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REAL AND COMPLEX STABILITY RADII IN AUTOMATIC LOAD-FREQUENCY CONTROL SYSTEMS VIA LQG/LTR AND LMI

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Abstract– In this work, two techniques of robust control (LQG/LTR and LMI), applied to a power electric system, are available via stability radii of the system. The structured uncertainties of the nominal model are considered in both designs. A set of models is generated considering the combinations of the parametric uncertainties. The structured singular values of the both systems are analysed.

Key Words– Robust control; LQG/LTR; LMI; real and complex stability radii; μ-analysis and power systems .

1 - Introduction

In this work, two techniques of robust control , LQG/LTR (*Linear Quadratic Gaussian/Loop Transfer Recovery*) and LMIs (*Linear Matrix Inequalities*) [1], [2], [5], [6], [9], [11], [12], [18], [25], [26], [27] and [28] are available via singular value structured of the system. The control of load-frequency in a two-area model of an electric system are considered. The structured uncertainties of the plant are considered too on the analysis of the real and complex radii.

The interconnection of electric power systems brings advantages from the operation point of view and, among these advantages, one of the most important is the possibility of power exchange in critical periods. In order to make this interconnected operation possible, a rigorous control of the frequency in the entire system, through a process called automatic load-frequency control, is necessary [28].

The controllers designed by the classic methods have been working in a satisfactory way. However, the growth of the load demand has lead the systems to operate frequently close to critical conditions, and more efficient controllers are needed to stabilise the systems at these points of operation.

The main contributions of this work are: analysis of the stability radii, structured singular values of the system (μ-analysis) and the uncertainty matrices that do the system unstable considering two methodologies of control design, LMI and LQG/LTR, in an electrical system. In [28] was done a model of one electrical system with two areas connected and two control systems were designed and compared. Here, they are compared considering the stability robustness of the system taking into account the parameter variations, in specific ranges, of the model. The main question that will be answered is: what are the distances of the instability of both systems?

2 - Power system modelling

The controllers were designed for a system with 5 buses and 2 generators, which can be obtained in [24]. By reducing this system to the constant e.m.f.'s behind the transient reactances of the generator buses, the non-linear dynamic equations that describe its dynamic behaviour are obtained:

$$\dot{\delta} = \dot{\delta}_1 - \dot{\delta}_2 = \omega_1 - \omega_2$$
$$m_1\dot{\omega}_1 = p_{m1} - E_1^2 G_{11} + C_{12}\,\mathrm{sen}\,\delta + D_{12}\cos\delta - d_1\omega_1$$
$$m_2\dot{\omega}_2 = p_{m2} - E_2^2 G_{22} + C_{21}\,\mathrm{sen}\,\delta + D_{21}\cos\delta - d_2\omega_2$$
$$\tau_1\dot{p}_{m1} = p_{ref1} - \frac{1}{r_1}\omega_1 - p_{m1}$$
$$\tau_2\dot{p}_{m2} = p_{ref2} - \frac{1}{r_2}\omega_2 - p_{m2}\,.$$

The data for this model are presented in table 1 (the basis values are 100 MVA and 138 kV).

Table 1.Values of the nominal model parameters.

Parameter	Nominal Value
Inertia constant of generator 1 (m_1)	0.2650 p.u. / rad/s^2
Inertia constant of generator 2 (m_2)	0.0050 p.u. / rad/s^2
Damping of load 1 (d_1)	1.0610 p.u. / rad/s
Damping of load 2 (d_2)	1.3263 p.u. / rad/s
Speed regulations of the generators (r_1,r_2)	0.0400 p.u.
Time constants of the turbines (τ_1 τ_2)	0.3000 s

Considering the linear system equations, the state space model of the nominal plant is obtained. This model is described in section 6, with integrators already introduced to the input. In this plant, the input variables are the reference powers of the speed regulators (Δp_{ref1} and Δp_{ref2}), the outputs are the angular speed of the generator ($\Delta\omega_1$) and the power transfer angle ($\Delta\delta$) and the state variables are the mechanical powers of the generators (Δp_{m1} and Δp_{m2}), the angular speeds ($\Delta\omega_1$ and $\Delta\omega_2$) and the power transfer angle between these generators ($\Delta\delta$). The constant P_{tie} comes from the linearization of the terms associated with the power transfer through the line ($C_{ij}\mathrm{sen}\delta+D_{ij}\cos\delta$). All the variables of the linearized model represent variations around a fixed operation point and, then, the objective of the controller is to keep the speed variations due to load variations and uncertainties in the system model, inside the specified limits.

2.1 - Uncertainty ranges

The variation ranges of the model parameters were obtained from the maximum and minimum values presented in [7] and [8] (see table 2), for damping, speed regulation and time constants of the turbines. For the line power, it was assumed a variation of 10% in the transmitted power, and this range was checked later with load flow simulations. Uncertainties in the inertia constants were not considered.

Table 2. Uncertainties in the nominal model parameters.

Parameter	Minimum	Maximum	Unit
d_1	1.0000	3.0000	p.u. (MVA) / rad/s
d_2	1.0000	3.0000	p.u. (MVA) / rad/s
τ_1,τ_2	0.1000	0.5000	S
p_{tie}	0.4462	0.5454	p.u.
r_1,r_2	0.0394	0.0406	p.u.

3 LQG/LTR controller design

After determining the uncertainties in the model, the post-multiplicative error is calculated, for a range of frequencies from 10^{-4} to 10^{2} rad/s, generating the stability robustness barrier. Then, three performance criteria are defined (where ω_{in} is the reference signal frequency) :

1. Reference signal tracking with maximum error of 1 % for $\omega_{in} \leq 10^{-2}$ rad/s ;

2. Perturbation rejection with maximum error of 1 % for $\omega_{in} \leq 10^{-2}$ rad/s ;

3. Plant variation sensibility inferior to 10 % for $\omega_{in} \leq 10^{-2}$ rad/s.

The Kalman Filter is included for loop shaping and, after that, the recovery procedure is applied, see this procedure in [5], [6], [26], [27] and [28]. The singular values generated by this process (for $\rho = 10^{-12}$) are shown in Fig. 1. The observer and controller gains obtained are

$$G_{lqg} = \begin{bmatrix} 456.9251 & 0.0120 & -15751 & 954800 & 23.1968 & 31317 & 5.3935 \\ 0.0120 & 1492.6 & 166430 & 258.2341 & 517250 & 1.6466 & 334190 \end{bmatrix}$$

and

$$H_{kf} = \begin{bmatrix} 119.9979 & -0.0466 & 0.9917 & 4.9483 & 0.0420 & 8.8539 & -0.4351 \\ 0.0505 & 119.9979 & -0.9917 & 0.0420 & 4.3885 & -0.3892 & 6.3587 \end{bmatrix}^T$$

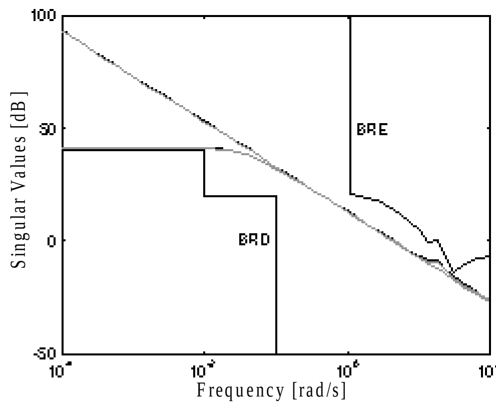


Figure 1. Target loop recovery.

