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Closed-loop model-free subspace-based LQG-design

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Abstract

When only input/output data of a system are available the classical way to design a linear quadratic Gaussian controller consists of mainly three separate parts. First a system identification step is performed to find the system parameters. With these parameters a Kalman filter is designed to find an estimate of the state of the system. Finally, this state is then used in an LQ-controller. In the literature these three steps are hardly ever considered as one joint identification/control problem. In (Favoreel *et al.*, 1998a), (Favoreel *et al.*, 1998b), (Favoreel *et al.*, 1998c) it was shown that, based on techniques from the field of subspace system identification, the three steps of the LQG-controller design can be replaced by a QR and a SV-decomposition. A drawback of the method is that the input and output data available for the LQG-design must be retrieved in open loop. In the present paper, a generalization of the results presented in (Favoreel *et al.*, 1998a), (Favoreel *et al.*, 1998b), (Favoreel *et al.*, 1998c) is presented for the case where the data is measured on a system working in closed-loop. It is shown that under mild conditions the closed-loop subspace-based controller and the classical LQG-controller are equivalent. The effectiveness of the method is illustrated by the hand of a simulation example. It is shown that the open-loop subspace-based LQG-controller gives biased results whereas the closed-loop version converges to the classical LQG-controller when the length of the backward horizon increases.

Keywords: *subspace identification, identification for control, LQG-control, optimal control, predictive control, Kalman filter, closed-loop.*

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1 Introduction

In this paper we will be interested in the design of an LQG-controller for linear time-invariant systems that can be described by the following state-space innovation form:

$$x_{k+1} = Ax_k + Bu_k + Ke_k, \tag{1}$$
$$y_k = Cx_k + Du_k + e_k \tag{2}$$

where the input $u_k \in \mathbb{R}^m$, the output $y_k \in \mathbb{R}^l$, the state $x_k \in \mathbb{R}^n$ and the stationary, ergodic, white Gaussian noise $e_k \in \mathbb{R}^l$ has the following covariance matrix:

$$\mathbf{E}[e_p e_q^T] = S\delta_{pq}.$$

This system is operating in closed-loop with a linear time-invariant controller that is described by the state-space equations:

$$x_{k+1}^c = A_c x_k^c + B_c y_k, \tag{3}$$
$$u_k = r_k - C_c x_k^c - D_c y_k. \tag{4}$$

We assume that the problem is well-posed in the sense that the output y_k should be uniquely determined by the reference input r_k, the disturbance input e_k and both the state of the controller x_k^c and the system x_k. This condition is satisfied when $I_l + DD_c$ is non-singular.

The main problem we are interested in is the following:

LQG-Control Problem

Given *a set of closed-loop measurements of the inputs u_k and the outputs y_k, $k \le 0$, of the unknown system (1)-(2) and the N first impulse response coefficients of the controller (3)-(4),* **find** *the input sequence $u_f = (u_1, \dots u_N)$ such that the following quadratic cost function J is minimized over the horizon N:*

$$J = \sum_{k=1}^{N} \widehat{y}_k^T Q_k \widehat{y}_k + u_k^T R_k u_k \tag{5}$$

where $\widehat{y}_k$ is the k-step-ahead predicted output given past inputs and outputs and future inputs up to time k. The reference output trajectory is described by r_k and the matrices $Q_k \in \mathbb{R}^{l\times l}$ and $R_k \in \mathbb{R}^{m\times m}$ are user-defined non-negative definite weightings of the outputs and the inputs.

We will call the *forward horizon* (length N) the number of time steps over which the system output is predicted and the control input is computed. The *backward horizon* (length M) is the set of past input/output data points that are used to predict the outputs on the forward horizon.

It should be noted that we use a receding-horizon approach which means that at every time-step the optimization of (5) is recalculated and only the first input u_1 of the calculated control sequence u_f is implemented.

