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A numerical algorithm for the design of a decentralized controller for open-channel networks

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Abstract

In this paper we propose the design of a decentralized constant-volume control law for open-channel networks. The decentralized control law enables us to maintain the stored volumes in the different reaches practically constant, even with variations in users withdrawals, by acting only on the upstream gate of the reach whose volume variation is detected. Control law is designed by solving a linear least squares problem in the frequency domain. The numerical algorithm adopted allows us to impose the desired structure to the feedback gain matrix by means of the optimization of the controller parameters. It makes the closed-loop transfer function approach a target function as closely as possible over a specified frequency range.

1 Introduction

Many irrigation canal regulation methods have been developed in the world. These methods differ from a country or region to another. They range from the simplest methods, developed more than 2000 years ago, to the most sophisticated ones developed recently, or under development (Reddy, 1992; Sawadogo *et al.*, 1992; Seatzu, 1999). A very detailed classification on the subject has been proposed by Malaterre in his Ph.D. thesis (Malaterre, 1995). These methods differ in their choice of: controlled variables, measured variables, control variables and logic of control.

In this paper we consider the two models deduced from the Saint-Venant equations by Corriga *et al.* in (1982; 1983; 1989). The first one, denoted as the *reference model*, expresses the dynamic relationships, in terms of transcendental functions, between the gate opening sections and the corresponding stored water volume variations in the different canal reaches with respect to an initial reference configuration of uniform flow. The second one, denoted as *approximate nominal model*, is obtained from the previous one by means of a series expansion around $s = 0$. It is a state variable linear and time invariant model. The state variables and the control variables are equal to the output and input variable of the preceding model, respectively. In this paper we propose the design of a decentralized constant–volume control law that enables us to maintain the stored volumes in the different reaches practically constant, even with variations in users withdrawals, by acting only on the upstream gate of the reach whose volume variation is detected.

When considering the control of large scale systems, like hydraulic open-channel networks, decentralized decision making is essential. When dealing with linear state variable models valid solutions to this problem has been proposed in (Duan, 1994; Lu *et al.*, 1993) where complete

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parametric approaches for the eigenstructure assignment are suggested. However, even if these procedures are satisfactory from a theoretic point of view, the number of unknown feedback gains introduces a bound on their effectiveness. On the contrary, in this paper we use a really simple procedure which, even if it presents some drawbacks with respect to the more sophisticated ones cited above, it has the considerable advantage that its computational complexity weakly increases with the dimension of the problem. Furthermore, it is useful even when dealing with systems whose models are nonrational transfer functions.

In this paper the control law has been designed by means of a numerical algorithm firstly proposed by Edmund in (Edmund, 1979) and involves the solution of a linear least squares problem. As it will be shown later, it revealed to be particularly effective in the case at hand. The main advantage with respect to other procedures based on parameter optimization (Seatzu, 1999) is that it does not require a good initial parameter estimation and no problem related to local minima occur. So it can be useful when dealing with high dimensional problems. Furthermore, it enables us to use the more general reference model instead of its low frequency approximation. The approximate linear model is only used to design satisfactory and compatible closed loop target functions.

In this paper we also present the results of numerical simulations that demonstrate the satisfactory behaviour of the system when unknown disturbances occur and the decentralized control law is implemented. All numerical simulations have been carried out by means of the commercial SIC software (Malaterre *et al.*, 1997).

2 Dynamic model of the open channel

In this section we recall the fundamental steps in the deduction of both the reference and the approximate nominal models used for the synthesis of the controller. These models are the result of much research effort by Corriga *et al.* (1982; 1983; 1989). The section provides all the analytical expressions of the basic transfer function matrices and summarizes the main simplifying assumptions introduced.

2.1 Reference model

Consider the system shown in Figure 1, consisting of a channel of N reaches joined by $N+1$ gates, where the last gate (the $(N+1)$-st) is fixed and the others are controlled. Let us suppose that water is conveyed to the first reach from a reservoir with constant level and that the level downstream from the final reach is also constant.

All the other variables considered, apart those that define the geometry, represent the variations with respect to a reference configuration, assumed to be of uniform flow.

Taking the Laplace transform of all the variables (we shall use capital letters for the L-transformed variables), we obtain the following vectors:

$$\boldsymbol{H} = [H_{A1} \;\; H_{B1} \;\; \cdots \;\; H_{Ai} \;\; H_{Bi} \;\; \cdots \;\; H_{AN} \;\; H_{BN}]^T$$

where H_{Ai} and H_{Bi} denote the upstream and downstream water level variations, respectively, in the i-th reach;

$$\boldsymbol{Q} = [Q_{A1} \;\; \cdots \;\; Q_{Ai} \;\; \cdots \;\; Q_{BN}]^T$$

where Q_{Ai} is the flow rate variation through the i-th gate, which is equal to the downstream flow rate variation in the previous reach ($Q_{Ai} = Q_{B(i-1)}, \forall i$) assuming constant users flow rates,

