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Decentralized adaptive controller with zero residual tracking errors

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Abstract

In this paper we develop an unified approach to the solution of the adaptive decentralized tracking problem. First we propose a decentralized information setup of control with reference model coordination, which allows to use coordinating information about reference signals of the other subsystems in all local control laws. This setup guarantees zero residual tracking errors for unmodeled interconnections and the local dynamics. We proposed a modified local adaptive control scheme with an additional delayed signal,which improves the transient performance. We use our new setup for the decentralized adaptive control of hybrid systems in which the control parameters are updated at discrete instants.

Keywords: Adaptive decentralized control, large-scale systems, model coordination

1 Introduction

In recent years there has been considerable interest to the study of decentralized adaptive control of large-scale multivariable systems. A variety of decentralized adaptive techniques have been developed (P.A.Ioannou and P.Kokotovich, 1983; B.M.Mirkin, 1986; D.T.Gavel and D.D.Siljak, 1989; R.Ortega and A.Herrera, 1993; A.Datta, 1993; Mirkin, 1995).

A specific class of this technique is the model reference adaptive control (MRAC). However the essential disadvantage of the known model reference adaptive decentralized control laws is that the local tracking errors converge only to a bounded residual set.

However *the best it can achieve in the known model reference adaptive decentralized control laws is the convergence of errors to some bounded residual set.* Besides the bounds of this set are unknown apriori and the size depends upon the bound for the strength of the unmodeled interconnections, so such adaptive schemes may be unsuitable for applications and one needs to develop new methods which would allow to avoid this disadvantage.

In this paper we develop an unified approach to the solution od the adaptive informationally decentralized tracking problem, based on the basic idea contained in (Mirkin, 1995).

There are several contributions in this paper. First we propose *a decentralized information setup of control with reference model coordination*, which allows to use coordinating information about reference signals of the other subsystems in all local control laws. This setup guarantees *zero residual tacking errors* for unmodeled interconnections and the local dynamics.

The second contribution of this work is that we present a modified model reference decentralized adaptive control scheme aimed at improving the performance of adaptive systems. In the scheme *proportional integral time delay (PITD) algorithms updating the parameters in the*

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local adaptive controller is used. This scheme has essentially the same stability properties of coordinated decentralized scheme of control but with the performance improved.

As a third contribution, we use our setup of control for *the decentralized adaptive control of hybrid systems* in which the control parameters are updated at discrete instants.

2 Model of system

The class of large-scale plants with M subsystems and parametric uncertainty that we shall consider in this paper is of the form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + \sum_{j=1}^{M} A_{ij} x_j(t), \\ y_i(t) &= h_i' x_i(t), \quad i = 1, 2, \dots, M, \end{aligned} \tag{1}$$

where for the i-th subsystem $x_i \in \mathbb{R}^{n_i}$ is the state vector $u_i(t) \in \mathbb{R}$ is the input control vector; $y_i(t) \in \mathbb{R}$ is the output vector, the constant matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, are not specified; $\sum_{j=1}^{M} n_i = n$.

The following assumptions are made:

(A1): All subsystems are completely controllable and system (1) is decentralizely stabilizable.

(A2): The matrices of subsystems A_i, B_i and matrices of interaction A_{ij} have the form

$$A_i = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -a_1^i & -a_2^i & \dots & -a_{n_i}^i \end{bmatrix},$$

$$a_i' = [-a_1^i, \dots, -a{n_i}^i], \quad B_i = [0, \dots, b_i],$$

$$A_{ij} = B_i {a_{ij}^*}', \quad {a_{ij}^*}' = [a_1^{ij}, \dots, a_{n_j}^{ij}],$$

(A3): The signs of b_i are assumed to be positive.