As a lot of the modern control methods, LQG-control uses a state space model to design a control system. When such a model is not available, the LQG-design thus requires a first step of system identification from input/output data. In addition, once a state-space model is available, the

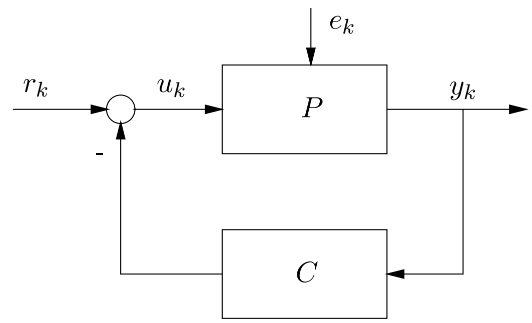

Figure 1: **The purpose of the present paper is to design an LQG-controller for the plant P which is operating in closed-loop with a controller C that is assumed to be time-invariant and not necessarily an LQG-controller. For the LQG-design we only dispose of a set of measurements of the controlled inputs u_k, the outputs y_k and the impulse response of the controller C.**

state of the system must be estimated using a step of Kalman filter design. Finally the third step then consists of designing an LQ-controller. In the literature the solution to the above problem is hardly ever considered as one identification/control problem. Most authors concentrate on either the identification of the unknown system given a set of input and output measurements or on the LQG-design given the system parameters A, B, C, D, K and S. In (Favoreel *et al.*, 1998b) it was shown that one can bypass these three steps by designing an LQG-controller directly from input/output data using techniques from the field of subspace system identification. This idea is illustrated in Figure 2. A constraint of the method presented in (Favoreel *et al.*, 1998b) is that the data is assumed to be retrieved in open-loop conditions. In this paper we will remove this restriction and generalize the results of (Favoreel *et al.*, 1998b) for systems that can operate in open as well as in closed-loop.

The idea of computing an LQG-controller directly from data, without any use of a model, has been developed by Hjalmarsson and collaborators (see (Hjalmarsson *et al.*, 1994), (Hjalmarsson *et al.*, 1998)) using a technique called Iterative Feedback Tuning (IFT) that is entirely different from the subspace-based technique developed here. In some ways, the fundamental difference between IFT-controller design and the subspace-based controller design developed here parallels the fundamental difference between parametric identification based on criterion minimization, and subspace-based identification based on subspace projections. Other recent work that is related to the results presented here was done by Furuta et al. (Furuta and Wongsaisuwan, 1995) where starting from the Markov parameters of the system, i.e. without knowledge of the state space matrices of the system thus without Kalman filter nor LQR-design, the Markov parameters of the LQG-controller are calculated. The problem of obtaining consistent estimates of the Markov parameters however is not addressed. A similar result was obtained by Kawamura (Kawamura, 1997), (Kawamura, 1998) which allows for the calculation of an LQR-controller based on a free response and an impulse response of the system. That method differs from the Furuta method in that it allows for the calculation of the LQR state feedback gain directly instead of a Markov parameter representation of the controller. A restriction of this approach is that it only calculates the LQR feedback gain and does not pay attention to the problem of finding an estimate of the state directly from the data without knowledge of the state space matrices. Moreover the method is iterative, which means that the user has to choose an initial value for the LQR state feedback gain. Only for an infinite control horizon the authors could prove that this feedback gain converges to the optimal value. As in (Furuta and Wongsaisuwan,

1995) the problem of obtaining a consistent estimate of the system impulse and free response is avoided.

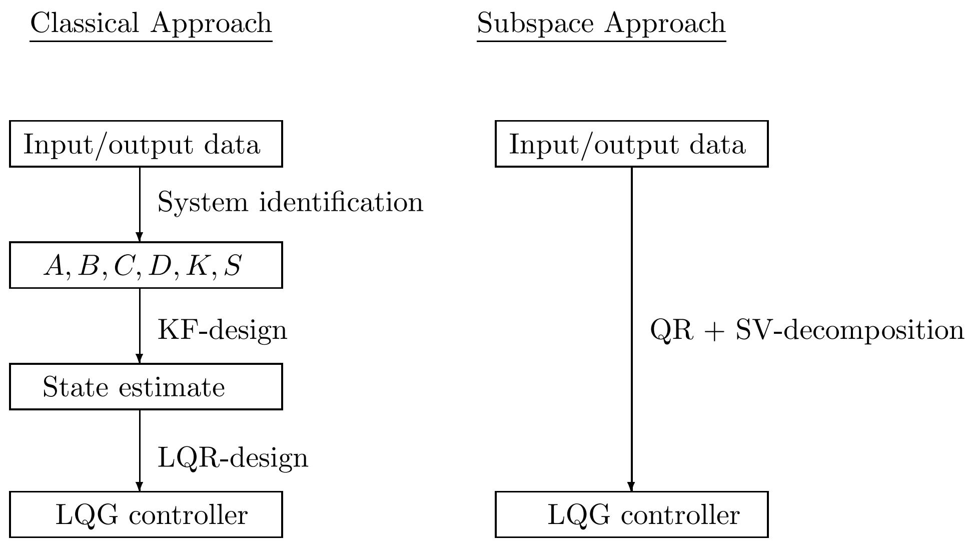

Figure 2: **Main result of the paper: based on techniques from the field of subspace system identification, a new algorithm is proposed for direct implementation of LQG-controllers starting from data. In the classical LQG-framework, first a system identification step is performed to find the state space matrices A, B, C, D, K and S. In a second step these matrices are used to design a Kalman filter which gives an estimate of the state. Finally, this state estimate is used in a linear quadratic controller. The main result of the paper is that these three steps can be short-circuited and replaced by one single QR and a SV-decomposition of matrices constructed out of input and output data of the system.**

