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Geometric and system decomposition techniques in application to control of a mobile robot with trailer *

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Abstract

The trajectory interception approach in its original form, as previously introduced in (Michalska, 1996) primarily applies to systems whose controllability Lie algebra is nilpotent and involves only Lie brackets of relatively low order. High order Lie brackets in the controllability Lie algebra of the system lead to excessively complex formulations of the open-loop trajectory interception problem which can no longer be solved analytically (in terms of the parameters which represent the values of a feedback control for an extended system). The purpose of this paper is to demonstrate that even in such difficult cases the trajectory approach can still be made use of. The model of a mobile robot with trailer used in this paper is not nilpotent and requires system motion in the directions of third order Lie brackets. To compensate for the lack of nilpotency of the original model, a nilpotent approximation of the system is introduced. System decomposition is further employed to obtain an analytically solvable trajectory interception problem formulation. The example of the mobile robot with trailer has the most complex algebraic structure of all the systems to which the trajectory interception problem was ever applied.

Keywords: vehicle control, nonholonomic systems, stabilization

1 Introduction

The purpose of this article is to demonstrate how the novel approach for the synthesis of time-varying stabilizing feedback for drift free systems , presented in (Michalska, 1996; Michalska *et al.*, 1998) , can be utilized to construct stabilizing feedback controls for systems whose complexity was previously thought to inhibit its application. The system considered here is a kinematic model of a mobile robot with a trailer which is characterised by the presence of non-integrable velocity constraints. It is well known that systems of this type cannot be stabilized by continuous static feedback, see (Brockett, 1983), and that the dependence of a stabilizing feedback control on time is essential, see (Coron, 1992). Many synthesis approaches have been proposed, see for example (Pomet, 1992), but rely heavily on the existence of suitable time-varying Lyapunov functions, which are often difficult to find, or else rely on the existence of specific coordinate

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transformations which bring such systems to chained or power forms. On the other hand, the approach which employs the "trajectory interception" concept of (Michalska, 1996; Michalska *et al.*, 1998), does not rely on the construction of a suitable control-Lyapunov function nor coordinate transformation. In this approach, a standard stabilizing feedback control for a Lie bracket extension of the system model is constructed first. The basic idea is then to formulate an open loop control problem whose task is to deliver a control which steers the original system to a point along the trajectory of the extended system at the end of a given time horizon $[0, T]$, provided that both trajectories evolve from the same initial condition. For the open-loop problem to be solved only once, it is essential that its solution is independent of the actual value of this initial condition. This requires the open-loop control problem to be restated in terms of flows of both systems and solved in suitable (logarithmic) coordinates on an associated Lie group. A periodic continuation of the solution to the open-loop problem is finally combined with the feedback control for the extended system to yield a time-varying feedback control which stabilizes the original system by insuring that its trajectory intercepts with a corresponding trajectory of the extended system with frequency $1/T$. The approach of (Michalska, 1996; Michalska *et al.*, 1998), however, primarily applies to systems which are nilpotent and whose controllability Lie algebras contain only Lie brackets of order one. The presence of higher order Lie brackets in the system's controllability Lie algebra results in an excessively complex (impossible to solve analytically) formulation of the trajectory interception problem with a very large number of equations describing the evolution of the flow of such systems.

In the above context, the contributions of this paper are listed as follows:

- Using a model of a mobile robot with trailer, which is a system which fails to be nilpotent and whose controllability Lie algebra involves brackets of order three, it is demonstrated that the application of the trajectory interception approach of (Michalska, 1996; Michalska *et al.*, 1998) is not limited to systems with simple structures. The primary purpose of this paper is hence not to demonstrate superiority of a specific control system design but to explore the feasibility of the trajectory interception approach in designing feedback controls for systems with complicated algebraic structures by breaking down the original models into simpler sub-systems and working with approximations.
- To compensate for the lack of nilpotency, the introduction of an approximate model, which generates a nilpotent controllability Lie algebra, is shown to be possible. A further decomposition of the approximate model permits huge simplification of the differential equations describing the evolution of the logarithmic coordinates in the open-loop problem formulation, enabling the application of the trajectory interception approach.

