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A Model-based Detection Observer of Component Failures for Distributed Parameter Systems*

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Abstract

In this note, fault detection techniques based on finite dimensional results are extended and applied to a class of infinite dimensional dynamical systems. This special class of systems assumes linear plant dynamics having an abrupt additive perturbation as the fault. This fault is assumed to be linear in the (unknown) constant (and possibly functional) parameters. An observer-based model estimate is proposed which serves to monitor the system's dynamics and its well posedness is summarized. Using a Lyapunov synthesis approach applied to infinite dimensional systems, a stable parameter learning scheme is developed. The resulting parameter adaptation rule is able to "sense" the instance of the fault occurrence. In addition, it identifies the fault parameters using the additional assumption of persistence of excitation. Simulation studies are used to illustrate the applicability of the theoretical results.

1 Introduction

The motivation of this work has come from recent developments in the use of neural networks for failure detection and diagnosis of finite dimensional dynamical systems using a model-based scheme (Polycarpou and Helmicki, 1995; Polycarpou and Vemuri, 1995). Extensions of these model-based schemes to infinite dimensional systems have not received considerable attention as in the finite dimensional case.

In this paper an abstract framework for the on-line fault detection and diagnosis for a class of infinite dimensional dynamical systems (plants) is developed. The fault is modeled as an additive perturbation of the dynamics that is expressed as a parametrized operator evaluated at an unknown parameter. The fault (i.e. the additive perturbation) is assume to commence at an unknown time instance. The nature of the additive perturbation in the dynamics is assumed to be known, but the parameter at which is evaluated is unknown and desired to be identified. The state estimator, or observer, takes the form of an infinite dimensional linear evolution system with time varying coefficients. This state estimator uses as its inputs the state of the plant (plant output) and the plant's adjustable parameters estimates (adaptive estimates). Using an argument based on Lyapunov redesign method (Ioannou and Sun, 1995; Krstic *et al.*, 1995; Khalil, 1992), which essentially forces the time derivative of a Lyapunov functional to be non positive, the update laws (adaptation rules) for parameter adjustment are derived. The right choice of the online parameter laws guarantees the convergence of the state error to zero with

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no additional conditions imposed either on the state of the plant or the input to the system. By imposing additional conditions on the state of the plant, and implicitly on the input signal, parameter convergence can be established, and hence *failure isolation.*

The combined state and parameter estimator purpose is twofold: (i) to serve as a monitor of the system dynamics and detect the time instance the failure occurs, and (ii) to diagnose the nature of the failure which in this case is assumed to be either a perturbation of the nominal dynamics or another operator whose structure is known but the parameter at which it is evaluated is unknown. Specifically, it is assumed that the failures are *additive* and are *linear* with respect to the parameters. No failures in the input term (actuator failure), commonly denoted by $Bu(t)$ in the literature, are considered at this stage as this would be more relevant in the context of actuator failure and plant accommodation.

The approach here represents an infinite dimensional analogue of an automated fault detection scheme developed for finite dimensional systems in (Polycarpou and Helmicki, 1995; Polycarpou and Vemuri, 1995) and more recently in (Demetriou and Polycarpou, November, 1998). The design of the diagnostic observers falls under the category of *model-based analytical redundancy* approach. The survey papers by Frank (Frank, 1990), Gertler (Gertler, 1988), Isermann (Isermann, 1984) and Patton (Patton, 1994) provide detailed overviews of the various model-base fault detection algorithms. For an in-depth exposure the reader is directed to the books (Basseville and Nikiforov, 1993; Gertler, 1998; Liu and Patton, 1998). The convergence of the state error is obtained using a Lyapunov estimate in a fashion similar to the finite dimensional case. Due to the linearity of the parameters with respect to the additive failures, parameter convergence can be guaranteed using the notion of *persistence of excitation.*

An outline of the remainder of the paper is as follows. In Section 2, the problem is formulated in an abstract setting and the mathematical preliminaries are provided. The detection observer (estimated model) and the fault (parameter) estimator are defined in a variational form. Convergence of the proposed adaptive monitoring scheme is investigated in Section 3. Examples and results of numerical simulations are presented in Section 4. Conclusions with directions for future research on fault detection and accommodation of systems governed by partial differential equations are presented in Section 5.

In general all notation is standard. For X and Y Banach spaces, $\mathcal{L}(X,Y)$ denotes the space of *bounded* linear operators from X into Y. Also, for X a linear space and Y a space of linear functionals on X, $\langle\varphi,x\rangle_{X,Y}$ denotes the action of the linear functional $\varphi\in Y$ on the element $x\in X$.

2 Problem statement and formulation

In this section a procedure for designing a fault detection scheme for a class of infinite dimensional systems is outlined. Specifically, we will be concerned with the following class of dynamical systems

$$\dot{x}(t)+Ax(t)+\beta(t-t^*)D(\theta)x(t)=Bu(t),\qquad x(0)=x_0\in H \tag{1}$$

where H is an infinite dimensional space, x denotes the state, A,D,B denote the system operator, the failure operator and the input operator, respectively. In this case, the failure is assumed to be *abrupt* (Polycarpou and Helmicki, 1995), and specifically the function $\beta(t-t^*)$ that represents the *time profile* of the failure is assumed to be a step function that is given by

$$\beta(t-t^*)=\begin{cases}1 & \text{for } t\geq t^*\\ 0 & \text{for } t<t^*.\end{cases} \tag{2}$$

The *nominal* system dynamics given in (1) via the term $Ax(t)$, i.e. the system

$$\dot{x}(t) + Ax(t) = Bu(t), \qquad x(0) = x_0 \in H, \tag{3}$$

are assumed to be known. The anticipated failure is modeled by the $\theta-$*parameterized* operator $D(\theta)$ and it is assumed that the structure of the failure is known, i.e. for a given parameter θ the operator $D(\cdot)$ is known, but the parameter θ is unknown. Below, we summarize the mathematical preliminaries required for the analysis and well-posedness of the plant and the derivation of the on-line estimated model of (1). This estimated model, or state observer, will use as its inputs the output of the plant $x(t)$ and the adjustable (on-line) estimates $\hat{\theta}(t)$ of the (unknown) parameter θ.

It will be shown in the next section that the proposed estimated model can detect the time of failure t^* and, by imposing the additional condition of *persistence of excitation*, the parameter θ in the additive term $D(\theta)x(t)$ will be identified asymptotically with time. This will also be demonstrated via some numerical simulations in Section 4 , where the time t^* of the failure will be *"sensed"* by this detection observer and the *parameter error* $\hat{\theta}(t) - \theta$ will asymptotically converge to zero in an appropriate norm.

2.1 Plant in Variational Form

We will consider the above equation (1) in weak or variational form. Towards this end, let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and corresponding norm $|\cdot|$. We also let V be a reflexive Banach space with norm denoted by $\|\cdot\|$, and assume that V is embedded densely and continuously in H. We let V^* denote the conjugate dual of V (i.e. the space of continuous conjugate linear functionals on V) with norm denoted by $\|\cdot\|_*$ (i.e the usual uniform operator norm). It then follows that $V \hookrightarrow H \hookrightarrow V^*$ with both embeddings dense and continuous. We then have that

$$|\varphi| \leq K\|\varphi\|, \qquad \varphi \in V, \tag{4}$$

for some positive (embedding) constant K, (Lions and Magenes, 1972; Showalter, 1977; Tanabe, 1979; Wloka, 1987). The notation $\langle \cdot, \cdot \rangle$ will also be used to denote the duality pairing between V^* and V induced by the continuous and dense embeddings in (4).