The results in this paper are heavily based on subspace system identification theory (Van Overschee and De Moor, 1996b), (Verhaegen and Dewilde, 1992), (De Moor *et al.*, 1998). Most of the subspace system identification techniques one can find in the literature have been developed for open-loop applications. This implies that they give biased results when applied on data measured in closed-loop. Recently, a subspace system identification algorithm was proposed (Van Overschee and De Moor, 1996a), (Van Overschee and De Moor, 1997) that gives consistent estimates of systems operating in open as well as in closed-loop. Those results are used here to generalize the algorithms presented in (Favoreel *et al.*, 1998b) to find an LQG-controller for systems operating in open- as well as in closed-loop.

We start the paper by giving a short overview of the basic results of closed-loop subspace system identification (Van Overschee and De Moor, 1997). The basic idea of open and closed-loop subspace identification are recalled. Section 3 contains the main results of the paper. First the LQG-problem is presented, the classical framework of solving it is recalled and the new subspace-based approach is given. Further it is shown under what conditions both approaches give the same control law. In Section 4 we show by the hand of an example that the open-loop subspace-based algorithm gives indeed biased results under closed-loop conditions. We also illustrate that the closed-loop algorithm is unbiased and converges to the solution of the classical LQG-controller. We end the paper in Section 5 with some conclusions and open problems.

2 closed-loop subspace system identification

The problem treated in closed-loop linear subspace identification is the following:

Given *measurements of the inputs* u_k *and the outputs* y_k *of the unknown system (1)-(2) and the first* N *impulse response coefficients of the controller (3)-(4),* **find** *an estimate of the system matrices* A, B, C, D *and the noise related matrices* S *and* K.

The starting point of all subspace identification algorithms is the following set of matrix input-output equations:

$$Y_f \quad = \quad \Gamma_N X_f + H_N U_f + H_N^s E_f. \tag{6}$$

It represents the effect of the state x_k, the deterministic input u_k and the unknown stochastic input e_k on the outputs y_k and can be directly derived from the state-space equations (1)-(2). For the controller (3)-(4), an analogue equation holds:

$$U_f \quad = \quad R_f - \Gamma_N^c X_f^c - H_N^c Y_f. \tag{7}$$

In what follows, the different terms in (6) and (7) are defined.

First of all there are the data block-Hankel matrices:

$$Y_p = \begin{pmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_{M+2} & \cdots & y_{M+j} \\ \cdots & \cdots & \cdots & \cdots \\ y_{M-1} & y_{2M} & \cdots & y_{2M+j-2} \end{pmatrix}, \; Y_f = \begin{pmatrix} y_M & y_{M+1} & \cdots & y_{M+j-1} \\ y_{M+1} & y_{M+2} & \cdots & y_{M+j} \\ \cdots & \cdots & \cdots & \cdots \\ y_{M+N-1} & y_{M+N} & \cdots & y_{M+N+j-2} \end{pmatrix}. \tag{8}$$

The indices p and f stand for past and future. In a similar way the block-Hankel matrices U_p, U_f, E_p, E_f and R_p, R_f can be defined for the inputs u_k, the output noise e_k and the reference signal r_k. We will also use the following short-hand notation:

$$W_p = \begin{pmatrix} Y_p \\ U_p \end{pmatrix}.$$

It should be noted that, for statistical reasons, it is assumed that the number of columns j goes to infinity. The state sequences X_f and X_f^c of the system and the controller are defined as:

$$X_f = \begin{pmatrix} x_M & x_{M+1} & \ldots & x_{M+j-1} \end{pmatrix}, \; X_f^c = \begin{pmatrix} x_M^c & x_{M+1}^c & \ldots & x_{M+j-1}^c \end{pmatrix}. \tag{9}$$