The composite system can be written as

$$\begin{aligned} \dot{x}(t) &= A_d x(t) + B_d u(t) + B_d A^* x(t), \\ y(t) &= C_d x(t), \end{aligned} \tag{2}$$

$x_(t) \in \mathbb{R}^{n_1 + \dots + n_M}, u_(t) \in \mathbb{R}^M$, $y(t) \in \mathbb{R}^M$ are the overall state, control and output vector respectively, the matrices $A_d \in \mathbb{R}^{n \times n}$, $B_d \in \mathbb{R}^{n \times M}$ and $C_d \in \mathbb{R}^{n \times M}$ are block diagonal with blocks $A_1, \dots, A_M$, $B_1, \dots, B_M$ and $C_1, \dots, C_M$.

The subscript "d" denotes that matrices are block-diagonal and prime denotes transpose.

The block matrix $B_d A^*$ where

$$A^* \in \mathbb{R}^{M \times n} = \begin{bmatrix} {A'}_{11}^* & \dots & {A'}_{1M}^* \\ \vdots & \ddots & \vdots \\ {A'}_{M1}^* & \dots & {A'}_{MM}^* \end{bmatrix}.$$

represents the interconnection pattern.

3 Problem formulation

Let M reference models are given by

$$\begin{aligned} \dot{x}_{mi}(t) &= A_{mi}x_{mi}(t) + B_{mi}u_{mi}(t), \\ x_{mi}(t_0) &= x_{mi0}, \quad i = 1, \dots, M, \end{aligned} \tag{3}$$

where for the i-th model, $x_{mi}(t) \in \mathbb{R}^{n_i}$ is the state vector, $u_{mi} \in \mathbb{R}$ is the input vector. The matrices A_{mi}, B_{mi} are known constant matrices of appropriate dimensions.

The composite reference model can be written as

$$\dot{x}_m(t) = A_{md}x_m(t) + B_{md}u_m(t), \quad x_m(t_0) = x_{m0},$$

where $x_m(t) \in \mathbb{R}^{n_1+\dots+n_M}$ and $u_m(t) \in \mathbb{R}^M$ are the overall state vector and the input vector respectively and the matrices $A_{md} \in \mathbb{R}^{n\times n}$ and $B_{md} \in \mathbb{R}^{n\times m}$ are block diagonal.

Coordinates for each local model are chosen so that the pairs (A_{mi}, B_{mi}) are in canonical form as in (1).

With this choice of coordinates, it is clear also, that there exists constant unknown vectors A_i^*, A_{ij}^*, b_i^* so that $A_{ij} = B_i A_{ij}^{*}{}'$, $A_i = A_{mi} + B_i A_i'^{*}$ $B_{mi} = B_i b_i^*$.

Then the *control objective* is to design decentralized controllers for system (1) and (2) such that the closed-loop system is stable and the states $x_i(t)$ track the states of M stable local reference models x_{mi} with the conditions

$$\begin{aligned} \lim_{t\to\infty} e_i(t) &= \lim_{t\to\infty} (x_i(t) - x_{mi}(t)) = 0, \\ i &= 1, 2, \dots, M., \end{aligned} \tag{4}$$

We demand that the tracking errors converges to zero asymptotically with time.

4 Decentralized Controller Synthesis

In this section, we first design a state feedback decentralized controller with reference model coordination that ensures the boundendness of all signals and yields zero steady-state tracking error. Second, this same controller is used in the state feedback case with using of combined proportional, integral and time delay adaptive control. The time delay is artificially introduced in order to improve transient response.

4.1 Proposed controller structure

Basically each local controller consist of two loops, i.e. the controller for the i-th subsystem we define as sum of two components

$$u_i(t) = u_{li}(t) + u_{gi}(t). \tag{5}$$

The local component $u_{li}(t)$ is defined as a linear combination of the tracking error vector and the input vector as follows

$$u_{li}(t) = -K_i'\omega_i(t), \tag{6}$$

where $K_i'(t) = [K_{ei}'(t), -K_{umi}(t)]$ is the vector of local adaptive parameters, the vector $\omega_i'(t) = [e_i'(t), u_{mi}(t)]$ is the "regressor" vector, $e_i(t) = x_i(t) - x_{mi}(t)$ is the tracking error.