The parameter space is denoted by Q and it is assumed to be a Hilbert space with inner product $\langle \cdot, \cdot \rangle_Q$ and norm $|\cdot|_Q$. The dual Q^* of Q is identified by $Q^* = Q$. For each $\theta \in Q$, let $D(\theta) : V \to V^*$ be an operator satisfying the following assumptions

(D1) (Q-Linearity) The map $\theta \to D(\theta)\varphi$ is linear from Q into V^* for each $\varphi \in V$.

(D2) ($V - V^*$-Boundedness) There exists a scalar $\alpha_d > 0$ such that for each $\theta \in Q$

$$|\langle D(\theta)\varphi, \psi \rangle| \leq \alpha_d |\theta|_Q \|\varphi\| \|\psi\|, \qquad \varphi, \psi \in V.$$

Remark 1 *The fact that the parameter space is chosen to be a Hilbert space (as opposed to a Euclidean space), enables the proposed scheme to identify functional parameters in the failure term $D(\theta)x(t)$.*

Continuing, we consider the rather standard assumptions on the nominal plant operator A that are required for the existence and uniqueness of solutions to the nominal system (3).

(A1) ($V - V^*$-Boundedness) There exists a scalar $\alpha_a > 0$ such that

$$|\langle A\varphi, \psi \rangle| \leq \alpha_a \|\varphi\| \|\psi\|, \qquad \varphi, \psi \in V.$$

(A2) ($V-H$-Coercivity) There exists a $\lambda_a \in R$ and a scalar $\beta_a > 0$ such that

$$\mathcal{R}e\,\langle A\varphi, \varphi\rangle + \lambda_a|\varphi|^2 \geq \beta_a\|\varphi\|^2, \qquad \varphi \in V.$$

In addition, we consider the operator $L : V \to V^*$, appearing in the diagnostic observer model below, which satisfies the following assumptions.

(L1) ($V-V^*$-Boundedness) There exists a scalar $\alpha_l > 0$ such that

$$|\langle L\varphi, \psi\rangle| \leq \alpha_l\|\varphi\|\|\psi\|, \qquad \varphi, \psi \in V.$$

(L2) (V-Coercivity) There exists a scalar $\beta_l > 0$ such that

$$\mathcal{R}e\,\langle L\varphi, \varphi\rangle \geq \beta_l\|\varphi\|^2, \qquad \varphi \in V.$$

In addition, for $\varphi \in V$ we define the linear operator $G(\varphi) : V \to Q$ by

$$\langle G(\varphi)\psi, \theta\rangle_Q = \langle D(\theta)\varphi, \psi\rangle, \quad \psi \in V, \quad \theta \in Q. \tag{5}$$

Using Assumption ($D2$), it is clear that for $\varphi \in V$, we have $G(\varphi) \in \mathcal{L}(V, Q)$ with

$$\|G(\varphi)\|_{\mathcal{L}(V,Q)} \leq a_d\|\varphi\|. \tag{6}$$

Using equation (5) we define, for $\varphi \in V$, the Banach space adjoint of the operator $G(\varphi)$, denoted here by $G^*(\varphi) \in \mathcal{L}(Q, V^*)$, as

$$\langle G^*(\varphi)\theta, \psi\rangle_{V^*,V} = \langle G(\varphi)\psi, \theta\rangle_Q, \quad \psi \in V, \quad \theta \in Q.$$

Let the initial data $x(0) = x_0 \in H$ and the input, or control, term $Bu(t) \in L_2(0, T; V^*)$, and consider the initial value problem

$$\dot{x}(t) + Ax(t) + \beta(t - t^*)D(\theta)x(t) = Bu(t), \qquad a.e.\, t > 0, \tag{7}$$

$$x(0) = x_0. \tag{8}$$

We establish the well-posedness of the above system via the existence of a weak solution. First, we show the well posedness of the (nominal) system for $t < t^*$, i.e. prior to the failure, and then establish the well posedness of the system with the failure incorporated into its dynamics. Specifically, we first consider the system

$$\text{(I)} \qquad \begin{cases} \dot{x}(t) + Ax(t) = Bu(t), & 0 < t < t^*, \\ x(0) = x_0, \end{cases} \tag{9}$$

and then (using (2)) the system

$$\text{(II)} \qquad \begin{cases} \dot{x}(t) + Ax(t) + D(\theta)x(t) = Bu(t), & t > t^*, \\ x(t^*) = x_*. \end{cases} \tag{10}$$

By a *weak solution* to the initial value problem (9) we mean a function $x \in L_2(0, T; V)$ with $\dot{x} \in L_2(0, T; V^*)$ for all $0 < T < t^*$ that satisfies (9). Similarly, by a weak solution to (10) we mean a function $x \in L_2(t^*, T; V)$ with $\dot{x} \in L_2(t^*, T; V^*)$ for all $T \geq t^*$ that satisfies (10). Sufficient conditions that guarantee the existence of a unique solution are presented in (Banks *et al.*, 1996; Lions, 1971; Lions and Magenes, 1972; Pazy, 1983; Showalter, 1977; Tanabe, 1979; Wloka, 1987). Specifically, the operator A being coercive and bounded with $Bu(t) \in L_2(0,T;V^*)$, are sufficient conditions to guarantee the existence of a unique solution to (9). Similarly, if the operator $[A + D(\theta)]$ is coercive and bounded for all $\theta \in Q$ with $Bu(t) \in L_2(0,T;V^*)$, are sufficient conditions for the existence of solutions to (10). The former condition can for example be satisfied if

$$\begin{aligned} \mathcal{R}e\,\langle (A + D(\theta))\varphi, \varphi\rangle + \lambda_a|\varphi|^2 &\geq \mathcal{R}e\,\langle A\varphi,\varphi\rangle - |\langle D(\theta)\varphi,\varphi\rangle| + \lambda_a|\varphi|^2 \\ &\geq \beta_a\|\varphi\|^2 - a_d|\theta|_Q\|\varphi\|^2 \\ &\geq (\beta_a - a_d|\theta|_Q)\,\|\varphi\|^2, \end{aligned}$$

which would require $\beta_a > a_d|\theta|_Q$.

2.2 Estimated Model and a Learning Scheme

Before we present the state estimator, we give the definition of a *bounded plant*, which in a way is a uniform boundedness condition on $\|x(t)\|$.

Definition 2 (Bounded plant) *A bounded plant is a pair* (θ, x) *with* x *a solution to* (7), (8) *for which there exists a constant* $\gamma = \gamma(x)$ *such that*

$$|\langle G(x(t))\varphi, \theta\rangle_Q| \leq \gamma(x(t))|\theta|_Q\|\varphi\|, \quad t > t^*, \quad \theta \in Q,\ \varphi \in V.$$

It should be noted that if the pair (θ, x) is a bounded plant, then we have that $G(x(\cdot)) \in L_2(t^*, T; \mathcal{L}(V,Q))$ for all $T > t^*$. It also follows from equation (6) that if (θ, x) is such that $\|x(t)\| \leq \gamma < \infty$, a.e. $t > t^*$, for some $\gamma > 0$, then (θ, x) is indeed a bounded plant. In (Baumeister *et al.*, 1997), it was shown that it is possible to provide sufficient conditions for the uniform boundedness of $\|x(t)\|$ for $t > t^*$.