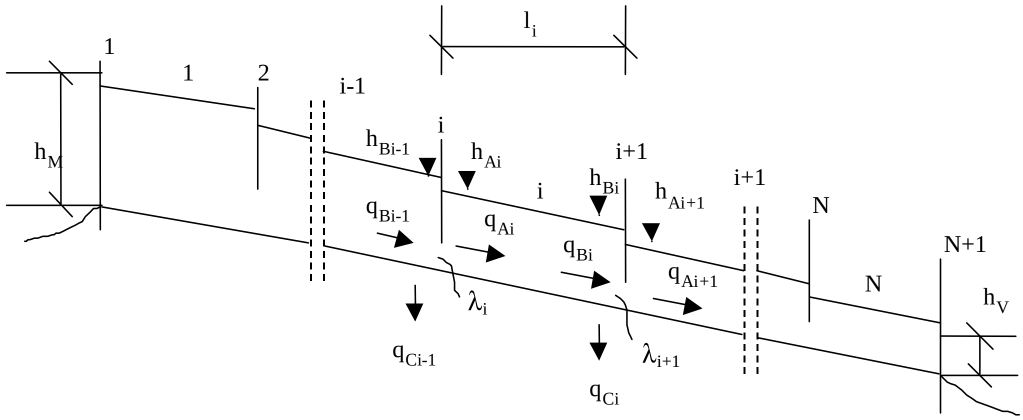

Figure 1: *Scheme of system composed by a cascade of N canal reaches.*

that is, their variations $Q_{Ci} = 0, \forall i$ (they will subsequently be introduced as disturbances);

$$\boldsymbol{\Lambda} = [\Lambda_1 \ \cdots \ \Lambda_i \ \cdots \ \Lambda_N]^T$$

where Λ_i is the variation in the i-th gate opening section, and finally,

$$\boldsymbol{V} = [V_1 \ \cdots \ V_i \ \cdots \ V_N]^T,$$

V_i being the water volume variation in the i-th reach.

It has been shown by Corriga *et al.* (1989) that the dynamics of such a system can be described by the following equations:

$$\boldsymbol{H}(s) = \frac{1}{s}\tilde{\boldsymbol{A}}(s)\boldsymbol{Q}(s) \tag{1}$$

$$\boldsymbol{Q}(s) = \boldsymbol{\Gamma\Lambda}(s) + \boldsymbol{\Delta H}(s) \tag{2}$$

$\tilde{\boldsymbol{A}}(s)$ being a $2N \times (N+1)$ matrix with the following structure:

$$\tilde{\boldsymbol{A}}(s) = \begin{bmatrix} A_{11}(s) & A_{21}(s) & 0 & 0 & \cdots & \cdots & 0 \\ A_{31}(s) & A_{41}(s) & 0 & 0 & \cdots & \cdots & 0 \\ 0 & A_{12}(s) & A_{22}(s) & 0 & \cdots & \cdots & 0 \\ 0 & A_{32}(s) & A_{42}(s) & 0 & \cdots & \cdots & 0 \\ \vdots & & & & & & \\ 0 & & \cdots & \cdots & & A_{4(N-1)}(s) & 0 \\ 0 & & \cdots & \cdots & & A_{1N}(s) & A_{2N}(s) \\ 0 & & \cdots & \cdots & & A_{3N}(s) & A_{4N}(s) \end{bmatrix}.$$

The expressions of $A_{ji}(s)$ are given in (Seatzu, 1999).

The $(N+1) \times N$ matrix $\boldsymbol{\Gamma}$ and the $(N+1) \times 2N$ matrix $\boldsymbol{\Delta}$ are written as:

$$\boldsymbol{\Gamma} = \begin{bmatrix} a_1 & 0 & & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & & \cdots & & a_N \\ 0 & & \cdots & & 0 \end{bmatrix}, \quad \boldsymbol{\Delta} = \begin{bmatrix} c_1 & 0 & & & \cdots & & & 0 \\ 0 & b_2 & c_2 & 0 & & \cdots & & 0 \\ 0 & 0 & 0 & b_3 & c_3 & 0 & \cdots & 0 \\ \vdots & & & & & & & \\ 0 & & & \cdots & & & 0 & b_{N+1} \end{bmatrix}.$$

The constants a_i, b_i and c_i are coefficients of the Taylor series expansion of the laws governing flow through the i-th gate (which depend of course on the reference situation). They are a function of the discharge coefficient η. Their expressions are given in (Seatzu, 1999).

To obtain an expression of equation (1) that also holds for $s = 0$, it can be rewritten as

$$\boldsymbol{H}(s) = \boldsymbol{A}_1(s)\boldsymbol{V}(s) + \boldsymbol{A}_2(s)\boldsymbol{Q}(s) \tag{3}$$

where $A_1(s)$ and $A_2(s)$ are given in (Seatzu,1999). Substituting equation (2) into (3) gives

$$\begin{aligned} \boldsymbol{Q}(s) &= [\boldsymbol{I} - \boldsymbol{\Delta A}_2(s)]^{-1}\boldsymbol{\Delta A}_1(s)\boldsymbol{V}(s) + [\boldsymbol{I} - \boldsymbol{\Delta A}_2(s)]^{-1}\boldsymbol{\Gamma\Lambda}(s) \\ &= \boldsymbol{A}_3(s)\boldsymbol{V}(s) + \boldsymbol{B}_3(s)\boldsymbol{\Lambda}. \end{aligned} \tag{4}$$