The global feedforward component $u_{ig}(t)$ is defined as a linear combination of the states of *all reference models* as follows

$$u_{gi}(t) = -\sum_{j=1}^{M} K'_{ij}(t)x_{mj}(t), \tag{7}$$

where $K_{ij}(t) \in \mathbb{R}^{nj}$ are the time-varying adaptation gain vectors.

The main difference from standard DMRAC schemes used in decentralized adaptive control is defined by the global component.

This is the main contribution of our approach. We assume that every local controller uses *the reference trajectories of all subsystems.* Such control law allows to get a result having a new quality connected with *the zero tracking error* under the unknown coefficients in the interconnection matrices. And in this way the totally decentralized structure of the current information update is saved.

The proposed setup with reference model coordination for decentralized model reference adaptive control uses decentralized feedback but centralized feedforward and provides zero tracking errors.

In summary, the total control input $u_i(t)$ to the i-th local system is

$$u_i(t) = u_{li}(t) + u_{gi}(t) = -K'_i\omega_i(t) - \sum_{j=1}^{M} K'_{ij}(t)x_{mj}(t). \tag{8}$$

4.2 Error model.

Following similar steps (B.M.Mirkin, 1986) we can write the error of the system as

$$\begin{aligned} \dot{e}_i(t) &= A_{mi}e_i(t) + B_i\sum_{j=1}^{M} {A'_{ij}}^{*}e_j(t) + \\ &\quad +B_i\Delta K'_i(t)\omega_i(t) + B_i\sum_{j=1}^{M}\Delta K'_{ij}(t)x_{mj}(t), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Delta K'_i(t) &= \left[(A_i^* - K_{ei}(t))', \quad (K_{umi}(t) - b_i^*)\right], \\ \Delta K_{ij}(t) &= \left[A_{ij}^{**} - K_{ij}(t)\right], \\ A_{ij}^{**} &= \begin{cases} A_{ij}^*, & \text{if } i \neq j, \\ A_{ii}^* + A_i^*, & \text{if } i = j. \end{cases} \end{aligned}$$

The composite system error can be written as

$$\dot{e}(t) = A_{md}e(t) + B_dA^*e(t) + B_d\Delta K_d(t)\omega(t) + B_d\Delta K(t)x_m(t), \tag{10}$$

where the block vectors $e(t), \omega(t)$ and matrices $A^*, A^{**}, \Delta K_d(t), \Delta K(t)$, have the form

$$\begin{aligned}
e'(t) &= [e'_1(t), \dots, e'_M(t)], \\
\omega'(t) &= [\omega'_1(t), \dots, \omega'_M(t)], \\
\Delta K_d(t) &= bdiag[\Delta K'_1, \dots, \Delta K'_M], \\
\Delta K(t) &= A^{**} - K(t), \\
A^* &= \begin{bmatrix} A'^*_{11} & \dots & A'^*_{1M} \\ \vdots & \ddots & \vdots \\ A'^*_{M1} & \dots & A'^*_{MM} \end{bmatrix}, \\
A^{**} &= \begin{bmatrix} A'^{**}_{11} & \dots & A'^{**}_{1M} \\ \vdots & \ddots & \vdots \\ A'^{**}_{M1} & \dots & A'^{**}_{MM} \end{bmatrix}, \\
K(t) &= \begin{bmatrix} K'_{11}(t) & \dots & K'_{1M}(t) \\ \vdots & \ddots & \vdots \\ K'_{M1}(t) & \dots & K'_{MM}(t) \end{bmatrix},
\end{aligned}$$

and other notations were introduced earlier.

The decision of the formulated problem is given with the below theorems.

4.3 Integral adaptation algorithms

We denote the solutions of the system (10) by $(e, \Delta K_d, \Delta)K(t)$ and prove the following theorem.

