Proceedings of the 7th Mediterranean Conference on Control and Automation (MED99) Haifa, Israel - June 28-30, 1999

Multi-Drug Infusion Control Using a Robust Direct Adaptive Controller for Plants with Time Delays*

Selahattin Ozcelik†
Department of Mechanical Engineering
Texas A&M University-Kingsville
Campus Box 191, Kingsville, TX 78363

Cesar C. Palerm‡ and Howard Kaufman§
ECSE Department
Rensselaer Polytechnic Institute
Troy, NY 12180-3590

Abstract

The control of hemodynamic variables, particularly mean arterial pressure (MAP) and cardiac output (CO), is a challenging problem. A good controller is difficult to design, due to the complex, nonlinear behavior of the system. Adding to this are the significant changes in dynamics from one patient to another, and even variations in the patients response to the drugs as his condition evolves. A robust direct model reference adaptive controller (DM-RAC) is developed for such plants with uncertainty in both the time delay elements and in the transfer function coefficients. In order to satisfy the conditions for asymptotic model following, it is sufficient to satisfy certain passivity conditions for all possible values of the plant parameters. This is done by transforming the plant variations and time delays into a frequency dependent plant perturbation in the plant transfer function. Feedforward compensator design procedures are then developed using an optimization based robust stability analysis, so that the passivity conditions are satisfied.

1 Introduction

The control of physiological parameters has been of interest for several years. One of the particular problems that has been subject to considerable research is the control of hemodynamic variables, particularly mean arterial pressure (MAP) and cardiac output (CO).

The two main cases where these variables have to be controlled are for patients in critical care with cardiac failure and in operating room scenarios. Currently, it is a physician who has to monitor these variables and adjust the infusion levels of the corresponding drugs manually. Automating this process would reduce the work load of the physicians, allowing them to better monitor other secondary parameters that are not as amenable to automation.

Over the past several years, different approaches have been investigated. Many have focused on the SISO control problem of using sodium nitroprusside (SNP) to maintain a desired blood pressure (Xu *et al.*, 1982; He *et al.*, 1986). An adequate controller for this problem is difficult to design, due to the complex, nonlinear behavior of the system. Adding to this are the significant changes in dynamics from one patient to another, and even variations in the patients response to the drugs as his condition evolves.

*This paper is based upon research performed under NSF Grant BES-9522639. C. Palerm's work is supported by the Consejo Nacional de Ciencia y Tecnología (CONACYT) of México.

†Email: sozcelik@tamuk.edu

‡Email: palerm@rpi.edu

§Prof. Kaufman passed away on 31 January 1999. We would like to dedicate this paper in his memory; he will be deeply missed.

Others have explored the more challenging problem of multiple input multiple output (MIMO) systems. Among these, most have concentrated in controlling blood pressure and cardiac output with the use of SNP and dopamine (DP) (Voss *et al.*, 1987; Barney and Kaufman, 1990; Westvold, 1992).

Yu(Yu *et al.*, 1992) modeled the hemodynamic system by a 2-input 2-output first order system with delays. He proposed the following equation to represent the plant model:

$$\begin{bmatrix} \Delta CO \\ \Delta MAP \end{bmatrix} = \begin{bmatrix} \frac{K_{11}e^{-T_{11}s}}{\tau_{11}s+1} & \frac{K_{12}e^{-T_{12}s}}{\tau_{12}s+1} \\ \frac{K_{21}e^{-T_{21}s}}{\tau_{21}s+1} & \frac{K_{22}e^{-T_{22}s}}{\tau_{22}s+1} \end{bmatrix} \begin{bmatrix} DP \\ SNP \end{bmatrix} \tag{1}$$

with the gains K_{ij} representing the patient sensitivity to the drug, τ_{ij} the corresponding time constant and T_{ij} the corresponding time delay between drug infusion and the response of the system. Typical values and ranges are shown in Table 1.

Parameter	Typical	Range
K_{11}	5	1 to 12
τ_{11}	300	300
T_{11}	60	15 to 60
K_{12}	12	-15 to 25
τ_{12}	150	150
T_{12}	50	15 to 60
K_{21}	3	0 to 9
τ_{21}	40	40
T_{21}	60	15 to 60
K_{22}	-15	-1 to -50
τ_{22}	40	40
T_{22}	50	15 to 60

Table 1: Nominal values and ranges of model parameters

The controller developed is based on a simple adaptive control approach of MIMO plants first proposed by Sobel, Kaufman, and Mabius (Sobel *et al.*, 1979) in 1979. This control structure uses a linear combination of feedforward model states and command inputs and feedback of the error between plant and model outputs. This class of algorithms requires neither full state access nor adaptive observers. Other important properties of this class of algorithms include (1) their applicability to non-minimum phase systems and (2) the fact that the plant (physical system) order may be much higher than the order of the reference model. Its ease of implementation and inherent robustness properties make this simple adaptive control approach attractive.

Although this simple direct model reference adaptive control (DMRAC) algorithm has the above attractive features, it requires that the plant to be controlled satisfies a strictly positive real (SPR) condition. That is, for a plant to be controlled, there exists a feedback gain such that the resulting closed-loop system is strictly positive real. Note that the plant satisfying the above condition is called almost strictly positive real (ASPR). One way to satisfy this positive real constraint is to design a parallel feedforward compensator.

Ozcelik et. al. (Ozcelik and Kaufman, 1995; Ozcelik, 1996; Ozcelik and Kaufman, 1997b,a), has developed and applied systematic design procedures that utilize optimization techniques for both the SISO and MIMO systems with parametric and/or frequency domain uncertainties.

However, feedforward compensator design methods were not addressed for plants with uncertain time delay elements, of which the drug infusion control problem is a specific case (Yu *et al.*, 1992).

Therefore, considering both the SISO and MIMO plants, this paper focuses on the design of parallel robust feedforward compensators for the DMRAC algorithm proposed in (Kaufman *et al.*, 1998), so that the strictly positive real (SPR) condition is satisfied in the presence of plant uncertainty which is modeled as variations in both the plant time delay elements and transfer function coefficients.

