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Decomposition-coordinated optimization of large-scale discrete systems with parallel-sequential coordinated scheme

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Abstract

Hierarchical algorithms are developed for optimal control of interconnected discrete dynamic large-scale systems with control and state constraints. Synthesis of algorithms based on goal function adaptation in a specially formulated intermediate equivalent optimization problem in three (or two) levels. New algorithms are used iterative parallel-sequential coordination scheme which take in to account information about subsystem states in the calculation coordinated parameters. One feature of this is that fixing state and control prediction trajectories are not common for all subsystems but update for them. This algorithms have shown computational benefits.

Keywords: Optimal control, large-scale interconnected systems, decomposition–coordinated methods.

1 Introduction

The application of traditional optimal control technique to complex large-scale systems gives rise to difficulties of a computational nature.

In recent years much effort has been devoted to hierarchical methods for optimization and control by large-scale systems (M.Jamshidi, 1983), (M.S.Mahmoud and A.S.Fawsy, 1984).

This has been prompted by the complexity of modern manufacturing technologies, distributed structure of automation objects, complication of connections between separate complexes and processes, and also, application in automatic control practice of distributed computing systems.

Decomposition-coordination methods are widely used for solving such problems, since on the one hand models of such large-scale systems consist of connected subsystems and, on the other, those models accord well with the modern tendency towards distributed data processing (M.Singh and A.Titli, 1978).

The important task here is the development of hierarchical computational algorithms with a simple computational structure, requiring less computational memory and allowing control and state bound constraints to be taken into account.

In this paper a method of optimization (B.M.Mirkin, 1986) based on goal function adaptation in a specially formulated intermediate equivalent problem in three (or two) levels is extended to develop novel hierarchical algorithms for solving discrete dynamic optimal control problems of large-scale systems with control and state constraints.

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A large scale system is considered as consisting of a finite number of interconnected subsystems and the hierarchical control consists of an individual controller for each subsystem together with a coordinating function at a higher level.

This approach provides state and control prediction from a higher level, which fixes unseparable components in the performance index and nonlinear interaction components among subsystems by adding corresponding constraints to the equality-type.

We propose solving large-scale non-linear optimization problems technique with parallel-sequential coordination scheme, in particular, we consider the optimal control problem of large-scale systems with control and state inequality constraints.

One feature of this is that fixing state and control prediction trajectories are not common for all subsystems but update for them. Computing property is improving.

Criterion adaptation in the equivalent auxiliary problem is achieved by a special choice of weighting penalty matrices in order to simplify calculation.

It present algorithms for solving optimization problem without control and state bound constraints (two-level solution structure) and with those constraints (three-level solution structure).

2 Model system and statement of the problem

Consider a large-scale dynamic system consisting of a set of M interconnected subsystems, whose structure is assumed to be represented by a model in interconnected form

$$\begin{aligned} x_i[k+1] &= A_i x_i[k] + B_i u_i[k] + \varphi_i(x,u,k), \\ x_i[0] &= x_{i0}, \qquad i = 1,2,...,M, \end{aligned} \tag{1}$$

or in equivalent composite form

$$\begin{aligned} x[k+1] &= A_d x[k] + B_d u[k] + \varphi(x,u,k), \\ x[0] &= x_0. \end{aligned} \tag{2}$$

Here $x_i \in \mathbb{R}_i^n$ and $x \in \mathbb{R}^n$ are the state vectors of the i-th subsystems and the composite system respectively; $u_i \in \mathbb{R}_i^m$ and $u \in \mathbb{R}^m$ are the control of the subsystems and the composite system. $A_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i \in \mathbb{R}^{n_i \times m_i}$ are matrix coefficients characterizing the autonomous subsystems dynamics; $A_d \in \mathbb{R}^{n \times m}$ and $B_d \in \mathbb{R}^{n \times m}$ are block-diagonal matrices with blocks A_i and B_i . Function $\varphi(x,u,k)$ characterizes the nonlinear interconnection between subsystems and meets the necessary conditions of smoothness for the existence, uniqueness and continuity of the optimization problem solution under arbitrary initial conditions.

The state vector and control vector have inequality constraints:

$$x_m \leq x[k] \leq x_M, \qquad u_m \leq u[k] \leq u_M, \tag{3}$$

where x_m, x_M, u_m, u_M- are minimum and maximum allowable levels of state and control signals respectively.