Theorem 1. *Consider the closed-loop system consists of a plant described by (1) and (2), controllers with control law given by (8). Under assumptions (A1)-(A3), all the signals in the system are bounded and the tracking errors $e_i(t) \to 0$ as $t \to \infty$ $(i = 1, \dots, M)$, if we choose the local adaptive laws as*

$$\begin{aligned}
\dot{K}_i(t) &= \Gamma_{1i}[B'_{mi}P_ie_i(t)]\omega_i(t), \\
\dot{K}_{ij}(t) &= \Gamma_{2i}[B'_{mi}P_ie_i(t)]x_{mj}(t), \\
i, j &= 1, \dots, M, \qquad (11)
\end{aligned}$$

where $\Gamma_{1i} = \Gamma'_{1i} > 0$, $\Gamma_{2i} = \Gamma'_{2i} > 0$ are constant matrices, and Pi is a positive definite solution for the following Lyapunov matrix equation

$$A'_{mi}P_i + P_iA_{mi} + Q_{0i} = 0, \quad i = 1, \dots, M, \qquad (12)$$

where $Q_{0i} = Q'_{0i} > 0$.

Proof: Consider a positive definite function V as Lyapunov function

$$V = \sum_{j=1}^{M} V_i,$$

$$\begin{aligned}
V_i &= e'_i\hat{P}_ie_i + (\Delta K_i - \bar{K}_{1i})'\Gamma_{1i}^{-1}(\Delta K_i - \bar{K}_{1i}) \\
&\quad - \sum_{j=1}^{M} \Delta K'_{ij}(t)\Gamma_{2i}^{-1}\Delta K_{ij}(t), \qquad (13)
\end{aligned}$$

where $\Gamma_{1i} = \Gamma'_{1i} > 0, \quad \Gamma_{2i} = \Gamma'_{2i} > 0$ the weighting matrices, $\hat{P}_i = b_i^* P_i$.

The variables $\bar{K}'_{1i} = [K'_{1i} \quad 0]$ are introduced as in (D.T.Gavel and D.D.Siljak, 1989) and they will be defined later.

The total time derivative of the function V_i which is computed with respect to (9) , is obtained as

$$\begin{aligned}
\dot{V}_i &= -e'_i(t) b_i^* Q_{0i} e_i(t) - 2\Delta\dot{K}_i(t)\Gamma_{1i}^{-1}\bar{K}_{1i} + \\
&\quad +2e'_i(t)P_i B_{mi}\Delta K'_i \omega_i(t) + 2\Delta K'_i(t)\Gamma_i^{-1}\Delta\dot{K}_i(t) \\
&\quad + \sum_{j=1}^{M} [e'_j(t) A'^{**}_{ij} B'_{mi} P_i e_i(t) + e'_i(t) P_i B_{mi} A^{**}_{ij} e_j(t)] \\
&\quad + \sum_{j=1}^{M} [x'_{mj}(t)\Delta K_{ij} B'_{mi} P_i e_i(t) + e'_i(t) P_i B_{mi} \Delta K'_{ij}(t) x_{mj}(t)].
\end{aligned} \tag{14}$$

Let K_{1i} be defined as

$$K_{1i} = -r_0 B'_{mi} P_i, \tag{15}$$

where the numbers r_0 is assumed positive. Substituting (15) and (11) in (14) yields

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{M} \dot{V}_i \\
&= e'(t)[A^* B'_{md} P_d + P_d B_{md} A'^* \\
&\quad -r_0 P_d B_{md} B'_{md} P_d - \hat{Q}_{0d}] e(t),
\end{aligned} \tag{16}$$

where the block diagonal matrices $P_d, \hat{Q}_{0d}$ have the form

$$\begin{aligned}
P_d &= bdiag[P_1, \ldots, P_M], \\
\hat{Q}_{0d} &= bdiag[b_1^* Q_{01}, \ldots, b_M^* Q_{0M}].
\end{aligned}$$

After completing the squares in (16) and dropping negative terms, we obtain

$$\dot{V} \leq -[r_0 \lambda_{min}(\hat{Q}_{0d}) - \lambda_{max}(A'^* A')] \|e\|^2, \tag{17}$$

where $\lambda_{min}(.)$ and $\lambda_{max}(.)$ are the minimum and maximum eigenvalues. By selecting a sufficiently large finite value r_0^* so that

$$r_0^* > \lambda min(A'^* A^*) \lambda_{min}^{-1}(\hat{Q}_{0d}), \tag{18}$$

we get $\dot{V} \leq 0$.