4 - Linear Matrix Inequalities (LMIs) applied to the observer-based controller

The application of Linear Matrix Inequalities in the problem of controlling a linear system subjected to uncertainties is growing considerably in the last years [1, 9, 11, 12, 18]. In this design methodology, the observer-based controller is presented in a LMI structure, with the objective of stabilizing a control system subjected to structured uncertainties by the optimization of LMIs. A more detailed description of this problem can be seen in [1].

Consider the linear system, subjected to uncertainties,

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) \\ y(t) &= (C + \Delta C(t))x(t) \end{aligned} \tag{1}$$

where *x(t)*, *u(t)*, *y(t)*, A, *B* and *C* are the states, inputs, outputs and their respective constant matrices with appropriate dimensions, defined in equation 1.

$$\Delta A(t) = \sum_{i=1}^{p} \alpha_i(t) A_i \text{ , } \Delta B(t) = \sum_{i=1}^{q} \beta_i(t) B_i \text{ ,}$$
$$\Delta C(t) = \sum_{i=1}^{r} \chi_i(t) C_i \text{ .} \tag{2}$$

The scalar functions $\alpha_i(t)$, $\beta_i(t)$ and $\chi_i(t)$ are Lebesgue measurable and

$$|\alpha_i(t)|,\ |\beta_i(t)|,\ |\chi_i(t)| \le 1 \text{ .} \tag{3}$$

A_i, B_i and C_i are matrices with known uncertainties, which are assumed to be constant and to have rank 1, given by

$$A_i = d_i\, o_i' , \qquad B_i = f_i\, g_i' , \qquad C_i = h_i\, j_i' \text{ .} \tag{4}$$

If these matrices do not have unitary rank, it is possible to decompose them in order to obtain a sequence of rank 1 matrices. Scalar v_i and s_i are defined for B_i and Ci, respectively. Constant matrices T, W, S, U, V and Y represent the time-varying uncertainties, which are the upper bound of these uncertainties.

$$\mathrm{T} \stackrel{\Delta}{=} \sum_{i=1}^{p} l_i\, d_i\, d_i' = \mathrm{D}\,\hat{\mathrm{L}}\,\mathrm{D}' \qquad \mathrm{W} \stackrel{\Delta}{=} \sum_{i=1}^{q} v_i\, f_i\, f_i' = \mathrm{F}\,\hat{\mathrm{V}}\,\mathrm{F}'$$
$$\mathrm{S} \stackrel{\Delta}{=} \sum_{i=1}^{r} s_i\, h_i\, h_i' = \mathrm{H}\,\hat{S}\,\mathrm{H}' \qquad \mathrm{U} \stackrel{\Delta}{=} \sum_{i=1}^{p} l_i^{-1} o_i\, o_i' = \mathrm{O}'\,\hat{\mathrm{L}}^{-1}\mathrm{O}$$
$$\mathrm{V} \stackrel{\Delta}{=} \sum_{i=1}^{q} v_i^{-1} g_i\, g_i' = \mathrm{G}'\,\hat{\mathrm{V}}^{-1}\mathrm{G}$$
$$\mathrm{Y} \stackrel{\Delta}{=} \sum_{i=1}^{r} s_i^{-1} j_i\, j_i' = \mathrm{J}'\,\hat{S}^{-1}\mathrm{J} \tag{5}$$

where

$$\mathrm{D} \stackrel{\Delta}{=} [\, d_1 \ \dots\ d_p], \qquad \mathrm{F} \stackrel{\Delta}{=} [\, f_1 \ \dots\ f_q], \quad \mathrm{H} \stackrel{\Delta}{=} [\, h_1 \ \dots\ h_r],$$
$$\mathrm{O} \stackrel{\Delta}{=} [\, o_1 \ \dots\ o_p]', \qquad \mathrm{G} \stackrel{\Delta}{=} [\, g_1 \ \dots\ g_q]', \qquad \mathrm{J} \stackrel{\Delta}{=} [\, j_1 \ \dots\ j_r],$$
$$\hat{\mathrm{L}} \stackrel{\Delta}{=} diag\,(\hat{l}_1 \ \dots\ \hat{l}_p), \qquad \hat{\mathrm{V}} \stackrel{\Delta}{=} diag\,(\hat{v}_1 \ \dots\ \hat{v}_q),$$
$$\hat{S} \stackrel{\Delta}{=} diag\,(\hat{s}_1 \ \dots\ \hat{s}_r). \tag{6}$$

Consider the state observer with the form

$$\dot{z}(t) = Az(t) - Bu(t) - L_{lmi}(Cz(t) - y(t)) \tag{7}$$

where $z(t) \in R^n$ is the state observer, L_{lmi} (n×q) is the gain matrix of the observer, $u(t) \in R^m$ is the input signal defined by $u(t) = -K_{lmi}\ z(t)$ and K_{lmi} (m×n) is the state feedback gain matrix. The stability of the system can be analyzed looking at the dynamics of the error $e(t) \stackrel{\Delta}{=} x(t) - z(t)$ and of the states, respectively given by the following system of equations :

$$\dot{x}(t) = [A + \Delta A - (B + \Delta B)K_{lmi}]x(t) + (B + \Delta B)K_{lmi}e(t)$$
$$\dot{e}(t) = (\Delta A - \Delta BK_{lmi} - L_{lmi}\Delta C)x(t) + (A - L_{lmi}C + \Delta BK_{lmi})e(t) \tag{8}$$

The quadratic Lyapunov function $V(x,e) = x'P_c x + e'P_o e$ is used to verify asymptotic stability for the system of eq. (8). P_c and P_o are (nxn) positive definite matrices.

