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On Readily Available Supervisory Control Policies that Enforce Liveness in a Class of Completely Controlled Petri Nets

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Abstract

A *Petri Net* (PN) is said to be *live* if it is possible to fire any transition from every reachable marking, although not necessarily immediately. Under appropriate conditions, a non-live PN can be made live via supervision. Under this paradigm an external-agent, the supervisor, prevents the firing of certain transitions at each reachable marking so as to enforce liveness. A PN is said to be completely controllable if the supervisor can prevent the firing of any transition. Testing the existence of a supervisory policy that enforces *liveness* in a completely controlled Petri net can be computationally expensive [8]. In this paper we present a new class of PNs for which there is a readily available supervisory policy that enforces liveness. This observation obviates the aforementioned test for the specific class of PNs introduced in this paper.

1 Introduction

A *Petri net* (PN) [3, 5, 4] is said to be *live* if it is possible to fire any transition from every reachable marking, although not necessarily immediately. In this paper we concern ourselves with the problem of enforcing liveness in a non-live PN via supervision. Essentially, we have a non-live PN where some, maybe all, transitions can be individually prevented from firing by an external-agent, the supervisor. A PN where every transition can (cannot) be individually prevented from firing by the supervisor is called a *completely controlled PN* (*partially controllable PN*). In reference [8] it is shown that the existence of a supervisory policy that enforces liveness in a partially controllable PN is undecidable in general. However, in the same reference it

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is shown that there exists a testable condition for the existence of a supervisory policy that enforces liveness in an arbitrary completely controllable PN. Additionally, the computational requirements for this test can be exorbitant in the general case.

In deciding the existence, and the synthesis of a supervisory policy that enforces liveness, significant savings in computation can be gained if the PN satisfies specific structural requirements [10, 11]. In a different approach to circumventing the aforementioned complexity issue, in reference [6] it is shown that there is a supervisory policy that enforces liveness in a completely controlled PN if and only if there is a corresponding policy for its *Free-Choice Equivalent*. A *Free-Choice PN* (FCPN) is a PN where every arc from a place to a transition is either the unique output arc from that place, or, it is the unique input arc to the transition. Supervisory policies that enforce liveness in an FCPN are characterized in reference [7].

In this paper we present a new class of PNs for which there is a readily available policy that enforces liveness. This observation eliminates the computationally expensive test for the existence of a supervisory policy that enforces liveness, when a plant PN is known to belong to this class of PNs.

The next section contains notations and definitions of the various concepts used in this paper. We then present the main results of this paper along with an illustrative example. The concluding section presents a potential future research topic.

2 Notations and Definitions

A *Petri net* (PN) $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is an ordered 4-tuple, where $\Pi = \{p_1, p_2, \ldots, p_n\}$ is a set of n *places*, $T = \{t_1, t_2, \ldots, t_m\}$ is a set of m *transitions*, $\Phi \subseteq (\Pi \times T) \cup (T \times \Pi)$ is a set of *arcs*, $\mathbf{m}^0 : \Pi \to \mathcal{N}$ is the *initial marking function* (or the *initial marking*), and $\mathcal{N}$ is the set of nonnegative integers. The *state* of a PN is the marking $\mathbf{m} : \Pi \to \mathcal{N}$ that identifies the number of *tokens* in each place. For a given marking $\mathbf{m}$ a transition $t \in T$ is said to be *enabled* if $\forall\, p \in ({}^\bullet t)_N$, $\mathbf{m}(p) \geq 1$, where $({}^\bullet x)_N := \{y \mid (y, x) \in \Phi\}$. The set of enabled transitions is denoted by the symbol $T_e(\mathbf{m})$. An enabled transition $t \in T_e(\mathbf{m})$ can *fire*, which changes the marking $\mathbf{m}$ to $\widehat{\mathbf{m}}$ according to the equation

$$\widehat{\mathbf{m}}(p) = \mathbf{m}(p) - card((p^\bullet)_N \cap \{t\}) + card(({}^\bullet p)_N \cap \{t\}), \tag{1}$$

where $(x^\bullet)_N := \{y \mid (x, y) \in \Phi\}$, and the symbol $card(\bullet)$ is used to denote the cardinality of the set argument. This notation is also used to denote the predecessor or successor set of a set of places or transitions. Sometimes, $(x^\bullet)_N$ (or, $({}^\bullet x)_N$) is just represented as $x^\bullet$ (${}^\bullet x$) when there is no confusion as to the identity of PN N in its definition.