Using standard arguments from the Lyapunov theory (Narendra and Annaswamy, 1989), we conclude that the solutions $(e, \Delta K_d, \Delta K)(t)$ are bounded and $e(t) \to 0$ as $t \to \infty$ and the proof is complete.

4.4 PITD adaptation algorithms

In this subsection we seek ways of improving transient performance. We propose a modified local adaptive control scheme which improves the transient performance. The local control laws is kept the same as the one used before, whereas the adaptive algorithm used for updating gains is modified with an additional delayed signal.

In nonadaptive control design some authors (for example, (I.H.Suh and Z.Bien, 1980)) already have used intentional time delays. These studies have shown that the performance of the controlled system may be improved by the judicious use of time-delay actions. For example, it has been shown that an appropriate controller with time delay performs an averaged derivative action and can thus replace the conventional proportional plus derivative (PD) controller.

An appropriate time-delay action in the centralized adaptive control (B.M.Mirkin, 1991) also may improve the performance on the controlled system.

Below we present a local adaptive controller with time delays for decentralized adaptive systems, which guarantees stability and performance improvements.

Theorem 2. *Consider the closed-loop system consists of a plant described by (1) and (2), controllers with control law given by (8). Under assumptions (A1)-(A3), all the signals in the system are bounded and the tracking errors $e_i(t) \to 0$ as $t \to \infty$ $(i = 1, \dots, M)$, if we choose the local adaptive laws as*

$$\begin{aligned}
\dot{K}_i(t) &= \Gamma_{1i}[B'_{mi}P_ie_i(t)]\omega_i(t) + \\
&\quad +\Gamma_{1i}\frac{d}{dt}[B'_{mi}P_ie_i(t)]\omega_i(t)] + \Gamma_{1i}\frac{d}{dt}[B'_{mi}P_ie_i(t-\tau_i)]\omega_i(t-\tau_i)], \\
\dot{K}_{ij}(t) &= \Gamma_{2i}[B'_{mi}P_ie_i(t)]x_{mj}(t) + \\
&\quad +\Gamma_{2i}\frac{d}{dt}[B'_{mi}P_ie_i(t)]x_{mj}(t)] + \Gamma_{2i}\frac{d}{dt}[B'_{mi}P_ie_i(t-\tau_{ij})]x_{mj}(t-\tau_{ij})], \\
i,j &= 1, \dots, M.
\end{aligned} \tag{19}$$

or

$$\begin{aligned}
K_i(t) &= K_i(0) + \Gamma_{1i}[B'_{mi}P_ie_i(t)]\omega_i(t) \\
&\quad +\Gamma_{1i}\int_0^t [B'_{mi}P_ie_i(t)]\omega_i(t)dt + \Gamma_{1i}[B'_{mi}P_ie_i(t-\tau_i)]\omega_i(t-\tau_i), \\
K_{ij}(t) &= K_{ij}(0) + \hat{\Gamma}_{2i}[B'_{mi}P_ie_i(t)]x_{mj}(t) \\
&\quad +\Gamma_{2i}\int_0^t [B'_{mi}P_ie_i(t)]x_{mj}(t)dt + \Gamma_{2i}[B'_{mi}P_ie_i(t-\tau_{ij})]x_{mj}(t-\tau_{ij}), \\
i,j &= 1, \dots, M.
\end{aligned} \tag{20}$$

