AN ANTIWINDUP CONTROL USING μ-SYNTHESIS

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Abstract

This paper deals with a systematic approach to design a controller which, on one hand satisfies objectives of performance and on the other hand, prevents from noxious influence of saturation. These different constraints leads to an augmented plant on which the optimisation uses μ-synthesis and its D-K iteration procedure, judiciously initialised by a pre-scaling.

1 Introduction

Input saturation is the most commonly problem encountered in real systems. Physical limitations impose constraints, for example voltage and currents amplitudes are limited, pumps and compressors have finite throughput capacity. So a nicely designed linear closed loop can exhibit poor performance with apparition of long lasting and badly damped overshoots, the plant can even drop in instability.
Anti-windup controllers are usually based on the following two-step design :
-first design the linear controller $C_T(s)$ without any consideration for input saturation.

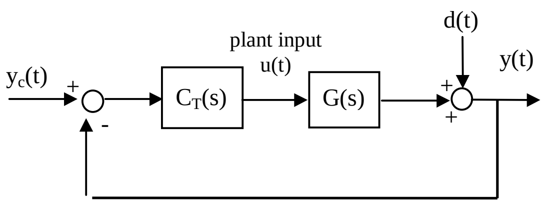

Figure 1:Ideal linear design-error feedback case

- then design the anti-windup system to overcome the effect of the nonlinearity on the behaviour of the closed loop.

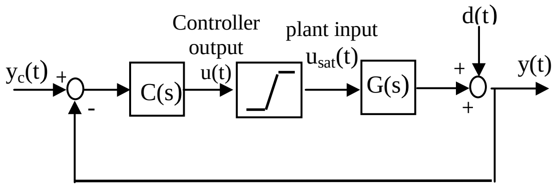

Figure 2: Controlled process with saturation

The anti-windup control (AWC) has to deal with two distinct objectives. It must preclude the emergence of undesirable effects as oscillations, limit cycles or instability when the actual control input reaches the saturation level. In this case, AWC should provide graceful degradation of the closed loop performance .
In linear mode, the saturation behaves like an unit gain , the AWC controller *C(s)* must ensure same performance for the closed loop as with the controller $C_T(s)$ of Fig.1.

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All AWC structures take into account the signal $u_{sat}(t)$ as an auxiliary input to design *C(s)*. However, $u_{sat}(t)$ is generally not measurable so classical approaches introduce a saturation model in the controller to rebuild this signal.

Various methods exist to resolve windup effects. Horowitz (1983) proposes to apply feedback so that the open loop around the saturation model behaves roughly like an integrator with gain k he adjusts . Next, comparison is made between the precedent open loop and the transfer *G(s).C(s).*

Morari and Zafiriou (1989) recommend the use of an internal model control scheme (IMC) with insertion of a model of the saturation at the input of the process model.

Wurmthaler and Hippe (1991) propose an observed state feedback solution. They show that plant input saturation can cause two different types of windup, the controller windup and the plant windup. The controller windup is related to integral action or more complex disturbance signal models incorporated in the controller. Once the controller windup is prevented, if the dynamic of the closed loop is too fast, plant windup shows itself in apparition of nonlinear oscillations or even limit cycles. In this case, provided the plant is stable, plant windup is avoided with a simple phase criterion for the frequency response of the transfer $\beta_{NL}(s)$, transfer function between $u_{sat}(t)$ and -u(t) when the saturation is removed.

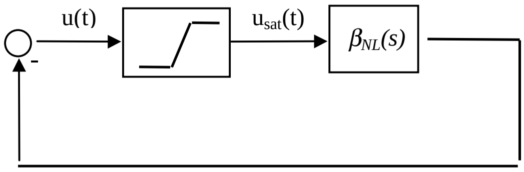

Figure 3: Control loop with isolated nonlinearity

In Ygorra *et al* (1998), we present a systematic method where exposition of the objectives and associated trade-offs is clear. The scheme of fig.4 is considered and an augmented plant based on H_∞ iteration to design the controller *C(s)* is proposed.

