2-Sliding Mode with Adaptation*

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Abstract

Sliding mode is used in order to retain a dynamic system accurately at a given constraint and is the main operation mode in variable structure systems. Such mode is a motion on a discontinuity set of a dynamic system and features theoretically-infinite-frequency switching. The standard sliding modes are known to feature finite-time convergence, precise keeping of the constraint and robustness with respect to internal and external disturbances. In realization their sliding precision is proportional to the time interval between measurements. Having generalized the notion of sliding mode, higher order sliding modes preserve or generalize its main properties and remove the chattering effect. With discrete measurements they may provide for up to the rth order of sliding precision with respect to the measurement interval. The main implementation problem of these modes is the information demand growing with the sliding order. If the aim is to nullify some output variable σ then r-sliding mode realization generally requires measurements of the time derivatives of up to the (r-2)th order of σ to be available. A new approach demonstrated in the paper provides for 3-sliding accuracy realization while only σ itself is available. That is the first controller of such kind.

1 Introduction

One of the most important control problems is control under heavy uncertainty conditions. While there are a number of sophisticated methods like adaptation based on identification and observation, or absolute stability methods, the most obvious way to withstand the uncertainty is to keep some constraints by "brutal force". Indeed any strictly kept equality removes one "uncertainty dimension". The most simple way to keep a constraint is to react immediately to any deviation of the system stirring it back to the constraint by a sufficiently energetic effort. Directly implemented the approach leads to so-called sliding modes, which became main operation modes in the variable structure systems (VSS) (Utkin, 1992; Zinober, 1994). Having proved their high accuracy and robustness with respect to various internal and external disturbances, they also reveal their main drawback: the so-called chattering effect, i.e. dangerous high-frequency vibrations of the controlled system. Such an effect was considered as an

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obvious intrinsic feature of the very idea of immediate powerful reaction to any miserable deviation from the chosen constraint. Another important feature is proportionality of the maximal deviation from the constraint to the time interval between the measurements (or to the switching delay).

To avoid chattering some approaches were proposed. The main idea is to change the dynamics in a small vicinity of the discontinuity surface in order to avoid real discontinuity and at the same time to preserve the main properties of the whole system. Recently invented higher order sliding modes (HOSM) generalize the basic sliding mode idea acting on the higher order time derivatives of the system deviation from the constraint instead of influencing the first deviation derivative like it happens in standard sliding modes. Keeping the main advantages of the original approach, at the same time they totally remove the chattering effect and provide for higher accuracy in realization. A number of such controllers were described in the literature (Emelyanov *et al.*, 1986; Levant, 1993, 1998b; Bartolini *et al*, 1998a,b).

HOSM is actually a movement on a discontinuity set of a dynamic system understood in Filippov's sense (1988). The sliding order characterizes the dynamics smoothness degree in the vicinity of the mode. If the task is to provide for keeping a constraint given by equality of a smooth function σ to zero, the sliding order is a number of continuous total derivatives of σ (including the zero one) in the vicinity of the sliding mode. Hence, the *r*th order sliding mode is determined by the equalities

$$\sigma = \dot{\sigma} = \ddot{\sigma} = ... = \sigma^{(r-1)} = 0. \qquad (1)$$

forming an *r*-dimensional condition on the state of the dynamic system. The words "*r*th order sliding" are often abridged to "*r*-sliding".

The standard sliding mode on which most variable structure systems (VSS) are based is of the first order ($\dot{\sigma}$ is discontinuous). While the standard modes feature finite time convergence, convergence to HOSM may be asymptotic as well. *r*-sliding mode realization may provide for up to the *r*th order of sliding precision with respect to the measurement interval (Levant, 1993, 1998b). In that sense *r*-sliding modes play the same role in sliding mode control theory as Runge - Kutta methods in numerical integration. Note that such an utmost accuracy is observed only for HOSM with finite-time convergence.