2 The exact and approximate kinematic models and their decomposition into subsystems

The model of a mobile robot with trailer considered below has the most complex algebraic structure of all systems to which the trajectory interception approach was ever applied. It

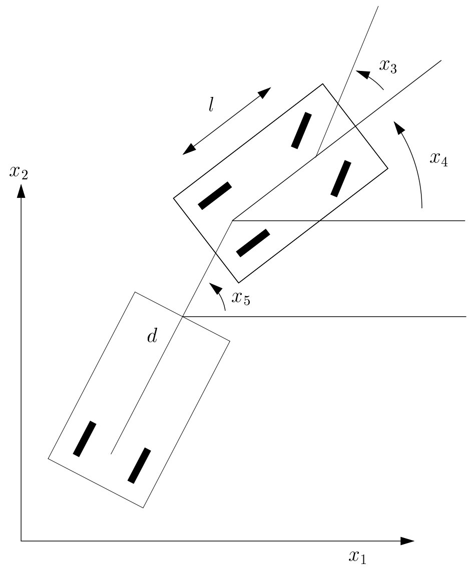

Figure 1: Model of a mobile robot with trailer

represents a five dimensional systems with control deficiency order three, possessing a non-nilpotent controllability Lie algebra which contains Lie brackets of depth one, two, and three.

The kinematic model of the robot, see (Lafferriere *et al.*, 1993), can be formulated in terms of the following state space equations :

$$\begin{aligned} \dot{x_1} &= cos\ x_3\ cos\ x_4\ u_1 \\ \dot{x_2} &= cos\ x_3\ sin\ x_4\ u_1 \\ \dot{x_3} &= u_2 \\ \dot{x_4} &= \frac{1}{l}\ sin\ x_3\ u_1 \\ \dot{x_5} &= \frac{1}{d}\ sin\ (x_4 - x_5)\ cos\ x_3\ u_1 \end{aligned} \tag{1}$$

where x_1, x_2 are the Cartesian coordinates of the centre of mass of the car, x_3 is the steering angle, x_4 and x_5 are the angles which the main axes of the car and trailer make with the x_1 axis, respectively, see Figure 1. Assuming for simplicity that $l = d = 1$ and re-defining variables $(x_1, x_2, x_3, x_4, x_5) = (z_1, z_4, z_3, z_2, z_5)$ the model can be written in a compact form :

$$\dot{z} = g_1(z)u_1 + g_2(z)u_2, \quad z \in I\!R^5 \tag{2}$$

$$\begin{aligned} g_1(z) &= cos\ z_3\ cos\ z_2\ \frac{\partial}{\partial z_1} + sin\ z_3\ \frac{\partial}{\partial z_2} + cos\ z_3\ sin\ z_2\ \frac{\partial}{\partial z_4} + \ cos\ z_3\ sin\ (z_2 - z_5)\ \frac{\partial}{\partial z_5} \\ g_2(z) &= \frac{\partial}{\partial z_3} \end{aligned}$$

The following Lie brackets:

$$g_3(z) \stackrel{def}{=} [g_1, g_2](z) = sinz_3\ cosz_2\ \frac{\partial}{\partial z_1} - cosz_3\ \frac{\partial}{\partial x_2} + sinz_3\ sinz_2\ \frac{\partial}{\partial z_4} + sinz_3\ sin(z_2 - z_5)\ \frac{\partial}{\partial z_5}$$

$$g_4(z) \stackrel{def}{=} [g_1, g_3](z) = -sinz_2\ \frac{\partial}{\partial z_1} + cosz_2\ \frac{\partial}{\partial z_4} + cos(z_2 - z_5)\ \frac{\partial}{\partial z_5}$$

$$g_5(z) \stackrel{def}{=} [g_1, g_4](z) = -sinz_3\ cosz_2\ \frac{\partial}{\partial z_1} - sinz_3\ sinz_2\ \frac{\partial}{\partial z_4} - (sinz_3\ sin(z_2 - z_5) - cosz_3)\ \frac{\partial}{\partial z_5}$$

show that the LARC condition for complete controllability is satisfied :

$$span\{g_i(z), i = 1, ..., 5\} = \mathbb{R}^5, \quad \text{for all} \ \ z \in \mathbb{R}^5 \tag{3}$$