Definition [9]: The system of eq. (1) is asymptotically stable if there exists a constant $\alpha \in R$ such that the derivative of the Lyapunov function $V(x,e)$, related to the system of eq. (8), satisfies the limit $\dot{V}(x,e,t) \leq -\alpha(\|x\|^2+\|e\|^2)$ for all x, $e \in R^n$ and $t \in R$ given any admissible $\alpha_i(\cdot)$, $\beta_i(\cdot)$ and $\chi_i(\cdot)$. Let

$$K_{lmi} = \frac{1}{\varepsilon_c} R_c^{-1} B' P_c \qquad \text{and} \qquad L_{lmi} = \frac{1}{\varepsilon_o} P_o^{-1} C' R_0^{-1}, \tag{9}$$

where ε_c, $\varepsilon_o \in R$ are positive constants, $R_c \in R^{m\times m}$ and $R_o \in R^{q\times q}$ are chosen constant matrices. Using eqs. (2) to (6) and (8) - (9) and the fundamental inequality $2|ab| \leq a^2+b^2$ for any a, b real scalars, the following equation can be obtained :

$$\dot{V}(x,e,t) \leq -[x' \quad e'] \begin{bmatrix} \Omega_c & -\frac{1}{\varepsilon_c} P_c B R_c^{-1} B' P_c \\ -\frac{1}{\varepsilon_c} P_c B R_c^{-1} B' P_c & \Omega_o \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} \Omega_c &= -A'P_c - P_c A + \frac{2}{\varepsilon_c} P_c B R_c^{-1} B' P_c - \frac{2}{\varepsilon_c} P_c B R_c^{-1} G' \hat{V}^{-1} G R_c^{-1} B' P_c - \frac{2}{\varepsilon_c} P_c F \hat{V} F' P_c - P_c D \hat{L} D' P_c - 2O' \hat{L}^{-1} O - \frac{1}{\varepsilon_o} J' \hat{S}^{-1} J \\ &= -A'P_c - P_c A + P_c \left\{ \frac{2}{\varepsilon_c} \left[B(R_c^{-1} - R_c^{-1} G' \hat{V}^{-1} G R_c^{-1}) B' - F \hat{V} F' \right] - D \hat{L} D' \right\} P_c - 2O' \hat{L}^{-1} O - \frac{1}{\varepsilon_o} J' \hat{S}^{-1} J \end{aligned} \tag{11}$$

$$\begin{aligned} \Omega_o &= -A'P_o - P_o A - P_o D \hat{L} D' P_o - \frac{2}{\varepsilon_c} P_o F \hat{V} F' P + \frac{2}{\varepsilon_o} C' R_o^{-1} C - \frac{1}{\varepsilon_o} C' R_o^{-1} H \hat{S} H' R_o^{-1} C - \frac{2}{\varepsilon_c} P_c B R_c^{-1} G' \hat{V}^{-1} G R_c^{-1} B' P_c \\ &= -A'P_o - P_o A - P_o \left(D \hat{L} D' - \frac{2}{\varepsilon_c} F \hat{V} F' \right) P_o + \frac{1}{\varepsilon_o} C' \left(2R_o^{-1} - R_o^{-1} H \hat{S} H' R_o^{-1} \right) C - \frac{2}{\varepsilon_c} P_c B R_c^{-1} G' \hat{V}^{-1} G R_c^{-1} B' P_c \end{aligned} \tag{12}$$

Adding ($\varepsilon_c Q_c$ - $\varepsilon_c Q_c$) in Ω_c (Q_c is a symmetric positive definite matrix), the right side of eq. (10) can be divided in two parts,

$$\dot{V}(x,e,t) \leq -x' \, \Theta_1 \, x \;\; -[x' \; e'] \; \Theta_2 \begin{bmatrix} x \\ e \end{bmatrix} \tag{13}$$

where $\Theta_1 \triangleq [\Omega_c - \varepsilon_c Q_c]$ $\Theta_2 \triangleq \begin{bmatrix} \varepsilon_c Q_c & -\frac{1}{\varepsilon_c} P_c B R_c^{-1} B' P_c \\ -\frac{1}{\varepsilon_c} P_c B R_c^{-1} B' P_c & \Omega_o \end{bmatrix}$.

***Theorem 1**: If there are positive constants $\varepsilon_c, \varepsilon_o$, symmetric positive definite matrices P_c, P_o and diagonal positive definite matrices $\hat{L}$, $\hat{V}$, $\hat{S}$ so that $\Theta_1 > 0$ and $\Theta_2 > 0$, the linear system (8), with K_{lmi} and L_{lmi} defined in (9), is asymptotically stable.*

Proof: See [9].

Theorem 1 states sufficiency conditions for the robustness of the controller through feedback of all states. Theorem 2 below presents an adaptation of theorem 1 to an LMI form.

***Theorem 2**: If there are positive constants δ_c and δ_o, symmetric positive definite matrices W_c, W_o, R_c, R_o, Q_c, and Q_o and diagonal positive definite matrices $\tilde{L}$, $\tilde{S}$, $\tilde{V}$ so that the following conditions are satisfied*

$$\text{i) } \Lambda_c \overset{\Delta}{=} \begin{bmatrix} \Phi_c & 2\delta_c B R_c^{-1} G' & W_c O' & W_c J' & W_c \\ G R_c^{-1} B' \delta_c 2 & \tilde{V} & 0 & 0 & 0 \\ O W_c & 0 & \tilde{L} & 0 & 0 \\ J W_c & 0 & 0 & \tilde{S} & 0 \\ W_c & 0 & 0 & 0 & \delta_c Q_c^{-1} \end{bmatrix} > 0 \tag{14}$$

and

$$\text{ii) } \Lambda_o \overset{\Delta}{=} \begin{bmatrix} \Phi_o & \delta_0 C' R_o^{-1} H & P_o D & P_o F & P_o \\ H' R_o^{-1} C \delta_0 & \bar{S} & 0 & 0 & 0 \\ D' P_o & 0 & \bar{L}/2 & 0 & 0 \\ F' P_o & 0 & 0 & \bar{V} & 0 \\ P_o & 0 & 0 & 0 & \delta_o Q_o^{-1} \end{bmatrix} > 0, \tag{15}$$

where

$$\Phi_c = -W_c A' - A W_c + 2\delta_c B R_c^{-1} B' - F\tilde{V}F' - 2D\tilde{L}D',$$

$$\Phi_o = -A' P_o - P_o A + 2\delta_o C' R_o^{-1} C - 4\delta_c^2 P_c B R_c^{-1} G' \bar{V} G R_c^{-1} B' P_c - \delta_c^3 P_c B R_c^{-1} B' P_c Q_c^{-1} P_c B R_c^{-1} B' P_c$$

and $\bar{L} \overset{\Delta}{=} \tilde{L}^{-1}$, $\bar{V} \overset{\Delta}{=} \tilde{V}^{-1}$, $\bar{S} \overset{\Delta}{=} \tilde{S}^{-1}$ *then, the linear system with uncertainties, where K and L are defined in eq. (9), is asymptotically stable.*

Proof: See [9].

The following design procedure can be established.

