Optimal Design of Transfer Lines and Multi-Position Machines*

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Abstract

Automatic transfer lines and multi-position equipment for machining large parts in mass production are investigated. Mathematical models and methods for optimal cost design of these production systems are considered. The following designed system parameters are determined: number of workstations; assignment of operations to the workstations; orientation of the parts, number of positions and cutting modes for each workstation. For solving the obtained optimization problem, a special multilevel decomposition scheme is proposed. It uses the decomposition approaches in combination with the methods of nonlinear and discrete programming.

1 Introduction

In this paper, the problem of optimal design of transfer lines and multi-position machines is considered. The studied systems represent "hard automation" where the line is designed for mass production of a single product. Two main classes of optimization problems are generally investigated in literature.

The first problem is the line balancing problem (Scholl, 1999). For this problem, an annual volume of production as well as an annual available operating time, a set of operations and operation times are known. The problem is to minimize the number of stations under a given partial order of operations and the desired *cycle time*. The cycle time t is calculated as the ratio of annual available operating time and annual production volume. Thus, for the given cycle time, each operation has to be assigned to one station so that the number n of stations is minimized and no precedence constraint is violated. Equivalently, a partition of the set $\mathbf{N}$ of all operations into a minimum number of disjoint sets N_k, $k=1,2,\ldots,n$ has to be found. The station time $u_0(N_k)$, equal to the sum of times of all operations in N_k, must not exceed the cycle time t.

This problem is typical to assembly systems. Generally, deterministic models are formulated (integer linear programming model with deterministic operation times and absence of machine failures, etc.). To solve them, exact or heuristic methods are used (Baybars, 1986; Talbot et al., 1986). The exact methods are mainly based on branch and bound algorithms. The most effective branch and bound procedures are FABLE, OptPack, Eureka and SALOME (Schcoll and Klein, 1998). Since the problem is known to be NP-hard, exact solution of large problems will often require an inordinate amount of time. In this case, the heuristics are used. The most old heuristic method is COMSOAL (Arcus, 1966), other heuristics have been also developed (Askin, 1993).

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For transfer lines, which are characterized by small cycle time and non negligible failures rate, another class of models is used. In this case, the machines have been selected, and operation have been already assigned to the machines. The goal is to determine the smallest amount of in-process inventory space so that the line meets the target mean cycle time. A Markov model has been proposed by Vladzievskii A.P. (1952) for lines with two machines. Further developments of this model have been done in (Levin and Pasjko, 1969; Gershwin, 1979; Coillard and Proth, 1983). A generalization of this model for *m* machines is based on various decomposition or aggregation techniques, for example (Dallery et al., 1989; Dolgui, 1993). These models are mostly applied in car and electronic industries.

Here we investigate transfer lines and multi-position machines with complex equipment designed for machining of large parts. The parts are characterized by a great number of operations and surfaces to be machined, by several types of machining and by relative big cost. Due to these reasons, the problem of buffers size determination is not actual and these production lines are designed as synchronized lines (all stations start simultaneously to perform their operations, maximum station time defines the real cycle time).

All operations N_k of each station k ($k=1,2, \ldots,n$) are machined simultaneously. The cutting modes X_k for station k depend on the assigned set N_k. Therefore, each operation time and the station time $u_0(N_k)$ are determined by the set of assigned operations N_k. The value $u_0(N_k)$ is not equal to the sum of fixed times of all operations in N_k as in the assembly line balancing case. Another difference is that the cost of the equipment needs to be factored into design decision analysis. A number of technological factors (such as the sequence of machining part surfaces; a mutual position of surfaces; the necessity of intermediate transitions during machining, etc.) determines an additional cost for each design decision.

The design decision defines the system life cycle cost. The quality of accepted decision must be rather high, because these systems are very expensive and slight improvement of the design decision allows to get an substantial economic benefit.

In this paper, the searching for a rational variant of structure and parameters of these systems, concerning preliminary design stages, is discussed. Several aspects of the considered problem were investigated in (Guschinsky and Levin, 1990).

2 Statement of the problem

The studied systems (see Fig.1) represent a sequence of synchronized workstations. Each station can have several identical work positions. The number of work positions determines the number of simultaneously machined parts. Between two workstations is located an auxiliary station which is used for transportation and reorientation (if necessary) of the part.