It should be noted that the span (3) now contains a vector field g_5 which is a Lie bracket of depth three. Theoretically, the trajectory interception approach, see (Michalska, 1996; Michalska *et al.*, 1998), could be applied directly to the model (2) and would produce a time varying feedback which stabilizes the system to the origin (without the loss of generality, also to any desired set point). However, due to the presence of a third order Lie bracket in the span (3), the equations governing the evolution of the flows of (2) would be exceedingly difficult to solve. In attempt to facilitate the solution of the trajectory interception problem, we thus consider the following decomposition of the original model in which the trajectory interception approach needs only be applied to subsystem S1:

$$S1: \quad \begin{bmatrix} \dot{z_1} \\ \dot{z_2} \\ \dot{z_3} \end{bmatrix} = \begin{bmatrix} cosz_2\ cosz_3 \\ sin\ z_3 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 \tag{4}$$

$$S2: \quad \begin{bmatrix} \dot{z_4} \\ \dot{z_5} \end{bmatrix} = \begin{bmatrix} sin\ z_2\ cos\ z_3 \\ cos\ z_3\ sin\ (z_2 - z_5) \end{bmatrix} u_1 \stackrel{def}{=} \begin{bmatrix} f_1(z) \\ f_2(z) \end{bmatrix} u_1 \tag{5}$$

By defining $x \stackrel{def}{=} (z_1, z_2, z_3)$, subsystem S1 can be written as:

$$\dot{x} = f_1(x)u_1 + f_2(x)u_2, \quad x \in \mathbb{R}^3 \tag{6}$$

$$\begin{aligned} f_1(x) &= cos\ z_2\ cos\ z_3\ \frac{\partial}{\partial z_1} + sin\ z_3\ \frac{\partial}{\partial z_2} \\ f_2(x) &= \frac{\partial}{\partial z_3} \end{aligned}$$

Subsystem $S1$ is controllable as it satisfies:

$$span\{f_1(x), f_2(x), f_3(x)\} \ = \ \mathbb{R}^3, \ \text{for all} \ \ x \in \mathbb{R}^3$$

where

$$f_3(x) \stackrel{def}{=} [f_1, f_2](x) = sin\ z_3\ cos\ z_2\ \frac{\partial}{\partial z_1} - cos\ z_3\ \frac{\partial}{\partial z_2}$$

However, it can be easily verified that the Lie algebra $L(f_1, f_2)$ is not nilpotent. Since the trajectory interception approach applies primarily to nilpotent systems, we next consider the following approximation to subsystem S1, valid in some neighbourhood of the origin (which, without the loss of generality, is assumed to be the desired set point):

$$\dot{x} = \tilde{f}_1(x)u_1 + \tilde{f}_2(x)u_2, \quad x \in I\!R^3 \tag{7}$$

$$\tilde{f}_1(x) \;=\; \frac{\partial}{\partial z_1} + z_3\,\frac{\partial}{\partial z_2}, \qquad \tilde{f}_2(x) = \frac{\partial}{\partial z_3}$$

and verify that it satisfies the LARC controllability condition, as necessary for the control construction:

$$span\{\tilde{f}_1(x), \tilde{f}_2(x), \tilde{f}_3(x)\} \;=\; I\!R^3, \qquad \text{for all} \qquad x \in I\!R^3$$

$$\text{where,} \qquad \tilde{f}_3(x) \stackrel{def}{=} [\tilde{f}_1, \tilde{f}_2](x) = -\frac{\partial}{\partial z_2}$$

The Lie brackets multiplication table for $L(\tilde{f}_1, \tilde{f}_2)$:

$$[\tilde{f}_1, \tilde{f}_2] = \tilde{f}_3 \qquad [\tilde{f}_1, \tilde{f}_3] = [\tilde{f}_2, \tilde{f}_3] = 0$$

shows that the controllability Lie algebra $L(\tilde{f}_1, \tilde{f}_2)$ is now nilpotent. The trajectory interception approach can thus be applied directly to steer the approximate subsystem $S1$, as explained below.

