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The J-Spectral Interactor Matrix in the Discrete-Time Singular H_∞ Filtering Problem*

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Abstract

This paper introduces the concept of J-spectral interactor matrix (JSIM) which is intimately associated with the singular filtering problem in H_∞. An algorithm for the computation of a JSIM is proposed and the role of the system delays in the solution of the singular filtering problem is eventually clarified.

1 Introduction

The role of the interactor matrix (Goodwin and Sin, 1984) is well recognized in the theory of systems and control. Typical problems where this matrix plays an important role are exact model-matching (Wolovich and Falb, 1976), system inversion (Silverman, 1969), adaptive control (Tsiligiannis and Svoronos, 1986; Mutoh and Ortega, 1993), LQG/LTR techniques (Maciejowski, 1985) and inner-outer factorization problems (Copeland and Safonov, 1992). Recently, a generalization of the interactor matrix has been proposed in (Bittanti *et al.*, 1995) in order to cope with the singular H_2 filtering problem. This brought to the definition of spectral interactor matrix (SIM) which is a polynomial matrix capable of preserving the spectral properties of the system.

The aim of this paper is to extend the concept of SIM to tackle the singular H_∞ filtering problem. Such a problem has been considered as a part of the general H_∞ output control problem in (Stoorvogel *et al.*, 1994). In that paper the solution of the singular filtering problem hinges on the solution of a suitable singular Riccati equation. Here, we move on a different path which consists in transforming the singular problem into a nonsingular one by means of the definition of the J-spectral interactor matrix (JSIM). This is defined as a polynomial interactor matrix which preserves the J-spectrum of the system generating the measurements and the output to be estimated. Using this concept we show that there exists a strict correspondence between the solutions of the singular Riccati equation and the ones of a suitably defined nonsingular Riccati equation. In particular we show that the stabilizing solution of the former equation can be viewed as the sum of two matrices related to the system delays and the finite zeros structure, respectively.

The paper is organized as follows. In Section 2 we introduce the definition of the JSIM, and provide a state-space realization of the J-equivalent delay-free system. In Section 3 the solution

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of the H_∞ singular problem is stated based on the property of a suitable nonsingular Riccati equation. The structure of the stabilizing solution of such an equation is characterized in terms of nonminimum phase zeros (both finite and infinite). Finally an illustrative example is shown in Section 4.

2 The J-Spectral Interactor Matrix

Consider the following time-invariant discrete-time linear system:

$$x(t+1) = Ax(t) + Bw(t) \tag{1a}$$
$$y(t) = Cx(t) + Dw(t) \tag{1b}$$
$$z(t) = Lx(t) \tag{1c}$$

where $x(\cdot) \in \Re^n$ is the state, $w(\cdot) \in \Re^m$ is a disturbance, $y(\cdot) \in \Re^m$ is the vector of measurements and $z(\cdot) \in \Re^q$ is the output vector to be estimated. Notice that we consider here the case where the transference

$$G_1(z) = D + C(zI - A)^{-1}B$$

is square. Moreover we also assume that $G_1(z)$ has full normal rank, but no assumption on the rank of the matrix D is made.

For the filtering problem in H_∞ a major role is played by the transference

$$H(z) = \begin{bmatrix} G_1(z) & 0 \\ G_2(z) & I \end{bmatrix}$$

where $G_2(z) = L(zI - A)^{-1}B$. Matrix $H(z)$ has full normal rank and is square. However,

$$\widehat{D} \stackrel{\triangle}{=} \lim_{z \to \infty} H(z)$$

may be well a singular matrix. This fact characterizes the so-called singular H_∞ filtering problem.

It is well known that an interactor matrix is any polynomial matrix $L(z)$ such that the product $H(z)L(z)$ turns out to be delay-free, namely $\lim_{z\to\infty} (H(z)L(z))$ is nonsingular. Among all the interactor matrices we focus our attention on special ones that preserve the J-spectral properties of the system $H(z)$. More precisely, denoting with $H(z)^{\tilde{}} = H(z^{-1})'$ the transference of the adjoint system, we consider the J-spectrum of $H(z)$ defined as

$$\Psi(z) = H(z)JH(z)^{\tilde{}}$$

where

$$J = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$$

and $\gamma > 0$ is a given constant.