Trivial cases of asymptotically stable HOSM are easily found in many classic VSSs. For example there is an asymptotically stable 2-sliding mode with respect to the constraint $x = 0$ at the origin $x = \dot{x} = 0$ (at one point only) of a 2-dimensional VSS keeping the constraint $x + \dot{x} = 0$ in a standard 1-sliding mode. Asymptotically stable or unstable HOSMs inevitably appear in VSSs with fast actuators (Fridman *et al.*, 1996). Stable HOSM reveals itself in that case by spontaneous disappearance of the chattering effect. Thus, examples of asymptotically stable or unstable sliding modes of any order are well known (Emelyanov *et al.*, 1986; Elmali *et al.*, 1992; Sira-Ramírez, 1993; Levant, 1993; Fridman *et al.*, 1996). On the contrary, examples of *r*-sliding modes attracting in finite time are known for $r = 1$ (which is trivial), for $r = 2$ (Levantovsky, 1985; Emelyanov *et al.*, 1986; Levant (Levantovsky), 1993; Bartolini *et al.*, 1998a,b) and for $r = 3$ (Fridman *et al.*, 1996). Arbitrary order sliding controllers with finite-time convergence were only recently presented (Levant, 1998b). Any new type of higher-order sliding controller with finite-time convergence is unique and requires thorough investigation.

The main problem in implementation of HOSMs is increasing information demand. Generally speaking, any *r*-sliding controller needs $\sigma, \dot{\sigma}, ..., \sigma^{(r-1)}$ to be available. The only known exclusion is a 2-sliding controller (Levant, 1993, 1998a) which needs only measurements of σ. First differences of $\sigma^{(r-2)}$ having been used, measurements of σ, $\dot\sigma$, ... , $\sigma^{(r-2)}$ turned out to be sufficient, which solves the problem only partially. A recently published robust exact differentiator with finite-time convergence (Levant, 1998a) allows to solve that problem in theoretical way. In practice, however, the differentiation error occurs to be proportional to $\varepsilon^{(2^{-k})}$, where $k < r$ is the differentiation order and ε is the maximal measurement error of σ. Yet the optimal one is proportional to $\varepsilon^{(r-k)/r}$ ($\sigma^{(r)}$ is supposed to be discontinuous, but bounded (Levant, 1998a)). Nevertheless, there is another way to approach HOSM.

It was mentioned above that r-sliding mode realization provides for up to the rth order of sliding precision with respect to the switching delay τ, but also the opposite is true (Levant, 1993): keeping $|\sigma| = O(\tau^r)$ implies $|\sigma^{(i)}| = O(\tau^{r-i})$, i = 0, 1, ..., r-1, to be kept, if $\sigma^{(r)}$ is bounded. Thus, keeping $|\sigma| = O(\tau^r)$ corresponds to approximate r-sliding. Algorithm providing for fulfillment of such relation in finite time, independent on τ, is called rth order real sliding algorithm (Levant, 1993). Few second order real sliding algorithms (Levant, 1993; W.-C. Su *et al.*, 1994) differ from 2-sliding controllers with discrete measurements. All known till now rth order real sliding algorithms require measurements of σ, $\dot\sigma$, ... , $\sigma^{(r-2)}$ with $r > 2$.

A third order real sliding controller utilizing only measured values of σ is first time presented in this paper. The main idea is to derive it from a known 2-sliding controller with discontinuous time derivative of control. Using the differentiator (Levant, 1998a), an estimation of the equivalent-control time derivative is achieved in real time. Having represented time derivative of control as a sum of that estimation and a discontinuous component, the magnitude of the latter one may be made infinitesimally small by persistent adaptation of the parameters. While only one controller - twisting algorithm (Levant, 1993)- is used in the present paper, the results are easily extended to sub-optimal 2-sliding controller (Bartolini *et al.*, 1998a,b).