Further we also have the following system related matrices:

$$H_N \quad = \quad \begin{pmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ CA^{N-2}B & CA^{N-3}B & \ldots & D \end{pmatrix}, \; H_N^s = \begin{pmatrix} I_l & 0 & \ldots & 0 \\ CK & I_l & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ CA^{N-2}K & CA^{N-3}K & \ldots & I_l \end{pmatrix},$$

$$\Gamma_N \quad = \quad \begin{pmatrix} C \\ CA \\ \ldots \\ CA^{N-1} \end{pmatrix}$$

where Γ_N is the extended observability matrix, H_N and H_N^s the matrices containing the impulse response of the system due to the deterministic input u_k and the unknown stochastic input e_k respectively. The analogue of these matrices for the controller are defined as:

$$\Gamma_N^c = \begin{pmatrix} C_c \\ C_c A_c \\ \dots \\ C_c A_c^{N-1} \end{pmatrix}, \; H_N^c = \begin{pmatrix} D_c & 0 & \dots & 0 \\ C_c B_c & D_c & \dots & 0 \\ \dots & \dots & \dots & \dots \\ C_c A_c^{N-2} B_c & C_c A_c^{N-3} B_c & \dots & D_c \end{pmatrix}$$

where Γ_N^c is the extended observability matrix and H_N^c the block Toeplitz matrix containing the impulse response coefficients of the controller.

Although there exist several subspace identification methods in the literature (Van Overschee and De Moor, 1996b), (Verhaegen and Dewilde, 1992), they all have the following three main steps in common:

Step 1: The first step of subspace identification problem can be interpreted as follows: given the past inputs and outputs W_p and the future inputs U_f, find a prediction of the future outputs Y_f. If we use a linear predictor:

$$\widehat{Y}_f = L_w W_p + L_u U_f \tag{10}$$

the least squares prediction $\widehat{Y}_f$ of Y_f can be found from the following least squares problem:

$$\min_{L_w, L_u} ||Y_f - \begin{pmatrix} L_w & L_u \end{pmatrix} \begin{pmatrix} W_p \\ U_f \end{pmatrix} ||_F^2. \tag{11}$$

Usually, the problem of finding $\widehat{Y}_f$ is formulated in terms of the orthogonal projection of the row space of Y_f into the row space spanned by W_p and U_f defined as:

$$\widehat{Y}_f = Y_f / \begin{pmatrix} W_p \\ U_f \end{pmatrix} = Y_f \begin{pmatrix} W_p \\ U_f \end{pmatrix}^\dagger \begin{pmatrix} W_p \\ U_f \end{pmatrix} \overset{\text{QR}}{=} \begin{pmatrix} L_w & L_u \end{pmatrix} \begin{pmatrix} W_p \\ U_f \end{pmatrix}. \tag{12}$$

The implementation of this projection can be done in a very fast and numerically robust way with a QR-decomposition (which is a standard Matlab command).

Step 2: The second step then consists in calculating the SVD of L_w, which is a rank deficient term (of order n) if the number of columns in the data block Hankel matrices is infinite ($j = \infty$). Due to the noise, L_w will not be a rank-deficient matrix in practise:

$$L_w \overset{\text{SVD}}{=} \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}.$$

To get rid of a part of the noise, L_w is approximated by a rank deficient matrix:

$$L_w \approx U_1 S_1^T V_1^T$$

where the rank is determined by inspecting the number of dominant singular values S_1. This number is an approximation of the order n of the system. Important is that, under the assumption that the number of columns in the data matrices is infinite ($j = \infty$), $U_1 S_1^{1/2}$ equals the observability matrix Γ_N while $S_1^{1/2} V_1^T W_p$ is a Kalman filter estimate of the state sequence X_f.

Step 3: The last step consists in finding the state space matrices A, B, C, D and K, S from Γ_N and/or $\widehat{X}_f$. One can distinguish three classes of algorithms: those that use the observability matrix Γ_N, those that use the estimate of the state sequence $\widehat{X}_f$ and finally those that use both. Since in this paper we only need the first two steps we do not go into details here. The interested reader is referred to (Van Overschee and De Moor, 1996b).

If the data is measured in open-loop, the above subspace identification scheme will work fine. However, if the data is recovered under closed-loop conditions, the results will be biased. The reason for this is that, due to the feedback, the regressors U_f are correlated with the residuals E_f. In that case the solution L_w, L_u of the least squares problem (11) will be asymptotically ($j \to \infty$) biased. This was shown in more details and illustrated with simulation examples in (Van Overschee and De Moor, 1996a), (Van Overschee and De Moor, 1997).