The optimization problem (OP) consists of finding controls which minimize the functional

$$\begin{aligned} 2J &= \| x[N] \|^2_{Q_{1d}} + \sum_{k=0}^{N-1} \| x[k] \|^2_{Q_{1xd}} \\ &+ \sum_{k=0}^{N-1} \| u[k] \|^2_{R_d} \\ &+ \sum_{k=0}^{N-1} \psi(x,u,k). \end{aligned} \tag{4}$$

under restriction (1) and (3). It is assumed that Q_{1d}, Q_{1xd}, R_d are block-diagonal matrices with M blocks Q_{1i}, Q_{1xi}, R_i.

The function $\psi(x,u,k)$ is convex.

3 Parallel-sequential coordinated scheme

To synthesis optimal coordinated decentralised algorithms we will solve the problem (1)-(4) by using hierarchical solution structure.

It's assumed (M.Singh and A.Titli, 1978) that interconnections are fixed by the higher level and used at the lower levels as known time functions:

$$\overline{x}[k] = x[k], \tag{5}$$

$$\overline{u}[k] = u[k]. \tag{6}$$

In this case, the whole system model can be decomposed into a set of small subsystems model and optimization problem OP can be decomposed into a set of independent subproblems by fixing nonseparable part $\psi(x,u,k)$ in criterion and interaction term in the system model.

The inequalities (3) can be converted into the equality

$$\begin{aligned} (x[k]-x_m)^T (x_M - x[k]) &= \alpha, \\ (u[k]-u_m)^T (u_M - u[k]) &= \beta, \end{aligned} \tag{7}$$

following the penalty-function method (M.S.Mahmoud and A.S.Fawsy, 1984).

As a result we have a set of simple dynamic optimization problems and can write the *equivalent intermediate optimization problem* (EOP):

to minimize functional

$$\begin{aligned} 2J &= \| x[N] \|^2_{Q_{1d}} + \| x[N] - \overline{x}[N] \|^2_{Q_{2d}} \\ &+ \sum_{k=0}^{N-1} \| x[k] \|^2_{Q_{1xd}} + \| u[k] \|^2_{R_d} \\ &+ \sum_{k=0}^{N-1} \psi(\overline{x},\overline{u},k) \\ &+ \sum_{k=0}^{N-1} \| x[k] - \overline{x}[k] \|^2_{Q_{2xd}} \\ &+ \sum_{k=0}^{N-1} \| u[k] - \overline{u}[k] \|^2_{Q_{2ud}} \end{aligned} \tag{8}$$

under constraints (5)-(7) and

$$\begin{aligned} x[k+1] &= A_d x[k] + B_d u[k] + \varphi(\overline{x}, \overline{u}, k), \\ x[0] &= x_0. \end{aligned} \tag{9}$$

Here adaptation components (block-diagonal weighted matrices Q_{2d}, Q_{2xd} and Q_{2ud}) are introduced into the criterion so as to simplify computation. At shown below their special choice enables us to obtain different variants of numerically simplification.

Traditionally (M.Singh and A.Titli, 1978) the optimization problem EOP is resolved with the help of an iteration multilevel procedure, during which higher level (coordinator) sends the lower ones *common for all subsystems* coordinating parameters $\overline{x}[k]$, $\overline{u}[k]$. Optimization problems are solved independently at the lower level for each of subsystems. The results of their solutions are sent to the higher level, then coordinating parameters $\overline{x}[k]$, $\overline{u}[k]$ are updated and again sent to the lower ones.

Thus, information exchange between subsystems is absent on each l $-$th iteration. It is easy to notice that subsystem state information will appear during the solution of independent optimization problems. Therefore, using this information may be useful in subsystems, where solution processing is not yet finished (M.Hassan, 1988). Subsystem states information will be sent to the higher level, where coordinated parameters are determinated.

We shall call this hierarchical solution scheme a *parallel-sequential coordinated scheme (PSCS)*.

A feature of this procedure is that coordination parameters $\overline{x}$ and $\overline{u}$ are updated at higher level for each subsystem and EOP should be solved in parallel-sequentially for each subsystem.

Hierarchical solution scheme consists of two levels, the coordinator and M local optimization units. Each local optimization unit is made up of the $i-$th local optimal control problem. The coordinator consists of updating the coordinated parameters and modifying them for each subsystem. Note that each local optimization problem can be solved independently. Thus the structure is suitable for the application of parallel processing methods.

The proposed procedure is more attractive since it takes complete advantage of parallel computation and can give faster convergence.

We apply $PSCS$ to decomposition-coordination optimization method with adaptation of criterion (B.M.Mirkin, 1986).

4 Problem without constraints

Consider the problem OP without constraints (3) in the framework of a two-level solution structure.

To solve this problem, form the Hamiltonian and write the necessary conditions of optimality.