Definition: Any polynomial matrix $L(z)$ such that:
i) $\lim_{z\to\infty} (H(z)L(z))$ is nonsingular
ii) $L(z)JL(z)^{\tilde{}} = J$

is called a J-Spectral Interactor Matrix (JSIM). ■

Our aim is now that of providing an algorithm to build a JSIM. To this end we preliminarily have to define the notions of null space and orthogonal projection in the formalism of Krein spaces. Precisely, given a matrix F with $q+m$ columns, the J-null space of F will be denoted by $KKer(F)$ and computed as follows:

$$KKer(F) = Ker(FJ)$$

Moreover, given a subspace $\mathcal{X}$, its orthogonal projection $\mathcal{X}^{J\perp}$ with respect to J is simply defined by:

$$\mathcal{X}^{K\perp} = J^{-1}\mathcal{X}^{\perp}$$

Consequently, two subspaces $\mathcal{X}_1$, $\mathcal{X}_2$ are said to be J-orthogonal if $\forall \xi_1 \in \mathcal{X}_1, \forall \xi_2 \in \mathcal{X}_2$, it results $\xi_1' J \xi_2 = 0$.

Now we are in the position to provide an algorithm to build a JSIM for a generic $(q+m)$ square transference

$$H^{(k)}(z) = \sum_{i=0}^{\infty} M_i^{(k)} z^{-i}$$

where $M_i^{(k)}$ are the associated Markov parameters , $i \geq 0$. Define a sequence of subspaces as follows:

$$\begin{aligned} X_0^{(k)} &= KKer(M_0^{(k)})^{K\perp} && \text{(2a)} \\ X_i^{(k)} &= \left(\bigoplus_{j=0}^{i-1} X_j^{(k)} \right)^{K\perp} \cap \left(\bigcap_{j=0}^{i} KKer(M_j^{(k)}) \right)^{K\perp}, \qquad i \geq 1 && \text{(2b)} \end{aligned}$$

This sequence of subspaces is initialized, for $k=0$, by the Markov parameters associated with $H^{(0)}(z) = H(z)$, i.e.,

$$\begin{aligned} M_0^{(0)} &= \widehat{D}, \quad M_i^{(0)} = \widehat{C} A^{i-1} \widehat{B}, \qquad i \geq 1 && \text{(3a)} \\ \widehat{B} &= \begin{bmatrix} B & 0 \end{bmatrix}, \widehat{C} = \begin{bmatrix} C \\ L \end{bmatrix}, \widehat{D} = \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix} && \text{(3b)} \end{aligned}$$

Remark: notice that any two distinct subspaces $X_i^{(k)}, X_j^{(k)}$, $i \neq j$ are J orthogonal, i.e. if $\zeta \in X_i^{(k)}$, $\eta \in X_j^{(k)}$, then it results $\zeta' J \eta = 0$. ■

The main assumption that we make throughout the paper is that there exists a positive scalar γ such that equations

$$\begin{aligned} X_0^{(k)} \oplus X_1^{(k)} \oplus X_2^{(k)} ... \oplus X_n^{(k)} &= \Re^{q+m} && \text{(4a)} \\ X_{n+i}^{(k)} &= \{0\}, \forall i > 0 && \text{(4b)} \end{aligned}$$

are verified for each $k \geq 0$. This assumption is experimentally satisfied for each γ ensuring the existence of a causal filter $F(z)$ such that $\|G_1(z) - F(z)G_2(z)\|_\infty < \gamma$. Analytical proof of this claim is still missing and will be subject of future investigations.