3 Time-varying stabilizing feedback synthesis for subsystem $S1$

The trajectory interception approach, as first presented in (Michalska, 1996), requires the construction of an extended system for the approximation to subsystem $S1$:

$$\dot{x} \;=\; \tilde{f}_1(x)v_1 + \tilde{f}_2(x)v_2 + \tilde{f}_3(x)v_3, \qquad x \in I\!R^3 \tag{8}$$

The importance of this algebraic extension lies in the fact that, unlike the original subsystem $S1$, it permits instantaneous motion in the "missing" Lie bracket direction $\tilde{f}_3 = [\tilde{f}_1, \tilde{f}_2]$.

Once the extended system is constructed, the solution of the stabilization problem for the approximation to $S1$ involves two steps:

1). The construction of a time-invariant feedback law which stabilizes the extended system (8).

2). The solution of a parametrized trajectory interception problem in the logarithmic parameters which effectively provides for 'pointwise equivalence' of the flows of the original and extended systems, (7) and (8). A periodic continuation of this solution is then composed with the feedback for the extended system to produce the final stabilizing control law for both the approximate and the original subsystem $S1$.

3.1 Stabilization of the extended system

Without the loss of generality, we first choose a quadratic Lyapunov function $V : I\!R^3 \to I\!R$, so that $V(x) \stackrel{def}{=} \frac{1}{2}\sum_{i=1}^{3} x_i^2$. It can be shown that the extended system (8) can be made (locally) asymptotically stable by introducing the following feedback control:

$$\tilde{v}_i(x) \stackrel{def}{=} -L_{\tilde{f}_i} V(x), \quad i = 1,2,3 \tag{9}$$

because, along the controlled extended system trajectories

$$\frac{d}{dt}V(x) = \sum_{i=1}^{3} [L_{\tilde{f}_i} V(x)]^2 < 0$$

unless $dV(x) = 0$, i.e. unless $x = 0$, which is due to the fact that $span\{\tilde{f}_1, ..., \tilde{f}_3\} = I\!R^3$.

It can be shown that discretization of the above control in time, with sufficiently high sampling frequency $\frac{1}{T}$, does not prejudice stabilization in that if the feedback control (9) is substituted by the discretized "sample and hold" feedback control:

$$\begin{aligned} \tilde{v}_i^T(x(t)) \stackrel{def}{=} \tilde{v}_i(x(nT)),\ t \in [nT, (n+1)T), \\ n = 0, 1, .., \quad i = 1, ..., 3 \end{aligned} \tag{10}$$

then the latter also stabilizes the system if T is small enough. This leads to a parametrized, asymptotically stable, controlled extended system:

$$\dot{x} = \tilde{f}_1 a_1 + \tilde{f}_2 a_2 + \tilde{f}_3 a_3 \tag{11}$$

where $a_i \stackrel{def}{=} \tilde{v}_i^T(x(t))$, $i = 1, ..., 3$, are constant over each interval $[nT, (n+1)T)$, $n = 0, 1, ...$.

Proposition 1 *(Michalska et al., 1998) Suppose the controlled extended system (8) is exponentially stable. Then, for any compact region $\mathcal{R} \subset \mathcal{M}$ which contains the origin, there exists a constant $T > 0$ such that the extended system with sample-and-hold feedback, (11), is also exponentially stable with region of attraction $\mathcal{R}$.*

3.2 The trajectory interception problem (TIP) for subsystem $S1$

To construct a time-varying control law such that the trajectories of the controlled approximate model (7) intersect the trajectories of the controlled extended system (8), with a period T, we state the following problem:

TIP: Find control functions $m_i(a, t)$, $i = 1, 2$, in the class of functions which are Holder continuous in $a \stackrel{def}{=} [a_1, ..., a_3]$, and piece-wise continuous and locally bounded in t, such that for *any initial condition* $x(0) = x$ the trajectory $x^a(t; x, 0)$ of the extended, parametrized system (11) intersects the trajectory $x^m(t; x, 0)$ of the approximate model (7) with controls m_i, $i = 1, 2$:

$$\dot{x} = \sum_{i=1}^{2} \tilde{f}_i m_i(a, t) \tag{12}$$

precisely at time T, so that $x^a(T; x, 0) = x^m(T; x, 0)$.