However, there is a way to get around this problem (Van Overschee and De Moor, 1997). The "trick" is to replace the future input Hankel matrix U_f by a new matrix M_f that is uncorrelated with the residuals E_f. The philosophy behind this approach is very similar to that of the well known instrumental variable method (See e.g. (Gustafsson, 1997)). Substituting the future input block Hankel matrix U_f in the system matrix input output equation (6) with U_f from the controller matrix input output equation (7) we have:

$$Y_f \quad = \quad T_N \left\{ \Gamma_N X_f + H_N M_f + H_N^s E_f \right\} \tag{13}$$

where we define $T_N = (I_{Nl} + H_N H_N^c)^{-1}$ and $M_f = U_f + H_N^c Y_f$. The matrix M_f can be directly calculated from its definition since we assumed that the first N impulse response coefficients of the controller (3)-(4), and therefore H_N^c, are known. Since M_f can also be written as $M_f = R_f - \Gamma_N^c X_f^c$ and $R_f E_f^T = 0, X_f^c E_f^T = 0$ it is easy to see (Van Overschee and De Moor, 1996a) that:

$$M_f E_f^T \quad = \quad 0.$$

In this sense M_f can be considered as a matrix containing the instrumental variables. The closed-loop subspace identification problem then consists of finding the linear prediction of the future outputs Y_f given the past inputs and outputs W_p and the instrumental variables M_f:

$$\begin{aligned} \widehat{Y}_f \quad &= \quad Y_f / \begin{pmatrix} W_p \\ M_f \end{pmatrix} \\ & L_w^c W_p + L_u^c M_f. \end{aligned} \tag{14}$$

A prediction $\widehat{Y}_f$ of the future output data Hankel matrix Y_f is provided by the solution L_w^c, L_u^c of the following least squares problem:

$$\min_{L_w^c, L_u^c} ||Y_f - \begin{pmatrix} L_w^c & L_u^c \end{pmatrix} \begin{pmatrix} W_p \\ M_f \end{pmatrix} ||_F^2. \tag{15}$$

which leads to asymptotically ($j \to \infty$) unbiased estimates of L_w^c and L_u^c. However, as we will see in Section 3.2, the matrices we are really interested in for the LQG-controller design are L_u and L_w of which only an asymptotically biased estimate could be found from the least squares problem (11) are asymptotically biased. It is however possible to calculate consistent estimates of L_w and L_u based on the knowledge of L_w^c, L_u^c and H_N^c. It can be seen from the results presented in (Favoreel *et al.*, 1998b) that if the backward horizon $M \to \infty$ we have that

$L_u = H_N$. It can be seen from the closed- and open-loop matrix input output equations (6), (10), (13) and (14) that L_w^c, L_u^c and L_w, L_u are related by the following equations:

$$\begin{aligned} L_w^c &= T_N L_w, \\ L_u^c &= T_N L_u. \end{aligned} \tag{16}$$

From the definition of T_N it can then be seen that the open loop parameters can be calculated as:

$$L_u = L_u^c(I_{Nl} - H_N^c L_u^c)^{-1}, \tag{17}$$
$$L_w = (I_{Nl} + L_u H_N^c)L_w^c. \tag{18}$$

3 LQG: classical vs. subspace approach

3.1 LQG - The classical approach

The classical way of solving the LQG-problem (5) as presented in the Introduction is to split it up into three separate subproblems: system identification, Kalman filter design and LQ-controller design (Kwakernaak and Sivan, 1972), (Bitmead *et al.*, 1990). These three steps can be summarized as follows:

System Identification: Given measurements of the inputs u_k and the outputs y_k of the unknown system (1)-(2), **find** an estimate of the system matrices A, B, C, D and the noise related matrices S and K. For the present paper it does not really matter what system identification has been used as long as an asymptotically unbiased estimate of A, B, C, D, K and S is found. This can be achieved for instance with subspace identification.