A collection of places $P \subseteq \Pi$ is said to be a *siphon* (*trap*) if ${}^\bullet P \subseteq P^\bullet$ ($P^\bullet \subseteq {}^\bullet P$). A trap (siphon) P is said to be *minimal* if $\not\exists\ \widetilde{P} \subset P$, such that $\widetilde{P}^\bullet \subseteq {}^\bullet\widetilde{P}$ (${}^\bullet\widetilde{P} \subseteq \widetilde{P}^\bullet$).

A string of transitions $\sigma = t_{j_1} t_{j_2} \cdots t_{j_k}$, where $t_{j_i} \in T$ $(i \in \{1, 2, \ldots, k\})$ is said to be a *valid firing string* starting from the marking $\mathbf{m}$, if,

- the transition t_{j_1} is enabled at the marking $\mathbf{m}$, and
- for $i \in \{1, 2, \ldots, k-1\}$ the firing of the transition t_{j_i} produces a marking at which the transition $t_{j_{i+1}}$ is enabled.

The set of *reachable markings* from $\mathbf{m}^0$, denoted by $\Re(N, \mathbf{m}^0)$, is the set of markings generated by all valid firing strings starting with marking $\mathbf{m}^0$ in the PN N. At a marking $\mathbf{m}^1$, if the firing of a valid firing string σ results in a marking $\mathbf{m}^2$, we represent it as $\mathbf{m}^1 \to \sigma \to \mathbf{m}^2$. A transition $t \in T$ is *live* if

$$\forall \mathbf{m}^1 \in \Re(N, \mathbf{m}^0), \exists \text{ a } \mathbf{m}^2 \in \Re(N, \mathbf{m}^1) \text{ such that } t \in T_e(\mathbf{m}^2).$$

A *supervisory policy* $\mathcal{P}: \mathcal{N}^n \to \{0,1\}^m$, is a total map that returns an m-dimensional binary vector for each reachable marking. The supervisory policy $\mathcal{P}$ permits the firing of transition t_i at marking $\mathbf{m}$, only if $\mathcal{P}(\mathbf{m})_i = 1$. If at a marking $\mathbf{m}$ all input places to a transition t_i contain a token, we say the transition t_i is *state-enabled* at $\mathbf{m}$. If $\mathcal{P}(\mathbf{m})_i = 1$, we say the transition t_i is *control-enabled* at $\mathbf{m}$. A transition has to be state-enabled and control-enabled before it can fire. A string of transitions $\sigma = t_{j_1} t_{j_2} \cdots t_{j_k}$, where $t_{j_i} \in T$ $(i \in \{1, 2, \ldots, k\})$ is said to be a *valid firing string* starting from the marking $\mathbf{m}$, if,

- the transition t_{j_1} is enabled at the marking $\mathbf{m}$, $\mathcal{P}(\mathbf{m})_{j_1} = 1$, and
- for $i \in \{1, 2, \ldots, k-1\}$ the firing of the transition t_{j_i} produces a marking $\widetilde{\mathbf{m}}$ at which the transition $t_{j_{i+1}}$ is enabled and $\mathcal{P}(\widetilde{\mathbf{m}})_{j_{i+1}} = 1$.

The set of reachable markings under the supervision of $\mathcal{P}$ in N from the initial marking $\mathbf{m}^0$ is denoted by $\Re(N, \mathbf{m}^0, \mathcal{P})$. A transition t_{j_i} is *live* under the supervision of $\mathcal{P}$ if

$$\forall \mathbf{m}^1 \in \Re(N, \mathbf{m}^0, \mathcal{P}), \exists \mathbf{m}^2 \in \Re(N, \mathbf{m}^1, \mathcal{P}) \text{ such that } t_{j_i} \in T_e(\mathbf{m}^1) \text{ and } \mathcal{P}(\mathbf{m}^2)_{j_i} = 1.$$

