STABILITY OF DYNAMICAL SYSTEMS WITH PARAMETER PERTURBATIONS

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ABSTRACT.

New approach for the absolute stability analysis of nonstationary control systems is used for investigation of one special kind of dynamical systems. This approach is based on results from the inner theory, stability theory, optimal control theory, variational methods. Inner approach allows to obtain sufficient algebraic conditions of absolute stability for different kinds of dynamical systems. The way to obtain necessary and sufficient conditions for absolute stability of systems with parameter perturbations is shown in this paper. Inner approach is combined with use of the Poutriagin's Maximum Principle and solving of the Cauchy problem. This method leads in some cases to algebraic necessary and sufficient conditions of absolute stability, but in some cases the question about necessary and sufficient conditions of absolute stability obtained by use of developed method is open.

Let us consider the following differential equation [1]

$$x^{(n)}+a_1 x^{(n-1)}+ \ldots +a_{n-1} x¢+ a_n(n)x = 0 \qquad (1)$$

where $a_n (n) = a_n^0 + n (t), n=n (t)$ is bounded function of time:
$|n (t)|\le V_0,\ a_1, \ldots, a_{n-1}, a_n^0, V_0$ are numbers and $a_1 l^{n-1} + \ldots +a_{n-1} l+ a_n(n)$ is the Hurwitz polynomial for an arbitrary constant function v.

The existence and uniqueness of a solution of Eq. (1) are guaranteed.

Absolute stability of Eq. (1) will be understood as the asymptotic stability in global of the trivial solution $x=0$ for any choice of permissible functions $n= n (t)$.

The problem is to find conditions whose fulfillment would lead to the absolute stability of Eq. (1).

Let us divide the set of Eqs. (1) which is given by the functional set

$$V = \{n (t)|\ |n(t)|\le V_0\}$$

into the following subsets.

#1. Equations all solutions of which are such that their (n-2) derivative does not oscillate.

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#k. Equations all solutions of which are such that their (n-k-1) derivative does not oscillate, but there is at least one solution (n-k) derivative of which does oscillate (k=2, 3, ….., n-2).

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#(n-1). Equations which have at least one solution first derivative of which oscillates, and at least one solution first derivative of which does not oscillate.

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#n. Equations all solutions of which are such that their first derivative oscillates.

Using the coefficients of the Eq.(1) we construct the square inner matrix Δ of order 2n-1 [2-4]:

$$\Delta = \left\| \begin{array}{ccccccccc} 1 & a_1 & \text{¼} & a_{n-2} & a_{n-1} & a_n(n) & \text{¼} & 0 & 0 \\ 0 & 1 & \text{¼} & a_{n-3} & a_{n-2} & a_{n-1} & \text{¼} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \text{¼} & 1 & a_1 & a_2 & \text{¼} & a_{n-1} & a_n(n) \\ 0 & 0 & \text{¼} & n & \boxed{n} & (n-1)a_1 & \text{¼} & 2a_{n-2} & a_{n-1} \\ 0 & 0 & \text{¼} & n & (n-1)a_1 & (n-2)a_2 & \text{¼} & a_{n-1} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & n & \text{¼} & 3a_{n-3} & 2a_{n-2} & a_{n-1} & \text{¼} & 0 & 0 \\ n & (n-1)a_1 & \text{¼} & 2a_{n-2} & a_{n-1} & 0 & \text{¼} & 0 & 0 \end{array} \right\|$$

The inners of the matrix Δ are the square matrices of orders 1,3, …, *2n-3* shown in Δ. A constant square matrix Δ is called inner-positive if the determinants of all its inners and also the determinant of the matrix Δ are positive.

Similarly to [3,4], the following propositions are proved.

Eqs. (1) from the subset #1 are absolutely stable.

For Eq. (1) to belong to the subset #1 it is necessary and sufficient that the following conditions (A) and (B) are satisfied for all numbers v **Î [**$-v_0$, v_0**].**

(A) Coefficients of the Eq. (1) are positive
(B) The matrix Δ is inner-positive

Inner indication. If conditions (A) and (B) are satisfied for all numbers v **Î [**$-v_0$, v_0**]*, then Eq. (1) is absolutely stable.***

In the Euclidean phase space {$x_1 = x$, $x_2=x'$, …, $x_n = x^{(n-1)}$} a straight line *l* which crosses the origin is called attainable if there exist such *n (t)* Î *V*, *t* Î [t_0, t_1] that the corresponding phase curve leaves *l* at the moment of time t_0 and comes back to the line *l* at the moment of time t_1 <¥ .

The following propositions are held.

If the Eq.(1) from the subset #(n-1) is absolutely stable then

$$\max \quad x^2(t_1)<1 \qquad (2)$$

$\nu(t) \in V$

where $x(t_0) \in l$, $x(t_1) \in l$, $x(t) \notin l$, ***for all*** $t \in (t_0, t_1)$;
$x^2_1(t_0)=1$ ***for all*** $l \in L=\{l \mid x_2=0, x_1x_2<0\}$.

The condition (2) is necessary and sufficient for the absolute stability of Eqs.(1) from the subset #n.

Let us consider Eq.(1) of order 3. Following the approach from [5] it is necessary to find

$$\max_{v(t) \in V} [x^2_1(t) + x^2_3(t)] \tag{3}$$

(this variational problem is equivalent to the problem (2)).

To solve the problem (3) we can use the Pontriagin's Maximum Principle which leads to the Cauchy problem:

$$\begin{array}{ll}
x'_1 = x_2 & x_1(t_1) = -\cos\gamma \\
x'_2 = x_3 & x_2(t_1)=0 \\
x'_3 = -(a_1x_3+a_2x_2+a_3^0x_1)+v_0x_1\,\mathrm{sign}(\psi_3 x_1) & x_3(t_1)=\sin\gamma \\
\psi'_1 = a^0_3\psi_3 - v_0\psi_3\,\mathrm{sign}(\psi_3 x_1) & \psi_1(t_1)=-\cos\gamma \\
\psi'_2 = -\psi_1 + a_2\psi_3 & \psi_2(t_1)=a_1\sin\gamma - a^0_3\cos\gamma + \nu_0|\cos\gamma| \\
\psi'_3 = -\psi_2 + a_1\psi_3 & \psi_3(t_1)=\sin\gamma \qquad \gamma \in (0,\pi)
\end{array}$$

The Cauchy problem has a solution in our case. On the basis of results from [3-5] the following criterion has been proved.

For the absolute stability of Eq.(1) of order 3 it is necessary and sufficient that condition (A) is satisfied and, besides that, either condition (B) or condition

$$\max_{\gamma \in \Gamma} [x^2_1(t_0) + x^2_3(t_0)] > 1 \tag{4}$$

is satisfied.

Here $x(t)$ is a solution of Cauchy problem, t_0 is the second zero of function $x_2(t)$, the parameter γ belongs to the special point set Γ which corresponds to the set of attainable lines l.

All relations (A), (B) and (4) are algebraic and are imposed on the coefficients of the Eq.(1) only.

This approach to the absolute stability analysis allows us to investigate Eq. (1) which belong to the classes #1, #3(n-1) and #n from the division shown above. The mentioned classes are the only classes that occur in the division of the set of Eq.(1) for n=3. This is why necessary and sufficient conditions of absolute stability for Eq.(1) of order 3 are obtained by using this method. The question about absolute stability of Eq.(1) from classes #2, #3,…, #(n-2) in the common case is still open. The cases of systems with time delay are investigated in [6,7].

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