Let $\rho_i^{(k)} = \dim(X_i^{(k)}) \leq q+m$, and define a matrix $\theta_i^{(k)}$ whose $\rho_i^{(k)}$ columns are J-orthogonal and span $X_i^{(k)}$, i.e.

$$\mathrm{Im}[\theta_i^{(k)}] = X_i^{(k)}, \qquad (\theta_i^{(k)})' J \theta_i^{(k)} = diagonal \tag{5}$$

In so doing we can define the matrix

$$\overline{N}^{(k)} = \left[\begin{array}{cccc} \theta_0^{(k)} & \theta_1^{(k)} & ... & \theta_n^{(k)} \end{array} \right] \tag{6}$$

which is square and invertible in view of assumptions (4) and, moreover, is such that $\overline{N}^{(k)} J \overline{N}^{(k)}$ is a diagonal matrix with m positive and q negative entries. Therefore there exists a permutation and normalization matrix $R^{(k)}$ such that

$$(\overline{N}^{(k)} R^{(k)})' J \overline{N}^{(k)} R^{(k)} = J \tag{7}$$

This means that $N^{(k)} = \overline{N}^{(k)} R^{(k)}$ is J^{-1}-unitary, that is $N^{(k)} J^{-1} (N^{(k)})' = J^{-1}$. Now define the polynomial matrix

$$\overline{L}^{(k)}(z) = \overline{N}^{(k)} \overline{\Delta}(z) R^{(k)} \tag{8}$$

where

$$\overline{\Delta}(z) = diag(I_{\rho_0} z^0, I_{\rho_1} z^1, ..., I_{\rho_n} z^n) \tag{9}$$

It turns out that $\overline{L}(z)$ is J^{-1}-paraunitary, i.e. $\overline{L}^{(k)}(z) J^{-1} \overline{L}^{(k)}(z)^{\tilde{}} = J^{-1}$. Indeed, since $R^{(k)}$ is a permutation and normalization matrix, it results $(R^{(k)})^{-1} \overline{\Delta}(z) R^{(k)} = \Delta(z)$, where $\Delta(z)$ is still a diagonal matrix whose entries are powers of z. Hence, $\overline{L}^{(k)}(z) = N^{(k)} (R^{(k)})^{-1} \overline{\Delta}(z) R^{(k)} = N^{(k)} \Delta(z)$. Finally define the polynomial matrix

$$L^{(k)}(z) = J \overline{L}^{(k)}(z) J^{-1} \tag{10}$$

This matrix is indeed J-paraunitary, i.e. $L^{(k)}(z) J L^{(k)}(z)^{\tilde{}} = J$. It can be shown, by mimicking the arguments of ((Bittanti *et al.*, 1995)), that for each $k \geq 0$, the rational matrix $H^{(k)}(z) L^{(k)}(z)$ is proper. For the updating of the step k, let

$$H^{(k+1)}(z) = H^{(k)}(z) L^{(k)}(z) \tag{11}$$

Now, again resorting to the results of ((Bittanti *et al.*, 1995)), it is possible to show that there exists an integer $0 < h \leq n$ such that the matrix $L(z)$ defined as

$$L(z) = \prod_{i=0}^{h} L^{(i)}(z) = L^{(0)}(z) L^{(1)}(z) \cdots L^{(h)}(z)$$

is a J-Spectral Interactor Matrix for $H(z)$.

Remark: we are well advised to summarize the main steps of the algorithm for the construction of the JSIM matrix $L(z)$. Consider $k = 0$ and the definition of subspaces (2), for the Markov parameters (3) of the system $H^{(k)}(z)$. Under assumptions (4), compute the matrices $\theta_i^{(k)}$ as in (5) and the corresponding matrix $\overline{N}^{(k)}$ defined in (6). Now find the permutation and normalization matrix $R^{(k)}$ satisfying (7) and compute $\overline{L}^{(k)}(z)$ according to (8),(9). The

J-paraunitary matrix $L^{(k)}(z)$ is given by (10). Now take $H^{(k+1)}(z)$ according to (11) and repeat the above sequence of operations till $k = h$. ■

Notice that if matrix $M_0^{(k)}$ is nonsingular, the sequence of subspaces $X_i^{(k)}$ reduce to $X_0^{(k)} = \Re^{q+m}$, $X_i^{(k)} = \{0\}$, $i > 0$. Also notice that as γ tends to infinity, the algorithm proposed in (Bittanti *et al.*, 1995) is eventually recovered.