A supervisory policy $\mathcal{P}$ enforces liveness if all transitions in N are live under $\mathcal{P}$. A supervisory policy $\mathcal{P}$ that enforces liveness in N essentially eliminates those markings reachable under the absence of supervision from which some transition in the PN can never be enabled. The above definition of a supervisory policy that enforces liveness in completely controlled PNs is simpler than that in reference [8] which addresses the general case when some transitions in the PN cannot be prevented from firing by the supervisor. Reference [8] presents a necessary and sufficient condition for the existence of a supervisory policy that enforces liveness in an arbitrary PN. This condition is not testable in general, but it is testable if the PN is bounded, or if each transition in the PN can be prevented from firing by the supervisory policy. For completely controlled PNs there is a supervisory policy that enforces liveness if and only if $\exists\ \mathbf{m}^1, \mathbf{m}^2 \in \Re(N, \mathbf{m}^0)$, $\exists\ \sigma_1, \sigma_2 \in T^*$, such that $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1 \rightarrow \sigma_2 \rightarrow \mathbf{m}^2$, where $\mathbf{m}^2 \geq \mathbf{m}^1$ and all transitions in T appear at least once in σ_2 [8]. The class of PNs we consider in this paper are not necessarily bounded, but we assume each transition can be prevented from firing by the supervisory policy. Testing the existence of a supervisory policy that enforces liveness in an arbitrary bounded PN is at least PSPACE-complete [8]. In this paper we present a class of PNs for which there is a readily available supervisory policy that enforces liveness. This class of PNs is in some sense a subset of a class of PNs called *Free-Choice* PNs, which is defined below.

A PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is a *Free-Choice* PN (FCPN) if

$$\forall p \in \Pi, card\ (p^\bullet) > 1 \Rightarrow {}^\bullet(p^\bullet) = \{p\}.$$

In other words, a PN is Free-Choice if and only if an arc from a place to a transition is either the unique output arc from that place, or is the unique input arc to the transition. An FCPN N is live if and only if every minimal siphon of N contains a marked trap (cf. *Commoner's Liveness Theorem* [1]). Reference [7] characterizes the class of policies that enforce liveness in FCPNs that violate Commoner's Liveness Theorem. This characterization is presented in theorem 2.1.

Theorem 2.1 *[7] A supervisory policy* $\mathcal{P}: \mathcal{N}^n \rightarrow \{0,1\}^m$, *enforces liveness in an FCPN* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ *if and only if the following conditions are satisfied:*

1. *If the supervisory policy* $\mathcal{P}$ *prevents the firing of a state-enabled transition* $t \in T$ *at some marking* $\mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P})$, *then* $\exists\ \widehat{m} \in \Re(N, \widehat{\mathbf{m}}^0, \mathcal{P})$, *such that the transition* t *is both state and control-enabled at the marking* $\widehat{\mathbf{m}}$.
2. $\forall\ \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P})$, $\forall\ P \subseteq \Pi$, *such that* ${}^\bullet P \subseteq P^\bullet$, $\sum_{p \in P} \mathbf{m}(p) \neq 0$.

An arbitrary PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ can be converted to an equivalent FCPN $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ with the addition of a few extra places and transitions (cf. [2,6]). Additionally, in

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(PN) `Compute_Subnet` $((PN)\ N = (\Pi, T, \Phi, \mathbf{m}^0)$, *(Subset of places)* $\Pi_1 \subseteq \Pi$, *(Subset of transitions)* $T_1 \subseteq T)$
{

  Create a set of places $\widehat{\Pi} = \Pi_1$;
  Create a set of transitions $\widehat{T} = T_1$;
  while $(({}^\bullet\widehat{T})_N \neq \widehat{\Pi})$ do {

    $\widehat{\Pi} = \widehat{\Pi} \cup ({}^\bullet\widehat{T})_N$;
    $\widehat{T} = \widehat{T} \cup ({}^\bullet\widehat{\Pi})_N$;