Formulation of the DMRAC algorithm is discussed in section 2, robust feedforward compensator design is formulated in section 3, the design of such a controller for the drug infusion control problem and the corresponding simulation results are given in Section 4. Finally, results are discussed, and conclusions are drawn in section 5.

2 Formulation of the DMRAC Algorithm

The linear time invariant model reference adaptive control problem is considered for the plant

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{2}$$

where $x_p(t)$ is the $(nx1)$ state vector, $u_p(t)$ is the $(mx1)$ control vector, $y_p(t)$ is the $(qx1)$ plant output vector, and A_p, B_p are matrices with appropriate dimensions. The range of the plant parameters is assumed to be known and bounded with

$$\begin{aligned} \underline{a}_{ij} \le a_p(i,j) \le \overline{a}_{ij}, i,j = 1,\cdots,n \\ \underline{b}_{ij} \le b_p(i,j) \le \overline{b}_{ij}, i,j = 1,\cdots,n \end{aligned} \tag{3}$$

The objective is to find, without explicit knowledge of A_p and B_p, the control $u_p(t)$ such that the plant output vector $y_p(t)$ follows the reference model

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{4}$$

The model incorporates the desired behaviour of the plant, but its choice is not restricted. In particular, the order of the plant may be much larger than the order of the reference model.

The adaptive control algorithm being presented is based upon the command generator tracker concept (CGT) developed by O'Brien and Broussard (Broussard and O'Brien, 1979). In the CGT method, it is assumed that there exists an ideal plant with ideal state and control trajectories, $x_p^*(t)$ and $u_p^*(t)$, respectively, which corresponds to perfect output tracking (i.e., when $y_p(t) = y_m(t)$ for $t \ge 0$). By definition, this ideal plant satisfies the same dynamics as the real plant, and the ideal plant output is identically equal to the model output. Thus,

$$\dot{x}_p^* = A_p x_p^* + B_p u_p^* \quad for \;\; all \quad t \ge 0 \tag{5}$$

and

$$y_p^* = y_m = C_p x_p^* = C_m x_m \tag{6}$$

Hence, when perfect tracking occurs, the real plant trajectories become the ideal plant trajectories, and the real plant output becomes the ideal plant output, which is defined to be the model output.

The ideal control law $u_p^*(t)$, generating perfect output tracking and the ideal state trajectories x_p^* is assumed to be a linear combination of the model states and model input:

$$\begin{bmatrix} x_p^* \\ u_p^* \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{7}$$

where the S_{ij} submatrices satisfy the following conditions

$$\begin{aligned} S_{11}A_m &= A_pS_{11} + B_pS_{21} \\ S_{11}B_m &= A_pS_{12} + B_pS_{22} \\ C_m &= C_pS_{11} \\ 0 &= C_pS_{12} \end{aligned} \tag{8}$$

In summary, when perfect output tracking occurs, $x_p(t) = x_p^*(t)$, and the ideal control is given by

$$u_p^*(t) = S_{21}x_m(t) + S_{22}u_m \tag{9}$$

If when perfect output tracking does not occur, $y_p(t) \neq y_m(t)$, asymptotic tracking is achievable provided stabilizing output feedback is included in the control law

$$u_p(t) = S_{21}x_m(t) + S_{22}u_m + K_e(y_m(t) - y_p(t)) \tag{10}$$

Then the adaptive control law based on this command generator tracker (CGT) approach is given as (Kaufman *et al.*, 1998)

$$u_p(t) = K_e(t)[y_m(t) - y_p(t)] + K_x(t)x_m(t) + K_u(t)u_m(t) \tag{11}$$

where $K_e(t)$, $K_x(t)$, and $K_u(t)$ are adaptive gains and concatenated into the matrix $K(t)$ as follows

$$K(t) = [K_e(t) \quad K_x(t) \quad K_u(t)] \tag{12}$$

Defining the vector $r(t)$ as

$$r(t) = \begin{bmatrix} y_m(t) - y_p(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \tag{13}$$

the control $u_p(t)$ is written in a compact form as follows

$$u_p(t) = K(t)r(t) \tag{14}$$

The adaptive gains are obtained as a combination of an integral gain and a proportional gain as shown below (Kaufman *et al.*, 1998)

$$\begin{aligned} K(t) &= K_p(t) + K_i(t) \\ K_p(t) &= [y_m(t) - y_p(t)]r^T(t)T_p, \quad T_p \geq 0 \\ \dot{K}_i(t) &= [y_m(t) - y_p(t)]r^T(t)T_i, \quad T_i > 0 \end{aligned} \tag{15}$$

The sufficiency conditions for asymptotic tracking are

1. There exists a solution to the CGT problem (6).

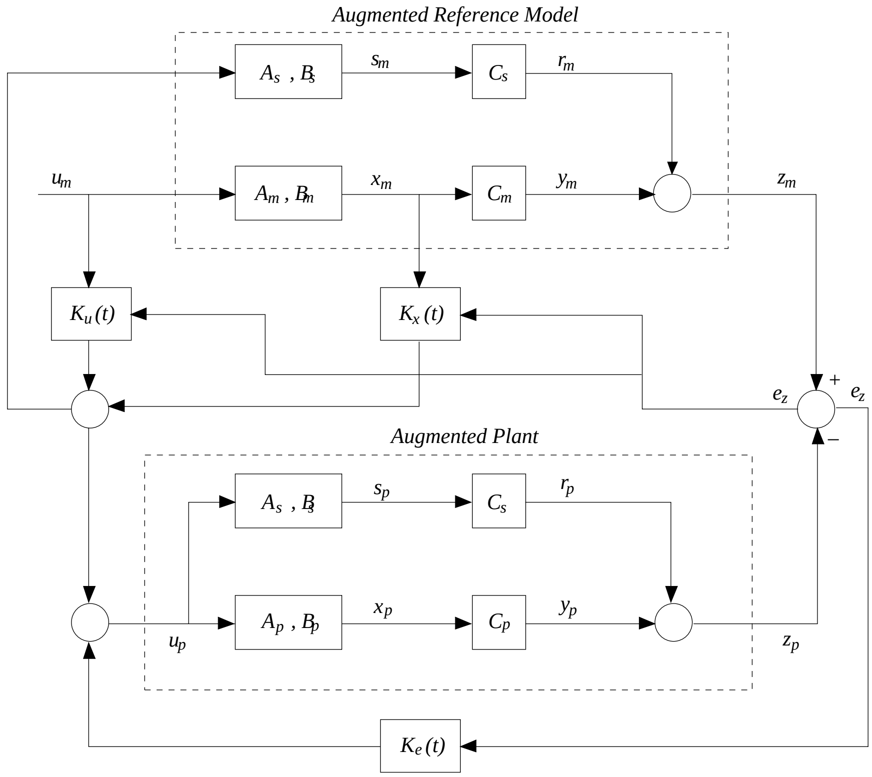

Figure 1: DMRAC with plant and reference model feedforward

2. The plant is ASPR; that is there exists a positive definite constant gain matrix K_e, not needed for implementation, such that the closed loop transfer function