Kalman filter: Given the system related matrices A, B, C, D, K, S and measurements of the inputs u_k and outputs y_k for $k \in \{-M+1, \ldots, 0\}$ and the initial condition $\widehat{x}_{-M+1}$ for the Kalman filter **then** the steady-state Kalman filter state estimate $\widehat{x}_1$ is the solution for $q = 0$ to the following equations:

$$\widehat{x}_{q+1} = A\widehat{x}_q + Bu_q + K_f(y_q - C\widehat{x}_q - Du_q), \tag{19}$$
$$K_f = (KS + A\Sigma C^T)(S + C\Sigma C^T)^{-1}, \tag{20}$$
$$\Sigma = A\Sigma A^T + KSK^T - (KS + A\Sigma C^T)(S + C\Sigma C^T)^{-1}(KS + A\Sigma C^T)^T. \tag{21}$$

LQ-controller: Given the system related matrices A, B, C, D, the weighting matrices Q_k, R_k, the steady-state Kalman filter state prediction $\widehat{x}_1$ and the initial condition $P_0 = 0$ for LQR Riccati equation, **then** the input u_1 that minimizes the performance criterion (5) is the solution to the following recursive LQR equations:

$$u_1 = L\widehat{x}_1, \tag{22}$$
$$L^T = -(C^T Q_1 D + A^T P_N B)(R_1 + D^T Q_1 D + B^T P_N B)^{-1}, \tag{23}$$
$$\begin{aligned} P_{q+1} = {}& A^T P_q A + C^T Q_{N-q} C - (C^T Q_{N-q} D + A^T P_q B) \\ & (R_{N-q} + D^T Q_{N-q} D + B^T P_q B)^{-1}(C^T Q_{N-q} D + A^T P_q B)^T. \end{aligned} \tag{24}$$

3.2 LQG - A subspace approach

In (Favoreel *et al.*, 1998b) the LQG-problem was studied from another point of view. There it was shown that the LQG-control law (19)-(24) can be written as:

$$u_f = -(R + L_u^T Q L_u)^{-1} L_u^T Q L_w w_p. \tag{25}$$

where

$$w_p = \begin{pmatrix} y_p \\ u_p \end{pmatrix}$$

with $u_p \in \mathbb{R}^{Mm}$ and $y_p \in \mathbb{R}^{Ml}$ the M last known values of the inputs and the outputs, and u_f the calculated N first control steps (of which only the first one will be implemented):

$$y_p = \begin{pmatrix} y_{-M+1} \\ \vdots \\ y_{-1} \\ y_0 \end{pmatrix}, \; u_p = \begin{pmatrix} u_{-M+1} \\ \vdots \\ u_{-1} \\ u_0 \end{pmatrix}, \; u_f = \begin{pmatrix} u_1 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix}.$$

The controller parameters L_u and L_w are calculated as in (11) for the open-loop case, or (17)-(18) for the closed-loop case. The matrices $Q \in \mathbb{R}^{Nl \times Nl}$ and $R \in \mathbb{R}^{Nm \times Nm}$ are defined as:

$$Q = \begin{pmatrix} Q_1 & 0 & \dots & 0 \\ 0 & Q_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_N \end{pmatrix} \quad , \quad R = \begin{pmatrix} R_1 & 0 & \dots & 0 \\ 0 & R_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R_N \end{pmatrix}.$$

If we look at the subspace-based LQG control law (25) on the one hand and the classical LQG control laws (21)-(22) on the other hand, one might wonder if they are equivalent. The answer is given in the following theorem of which a proof can be found in (Favoreel *et al.*, 1998b).

Theorem 1
The subspace-based LQG-controller (25) and classical LQG-controller equations (19)-(24) produce the same value for u_1 if the **backward** *horizon is infinite ($M \to \infty$).*

The final subspace-based LQG-algorithm can now be implemented as in Figure 3.

We make the following final remarks:

- Although the results presented here are clearly based on techniques from the field of subspace system identification, we deliberately avoided the term "subspace system identification". Indeed, the system identification step is inherent in the way of solving the control problem in the sense that the system parameters A, B, C, D, K and S are never explicitly calculated.

- Except for the user-defined parameters Q and R, the different steps in the design of the LQG-controller, i.e. the system identification step, the Kalman filter and the LQ-controller, are replaced by a QR- and a SV-decomposition (see also Figure 2).

Closed-loop subspace-based LQG-controller

1. Construct the data block-Hankel matrices Y_f, U_f and $W_p = \binom{Y_p}{U_p}$ from the data. Also calculate the matrix $M_f = U_f + H_N^c Y_f$.

2. Make the following projection and derive (preferably with a QR-decomposition) the parameters L_w^c and L_u^c:

$$Y_f / \begin{pmatrix} W_p \\ M_f \end{pmatrix} \overset{\text{QR}}{=} L_w^c W_p + L_u^c M_f.$$

3. Derive the open-loop predictor parameters:

$$\begin{aligned} L_u &= L_u^c (I_{Nl} - H_N^c L_u^c)^{-1}, \\ L_w &= (I_{Nl} + L_u H_N^c) L_w^c. \end{aligned}$$

4. Approximate L_w by a rank-n matrix by taking the singular value decomposition i.e.:

$$L_w \overset{\text{SVD}}{=} U_1 S_1 V_1^T.$$

An estimate of the system order n can be found by inspecting the number of dominant singular values.

