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Aspects of Traction Control

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Abstract

Propulsion by traction raises several issues, including modeling of the friction force that produces traction and the design of appropriate control laws. The traditional "adhesion" model and several other static and dynamic friction models are described. Control laws that account in some manner for the severe nonlinearity of traction are investigated by simulation. It is shown that ignoring the nonlinear effects can result in an unstable system, but that the instability can be avoided by an appropriate control law design including an observer that accounts for the nonlinear friction model.

1 Introduction

Traction is the most common means of propulsion. It is the means of propulsion in automobiles, bicycles, and walking. It is also used in belt drives, clutches, brakes, and other systems too numerous to list. Paradoxically, however, from the viewpoint of control, it may well be the least understood. Underlying the lack of understanding of the control aspects of traction is that the propulsive force is produced by friction between the rolling wheel and the surface upon which the vehicle moves. And the physical mechanism of the friction present between the wheel and the surface is still problematical.

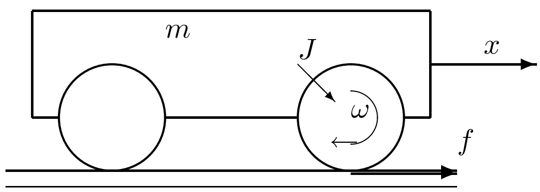

Figure 1: Vehicle propelled by friction

The issue of friction is avoided in elementary dynamics texts by using the concept of "rolling without slipping," in which the friction force at the interface between the wheel and the surface upon which it rolls adjusts to whatever value is necessary to maintain the rim velocity of the wheel equal to the linear velocity of the vehicle, shown schematically in Figure 1. The former is given by $r\omega$, where r is the radius of the wheel and ω is its angular velocity. Hence the condition for rolling without slipping is

$$v_s := r\omega - v = 0 \tag{1}$$

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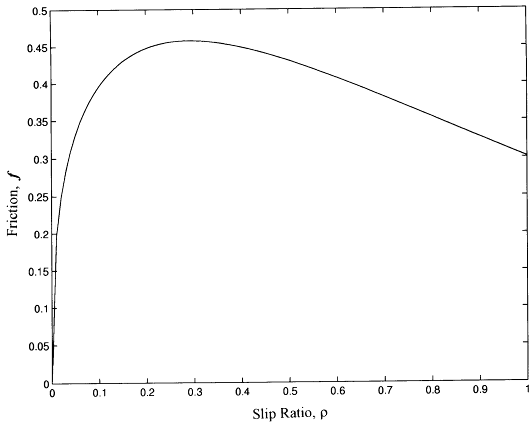

Figure 2: Conventional adhesion model.

The difference v_s between the rim velocity of the wheel the velocity of the vehicle is frequently called the *slip velocity*; hence rolling without slipping is the same as having a slip velocity of zero.

Friction is the mechanism responsible for making the wheel "adhere" to the surface on which it rolls. Thus, unlike cases in which friction is an undesirable nuisance, for traction it is a necessity. Understanding the dynamics of vehicles propelled by friction necessitates an understanding of the mechanism of friction.

The goals of this paper are to review some of the friction models that may be useful in the design of closed-loop traction control systems and to investigate their impact on the performance of such systems.

2 Friction Models

Modeling friction for purposes of analyzing and designing control systems in which friction is a nuisance force has emerged as an important research area. It has not achieved equal prominence as a research topic, however, in applications where friction is not a nuisance but is the source of the propulsive force or torque. Some of the issues in friction modeling for purposes of traction control are addressed in this section.

2.1 Adhesion Model

A popular model (Olsson, 1996) for the propulsive friction force is expressed by

$$f = \phi(\rho) \tag{2}$$

where f is the traction force,

$$\rho = \frac{v_s}{v}$$

is the *slip ratio*, and $\phi(\ )$ is an empirically determined function having the shape illustrated in Figure 2. Note that this function has a peak value at a slip ratio around 0.2.

Notwithstanding its popularity, this model is problematical for several reasons:

- The friction force f is zero when the slip ratio is zero. Thus, when the vehicle velocity is nonzero, the traction force is zero. Thus the model does not support the ideal case of rolling without slipping in any situation where non-zero traction is needed (such as acceleration, or maintaining constant speed on an incline).
- While the slip ratio may be a convenient and empirically justified variable for a constant or nearly constant (nonzero) operating velocity, it poses problems in applications in which the vehicle velocity may hover around or frequently cross zero, when the slip ratio may become very large. The model does not predict the behavior of the adhesion force for a situation in which both the wheel speed and the vehicle speed are zero, a situation that can readily occur in practical applications.