$$G(s) = [I + G_p(s)K_e]^{-1}G_p(s) \tag{16}$$

is strictly positive real (SPR).

In general, the ASPR condition is not satisfied by most real systems. Therefore, Bar Kana and Kaufman (Bar-Kana and Kaufman, 1985) have shown that a non-ASPR plant of the form $G_p(s) = C_p(sI - A_p)^{-1}B_p$ can be augmented with a feedforward compensator $H(s)$ such that the augmented plant transfer function

$$G_a(s) = G_p(s) + H(s) \tag{17}$$

is ASPR. However the resulting adaptive controller will in general result in a model following error that is bounded but not zero in steady state. To eliminate this problem, a modification that incorporates the supplementary feedforward into the reference model output as well as the plant output has been developed by Kaufman and Neat (Kaufman and Neat, 1993). This configuration is shown in Figure 1.

3 Feedforward Compensator Design

This section presents the design of feedforward compensators for plants with plant uncertainty and time delay. Development of design procedures for feedforward compensators will be performed using the transfer function representation of a plant. Plant uncertainty will be represented as variations in the coefficients of the numerator and denominator polynomials of the

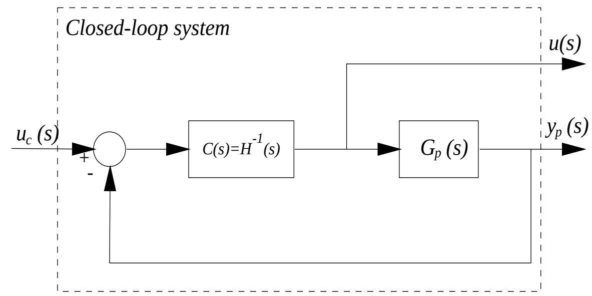

Figure 2: Closed-Loop System

plant transfer function. Design conditions for a feedforward compensator will be developed utilizing an optimization procedure for robust stability.

The objective is to develop a feedforward compensator design procedure such that the ASPR conditions for the augmented system are satisfied over the given range of parameter variations of the plant. In (Iwai and Mizumoto, 1994), Iwai and Mizumoto showed existence of a feedforward compensator in the presence of plant unmodeled dynamics only and did not take time delays into account. In the following, design conditions are developed using an optimization based method for robust stability, in which parametric variations and the time delay of the plant are explicitly taken into account.

In the following, the ASPR Lemma (Bar-Kana, 1991), which will be needed in the development of a design procedure for a feedforward compensator, is given.

Lemma 1 *Let $G_p(s)$ be any $m \times m$ transfer matrix of arbitrary McMillan degree. $G_p(s)$ is not necessarily stable or minimum phase. Let $H(s)^{-1}$ be any dynamic stabilizing controller as shown in Figure 2. Then*

$$G_a(s) = G_p(s) + H(s) \tag{18}$$

is ASPR if the relative McMillan degree of $G_a(s)$ is zero or m.

Proof 1 *See (Bar-Kana, 1991)*

An equivalent statement of the ASPR Lemma is the following (see (Kaufman *et al.*, 1998)): *Let $G_p(s)$ be a strictly minimum phase $m \times m$ transfer matrix of relative McMillan degree m or zero. Let $G_p(s)$ have the minimal realization (A_p, B_p, C_p) with the high frequency gain $C_pB_p > 0$ (positive definite). Then, $G_p(s)$ is ASPR.*

Now, consider a non-ASPR SISO plant of the form

$$G_p(s) = G(s)T(s) \tag{19}$$

where

$$G(s) = \frac{C_m s^m + C_{m-1}s^{m-1} + \cdots + C_0}{B_n s^n + B_{n-1}s^{n-1} + \cdots + B_0} \tag{20}$$

in which the coefficients B_{n-j} and C_{m-j} can take any values within the given bounds:

$$\begin{aligned} \underline{C}_{m-j} \le C_{m-j} \le \overline{C}_{m-j}, j = 0, 1, \cdots, m \\ \underline{B}_{n-j} \le B_{n-j} \le \overline{B}_{n-j}, j = 0, 1, \cdots, n \end{aligned} \tag{21}$$

and $T(s)$ is the time delay of the plant and is of the form

$$T(s) = e^{-T_d s} \tag{22}$$

Let $G_0(s)$ be the nominal plant obtained from $G(s)$ using the nominal values of parameters. Then, defining

$$\Delta_a(s) = G(s) - G_0(s) \tag{23}$$

and

$$\Delta_m(s) = T(s) - 1 \tag{24}$$

The actual plant $G_p(s)$ can be rewritten as

$$G_p(s) = (G_0(s) + \Delta_a(s))(1 + \Delta_m(s)) \tag{25}$$

Defining

$$\Delta(s) = \Delta_m(s) + G_0^{-1}(s)\Delta_a(s)(1 + \Delta_m(s)) \tag{26}$$

The actual plant $G_p(s)$ takes the following form

$$G_p(s) = G_0(s)(1 + \Delta(s)) \tag{27}$$

From (23) and (26), it is seen that the uncertainty $\Delta(s)$ is a function of plant parameters which vary in a given range. Thus, in the design of a feedforward compensator, the worst case uncertainty should be taken into account. To this effect, the following optimization procedure is considered to determine the worst case uncertainty at each frequency.