Left-multiplying both sides of equation (4) by the following $N \times (N+1)$-dimensional operator:

$$\boldsymbol{I}^* = \begin{bmatrix} 1 & -1 & 0 & \cdots & & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ \vdots & & & & & \\ 0 & \cdots & & 0 & 1 & -1 \end{bmatrix}$$

gives

$$\boldsymbol{I}^*\boldsymbol{Q}(s) = s\boldsymbol{V}(s) = \boldsymbol{I}^*\boldsymbol{A}_3(s)\boldsymbol{V}(s) + \boldsymbol{I}^*\boldsymbol{B}_3(s)\boldsymbol{\Lambda}(s)$$

and setting

$$\tilde{\boldsymbol{A}}(s) = \boldsymbol{I}^*\boldsymbol{A}_3 \qquad \tilde{\boldsymbol{B}}(s) = \boldsymbol{I}^*\boldsymbol{B}_3(s)$$

we get

$$s\boldsymbol{V}(s) = \tilde{\boldsymbol{A}}(s)\boldsymbol{V}(s) + \tilde{\boldsymbol{B}}(s)\boldsymbol{\Lambda}(s)$$

from which the 'exact' transfer matrix is obtained:

$$\boldsymbol{G}(s) = [s\boldsymbol{I} - \boldsymbol{A}(s)]^{-1}\boldsymbol{B}(s) \tag{5}$$

whose elements are transcendental functions of s.

2.2 The approximate nominal model

To obtain an approximate model of the channel dynamics of the form

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{A}\boldsymbol{v}(t) + \boldsymbol{B}\boldsymbol{\lambda}(t) \tag{6}$$

where matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant, equation (1) can be expanded in Taylor series, the elements of $\tilde{\boldsymbol{A}}(s)$ being analytic functions. Since the model needs to hold mainly in the low-frequency range, where the most significant phenomena take place, $s = j\omega = 0$ is taken as initial point. By truncating the series expansion to the second term, equation (1) becomes

$$\begin{aligned} \boldsymbol{H}(s) &\cong \frac{1}{s}\left[\tilde{\boldsymbol{A}}(0) + s\left(\frac{d}{ds}\tilde{\boldsymbol{A}}(s)\right)_{s=0}\right]\boldsymbol{Q}(s) \\ &= \frac{1}{s}\tilde{\boldsymbol{A}}(0)\boldsymbol{Q}(s) + \tilde{\boldsymbol{A}}'(0)\boldsymbol{Q}(s). \end{aligned} \tag{7}$$

As shown in (Corriga *et al.*, 1989) equation (7) can be rewritten as

$$\boldsymbol{H}(s) \cong \boldsymbol{A}_1(0)\boldsymbol{V}(s) + \tilde{\boldsymbol{A}}'(0)\boldsymbol{Q}(s). \tag{8}$$

Equations (2) and (8) define the approximate model.

Following the same procedure as for the reference model, the expressions for the constant matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of equation (6) are obtained:

$$\begin{aligned}\boldsymbol{A} &= \boldsymbol{I}^*[\boldsymbol{I} - \boldsymbol{\Delta}\tilde{\boldsymbol{A}}'(0)]^{-1}\boldsymbol{\Delta A}_1(0)\\ \boldsymbol{B} &= \boldsymbol{I}^*[\boldsymbol{I} - \boldsymbol{\Delta}\tilde{\boldsymbol{A}}'(0)]^{-1}\boldsymbol{\Gamma}\end{aligned}$$

and the corresponding transfer matrix is

$$\boldsymbol{G}_A(s) = [s\boldsymbol{I} - \boldsymbol{A}]^{-1}\boldsymbol{B} \tag{9}$$

whose elements are rational functions of s, which is an approximation of (5).

Finally, taking into account the variations of the users flow rates $\boldsymbol{q}_C$, equation (6) can be rewritten as:

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{A}\boldsymbol{v}(t) + \boldsymbol{B}\boldsymbol{\lambda}(t) - \boldsymbol{I}\boldsymbol{q}_C(t) \tag{10}$$

where $\boldsymbol{I}$ is the N order identity matrix.

3 Edmund's algorithm

We now describe the numerical algorithm proposed by Edmund (1979) and clearly summarized in (Maciejowski, 1989). Suppose that our plant is represented by transfer function matrix $\boldsymbol{G}$, with l columns (inputs) and m rows (outputs), and that we are to design a controller with transfer-function matrix $\boldsymbol{K}$ (with m columns and l rows). Let the closed-loop transfer function actually achieved by a controller $\boldsymbol{K}$ be $\boldsymbol{T} = \boldsymbol{GK}(\boldsymbol{I}+\boldsymbol{GK})^{-1}$, and let the 'target' transfer function which we would like to achieve be $\boldsymbol{T}_t$. Corresponding to $\boldsymbol{T}_t$, there is a 'target' controller $\boldsymbol{K}_t$ such that

$$\boldsymbol{GK}_t = \boldsymbol{T}_t(\boldsymbol{I} - \boldsymbol{T}_t)^{-1}. \tag{11}$$

We define an error function

$$\boldsymbol{E} = \boldsymbol{T}_t - \boldsymbol{T}. \tag{12}$$

Then it can be shown, with a little manipulation, that

$$(\boldsymbol{I} - \boldsymbol{T})(\boldsymbol{GK}_t - \boldsymbol{GK})(\boldsymbol{I} - \boldsymbol{T}_t) = \boldsymbol{E}. \tag{13}$$

If we suppose that $\|\boldsymbol{E}\|$ is sufficiently small, which will be the case if $\boldsymbol{K}$ is sufficiently close to $\boldsymbol{K}_t$, then, by replacing $\boldsymbol{I} - \boldsymbol{T}$ by $\boldsymbol{I} - \boldsymbol{T}_t$ in (13), we obtain

$$(\boldsymbol{I} - \boldsymbol{T}_t)(\boldsymbol{GK}_t - \boldsymbol{GK})(\boldsymbol{I} - \boldsymbol{T}_t) \approxeq \boldsymbol{E} \tag{14}$$

since

$$(\boldsymbol{I} - \boldsymbol{T})(\boldsymbol{GK}_t - \boldsymbol{GK})(\boldsymbol{I} - \boldsymbol{T}_t) = (\boldsymbol{I} - \boldsymbol{T}_t)(\boldsymbol{GK}_t - \boldsymbol{GK})(\boldsymbol{I} - \boldsymbol{T}_t) + O(\|\boldsymbol{E}\|^2). \tag{15}$$