  }
  Construct/draw the arcs in the set $\widehat{\Phi}$, where $\widehat{\Phi} = \{(\widehat{\Pi} \times \widehat{T}) \cup (\widehat{T} \times \widehat{\Pi})\} \cap \Phi$;
  Define the initial-marking $\widehat{\mathbf{m}}^0$ as follows: $\forall p \in \widehat{\Pi}$, $\widehat{\mathbf{m}}^0(p) = \mathbf{m}^0(p)$;

return($\widehat{N} = (\widehat{\Pi}, \widehat{T}, \widehat{\Phi}, \widehat{\mathbf{m}}^0)$);
}

Figure 1: The procedure $\widehat{N} =$ `Compute_Subnet`(N, Π_1, T_1), where $\widehat{N} = (\widehat{\Pi}, \widehat{T}, \widehat{\Phi}, \widehat{\mathbf{m}}^0)$.

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reference [6] it is shown that there exists a supervisory policy that enforces liveness in the PN N if and only if there exists a corresponding policy for the FCPN $\widetilde{N}$. The following result from [6] establishes the fact that there is a supervisory policy that enforces liveness in the PN N if and only if there is a corresponding policy for the FCPN $\widetilde{N}$.

Theorem 2.2 *[6] Let $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ be the free-choice equivalent of PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$. There exists a supervisory policy that enforces liveness in the PN N if and only if there exists a corresponding policy for the PN $\widetilde{N}$.*

Given an arbitrary PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$, and a subset of places $\Pi_1 \subseteq \Pi$, and transitions $T_1 \subseteq T$, we define a subnet $\widehat{N}(\Pi_1, T_1) = (\widehat{\Pi}, \widehat{T}, \widehat{\Phi}, \widehat{\mathbf{m}}^0)$, of the PN N, as described in the procedure outlined in figure 1. The PN $\widehat{N}$ is the subnet induced by the transitive closure of the input places and input transitions to T_1 and Π_1. Since the T and Π are finite-sets, the while-loop in the procedure of figure 1 is guaranteed to halt. We now present the main results of this paper.

3 Main Results

Let $N = (\Pi, T, \Phi, \mathbf{m}^0)$ be an arbitrary PN and let $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ be its Free-Choice equivalent. Theorem 3.1 implies there is a supervisory policy that enforces liveness in N only if every minimal siphon ${}^\bullet P \subseteq P^\bullet$ $(P \subseteq \widetilde{\Pi})$ that does not contain a trap, is not empty at the initial marking, and there is an input transition $\widehat{t}$ to some place in P (i.e. $\widehat{t} \in {}^\bullet P$) that has more than one output place in P (i.e. $card(\widehat{t}^\bullet \cap P) > 1$).

Theorem 3.1 *Let $N = (\Pi, T, \Phi, \mathbf{m}^0)$ be an arbitrary PN and let $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ be its Free-Choice equivalent. There exists a supervisory policy that enforces liveness in N only if:*

1. $\sum_{p \in P} \mathbf{m}^0(p) > 0$, *and*
2. $\exists\ \widehat{t} \in {}^\bullet P$ *such that* $card(\widehat{t}^\bullet \cap P) > 1$.

Proof: From theorem 2.2 we know there is a supervisory policy that enforces liveness in the completely controlled PN N if and only if there is a corresponding policy for its Free-Choice equivalent $\widetilde{N}$. To complete the proof, using a contrapositive argument we show that the conditions of the theorem are necessary for the existence of a policy that enforces liveness in $\widetilde{N}$.

Since ${}^\bullet P \subseteq P^\bullet$, if $\sum_{p \in P} \mathbf{m}^0(p) = 0$, then under any supervisory policy $\mathcal{P}$, it follows that $\forall\ \mathbf{m}^1 \in \Re(N, \mathbf{m}^0, \mathcal{P})$, $\sum_{p \in P} \mathbf{m}^1(p) = 0$. In turn this implies none of the transitions in $P^\bullet$ can be live under the supervision of $\mathcal{P}$.