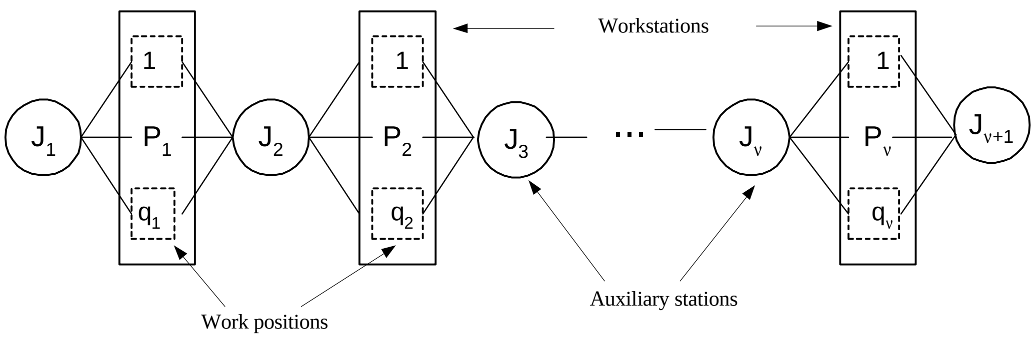

Fig. 1 Multistage and multi-position production system

The set of operations **N** for the part is given. A number of known technological factors determines the partial order of the operations, which regulates the possible sequences of machining.

A high precision of certain operations and small relative tolerances demand machining of the corresponding operations simultaneously, imply that these operations must be assigned to the same workstation. The mutual influence of the combined operations during machining, the impossibility of spatial overlapping, etc. determines also a relation of mutual exclusion between the operations.

The several factors impose rigid restrictions upon an orientation of the part on the workstation, for instance: mutual orientation of the part surfaces; possible orientation of the executive bodies.

The cutting modes of the work position must also satisfy a system of constructive and technological constraints which determine the area of feasible values of these parameters.

The problem is to determine:

- the number n of workstations;
- the assignment of the given set **N** of operations to the workstations $\{N_k | k = \overline{1,n}\}$;
- the orientation h_k of the part for each workstation;
- the number of positions q_k for each workstation;
- the cutting modes X_k of each workstation.

For the considered preliminary stage of design, the set $P_k=(N_k, q_k, h_k, X_k)$ defines the *k-th* workstation, and the set $J_k=(q_{k-1}, h_{k-1}, q_k, h_k)$ specifies the *k-th* auxiliary station. Therefore, the collection $P=(P_1,..., P_n)$ determines the design system and the manufacturing process.

The station cost depends on the number of work positions and on the set of operations assigned to this station. The auxiliary station cost depend on parameters of adjacent workstations (number of work positions, spatial part orientation). The optimization problem is to determine the set of parameters, which satisfy the above constraints and provide the given productivity, minimizing the system life cycle cost.

3 Mathematical model

The production system is designed for the given annual productivity T. The system life cycle cost per part (variable cost) can be estimated by the following formula (Vladzievskii, 1958):

$$\Theta_1 = a_1\, t(1 + B) + t(R + Q) + I + (a_{12} A + a_{13} F)/T, \tag{1}$$

where

t is the cycle time;
a_1 is the labor cost (wages of the workers, including the overhead, for a time unit);
B is the relative machine set-up time for a time unit of non-interrupted work;
R is the relative maintenance cost for a time unit of non-interrupted work;
Q is the relative electric power cost for a time unit of non-interrupted work;
I is the relative tool cost for one part;
A is the investment volume for the system;
a_{12} is the investment amortization factor;
F is the area occupied by the system;
a_{13} is the area amortization factor.

The cycle time t for the considered system can be defined by the following relationship:

$$t = \max\{u_0(N_k, h_k, q_k, X_k)/q_k \mid k = \overline{1,n}\},$$

where $u_0(N_k, h_k, q_k, X_k)$ is the *k-th* workstation load time (time of machining of one part).

Each component of system life cycle cost Θ_1 in (1) can be estimated (Levin, 1978) as a sum of two elements: the first element corresponds workstations, the second element concerns auxiliary stations. In the same way, these elements are calculated as the sum of two items, the first one is proportional to cycle time, and the second item does not depend on it.

Therefore, the cost $\Theta_1(P)$ can be expressed by the following equation:

$$\Theta_1(P) = \max\{u_0(N_k, h_k, q_k, X_k)/q_k \Big|\ k = \overline{1,n(P)}\} \times [a_1 + \textstyle\sum_{k=1}^{n(P)} u_{11}(N_k, h_k, q_k) + \quad (2)$$

$$\textstyle\sum_{k=1}^{n(P)+1} u_{12}(h_{k-1}, q_{k-1}, h_k, q_k)] + \sum_{k=1}^{n(P)} [u_{13}(N_k, h_k, q_k) + u_{14}(N_k, h_k, X_k)] +$$

$$\textstyle\sum_{k=1}^{n(P)+1} u_{15}(h_{k-1}, q_{k-1}, h_k, q_k),$$

where

$u_{1j}(\circ)$ characterize the items of cost determined by the workstations (for j=*1,3*) or auxiliary stations (for *j=2,5*); these items are proportional to cycle time (for *j=1,2*) or not dependent on it (for *j=3,5*);

$u_{14}(\circ)$ characterizes the item determined by the tools of the *k-th* workstation.