Now let us write

$$\boldsymbol{K}(s) = \frac{1}{d(s)}\boldsymbol{N}(s) \tag{16}$$

where $d(s)$ is a common-denominator polynomial which is assumed to be known, and $\boldsymbol{N}(s)$ is a matrix of polynomials of known degrees but with unknown coefficients. Finally, we define

$$\boldsymbol{L}(s) = \boldsymbol{I} - \boldsymbol{T}_t(s) \tag{17}$$

$$\boldsymbol{M}(s) = \frac{1}{d(s)}\boldsymbol{L}(s)\boldsymbol{G}(s) \tag{18}$$

and

$$\boldsymbol{Y}(s) = \boldsymbol{L}(s)\boldsymbol{G}(s)\boldsymbol{K}_t(s)\boldsymbol{L}(s). \tag{19}$$

Then (14) becomes

$$\boldsymbol{Y}(s) \approxeq \boldsymbol{M}(s)\boldsymbol{N}(s)\boldsymbol{L}(s) + \boldsymbol{E}(s). \tag{20}$$

The noteworthy features here are that the unknown coefficients in $\boldsymbol{N}(s)$ appear linearly in this expression, that $\boldsymbol{M}(s)$, $\boldsymbol{L}(s)$ and $\boldsymbol{Y}(s)$ are well known and can be evaluated at particular values of s when required and, hence, that the problem of finding $\boldsymbol{N}(s)$ which minimizes

$$\|\boldsymbol{E}\|_2^2 = \int_{-\infty}^{\infty} tr[\boldsymbol{E}^T(-j\omega)\boldsymbol{E}(j\omega)]d\omega$$

is a linear least-squares problem if the approximate equality in (20) is replaced by exact equality.

To put (20) into the more familiar standard form in which linear least-squares problems are usually seen, we need to 'stack' the columns of $\boldsymbol{Y}$, $\boldsymbol{N}$ and $\boldsymbol{E}$ on top of each other. For this purpose we define their columns by

$$\boldsymbol{Y}(s) = \begin{bmatrix} y_1(s) & \cdots & y_m(s) \end{bmatrix} \tag{21}$$

$$\boldsymbol{N}(s) = \begin{bmatrix} n_1(s) & \cdots & n_m(s) \end{bmatrix} \tag{22}$$

$$\boldsymbol{E}(s) = \begin{bmatrix} e_1(s) & \cdots & e_m(s) \end{bmatrix}. \tag{23}$$

We also need to use the $\otimes$ notation for the **Kronecker** or **tensor product** of two matrices: if $\bar{\boldsymbol{P}}$ has p rows and q columns, and $\bar{\boldsymbol{Q}}$ has r rows and s columns, then $\bar{\boldsymbol{P}} \otimes \bar{\boldsymbol{Q}}$ is the $pr \times qs$ matrix:

$$\bar{\boldsymbol{P}} \otimes \bar{\boldsymbol{Q}} = \begin{bmatrix} p_{11}\bar{\boldsymbol{Q}} & p_{12}\bar{\boldsymbol{Q}} & \cdots & p_{1s}\bar{\boldsymbol{Q}} \\ p_{21}\bar{\boldsymbol{Q}} & p_{22}\bar{\boldsymbol{Q}} & \cdots & p_{2s}\bar{\boldsymbol{Q}} \\ \vdots & \vdots & & \vdots \\ p_{r1}\bar{\boldsymbol{Q}} & p_{r2}\bar{\boldsymbol{Q}} & \cdots & p_{rs}\bar{\boldsymbol{Q}} \end{bmatrix}. \tag{24}$$

In this notation, (20) can be written as

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_m(s) \end{bmatrix} \approxeq \left[\boldsymbol{L}^T(s) \otimes \boldsymbol{M}(s)\right] \begin{bmatrix} n_1(s) \\ n_2(s) \\ \vdots \\ n_m(s) \end{bmatrix} + \begin{bmatrix} e_1(s) \\ e_2(s) \\ \vdots \\ e_m(s) \end{bmatrix}. \tag{25}$$

Remember that $n_i(s)$ represents a vector of polynomials:

$$n_i(s) = \begin{bmatrix} n_{1i}(s) & \cdots & n_{1i}(s) \end{bmatrix}^T \tag{26}$$

and suppose that

$$n_{ij}(s) = \nu_{ij}^0 s^p + \nu_{ij}^1 s^{p-1} + \cdots + \nu_{ij}^p \tag{27}$$

for some positive integer p, assuming for notational convenience that each ν_{ij} has the same degree. This is not a real restriction, since $\nu_{ij}^x = 0$ is allowed, and can be forced if desired. Then $\{\nu_{ij}^x\}$ is the set of controller parameters to be optimized; if each n_{ij} has degree p, there are $lm(p+1)$ of them. We need to introduce one more new notation: let $\boldsymbol{\sigma}(s)$ be the matrix (with lm rows and $lm(p+1)$ columns)