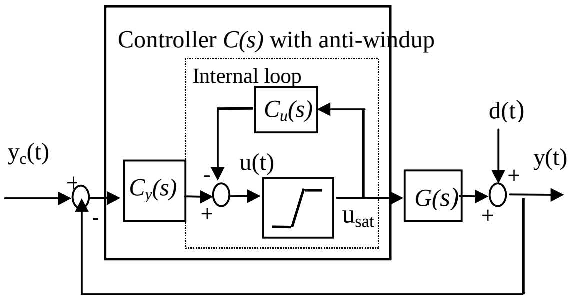

Figure 4: Antiwind-up system

Nevertheless, in this paper, we consider a different approach of the AWC design because it is an only one-step design. An augmented plant ,which simultaneously cope with the constraints of performance in linear case and the necessity to overcome the effects of the saturation on the behaviour of the closed loop, is presented. So the controller “target” $C_T(s)$. is no longer necessary. We will expose the procedure to resolve this multiobjective optimisation using μ-synthesis. In section 3, the example proposed by Doyle et al (1987) is treated, results simulations will be commented and compared.

Notation

R is the set of real numbers. For a matrix *A*, A^* denotes its complex conjugate transpose, A^T denotes its transpose and A^{-1} denotes its inverse. *A*>*B* (*A*≥*B*) means that *A* and *B* are square hermitian matrices and *A*-*B* is positive (semi-) definite.

The structured singular value of a complex matrix M for a given block-diagonal structure $\overline{\Delta} = blocdiag(\Delta_1, ..., \Delta_n)$ is defined as:

$$\mu_{\overline{\Delta}}(M) = \frac{1}{\min\limits_{\Delta \in \overline{\Delta}} \left(\overline{\sigma}(\Delta) / \det(\mathrm{I} + \mathrm{M}\Delta) = 0\right)}$$

and

$$\mu_{\overline{\Delta}}(M) = 0 \text{ if no } \Delta \text{ in } \overline{\Delta} \text{ makes } \mathrm{I} + \mathrm{M}\Delta \text{ singular.}$$

2 Anti-Windup Control

2.1 Objectives

The basic idea is that the designer has to consider two distinct objectives to achieve his goal .

- In linear mode, when there is no input saturation , the saturation behaves like an unit gain transfer function. So the design of *C(s)* must take account of objectives of performance for a good behaviour of the closed loop.
- When input saturation occurs, *C(s)* must cancel instability effects. The nonlinear loop system, figure 4, must be stable.

An analysis based on describing function, see (Wurmthaler and Hippe, 1991), leads us to maintain a minimal phase distance between $\beta_{NL}(j\omega)$ and the negative inverse amplitude-dependent describing function $N(u_1, u0)$. In the case of a saturation nonlinearity, $N(u_1, u_0)$ is a half straight-line in the Nichols chart, with an argument equal to -180°, and a modulus greater than unity.

The goal is to have a sufficient phase distance between $\beta_{NL}(j\omega)$ and the axis(-180°). The classical Nichols charts will be used because it allows to have a correlation between the open loop $\beta_{NL}(j\omega)$ and the closed loop $\dfrac{\beta_{NL}}{1+\beta_{NL}}$.

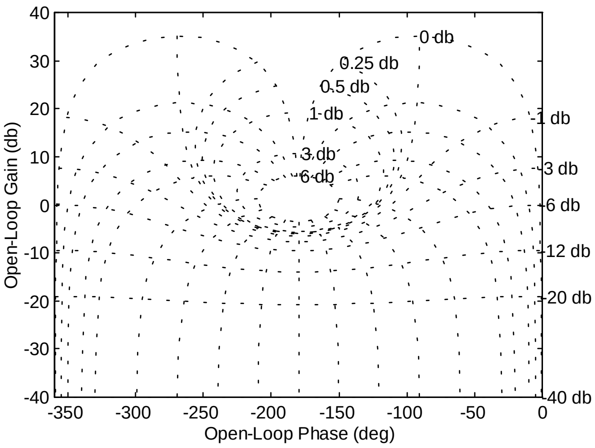

Figure 5: Contours of modulus $\left|\dfrac{z}{1+z}\right| = C^{te}$ are represented in Nichols plane, z is a complex number .