  For applications in which the vehicle velocity can be both positive and negative, the adhesion model is sometimes "fixed" by using $|v|$ in place of v in the definition of ρ. This only exacerbates the problem, however, by introducing a discontinuity at $v = 0$.
- The adhesion model is inconsistent with other friction models, and with accepted physical theory which hold that the force of friction depends on the relative velocity of the two bodies at the interface, and not on the velocities of the individual bodies.

2.2 Coulomb-Stiction Model

Consistent with the elementary accepted physical theory of friction is the Coulomb friction model

$$f = -K\mathrm{sgn}(v_s) \tag{3}$$

where

$$\mathrm{sgn}(v) = \begin{cases} 1, & v > 0 \\ \text{undefined}, & v = 0 \\ -1, & v < 0 \end{cases}$$

Leaving sgn( ) undefined at $v = 0$ supports the rolling-without-slipping condition, since the friction force f can adjust to whatever value it needs to be in order to maintain the condition. Since the friction force is limited, however, to its "stiction" magnitude $K_s > K$, it is necessary to postulate some mechanism at $v = 0$ to verify that the force that maintains rolling-without-slipping is less than its maximum value.

This model raises some difficulties for use in simulation. Except possibly as an initial condition, it is unlikely that the slip velocity as obtained by numerical integration will exactly equal "zero", i.e., the smallest magnitude number that can be computed. Thus, as a practical matter you have to replace the condition $v = 0$ with the condition

$$|v| < \epsilon \tag{4}$$

In addition, when (4) is satisfied, you have to compute the acceleration and check whether its magnitude is less than the stiction level.

To avoid the difficulties with the discontinuity at the origin, a "soft" friction model of Coulomb friction, for example,

$$f = K\tanh(cv_s) \tag{5}$$

where c is a large number.

The soft Coulomb friction model shares a difficulty with the adhesion model, namely, that a nonzero friction force requires a nonzero slip velocity. But, unlike the adhesion model, the soft model can produce nearly its maximum force with only a tiny value of slip velocity.

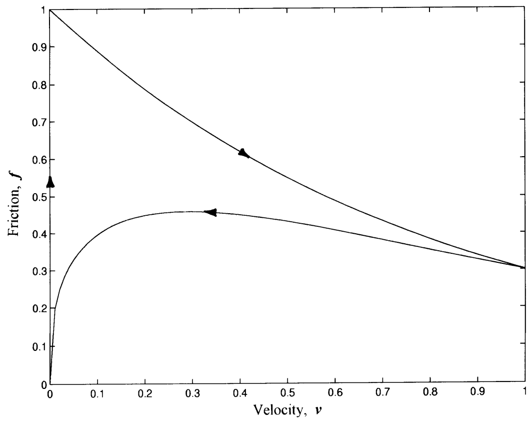

Figure 3: Hysteresis model.

A number of recent experimental investigations support the hypothesis that friction is not a static (*zero-memory*) phenomenon, but rather that it is dynamic in the sense that the present value of the friciton force depends not only on the current (slip) velocity but also on the past history of the process. Various dynamic models have been proposed; our discussion here will be limited to two of these.

2.3 Friction Models with Hysteresis

A friction model that includes hysteresis, such as shown in Figure 3, can be consistent with both the conventional adhesion model and with the assumption of rolling-without-friction. In a situation in which the slip velocity is decreasing, the friction force follows the lower curve, and hence can have a peak at a slip velocity not close to zero. But when the vehicle is accelerating, starring from rest, friction follows the upper curve, and hence can produce a nonzero friction force at zero slip, thus supporting the assumption of rolling without slipping.

The existence of hysteresis in lubricated contacts has been established in numerous experiments. The conditions, if any, under which appears in traction control, in which the velocity of the driven vehicle hovers about zero, remain to be investigated.

Implementation of the hysteresis model requires entails a side calculation of the slip acceleration, usually not a difficult calculation, but which prevents representation of friction as a self-contained "object". A state-variable approximation to hysteresis can avoid this problem and perhaps represent a more faithful representation of the physics of the situation.