5. Construct the controller inputs:

$$w_p = \left(\begin{array}{ccc|ccc} y_{-M+1}^T & \dots & y_{-1}^T \;\; y_0^T & u_{-M+1}^T & \dots & u_{-1}^T \;\; u_0^T \end{array} \right)^T.$$

6. Implement the first input u_1 of the LQG-control sequence u_f:

$$u_f = -(R + L_u^T Q L_u)^{-1} L_u^T Q L_w w_p.$$

7. To implement the following control step, measure the system output y_1 and repeat from step 5 i.e. calculate the new controller inputs with y_1, which are simply the previous controller inputs shifted one time step.

Figure 3: **Algorithm for the subspace-based LQG-controller. The first 4 steps, from which the controller parameters L_w and L_u are derived, only have to be performed once. The next 3 steps represent the implementation of the controller.**

- Contrary to the classical LQG-framework the extension to output tracking is straightforward. The control law (25) simply becomes:

$$u_f = (R + L_u^T Q L_u)^{-1} L_u^T Q (r_f - L_w w_p)$$

where r_f is the a vector containing the output reference trajectory $r_f = [r_1^T \; r_2^T \; \dots \; r_N^T]$.

4 Application

Let us now use an example to compare the presently discussed closed loop algorithm with the previously presented open-loop algorithm (Favoreel *et al.*, 1998b). The example is partially borrowed from (Hakvoort, 1990) and is also used in (Verhaegen, 1993) and (Van Overschee and De Moor, 1996a). The plant corresponds to a discrete time model of a laboratory plant setup of two circular plates rotated by an electrical servo motor with flexible shafts. The state space matrices of the model (1)-(2) are:

$$A = \begin{pmatrix} 4.40 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4.00 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.0033 \\ -0.00002 \end{pmatrix}, \; C^T = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \; K = \begin{pmatrix} 2.3 \\ -6.64 \\ 7.515 \\ -4.0146 \\ 0.86336 \end{pmatrix},$$

$D = 0$ and e_k is a Gaussian zero mean white noise sequence with $\mathbf{E}[e_k^2] = 1/9$. Note that the plant has an integrator and is therefore marginally stable. The configuration of model and controller is the one depicted in Figure 1. The controller has a state space description as in (3)-(4) with:

$$A_c = \begin{pmatrix} 2.65 & -3.11 & 1.75 & -0.39 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \; B_c = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \; C_c^T = \begin{pmatrix} -0.4135 \\ 0.8629 \\ -0.7625 \\ 0.2521 \end{pmatrix}$$

and $D_c = 0.61$. The reference signal r_k is a Gaussian zero mean white noise sequence with variance 1 (Figure 4). We take the number of data points $j = 1000$ and the future horizon in the LQG criterion $N = 10$. Note that N is also the number of block rows in the Hankel matrices of future data Y_f and U_f. Different values for the backward horizon M where used. On this data set the following three algorithms were applied:

Classical: the classical algorithm to solve the LQG-control problem as described in Section 3.1. An important note is that, for the classical LQG-controller, we did not identify the state space matrices A, B, C, D, K, S from the data but assumed that they were known. This was done since Theorem 1 assumes assymptotic conditions ($M, j \to \infty$).

Open-loop: the open-loop version of the closed-loop algorithm presented in this paper (Favoreel *et al.*, 1998b). The closed-loop data was used in the open-loop algorithm.

Closed-loop: the closed-loop algorithm of Figure 3.

Figure 5 compares the response of the controlled system with the classical, the open-loop subspace-based and the closed-loop subspace-based LQG controller.