2 Generalized constraint fulfillment problem

Our intention is to replace the standard relay algorithm u = -sign σ by a continuous output of some dynamic subsystem. To simplify and detail the constraint fulfillment problem, consider the dynamic system given by the equation

$$\dot x = f(t,x,u) \tag{2}$$

where $x\in X$ is a state variable, X is a smooth finite dimensional manifold, t is time, $u\in \boldsymbol{R}$ is control, f is a C^2-function. Let $\sigma(t,x)\in \boldsymbol{R}$ be a C^3-function. The only available current information consists of the current values of t, $u(t)$ and $\sigma(t)$ ($\sigma(t) = \sigma(t,x(t))$). There is also a number of known constants defined below. The goal is to force the constraint function σ to vanish in finite time by means of control continuously dependent on time.

It is easy to check that $\dot\sigma(t)$, $\ddot\sigma(t)$ and $\dddot\sigma(t)$ may be expressed in the form

$$\dot\sigma(t,x,u) = \sigma'_t(t,x) + \sigma'_x(t,x)f(t,x,u) ;$$
$$\ddot\sigma = \xi_0(t,x,u) + \dot\sigma'_u(t,x,u)\dot u, \quad \dot\sigma'_u = \sigma'_x f'_u ;$$
$$\dddot\sigma = \xi_1(t,x,u) + \xi_2(t,x,u)\dot u + \xi_3(t,x,u)\dot u^2 + \dot\sigma'_u(t,x,u)\ddot u .$$

Let K_m , K_M , C_i , i = 0, 1, 2, 3, be positive constants, $K_m < K_M$, and assume the following:

1. $|u| \le \kappa$, κ = const > 1. Any solution of (2) is well defined for all t, provided $u(t)$ is continuous and $|u(t)| \le \kappa$ for each t.

2. There exists $u_1 \in (0,1)$ such that for any continuous function $u(t)$ with $|u(t)| \ge u_1$, there is t_1, such that $\sigma(t)u(t) > 0$ for each $t > t_1$. Hence, the control $u(t)$ = - sign $\sigma(t_0)$, where t_0 is the initial value of time, provides hitting the manifold $\sigma = 0$ in finite time.

3. There are positive constants σ_0, $u_0 < 1$, K_m, K_M such that if $|\sigma(t,x)| < \sigma_0$ then

$$0 < K_m \le \frac{\partial}{\partial u}\, \dot{\sigma}\,(t,x,u) \le K_M$$

for all u, $|u| \le \kappa$, and the inequality $|u| > u_0$ entails $\dot{\sigma}\, u > 0$.

4. Within the region $|\sigma| < \sigma_0$ for all t, x, and u the inequality $|\xi_0| < C_0$ holds. It means that the second time derivative of constraint function σ which is calculated with fixed values of control u, is uniformly bounded.

5. It is required for $|\xi_1|$, $|\xi_2|$, $|\xi_3|$ to be uniformly bounded by the constants C_1, C_2, C_3 within the region $|\sigma| < \sigma_0$. In particular, the third time derivative of constraint function σ is uniformly bounded, when $\dot{u}$, $\ddot{u}$ are bounded.

It follows from the theorem on implicit function that there is a function $u_{eq}(t,x)$ (equivalent control, Utkin, 1992) satisfying the equation $\dot{\sigma}$ = 0. Once $\sigma = 0$ is already achieved, the control $u = u_{eq}(t,x)$ would provide for exact constraint fulfillment. Condition 3 means that $|\sigma| < \sigma_0$ implies $|u_{eq}| < u_0 < 1$, and that the velocity of u_{eq} changing is bounded. It follows from conditions 4, 5 that $\dot{u}_{eq}$ is bounded and $\ddot{u}_{eq}$ is bounded with bounded $\dot{u}$. That provides for a possibility to approximate u_{eq} by a Lipschitzian control, which in its turn may undergo real-time differentiation (Levant, 1998a). Note, also, that linear dependence on control u is not required.