2.4 State-Variable Friction Models

The discontinuity at the origin remains for the hysteresis model. Moreover, it may difficult in simulation to calculate the derivative of velocity, as needed to calculate which branch of the hysteresis function to use. These reasons, supported by physical considerations, suggest use of a state variable model:

$$f = \phi(v, z) \tag{6}$$

$$\dot{z} = \gamma(v, z) \tag{7}$$

The state z in (7) is a rapidly changing variable which is intended to capture the behavior in the vicinity of zero velocity. Popular state-space models are the "reset-integrator" model (Haessig and Friedland, 1991) and the so-called "LuGre" model (Canudas de Wit *et al.*, 1995) in both of which z is a scalar.

A state-variable model that produces a hysteresis characteristic such as shown in Figure 3 can be generated by the following dynamic model:

$$f = F(1 + \mathrm{sgn}(z))e^{-c|v|}\mathrm{sgn}(v) \tag{8}$$

$$\dot{z} = -Kz + v \tag{9}$$

3 Case Study

3.1 Equations of Motion

Some of the aspects of the traction control problem can be appreciated by examining the dynamics of the system illustrated in Figure 1. The vehicle as assumed to be driven by one wheel (actually a pair of wheels rigidly attached to a single axle); the other wheel (pair of wheels) is present only to balance the vehicle and is assume to have no significant inertia.

The equations of motion of the system are:

$$\dot{x} = v \tag{10}$$

$$m\dot{v} = \phi(v_s) \tag{11}$$

$$J\dot{\omega} = -r\phi(v_s) + \tau \tag{12}$$

where m is the mass of the vehicle (including the wheels), J is the moment of inertia of the drive wheels (and any other rotating objects connected to them, such as the rotor of the motor), r is the wheel radius, and τ is the drive torque. To simplify matters, assume that the torque can be controlled directly.

The dynamics for rolling without slipping can be obtained by eliminating the adhesion force between (11) and (12):

$$m\dot{v} + \frac{J}{r}\omega = \frac{\tau}{r}$$

Thus, using the condition (1) for rolling without slipping,

$$\left(m + \frac{J}{r^2}\right)\dot{v} = \frac{\tau}{r} \tag{13}$$

replaces (11) and (12).

To investigate the effect of friction, however, we cannot assume rolling without slipping and must consider the third-order system of (10) – (12). Analysis is facilitated by using the slip velocity rather than the angular velocity of the wheel as as state variable. Whence

$$\dot{v}_s = -\frac{1}{M}v_s + u \tag{14}$$

where

$$\frac{1}{M} = \frac{1}{m} + \frac{J}{r^2}, \quad u = \frac{\tau}{J}$$

Using the convention of representing the friction force as the product of the normal force and the "coefficient of friction"

$$\phi(v_s) = F\mu(v_s)$$

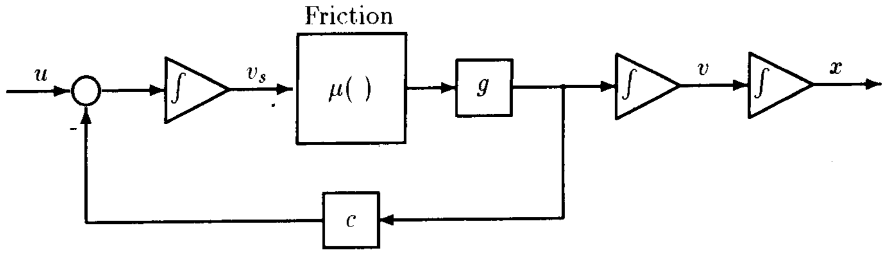

Figure 4: Traction control dynamics.

and recognizing that the normal force in this situation is the weight $w = mg$ of the vehicle, we can express (10) – (12) as

$$\begin{aligned} \dot{x} &= v && (15) \\ \dot{v} &= g\mu(v_s) && (16) \\ \dot{v}_s &= -cg\mu(v_s) + u && (17) \end{aligned}$$

where $\mu(v_s)$ is the coeffiecient of friction rather than the friction force itself, and

$$c = 1 + \frac{mr^2}{J}$$

A block diagram representation of the system is given in Figure 4.