1. Choose the matrices Q_c and R_c, so that the optimization problem P1 has a non-empty set of feasible solutions (M_c, W_c, $\tilde{V}$, $\tilde{L}$, $\tilde{S}$, δ_c), where M_c and W_c are symmetric positive definite matrices, $\tilde{V}$, $\tilde{L}$ and $\tilde{S}$ are diagonal positive definite matrices and δ_c is a scalar.

$$\text{P1: } \min_{M_c, W_c, \tilde{V}, \tilde{L}, \tilde{S}, \delta_c} f_c(M_c, W_c, \tilde{V}, \tilde{L}, \tilde{S}, \delta_c) = \mathrm{tr}(M_c) \tag{16}$$

$$\text{subjected to:} \begin{bmatrix} M_c & I \\ I & W_c \end{bmatrix} > 0,\ \Lambda_c > 0,\ M_c, W_c, \tilde{V}, \tilde{L}, \tilde{S}, \delta_c > 0 \tag{17}$$

2. Calculate $\bar{V}$, $\bar{L}$, $\bar{S}$ and $P_c \overset{\Delta}{=} W_c^{-1}$. Choose symmetric positive definite matrices Q_o and R_o so that the optimization problem P2 has a non-empty set of feasible solutions (M_o, P_o, δ_o) with matrices P_o and M_o and a scalar δ_o.

$$\text{P2: } \min_{M_o, P_o, \delta_o} f_o(M_o, P_o, \delta_o) = \mathrm{tr}(M_o) \tag{18}$$

$$\text{subject to :} \begin{bmatrix} M_o & I \\ I & P_o \end{bmatrix} > 0,\ \Lambda_o > 0,\ M_o, P_o, \delta_o > 0. \tag{19}$$

3. Calculate K_{lmi} and L_{lmi} by eq. (9). Theorem 2 guarantees asymptotic stability for the system (7).

The choices of Q_c and R_c are similar to the choices of the weighting matrices of the algebraic Riccati equations (AREs). In the optimization problem P1, there is the advantage of choosing the weighting matrices $\tilde{V}$, $\tilde{L}$, $\tilde{S}$ and the constant ε_c. In a similar way, this freedom of choice is also valid for problem P2. Comparatively, the formulation of these problems via LMI has more flexibility than the formulation via ARE. This flexibility is related to the rank 1 decomposition of the weighting matrices and with the choice of the constant ε_c and ε_o. When the solutions via ARE are chosen, these decompositions must be made in such a way that simultaneous solutions for both AREs do exist, and this can be an exhaustive task. LMIs overcome this problem and there's no need to choose the constants ε_c and ε_o.

5 - LMI controller design

Eqs. (1) and (2) give the nominal plant and the uncertainty matrices, considering the system with integrators we have:

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1.8709 & -4.0038 & 0 & 3.7736 & 0 \\ 0 & 0 & 99.16 & 0 & -212.2 & 0 & 200 \\ 3.3333 & 0 & 0 & -83.3333 & 0 & -3.3333 & 0 \\ 0 & 3.3333 & 0 & 0 & -83.3333 & 0 & -3.3333 \end{bmatrix}$$

$$B=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad C=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Delta A(t)=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1873\alpha(t) & -7.3169\alpha(t) & 0 & 0 & 0 \\ 0 & 0 & 9.916\alpha(t) & 0 & -387.7955\alpha(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$D=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}'$, $O=\begin{bmatrix} 0 & 0 & -0.1873 & -7.3167 & 0 & -6.6666 & 0 \\ 0 & 0 & 9.9160 & 0 & -387.7955 & 0 & -6.6666 \end{bmatrix}$, and the matrices *F*, *G*, *H* and *J* are zeroes matrices. The same observer of the LQG/LTR design is used here. The gain of this controller is

$$K_{lmi}=\begin{bmatrix} 40.740 & 0.0016 & -163.75 & 2370.7 & 0.9549 & 494.17 & 0.1755 \\ 0.0016 & 1287.5 & 138520 & 139.23 & 537770 & 0.1078 & 325370 \end{bmatrix}.$$

6 - Real and complex stability radii

6.1 - LQG/LTR Controller

The stability real radius is a problem that has been considered by several researchers of the control theory [2], [3], [4], [10], [13], [14], [15], [16], [17]. This radius measures the capacity of a matrix in preserving her stability when occur real perturbations.

In [22] is presented a general definition of the stability radius in the field K (i.e. K=C or K=R) taking into account the structured singular value (ssv) of the system via H_∞-norm. It is possible to calculate singular vectors to generate a perturbation matrix in an appropriated mapping, with a real parameter γ that belongs to (0,1]. Two algorithms to determine the frequency range where the maximum of the singular values of a transference matrix $M\in \mathbb{C}^{pxm}$ is contained and to calculate the perturbation matrix, $\Delta\in \mathbb{R}^{mxp}$, are used, see for more details [3] and [22]. The first algorithm determines, too, the frequency that this maximum is given. The second algorithm utilizes the singular vectors of a determined real matrix, of the transfer matrix M of the system, denoted by $P(\gamma)$. With this, the real stability radius can be calculated.

For the construction of the uncertainty matrix, the real and complex parts of the matrix $M\in \mathbb{C}^{mxm}$, $M=X+jY$, are utilized. Three cases are implemented in the algorithm: when the matrix Y is equal zero, when the rank of Y is equal one, and when the rank of Y is grater than 1.

Here, the limits of the ssv and the real and complex stability radii of one electric system controlled via two techniques LQG/LTR and LMI, are given. We display too, the perturbation matrices that can cause the instability of the system. The complex stability radius is

$$r_C(A,B,C) = \left(\max_{\omega^* \in [\omega_{\min}, \omega_{\max}]} \mu_C [C(j\omega^* I - A)^{-1} B] \right)^{-1} = \left(\max_{\omega^* \in [\omega_{\min}, \omega_{\max}]} \bar{\sigma}[C(j\omega^* I - A)^{-1} B] \right)^{-1}$$

where ω^* is the frequency where the maximum of the greatest complex ssv of the system is given.

The real structured singular value is

$$\mu_R[C(j\omega^* I-A)^{-1}B] = \inf_{\gamma\in(0,1]} \sigma_2\left(\begin{bmatrix} \mathrm{Re}M & -\gamma\ \mathrm{Im}M \\ \gamma^{-1}\mathrm{Im}M & \mathrm{Re}M \end{bmatrix} \right)$$

where the $\sigma 2(.)$ is the second singular value. The real stability radius is given by

$$r_R(A,B,C) = \left(\max_{\omega^* \in [\omega_{\min}, \omega_{\max}]} \mu_R [C(j\omega^* I - A)^{-1} B] \right)^{-1} .$$