First, we note that if P is minimal, $card({}^\bullet\widehat{t} \cap P) = 1, \forall \widetilde{t} \in {}^\bullet P$ (cf. observation 3 [7]). If $card(\widehat{t}^\bullet \cap P) = 1, \forall \widetilde{t} \in {}^\bullet P$, then the sum of the token loads of the places in P will remain unaltered following the firing of any transition $\widetilde{t} \in {}^\bullet P$. That is, $\sum_{p \in P} \mathbf{m}^1(p) = \sum_{p \in P} \mathbf{m}^2(p)$, where $\mathbf{m}^1 \rightarrow \widehat{t} \rightarrow \mathbf{m}^2$ and $\widehat{t} \in {}^\bullet P$. However, since $\widetilde{N}$ is a FCPN, this sum will decrease by unity every time a transition in $P^\bullet \Leftrightarrow {}^\bullet P$ fires. It follows that under any supervisory policy $\mathcal{P}$ the transitions in $P^\bullet \Leftrightarrow {}^\bullet P$ can fire only a finite number of times. Consequently, there can be no policy that enforces liveness in $\widetilde{N}$. Hence the second observation.

♣

The conditions of theorem 3.1 are not sufficient in general. The PN N shown in figure 2 is a FCPN. So, $\widetilde{N} = N$, and $\widetilde{N}$ has two minimal siphons P_1 $(= \{p_1, p_3, p_4, p_7, p_9\})$ and P_2 $(= \{p_2, p_5, p_6, p_8, p_9\})$ that do not contain traps. Additionally, $t_1 \in {}^\bullet P_1$, $t_2 \in {}^\bullet P_2$, $card(t_1^\bullet \cap P_1) > 1$, $card(t_2^\bullet \cap P_2) > 1$, and both minimal siphons are non-empty at the initial marking. From reference [8] we know there is a supervisory policy that enforces liveness in this FCPN if and only if $\exists\ \mathbf{m}^1, \mathbf{m}^2 \in \Re(N, \mathbf{m}^0)$, $\exists\ \sigma_1, \sigma_2 \in T^*$, such that $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1 \rightarrow \sigma_2 \rightarrow \mathbf{m}^2$, $\mathbf{m}^2 \geq \mathbf{m}^1$

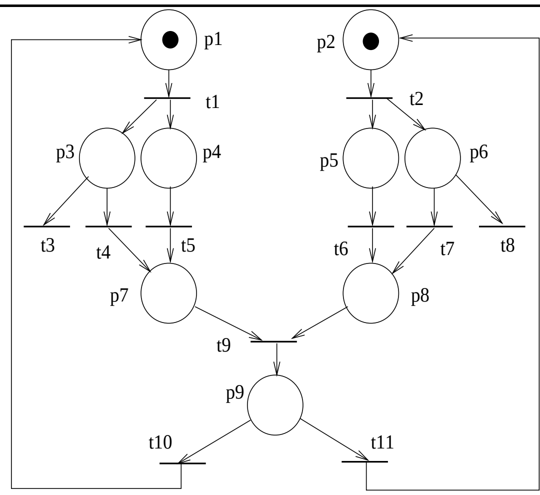

Figure 2: An illustration of the insufficiency of the conditions of theorem 3.1 for the existence of a supervisory policy that enforces liveness.

and all transitions in T appear at least once in σ_2. Since, $\mathbf{m}^2 = \mathbf{m}^1 + \mathbf{C}\mathbf{x}(\sigma_2)$, if $\mathbf{m}^2 \geq \mathbf{m}^1$ it follows that $\mathbf{C}\mathbf{x}(\sigma_2) \geq \mathbf{0}$. For the EPN shown in figure 2 however there does not exist a $\sigma_2 \in T^*$ with the required properties, therefore there can be no supervisory policy that enforces liveness even though the conditions of theorem 3.1 are satisfied. The fact that there is no string σ_2 with the desired properties can be inferred from Farkas Lemma: $\not\exists\ \mathbf{x} > 0$, such that $\mathbf{C}\mathbf{x} \geq 0 \Leftrightarrow \exists\ \mathbf{y} \geq 0$, such that $\mathbf{y}^T\mathbf{C} \leq 0$, and some element of $\mathbf{y}^T\mathbf{C}$ is strictly less than zero. For the incidence matrix $\mathbf{C}$ of the EPN N, we note that $\mathbf{y}^T\mathbf{C} = (0\ 0\ -1\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 0)$ if $\mathbf{y}^T = (2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 2)$. We note that there might be classes of EPNs, consequently classes of PNs whose Free-Choice equivalents belong to this class of EPNs, where the conditions of theorem 3.1 are also sufficient for the existence of a supervisory policy that enforces liveness. We suggest the identification of this class as a future research topic. We now state the main result of this paper.