3.2 Full-State Feedback Control Laws

A natural starting point for the design of a control law for the system would be the assumption of rolling without slipping, $v_s = 0$. For this case an appropriate control law for tracking a step position command would be

$$u = G_1(x_r - x) - G_2 v \tag{18}$$

This control law works for step commands that generate control signals that do not require the vehicle to exceed its maximum acceleration capability, i.e., to make the wheels slip. For a sufficently large command signal, however, no choice of control gains G_1 and G_2 in the control law (18) can avoid causing the wheels to slip and consequent vehicle instability.

A describing function analysis of the situation is instructive: The control law of (18) is used for the dynamic system (15)–(17), and the nonlinearity $\mu(v_s)$ is represented by its describing function $N(A, \Omega)$

To determine the possibility of a limit cycle and its character, the describing function is treated as a linear element with a transfer function N. The corresponding characteristic equation of the closed loop system is

$$\Delta(s) = s^3 + cgNs^2 + gNG_2 s + gNG_1 = 0 \tag{19}$$

For a static nonlinearity, the describing function lies on the negative real axis and N is treated as a simple gain. For the characteristic equation (19) the stability requirement is that

$$N(A, \Omega) > \frac{G_1}{cgG_2} \tag{20}$$

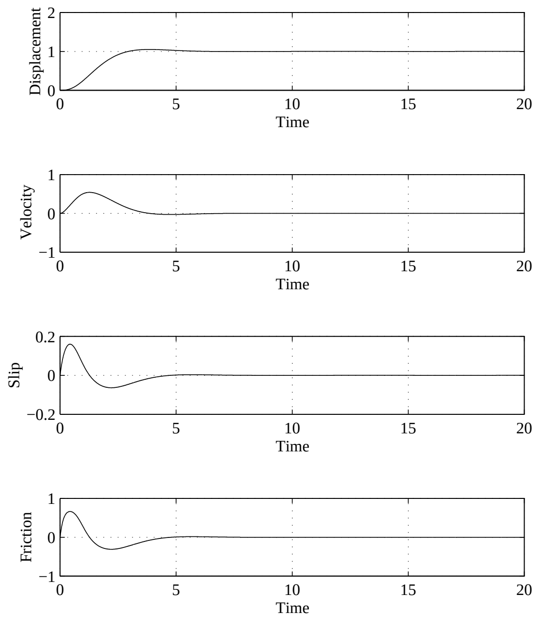

Figure 5: Step response of closed-loop system designed for rolling-without slipping. Small amplitude command

Since the describing function of a nonlinearity, such as friction, that saturates at a finite value, goes to zero as the amplitude approaches ∞, the describing function method predicts that a limit cycle is inevitable if the initial condition or the reference input is sufficiently large. Further describing function analyis predicts that the limit cycle is unstable and occurs at a frequency

$$\Omega = \sqrt{G_1/c} \tag{21}$$

The describing function analysis thus suggests that the rolling-without-slipping assumption cannot safely be used unless it can be assured that large reference signals or initial conditions will not occur in practice.

The validity of the describing function analysis is substantiated by the simulation results shown in Figures 5 and 6 with the following parameters:

$$c = 1., \quad g\mu_{\max} = 1.0, \quad G_1 = 1.0, \quad G_2 = 1.414$$

and a Coulomb friction model. The former shows the behavior of the closed loop control system for a step reference input of 1.0 , well within the region of convergence, and the latter, with a step input command of 3.1 which is slightly too large to be accommodated by the coefficient of friction.

The unsatisfactory behavior of the control system for large errors under the rolling-without-slipping hypothesis has two possible explanations:

1. Failure to account for the possibility of slip.

2. Failure to account for the physical limit on the maximum acceleration magnitude.

To test the first possibility, assume that the slip velocity can be measured and consider the control law