$$\boldsymbol{\Sigma}(s) = \begin{bmatrix} s^p & s^{p-1} & \cdots & 1 & & & & & & & & \mathbf{0} \\ & & & & s^p & s^{p-1} & \cdots & 1 & & & & \\ & \mathbf{0} & & & & & & & \ddots & & & \\ & & & & & & & & s^p & s^{p-1} & \cdots & 1 \end{bmatrix} \tag{28}$$

then

$$\begin{bmatrix} n_1(s) \\ \vdots \\ n_m(s) \end{bmatrix} = \boldsymbol{\Sigma}(s)\boldsymbol{\nu} \tag{29}$$

where

$$\boldsymbol{\nu} = \begin{bmatrix} \nu_{11}^0 & \nu_{11}^1 & \cdots & \nu_{ml}^p \end{bmatrix}^T. \tag{30}$$

so if we let

$$\boldsymbol{X}(s) = \left[\boldsymbol{L}^T(s) \otimes \boldsymbol{M}(s)\right] \boldsymbol{\Sigma}(s) \tag{31}$$

$$\boldsymbol{\varrho}(s) = \begin{bmatrix} y_1^T(s) & \cdots & y_m^T(s) \end{bmatrix}^T \tag{32}$$

and

$$\boldsymbol{\varepsilon}(s) = \begin{bmatrix} \varepsilon_1^T(s) & \cdots & \varepsilon_m^T(s) \end{bmatrix}^T \tag{33}$$

then (25) becomes

$$\boldsymbol{\varrho}(s) \approxeq \boldsymbol{X}(s)\boldsymbol{\nu} + \boldsymbol{\varepsilon}(s) \tag{34}$$

which is in a standard form; $\boldsymbol{\varrho}(s)$ is a known matrix, $\boldsymbol{\nu}$ is a vector of unknown parameters and $\boldsymbol{\varepsilon}(s)$ is a vector of 'errors'.

To obtain a practical algorithm, we need to evaluate $\boldsymbol{\varrho}(s)$ and $\boldsymbol{X}(s)$ at a number of points on the imaginary axis, say $\{s = j\omega_i : i = 1, 2, \cdots, \mu\}$, and approximate $\|\boldsymbol{E}\|_2$ (which is the same as $\|\boldsymbol{\varepsilon}\|_2$) by

$$\|\boldsymbol{E}\|_2^2 \approxeq \sum_{i=1}^{\mu} \varepsilon^T(j\omega_i)\varepsilon(j\omega_i). \tag{35}$$

Assembling data from all these points, we obtain

$$\begin{bmatrix} \boldsymbol{\varrho}(j\omega_1) \\ \vdots \\ \boldsymbol{\varrho}(j\omega_\nu) \end{bmatrix} \approxeq \begin{bmatrix} \boldsymbol{X}(j\omega_1) \\ \vdots \\ \boldsymbol{X}(j\omega_\nu) \end{bmatrix} \boldsymbol{\nu} + \begin{bmatrix} \boldsymbol{\varepsilon}(j\omega_1) \\ \vdots \\ \boldsymbol{\varepsilon}(j\omega_\nu) \end{bmatrix}. \tag{36}$$

The standard least-squares solution to this would be (Lawson *et al.* 1974):

$$\begin{aligned} \hat{\boldsymbol{\nu}} = &\left\{ \begin{bmatrix} \boldsymbol{X}^T(-j\omega_1) & \cdots & \boldsymbol{X}^T(-j\omega_\mu) \end{bmatrix} \begin{bmatrix} \boldsymbol{X}(j\omega_1) \\ \vdots \\ \boldsymbol{X}(j\omega_\mu) \end{bmatrix} \right\}^{-1} \\ &\times \left\{ \begin{bmatrix} \boldsymbol{X}^T(-j\omega_1) & \cdots & \boldsymbol{X}^T(-j\omega_\mu) \end{bmatrix} \begin{bmatrix} \boldsymbol{\varrho}(j\omega_1) \\ \vdots \\ \boldsymbol{\varrho}(j\omega_\mu) \end{bmatrix} \right\} \end{aligned} \tag{37}$$

but in general this would give *complex* parameter values. A solution to this problem can be obtained by means of the following lemma:

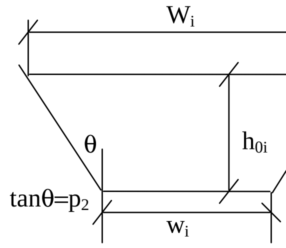

Figure 2: *Canal cross section.*

Lemma 1. *If* $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\theta} + \boldsymbol{E}$*, the value of* $\boldsymbol{\theta}$ *which minimizes* $\|\boldsymbol{E}\|_2$*, given* $\boldsymbol{X}$ *and* $\boldsymbol{Y}$*, and subject to the constraint* $Im\{\boldsymbol{\theta}\} = 0$*, is*

$$\hat{\boldsymbol{\theta}} = \left[Re\left\{\boldsymbol{X}^H \boldsymbol{X}\right\}\right]^{-1} Re\left\{\boldsymbol{X}^H \boldsymbol{Y}\right\}. \tag{38}$$