It is possible to show the following result.

Theorem 1 *(Michalska et al., 1998) Suppose that a solution to the TIP problem can be found. Then, the controls:*

$$\tilde{u}_i(x) \stackrel{def}{=} m_i(\tilde{v}^T(x), t), \quad i = 1, 2 \quad \tilde{v}^T \stackrel{def}{=} [\tilde{v}_1^T, ..., \tilde{v}_3^T]$$

provide a time varying asymptotically stabilizing feedback for the approximate system (7), with region of attraction $\mathcal{R}$ of Proposition 1.

Since the algebra $L(\tilde{f}_1, \tilde{f}_2, \tilde{f}_3)$ is finite dimensional, it is possible to employ the formalism of (Wei *et al.*, 1964) to obtain a solution to TIP. The latter is based on considering a formal equation for the evolution of flows of both (7) and (8) :

$$\dot{S}(t) = S(t) \; (\sum_{i=1}^{3} X_i w_i), \tag{13}$$

$$S(0) = I \tag{14}$$

with $w_3 = 0$ in the case of (7).

It is well known that the solution of (13)-(14) describes, via an evaluation homomorphism which maps each $\tilde{X}_i$ into the corresponding $\tilde{f}_i$, $i = 1, ..., 3$, the flows of both : system (7) with controls $u_i = w_i$, $i = 1, 2$, and system (8) with $v_i = w_i$, $i = 1, ..., 3$. It is also a basic fact from algebra that this solution and can be expressed locally by

$$S(t) = \prod_{i=1}^{3} exp(\gamma_i(t) X_i) \tag{15}$$

where the functions γ_i, $i = 1, .., 3$ are called the logarithmic coordinates of the corresponding flow. Although this representation is not necessarily global, the differential equations determining the evolution of these parameters can be obtained easily, see for example (Wei *et al.*, 1964) for such a calculation and, in our particular case, read :

$$\begin{aligned} \dot{\gamma}_1 &= a_1 \\ \dot{\gamma}_2 &= a_2 \\ \dot{\gamma}_3 &= -\gamma_1 a_2 + a_3 \end{aligned}$$

The TIP in logarithmic coordinates now takes the form of a trajectory interception problem for the following two "control systems":

$$CS1: \quad \begin{cases} \dot{\gamma}_1 = a_1 \\ \dot{\gamma}_2 = a_2 \\ \dot{\gamma}_3 = -\gamma_1 a_2 + a_3 \end{cases} \qquad CS2: \quad \begin{cases} \dot{\gamma}_1 = m_1 \\ \dot{\gamma}_2 = m_2 \\ \dot{\gamma}_3 = -\gamma_1 m_2 \end{cases}$$

with common initial conditions $\gamma_i(0) = 0$, $i = 1, .., 3$. The TIP in the logarithmic coordinates can thus be re-stated as follows :

TIP in logarithmic coordinates: Find control functions $m_i(a, t)$, $i = 1, 2$, in the class of functions which are Holder continuous in $a \stackrel{def}{=} [a_1, ..., a_3]$ and piece-wise continuous, and locally bounded in t, such that the trajectory $t \mapsto \gamma^a(t)$ of $CS1$ intersects the trajectory $t \mapsto \gamma^m(t)$ of $CS2$ in which

$$m(a,t) \stackrel{def}{=} [m_1(a,t), m_2(a,t), 0]^T \tag{16}$$

at time T, so that

$$\gamma^a(T) = \gamma^m(T) \tag{17}$$

Complete controllability of CS1 and CS2 guarantees existence of solutions to the TIP. One such solution can be calculated as follows.