The proposed controllers depend on few constant parameters. These parameters are to be tuned in order to control the whole class of processes and constraint functions defined by the concrete values of σ_0, K_M, K_m, C_i. By increasing the constants K_M, K_m, C_i and reducing σ_0 at the same time, we enlarge the controlled class too. Such algorithms are obviously insensitive to any model perturbations and external disturbances which do not stir the dynamic system from the given class.

The variable structure system theory deals usually with systems of the form $\dot{x} = a(t,x) + b(t,x)\, v$, where $x \in \mathbf{R}^n$, v is control. Under conventional assumptions the task of keeping the constraint $\varphi(t,x)$=0 is reduced to the task stated above. A new control u and a constraint function σ are to be defined in this case by the transformation

$$v = k\Phi(x)u, \quad \sigma = \varphi(t,x)/\Phi(x), \quad \Phi(x) = \sqrt{xDx^t + h}\,, \tag{3}$$

where k, h > 0 are constants, D is a non-negative definite matrix.

In the simple case when $\dot{x} = A(t)x + b(t)u$, $\varphi = c(t)x + \xi(t)$ all conditions are reduced to the boundedness of $c, \dot{c}, \ddot{c}, \dddot{c}, \dot{\xi}, \ddot{\xi}, \dddot{\xi}, A, \dot{A}, \ddot{A}, b, \dot{b}, \ddot{b}$ and to the inequality $cb > \text{const} > 0$ (i.e., the relative degree equals 1). The corresponding constants determine the controlled class.

3 2-sliding adaptation

Consider some 2-sliding algorithm of the form $\dot{u} = R\cdot S(\sigma, \dot{\sigma})$ where $R > 0$ characterizes the magnitude of $\dot{u}$, S is some discontinuous function. In particular, two specific controllers are considered: the twisting algorithm (Levant, 1993) and sub-optimal algorithm (Bartolini *et al.*, 1998a,b). The twisting controller is given by

$$\dot{u} = R\cdot S(\sigma, \dot{\sigma}), \tag{4}$$

$$S = \begin{cases} -u, & |u| > 1, \\ -\alpha_M \operatorname{sign} \sigma, & \sigma\dot{\sigma} > 0, |u| \le 1, \\ -\alpha_m \operatorname{sign} \sigma, & \sigma\dot{\sigma} \le 0, |u| \le 1, \end{cases} \tag{5}$$

where $\alpha_M > \alpha_m > 0$. It is known that finite-time convergence into 2-sliding mode $\sigma = \dot{\sigma} = 0$ is ensured if $R\cdot\alpha_m > 4K_M/\sigma_0$, $R\cdot\alpha_m > C_0/K_m$, $R\cdot K_m\alpha_M - C_0 > R\cdot K_m\alpha_M + C_0$. Any admissible value of u may be taken here as an initial value. Trajectories twist around the second order sliding manifold $\sigma = \dot{\sigma} = 0$ converging to it in finite time.

Suppose that the above controller caused convergence of the system into 2-sliding mode. In that mode the identity $u = u_{eq}$ holds. According to condition 5, $\ddot{u}_{eq}$ is bounded, if R is *a priori* less than some maximal value R_M. That allows real-time robust differentiation (Levant, 1998a) of u producing some signal v, $v = \dot{u}_{eq}$ after a finite-time transient process. Let now the control be given by

$$\dot{u} = R\cdot S(\sigma, \dot{\sigma}) + v, \tag{6}$$

where v is zero before the real 2-sliding is attained and is above-described estimation of $\dot{u}_{eq}$ otherwise. It follows from conditions 3-5 that $\ddot{\sigma} = \dot{\sigma}'_u \cdot(\dot{u} - \dot{u}_{eq})$. Thus, any small R will keep the 2-sliding when $v = \dot{u}_{eq}$, and, with infinitesimally small R, $\ddot{\sigma} \approx 0$. That actually fulfills approximately 3-sliding condition $\sigma = \dot{\sigma} = \ddot{\sigma} = 0$. In practice, the differentiator is to be applied already in some small vicinity of the 2-sliding mode. Thus, $u(t)$ is to be considered as a real-time approximation of unknown signal $u_{eq}(t)$, obtained with an infinitesimal ”measurement error”. Consequently, the differentiator produces a robust evaluation of $\dot{u}_{eq}(t)$ being exact in the absence of the ”measurement errors”.