For any $M\in \mathbf{C}^{nxn}$ we have the following inequalities

$$\rho_R(M) \le \mu_K(M) \le \bar{\sigma}(M)$$

where $\rho_R(.)$ and $\bar{\sigma}(.)$ denote spectral radius and maximum singular value. When $\gamma=1$ in

$$\sigma_2 \underset{\substack{\omega\in[\omega_m,\omega_M] \\ \gamma\in(0,1]}}{[C\omega jI - A)^{-1}B]} = \sigma_2\left(\begin{bmatrix} \mathrm{Re}\left(C(\omega jI - A)^{-1}B\right) & -\gamma \mathrm{Im}\left(C(\omega jI - A)^{-1}B\right) \\ \gamma^{-1}\mathrm{Im}\left(C(\omega jI - A)^{-1}B\right) & \mathrm{Re}\left(C(\omega jI - A)^{-1}B\right) \end{bmatrix} \right)$$

we have the complex ssv. The function that describe the real ssv in the frequency, with minimization in γ, is

$$\mu_R[C\omega jI - A)^{-1}B]_{\omega\in[\omega m,\omega M]} = \inf_{\gamma\in(0,1]} \sigma_2\left(\begin{bmatrix} \mathrm{Re}\left(C(\omega jI - A)^{-1}B\right) & -\gamma\,\mathrm{Im}\left(C(\omega jI - A)^{-1}B\right) \\ \gamma^{-1}\,\mathrm{Im}\left(C(\omega jI - A)^{-1}B\right) & \mathrm{Re}\left(C(\omega jI - A)^{-1}B\right)\end{bmatrix}\right).$$

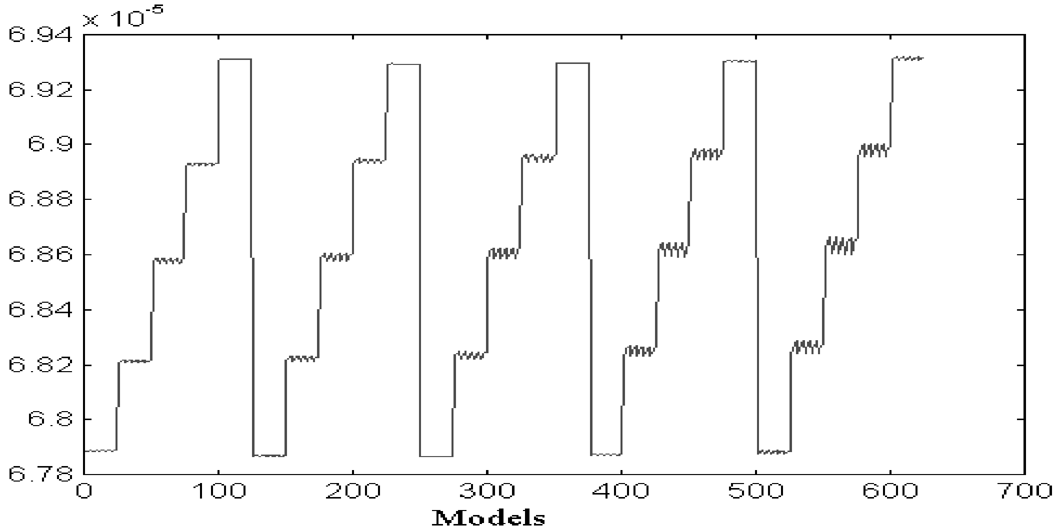
The first controller analyzed is the designed via LQG/LTR methodology. We consider the nominal plant with integrators, the uncertainty matrix A_{del}, given bellow, the observer gain (designed via Kalman filter), H_{kf}, and the controller gain (designed via Linear Quadratic Gaussian regulator), G_{lqg}, given above.

$$A_{del} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1873a & -7.3169b & 0 & 0 & 0 \\ 0 & 0 & 9.916c & 0 & -387.7955d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a, b, c e d belong to [-1,1] with appropriated combinations. In the following, the real and complex stability radius for the controller K_{LQG} are given

$$K_{LQG}(\omega)=G_{lqg}\,(\omega I-A_n-Adel+B_nG_{lqg}+H_{kf}C_n)^{-1}H_{kf}.$$

We have 625 combinations of the system uncertainties, for these models we fix a determined parametric uncertainty changing the resting, this procedure was done for all parametric uncertainty. For each combination we have one model. The following results were obtained a)



b)

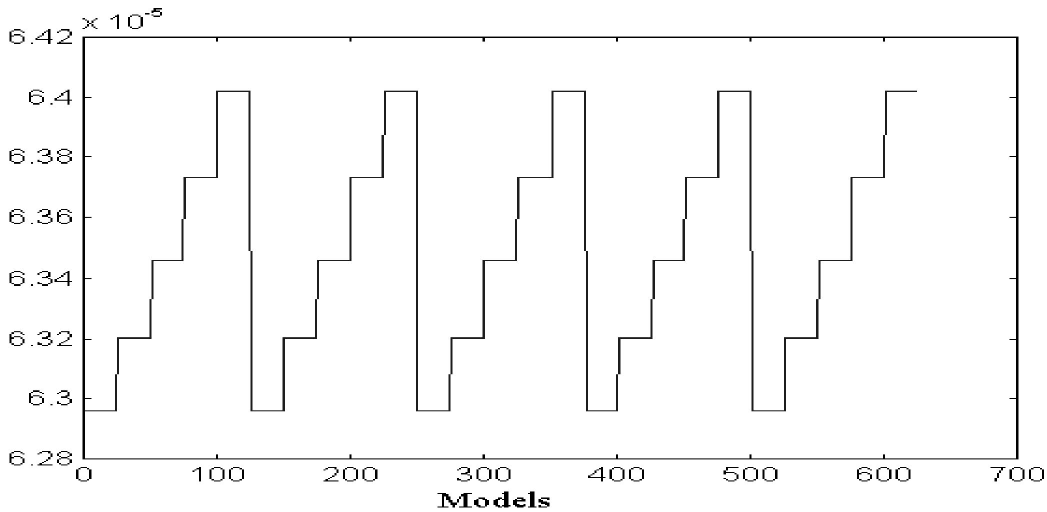


Figure 5 - Real (a) and complex (b) stability radius for 625 models, LQG/LTR controller for the power electric system.

The figure 5 displays that the real radius has more variations (fig. 5 a), it is more sensible to the parametric uncertainties, than the complex radius. The complex radius are always smaller or equal than the real radius.

Table 3

Real and complex SSV and real and complex stability radii - LQG/LTR controller			
ω_C^*	256.8650	ω_R^*	319.7072
$\mu_C[C_{lqg}(j\,\omega_C^*\,I\text{-}A_{lqg})^{-1}B_{lqg}]$	1.5883e+004	$\mu_R[C_{lqg}(j\,\omega_R^*\,I\text{-}A_{lqg})^{-1}B_{lqg}$	1.4625e+004
$r_C(A_{lqg},B_{lqg},C_{lqg})$	6.2962e-005	$r_R(A_{lqg},B_{lqg},C_{lqg})$	6.8374e-005

Matrix M in the frequency ω_R^*

$$M=\begin{bmatrix} 14725.5141-25.9620\,i & 126.1546-44.8778\,i \\ -5.8840+14.4441\,i & 16.2713+1731.3838\,i \end{bmatrix}.$$