We end this section by recalling the formulas for the construction of a state-space realization of the delay-free system $H_f(z) = H^{(h+1)}(z) = H(z)L(z)$. To this aim write the polynomial matrix $L(z)$ as:

$$L(z) = L_0 + L_1 z + ... + L_r z^r \tag{12}$$

Notice that the coefficients of the polynomial matrix $L(z)$ enjoy the property

$$\sum_{i=0}^{r-j} L_i J L'_{i+j} = \begin{cases} J & j = 0 \\ 0 & j = 1, 2, ..r \end{cases} \tag{13}$$

which follows from $L(z)$ being J-paraunitary.
It is easy to verify (see the derivations in point D of ((Bittanti *et al.*, 1995))) that

$$H_f(z) = \widehat{C}(zI - A)^{-1}\widehat{B}_f + \widehat{D}_f$$

where A,C were defined in (1),(3) and

$$\widehat{B}_f = \sum_{j=0}^{r} A^j \widehat{B} L_j, \qquad \widehat{D}_f = \widehat{D} L_0 + \widehat{C} \sum_{j=1}^{r} A^{j-1} \widehat{B} L_j \tag{14}$$

Now, define the following matrices

$$\Gamma_j = \sum_{i=1}^{r-j} A^{i-1} \widehat{B} L_{i+j}, \qquad P_d = \sum_{j=0}^{r-1} \Gamma_j J \Gamma'_j$$

The next results brings into light the relation between the system matrices of the state-space description of $H(z)$ and $H_f(z)$. Its proof is omitted for simplicity and it is straightforwardly obtained by using (13) and the fact that $H_f(z)$ is a proper rational transference.

Lemma 1: *The following identities hold:*

$$\begin{aligned} \widehat{B}_f J \widehat{B}'_f &= \widehat{B} J \widehat{B}' - P_d + A P_d A' \\ \widehat{B}_f J \widehat{D}'_f &= \widehat{B} J \widehat{D}' + A P_d \widehat{C}' \\ \widehat{D}_f J \widehat{D}'_f &= \widehat{D} J \widehat{D}' + \widehat{C} P_d \widehat{C}' \end{aligned}$$

■

Example: consider the discrete-time system (1) where:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1/2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; D = 0, \; L = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{15}$$

so that

$$H^{(0)}(z) = H(z) = \begin{bmatrix} \dfrac{1}{z(z+0.5)} & 0 \\ \dfrac{z+1}{z(z+0.5)} & 1 \end{bmatrix}$$

We apply the procedure (2) for computing the subspaces $X_i^{(k)}$. It is easily seen that it works for $\gamma > 1$. Since $n = 2$, we are interested in the first two Markov parameters $M_i^{(0)}$ associated with $H(z)$. By using (3) we get

$$M_0^{(0)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ M_1^{(0)} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

and, applying (2)

$$X_0^{(0)} = \mathrm{Im}\left(\begin{vmatrix} 0 \\ 1 \end{vmatrix} \right),\ X_1^{(0)} = \mathrm{Im}\left(\begin{vmatrix} 1 \\ 0 \end{vmatrix} \right)$$

Hence, in the first step we obtain:

$$\overline{N}_0^{(0)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ R_0^{(0)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \overline{L}^{(0)}(z) = L^{(0)}(z) = \begin{bmatrix} z & 0 \\ 0 & 1 \end{bmatrix}$$

Now, compute

$$H^{(1)}(z) = H^{(0)}(z)L^{(0)}(z) = \begin{bmatrix} \dfrac{1}{z+0.5} & 0 \\ \dfrac{z+1}{z+0.5} & 1 \end{bmatrix}$$

which is still strictly proper. Hence, take the Markov parameters

$$M_0^{(1)} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\ M_1^{(1)} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \end{bmatrix}$$

and the corresponding subspaces

$$X_0^{(1)} = \mathrm{Im}\left(\begin{vmatrix} 1 \\ 1 \end{vmatrix} \right),\ X_1^{(1)} = \mathrm{Im}\left(\begin{vmatrix} 1 \\ \gamma^{-2} \end{vmatrix} \right)$$