Proof: Consider the positive definite function V as Lyapunov function

$$V = \sum_{i=1}^{M} V_i, \ (V_i = V_{1i} + V_{2i} + V_{3i}), \tag{21}$$

where

$$
\begin{aligned}
V_{1i} &= e_i' \hat{P}_i e_i, \\
V_{2i} &= (\Delta K_i(t) - \bar{K}_{1i} - K_{0i}(t) - K_{0i}(t-\tau))' \Gamma_i^{-1} (\Delta K_i(t) - \bar{K}_{1i} - K_{0i}(t) - K_{0i}(t-\tau_i)), \\
&\quad + \int_{t-\tau_i}^{t} K_{0i}'(s) \Gamma_i^{-1} K_{0i}(s) ds \\
V_{3i} &= \sum_{j=1}^{M} (\Delta K_{ij}(t) - K_{0ij}(t) - K_{0ij}(t-\tau_{ij}))' \Gamma_{2i}^{-1} (\Delta K_{ij}(t) - K_{0ij}(t) - K_{0ij}(t-\tau_{ij})) \\
&\quad \sum_{j=1}^{M} \int_{t-\tau_{ij}}^{t} K_{0ij}'(s) \Gamma_{2i}^{-1} K_{0ij}(s) ds.
\end{aligned} \tag{22}
$$

Taking the time derivative of (21) along (9) gives

$$
\begin{aligned}
\dot{V}_{1i} &= e_i'(A_{mi}' \hat{P}_i + \hat{P}_i A_{mi} + b_i^* Q_{0i}) e_i - e_i'(t) b_i^* Q_{0i} e_i(t) \\
&\quad + 2 e_i'(t) P_i B_{mi} \Delta K_i' \omega_i(t) \\
&\quad + \sum_{j=1}^{M} [e_j'(t) A'^{**}_{ij} B_{mi}' P_i e_i(t) + e_i'(t) P_i B_{mi} A_{ij}^{**} e_j(t)] \\
&\quad + \sum_{j=1}^{M} [x_{mj}'(t) \Delta K_{ij} B_{mi}' P_i e_i(t) + e_i'(t) P_i B_{mi} \Delta K_{ij}'(t) x_{mj}(t)].
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
\dot{V}_{2i} &= +2 \Delta K_i'(t) \Gamma_i^{-1} \Delta \dot{K}_i(t) - \dot{K}_{0i}(t) - \dot{K}_{0i}(t-\tau_i)) \\
&\quad -2[K_{0i}(t) + K_{0i}(t-\tau_i)] \Gamma_i^{-1} (\Delta \dot{K}_i(t) - \dot{K}_{0i}(t) - \dot{K}_{0i}(t-\tau_i)) \\
&\quad -2 \bar{K}'_{1i} \Gamma_i^{-1} (\Delta \dot{K}_i(t) - \dot{K}_{0i}(t) - \dot{K}_{0i}(t-\tau_i)) \\
&\quad + K_{0i}'(t) \Gamma_i^{-1} K_{0i}(t) - K_{0i}'(t-\tau_i) \Gamma_i^{-1} K_{0i}(t-\tau_i).
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\dot{V}_{3i} &= +2 \sum_{j=1}^{M} [\Delta K_{ij}'(t) \Gamma_{2i}^{-1} [\Delta \dot{K}_{ij}(t) - \dot{K}_{0ij}(t) - \dot{K}_{0ij}(t-\tau_{ij})] \\
&\quad -2 \sum_{j=1}^{M} [(\dot{K}_{0ij}(t) + \dot{K}_{0ij}(t-\tau_{ij})) \Gamma_{2i}^{-1} (\Delta \dot{K}_{ij}(t) - \dot{K}_{0ij}(t) - \dot{K}_{0ij}(t-\tau_{ij}))] \\
&\quad + \sum_{j=1}^{M} [K_{0ij}'(t) \Gamma_{ij}^{-1} K_{0ij}(t) - K_{0ij}'(t-\tau_{ij}) \Gamma_{2i}^{-1} K_{0ij}(t-\tau_{ij})].
\end{aligned} \tag{25}
$$

Let matrices $K_{0i}(t)$ and $K_{0ij}(t)$ be defined as

$$
\begin{aligned}
K_{0i}(t) &= -\hat{\Gamma}_{1i} [B_{mi}' P_i e_i(t)] \omega_i(t), \\
K_{0ij}(t) &= -\hat{\Gamma}_{2i} [B_{mi}' P_i e_i(t)] x_{mj}(t),
\end{aligned} \tag{26}
$$