For any $M\in \mathbf{C}^{nxn}$ we have the following inequalities:

$$\rho_R(M) \le \mu_K(M) \le \bar{\sigma}(M).$$

$$0\le\mu_R(M)= 1.4620e+004\le 1.4726e+004$$

and the perturbation matrix of the system is

$$\Delta=\begin{bmatrix} 0.000067956592 & -0.000007490390 \\ -0.000007642969 & -0.000067724530 \end{bmatrix}.$$

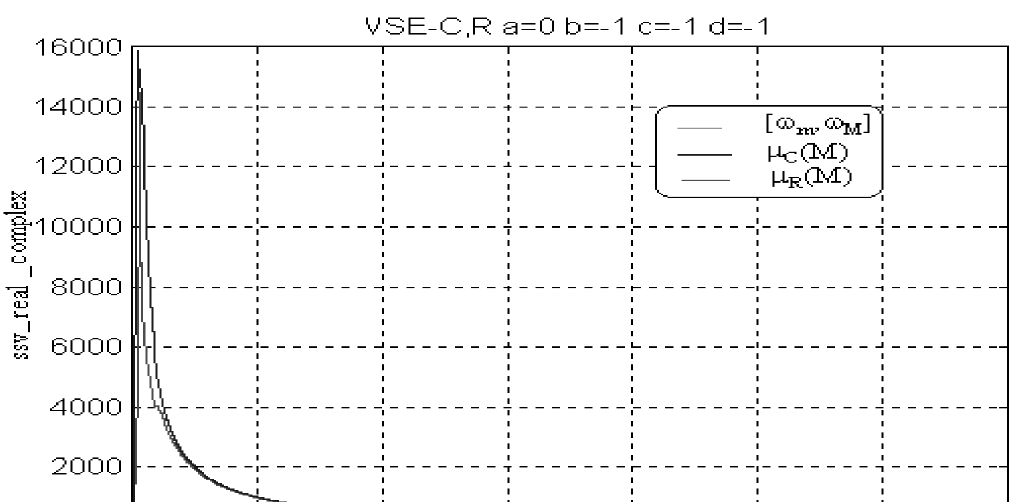


Figure 6 - Real and complex ssv for the LQG/LTR controller.

The minimum of the second singular value of the matrix P(γ) is given by:

$$\mu_R\underset{\omega\in[\omega_m,\omega_M]}{[C_{lqg}\,\omega jI-A_{lqg})^{-1}B_{lqg}]}=\inf_{\gamma\in(0,1]}\sigma_2\left(\begin{bmatrix} \mathrm{Re}\left(C_{lqg}(\omega jI-A_{lqg})^{-1}B_{lqg}\right) & -\gamma\,\mathrm{Im}\left(C_{lqg}(\omega jI-A_{lqg})^{-1}B_{lqg}\right) \\ \gamma^{-1}\,\mathrm{Im}\left(C_{lqg}(\omega jI-A_{lqg})^{-1}B_{lqg}\right) & \mathrm{Re}\left(C_{lqg}(\omega jI-A_{lqg})^{-1}B_{lqg}\right) \end{bmatrix}\right)$$

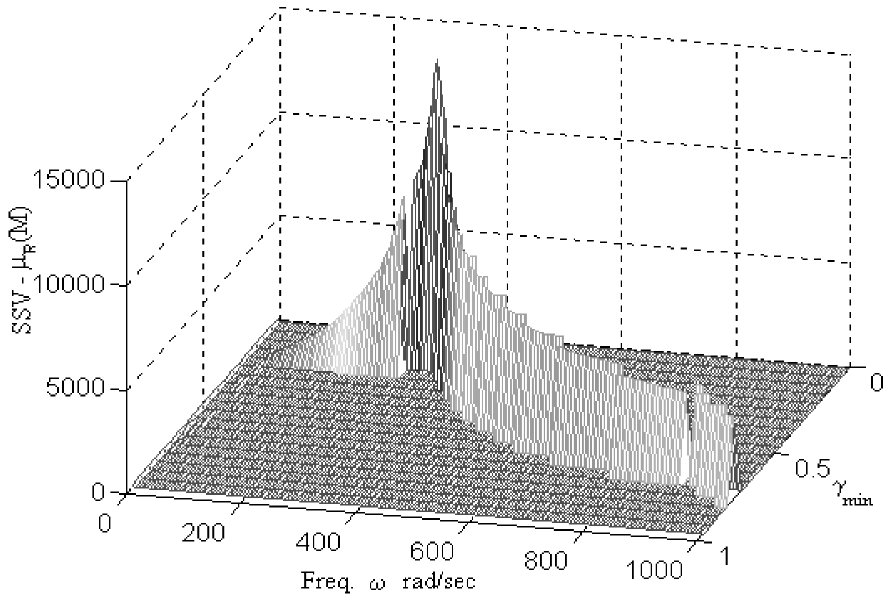


Figure 9: Real ssv for each frequency minimised in γ - LQG/LTR controller.

6.2 - LMI controller

The gain of the controller designed via Linear Matrix Inequality is given by:

$$K_{LMI} = \begin{bmatrix} 40.740 & 0.0016 & -163.75 & 2370.7 & 0.9549 & 494.17 & 0.1755 \\ 0.0016 & 1287.5 & 138520 & 139.23 & 537770 & 0.1078 & 325370 \end{bmatrix}.$$

The real and complex radius for the same uncertainty combinations given above, are given bellow for the following transfer function

$$K_{LMI}(\omega) = K_{dml}(\omega I - A_n + Adel - B_n K_{lmi} - H_{kf} C_n)^{-1} H_{kf}.$$

We have the following graphics

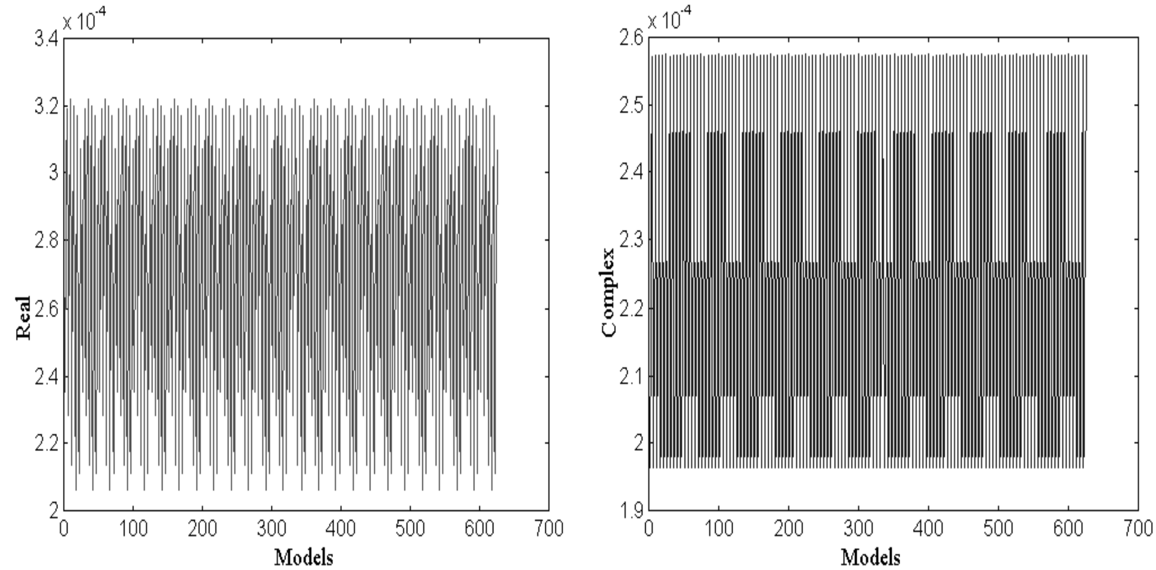


Figure 10 - Real and complex radii for 625 models - controller designed via LMI for the power electric system.