We can now propose the estimated model of (7), (8) along with the adaptive law for the adjustment of the parameter estimates. They take the form of an initial value problem and are given by

$$\dot{\hat{x}}(t) + L\hat{x}(t) + G^*(x(t))\hat{\theta}(t) = Bu(t) - Ax(t) + Lx(t), \tag{11}$$

$$\dot{\hat{\theta}}(t) - G(x(t))\hat{x}(t) = -G(x(t))x(t), \quad a.e. \quad t > 0, \tag{12}$$

$$\hat{x}(0) = x(0), \quad \hat{\theta}(0) = 0. \tag{13}$$

To establish the well posedness of the (state and parameter) estimator (11) - (13), we follow a procedure similar to the one taken for the adaptive parameter estimation of distributed parameter systems in (Demetriou, 1993). We let $X = H \times Q$ and $Y = V \times Q$. When X and Y are endowed with the usual product norm topologies, X becomes a Hilbert space and Y a reflexive Banach space. We then have the dense and continuous embeddings $Y \hookrightarrow X \hookrightarrow Y^*$. We define the operator $\mathcal{A}(t) : Y \to Y^*$ by

$$\mathcal{A}(t) = \begin{bmatrix} L & G^*(x(t)) \\ -G(x(t)) & 0 \end{bmatrix}, \tag{14}$$

and the input $\mathcal{F}(t) \in Y^*$ by

$$\mathcal{F}(t) = \begin{bmatrix} Bu(t) - Ax(t) + Lx(t) \\ -G(x(t))x(t) \end{bmatrix}, \tag{15}$$

for almost every $t > 0$.

The fact that (θ, x) is a bounded plant (Definition 2) and $Bu(t) \in L_2(0,T;V^*)$ implies that $\mathcal{F} \in L_2(0,T;Y^*)$ for all $T > 0$. Assumptions $(L1)$ and $(L2)$ together with (θ, x) being a bounded plant imply that $\mathcal{A}(t) \in \mathcal{L}(Y, Y^*)$, $t > 0$ and that for $t > 0$

$$\mathcal{R}e\langle \mathcal{A}(t)\varphi, \varphi\rangle_{Y^*,Y} + \rho|\varphi|_X^2 \geq \sigma\|\varphi\|_Y^2, \qquad \varphi \in Y,$$

where $|\cdot|_X$ and $\|\cdot\|_Y$ denote respectively, the norms on X and Y, and ρ, $\sigma > 0$. It follows (see, for example, (Lions, 1971; Lions and Magenes, 1972; Showalter, 1977; Tanabe, 1979; Wloka, 1987)) that the initial value problem

$$\begin{aligned} &\dot{\xi}(t) + \mathcal{A}(t)\xi(t) = \mathcal{F}(t), \qquad \text{a.e. } t > 0, \\ &\xi(0) \in X, \end{aligned}$$

admits a unique solution $\xi \in L_2(0,T;Y)$ with $\dot{\xi} \in L_2(0,T;Y^*)$, all $T > 0$. Consequently, the estimator (11) - (13) admits a unique solution $(\hat{\theta}, \hat{x}) \in L_2(0,T;Q) \times L_2(0,T;V)$ with $(\dot{\hat{\theta}}, \dot{\hat{x}}) \in L_2(0,T;Q) \times L_2(0,T;V^*)$, all $T > 0$. Moreover, for each $T > 0$, $\hat{\theta}$ and $\hat{x}$ agree almost everywhere with functions in $C([0,T];Q)$ and $C([0,T];H)$, respectively.

We denote the *output estimation error* or state error, by $e(t) = \hat{x}(t) - x(t)$ and the *parameter estimation error* or parameter error, by $r(t) = \hat{\theta}(t) - \theta$. Using the linearity assumption $(D1)$ and the fact that for $t < t^*$ the parameter $\theta \equiv 0$, we have

$$\begin{aligned} \beta(t-t^*)D(\theta)x(t) &= 0 \cdot D(\theta)x(t) \\ &= D(0)x(t) \end{aligned}, \qquad t < t^*,$$

and that for $t \geq t^*$

$$\begin{aligned} \beta(t-t^*)D(\theta)x(t) &= 1 \cdot D(\theta)x(t) \\ &= D(\theta)x(t) \end{aligned}, \qquad t \geq t^*,$$

we can then write the *error equations* as

$$\dot{e}(t) + Le(t) + G^*(x(t))r(t) = 0, \quad a.e.\ t > 0, \tag{16}$$

$$\dot{r}(t) - G(x(t))e(t) = 0, \quad a.e.\ t > 0, \tag{17}$$

where for $t < t^*$ the parameter error is given by $r(t) = \hat{\theta}(t) - 0$ and for $t \geq t^*$ it is given by $r(t) = \hat{\theta}(t) - \theta$. The initial conditions are given by

$$e(0) = \hat{x}(0) - x(0) = x(0) - x(0) = 0, \quad r(0) = \hat{\theta}(0) - 0 = 0. \tag{18}$$

Equivalently, the error equations (16), (17) can be written as

$$\frac{d}{dt}\begin{bmatrix} e(t) \\ r(t) \end{bmatrix} + \mathcal{A}(t)\begin{bmatrix} e(t) \\ r(t) \end{bmatrix} = 0, \qquad a.e.\ t > 0, \tag{19}$$

with the operator $\mathcal{A}(t)$ given by (14).

The choice $\hat{\theta}(0) = 0$ will be explained in the next section where it will be shown that for $t < t^*$ the parameter estimator $\hat{\theta}$ will estimate the zero parameter (i.e. $\theta(t) = 0$) and at $t \geq t^*$ will adaptively estimate the nonzero parameter θ. In addition, the choice $\hat{x}(0) = x(0)$ will be shown to guarantee that for $t < t^*$, $e(t) \equiv 0$ and for $t \geq t^*$, $|e(t)| \geq 0$. The latter is a means of sensing the failure in the system, i.e. when $e(t)$ becomes nonzero it means that the system dynamics changed from $Ax(t)$ to $[A + D(\theta)]x(t)$. With no additional assumptions, it will be shown that the state error $e(t)$ converges to zero asymptotically after the failure occurs. Additionally, the parameter estimate will be shown to estimate the zero parameter (i.e. $\hat{\theta}$ remain at zero) for $t < t^*$ and after the failure occurs, it will attempt to estimate the parameter θ. In order to quarantee that the parameter estimator will asymptotically estimate the parameter θ after the failure, we must impose the additional assumption of *persistence of excitation*, see (Krstic *et al.*, 1995; Narendra and Annaswamy, 1989; Sastry and Bodson, 1989).

3 Convergence of the Learning Scheme

In this section, we make the standing assumption that the pair (θ, x) is a bounded plant (Definition 2). We use a Lyapunov-like argument to show convergence of the state error $e(t)$ to zero. Toward this end, we define the function $E : [0, \infty) \to R$ by

$$E(t) = \frac{1}{2}\left\{|e(t)|^2 + |r(t)|_Q^2\right\}, \quad t \geq 0. \tag{20}$$

As a first result we get a bound on the *energy function* $E(t)$.