Depending on the constant C^{te} chosen, the open loop $\beta_{NL}(j\omega)$ will tangent the corresponding curve. By an augmented system, a judicious selection of the weighting function leads to have a good phase distance.

2.2 Augmented plant and weighting functions

We propose the following augmented system to design the controller $C := \lfloor C_y, C_u \rfloor$

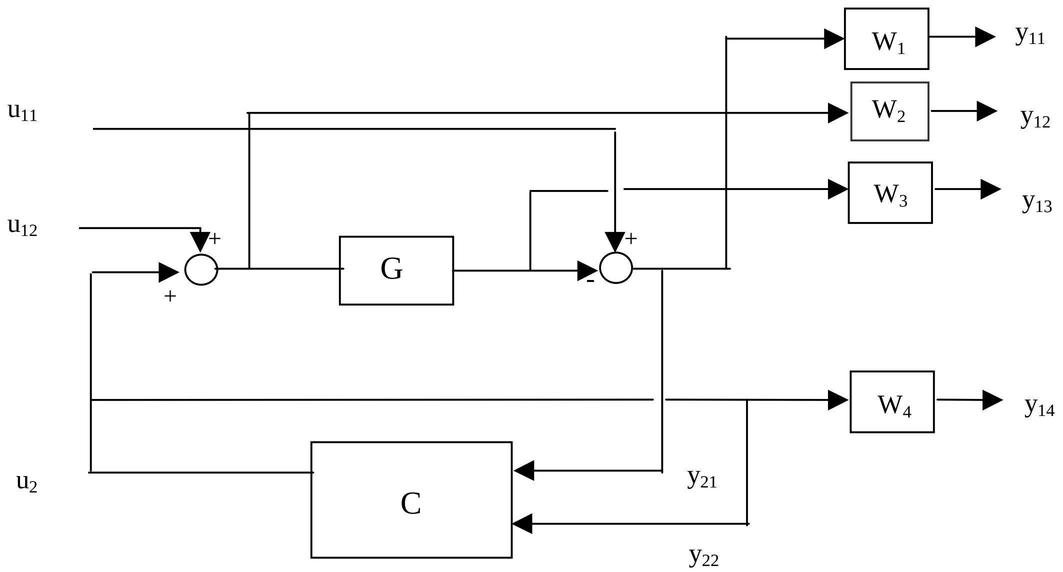

Figure 6: Augmented system

Let us now discuss the methodology to choice the weighting functions. Each weighting function copes with a particular design specification :

•In linear mode, $C = (I + C_u)^{-1} C_y$, so we have the following transfers

$$\frac{y_{11}}{u_{11}} = W_1 (I + GC)^{-1} \qquad 1)$$

$$\frac{y_{12}}{u_{11}} = W_2 (I + GC)^{-1} C \qquad 2)$$

$$\frac{y_{13}}{u_{11}} = W_3 (I + GC)^{-1} GC \qquad 3)$$

W_1, W_2 and W_3 respectively penalise the sensitivity ,the input sensitivity and the complementary sensitivity. These transfer functions deal with the objectives of performance for the closed loop, it is a classical mixed. sensitivity problem, see Kwakernaak (1993) for a good survey on this problem.

- $\frac{y_{14}}{u_{12}} = -W_4 (I + \beta_{NL})^{-1} \beta_{NL}$ 4)

The weighting function W_4 copes with the specification of avoiding the noxious influence of the saturation. Typically, W_4 will be set equal to a value close to 2(6dB) in order to maintain a sufficient phase distance for the open loop $\beta_{NL}(j\omega)$ in the interesting band of frequencies. Beyond, W_4 allows to make the modulus of $\beta_{NL}(j\omega)$ decreasing in high frequencies because it needs. a controller $C_u(s)$ strictly proper so that the internal anti windup loop formed by $C_u(s)$ and the saturation model is realisable. In this case, $C_y(s) = (I + C_u(s))C(s)$ is also realisable.