Define a vector whose elements are plant parameters, i.e.,

$$V = C_m \;\; C_{m-1} \cdots C_0 \;\; B_n \;\; B_{n-1} \cdots B_0 \tag{28}$$

Then

$$\underset{V}{\text{maximize}} \mid \Delta(jw) \mid \quad \text{at each } w$$

$$\text{subject to :} \begin{cases} \underline{C}_{m-j} \leq C_{m-j} \leq \overline{C}_{m-j} \\ \underline{B}_{n-j} \leq B_{n-j} \leq \overline{B}_{n-j} \end{cases} \tag{29}$$

where $\Delta(jw)$ is the perturbation given by (26). It is important to note that this optimization is performed for each frequency. Given the worst case uncertainty at each frequency, it is assumed that there exists a known rational function $W(s) \in RH_\infty$ such that

$$\mid W(jw) \mid \geq max \mid \Delta(jw) \mid, \; \forall \, w \tag{30}$$

Now, the following assumptions are imposed on the plant

Assumption 1

1. *Nominal plant $G_0(s)$ is known, minimum phase, and stable,*
2. *Actual plant $G_p(s)$ is stable,*
3. *$min(\rho_p) \geq \rho_m$, where ρ_p and ρ_m are the relative degrees of the actual plant and the nominal plant, respectively.*
4. *$\Delta(s)$ satisfies (30)*

Now consider the following augmented plant with the parallel feedforward compensator

$$G_a(s) = G_p(s) + H(s) \tag{31}$$

The following theorem gives the design conditions for a parallel feedforward compensator $H(s)$ so that the ASPR conditions of the augmented plant are satisfied in the presence of plant perturbations and time delay.

Theorem 1

If $H(s)$ is designed according to the following conditions, then the augmented plant $G_a(s) = G_p(s) + H(s)$ with plant perturbations will be ASPR.

1. *$H(s)$ is stable with relative degree one*
2. *The augmented nominal plant, $G_0(s) + H(s)$ is ASPR*
3. *$\tilde{\Delta}(s) \in RH_\infty$ and $\| \tilde{\Delta}(s) \|_\infty < 1$*
   where

   $$\tilde{\Delta}(s) = \frac{G_0(s)W(s)}{G_0(s) + H(s)}$$

   is the uncertainty of the augmented plant.

Proof 2 *See (Ozcelik, 1996)*

With regard to the design conditions for a feedforward compensator, the following design procedure is proposed

Design procedure 1

1. the order of a feedforward compensator can be determined from the fact that the sufficient order of a feedforward compensator is equal to the order of a plant.
2. compensator parameters are determined from the following optimization procedure:

$$\underset{X}{\text{minimize}} \| \tilde{\Delta}(jw) \|_\infty \tag{32}$$

$$\text{subject to: } Real[roots(z(s))] < 0$$

where $z(s)$ is the zero polynomial of the augmented nominal plant and X is a vector whose elements are compensator parameters. All the conditions of Theorem 1 will be satisfied using the design procedure given above.

Extension to MIMO plants

In this section, the feedforward compensator design procedure developed for SISO plants will be extended to MIMO plants. Now consider a non-ASPR (mxm) MIMO plant of the form

$$G_p(s) = G(s)T(s) \tag{33}$$

where

$$G(s) = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mm} \end{bmatrix} \tag{34}$$

in which each element g_{ij} of $G(s)$ is of the form

$$g_{ij}(s) = \frac{C_p^{ij} s^p + C_{p-1}^{ij} s^{p-1} + \cdots + C_0^{ij}}{B_r^{ij} s^r + B_{r-1}^{ij} s^{r-1} + \cdots + B_0^{ij}} \tag{35}$$

The coefficients B_{r-k}^{ij} and C_{p-k}^{ij} can take any values within the given bounds:

$$\begin{aligned} \underline{C}_{p-k}^{ij} \leq C_{p-k}^{ij} \leq \overline{C}_{p-k}^{ij}, k = 0, 1, \cdots, p \\ \underline{B}_{r-k}^{ij} \leq B_{r-k}^{ij} \leq \overline{B}_{r-k}^{ij}, k = 0, 1, \cdots, r \end{aligned} \tag{36}$$

$T(s)$ is the time delay of the plant and is of the form

$$T(s) = diag[e^{-T_{d1}s}, e^{-T_{d2}s}, ..., e^{-T_{dm}s}] \tag{37}$$

Let $G_0(s)$ be the nominal plant obtained from $G(s)$ using the nominal values of parameters. Then, defining

$$\Delta_a(s) = G(s) - G_0(s) \tag{38}$$

and

$$\Delta_m(s) = T(s) - I \tag{39}$$

The actual plant $G_p(s)$ can be rewritten as in the SISO case

$$G_p(s) = (G_0(s) + \Delta_a(s))(I + \Delta_m(s)) \tag{40}$$

Again, defining

$$\Delta(s) = \Delta_m(s) + G_0^{-1}(s)\Delta_a(s)(I + \Delta_m(s)) \tag{41}$$

The actual plant $G_p(s)$ takes the following form

$$G_p(s) = G_0(s)(I + \Delta(s)) \tag{42}$$

As in the SISO case, from (38) and (41), it is obvious that the uncertainty $\Delta(s)$ is a function of plant parameters which vary in a given range. Thus, the worst case uncertainty should be taken into account for the design of a feedforward compensator. The worst case uncertainty at each frequency can be determined from the following optimization procedure.