Lemma 3 *For all $t < t^*$ we have*

$$E(t) + \beta_l \int_0^t \|e(\tau)\|^2 d\tau \leq E(0), \tag{21}$$

and for $t > t^*$ we have

$$E(t) + \beta_l \int_{t^*}^t \|e(\tau)\|^2 d\tau \leq E(t^*). \tag{22}$$

Proof 1 *Using* (16), (17), (18) *and assumption* (L2) *we have that for $t < t^*$*

$$\begin{aligned} \frac{d}{dt}E(t) &= \langle \frac{d}{dt}e(t), e(t)\rangle + \langle \frac{d}{dt}r(t), r(t)\rangle = -\langle Le(t), e(t)\rangle \\ &\leq -\beta_l \|e(t)\|^2. \end{aligned} \tag{23}$$

When equation (23) *is integrated from* 0 *to some $t < t^*$ we obtain the desired result* (21). *Similarly, when we integrate from t^* to some $t \gg t^*$ we get* (22).

The above lemma is used to show that the state error $e(t)$ either remains at zero for $t < t^*$ or it converges asymptotically to zero for some $t \gg t^*$. This is stated as a theorem below.

Theorem 4 *The error equations* (16) - (18) *that result by combining the plant* (7), (8) *with the state and parameter estimator* (11) - (13), *satisfy:*

(i) for $t < t^*$ we have

$$E(t) \equiv e(t) \equiv r(t) \equiv 0,$$

(ii) for $t > t^*$ the energy function $E(t)$ is nonincreasing and

$$\lim_{t \to \infty} |e(t)| = 0.$$

Proof 2 Case (i), $t < t^*$. *Using the fact that at $t = 0$ the initial conditions $e(0) = \hat{x}(0) - x(0) = 0$ and $r(0) = \hat{\theta}(0) - 0 = 0$, and the result of Lemma 3, we have that*

$$E(t) + \beta_l \int_0^t \|e(\tau)\|^2 \, d\tau \le E(0) \equiv 0, \qquad t < t^*,$$

which implies that $E(t) \equiv 0$ for all $t < t^*$.
Case (ii), $t > t^*$. *Using equation* (22)

$$E(t) + \beta_l \int_{t^*}^t \|e(\tau)\|^2 \, d\tau \le E(t^*), \qquad t > t^*$$

we have that E (with $E(t^*) \neq 0$) is nonincreasing. The convergence of $|e(t)|$ to zero follows from the same arguments used in (Demetriou, 1993) for the adaptive parameter estimation of distributed parameter systems. It is essentially based on Barbălat's *lemma (Popov, 1973) often used in the adaptive estimation and control of finite dimensional systems, (Krstic* et al.*, 1995; Narendra and Annaswamy, 1989; Popov, 1973).*

Remark 5 *It can be observed from Theorem 4 that the estimator will* sense *the time t^* of failure, since the state error is identical to zero for all time t up to failure time t^*. The state error becomes nonzero after t^* and converges to zero afterwards. When the state error attains a nonzero value it indicates that the failure occurred and hence the time t^* can be detected by monitoring the state error $e(t)$. Another way to detect the failure is by monitoring the parameter estimate $\hat{\theta}(t)$, as it too remains at zero for $t < t^*$ and becomes nonzero thereafter.*

Remark 6 *In the above design, it was assumed that the initial condition $x(0)$ was known. If this is not known, one can actually built a monitoring observer of the healthy system in the infinitely remote past and assume that the state error $e(t)$ is relaxed at time t_0, (Green and Limebeer, 1995). Alternatively, if an upper bound on the norm of $e(t_0)$ is known, then a* dead-zone *adaptive law can be augmented in the design to ensure that false alarms due to nonzero $e(t_0)$ are avoided, see (Demetriou and Polycarpou, November, 1998; Polycarpou and Helmicki, 1995; Polycarpou and Vemuri, 1995) for the finite dimensional treatment.*

The convergence of $\hat{\theta}(t)$ to the actual parameter θ is established by imposing the additional assumption of *persistence of excitation*, (Krstic *et al.*, 1995; Narendra and Annaswamy, 1989). Below, we provide the equivalent definition of this persistence of excitation condition as it extends to infinite dimensional systems, (Demetriou, 1993).

Definition 7 (Persistence of Excitation) *A bounded plant (θ, x) is said to be persistently excited, if there exists $T_0, \delta_0, \epsilon_0 > 0$ such that for each $p \in Q$ with unit norm (i.e. $|p|_Q = 1$) and each $t > 0$ sufficiently large (in this case $t \gg t^*$), there exists a $\tilde{t} \in [t, t + T_0]$ such that*

$$\left\| \int_{\tilde{t}}^{\tilde{t}+\delta_0} G^*(x(\tau)) p \, d\tau \right\|_* \ge \epsilon_0, \tag{24}$$

where $G^*(x(t)) \in \mathcal{L}(Q, V^*)$ is the Banach space adjoint of the operator defined in (5).

Theorem 8 *If the plant (q, x) is persistently excited then*

$$\lim_{t \to \infty} |r(t)|_Q = \lim_{t \to \infty} |\hat{\theta}(t) - \theta|_Q = 0.$$

Proof 3 *The proof is identical to the one used for the adaptive parameter identification of infinite dimensional dynamical systems in (Baumeister* et al.*, 1997) and it is therefore omitted.*

4 Examples

In this section we present some examples to demonstrate the applicability of the proposed fault detection scheme. We first examine a one dimensional heat equation with spatially varying coefficients and then a second order (in time) hyperbolic pde (wave equation) with spatially varying fault of the stiffness parameter.

4.1 Example 1

As a first example, we consider the one dimensional diffusion equation with spatially varying parameter given by

$$\frac{\partial}{\partial t}x(t,\xi) = \frac{\partial}{\partial \xi}\left(a(\xi)\frac{\partial}{\partial \xi}x(t,\xi)\right) + f(t,\xi) + \beta(t-10)\frac{\partial}{\partial \xi}\left(\theta(\xi)\frac{\partial}{\partial \xi}x(t,\xi)\right) \tag{25}$$

and with boundary and initial conditions

$$x(t,0) = x(t,1) = 0, \quad t > 0, \qquad x(0,\xi) = 0, \quad 0 < \xi < 1. \tag{26}$$

The Hilbert space H is taken to be $H = L_2(0,1)$ and the Sobolev space V is $V = H_0^1(0,1)$; the reader is directed to the books of Adams (Adams, 1975) or Lions and Magenes (Lions and Magenes, 1972) for an exposition to Sobolev spaces. The parameter space in this case is $Q = H^1(0,1)$ endowed with the weighted inner product

$$\langle q,p\rangle_Q = \omega_1 \int_0^1 q(\xi)\cdot p(\xi)\,d\xi + \omega_2 \int_0^1 q'(\xi)\cdot p'(\xi)\,d\xi, \quad q,p \in H^1(0,1),$$

where the weights ω_1 and ω_2 are assumed to be positive.

