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On the Role of Invariance in the Theory of Systems and Control –An Intelligible Introduction for the Beginners–

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Abstract

A framework of the introductory course to the control system theory is presented from the viewpoint of nonlinear theory. In the first part, the description is based on the input-output relations of the input affine systems. The notions appearing first are the relative order and the invariance. Geometric notions are gradually introduced for the beginners. Feedback design methods are given for the nonlinear systems and then for the linear systems. In the second part, the Hamiltonian formulation and the variational method are used to derive conditions of the optimality and the invariance. Both parts are linked via the notion of invariance. This style is partly realized in the author's lectures.

1 Introduction

In this paper, we present an outline of an introductory course to the theory of systems and control, a little "peculiar" in its style and viewpoint. Since the long time and the considerable efforts are required for the study of control theory, it is not unusual that there remains only a short period for the nonlinear geometric theory (Isidori, 1995). In addition, the geometric theory (Isidori, 1995; Wonham, 1979) is difficult for the beginners without the necessary prerequisite knowledge. So, we devised and gave a lecture which starts from the nonlinear theory and especially from the input-output relations, because it seems to be indispensable for the students, i.e. future engineers, to have the applicable knowledge of nonlinear geometric theory. In our lecture, notions of geometry appear gradually one by one.

2 Part I. Input-Output Expressions

We start with the state space representation and the input-output relations, which is simple and plain. In order to derive the input-output relations of input affine systems of C^K class, we have used the partial integration and the iterated integrals of inputs (Shima, 1992). Obtained expressions are equivalent to the Wiener-Volterra-Fliess functional series expansion formula for input affine systems of class C^ω and are useful for feedback design. But, later, it has become clear that our method is only a revision or an extension of the procedure to derive the Taylor

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series expansion and its residual term (Mizohata, 1973). In the process of defining the integral along the trajectory, vector field, its flow and Lie derivative along the flow are introduced. There appears "naturally" the iterated integrals of inputs which are directly convolution integrals for linear systems. For C^∞ systems, the formal functional series expansion formula is derived from the Taylor series expansion and we obtain the functional series expansion of the system outputs for C^ω systems. If the system is linear and time-invariant as a special case of C^ω systems, it is easily seen that the exponential of the system matrix appears. The Peano-Baker series as a fundamental solution of the time-variant linear systems can be derived by means of the direct application of the partial integration methods and also yields the exponential of the system matrix of the linear time-invariant systems.

The Taylor series expansion formula for the state of systems can be obtained from the formula derived above. Therefore, in principle, we can study the attainability property of the system. But, as is well known, Lie brackets of vector fields of input affine system do not appear in the Taylor series expansion formula in a "natural" way (Matsushima, 1965). In this place, an exercise is given to see the Lie brackets via the special inputs (Nijmeijer and Van der Schaft, 1990) and Lie derivatives of vector field and covector field are explained. *For the C^K class system, the first notion of control theory which appears in the Taylor series expansion with residual terms is "relative order," which is "equivalent" to the invariance.* In this stage including the case of C^∞ systems, the invariance condition is expressed by the Lie derivatives of output functions, which is equivalent to the expression using the Lie brackets of vector fields. The latter is the same as the local strong accessibility conditions. But, these conditions are not necessary or sufficient if the global behavior is to be studied (Jurdgevic, 1997) or the functional independence is not assured (Nijmeijer and Van der Schaft, 1990).

Since the relative order is already defined and the invariance conditions are available, we can explain various state feedback design methods such as non-interacting control (Yamashita and Shima, 1988), model following control (Isurugi and Shima, 1985), nonlinear observer (Shima *et al.*, 1980), output regulation, many of them with "zero dynamics" or "hidden dynamics." In this place, in order to study the stability of the zero dynamics, the stability theorems using Lyapunov functions is to be explained. And Lyapunov functions are defined by means of the integral along the flow. In this way, the state feedback control for the linear systems can be treated as a special case of nonlinear systems of C^K class. The origin of the notion "zero dynamics" is a good exercise for students after we have introduced the notion "transfer function" using the input-output expression via the convolution integral and the operational calculus of Miksinski (Miksinski, 1963; Hukawa, 1987). In addition, the Taylor series expansion formula for the state of systems of C^K class can be used to obtain the discretization algorithms for the feedback control law, which was originally derived from the straightforward Taylor series expansion formula using the derivatives of inputs (Isurugi and Shima, 1986).