Substituting $K_{0i}(t)$ from (26) into (24) and using (20) and (12), it can be shown after lengthy manipulation that

$$
\begin{aligned}
\dot{V}_{2i} &= -2 \bar{K}'_{1i} \Gamma_i^{-1} K_{0i}(t) - 2 K_{0i}'(t) \Gamma_i^{-1} K_{0i}(t) - 2 K_{0i}'(t-\tau_i) \Gamma_i^{-1} K_{0i}(t) \\
&\quad + K_{0i}'(t) \Gamma_i^{-1} K_{0i}(t) - K_{0i}'(t-\tau_i) \Gamma_i^{-1} K_{0i}(t-\tau_i) \\
&= -2 \bar{K}'_{1i} \Gamma_i^{-1} K_{0i}(t) - (K_{0i}(t) - K_{0i}(t-\tau_i))' \Gamma_i^{-1} (K_{0i}(t) - K_{0i}(t-\tau_i))
\end{aligned} \tag{27}
$$

Now using ${K'}_{1i}$ as given by (15)

$$\bar{K'}_{1i} = [{K'}_{1i}, 0], \quad {K'}_{1i} = -r_{0i} B'_{mi} P_i \tag{28}$$

we obtain

$$\dot{V}_{2i} \;=\; -2r_{0i} e'_i P_i B_{mi} B'_{mi} P_i e_i - (K_{0i}(t) - K_{0i}(t-\tau_i))' \Gamma_i^{-1} (K_{0i}(t) - K_{0i}(t-\tau_i)) \tag{29}$$

Similarly it can be shown that

$$\begin{aligned} \dot{V}_{3i} &= -2K'_{0ij}(t)\Gamma_{2i}^{-1}K_{0ij}(t) - 2K'_{0ij}(t-\tau_{ij})\Gamma_{2i}^{-1}K_{0ij}(t) \\ &\quad + K'_{0ij}(t)\Gamma_i^{-1}K_{0ij}(t) - K'_{0ij}(t-\tau_{ij})\Gamma_{2i}^{-1}K_{0ij}(t-\tau_{ij}) \\ &= -(K_{0ij}(t) - K_{0ij}(t-\tau_{ij}))'\Gamma_{2i}^{-1}(K_{0ij}(t) - K_{0ij}(t-\tau_{ij})). \end{aligned} \tag{30}$$

From (23), (27) and (30), it follows that

$$\begin{aligned} \dot{V}_i &= -e'_i(t) b_i^* Q_{0i} e_i(t) \\ &\quad + \sum_{j=1}^{M} [e'_j(t) {A'}_{ij}^{**} B'_{mi} P_i e_i(t) + e'_i(t) P_i B_{mi} A_{ij}^{**} e_j(t) - 2r_{0i} e'_i(t) P_i B_{mi} B'_{mi} P_i e_i(t)] \\ &\quad - (K_{0i}(t) - K_{0i}(t-\tau_i))'\Gamma_i^{-1}(K_{0i}(t) - K_{0i}(t-\tau_i)) \\ &\quad - (K_{0ij}(t) - K_{0ij}(t-\tau_{ij}))'\Gamma_{2i}^{-1}(K_{0ij}(t) - K_{0ij}(t-\tau_{ij})) \end{aligned} \tag{31}$$

Substituting (31) in (21) yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{M} \dot{V}_i \\ &\leq e'(t)[A^* B'_{md} P_d + P_d B_{md} A'^{*} - r_0 P_d B_{md} B'_{md} P_d - \hat{Q}_{0d}] e(t). \end{aligned} \tag{32}$$

Further all as in the theorem 1.

5 Hybrid adaptive problem

In this section we extend our information scheme for the case of hybrid systems.

5.1 Controller structure.

For the hybrid adaptive control problem we use an identical structure for the controller as early but only adjust the controller parameters at discrete instants t_s, $t_s = s\delta t \;\; (s = 1, 2, \dots)$ and δt is the sampling interval.