The time per part is defined as follows: $\Theta_2 = t(1 + B)$. If we introduce $a_2=1$, an expression similar to the expression (2) can be obtained for $\Theta_2(P)$:

$$\Theta_2(P) = \max\{u_0(N_k, h_k, q_k, X_k)/q_k \Big|\ k = \overline{1,n(P)}\} \times [a_2 + \textstyle\sum_{k=1}^{n(P)} u_{21}(N_k, h_k, q_k) + \quad (3)$$

$$\textstyle\sum_{k=1}^{n(P)+1} u_{22}(h_{k-1}, q_{k-1}, h_k, q_k)] + \sum_{k=1}^{n(P)} [u_{23}(N_k, h_k, q_k) + u_{24}(N_k, h_k, X_k)] +$$

$$\textstyle\sum_{k=1}^{n(P)+1} u_{25}(h_{k-1}, q_{k-1}, h_k, q_k),$$

where

$u_{2j}(\circ)$ characterize the items of time determined by the workstations (for j=*1,3*) or auxiliary stations (for *j=2,5*); these items are proportional to cycle time (for *j=1,2*) or not dependent on it (for *j=3,5*);

$u_{24}(\circ)$ characterizes time determined by the tools of the *k-th* workstation.

The problem of searching an optimum collection *P* is a very complex optimization problem. This complexity is caused, in particular, by the discrete and combinatorial nature of a set of variables. The prior experience can assist to reduce the area of search before the use of optimization methods.

The maximum admissible value of time per part is equal $t_0 = \Phi / T$ where *T* is the given annual volume and *F* is the real value of annual operating time. The value *F* is calculated taking into account losses of productivity due to failures and to others organizational factors.

The analysis of equipment, which is similar to the designed one, allows to get the lower $\underline{B}$ and the upper $\overline{B}$ estimations of set-up time *B*. Marketing analysis gives the lower estimation $\overline{T}$ of productivity. These estimations allow to determine the lower bound $\underline{t}_0 = \Phi / \overline{T}$ of time per part, as well as both the lower $\underline{t} = \underline{t}_0 / (1 + \overline{B})$ and the upper $\bar{t} = t_0 / (1 + \underline{B})$ bounds of cycle time.

The lower $\underline{q}(N_k, h_k)$ and upper $\overline{q}(N_k, h_k)$ bounds of values q_k can be obtained for fixed values N_k and h_k, taking into account the limiting values of cutting modes $X_k(N_k, h_k)$.

One more essential opportunity of reducing the search area is to construct the operators $O(N^1, N^2, N^3, h, q)$ based on expert knowledge, which permit to estimate the preference of overlapping of different groups of operations in one workstation.

For (N^1, N^2, N^3, h, q) the operator $O(N^1, N^2, N^3, h, q)$ is equal to 0, if the assignment of a subset of operations N^1 to the workstation with parameters $(N^1 \cup N^2, h, q)$ is certainly preferable than its assignment to other workstation together with some subset of operations $N \subseteq N^3$. This operator is equal to 1, if the opposite condition is true, and is equal to -1, if no conclusions can be made. Here N^1, N^2, N^3 are disjoint subsets of **N**. The operator $O(\cdot)$ allows to exclude from consideration unpromising (from the engineering point of view) collections P.

The partial order of the operations is given by a digraph Ψ=(**N**, Ω). An arc $(i,j)\in\Omega$ if and only if the operation $j\in$**N** cannot precede the operation $i\in$**N**. An impossibility of overlapping groups of operations can be defined by a hypergraph K=(**N**, $\Re$). A hyperedge of the hypergraph K consists of operations which cannot be performed at one workstation.