We distinguish the notions of “ideal sliding mode” when the motions satisfy the strict equality $\sigma = 0$, and “real sliding mode” (Utkin, 1992), when the equality is kept only approximately. Respectively, real r-sliding mode corresponds to approximate r-sliding condition. On the other hand, rth order real sliding (Levant, 1993) with respect to ε corresponds to the condition $\sigma = O(\varepsilon^r)$, where $\varepsilon \to 0$ is a parameter of the controller.

Hence, a following scheme (Fig. 1) is received. A differentiator unit D yields an estimation v of $\dot{u}_{eq}$. Some adaptation unit A carefully reduces R keeping real 2-sliding mode and provides for fast increase of R if the sliding mode is lost. That scheme requires obviously some real-time criterion of real 2-sliding.

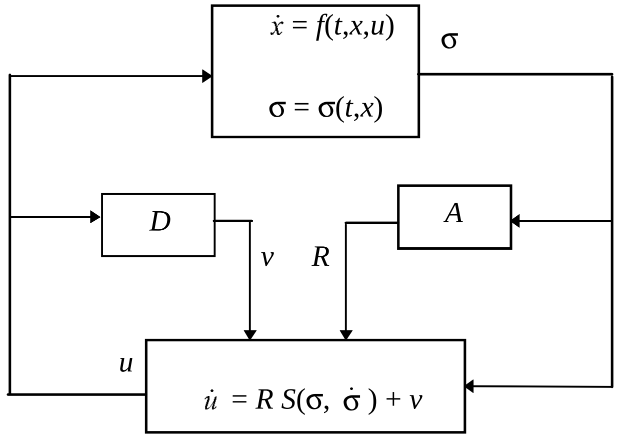

Fig. 1: Adaptation scheme

Real-sliding criterion. Let $T > 0$ and N_{sw} be the number of switchings of sign σ during the time interval T. It is easy to see that $N_{\mathrm{sw}} = 2n + 1$ corresponds to n trajectory-projection rotations around the origin in the plane $\sigma, \dot{\sigma}$ during the time T. Fix any integer $k > 2$. Real 2-sliding criterion is recognized as true if $N_{\mathrm{sw}} \geq k$.

Adaptation rule. Parameter R is changed at the end of every successive interval T:

$$R_{i+1} = \begin{cases} \max(R_i - \Lambda T, 0), & N_{\mathrm{sw}} \geq k, \\ \min(R_i + \Lambda T, R_{\mathrm{M}}), & N_{\mathrm{sw}} < k \end{cases} \tag{7}$$

Here $R_{\mathrm{M}}, \Lambda > 0$.

Differentiator (Levant, 1998a). Let

$$\dot{z} = v_1 = -\lambda |z-u|^{1/2} \operatorname{sign}(z-u) + z_1, \tag{8}$$

$$\dot{z}_1 = -\alpha \operatorname{sign}(z-u). \tag{9}$$

v is an output of the following nonlinear smoothing filter:

$$\dot{v} = \begin{cases} -v, & |v| > v_{\mathrm{M}} \\ -\beta \operatorname{sign}(v - v_1), & |v| \leq v_{\mathrm{M}} \end{cases} \tag{10}$$

where $v_{\mathrm{M}}, \beta > 0$.

Initial conditions: $R = R_{\mathrm{M}}$, $z = u$, $z_1 = 0$. Any admissible value of u may be taken.