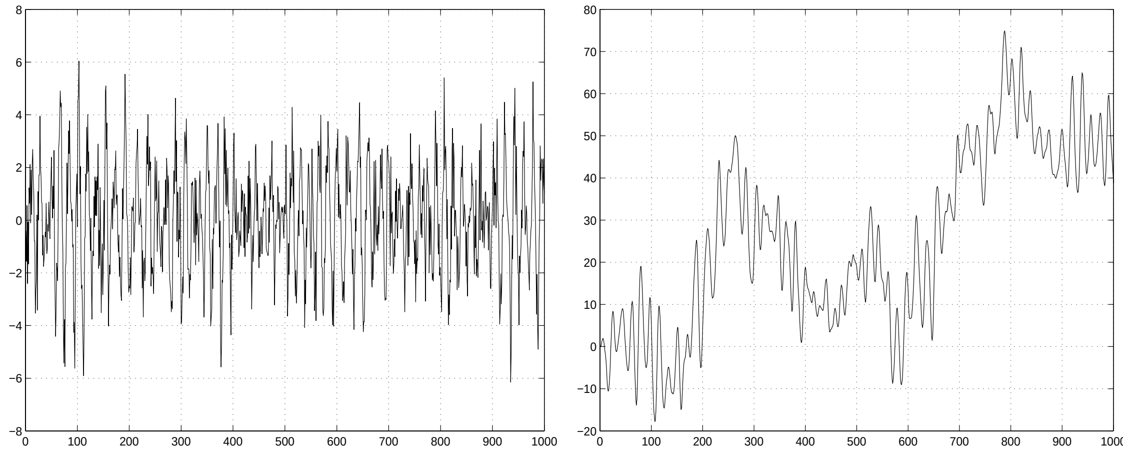

Figure 4: **Left: the reference input signal r_k is a white zero mean stationary signal, Right: the corresponding output signal y_k. One can clearly see a trend on the output which is due to the integrator in the system.**

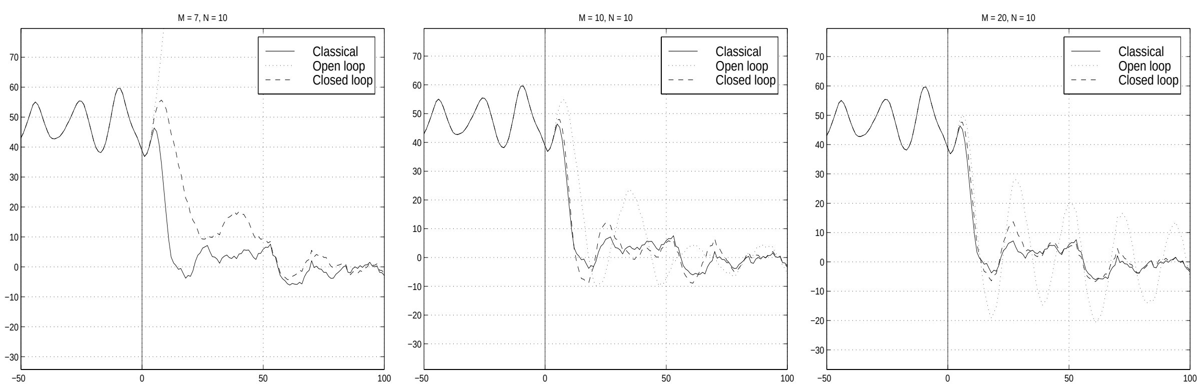

Figure 5: **The above figures compare the response of the controlled system with the classical (full line), the open-loop subspace-based (dotted line) and the closed-loop subspace-based (dashed line) LQG-controller. The comparison is made for different values of the backward horizon ($M = 7, 10, 20$) and a constant forward horizon ($N = 10$). The controller is calculated based on the data up to time step 0 (vertical line). From time step 1 on the LQG controller is implemented. The reference signal r_k for $k > 0$ is equal to zero. One can see that the closed-loop subspace-based controller converges to the classical LQG-controller as the size of the backward horizon M increases (See Theorem 1). It is clear that, due to the fact that the data was measured in closed-loop, the open-loop subspace-based LQG-controller is biased and may even become unstable when the backward horizon is not large enough.**

5 Conclusions

In this paper we have presented an algorithm for the calculation of LQG-controllers of linear systems directly from input/output data, i.e. without a plant model. It is a direct generalization of a previously presented algorithm (Favoreel *et al.*, 1998b) where it was assumed that the data is measured under open-loop conditions. Here, this assumption is removed and the algorithm now also holds for data measured on systems under time-invariant linear feedback. Even though the derivation is based on expressions from closed-loop subspace system identification theory, the algorithm bypasses the identification step altogether. The main result is that given input/output data of the system, one can directly derive the LQG-controller parameters from one single QR and a SV-decomposition. It was proven that in the case the backward horizon is infinite, the model-free subspace-based LQG-controller is equivalent to the classical LQG-controller. The theoretical results where confirmed when applied on a simulation example: the open-loop subspace-based control algorithm is biased (due to the feedback) whereas the closed-loop version converges to the classical LQG-controller as the backward horizon increases.

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