We consider the following mathematical model of the design problem:

$$\Theta_1(P) \to \min, \tag{4}$$

subject to:

$$\Theta_2(P) \le t_0, \tag{5}$$

$$\bigcup_{k=1}^{n(P)} N_k = \mathbf{N}, \tag{6}$$

$$N_{k_1} \cap N_{k_2} = \varnothing,\ k_1, k_2 = \overline{1, n(P)},\ k_1 \ne k_2, \tag{7}$$

$$M(i) \le M(j),\ (i,j) \in \Omega, \tag{8}$$

$$r \not\subset N_k,\ r \in \Re,\ k = \overline{1, n(P)}, \tag{9}$$

$$h_k \in H(N_k),\ k = \overline{1, n(P)}, \tag{10}$$

$$q_k \in [\underline{q}_k(N_k, h_k), \overline{q}_k(N_k, h_k)], \tag{11}$$

$$X_k \in \mathbf{X}(N_k, h_k) = 1,\ k = \overline{1, n(P)}, \tag{12}$$

$$O(N^1, N_k \setminus N^1, \mathbf{N} \setminus \bigcup_{r=1}^{k} N_r, h_k, q_k) \ne 1,\ \ N^1 \subseteq N_k, \tag{13}$$

$$O(N^1, N_k, \mathbf{N} \setminus \bigcup_{r=1}^{k} N_r \setminus N^1, h_k, q_k) \ne 0,\ \ N^1 \subseteq \mathbf{N} \setminus \bigcup_{r=1}^{k} N_r,\ \ r \not\subset (N_k \cup N^1), \tag{14}$$

where

$M(i)=k$ for $i\in$**N** if and only if $i\in N_k$;

X(N_k, h_k) is the set of feasible cutting modes for the workstation N_k with orientation h_k of the part;

$H(N_k)$ is the set of feasible spatial orientations of the part for the workstation N_k.

The objective function (4) is to minimize system life cycle cost per part; the constraint (5) provides the given productivity; the constraints (6-7) determine the condition of assignment of all operations of the set **N**, and each operation can be perform only at one workstation; (8) defines the precedence constraints over the set **N**; (9) corresponds to the condition of assignment of separate groups of operations from **N** at one workstation; (10-12) define the sets of feasible values of parameters h_k, X_k, q_k; (13-14) describe the operator of "preference" of assignment of set of operations $N^1 \subseteq \mathbf{N}$ to the workstation $(N^1 \cup N^2, h, q)$ or to another workstation together with $N \subseteq N^3$.

The problem (4-14) is considered as an initial problem **A**. Let designate by **P** the set of collections P, satisfying relations (6-14).

4 Method for solving problem

4.1 Decomposition scheme

The set $\{(\Theta_1(P), \Theta_2(P)) | P \in \mathbf{P}\}$ with sufficient accuracy is supposed effectively convex. Let us consider the Lagrangian function

$$\Theta(P,\lambda) = \lambda\, \Theta_1(P) + (1-\lambda)\, \Theta_2(P),$$

where $\lambda \in [0,1]$.

Let $P^*(\lambda)$ be a vector from **P** with minimum value of function $\Theta(P,\lambda)$ for fixed $\lambda \in [0,1]$. It is easy to prove that function $\Theta_1(P^*(\lambda))$ does not increase, and that the function $\Theta_2(P^*(\lambda))$ does not decrease over the segment [0,1]. It is obviously that $P^*(1)$ is the solution of the problem **A**, if $\Theta_2(P^*(1)) \leq t_0$, and that the problem **A** has no decision, if $\Theta_2(P^*(0)) > t_0$. Hence, the solution of the initial problem **A** can be obtained using the following two-level decomposition scheme (see Fig. 2).

On the upper level, the problem $\mathbf{A_1}$ of determination $\lambda^* = \max\{\lambda \in [0,1] \mid \Theta_2(P^*(\lambda)) \leq t_0\}$ is solved, and on the lower level the problem $\mathbf{A_2}(\lambda)$ of finding $P^*(\lambda) \in \mathbf{P}$ is solved for the fixed value λ. When the problem $\mathbf{A_1}$ is solved**,** monotony of the function $\Theta_2(P^*(\lambda))$ on λ over the segment [0,1] allows to use methods similar to existing methods for solving the equation with a monotonous left part (for instance, dichotomy method).

The problem $\mathbf{A_2}(\lambda)$ is more difficult and requires to develop special methods for its solving. It can be formulated in the terms of minimization of a superposition of recurrent-monotonic functions over a set of parameterized paths in digraphs (Guschinsky *et al.*, 1990).

Let us consider a state $\bar{s}_k$ of the part after machining at first k workstations according to $P \in \mathbf{P}$. Under the accepted assumptions, the state is uniquely defined by the set of operations from **N**, assigned to these positions, i.e. it is possible to consider, that $\bar{s}_k = \bigcup_{r=1}^{k} N_r$, $k = \overline{1, n(P)}$. Together with the state $\bar{s}_k$, the extended state determined by the vector $s_k = (\bar{s}_k, h_k, q_k)$ must also be considered.

Let S and **S** be the sets of possible states of the part and its extended states for all $P \in \mathbf{P}$. Into set **S,** the initial state $s_0 = (\emptyset, 0, 0)$ and the final state $s_N = (\mathbf{N}, 0, 0)$ are also included.