With the aid of this lemma we obtain the optimal *real* parameters:

$$\begin{aligned} \hat{\boldsymbol{\nu}} = &\left(Re\left\{ \left[\begin{array}{ccc} \boldsymbol{X}^T(-j\omega_1) & \cdots & \boldsymbol{X}^T(-j\omega_\mu) \end{array}\right] \left[\begin{array}{c} \boldsymbol{X}(j\omega_1) \\ \vdots \\ \boldsymbol{X}(j\omega_\mu) \end{array}\right] \right\} \right)^{-1} \\ &\times Re\left\{ \left[\begin{array}{ccc} \boldsymbol{X}^T(-j\omega_1) & \cdots & \boldsymbol{X}^T(-j\omega_\mu) \end{array}\right] \left[\begin{array}{c} \boldsymbol{\varrho}(j\omega_1) \\ \vdots \\ \boldsymbol{\varrho}(j\omega_\mu) \end{array}\right] \right\}. \end{aligned} \tag{39}$$

The validity of this algorithm depends on the validity of (14), which in turn depends on the size of $\|\boldsymbol{E}\|_2$ obtained with the parameter vector $\hat{\boldsymbol{\nu}}$.

It is imperative that the actual algorithm employed solves (36) by using a numerically stable procedure and does not use (39), since in the neighborhood of the true solution the matrix

$$Re\left\{ \left[\begin{array}{ccc} \boldsymbol{X}^T(-j\omega_1) & \cdots & \boldsymbol{X}^T(-j\omega_\mu) \end{array}\right] \left[\begin{array}{c} \boldsymbol{X}(j\omega_1) \\ \vdots \\ \boldsymbol{X}(j\omega_\mu) \end{array}\right] \right\}$$

approaches singularity. A numerically stable algorithm is obtained as follows. In the notation of Lemma 7.1, let $\boldsymbol{X} = \boldsymbol{X}_{Re} + j\boldsymbol{X}_{Im}$, $\boldsymbol{Y} = \boldsymbol{Y}_{Re} + j\boldsymbol{Y}_{Im}$, then $\hat{\boldsymbol{\theta}}$, defined by (38), is also obtained as the least squares solution of the equation

$$\left[\begin{array}{c} \boldsymbol{Y}_{Re} \\ \boldsymbol{Y}_{Im} \end{array}\right] = \left[\begin{array}{c} \boldsymbol{X}_{Re} \\ \boldsymbol{X}_{Im} \end{array}\right] \boldsymbol{\theta} + \left[\begin{array}{c} \boldsymbol{E}_{Re} \\ \boldsymbol{E}_{Im} \end{array}\right].$$

This can be solved in a numerically stable way by using the Householder transformation and QR factorization (Stoer *et al.* 1992).

4 Applicative example

The procedure was applied to a two-reach canal, corresponding to the general scheme shown in Figures 1-2, with the following characteristics (Corriga *et al.*, 1989):

- length of the first reach: $l_1 = 4000m$;

- length of the second reach: $l_2 = 5000m$;
- canal bottom slope: $p_1 = 0.0003$;
- water level depth in upstream reservoir measured from the canal bottom in the upper end section: $h_M = 2.5m$;
- water level depth in downstream reservoir measured from the canal bottom in the lower end section: $h_V = 1m$;
- trapezoidal cross section (see Figure 2) with $w = 1.7m$, $\theta = 45^o$;
- constant opening section of the third gate: $\lambda_3 = \lambda_{03} = 2.41m^2$;
- discharge coefficient: $\eta = 0.6$;
- roughness coefficient: $\gamma = 0.36$.

The nominal configuration of uniform flow is characterized by the following levels and discharge values:

- water level depth in the 1-st reach: $h_{01} = 1.70m$;
- water level depth in the 2-nd reach: $h_{02} = 1.20m$;
- flow rate in the 1-st reach : $q_{01} = 5.94m^3/s$;
- flow rate in the 2-nd reach : $q_{02} = 3.02m^3/s$;
- user flow rate at the 1-st reach lower end: $q_{0c1} = 2.92m^3/s$;
- user flow rate at the 2-nd reach lower end: $q_{0c2} = 0.15m^3/s$;
- opening section of the 1-st gate: $\lambda_{01} = 2.50m^3/s$;
- opening section of the 2-nd gate: $\lambda_{02} = 1.61m^3/s$.

In the following subsection we show how the above numerical procedure can be satisfactorily applied to design decentralized controller.

4.1 Decentralized control law

As already specified in the previous section, the effectiveness of the Edmund's algorithm depends on the choice of the target closed loop function which should be compatible with both the model and the feedback gain matrix structure. At this purpose, the target closed loop matrix function $\boldsymbol{T}_t$ has been designed by means of the LQR technique (Kwakernaak, 1972) applied to the linear-time invariant model (6). In fact, the feedback gain matrix $\boldsymbol{K}_t$ has been obtained as the solution of the following linear optimization problem by solving a Riccati equation (Kwakernaak, 1972):

$$\begin{aligned}
&\min J = \int_0^\infty [\boldsymbol{v}(t)^T \boldsymbol{Q} \boldsymbol{v}(t) + \boldsymbol{\lambda}(t)^T \boldsymbol{R} \boldsymbol{\lambda}(t)]dt \\
&s.t. \\
&\dot{\boldsymbol{v}}(t) = \boldsymbol{A}\boldsymbol{v}(t) + \boldsymbol{B}\boldsymbol{\lambda}(t)
\end{aligned}$$

where

$$\boldsymbol{A} = \begin{bmatrix} -1.50 & 0.56 \\ 1.10 & -1.90 \end{bmatrix} \cdot 10^{-4}, \quad \boldsymbol{B} = \begin{bmatrix} 1.81 & -0.92 \\ -0.33 & 1.31 \end{bmatrix},$$