2.3 μ-synthesis, DK iterations and pre-scaling

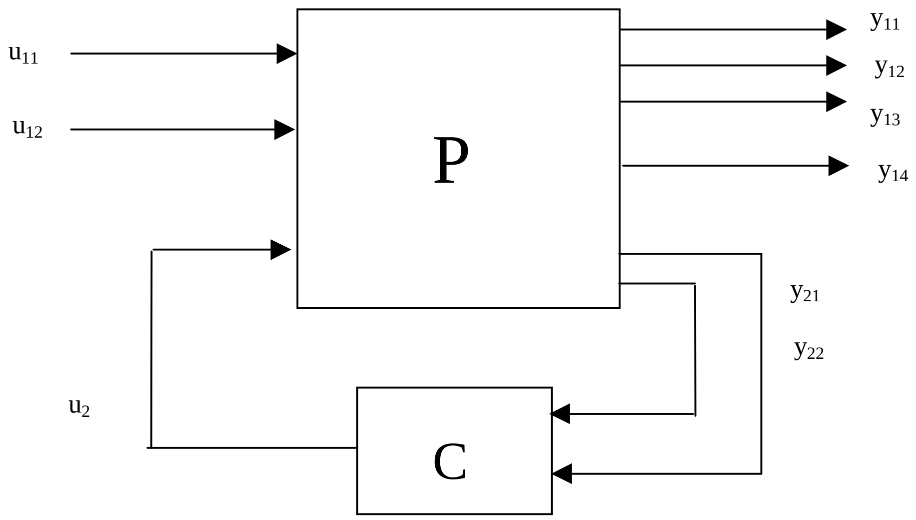

Figure 7: Augmented plant

The problem is a multiobjective problem because we are concerned with two distinct objectives:

$$\left\| \begin{matrix} W_1 T_{y_{11}u_{11}} \\ W_2 T_{y_{12}u_{11}} \\ W_3 T_{y_{13}u_{11}} \end{matrix} \right\|_\infty \leq 1 \qquad 5)$$

and

$$\left\| W_4 T_{Y_{14}} u_{12} \right\| \leq 1 \qquad 6)$$

If T_{ez} represent all the transfers that can be found from exogenous inputs e and weighting outputs z, 8) and 9) are satisfied if

$$\left\| T_{ez} \right\|_\infty \leq 1 \qquad 7)$$

Relation 10) is equivalent to the robust stability condition of the closed loop, including the controller C(s) and a fictitious uncertainty block, whose H_∞ norm is lower than unity, see fig.8.

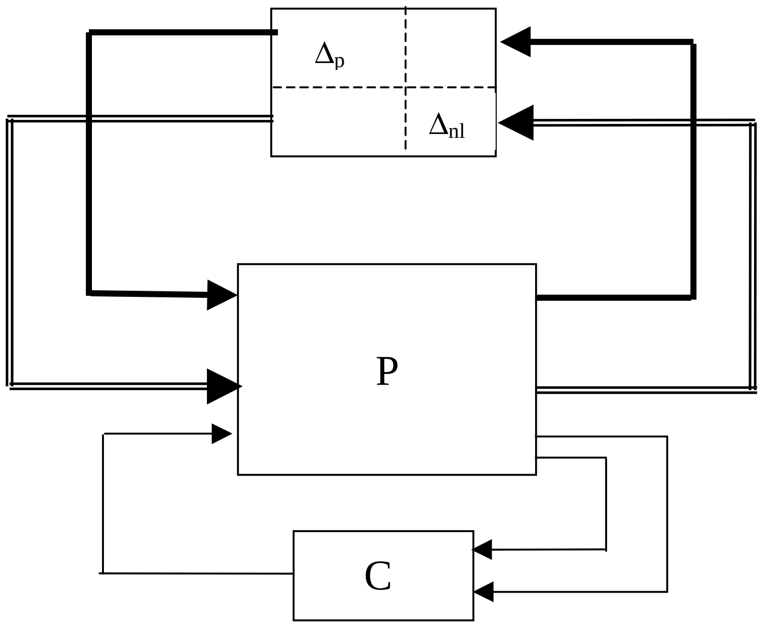

Figure 8: Equivalent closed loop plant

The matrix Δ is block diagonal and has the following structure:

$$\Delta = \left[\begin{array}{ccc|c} \Delta_{y_{11}} & \Delta_{y_{12}} & \Delta_{y_{13}} & 0 \\ \hline 0 & 0 & 0 & \Delta_{y_{14}} \end{array}\right] \qquad 8)$$