Define a vector whose elements are plant parameters, i.e.,

$$v = [C_p^{ij} \;\; C_{p-1}^{ij} \cdots C_0^{ij} \quad B_r^{ij} \;\; B_{r-1}^{ij} \cdots B_0^{ij}] \tag{43}$$

Then

$$\underset{v}{\text{maximize}} \; \| \Delta(jw) \| \quad \text{at each } w$$

$$\text{subject to :} \begin{cases} \underline{C}^{ij}_{p-j} \le C^{ij}_{p-j} \le \overline{C}^{ij}_{p-j} \\ \underline{B}^{ij}_{r-j} \le B^{ij}_{r-j} \le \overline{B}^{ij}_{r-j} \end{cases} \tag{44}$$

where $\Delta(jw)$ is the perturbation given by (41). Note that this optimization is performed for each frequency. Given the worst case uncertainty at each frequency, it is assumed that there exists a known rational function $W(s) \in RH_\infty$ such that

$$\| W(jw) \| \ge \ max \| \Delta(jw) \| \quad \forall w \tag{45}$$

Now, the following assumptions similar to Assumption 1 are made on the plant,

Assumption 2

1. *The nominal plant $G_0(s)$ is known, minimum phase, and stable,*
2. *Actual plant $G_p(s)$ is stable and the degrees of its diagonal elements are known,*
3. *The off-diagonal elements of $G_{p0}(s)$ and $\Delta(s)$ are strictly proper.*
4. *$min(\rho_p) \ge \rho_m$, where ρ_p and ρ_m are the relative McMillan degrees of the actual plant and the nominal plant, respectively.*
5. *$\Delta(jw)$ satisfies (45).*

Now, consider the following augmented plant with the parallel feedforward compensator

$$G_a(s) = G_p(s) + H(s) \tag{46}$$

where the feedforward compensator $H(s)$ is defined as follows in Lemma 2

Lemma 2

Let the feedforward compensator $H(s)$ be of the form,

$$H(s) = \begin{bmatrix} h_{11} & 0 & \dots & 0 \\ 0 & h_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_{mm} \end{bmatrix} \tag{47}$$

with each element $h_{ii}(s)$ of a feedforward compensator being relative degree zero, then the augmented plant $G_a(s) = G_p(s) + H(s)$ will have positive definite high frequency gain and relative McMillan degree zero.

Proof 3 *See (Ozcelik, 1996)*

It should noted that the above Lemma 2 gives the sufficiency condition for the satisfaction of the positive definite high frequency gain and the relative degree conditions of Lemma 1. It is possible, in some cases, that using relative degree one elements in the feedforward compensator may also satisfy the above conditions. For detailed discussion on the feedforward compensator, interested readers are referred to (Ozcelik, 1996; Kaufman *et al.*, 1998).

Now, the following theorem gives the design conditions for a parallel feedforward compensator $H(s)$, so that the ASPR condition of the augmented plant is satisfied in the presence of plant perturbations.

Theorem 2

If $H(s)$ is designed according to the following conditions, then the augmented plant $G_a(s) = G_p(s) + H(s)$ with plant perturbations will be ASPR.

1. *$H(s)$ is stable with each $h_{ij}(s)$ being relative degree zero*
2. *$H(s)^{-1}$ stabilizes the nominal closed-loop system*
3. *$\tilde{\Delta}(s) \in RH_\infty$ and $\| \tilde{\Delta}(s) \|_\infty < 1$*

where $\tilde{\Delta}(s)$ is the uncertainty of the augmented plant and defined as follows:

$$\tilde{\Delta}(s) = (G_{p0}(s) + H(s))^{-1} G_{p0}(s) W(s)$$

Proof 4 *See (Ozcelik, 1996)*

With regard to the design conditions for the feedforward compensator, the following design method is proposed to determine the necessary coefficients of the compensator.

Design method 2

1. the order of each element $h_{ii}(s)$ of a feedforward compensator is chosen to be equal to the order of the corresponding diagonal element of the nominal plant $G_0(s)$.
2. compensator parameters are determined from the following optimization procedure:

$$\underset{X}{\text{minimize}} \| \tilde{\Delta}(jw) \|_\infty \tag{48}$$

$$\text{subject to: } Real[roots(Z(s))] < 0$$

where $Z(s)$ is the characteristic polynomial of the nominal closed-loop system matrix and X is a vector composed of the parameters of each $h_{ij}(s)$. All the conditions of Theorem 2 will be satisfied using the design procedure given above. Thus, the augmented plant satisfies the almost strictly positive real condition over a wide range of plant parameter variations.

Having proposed a feedforward compensator design procedure for MIMO plants with uncertainties, the following example demonstrates the use of this design procedure and shows its effectiveness.

4 Simulation Results

To facilitate the illustration of the feedforward compensator design procedures presented in section 3 both a SISO and MIMO example will be considered. The SISO transfer function between MAP and SNP is

$$G_p(s) = \frac{K_{22} e^{-T_{22} s}}{\tau_{22} s + 1} \tag{49}$$

The nominal transfer function, defined with the typical gain and time constant from Table 1, but without time delay, is given by

$$G_0(s) = \frac{-15}{40s+1} \tag{50}$$

Of interest is an MAP response with a settling time less than 10 min; thus a reasonable choice for a reference model is:

$$G_m(s) = \frac{1}{90s+1} \tag{51}$$

The additive uncertainty is then obtained, using equation (23), to be

$$\Delta_a(s) = -\frac{K_{22}+15}{40s+1} \tag{52}$$

The multiplicative uncertainty, as defined in equation (24), then becomes

$$\Delta_m(s) = e^{-T_{22}s} - 1 \tag{53}$$

Then the combined uncertainty, using equation (26), is

$$\Delta(s) = e^{-T_{22}s}\frac{K_{22}+30}{15} - 1 \tag{54}$$

Figure 3 shows the maximum at each frequency of $\Delta(jw)$, as well as the bounding rational function given by

$$W(s) = \frac{125s+1}{35s+1} \tag{55}$$

In this case a first order compensator is adequate; the denominator can be predetermined, taking into account that the compensator should be faster than the reference model. Thus consider,

$$H(s) = \frac{b_0}{s+2} \tag{56}$$

The uncertainty of the augmented plant is then given by

$$\tilde{\Delta}(s) = \frac{-45(s+2)}{(40b_0-15)s+b_0-30} \tag{57}$$

In this case we can easily solve for $\| \tilde{\Delta}(jw) \|_\infty$ analytically. For all conditions in Theorem 1 to be satisfied we find that we need either $b_0 \leq -60$ or $b_0 \geq 120$; the larger $| b_0 |$, the smaller $\| \tilde{\Delta}(jw) \|_\infty$ will be. Since a high gain in the feedforward compensator will decrease performance, we choose the smallest positive gain possible. This resulted in

$$H(s) = \frac{120}{s+2} \tag{58}$$

The values for the adaptation gains T_p and T_i were obtained by tuning for the nominal system and fine tuning to minimize the overshoot for the extreme high gain of -50. The initial gains were also adjusted to improve the response of the system. These values are

$$\begin{aligned} T_p &= \begin{bmatrix} 1e\text{–}5 & 1e\text{–}8 & 1e\text{–}6 \end{bmatrix} \\ T_i &= \begin{bmatrix} 1e\text{–}8 & 1e\text{–}12 & 1e\text{–}6 \end{bmatrix} \\ K_0 &= \begin{bmatrix} 5e\text{–}3 & 5e\text{–}5 & 2e\text{–}2 \end{bmatrix} \end{aligned} \tag{59}$$