The controls $m_i(a,t)$, $i = 1,2$, can be sought in the form

$$\begin{aligned} m_1 &= b_1 + b_3 \; sin(\frac{2\pi}{T} t) \\ m_2 &= b_2 + b_4 \; cos(\frac{2\pi}{T} t) \end{aligned}$$

where b_i, $i = 1,..,4$ are some unknown coefficients. The above are substituted into CS2, and the systems CS1 and CS2 are integrated symbolically, to yield respective solutions $\gamma^a(T)$ and $\gamma^m(T)$ in terms of parameters a and b. The equation $\gamma^a(T) = \gamma^m(T)$ is then also solved symbolically to deliver the values for the unknown coefficients $b_i(a)$, $i = 1,...,4$ as functions of the control parameters $a = [a_1, a_2, a_3]$ and T:

$$\begin{aligned} &b_1 = a_1, \quad b_2 = a_2, \\ &b_3 = b_4 = \pm 3.54491 \sqrt{a_3} / \sqrt{T}. \end{aligned}$$

which reflects that two solutions were found.

At the implementation stage of the final feedback control, all the terms involving square roots of the extended discretized controls a_i, such as $\sqrt{a_i}$, must naturally be substituted by $sign(a_i)\sqrt{|a_i|}$.

3.3 Time varying stabilizing controls for subsystem $S1$

Using the solution to the TIP problem, the time varying stabilizing controls for subsystem $S1$ are finally given by

$$\begin{aligned} u_1(x) &= (\tilde{v}_1^T(x) + b_3(\tilde{v}_3^T(x)) \; sin(\frac{2\pi}{T} t)) \\ u_2(x) &= (\tilde{v}_2^T(x) + b_3(\tilde{v}_3^T(x)) \; cos(\frac{2\pi}{T} t)) \end{aligned} \tag{18}$$

where, $\quad b_3 = \pm 3.54491 \sqrt{\tilde{v}_3^T(x)} / \sqrt{T}$.

Since the algebraic structure of the system model is preserved during the approximation of this model, as can be seen by comparing Lie brackets of f_1, f_2, f_3, with those of $\tilde{f}_1, \tilde{f}_2, \tilde{f}_3$ and the Lie algebraic controllability conditions for the original and approximated subsystem $S1$, then the control strategy constructed for the approximate model of $S1$ is also stabilizing when applied to the original subsystem $S1$ provided that $\tilde{v}^T(x)$ in (18), is replaced by the sample-and-hold control, $v^T(x)$, for the original $S1$.

For faster convergence, v_i^T can be replaced by $k\ v_i^T$, $i = 1, 2, 3$, where k is any positive constant. In fact, $k = 3$ was used in simulations. Finally, since the time-varying feedback so constructed turns out to be very robust with respect to model-system error, continuous feedback can be employed instead of the piece-wise constant sample-and-hold.

4 Stabilizing control algorithm for the robot with trailer

The time varying feedback control constructed for subsystem $S1$ must now be complemented by a control which also takes account of subsystem $S2$. Stabilization of $S2$ requires generating motion of the entire system in the direction of the second order Lie bracket $[g_1, [g_1, g_2]]$, which can be achieved by application of sinusoidal controls such as, for example : $u_1(t) = sin(\frac{2\pi}{T}t)$, $u_2(t) = cos(\frac{4\pi}{T}t)$, with $u_3(t) = 0$, see (Murray *et al.*, 1993). This observation leads to a control algorithm for the entire system consisting of subsystems $S1$ and $S2$, in which the symbol $\mathcal{N}(\mathcal{S}; \epsilon)$ denotes the ϵ-neighbourhood of a set $\mathcal{S}$.