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 0 \\ 0 & v_{10}/v_{20} \end{bmatrix}, \quad R = 50000.$$

The structure of $\boldsymbol{Q}$ has been chosen such that the volume variations in each reach, with respect to the initial volume, are weighted in the same manner. R has been assumed to be scalar so as to control all the gates with the same energy. $R = 50000$ is an appropriate numerical value determined by a trial and error procedure. In such a way we obtained

$$\boldsymbol{K}_t = \begin{bmatrix} -4.31 & -1.12 \\ 0.91 & -4.74 \end{bmatrix} \cdot 10^{-3}$$

and

$$\boldsymbol{T}_t(s) = \boldsymbol{G}_A(s)\boldsymbol{K}_t(\boldsymbol{I} + \boldsymbol{G}_A(s)\boldsymbol{K}_t)^{-1}$$

where $\boldsymbol{G}_A(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}$.

Now, we want to design a proportional decentralized control law

$$\boldsymbol{\lambda}(t) = \boldsymbol{K}\boldsymbol{v}(t)$$

by applying the Edmund's algorithm. Therefore we assumed $d(s) = 1$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

In such a way all the off-diagonal elements of the feedback gain matrix are constrained to be zero.

Since the most important hydraulic phenomena occur in a low frequency range, we assumed $w_1 = 10^{-9}$ and $w_\mu = 10^{-2}$. We further evaluated that a good choice for μ is 250.

In such a way the diagonal gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} -6.91 & 0 \\ 0 & -4.42 \end{bmatrix} \cdot 10^{-3},$$

while the corresponding closed loop transfer matrix is

$$\boldsymbol{T}(s) = \boldsymbol{G}(s)\boldsymbol{K}(\boldsymbol{I} + \boldsymbol{G}(s)\boldsymbol{K})^{-1},$$

where $\boldsymbol{G}(s)$ is defined by relationship (5).

To evaluate the effectiveness of the applied procedure we compared the Bode diagrams of all the elements of $\boldsymbol{T}$ and $\boldsymbol{T}_t$. The results of such a comparison are reported in Figure 3. As it can be clearly seen, the Bode diagrams are really close even in a wider frequency range than that considered for the controller parameter identification procedure.

In this subsection we also present the results of a numerical simulation that demonstrate the good performance of the system in the presence of unknown disturbances, when the above decentralized control law is implemented. All numerical simulations have been carried out using the commercial SIC software (Malaterre *et al.* 1997), a completely nonlinear model developed at Cemagref (Montpellier, France).

The unknown disturbances are those given in Figure 4. The results of simulations are shown in Figure 5: a) shows the volume percentage variations; the gate opening variations are shown in b). As it can be seen, the system's behaviour is rather satisfactory.

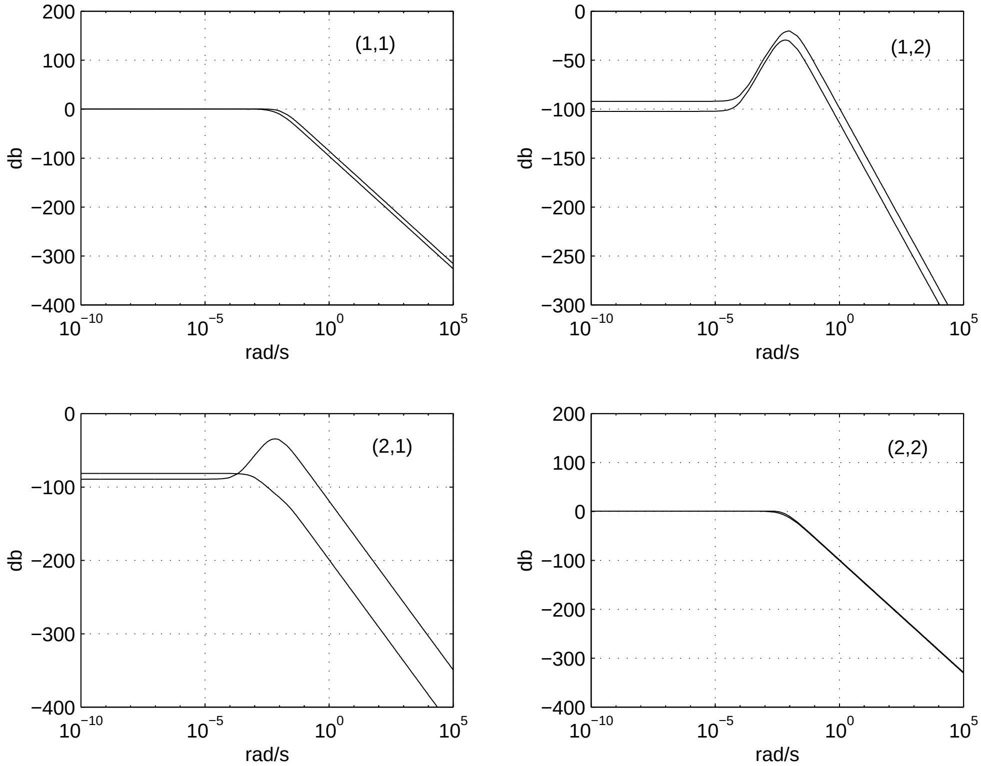

Figure 3: *Bode diagrams of all the elements of* $\boldsymbol{T}$ *and* $\boldsymbol{T}_t$*.*