Parameter choice. R_{M} is to be chosen first. It may be taken much larger than needed in order to ensure convergence of controller (4), (5). Also $v_{\mathrm{M}} > \sup|\dot{u}_{\mathrm{eq}}|$, $\beta > \sup|\ddot{u}_{\mathrm{eq}}|$, $\Lambda\alpha_{\mathrm{m}} > \sup|\ddot{u}_{\mathrm{eq}}|$ are to be ensured. One of the many ways to choose λ and α (Levant, 1998a) is to take $\alpha = 1.1 \sup|\ddot{u}_{\mathrm{eq}}|$, $\lambda = 1.5 \sup|\ddot{u}_{\mathrm{eq}}|^{\frac{1}{2}}$. The corresponding inequalities may be trivially developed from conditions 1 - 5, but they are practically useless, for the parameters would be redundantly large. The best way is to find them by computer simulation.

Theorem 1. *There are such constants* $a_i > 0$ *that with properly chosen parameters and T sufficiently small, controller* (5) - (10) *provides for the following inequalities after a finite-time transient process:*

$$|\sigma| \le a_1 T^3, \quad |\dot\sigma| \le a_2 T^2, \quad |\ddot\sigma| \le a_3 T, \quad |R| \le a_4 T.$$

Theorem 1 means that controller (5) - (10) is a third order real sliding algorithm with respect to $T \to 0$ (Levant, 1993). The exact value of $\dot\sigma$ is not available in practice, therefore its first difference may be used. Let the measurements be carried out at discrete times t_j, j = 0, 1, 2, ..., t be the current time, $t \in [t_j, t_{j+1})$, $t_{j+1} - t_j = \tau = const > 0$. Denote $\sigma(t_j) = \sigma(t_j, x(t_j))$, and $\Delta\sigma_j = \sigma(t_j) - \sigma(t_{j-1})$. Then S is transformed into

$$S_{\mathrm{d}} = \begin{cases} -u(t_j), & |u(t_j)| > 1, \\ -\alpha_{\mathrm{M}} \operatorname{sign} \sigma(t_j), & \sigma(t_j)\Delta\sigma_j > 0, |u(t_j)| \le 1, \\ -\alpha_{\mathrm{m}} \operatorname{sign} \sigma(t_j), & \sigma(t_j)\Delta\sigma_j \le 0, |u(t_j)| \le 1, \end{cases} \tag{11}$$

Substituting S_{d} for S and discretizing the measurements achieve

$$\dot u = R \cdot S_{\mathrm{d}}(\sigma, \Delta\sigma_j) + v(t_j).$$

Equations (7)-(10) are similarly modified. $T = N\tau$ is taken, where $N > 0$ is some integer constant.

Theorem 2. *There are such constants* $b_i > 0$ *that, provided properly chosen parameters and N sufficiently large, the discretized controller provides for the following inequalities after a finite-time transient process:*

$$|\sigma| \le b_1 \tau^3, \quad |\dot\sigma| \le b_2 \tau^2, \quad |\ddot\sigma| \le b_3 \tau, \quad |R| \le b_4 \tau.$$

The choice of N depends on $\alpha_{\mathrm{M}}/\alpha_{\mathrm{m}}$ and k. For example, with $\alpha_{\mathrm{M}}/\alpha_{\mathrm{m}} = 5$ and $k = 3$ $N = 50$ is sufficient.

4 Simulation example

Consider a model example of a tracking system. Let the controlled system be

$$\dot x = u.$$

The task is to track by x some real-time signal y, hence, $\sigma = x - y$ is taken. It was taken

$y(t) = \sin 0.5t + \cos t$,

$R_{\mathrm{M}} = 10$, $\Lambda = 5$, $\alpha_{\mathrm{m}} = 1$, $\alpha_{\mathrm{M}} = 4$, $\lambda = 2$, $\alpha = 2.2$, $\beta = 3$, $v_{\mathrm{M}} = 2$, $N = 50$, $k = 5$.