Stabilizing algorithm for a mobile robot with trailer:

Repeat the following steps until sufficient accuracy is achieved in reaching the origin:

Data : $\epsilon > 0$

Step a: Apply the controls (18) to original system (2) until its trajectories converge to $\mathcal{N}(\mathcal{S}_1; \epsilon)$, where :

$$\mathcal{S}_1 \stackrel{def}{=} \{z \in I\!R^5 : z_1 = z_2 = z_3 = 0, \quad z_4 \neq 0,\ z_5 \neq 0\}$$

(b): To generate motion along $g_4 = [g_1, [g_1, g_2]]$, apply the following controls

$$\begin{aligned} u_1 &= k_1\ sin(\frac{2\pi}{T}t) \\ u_2 &= k_2\ cos(\frac{4\pi}{T}t) \end{aligned} \tag{19}$$

until the system trajectories converge to $\mathcal{N}(\mathcal{S}_2; \epsilon)$, where :

$$\begin{aligned} \mathcal{S}_2 &\stackrel{def}{=} \{z \in I\!R^5 : z_4 = 0\ \&\ sin\ z_2\ cos\ z_3 = 0\} \\ &= \{z \in I\!R^5 : z_4 = z_2 = 0\} \end{aligned}$$

(c): Again apply the control (18) until the system trajectories converge to $\mathcal{N}(\mathcal{S}_3; \epsilon)$:

$$\mathcal{S}_3 \stackrel{def}{=} \{z \in I\!R^5 : z_1 = z_2 = z_3 = z_4 = 0, \quad z_5 \neq 0\}$$

(d): To generate motion along $g_5 = [g_1, [g_1, [g_1, g_2]]]$, apply the following controls

$$\begin{aligned} u_1 &= k_3 \; sin(\frac{2\pi}{T} t) \\ u_2 &= k_4 \; cos(\frac{6\pi}{T} t) \end{aligned} \tag{20}$$

until its trajectories converge to $\mathcal{N}(\mathcal{S}_4; \epsilon)$:

$$\begin{aligned} \mathcal{S}_4 &\stackrel{def}{=} \{z \in I\!R^5 : z_5 = 0 \;\&\; f_2(z) = 0\} \\ &= \{z \in I\!R^5 : z_5 = 0 \;\&\; sin\; (z_2 - z_5)\; cos\; z_3 = 0\} = \{z \in I\!R^5 : z_5 = z_2 = 0\} \end{aligned}$$

(e): Set $\epsilon := \frac{\epsilon}{2}$.

Simulation results are depicted in Figures 2 - 3 which confirm the applicability of combined decomposition and trajectory interception approach. In simulations, the values $k_1 = -2$, $k_1 = -3$, $k_3 = -2.8$, $k_4 = 5$, and $T = 1.2$ were used.