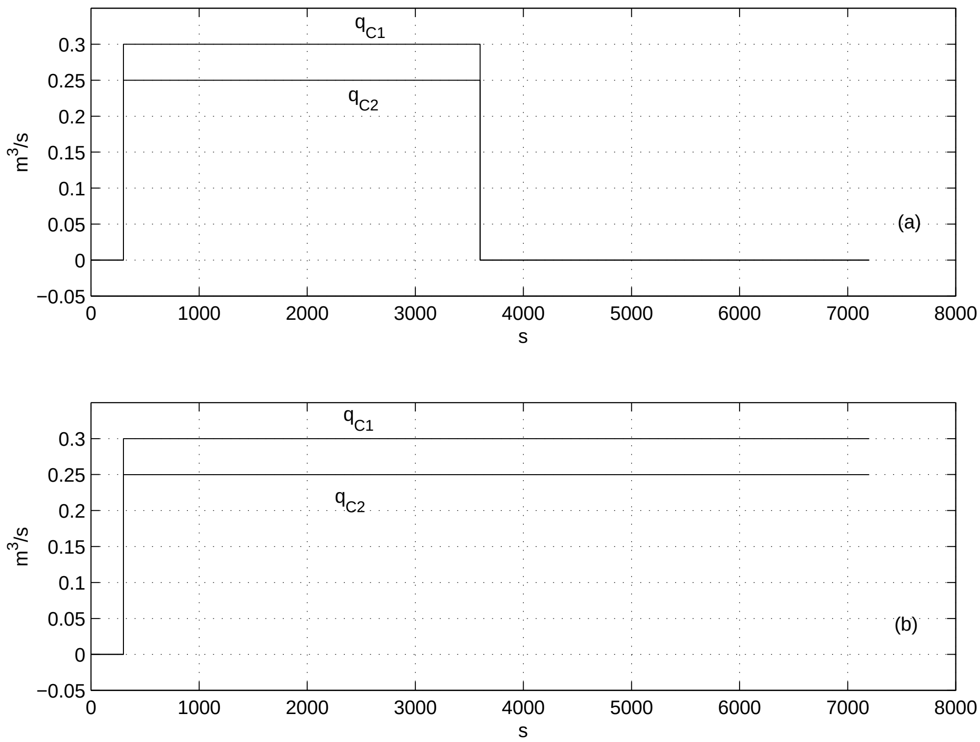

Figure 4: *Unknown disturbances.*

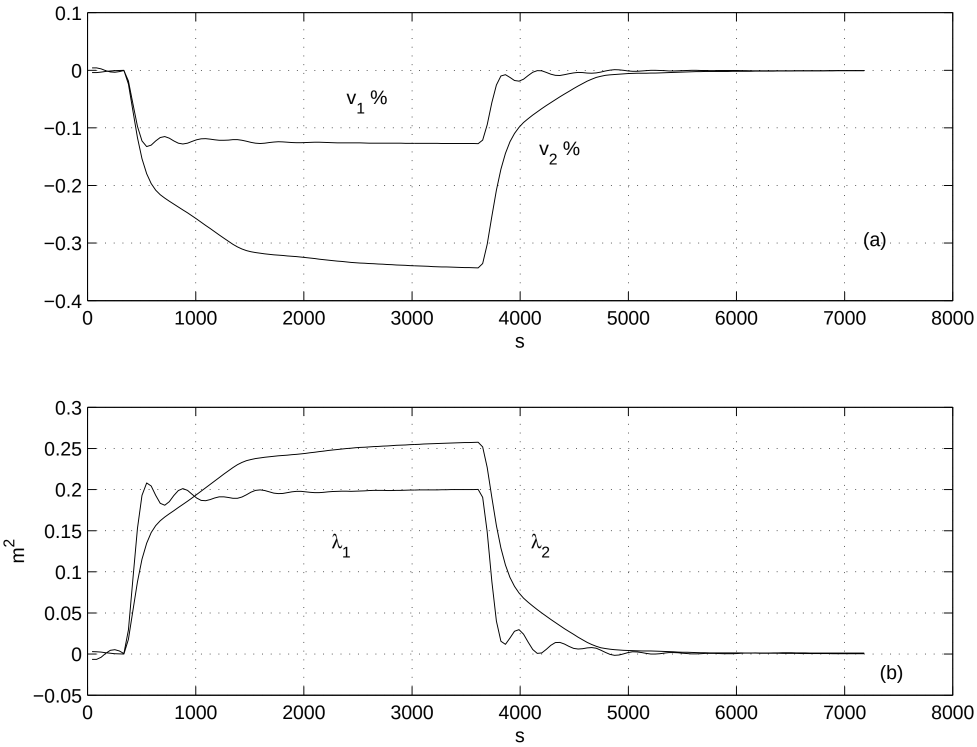

Figure 5: *Results of the numerical simulation.*

5 Conclusions

In this paper a decentralized constant-volume controller has been designed by solving a linear least squares problem in the frequency domain. The solution has been obtained by means of the Edmund's algorithm which enables us to impose the desired structure to the feedback gain matrix. The controller parameters are evaluated so as to make the closed-loop transfer function approach a target function as closely as possible over a specified frequency range.

The main advantage of this procedure is that it does not require a good initial parameter estimation and no problem related to the local minima occur. Finally, it allows us to use a more general model than that considered in previous approaches.

A comparison between the transfer function elements of the target and of the resulting closed loop transfer function is proposed.

Finally, numerical simulations, carried out by means of the commercial SIC software, demonstrated the satisfactory behaviour of the system when the proposed decentralized control law is implemented.

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