Nevertheless, relation 10) can be too conservative so we introduce the following sufficient condition, using the structured singular value (Doyle,1982), to reduce the conservatism:

$$\mu_{\Delta}(T_{ez}) \leq 1 \qquad 9)$$

Our problem is now resulting in a problem of μ-synthesis, and DK iteration procedure of Balas will be used with a little change. In Prempain and Bergeon (1996), it has been shown that a good initialisation can improve the D-K iterations. So we introduce the following pre-scaling.

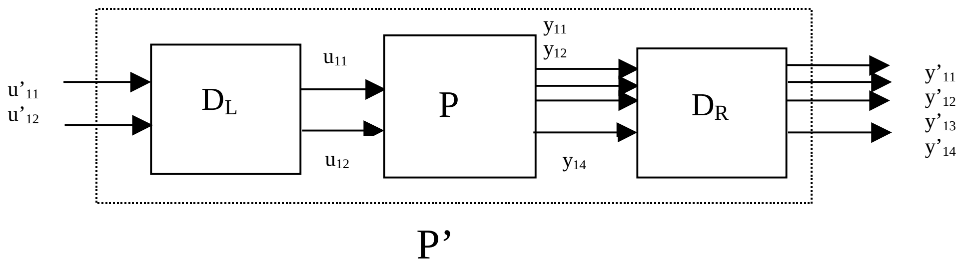

Figure 9: Modified augmented plant

With :

$$D_L = \begin{bmatrix} 1 & \rho_{sat} \end{bmatrix}$$

$$D_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{\rho} \end{bmatrix} \qquad 10)$$

This pre-scaling permits to reinforce the decoupling by minimising the effects of the anti-diagonal transfers. ρ_{sat} is set to a small constant value and increasing ρ leads to underline he importance of the transfers which cope with the non linear objectives. Typically, ρ is a value very close to ρ_{sat}. So the D-K iteration procedure begins with the modified augmented plant P'

3. Application

This example is taken from Doyle *et al.*(1987). It is interesting because it presents a little weird as underlined Doyle when he applied his CAW method.
The plant model *G(s)* and the controller $C_T(s)$ are given by

$$G(s) = 0.2\left\{\left[\frac{s^2+2\zeta_1\omega_1 s+\omega_1^2}{s^2+2\zeta_1\omega_2 s+\omega_2^2}\right]\left[\frac{s^2+2\zeta_2\omega_1 s+\omega_1^2}{s^2+2\zeta_2\omega_2 s+\omega_2^2}\right]\right\}$$

$\omega_1 = 0.2115 \qquad \omega_2 = 0.0473$

$\zeta_1 = 0.3827 \qquad \zeta_2 = 0.9239$

$C_T(s) = \frac{5}{s}$.

The Nichols plot of the open loop $\beta(j\omega) = C_T(j\omega)G(j\omega)$ is depicted

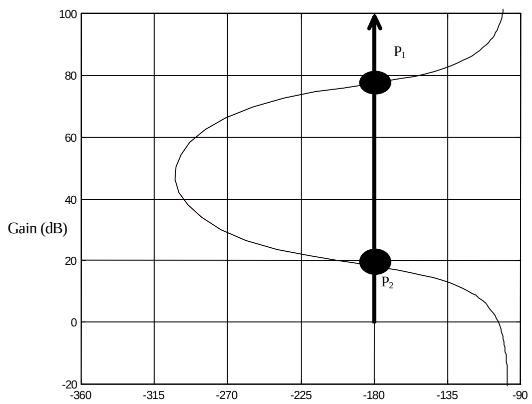

Figure 10: Open loop $C_T(j\omega)G(j\omega)$

There are two intersection points, P_1 and P_2. Only P_1, which corresponds to ω=0.03, rad/s can lead to stable limit cycle according to an describing function analysis.