The operators A and $D(\cdot)$ in (1) are given by

$$\begin{aligned} \langle A\phi,\psi\rangle &= \int_0^1 a(\xi)\cdot\phi'(\xi)\cdot\psi'(\xi)\,d\xi, \\ \langle D(\theta)\phi,\psi\rangle &= \int_0^1 \theta(\xi)\cdot\phi'(\xi)\cdot\psi'(\xi)\,d\xi. \end{aligned} \tag{27}$$

The operator $G(x(t))$ in (5) is given in weak form by

$$\langle G(x(t))\varphi,p\rangle_{H^1(0,1)} = \langle D(p)x(t),\varphi\rangle = \int_0^1 p(\xi)\frac{\partial}{\partial \xi}x(t,\xi)\cdot\varphi'(\xi)\,d\xi,$$

for $\varphi \in H_0^1(0,1)$, $p \in H^1(0,1)$, and the estimator operator L is given by

$$\langle L\phi,\psi\rangle = 2\int_0^1 \phi(\xi)\cdot\psi(\xi)\,d\xi, \quad \phi,\psi \in H_0^1(0,l).$$

In this case, the effect of the fault is a change in the thermal diffusivity from $a(\xi)$ to $a(\xi)+\theta(\xi)$. In other words,

$$D(\theta)|_{\theta(\xi)=a(\xi)}\, x(t) \equiv Ax(t)$$

or the θ-parameterized operator $D(\theta)$ is the same as the nominal operator A evaluated at a different diffusivity parameter. The nominal thermal diffusivity is given by

$$a(\xi) = 1.5\times 10^{-3}\left(1.5 - \sin(\pi\xi)\right), \qquad 0 \le \xi \le 1, \tag{28}$$

and the unknown perturbation $\theta(\xi)$ of the diffusivity is chosen as

$$\theta(\xi) = 1.5 \times 10^{-3} \left(1 - \frac{\sqrt{2}}{2} - \sin(3\pi\xi)\right) \chi_{[0.3,0.7]}(\xi), \qquad 0 \le \xi \le 1, \tag{29}$$

where $\chi_{[0.3,0.7]}(\xi)$ denotes the characteristic function over the interval $[0.3, 0.7]$. This is also illustrated in Figure 1 where both $a(\xi)$ and $a(\xi) + \theta(\xi)$ are depicted. It can be easily verified that assumptions $(D1)$, $(D2)$, $(A1)$, $(A2)$ and $(L1)$, $(L2)$ are satisfied with $\alpha_d = 1.0$, $\lambda_a = 0$, $\alpha_a = 2.25 \times 10^{-3}$, $\beta_a = 0.75 \times 10^{-3}$, $\alpha_l = \beta_l = 2$. The embedding constant in (4) is $K = \pi^{-1}$. It follows that

$$\langle [A + D(\theta)]\varphi, \varphi \rangle = \int_0^1 (a(\xi) - \theta(\xi)) \, [\varphi_\xi(\xi)]^2 \, d\xi \ge 1 \times 10^{-3} \|\varphi\|^2$$

will guarantee the existence of a unique solution to (25) as mentioned in Section 2.

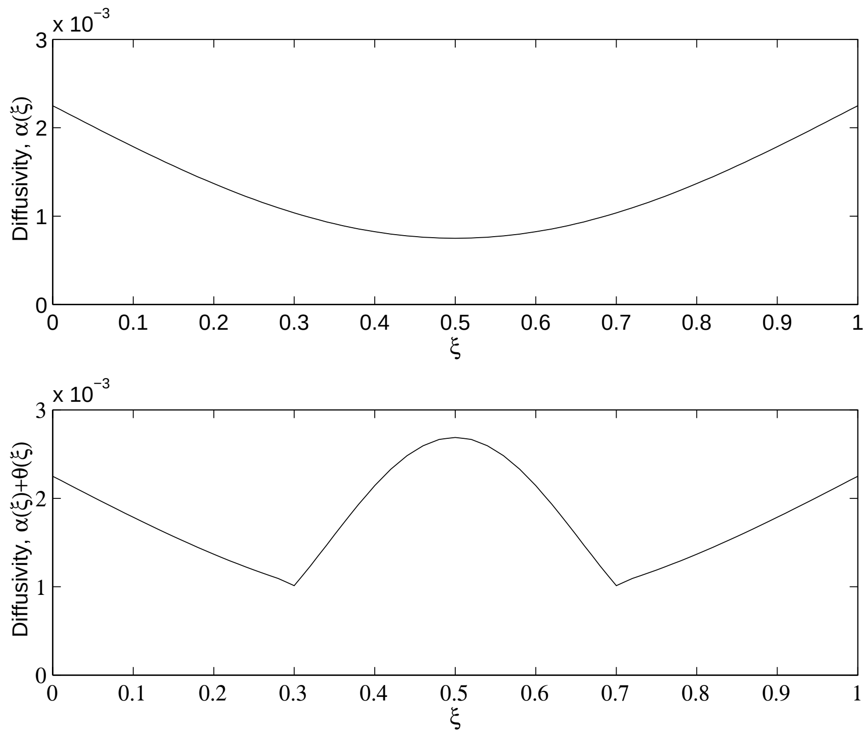

Figure 1: Example 1; Diffusivity parameter $\alpha(\xi)$ (before failure) and $\alpha(\xi) + \theta(\xi)$ (after failure).

The forcing function is given by

$$f(t, \xi) = 10^{-2} \left\{ \left[3 + 0.1 \sin\left(\frac{\pi t}{50}\right)\right] \times 2e^{-0.01t} + \sin\left(\frac{\pi t}{200}\right) \right\} \chi_{[0.3,0.7]}(\xi).$$

In Figure 2, we observe that both $|e(t)|$ and $|r(t)|$ remain at zero for $0 \le t < t^*$. The estimated model approximates the system after failure and the state increases at the time of the system failure at $t = 10$ seconds but converges to zero within 6 seconds. Additionally, we observe that the on-line parameter approximator can also serve as an indicator of the system's failure. Figure 3 shows the parameter approximator $\hat{\theta}(t, \xi)$ compared to the actual parameter

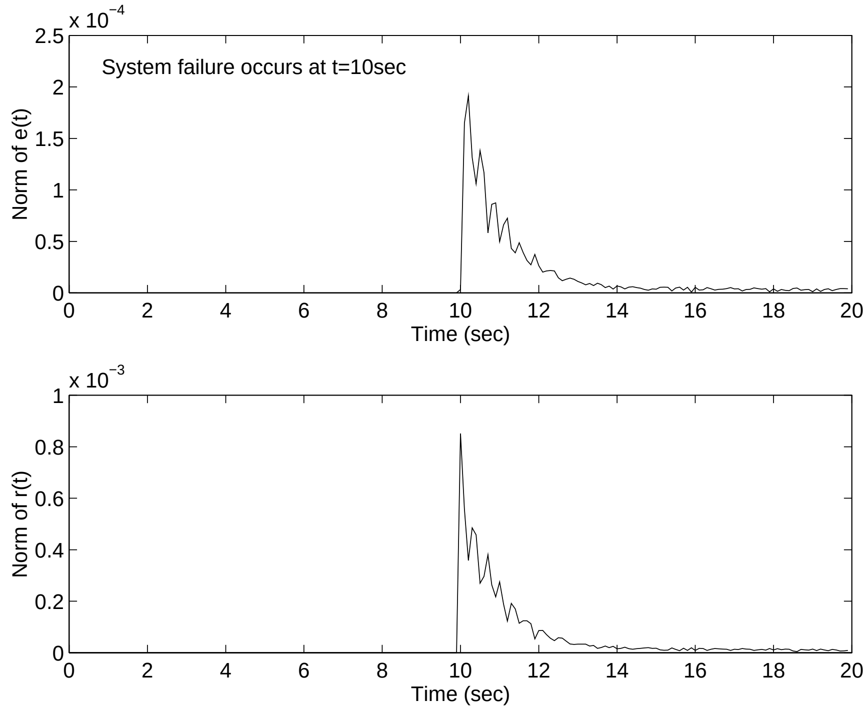

Figure 2: Example 1; Evolution of state error, $e(t) = \hat{x}(t) - x(t)$, and parameter error $r(t) = \hat{\theta}(t) - \theta$.