The invariance condition, or equivalently the strong local accessibility condition, is necessary and sufficient for C^ω systems, especially for the linear time-invariant systems (Jurdgevic, 1997). For the linear systems, we can derive the well known controllability condition, which together with its dual observability condition yields structure theorems of linear systems. The pole assignment theorem of Wonham (1967) is explained conveniently as a special case of the state linearization procedure for nonlinear systems (Shima, 1980). The input-output linearization is illustrated via relative order.

Explaining the stability theorems and its criteria for linear systems, we can introduce the frequency transfer function and the related techniques of analysis and design.

This is the first part of our lecture. Reviewing the process of inference, we know that the important notion of the theory of systems and control is relative order or equivalently invariance.

The notion of invariance can be used to study the structure of systems in place of controllability.

3 Part II. Hamiltonian Systems

If we adopt the frame of Part I as an introduction to the control theory, the invariance condition is an important component of the frame. Moreover, the invariance condition is also derived from the variational approach (Rozonoer, 1963; Shima *et al.*, 1979; Yamashita and Shima, 1990), which is closely related with the optimality principle (Pontryagin *et al.*, 1962; Rozonoer, 1959). So, we use the notion of invariance as a key notion to change the stage where the drama of theoretical development is played.

We study the general nonlinear control system of C^K class. Outputs of the system can be expressed as the Performance Indices in the Lagrange form. Hamiltonian and its auxiliary variables of Maximum Principle are introduced together with their dynamics and boundary conditions (Rozonoer, 1959). Thus, we have a Hamiltonian system. Following the procedures used by Rozonoer (1959), we can derive the expressions of the first and the second variations of performance index corresponding to input variations.

Using these expressions, we can obtain the necessary conditions for optimal control in the time domain. Substituting the auxiliary variables with the partial derivatives of the value function, we can obtain the partial differential equations of Hamilton-Jacobi Bellman type. If we put aside the problem of singular optimal control and the ambiguity of the substitution, we can study the regulator problem and the time optimal control problem.

Discussing the solution of the nonlinear regulator problem, we can explain the importance of numerical or approximate solution of the fundamental design equations of control systems. Then, we introduce the linear quadratic regulator problems and related topics as a special and very important design tool. It is shown that the controllability condition is not always necessary for the existence of its solution and that the problem of zero dynamics does not arise.

About the time optimal control problems, we explain simple examples and the difficulties concerning the frequent appearances of optimal singular control. In this place, it is also stressed that the problem of numerical solutions of the Hamilton-Jacobi-Bellman equation with the input constraints is very important. *If it is shown that there exists a solution of this problem, the attainability problem is solvable. In order to study the attainability problem from one point to another point, this procedure seems to be most simple and sure.*

Using the variational expressions, we can derive the saddle point conditions of differential games, in other words, the conditions to be satisfied by the "optimal" disturbances and the optimal controls. In the same way and always with the same ambiguities of substitution procedure, the Hamilton-Jacobi-Issacs partial differential equation is derived. Thus, we can explain the nonlinear H^∞ control problem, one of the terminal station of the control theory and make reference to the robust control of the linear systems.

The conditions of invariance, the key notion with which Part II is linked to Part I, is also derived from the variational expressions (Rozonoer, 1963; Shima *et al.*, 1979; Yamashita and Shima, 1990). The conditions of invariance for general nonlinear systems are studied in detail in (Yamashita and Shima, 1990; Shima and Yamashita, 1997). From this variational approach, we can obtain the condition expressed by means of the Lie brackets of the vector fields involved, which is equivalent to the local strong accessibility distributions. Explaining the distribution, we can discuss various distribution algorithms of design.

In Part II, the attainability problem as one of the most basic properties of controlled systems is reduced to the solvability of the related Hamilton-Jacobi-Bellman equation with input constraints and stress is put on its numerical and/or approximate solutions, which may be suitable for the computer age. The problem of appearance of singular optimal control can be solved in the time domain by Yamashita's method (Yamashita and Shima, 1997). There may be another approach or explanation using the viscosity solution of Hamilton-Jacobi-Bellman equation with input constraints.

4 Concluding Remarks

In this paper, an effort is reported to construct an introductory and intelligible course of the theory of systems and control from the viewpoint of nonlinear theory. In view of the frames deduced from Part I and Part II, the basic principles of the theory seems to be optimality, stability and invariance. Since all of these notions or concepts are familiar and important in the dynamical system theory, we recommend our students to study it from the standpoint of control and with these notions in mind. It is to be noted that three lectures in addition to this lecture are prepared for our students and cover classical control theory, linear system theory and "mechatronics." And, this lecture is also divided into two lectures: "Introduction to the theory of systems and control" and "Nonlinear system theory." In addition, according to the recent study, it is possible to start the Part I from the description of general nonlinear systems. Thus, the unified style of both Parts can be attained.

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