In the table 5 the real and complex radii and ssv of the LMI controller are given in the frequencies ω_R^* and ω_C^*. The matrices of the LMI controller are

$$A_{dml} = A_n + Adel - B_n K_{lmi} - H_{kf} C_n$$

$$= \begin{bmatrix} -40.7400 & -0.0016 & 43.7521 & -2370.7000 & -1.0054 & -494.1700 & -0.1755 \\ -0.0016 & -1287.5000 & -138519.9534 & -139.2300 & -537889.9979 & -0.1078 & -325370 \\ 0 & 0 & -0.9917 & 1.0000 & -0.0083 & 0 & 0 \\ 0 & 0 & -7.0065 & 3.3131 & -0.0420 & 3.7736 & 0 \\ 0 & 0 & 104.0760 & 0 & -22.6908 & 0 & 200.0000 \\ 3.3333 & 0 & -8.8539 & -83.3333 & 0.3892 & -3.3333 & 0 \\ 0 & 3.3333 & 0.4351 & 0 & -89.6920 & 0 & -3.3333 \end{bmatrix}$$

$$B_{dml}' = H_{kf}, \qquad C_{dml} = K_{dml};$$

Real and complex radii and ssv - LMI controller			
ω_C^*	893	ω_R^*	997.5
$\mu_C[K_{LMI}]$	5.008e+003	$\mu_R[K_{LMI}]$	4.8528e+003
$r_C(K_{LMI})$	1.9965e-004	$r_R(K_{LMI})$	2.0607e-004

Table 5.

Matrix M on the frequency ω_R^*

$$M=\begin{bmatrix} 0.7825-20.8895i & -0.0172-0.0735i \\ 22.0419-6.0551i & 4858.5187+0.3810i \end{bmatrix}.$$

In the following we display the perturbation matrix of the system for the frequency ω_R^*

$$\Delta=\begin{bmatrix} -0.0002046036 & -0.0000109908 \\ -0.0000109053 & -0.0002057741 \end{bmatrix}$$

and for any $M\in \mathbf{C}^{mxm}$ we have the inequalities

$$\rho_R(M) \le \mu_K(M) \le \bar{\sigma}(M).$$

$$0\le\mu_R(M)= 4.8528e+003\le 4.8586e+003$$

The graphic of the ssv $\mu_R(M)$ is given in the figure 11.

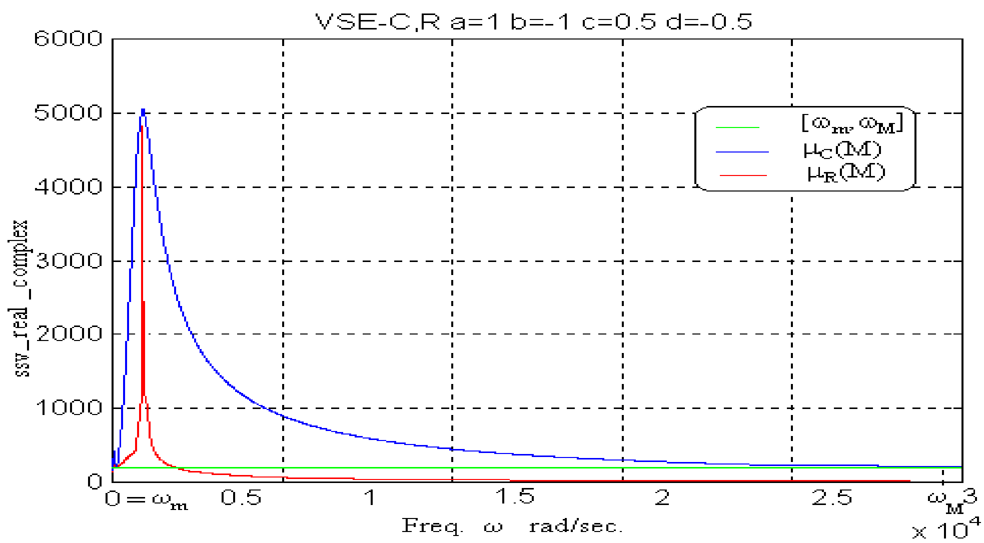


Figure 11 - Real and complex ssv - LMI controller.

In the following, the graphic of the minimum structured singular values, in the frequency domain, for the system controlled via Linear Matrix Inequalities is displayed in the figure 12.

$$\mu_R[C_{LMI}\omega jI-A_{LMI})^{-1}B_{LMI}]_{\omega\in[\omega_m,\omega_M]}= \inf_{\gamma\in(0,1]} \sigma_2\left(\begin{bmatrix} \mathrm{Re}\left(C_{LMI}(\omega jI-A_{LMI})^{-1}B_{LMI}\right) & -\gamma\,\mathrm{Im}\left(C_{LMI}(\omega jI-A_{LMI})^{-1}B_{LMI}\right) \\ \gamma^{-1}\mathrm{Im}\left(C_{LMI}(\omega jI-A_{LMI})^{-1}B_{LMI}\right) & \mathrm{Re}\left(C_{LMI}(\omega jI-A_{LMI})^{-1}B_{LMI}\right) \end{bmatrix}\right)$$

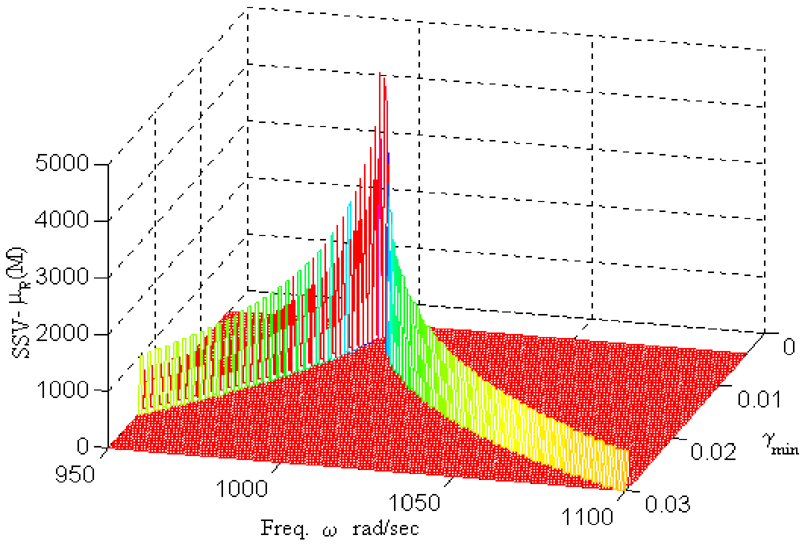


Figure 12 - Real ssv for each frequency minimised in γ - LMI controller.