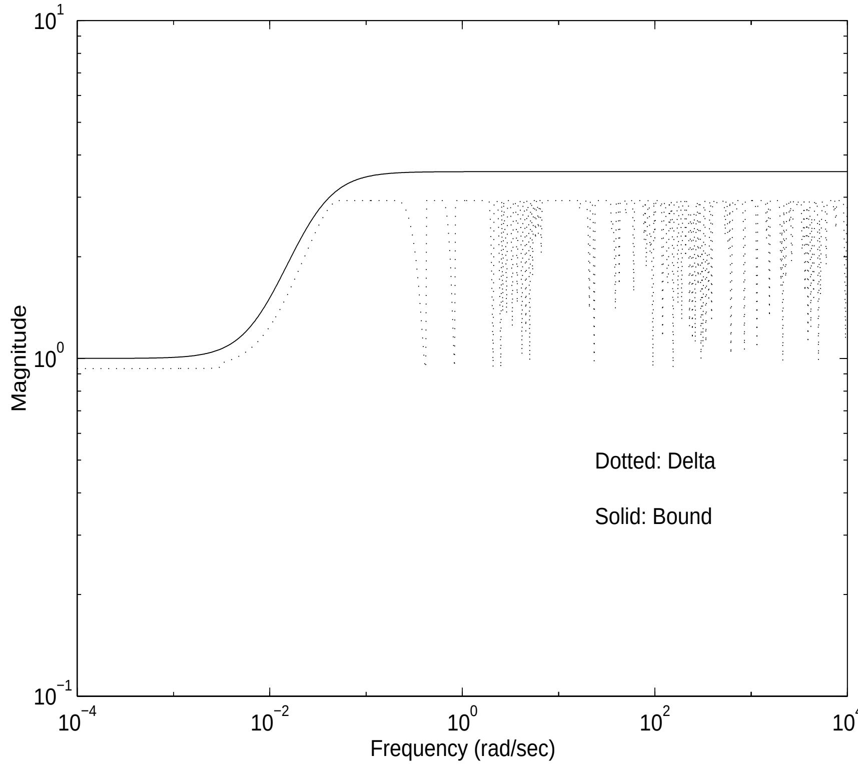

Figure 3: $max\|\Delta(jw)\|$ and its bound $\|W(jw)\|$ for SISO case

The results for the cases described below in Table 2 are shown in Figure 4. All cases were stable although the plant with the very low gain value of -1 took quite a bit longer to adapt. This could have been ameliorated at the expense of a larger overshoot in the first cycle for the high gain case.

Case	K_{22}	T_{22}
1	-1	15
2	-1	50
3	-1	60
4	-15	15
5	-15	50
6	-15	60
7	-50	15
8	-50	50
9	-50	60

Table 2: Values of model parameters for simulation cases

For the MIMO case, $G_0(s)$ was again selected using the typical value from Table 1, but without time delays. Thus,

$$G_0(s) = \begin{bmatrix} \frac{5}{300s+1} & \frac{12}{150s+1} \\ \frac{3}{40s+1} & \frac{-15}{40s+1} \end{bmatrix} \tag{60}$$

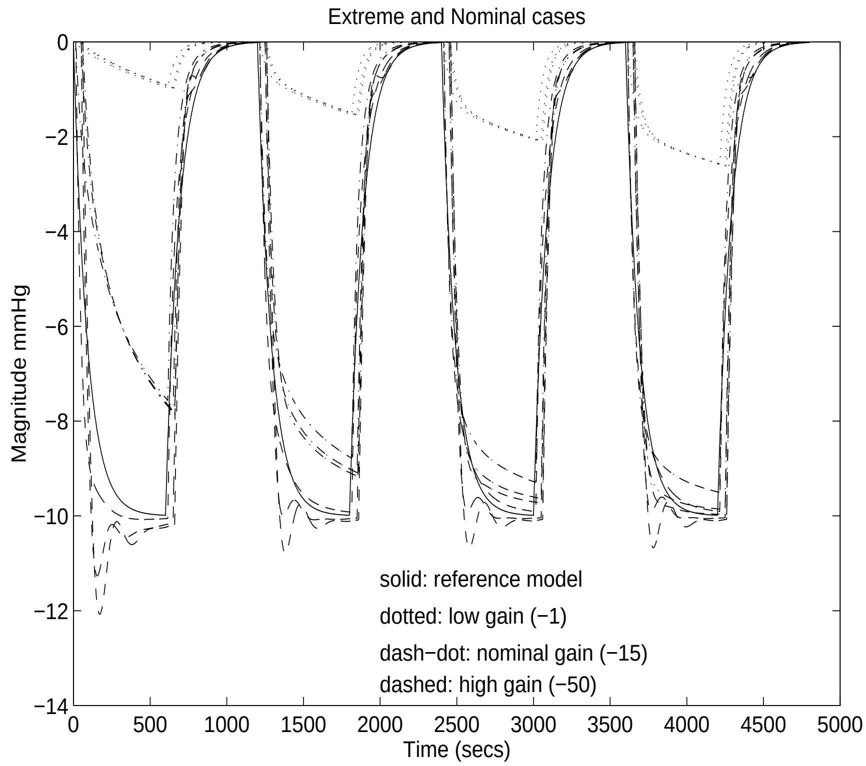

Figure 4: Simulation of DMRAC for SISO case.
$T_p = [1e\text{–}5,\ 1e\text{–}8,\ 1e\text{–}6]$ $T_i = [1e\text{–}8,\ 1e\text{–}12,\ 1e\text{–}6]$ $K_0 = [5e\text{–}3,\ 5e\text{–}5,\ 2e\text{–}2]$.

and

$$T(s) = \begin{bmatrix} e^{-T_{11}s} & 0 \\ 0 & e^{-T_{22}s} \end{bmatrix} \tag{61}$$

Once again, of interest is an MAP response with a settling time less than 10 min. For CO the desired settling time should be less than 20 min. Thus, a reasonable choice for a reference model is:

$$G_m(s) = \begin{bmatrix} \frac{1}{90s+1} & 0 \\ 0 & \frac{1}{300s+1} \end{bmatrix} \tag{62}$$

The combined uncertainty, based on equation (41), is computed numerically for each frequency when doing the maximization of $\Delta(jw)$ according to equation (44).