Initial conditions at time $t = 0$ are $x = 1$, $u = z = 1$, $z_1 = 0$, $v = 0$, $R = 10$. Mutual graphs of x and y, $\dot x$ and $\dot y$, $\ddot x$ and $\ddot y$ are presented in Fig. 2, 3, 4 respectively. Accuracies $|\sigma| \le 1.3 \cdot 10^{-4}$, $|\dot\sigma| \le 8.5 \cdot 10^{-3}$, $|\ddot\sigma| \le 2.04$ were received with $\tau = 10^{-3}$. They changed to $|\sigma| \le 1.4 \cdot 10^{-8}$, $|\dot\sigma| \le 4.4 \cdot 10^{-5}$, $|\ddot\sigma| \le 0.205$ with $\tau = 10^{-4}$.

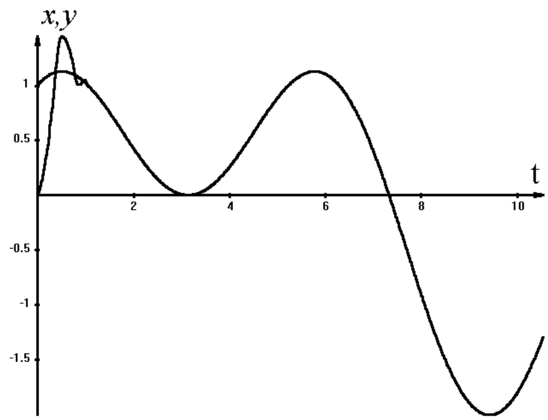

Fig. 2: Real 3-sliding tracking

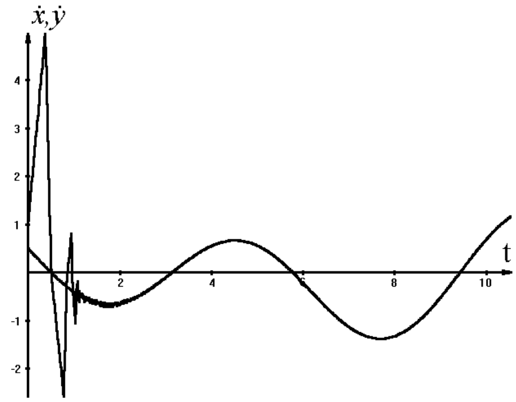

Fig. 3: Real 3-sliding tracking: first derivatives

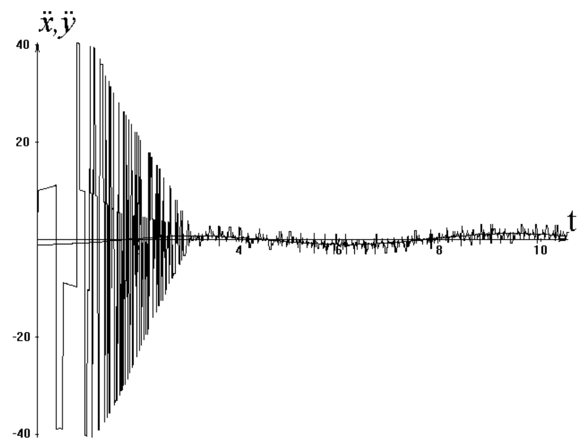

Fig. 4: Real 3-sliding tracking: second derivatives

5 Conclusions and remarks

An approach was proposed for real-time adaptation of 2-sliding controller parameters. The equivalent control is estimated by means of a special differentiator based on 2-sliding modes, which allows to make the magnitude of the discontinuous control component infinitesimally small.

An original criterion of the establishment of a real 2-sliding mode was developed for the purpose of real-time parameters adjustment keeping the real-2-sliding condition.

The resulting controller provides for the third order sliding precision of equality $\sigma = 0$ with respect to the measurement interval when only σ itself is available. That result opens a way to higher order sliding controller design based on minimal information requirements only. In particular, it may be used for effective real-time higher order differentiation with optimal asymptotics (Levant, 1998a) in the presence of noises.

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