$$u = G_1(x_r - x) - G_2 v - G_3 v_s \tag{22}$$

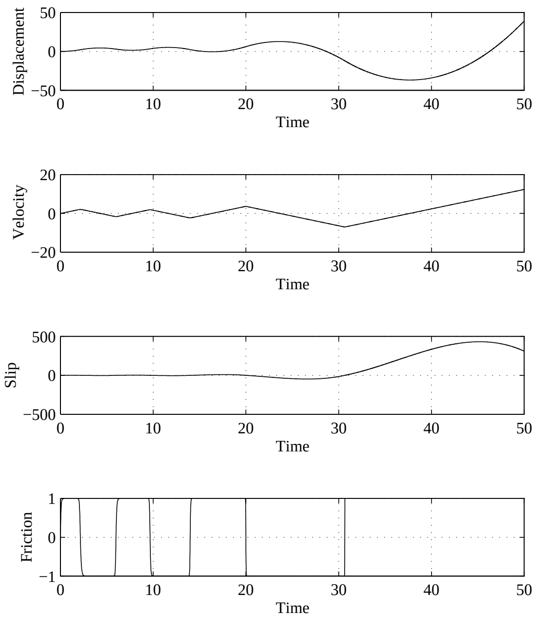

Figure 6: Step response of closed-loop system designed for rolling-without slipping. Large amplitude command

For purposes of describing function analysis the characteristic equation of the closed-loop system is

$$\Delta(s) = s^3 + cgNs^2 + (gNG_2 + G_3)s + gNG_1 = 0$$

and hence the criterion for stability is

$$gNG_2 + G_3 > G_1/c \tag{23}$$

Clearly, if $G_3 > G_1/c$, (23) is satisfied even when the friction force of traction has a describing function that approaches zero.

This result is substantiated by simulation. Figure 7 shows the performance for a step input of 50 (which is many times higher than the value of step input that causes instability without slip velocity feedback), a Coulomb friction model and the control law (22) with:

$$G_1 = 1.0, \quad G_2 = 4.0, \quad G_3 = 25.$$

Although the control torque saturates, the performance is excellent—close to time optimal, in fact.

3.3 Observer Design

The outstanding performance of the control law of the previous section is premised on the measurement of all the state variables. For position control it is reasonable to assume that the vehicle position is measurable. It is also reasonable to assume vehicle velocity is also measurable. Direct measurement of slip velocity, however, does not appear to be practical. (It is practical, but difficult, to measure the torque at the wheel and, using a friction model, infer the slip velocity.) Estimation of slip velocity using a (reduced-order) observer is an alternative to direct measurement of slip-velocity.

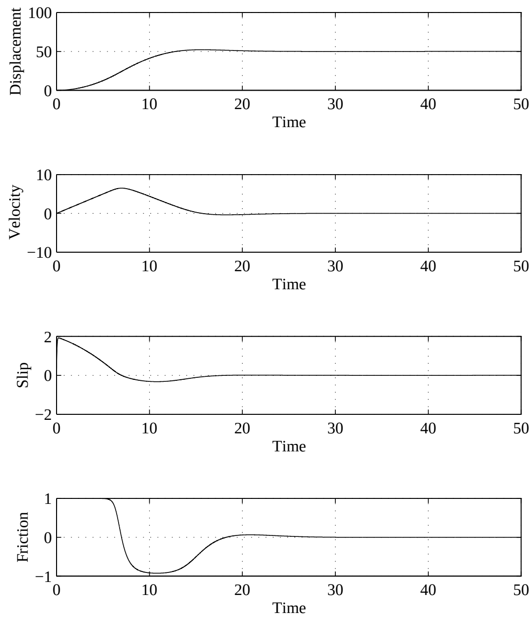

Figure 7: Step response for system design that accounts for slipping.

In accordance with the theory of nonlinear, reduced-order, observers (Friedland, 1995), the dynamics of an observer suitable for the estimation of the slip velocity is given by

$$\hat{v}_s = K_1 x + K_2 v + z \tag{24}$$

$$\dot{z} = -(c + K_2)\mu(\hat{v}_s) - K_1 v + u \tag{25}$$

In a simulation of performance of the control system using the estimated slip velocity v_s instead of the actual slip velocity, the substitution is scarcely noticeable, provided that the friction model used in the observer is matched to the actual friction in the system. As a practical matter, however, it is not possible to model the actual friction at the interface between the wheel and the surface it moves on. Any model used in the observer is at best an approximation to reality.

What is the effect of using an approximate friction model? A simulation was performed in an effort to learn the answer. The simplest case is that in which the shape of the actual friction functions are the same but their amplitudes are different. In this case our simulation revealed the following:

- When the friction is *underestimated* in the observer, i.e., its level is less than the true friction level, performance is quite good. (See Figure 8)
- But when the friciton is *overestimated* in the observer, the system becomes unstable for sufficiently large reference inputs.