Let $G = (\mathbf{S}, E)$ be a digraph such that a pair $(s' = (\bar{s}', h', q'),\ s'' = (\bar{s}'', h'', q''))$ of vertices from **S** belongs to set of arcs E if and only if (i) $\bar{s}' \subset \bar{s}''$ and (ii) the set (N'', h'', q'') where $N'' = \bar{s}'' \setminus \bar{s}'$, satisfies the conditions (8)-(11). For each arc $(s', s'') \in E$, we assign the set $\Gamma_{s's''} = \mathbf{X}(N'', h'')$, if $s'' \neq s_N$, and $\Gamma_{s's''} = \emptyset$ otherwise.

A pair $z = (w, g)$, where w is a path in the graph G and $g \in \Gamma(w) = \prod_{(s',s'') \in w} \Gamma_{s's''}$, is the parameterized path in the graph G. It is easy to prove that there exists one-to-one correspondence between the set **P** and the set **Z** of all parameterized paths in digraph G from the initial vertex s_0 to the final vertex s_N.

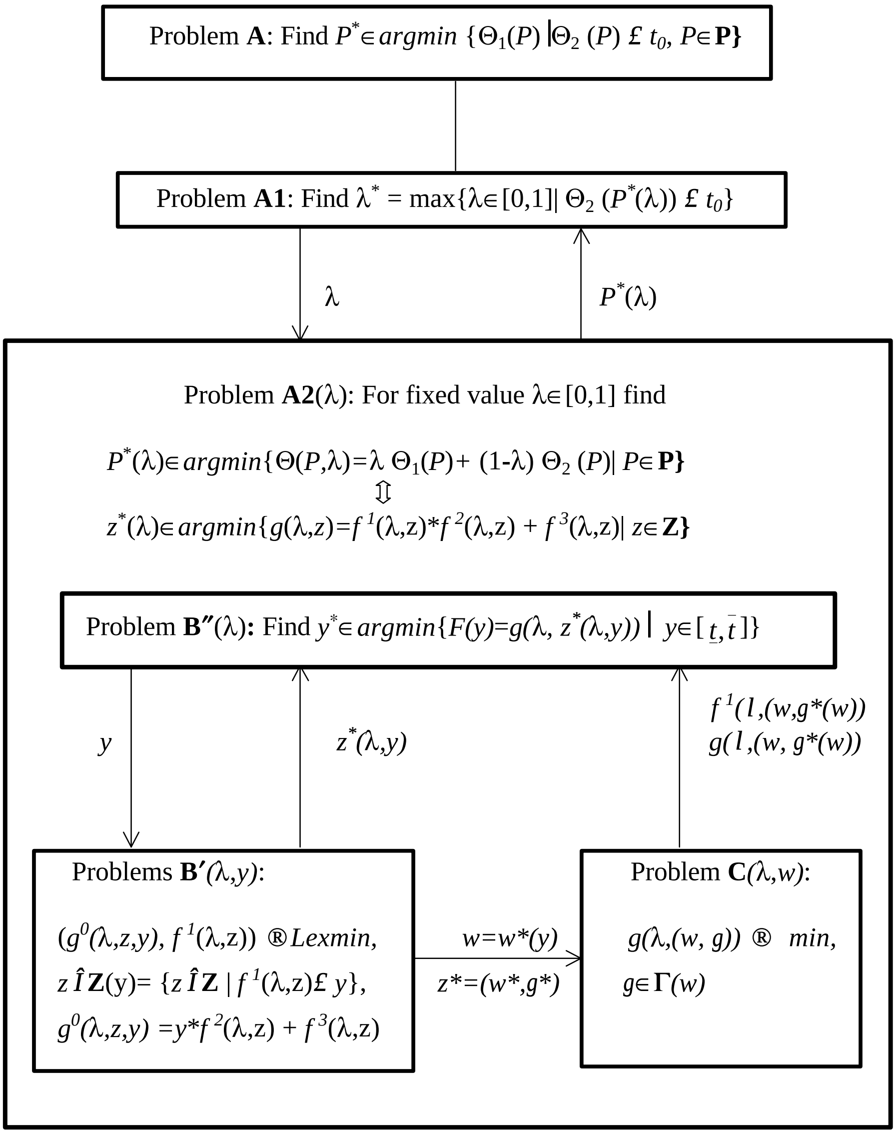

Fig. 2 Decomposition scheme

Each arc $(s',s'') \in E$ is assigned the following non-increasing in the second argument functions $\varphi^r_{s's''}(\lambda, a, x)$, $r = 1,2,3$, $a \in R$, $x \in \Gamma_{s's''}$, $s'' \ne s_N$ ($\varphi^r_{s's_N}(\lambda, a, x) = 0$):