We obtain

$$\begin{aligned} \overline{N}_0^{(1)} &= \begin{bmatrix} 1 & 1 \\ 1 & \gamma^{-2} \end{bmatrix},\ R_0^{(1)} = \frac{\gamma}{\sqrt{\gamma^2-1}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \\ \overline{L}^{(1)}(z) &= \frac{\gamma}{\sqrt{\gamma^2-1}} \begin{bmatrix} z & 1 \\ z\gamma^{-2} & 1 \end{bmatrix},\ L^{(1)}(z) = \frac{\gamma}{\sqrt{\gamma^2-1}} \begin{bmatrix} z & -\gamma^{-2} \\ -z & 1 \end{bmatrix} \end{aligned}$$

In conclusion,

$$L(z) = L^{(0)}(z)L^{(1)}(z) = \frac{\gamma}{\sqrt{\gamma^2-1}} \begin{bmatrix} z^2 & -\gamma^{-2}z \\ -z & 1 \end{bmatrix} \tag{16}$$

and

$$H_f(z) = H^{(2)}(z) = \begin{bmatrix} \dfrac{\gamma z}{z+0.5} & -\dfrac{\gamma^{-1}}{z+0.5} \\ \dfrac{\gamma z/2}{z+0.5} & \dfrac{(\gamma^2-1)z-1+\gamma^2/2}{\gamma(z+0.5)} \end{bmatrix} \tag{17}$$

is delay-free since $\gamma \neq 1$.

3 Singular H_∞ filtering

The H_∞ filtering problem for system (1) consists of finding a causal linear filter (whose transference is henceforth denoted by $F(z)$) yielding an estimate $\widehat{z}$ of z such that the transference $G_2(z) - F(z)G_1(z)$ from noise w to the filtering error $z - \widehat{z}$ is stable with H_∞ norm less than a prescribed positive constant γ. In (Stoorvogel *et al.*, 1994) it was proved the following theorem

Theorem 1: *Assume that (A, B, C, D) has no invariant zeros on the unit circle. Then, there exists a filter that solves the H_∞ filtering problem if and only if there exists a symmetric solution $P \geq 0$ of the Riccati equation:*

$$P = APA' + \widehat{B}\widehat{B}' - \left(AP\widehat{C}' + \widehat{B}\widehat{D}'\right)\left(\widehat{D}J\widehat{D}' + \widehat{C}P\widehat{C}'\right)^{\dagger}\left(AP\widehat{C}' + \widehat{B}\widehat{D}'\right)' \tag{18}$$

such that

$$rank \begin{bmatrix} zI - A & AP\widehat{C}' + \widehat{B}\widehat{D}' \\ -\widehat{C} & \widehat{D}J\widehat{D}' + \widehat{C}P\widehat{C}' \end{bmatrix} = n + m + q \quad \forall z : |z| \geq 1 \tag{19}$$

and

$$\gamma^2 I - LPL' + LPC'\left(DD' + CPC'\right)^{\dagger} CPL' > 0 \tag{20}$$

$\blacksquare$

In eqs. (18),(20) the symbol $\dagger$ denotes the Moore-Penrose pseudoinverse. In the case of nonsingular matrix D, condition (20) is the so-called feasibility condition whereas (19) reduces to the standard stabilizing property of P.

Here we exploit the properties of the JSIM to work out an alternative way to construct a filter via the formalism of J-spectral factorization (see e.g. (Green *et al.*, 1990)). The rationale behind such a proposed approach relies on the fact that a J-spectral factor of $H(z)JH(z)^{\sim}$ is also a J-spectral factor of $H_f(z)JH_f(z)^{\sim}$ where $H_f(z) = H(z)L(z)$ is the delay-free image of $H(z)$ under the action of the JSIM $L(z)$. In order to build a J-spectral factor of $H_f(z)JH_f(z)^{\sim}$, we consider the Riccati-type equation formally related to the extended system $H_f(z)$, i.e.

