)} \right)$$

does not encircle the origin any times, is identical to (19).

Sufficient part of the proof is as follows. Assume that

$$\left\| \frac{(K(s)G_{un}(s) + Q(s))G_s(s)W(s)}{1 + K(s)G_m(s)} \right\|_\infty \leq 1. \tag{20}$$

It is obvious that the Nyquist plot of

$$1 + \frac{(K(s)G_{un}(s) + Q(s))G_s(s)\Delta(s)}{1 + K(s)G_m(s)}$$

can encircle the origin no time for arbitraly $\Delta(s)$ included in Ω.

Necessary part is to show if

$$\left\| \frac{(K(s)G_{un}(s) + Q(s))G_s(s)W(s)}{1 + K(s)G_m(s)} \right\|_\infty > 1$$

, then $\Delta(s) \in \Omega$ exists to let the Nyquist plot of

$$1 + \frac{(K(s)G_{un}(s) + Q(s))G_s(s)\Delta(s)}{1 + K(s)G_m(s)}$$

encircle the origin. If

$$\left\| \frac{(K(s)G_{un}(s) + Q(s))G_s(s)W(s)}{1 + K(s)G_m(s)} \right\|_\infty > 1$$

, then some ω exists satisfying

$$\frac{(K(j\omega)G_{un}(j\omega) + Q(j\omega))G_s(j\omega)W(j\omega)}{1 + K(j\omega)G_m(j\omega)} = \epsilon \ \ (|\epsilon| > 1)$$

If

$$\Delta(j\omega) = -\frac{W(j\omega)}{\epsilon}, \tag{21}$$

$\Delta(s)$ is included in Ω on ω because of

$$\begin{aligned} |\Delta(j\omega)| &= \frac{|W(j\omega)|}{|\epsilon|} \\ &< |W(j\omega)|. \end{aligned} \tag{22}$$

We have

$$1 + \frac{(K(j\omega)G_{un}(j\omega) + Q(j\omega))G_s(j\omega)\Delta(j\omega)}{1 + K(j\omega)G_m(j\omega)} = 0. \tag{23}$$

From above discussion, the proof of this theorem is completed. ■

Sensitivity function of the system in Fig. 6 is written by

$$S(s) = \frac{1 - Q(s)G_s(s)}{1 + K(s)G_m(s)}. \tag{24}$$

Small difference between sensitivity function of Fig. 1 and that of Fig. 6 exists. But both sensitivity function have a tendency to be small by settng $1 - Q(s)G_s(s)$ small. Therefore modified Internal Model Control does not loose the advantage Intarnal Model Control. The following conclusion can be drawn from the above view. Modified Internal Model Control is a natural expanded system of Internal Model Control.

5 conclusion

In the present paper, we proposed a new modified Internal Model Control systems for unstable plants. This modification is simple and natural. It was shown that modified Internal Model Control did not loose advantage of Internal Model Control.

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