$\theta(\xi)$ at four different time instances. It is observed that the parameter approximator identifies the *location* (i.e. function is nonzero on the spatial interval $0.3 \leq \xi \leq 0.7$) and the *shape* of the failure (i.e. the function $1 - \frac{\sqrt{2}}{2} - \sin(3\pi\xi)$) within 4 seconds, from $t = 11$ seconds to $t = 14$ seconds.

4.2 Example 2

In this example we consider a second order system that can be written as a first order system. The plant is given by the wave equation with Kelvin-Voigt viscoelastic damping

$$\begin{aligned} &\frac{\partial^2}{\partial t^2} w(t,\xi) - \frac{\partial}{\partial \xi}\left(c_D I(\xi) \frac{\partial^2}{\partial \xi \partial t} w(t,\xi) + EI(\xi) \frac{\partial}{\partial \xi} w(t,\xi) \right) \\ &\qquad -\beta(t-10) \frac{\partial}{\partial \xi}\left(\theta(\xi) \frac{\partial}{\partial \xi} w(t,\xi) \right) = f(t,\xi), \qquad \text{in } \Omega \end{aligned} \tag{30}$$

where $w(t,\xi)$ denotes the displacement and $w_t(t,\xi)$ the velocity. The boundary and initial conditions are given by

$$w(t,\xi)|_{\partial\Omega} = w_x(t,\xi)|_{\partial\Omega} = 0, \qquad \begin{aligned} w(0,\xi) &= d_0(\xi) \in H_0^1(0,l), \\ w_t(0,\xi) &= v_0(\xi) \in L^2(0,l). \end{aligned}$$

Since equation (25) has strong damping, we can use the same techniques in (Demetriou, 1993) applied for the adaptive parameter estimation of hyperbolic distributed parameter systems, to write equation (30) as a first order system with $H = H_0^1(0,l) \times L^2(0,l)$, $V = H_0^1(0,l) \times H_0^1(0,l)$

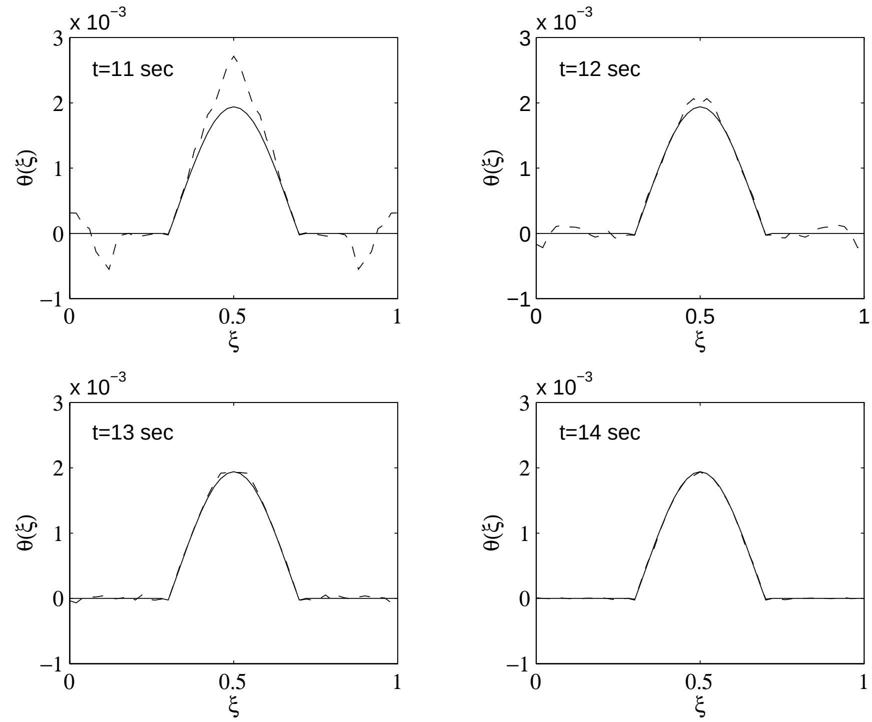

Figure 3: Example 1; Parameter $\hat{\theta}(t,\xi)$ (dashed) and $\theta(\xi)$ (solid) at different time epochs t=11,12,13 & t=14 seconds.

and the parameter space $Q = H^1(0,l)$. We briefly describe the procedure. When the above system is written as an abstract second order initial value problem, we arrive at

$$\begin{gathered} w_{tt}(t) + K_1 w_t(t) + K_2 w(t) + \beta(t-10) K_3(\theta) w(t) = f(t), \ a.e.\ t > 0 \\ w(0) = d_0, \quad w_t(0) = v_0. \end{gathered} \tag{31}$$

which when written in a weak (or variational) form becomes

$$\begin{gathered} \langle w_{tt}(t), \phi\rangle + \langle K_1 w_t(t), \phi\rangle + \langle K_2 w(t), \phi\rangle + \beta(t-10)\langle K_3(\theta) w(t), \phi\rangle \\ = \langle f(t), \phi\rangle \\ w(0) = d_0 \in H_0^1(0,l), \quad w_t(0) = v_0 \in L^2(0,l). \end{gathered} \tag{32}$$

The state and parameter estimators are given, for $\phi = (\phi_1, \phi_2) \in H_0^1(0,l) \times L^2(0,l)$, by

$$\begin{aligned} &\langle L_2 \frac{d}{dt}\hat{w}(t), \phi_1\rangle = \langle L_2 \hat{w}_t(t), \phi_1\rangle + \lambda\langle L_2 e(t), \phi_1\rangle, \\ &\langle \frac{d}{dt}\hat{w}(t), \phi_2\rangle + \langle L_1 e_t(t), \phi_2\rangle + \langle L_2 e(t), \phi_2\rangle + \lambda\langle e_t(t), \phi_2\rangle \\ &\qquad\qquad + \langle K_3(\hat{\theta}(t)) w(t), \phi_2\rangle = \langle f(t), \phi_2\rangle \end{aligned} \tag{33}$$

and

$$\langle \frac{d}{dt}\hat{\theta}(t), p\rangle_{H^1(0,l)} = \langle K_3(p) w(t), e_t(t)\rangle, \ \ p \in H^1(0,l). \tag{34}$$

The design operators $L_1, L_2 \in \mathcal{L}(H_0^1(0,l), H^{-1}(0,l))$ are chosen to satisfy assumptions $(L1)$ and $(L2)$ and the operators $K_i \in \mathcal{L}(H_0^1(0,l), H^{-1}(0,l))$, $i = 1,2$, $K_3(\theta) \in \mathcal{L}(H_0^1(0,l), H^{-1}(0,l))$,

$\theta \in Q$, are given by

$$
\begin{aligned}
\langle K_1 w_t(t), \psi \rangle &= \int_0^l c_D I(\xi) \cdot \frac{\partial}{\partial \xi} w_t(t,\xi) \cdot \psi'(\xi)\, d\xi, \\
\langle K_2 w(t), \psi \rangle &= \int_0^l EI(\xi) \cdot \frac{\partial}{\partial \xi} w(t,\xi) \cdot \psi'(\xi)\, d\xi, \\
\langle K_3(\theta) w(t), \psi \rangle &= \int_0^l \theta(\xi) \cdot \frac{\partial}{\partial \xi} w(t,\xi) \cdot \psi'(\xi)\, d\xi.
\end{aligned} \tag{35}
$$

The fault is modeled as a change (decrease) in the stiffness parameter EI which is given below and depicted in Figure 4,

$$
\theta(\xi) = -1.5 \times 10^{-4} \chi_{[0.3,0.7]}(\xi) \left(1 - \frac{\sqrt{2}}{2} - \sin(3\pi\xi)\right), \quad 0 < \xi < l. \tag{36}
$$

Since in this case $EI(\xi) + \theta(\xi) > 0 \ \forall \xi \in [0,l]$ and hence $\langle (K_1 + K_3(\theta))\varphi, \varphi \rangle \geq \alpha \|\varphi\|^2$, we can conclude well posedness using already established results on second order systems.