Figure 5 shows the maximum at each frequency of $\Delta(jw)$, as well as the bounding rational function given by

$$W(s) = \begin{bmatrix} \frac{3.75(300s+1)}{170s+1} & 0 \\ 0 & \frac{3.75(300s+1)}{170s+1} \end{bmatrix} \tag{63}$$

For this case a first order compensator is also adequate; the denominator can be predetermined as well. Thus consider,

$$H(s) = \begin{bmatrix} \frac{b_0}{5s+1} & 0 \\ 0 & \frac{b_1}{5s+1} \end{bmatrix} \tag{64}$$

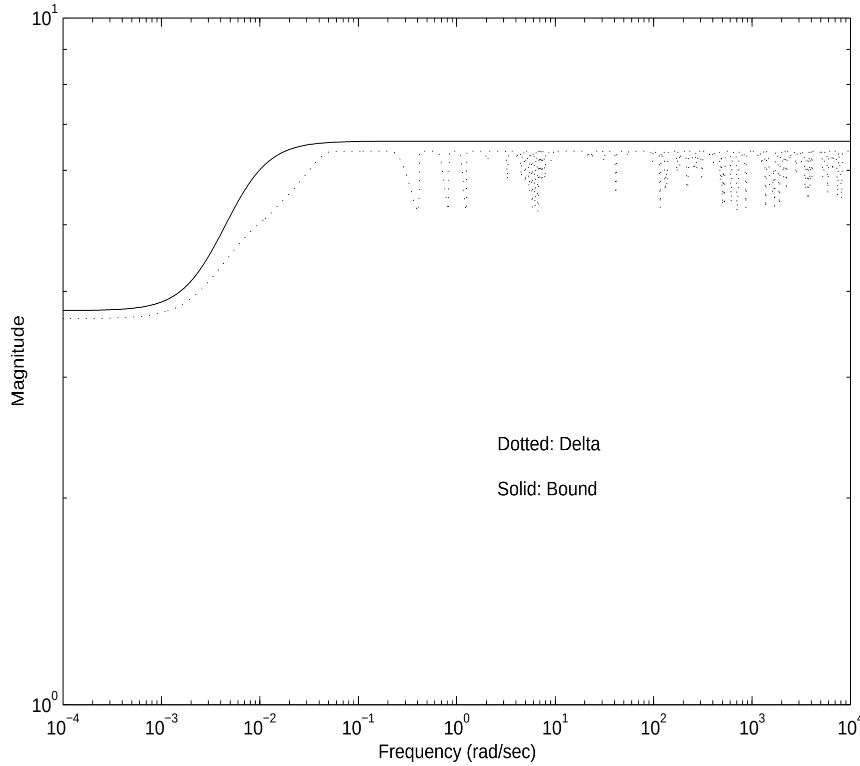

Figure 5: $max\|\Delta(jw)\|$ and its bound $\|W(jw)\|$ for MIMO case

The uncertainty of the augmented plant for the MIMO case is best computed numerically. The compensator was selected to be

$$H(s) = \begin{bmatrix} \frac{84}{5s+1} & 0 \\ 0 & \frac{120}{5s+1} \end{bmatrix} \tag{65}$$

The values for the adaptation gains T_p and T_i were obtained by tuning for the nominal system and fine tuning to minimize the overshoot for the extreme high gains. The initial gains were also adjusted to improve the response of the system. These values are

$$T_p = \begin{bmatrix} 1e–1 & 0 & 2.5e–8 & 0 & 1e–5 & 0 \\ 0 & 1e–1 & 0 & 2.5e–7 & 0 & 1e–4 \end{bmatrix}$$
$$T_i = \begin{bmatrix} 1e–3 & 0 & 1e–8 & 0 & 1e–6 & 0 \\ 0 & 1e–2 & 0 & 1e–8 & 0 & 1e–6 \end{bmatrix}$$
$$K_0 = \begin{bmatrix} -2.8598e–2 & 6.5843e–16 & 6.1421e–4 & 1.0842e–19 & 1.7798e–2 & -1.5405e–2 \\ -5.1605e–3 & -4.2026e–15 & 5.1594e–3 & -8.6736e–19 & -4.5475e–3 & 7.7027e–2 \end{bmatrix}$$

The results for the cases described below in Table 3 are shown in Figure 6. As with the SISO case, the system with the low gains takes a long time to adapt. Larger values for the initial gains and/or larger adaptation gains would improve this response significantly, but would cause larger overshoots and oscillations for the case with the high gains.