$$\varphi^1_{s's''}(\lambda, a, x) = \max[a, u_o(N'', h'', q'', x) / q''];$$

$$\varphi^2_{s's''}(\lambda, a, x) = a + \lambda \left(u_{11}(N'', h'', q'') + u_{12}(h', q', h'', q'')\right) + (1 - \lambda)\left(u_{21}(N'', h'', q'') + u_{22}(h', q', h'', q'')\right);$$

$$\varphi^3_{s's''}(\lambda, a, x) = a + \lambda \left(u_{13}(N'', h'', q'') + u_{14}(N'', h'', x) + u_{15}(h', q', h'', q'')\right) + (1 - \lambda)\left(u_{23}(N'', h'', q'') + u_{24}(N'', h'', x) + u_{25}(h', q', h'', q'')\right);$$

For the parameterized path $z=\big(w=(s_0=i_0,...,i_l=s_N),\ g=(g_1,...,g_l)\big)$, we define a vector-function $f(l,z)=(f^1(l,z),\ f^2(l,z),\ f^3(l,z))$ by the following recurrent relationships:

$$f^r(l,z)=f_l^r(l,z);$$

$$f_k^r(l,z)=j^r_{s_{k-1}s_k}(l,f_{k-1}^r(l,z),g_k),\ k=\overline{1,l},\ r=1,2,3;$$

$$f_0^1(l,z)=f_0^3(l,z)=0;$$

$$f_0^2(l,z)=l\,a_1+(1-l)a_2.$$

The problem $\mathbf{A}_2(\lambda)$ of finding the collection $P^*(\lambda)\in\mathbf{P}$ with minimum value of function $\Theta(P,\lambda)$ for a fixed $\lambda\in[0,1]$ is equivalent to the problem of finding the parameterized path $z^*(\lambda)$ in digraph G from the vertex s_0 to the vertex s_N with minimum value of the function

$$g(\lambda,z)=f^1(\lambda,z)*f^2(\lambda,z)+f^3(\lambda,z).$$

The solution of the problem $\mathbf{A}_2(\lambda)$ can also be obtained using two-level decomposition scheme. On the lower level, for the fixed $y\in[\underline{t},\bar{t}]$ the problems $\mathbf{B}'(\lambda,y)$ of lexicographic minimization of the vector-function

$$(g^0(\lambda,z,y)=(y*f^2(\lambda,z)+f^3(\lambda,z),f^1(\lambda,z)),$$

over a set $\{z\in\mathbf{Z}\mid f^1(\lambda,z)\le y\}$ are solved. For the second criterion $f^1(\lambda,z)$, an approximate solution is admissible. The solution $z^*(\lambda,y)$ of the problem $\mathbf{B}'(\lambda,y)$ can be "improved" by solving the problem $\mathbf{C}(\lambda,w)$ of minimization of the function $g(\lambda,(w,g))$ over a set $\Gamma(w)$. On the upper level, the problem $\mathbf{B}''(\lambda)$ of minimization of function $F(y)=g(\lambda,z^*(\lambda,y))$ for $y\in[\underline{t},\bar{t}]$ is solved.

4.2 Methods for solving sub-problems

For solving the problems $\mathbf{B}'(\lambda,y)$, we propose modified methods of finding shortest path in graph. These modifications allow to take into account parametric properties of problems $\mathbf{B}'(\lambda,y)$ and to use the obtained earlier data for solving the current problem $\mathbf{B}'(\lambda,y)$. For more details, the interested reader is addressed to (Guschinsky *et al*., 1990). While using these methods, we obtain the exact solution of the problem $\mathbf{B}'(\lambda,y)$ if it is possible to find the exact solution of the problem $\mathbf{D}(\lambda,s',s'',y)$ of lexicographic minimizing the vector-function

$$(l\,u_{14}(N'',h'',x)+(1-l)u_{24}(N'',h'',x),\ u_o(N'',h'',q'',x))$$

over the set $\{u_o(N'',h'',q'',x)\le y*q'',x\in\Gamma_{s's''}$ for any arc $(s',s'')\in E$ and fixed y.