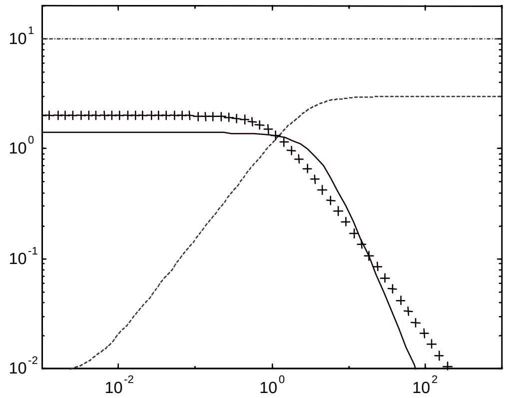

Figure.11: Inverse weighting functions

The weighting functions are chosen as:

$$W_1=\frac{\frac{p}{2}+1}{3\left(\frac{p}{2}+10^{-3}\right)} \qquad W_3=\frac{\left(\frac{p}{5}+1\right)^2}{1.4\left(\frac{p}{10^3}+1\right)^2} \qquad W_2=\frac{1}{10}$$

$$W_4=\frac{p+1}{2\left(\frac{p}{10^3}+1\right)}$$

$\rho_{sat}=10^{-3}=\rho$

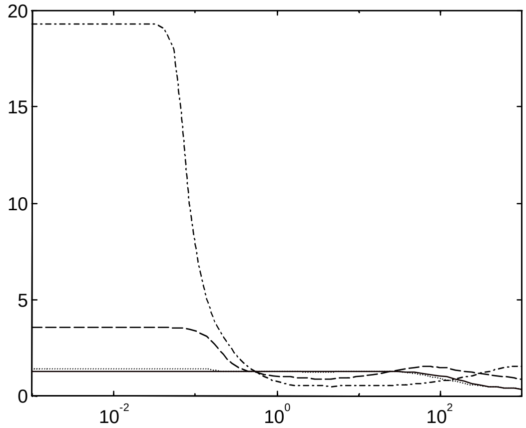

Figure 12: Evolution of $\mu(\omega)$

After the fourth iteration, the curve $\mu(\omega)$ is nearly flat and the greatest value is 1.3 which is satisfying. The order of the controller after reduction is 18.

The open loop obtained with the controller $C:=\lfloor C_y, C_u \rfloor$ designed is depicted .

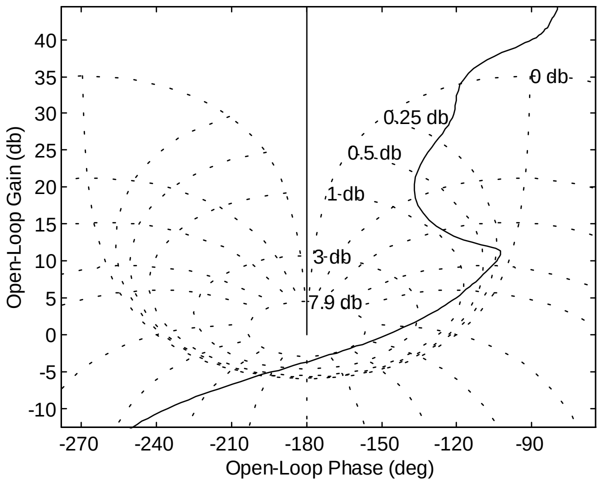

Figure.13:. Open loop $\beta_{NL}(j\omega)$

It can be seen that there is no intersection between $\beta_{NL}(j\omega)$ and $N(u_1, u_0)$. So, reset windup, limit cycles or instability effects have been cancelled.