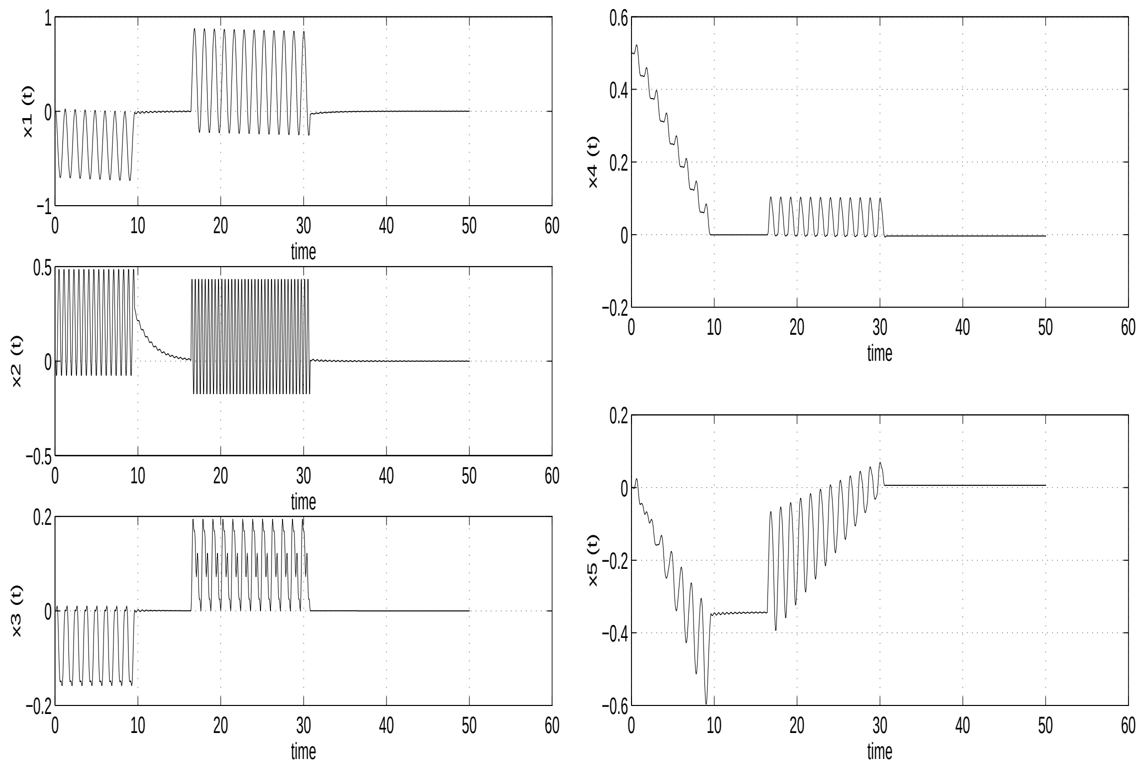

Figure 2: *Mobile robot with trailer* : Plots of the controlled state trajectories $t \mapsto ((z_1(t), ..., z_5(t))$ versus time.

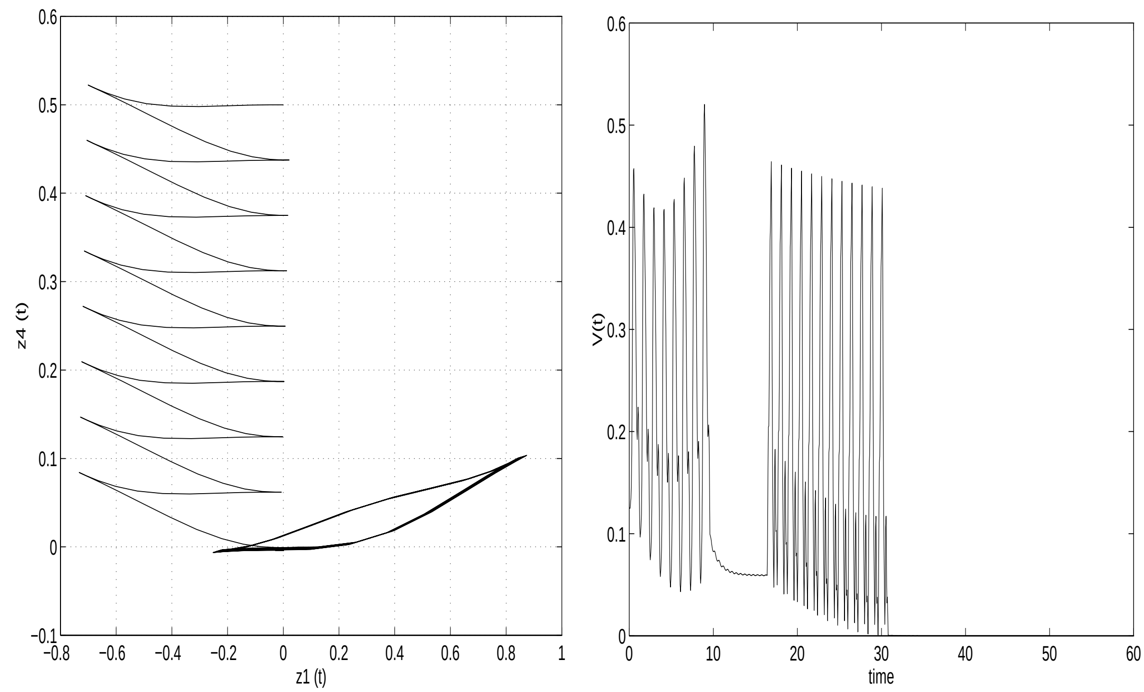

Figure 3: *Mobile robot with trailer* : Plots of the controlled state trajectories $z_1(t)$ versus $z_4(t)$, and Lyapunov function $V(z(t)) = \frac{1}{2}\sum_{i=1}^{5} z_i^2(t)$ along the controlled state trajectories.

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