For solving the problem $\mathbf{B}''(\lambda)$**,** special modification (Guschinsky *et al*., 1990) of "branch and bound" method is used. It takes into account that either lower bound σ_i of the function $g^0(\lambda,z^*(\lambda,y),y)$ over the set $Y_i=[y_i^-,y_i^+]\subseteq[\underline{t},\bar{t}]$ is the lower bound of the function $F(y)$ over Y_i or $\hat{y}$ exists such that $F(\hat{y})\le\min\{F(y)\mid y\in Y_i\}$. The value

$$s_i = \max[g' - (y' - y_i^-) b_i^2, g' y_i^- / y' + b_i^3 (1 - y_i^- / y')]$$

is considered as a lower bound of the function *F(y)* over Y_i. Here $g' \leq g^0(\lambda, z^*(\lambda, y'), y')$ for some $y' \geq y_i^+$, b_i^2 is a upper bound of the function $f^2(l, z)$ and b_i^3 is a lower bound of the function $f^3(l, z)$ over the set $\{z \in \mathbf{Z} \mid y_i^- < f^1(l, z) < y_i^+\}$. The segment $[\hat{y}_i, y_i^+]$ can be deleted from Y_i where

$$\hat{y}_i = \min[y' - (g' - F^0) / b_i^2, y'(F^0 - b_i^3)/(g' - b_i^3)]$$

and F^0 is current record.

The problem **C**(λ, w) is to minimize the function

$$\mathrm{g}(l, (w, g)) = \max\{u_0(N_k, h_k, q_k, g_k)/q_k \mid k = \overline{1, l}\} \times c_1(l, w) + l \sum_{k=1}^{l} u_{14}(N_k, h_k, g_k) + (1 - l)\sum_{k=1}^{l} u_{24}(N_k, h_k, g_k) + c_2(l, w)$$

for the fixed path $w=(s_0, ..i_0, i_l=s_N)$ over the set $\Gamma(w)$ where functions $c_i(l, w), i = 1,2$ do not depend on $\Gamma(w)$. Note that the function $g(\lambda,(w,g))$ is in fact a superposition of two recurrent-monotonic functions. Therefore the procedures (Guschinsky and Levin, 1988), reducing the area of search, can be applied. For solving the problem **C**(λ,w), we also use the above two-level decomposition scheme. In this case, the lower level problem is decomposed on *l* separate problems **D**(λ,s',s'',y) which are (as well as **C**(λ,w) as a whole) the problems of optimization of cutting modes of multi-tools machining.

5 Numerical example

In this section, we give a simple numerical example to illustrate the method (mainly the procedure for solving the problem $\mathbf{A}_2(\lambda)$). The set **N** contains 5 operations. The graph of precedence constraints Ψ=(**N**, Ω) is shown in Fig. 3.

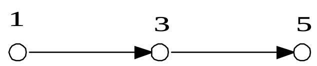

Fig. 3 Precedence graph

The hypergraph K=(**N**, $\mathfrak{R}$) defines an impossibility of overlapping groups of operations (see Fig.4.)

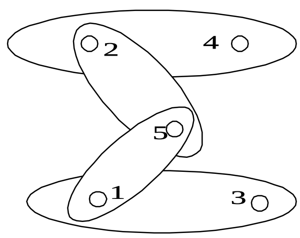

Fig. 4 Incompatibility constraints

The following operators of preference of operations overlapping groups are defined: O({i},{j},∅)=0 for all $i,j\in$ **N,** $i^{1}5$; O({$3,5$},{4},∅)=0; O({5},{j},∅)=-1 for all $j\in$ **N.**

Therefore, the following sets N_k may be considered: {$1,2$}, {$1,4$}, {$3,4$}, {$2,3$}, {5}, {$3,4,5$}, {$2,3,5$}. We suppose also that $q_k=1$ and $|H(N_k)|=1$ for all k. The obtained graph G is given in Fig. 5.

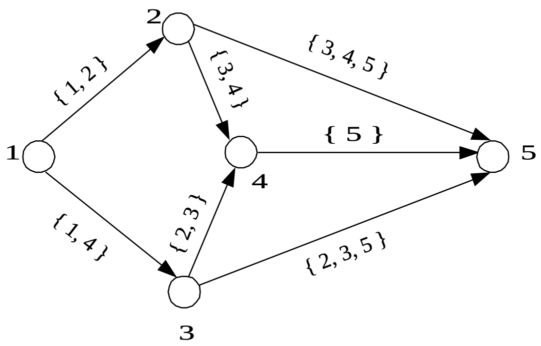

Fig. 5 Graph model

The sets **X**(N) and functions u_{ij} are defined according to the Table 1.