Theorem 3.2 *Let $N = (\Pi, T, \Phi, \mathbf{m}^0)$ be an arbitrary PN and let $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ be its F-Choice equivalent. There exists a supervisory policy that enforces liveness in N if every minimal siphon $P \subseteq \widetilde{\Pi}$ in $\widetilde{N}$ that does not contain a marked trap, satisfies the following properties:*

1. *$\exists \hat{t} \in^{\bullet} P$ such that $card(\hat{t}^{\bullet} \cap P) > 1$ (cf. theorem 3.1),*

2. *The EPN $\widehat{N} = \mathtt{Compute\_Subnet}(\widetilde{N}, ({}^{\bullet}\hat{t})_{\widetilde{N}}, \{\hat{t}\})$ is live,*

3. The place $^\bullet\hat{t} \cap P$ *is unbounded in* $\hat{N}$, *and*

4. $\hat{\Pi}^\bullet \Leftrightarrow (P^\bullet \cup \hat{T}) = \emptyset$, *where* $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$.

Proof: Consider a transition $t \in P^\bullet \Leftrightarrow \hat{T}$, where $P \subseteq \tilde{\Pi}$ is some minimal siphon of $\tilde{N}$ that does not contain a marked trap. Using theorem 2.1 we show that the supervisory policy that permits the firing of the transition t only if

- the marking resulting from the firing of t in $\tilde{N}$ keeps the subnet of $\tilde{N}$ that is identical to $\hat{N}(^\bullet\hat{t}, \{\hat{t}\})$ live, and
- • the place $p = (^\bullet\hat{t})_{\tilde{N}} \cap P$ unbounded in this subnet at the new marking,

enforces liveness in $\tilde{N}$. Any transition that does not fit the description of t as listed above, is permanently control-enabled under the policy.

Under this supervisory policy the minimal siphons are never emptied. This observation can be established by the contrapositive argument. If a minimal siphon is emptied at some marking $\mathbf{m}^i$, the subnet of $\tilde{N}$ that is identical to $\hat{N}(^\bullet\hat{t}, \{\hat{t}\})$ would not be live at marking $\mathbf{m}^i$ and the supervisory policy would not permit the firing of the transition that resulted in the marking $\mathbf{m}^i$.

If a transition $t \in \tilde{T}$ is prevented from firing under the prescribed supervisory policy at some marking $\mathbf{m}^i$, then one of the two conditions in the statement of the policy must be violated under the new marking $\mathbf{m}^{i+1}$ arising from the firing of the transition t (i.e. $\mathbf{m}^i \rightarrow t \rightarrow \mathbf{m}^{i+1}$). It can be shown that there is always a marking $\mathbf{m}^j$, reachable from the marking $\mathbf{m}^i$, at which the transition t is both control and state-enabled. This observation can be established using conditions 1, 2 and 3 in the statement of the theorem.

As a consequence of theorem 2.1 we infer that the supervisory policy mentioned above does enforce liveness. Hence the result.

♣

Consider the PN N_1 shown in figure 3(a), this PN is not a FCPN as $card(p_8^\bullet) = 2$ and $^\bullet(p_8^\bullet) = \{p_8, p_6\} (\neq \{p_8\})$. Figure 3(b) contains the free-choice equivalent, N_2 (cf. [6]) of the PN N_1 shown in figure 3(a). There are three minimal siphons in the FCPN shown in figure 2(b): $P_1 = \{p_5, p_6\}$, $P_2 = \{p_1, p_3, p_4, p_7\}$ and $P_3 = \{p_2, p_3, p_4, p_5, p_7, p_8, p_9\}$. The siphon P_1 is also a trap and it is marked. The siphons P_2 and P_3 do not contain a trap, and therefore this FCPN is not live in the absence of supervision.