$$P_f = AP_fA' + \widehat{B}_fJ\widehat{B}_f' - \left(AP_f\widehat{C}' + \widehat{B}_fJ\widehat{D}_f'\right)\left(\widehat{D}_fJ\widehat{D}_f' + \widehat{C}P_f\widehat{C}'\right)^{-1}\left(AP_f\widehat{C}' + \widehat{B}_fJ\widehat{D}_f'\right)' \tag{21}$$

Now, define the operator $blk\det(\cdot)$ as :

$$blk\det\left(\begin{bmatrix} S_{11} & S_{12} \\ S_{12}' & S_{22} \end{bmatrix}\right) = S_{22} - S_{12}'(S_{11})^{\dagger}S_{12}$$

where S_{11}, S_{22} are square matrices with dimension m and q, respectively.

The next results establishes a simple relation between the symmetric solutions of eqs. (18) and (21):

Theorem 2: *If the matrix P_f is a symmetric solution of eq. (21) then $P = P_f + P_d$ is a symmetric solution of eq. (18). Moreover the condition (19) is satisfied if and only if P_f is stabilizing, i.e.*

$$A - \left(AP_f\widehat{C}' + \widehat{B}_f J\widehat{D}'_f\right)\left(\widehat{D}_f J\widehat{D}'_f + \widehat{C}P_f\widehat{C}'\right)^{-1}\widehat{C}$$

is asymptotically stable. Finally the condition (20) holds if and only if P_f is feasible, i.e.

$$blk\det(\widehat{D}_f J\widehat{D}'_f + \widehat{C}P_f\widehat{C}') < 0$$

Proof: The proof consists of a simple substitution of $P = P_f + P_d$ in eq. (18) by using the identities of Lemma 1 and relations (13). ■

The above result highlights the structure of the solution to eq. (18) as given by the sum of two terms. Precisely, P_f takes into account of the finite unstable zeros of (A, B, C, D) whereas P_d reflects the infinite unstable zeros (system delays). In order to clarify this claim notice that eq. (21) always admits the solution $P_f = 0$. However such a solution is stabilizing if and only if the system $(A, \widehat{B}_f, \widehat{C}, \widehat{D}_f)$ has minimum-phase finite zeros. On the other hand these zeros coincide with those of $(A, \widehat{B}, \widehat{C}, \widehat{D})$ and, in turn, of (A, B, C, D). As for role of system delays, notice that if D is nonsingular, then the JSIM matrix $L(z)$ is the identity, so that $P_d = 0$.

A second observation concerns the construction of all the filters ensuring the given attenuation level. Indeed, once the stabilizing P_f is known, it is possible to parametrize all the J-spectral factors associated with the transference $H(z)$ and, correspondingly, the H_∞ filters.

Example: Consider the discrete-time system (1) where the matrices A, B, C, D are given by (15). In section 2 we showed that $H_f(z)$ provided in (17) is the transference of a J-spectral equivalent delay-free system. Recalling (16) and (12), we get:

$$L_0 = \begin{bmatrix} 0 & 0 \\ 0 & \alpha \end{bmatrix}, \quad L_1 = \alpha\begin{bmatrix} 0 & -\gamma^{-2} \\ -1 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix}, \quad \alpha = \frac{\gamma}{\sqrt{\gamma^2 - 1}}$$

By using relations (14) we write a state-space realization of $H_f(z)$. It results:

$$\widehat{B}_f = \alpha\begin{bmatrix} -1/2 & -\gamma^{-2} \\ 1/4 & 1/2\gamma^{-2} \end{bmatrix}, \quad \widehat{D}_f = \alpha\begin{bmatrix} 1 & 0 \\ 1/2 & 1 - \gamma^{-2} \end{bmatrix}$$

Consider $\gamma = 1.1$. By solving eq. (21) we get that the stabilizing solution is $P_f = 0$ that is also "feasible". Indeed the system (A, B, C, D) has no unstable zeros, thus confirming what said about the meaning of P_f. The feasible stabilizing solution of (18) turns out to be

$$P = P_d = \begin{bmatrix} 5.76 & -2.88 \\ -2.88 & 2.44 \end{bmatrix}$$

that takes into account the effects of the system delays.

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