Each local controller now we define as sum of three components

$$u_i(t) = u_{il}(t) + u_{ig}(t) + v_i(t). \tag{33}$$

The local component $u_{il}(t)$ is defined as a linear combination of the tracking error vector and the input vector as follows

$$u_{il}(t) = -{K_i}'[t_s]\omega_i(t), \tag{34}$$

where the vector $\omega_i'(t) = [e_i'(t), u_{mi}(t)]$ is the "regressor" vector, $e_i(t) = x_i(t) - x_{mi}(t)$ is the tracking error and the vector $K_i'[t_s] = [K_{ei}'[t_s], -K_{umi}[t_s]]'$ is the adaptively adjusted feedback gain vector of local parameters, which is constant over each sampling interval $[t_s, t_{s+1})$, but may change from one interval to another. The global feedforward component $u_{ig}[t_s]$ is defined as a linear combination of the states of all reference models as follows

$$u_i(t) = -\sum_{j=1}^{M} K_{ij}'[t_s] x_{mj}(t) \tag{35}$$

where $K_{ij}[t_s] \in \mathbb{R}^{nj}$ is the time-varying adaptation gain vectors, which also is constant over each sampling interval $[t_s, t_{s+1})$, but may change from one interval to another.

We also introduce auxiliary local feedback components $v_i(t)$ $i = 1, 2$ and they will be defined later.

Also as early the *control objective* is (4).

5.2 Error model and stability analysis.

Following similar steps as early we can write the error of the system instead of (9) and prove the next theorem.

Theorem 3. *Consider the closed-loop system consists of a plant described by (1) and (2), controllers with control law given by (33). Under assumptions (A1)-(A3), all the signals in the system are bounded and the tracking errors $e_i(t) \to 0$ as $t \to \infty$ $(i = 1, \dots, M)$, if we choose the local adaptive laws as*

$$\begin{aligned} K_i[t_{s+1}] &= K_i[t_s] + \frac{1}{\Delta t}\int_{t_s}^{t_{s+1}} (B_{mi}' P_i e_i)\omega_i \, dt \\ K_{ij}[t_{s+1}] &= K_{ij}[t_s] \\ &\quad + \frac{1}{\Delta t}\int_{t_s}^{t_{s+1}} (B_{mi}' P_i e_i) x_{mj} \, dt \end{aligned} \tag{36}$$

and auxiliary signals $v_i(t)$ as

$$v_i(t) = v_{1i}(t) + v_{2i}(t), \tag{37}$$

where

$$\begin{aligned} v_{1i}(t) &= -\alpha_1 (B_{mi}' P_i e_i)\omega_i'\omega_i, \\ v_{3i}(t) &= -\alpha_2 (B_{mi}' P_i e_i)\sum_{j=1}^{M} x_{mj}' x_{mj}, \end{aligned} \tag{38}$$

and P_i is a positive definite solution of the Lyapunov matrix equation (12).

The proof is based on the Lyapunov theory (K.S.Narendra *et al.*, 1985).

6 Conclusion

In this paper, we have developed coordinated decentralized adaptive controllers for a class of large-scale systems with unknown interconnected strengths.

We presented a modified DMRAC scheme which requires the signals exchange between the different reference models and does not involve the exchange of output signals between the different subsystems. Our scheme can be classified as *the decentralized adaptive control scheme with model coordination.* It can not only guarantee closed-loop stability but can also guarantee *the asymptotical zero of the tracking errors* under uncertainties in subsystems and interconnections.

Since the reference model signals can be easily exchanged between the subsystems this scheme is *feasible.*

We proposed a modified local adaptive control scheme which improves the transient performance. The local control laws is kept the same as the one used before, whereas the adaptive algorithm used for updating gains is modified with an additional delayed signal.

Next we presented a modified model reference decentralized adaptive control scheme for hybrid systems in which the control parameters are adjusted at discrete instants.

Proposing new control laws that sometimes are viewed as 'upgrades' to the existing schemes.

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