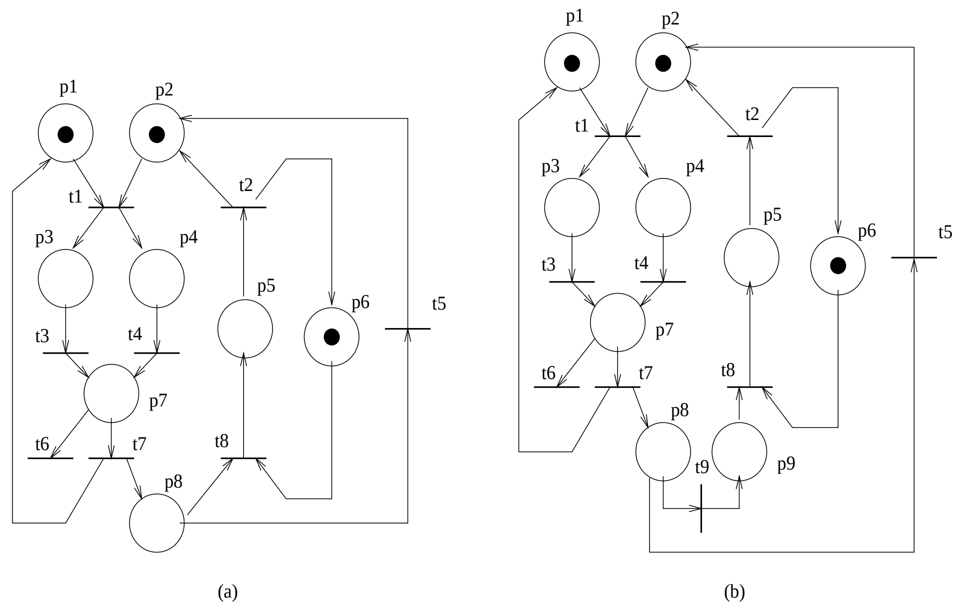

Figure 3: An illustration of theorem 3.2.

Since $t_1^\bullet = \{p_3, p_4\}$, the first requirement of theorem 3.2 is satisfied. The CPN $\widehat{N} = \texttt{Compute\_Subnet}(N_2, ({}^\bullet t_1)_{N_2}, \{t_1\})$ is the CPN N_2 without the transition t_6. The minimal siphons of this CPN are identical to that of N_2, but with the absence of the transition t_6, the siphon P_2 contains marked traps $\{p_1, p_3, p_7\}$ and $\{p_1, p_4, p_7\}$. Similarly, without the transition t_6, the siphon P_3 contains marked traps $\{p_2, p_3, p_5, p_7, p_8, p_9\}$ and $\{p_2, p_4, p_5, p_7, p_8, p_9\}$. By Commoner's Liveness Theorem we infer the CPN $\widehat{N}$ is live. Additionally, for P_1 (P_2) the place p_1 (p_2) is unbounded in $\widehat{N}$. This satisfies second and third requirements in the statement of theorem 3.2. It is also easy to verify the final condition of theorem 3.2. So, from theorem 3.2 we know the CPN N_2 can be made live via supervision, and from theorem 2.2 we infer the PN N_1 can also be made live via supervision. It is not hard to see that the supervisory policy that permits the firing of t_6 when the sum of the tokens in the place-set $\{p_1, p_3, p_4, p_7\}$ exceeds unity enforces liveness in N_1 and N_2. The CPN shown in figure 1, for which there is no supervisory policy that enforces liveness, does not satisfy the third requirement of theorem 3.2. We suggest investigations into specific CPN structures for which the conditions of theorem 3.2 are also necessary as a future research topic. In the following section we conclude with some additional future research directions.

4 Conclusions

Testing the existence of a supervisory policy that enforces liveness in an arbitrary, completely controllable *Petri net* (PN) is computationally expensive [8]. In this paper we identify a class of PNs for which there is a supervisory policy that enforces liveness. Having eliminated the need to test the existence of a supervisory policy that enforces liveness for this class of PNs, what remains is the synthesis of a policy that enforces liveness for these PNs. We suggest this synthesis as a future research topic. We surmise that the policy that enforces liveness in the free choice equivalent of the given PN, which is outlined in the proof of Theorem 3.2 of this paper, can be modified to enforce liveness in the original PN.

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