The stiffness and damping parameters $EI(\xi)$ and $c_D I(\xi)$ are chosen as $EI(\xi) = 3 \times 10^{-2}$ and $c_D I(\xi) = 5 \times 10^{-3}$, $0 < \xi < l$ respectively. The input $f(t,\xi)$ is given by

$$
f(t,\xi) = \chi_{[0.4,0.6]}(\xi) \left(\sin(\frac{\pi t}{200}) + 5(3 + 0.1 \sin(\frac{\pi t}{50})) e^{-0.01t} \right).
$$

The constant λ is set to $\lambda = 1$ and the initial conditions $d_0(\xi)$, $v_0(\xi)$ are given by

$$
d_0(\xi) = 0.01 \sin(\pi\xi/l), \ \ v_0(\xi) = 0.001 \sin(4\pi\xi/l), \ \ 0 < \xi < l.
$$

The design operators L_1, L_2 are chosen to have the same structure as the damping and stiffness operators K_1, K_2 evaluated at different (and constant) damping and stiffness parameters, and are given by

$$
\begin{aligned}
\langle L_1 w_t(t), \psi \rangle &= \int_0^l 0.02 w_{t\xi}(t,\xi) \cdot \psi'(\xi)\, d\xi, \quad \psi \in H_0^1(0,l) \\
\langle L_2 w(t), \psi \rangle &= \int_0^l 0.04 w_{\xi}(t,\xi) \cdot \psi'(\xi)\, d\xi, \quad \psi \in H_0^1(0,l),
\end{aligned} \tag{37}
$$

The adaptation rule for $\theta(t,\xi)$ is given by

$$
\langle \frac{d}{dt}\theta(t), p \rangle_{H^1(0,l)} = \int_0^l p(\xi) w_\xi(t,\xi) \cdot e_{t\xi}(t,\xi)\, d\xi, \quad p \in H^1(0,l). \tag{38}
$$

Both displacement and velocity state errors (in their respective norms) converge to zero after the failure occurs as depicted in Figure 5. The norm of the parameter estimate $\theta(t,\xi)$ converges to the norm of the actual parameter $\theta(\xi)$ as observed in Figure 6. The graph of the parameter estimate and its adaptive estimate are plotted (pointwise) in Figure 7. The pointwise convergence is established at 100 seconds.

5 Conclusions and Further Research

In this note the finite dimensional theory of model-based fault diagnosis was extended into a class of infinite dimensional dynamical systems. The proposed state estimator with the parameter

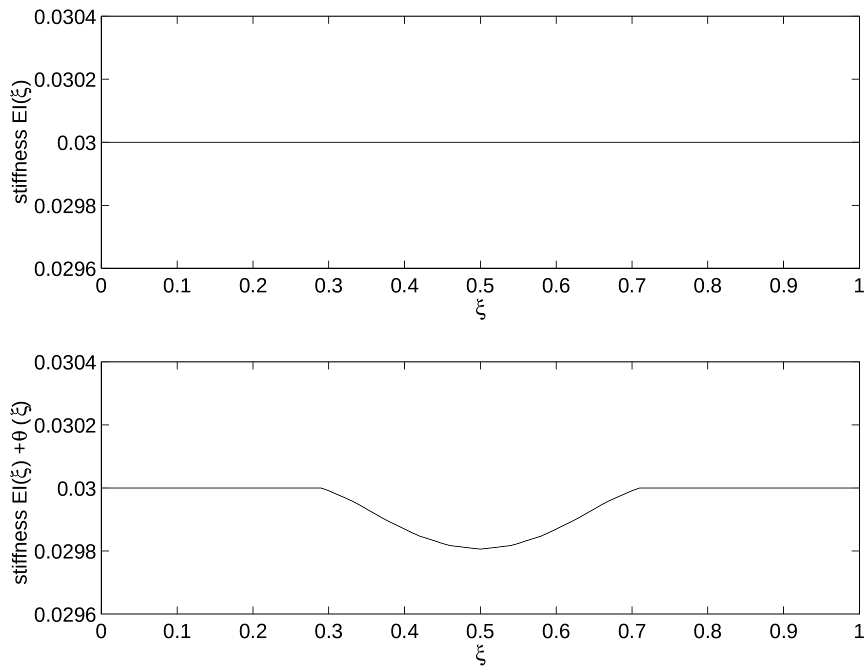

Figure 4: Example 2; Stiffness parameter $EI(\xi)$ (before failure) and $EI(\xi)+\theta(\xi)$ (after failure).

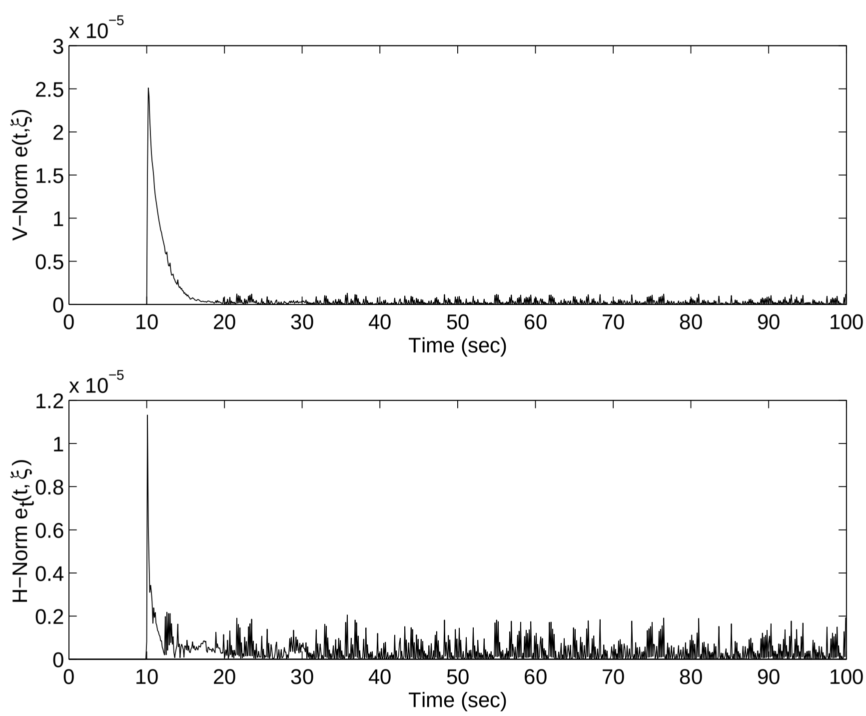

Figure 5: Example 2; Evolution of displacement state error $e(t,\xi)$ and velocity state error $e_t(t,\xi)$.