This result can be explained as follows: When the friction level is underestimated, the observer adjusts its estimate of the friction to to its maximum value, which is within the capability of the actual friction source to produce. On the other hand, when it is overestimated, the observer operates on the assumption that the friction level is higher than the source can actually produce, and hence the system can become unstable.

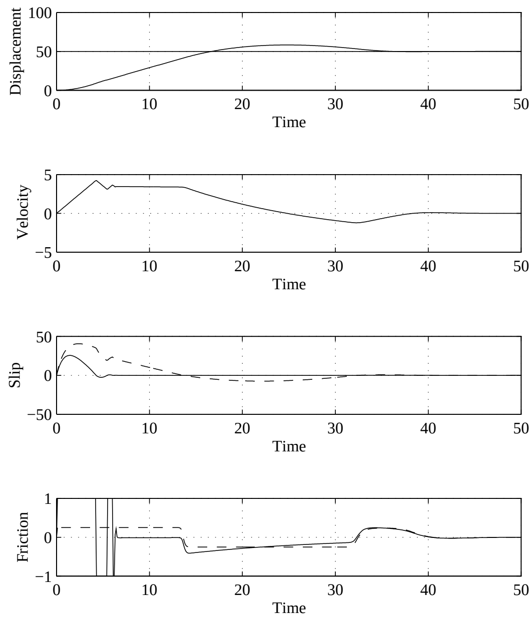

Figure 8: Performance of closed-loop control system with friction coefficient underestimated.

In the two simulations reported above, the slip observer and the "true" dynamic model used the soft Coulomb friction model–only the levels were different. A more realistic test of the capability of the observer is the case in which the observer does not use the true friction model, but an approximation instead. To test the behavior under this more demanding situation, the hysteresis model of (8)–(9) was used in place of the Coulomb model. The results are shown in Figure 9. As the figure shows, the effect on performance is scarecely noticeable, demonstrating that satisfactory performance can be achieved even when the friction model used in the observer is not accurately matched to the true situation.

4 Conclusions

Friction as the driving force in traction control systems makes the process dynamics highly nonlinear. Failure to account for the nonlinear effects can result in an unstable control system. On the other hand, a satisfactory control system can be designed if the friction nonlinearities are taken into account in the design.

The investigation suggests that the following considerations are important:

- Slip velocity must be considered when commands can require friction forces near the maximum that the source can produce.
- If slip velocity cannot be measured, it may be possible to estimate it by means of an observer that incorporates the nonlinearity of friction into its dynamics.
- If the maximum level of friction is not known, the observer should be based on an *under-estimate* of this level.
- The model of friction used in the observer need not be accurately matched to the true friction in the system.

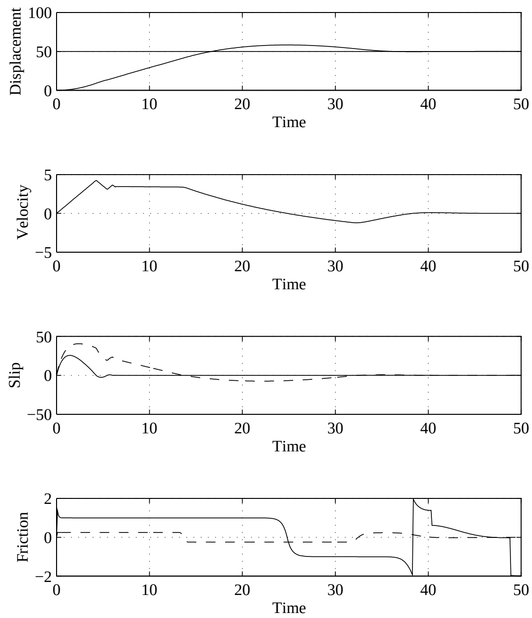

Figure 9: Performance of closed-loop control system with friction coefficient not matched to “true” model.

Although these factors are important, they are not difficult to take into account in a practical system design. Consequently, precise position control using traction as the driving force appears to be readily attainable, at least for a relatively simple plant that was the subject of the case study reported here. Whether observing these considerations is sufficient in the design of control systems for more complex plant dynamics, however, remains an open question.

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