CONCLUSION

In this paper we considered the real and complex stability radii and the structured singular values of a power electric system controlled via two methodologies LQG/LTR and LMI. The figures 5 and 10 display the behaviour of these radii for both systems. The variations of the radii considering LMI controller are more intensifies than the radii for the system controlled via LQG/LTR methodology. The real and complex radii of the LMI controller are bigger than the LQG/LTR controller radii. For this electric power system, the distance of the instability for the first controller is bigger than the second controller. For a further research it is interesting to investigate the generalisation for any system.

REFERENCES

[1] Boyd, S.; El-Ghaoui, L.; Feron, E. and Balakrishnan, V. - Linear Matrix Inequalities in System and Control Theory. *SIAM - Society for Industrial and Applied Mathematics, 1994.*

[2] Boyd, S; Balakrishnan, V. and Kabamba, P. "A bisection method for computing the H_∞-norm of a transfer matrix and related problems," Math. Contr., Sig., Syst., vol. 2, pp. 207-219, 1989.

[3] Byers, R. (1988). A bisection method for measuring the distance of a stable matrix to the unstable matrices. SIAM J. Sci. Stat. Comput., **9,** 875-881.

[4] Byers, R. (1988). A bisection method for measuring the distance of a stable matrix to the unstable matrices. SIAM J. Sci. Stat. Comput., **9,** 875-881.

[5] Cruz, J. J. - Controle Robusto Multivariável. *EDUSP - Editora da USP, 1996.*

[6] Doyle, J. C. and Stein G. - Multivariable Feedback Design: Concepts for a Classical/ Modern Synthesis. *IEEE Transactions on Automatic Control, vol. AC-25, no. 1, February 1981.*

[7] Elgerd, O. I. - Eletric Energy Systems Theory - An Introduction. *McGraw-Hill Book Company, second edition, 1982.*

[8] Fouad, A. A. and Anderson, P. M. - Power System Control and Stability. *IEEE Press - IEEE Power Systems Engineering Series -1994.*

[9] Costa, E. F.; Terra, M.H. and Oliveira, V. A. - Observer-based Controller for a Class of Uncertain Systems: a Linear Matrix Inequality Approach. *The IEEE Singapore International Symposium on Control Theory and Applications (SISCTA'97), Singapore, 29-30 July 1997.*

[10] Fan, M. K. H. and A L. Tist (1986). “Characterization and efficient computation of the structured singular values, “ IEEE Trans. Automat. Control, vol.AC-31,no.8, pp.734-743.

[11] Fishman, A.; Dion, J. M.; Duggard, L. and Troffino, A. - A Linear Matrix Inequality Approach for Guaranteed Cost Control. *Proc. of the 13th IFAC Congress,* pp. 197-202, San Francisco, USA, 1996.

[12] Gahinet, P.; Nemiroviski, A.; Laub, A. J. and Chilali, M. - LMI Control Toolbox. *The Math Works Inc., 1995. Also in Proc. 33rd IEEE Conf. On Decision and Control, Lake Buena Vista, FL, pp. 2038-2041*

[13] Hinrichsen; D.; Kelb, B. and Linnemann, A. “An algorithm for the computation of the structured stability radius, “Automatica, vol. 25, pp. 771-775, 1989.

[14] Loan, V. C. (1985). How near is a stable matrix to an unstable matrix? Contemp. Math., **47,** 465-477.

[15] Packard, A. and J. C. Doyle (1993), “The complex structured singular values,” Auto-matica Vol. 29, pp. 71-109.

[16] Packard, A. and P. Pandey (1993). “Continuity properties of the real/complex structured singular values,” IEEE Trans. Automatic Control, vol. 38 No. 3 pp. 415-428.

[17] Peter M. Young and John C. Doyle (1996). “Properties of the Mixed μ Problem and Its Bounds” IEEE Trans. Automatic Control, vol. 41 No. 1 pp. 155-159.

[18] Petersen, I.R. - A Riccati Equation Approach to the Design of Stabilizing Controllers and Observers for a Class of Uncertain Systems. *IEEE Transactions on Automatic Control, n. 30, p. 904-907, 1985.*

[19] Qiu, L., and E. J. Davison (1989). A simple procedure for the exact stability robus-tness computation of polynomials with affine coefficient perturbations. Syst. Control Lett. **13,** 413-420.

[20] Qiu, L., and E. J. Davison (1991). The stability robustness determination of state space models with real unstructured perturbations. Math. Control Signals Syst., **4,** 247-267.

[21] Qiu, L., and E. J. Davison (1992). Bounds on the real stability radius. In M. Mansour, S. Balemi and W. Truol, (Eds), Robustness of Dynamic systems with parameter Uncertainties, pp. 139-145. Bikhauser, basel.

[22] Qiu, L.; Bernhardsson, B.; Rantzer, A; Davison, E.J.; Young; P. M. and Doyle, J. C. (1995)“A formula for computation of the real stability radius,”Automatica, Vol. 31, no. 6, pp. 879-890, 1995.

[23] Qiu, L.; Tist, A L. and Yang, Y. (1995). “On the Computation of the Real Hurwitz-Stability Radius,” IEEE Trans. Autom. Control., vol. 40**,** no. 8, pp. 1475-1475.

[24] Stagg, G. W. and El-Abiad, A. H. - Computer methods in power system analysis. *McGraw-Hill Book Company, vol. 1, 1968.*

[25] Stein, G. and Athans, M. - The LQG/LTR Procedure for Multivariable Feedback Control Design. *IEEE Transactions on Automatic Control , vol. ac-32, n. 2, February 1987.*

[26] Terra, M. H. and Leite, V. M. P. L. - Loop Transfer Recovery (LTR) by Output Feedback Control For Non-Minimum Phase Systems. *IFAC'96 (13th World Congress International Federation of Automatic Control - 1996, San Francisco California, USA*).

[27] Terra, M. H. and Leite, V. M. P. L. - Recovery Surface on Robust Control - *2nd IFAC Symposium on Design Robust Control - Budapest - Hungary - June - 1997.*

[28] Terra, M. T., Rodrigo. Ramos e N. G. Bretas. ”Sistemas de controle automá-tico de carga e freqüência utilizando as metodologias LQG/LTR e desigualdade matriciais lineares” Congresso Brasileiro de Automática 1998.