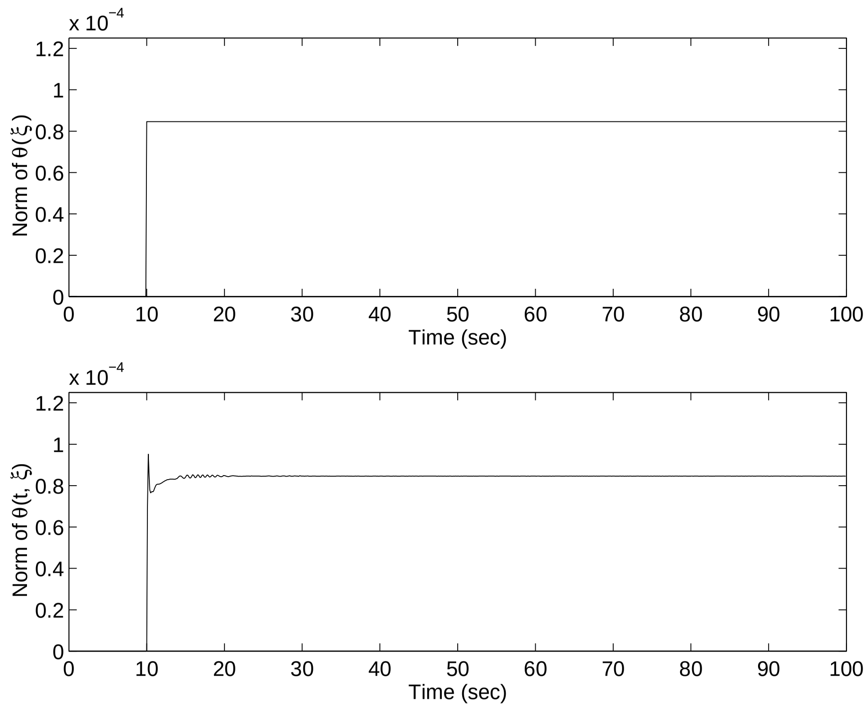

Figure 6: Example 2; Evolution of the parameter norm of $\beta(t-T)\theta(\xi)$ and its adaptive estimate $\widehat{\theta}(t,\xi)$.

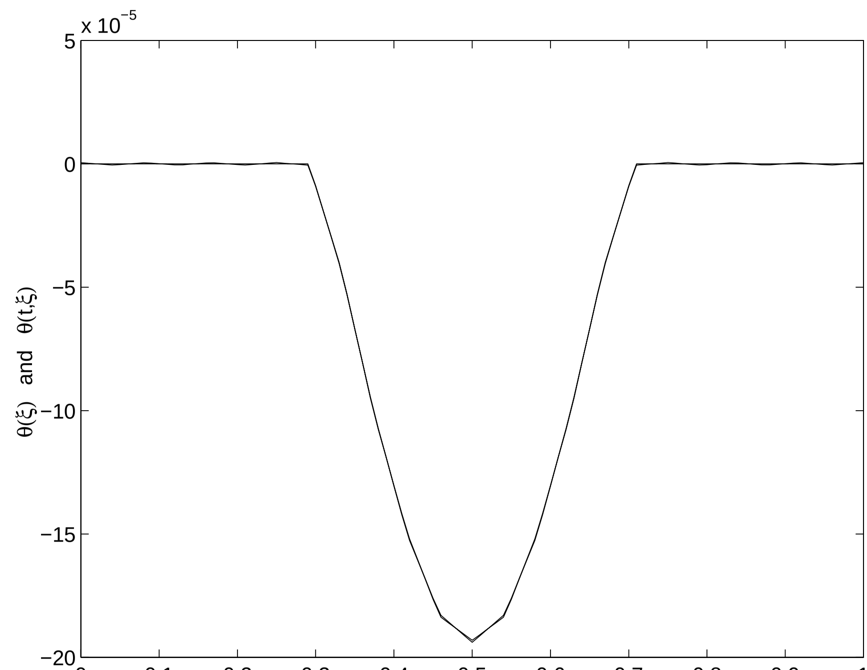

Figure 7: Example 2; Parameter $\theta(\xi)$ (dashed) and its adaptive estimate $\widehat{\theta}(t,\xi)$ (solid) at $t = 100$ sec for $0 < \xi < l$.

identifiers can detect the failure time. These on-line parameter estimators can identify (isolate) the location of the fault (in the spatial domain) and assess the nature of the fault thus allowing for the successful design of a control policy to accommodate such a system. The proposed scheme was designed with the applications for flexible structures in mind, but this general framework encompass systems governed by parabolic and hyperbolic partial differential equations. Delay differential equations can also be included in the proposed framework. Another application is in the aerospace applications and especially in the area of detection of air contamination in both enclosed and open environments (cavities), see (Skliar and Ramirez, 1997).

This scheme is by no means complete as it requires full state measurements, often impossible to acquire, and assumes known initial conditions with no modeling uncertainties and no external inputs present. It does however lay down the abstract framework for the study of a wide class of infinite dimensional systems with unbounded state and input operators. Such a class of systems includes the *Pritchard-Salamon* class (Curtain and Zwart, 1995; Keulen, 1993; Pritchard and Salamon, 1987) of infinite dimensional systems.

Future direction would involve failures in actuators and in the form of nonlinear dynamics (as opposed to the current case of linear perturbations with linearly parametrized operators) or exogenous failures. This could possibly employ neural networks as used in the finite dimensional case in (Polycarpou and Helmicki, 1995). As it is often the case, restricted plant information is available, which means that only a noise-corrupted system output is available to assess failures in the system. This type of failure would be studied in the context of flexible structures. Some attempts were made to detect the failure in a model for a nonlinear beam in (Demetriou and Fitzpatrick, 1995a,b, 1997) and in (Demetriou, 1996) to detect failures in thermal processes. Specifically, for an Euler-Bernoulli beam with Kelvin-Voigt viscoelastic damping

$$\frac{\partial^2}{\partial t^2} w(\xi,t) + \frac{\partial^2}{\partial \xi^2}\left[EI(\xi)\frac{\partial^2}{\partial \xi^2} w(\xi,t) + c_D I(\xi)\frac{\partial^3}{\partial \xi^2 \partial t} w(\xi,t)\right] = f(\xi,t)$$

the failure could be modelled as a (non)linear additive perturbation of stiffness, to give the following model

$$\begin{aligned}\frac{\partial^2}{\partial t^2} w(\xi,t) + \frac{\partial^2}{\partial \xi^2}\Bigg[EI(\xi)\frac{\partial^2}{\partial \xi^2} w(\xi,t) &+ c_D I(\xi)\frac{\partial^3}{\partial \xi^2 \partial t} w(\xi,t) \\ &+\beta(t-T)\mathcal{G}\left(\theta(\xi), \frac{\partial^2 w(\xi,t)}{\partial \xi^2}\right)\Bigg] = f(\xi,t).\end{aligned}$$

Using an additive perturbation of the stiffness parameter to model failures in structures has been suggested in (Chen and Garba, 1987) where for a series of interconnected masses with springs, the failure was modelled as a break of one of the connecting springs. The above proposed model of failure in the beam bares no similarity to the aforementioned paper but it uses the stiffness perturbation as in (Chen and Garba, 1987) to model the failure. The nonlinear function $\mathcal{G}(\theta, w_{\xi\xi})$ is desired to be identified in order to allow for the correct accommodation of the post-failure structure.

Another avenue of interest involves the detection of actuator failure and consequent controller design for the accommodation of the structure. Some preliminary studies on this appeared in (Demetriou and Polycarpou, 1997c,a,b, 1998; Ackleh *et al.*, 1998). Many flexible structures are using smart actuators and sensors for control and observation. These sensors and actuators need to be monitored in order to detect their failures. On-line schemes for actuator/sensor failure detection are thus needed to accommodate these intelligent structures.

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