N	{$1,2$}	{$1,4$}	{$3,4$}	{$2,3$}	{5}	{$3,4,5$}	{$2,3,5$}
|**X**(N)|	2	2	2	2	3	1	1
u_0	{$3,4$}	{$4,5$}	{$4,5$}	{$3,5$}	{$2,3,4$}	6	6
u_{11}	2	2	2	2	1	3	3
u_{12}	1	1	1	1	1	1	1
u_{13}	3	3	3	3	3	3	3
u_{14}	{$5,3$}	{$5,4$}	{$4,3$}	{$5,3$}	{$3,2,1$}	5	4
u_{15}	2	2	2	2	1	3	3
u_{21}	2	2	2	2	1	3	3
u_{22}	1	1	1	1	1	1	1
u_{23}	1	1	1	1	1	1	1
u_{24}	{$3,2$}	{$3,2$}	{$3,2$}	{$3,2$}	{$3,2,1$}	3	3
u_{25}	1	1	1	1	1	1	1

Table 1. The sets **X**(N) and functions u_{ij}.

Here |**X**(N)| is a cardinality of the set **X**(N), |**X**(N)| $=2$ and u_0 = {$3,4$} means that $u_0(X_1)=3$, $u_0(X_2)=4$. We accept α_1=0.8, α_2=1.0, $[\underline{t},\bar{t}]$=[4,6], t_0=49. The following functions $j^{r}_{s's''}(l,a,x)$, $r=1,2,3$ are obtained for λ=1 (see Table 2).

N	{1,2}	{1,4}	{3,4}	{2,3}	{5}	{3,4,5}	{2,3,5}
G={g(i)}	{1,2}	{1,2}	{1,2}	{1,2}	{1,2,3}	{1}	{1}
$j^1_{s's''}(l,a,x) = \max(a+$	{3,4}	{4,5}	{4,5}	{3,5}	{2,3,4}	6	6
$j^2_{s's''}(l,a,x) = a+$	3	3	3	3	2	4	4
$j^3_{s's''}(l,a,x) = a+$	5+{5,3}	5+{5,4}	5+{4,3}	5+{5,3}	4+{3,2,1}	11	10

Table 2. Functions $j^r_{s's''}(l,a,x)$, $r=1,2,3$ for λ=1.

The procedure of solving the problem A2(1) (and **B″**(1)) starts with solving the problem **B′**(λ,*y*) for y= $\underline{t}$ =4. The optimal solution of this problem is the parameterized path *z*(y)*=((1,2,4,5), (*g(2), g(1)* , *g(1)*)), $g^0(z^*(y))=g(z^*(y))=57.2$. The optimal solution of the problem **B′**(λ,*y*) for y=$\bar{t}$ =6 is the parameterized path *z*(y)*=((1,2,5), (*g(2), g(1)*)), $g^0(z^*(y))=g(z^*(y))=65.8$.

Calculation of the upper bound of the function $f^2(l,z)$ gives b_i^2 = 8.8 and of the lower bound of the function $f^3(l,z)$ gives b_i^3 =21. Then we obtain:

$$s_i = \max[g' - (y' - y_i^-)\beta_i^2,\ g' y_i^- / y' + \beta_i^3(1 - y_i^- / y')] = \max[65.8\text{-}(6\text{-}4)8.8,\ 65.8(4/6)+21(1\text{-}4/6)] = \max[48.2,\ 50.8] = 50.8,$$

$$\hat{y}_i = \min[y' - (g' - F^0)/\beta_i^2,\ y'(F^0 - \beta_i^3)/(g' - \beta_i^3)] = \min[6\text{-}(65.8 - 57.2)/8.8,\ 6(57.2\text{-}21)/(65.8\text{-}21)] = \min[5.02, 4.84] = 4.84.$$

The procedure stops, because the obtained $[y_i^-, y_i^+]$ = [4,4.84] and there exists no *y* such that $y \in (4,4.84)$ and $f^1(l,z) = y$.

Thus,

$$\Theta_2(P^*(1)) = \max\{u_0(N_k,h_k,q_k,g_k)/q_k \mid k=\overline{1,3}\} \times [1 + \textstyle\sum_{k=1}^3 u_{21}(N_k,h_k,q_k) + \sum_{k=1}^3 u_{22}(h_{k-1},q_{k-1},h_k,q_k)] + \sum_{k=1}^3 [u_{23}(N_k,h_k,q_k) + u_{24}(N_k,h_k,g_k)] + \sum_{k=1}^3 u_{25}(h_{k-1},q_{k-1},h_k,q_k) = \max\{4,4,4\}\ [1+(2+2+1)+(1+1+1)]+[(1+1+1)+(2+3+1)]+(1+1+1) = 48.$$

Since the obtained value $\Theta_2(P^*(1)) \le t_0$,we find the optimal solution.

6 Conclusion

The problem of optimal design of transfer lines and multi-position machines is very complex. A method for solving this problem is proposed. It is based on parametric decomposition and on graph optimization methods. The method allows to minimize the system life cycle cost under the given productivity.

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