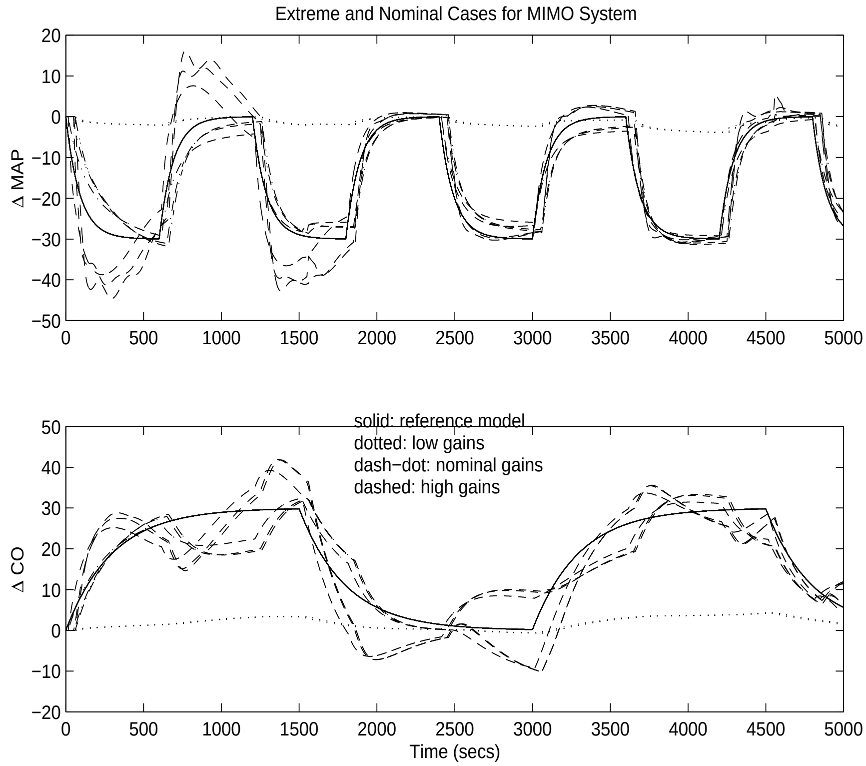

Figure 6: Simulation of DMRAC for MIMO case.

5 Conclusions

Considering both SISO and MIMO plants with uncertainties in both plant parameters and time delay elements, a systematic design method based on an optimization procedure for robust stability analysis for a feedforward compensator was developed. This easily implementable design procedure enables the augmented plant to satisfy the ASPR conditions in the presence of variations in plant parameters and time delay elements. Hence, the applicability of direct adaptive controllers has now been extended to systems with time delays. Simulation results demonstrate the viability of the DMRAC algorithm designed using this new method.

There is obviously a need to improve the response of the system over the whole range of possible plant variations. Due to their wide range, and the time delay factor, it is extremely difficult to have a single controller perform optimally. Work is in progress to incorporate the concepts presented in (Narendra *et al.*, 1995), to use multiple models. Command limiting is another issue that is being addressed, as in reality there are physical and safety constraints imposed on the drug infusion rates.

The design procedure outlined is also being used to design controllers for the same system, but using a nonlinear model. These designs will be tested on dogs.

References

Bar-Kana, I. (1991). "Positive realness in multivariable stationary linear systems," *J. Franklin Institute*, **328**, pp. 403–417.

Case	K_{11}	K_{12}	K_{21}	K_{22}	T_{22}	T_{22}
1	5	12	3	-15	60	50
2	5	12	3	-15	15	15
3	5	12	3	-15	60	60
4	1	0	0	-1	60	50
5	1	0	0	-1	15	15
6	1	0	0	-1	60	60
7	12	25	9	-50	60	50
8	12	25	9	-50	15	15
9	12	25	9	-50	60	60

Table 3: Values of model parameters for MIMO simulation cases

Bar-Kana, I. and H. Kaufman (1985). "Global stability and performance of a simplified adaptive control algorithm," *Int. J. Control*, **42**, no. 6, pp. 1491–1505.

Barney, E. and H. Kaufman (1990). "Model reference adaptive control of cardiac output and blood pressure through two drug infusions," in *5th IEEE Int. Symp. on Intelligent Control*, IEEE, Philadelphia.

Broussard, J. and O. O'Brien (1979). "Feedforward control to track the output of a forced model," in *17th IEEE Conference on Desicion and Control*, San Diego, CA, vol. 42, pp. 1149–1155.

He, W., H. Kaufman, and R. Roy (1986). "Multiple model adaptive control procedure for blood pressure control," *IEEE Trans. Biomed. Eng.*, **33**, pp. 10–19.

Iwai, Z. and I. Mizumoto (1994). "Realization of simple adaptive control by using parallel feedforward compensator," *Int. J. Control*, **59**, no. 6, pp. 1543–1565.

Kaufman, H., I. Bar-Kana, and K. Sobel (1998). *Direct Adaptive Control Algorithms*, Springer-Verlag.

Kaufman, H. and G. Neat (1993). "Asymptotically stable mimo direct mrac for processes not necessarily satisfying a positive real constraint," *Int. J. Control*, **58**, pp. 1011–1031.

Narendra, K. S., J. Balakrishnan, and M. K. Ciliz (1995). "Adaptation and learning using multiple models, switching, and tuning," *Control Systems*, **15**, no. 3, pp. 37–51.

Ozcelik, S. (1996). *Design of Robust Feedforward Compensators for Direct Model Reference Adaptive Controllers*, Ph.D. thesis, Rensselaer Polytechnic Institute, Troy, NY.

Ozcelik, S. and H. Kaufman (1995). "Robust direct model reference adaptive controllers," in *Proceedings CDC*, IEEE, pp. 3955–3960.

Ozcelik, S. and H. Kaufman (1997a). "Design of mimo robust direct model reference adaptive controllers," in *Proceedings CDC*, IEEE.

Ozcelik, S. and H. Kaufman (1997b). "Frequency domain design conditions for robust direct model reference adaptive controllers," in *Proceedings ACC*, IEEE, pp. 1652–1656.

Sobel, K., H. Kaufman, and L. Mabius (1979). "Model reference output adaptive control systems without parameter identification," in *18th IEEE Conference on Desicion and Control*, Ft. Lauderdale, FL.

Voss, G., P. Katona, and H. Chizeck (1987). "Adaptive multivariable drug delivery: Control of arterial pressure and cardiac output in anesthetized dogs," *IEEE Trans. Biomed. Eng.*, **34**, pp. 617–623.

Westvold, S. S. (1992). *Model Reference Adaptive Control of Cardiac Output and Mean Arterial Pressure in a Realistic Model of a Circulatory System*, Master's thesis, Rensselaer Polytechnic Institute.

Xu, X., H. Kaufman, and R. Roy (1982). "Model reference adaptive control of drug infusion rate," in *6th IFAC Symp. on Identificatoin*, Washington, D.C., pp. 216–221.

Yu, C., R. Roy, H. Kaufman, and B. Bequette (1992). "Multiple-model adaptive predictive control of meanarterial pressure and cardiac output," *IEEE Trans. in BME*, pp. 765–778.