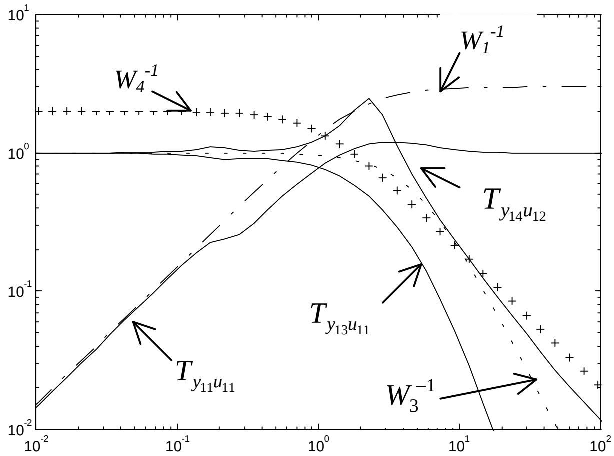

Figure.13: Frequency responses of the controlled plant and inverse weighting functions

Based on the precedent figure, we can see that the controller achieves performance requirements for the linear case.The time response to an output disturbance step of level one, at time t=0 s, is represented

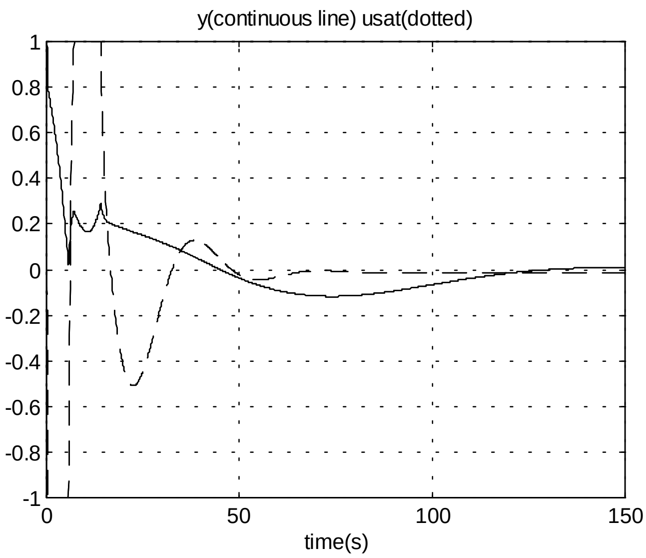

Figure 14: Time response

We obtain response similar to the one obtained by Doyle *et al*.(1987).
In next figure, response to an output disturbance with unit step at time t=0 and a switch to -1 at t=4s is depicted.

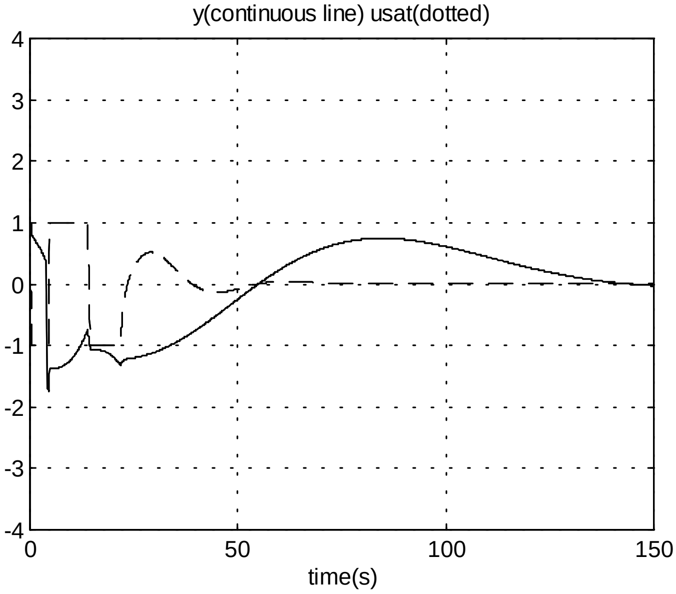

Fig.15. Time response

In this case, no limit cycle emerges as it is the case in Doyle *et al.*(1987) .

CONCLUSION

In this paper, an anti-windup design has been presented. The originality of the method is that it is only one step design, no target controller is needed since the performances for the linear case are also treated during the synthesis. An augmented plant has been proposed and an procedure of pre-scaling is introduced to improve the convergence of the DK iterations. Doyle's example shows that this method is particularly suitable for the plant windup. The method proposed in this paper is systematic, the exposition of the objectives and associated trade-offs is clear. MIMO systems and the LMIs tools are topics of ongoing research on this problem..

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