The necessary conditions of optimality can be written as:

$$\begin{aligned} \partial H/\partial \alpha &= 0: \\ x[k] - \overline{x}[k] &= 0, \end{aligned} \tag{10}$$

$$\begin{aligned} \partial H/\partial \beta &= 0: \\ u[k] - \overline{u}[k] &= 0, \end{aligned} \tag{11}$$

$$\begin{aligned} \partial H/\partial \overline{x} &= 0: \\ \hat{\alpha}[k] &= (\partial \psi/\partial \overline{x})^T - Q_{2xd}\,(x[k] - \overline{x}[k]) \\ &\quad -(\partial \varphi/\partial \overline{x})^T \lambda[k+1], \end{aligned} \tag{12}$$

$$\begin{aligned} \partial H/\partial \overline{u} &= 0 : \\ \hat{\beta}[k] &= (\partial\psi/\partial\overline{u})^T - Q_{2ud}\left(u[k] - \overline{u}[k]\right) \\ &\quad -(\partial\varphi/\partial\overline{u})^T \lambda[k+1]. \end{aligned} \tag{13}$$

From the above necessary condition of optimality we can obtain iterative algorithms of the second, highest level:

$$\begin{bmatrix} \alpha[k] \\ \beta[k] \\ \overline{x}[k] \\ \overline{u}[k] \end{bmatrix}^{l+1} = \begin{bmatrix} \hat{\alpha}[k] \\ \hat{\beta}[k] \\ x[k] \\ u[k] \end{bmatrix}^{l}. \tag{14}$$

Equations for the calculation of $x[k]$ and $u[k]$ can be obtained from:

$$\begin{aligned} \partial H/\partial x[k] &= \lambda[k] : \\ \lambda[k] &= A_d^T \lambda[k+1] + Q_{1xd} x[k] \\ &\quad + Q_{2xd}\left(x[k] - \overline{x}[k]\right) \\ &\quad + \alpha[k] + 2\mu[k]x[k] \\ &\quad - \mu[k]\left(x_m + x_M\right), \\ \partial H/\partial\lambda[k+1] &= x[k+1] : \\ x[k+1] &= A_d x[k] + B_d u[k] + \varphi(\overline{x},\overline{u},k), \\ \partial H/\partial u[k] &= 0 : \\ 0 &= R_d u[k] + Q_{2ud}(u[k] - \overline{u}[k]) \\ &\quad + B_d^T \lambda[k+1] + \beta[k]. \end{aligned} \tag{15}$$

The latter relationships with help of the transformation

$$\lambda[k] = P[k]x[k] + f[k]$$

allow us to obtain the equations for finding $u[k]$ and $f[k]$:

$$\begin{aligned} u[k] &= -DB_d' P[k+1] A_d x[k] - DB_d' P[k+1] \varphi(\overline{x},\overline{u},k) - \\ &- \; DB_d' f[k+1] + DQ_{2ud}\overline{u}[k] - D\beta[k]; \end{aligned} \tag{16}$$

$$\begin{aligned} f[k] &= A_d'\Big[I - P[k+1]B_d DB_d'\Big] f[k+1] - Q_{2xd}\overline{x}[k] + \alpha[k] + A_d' P[k+1] \times \\ &\times \; \Big[I - B_d DB_d' P[k+1]\Big]\varphi(\overline{x},\overline{u},k) + A_d' P[k+1] B_d D\Big[Q_{2ud}\overline{u}[k] - \beta[k]\Big]; \end{aligned} \tag{17}$$

$$f[N] = -Q_{2d}\overline{x}[k];$$

$$\begin{aligned} P[k] &= Q_{1xd} + Q_{2xd} + A_d' P[k+1] A_d - A_d' P[k+1] B_d DB_d' P[k+1] A_d; \\ P[N] &= Q_{1d} + Q_{2d}; \end{aligned} \tag{18}$$

$$x[k+1] = A_d x[k] + B_d u[k] + \varphi(\overline{x},\overline{u},k), \quad x[0] = x_0; \tag{19}$$

Relations (16)-(19) will produce lower level algorithms on condition that coordinated parameters $\overline{x}, \alpha, \overline{u}, \beta$ are fixed by the higher level. They will be updated using relationship (14) with the aid of (10)-(13).

The choice of the matrices $Q'_{2xd}[k]$ and $Q'_{2ud}[k]$ influences the speed of convergence of the computing procedure.

Besides, also as in (B.M.Mirkin and M.X.Gandelman, 1984) it can be shown that for computational advantage the special choice of these matrices allows us to solve a set of algebraic and not difference Riccati matrices equations.

It has been shown (B.M.Mirkin and N.M.Lychenko., 1996) that the proposed algorithm is convergent and that the *PSCS* is a rapid one and has fewer number of iterations than in the traditional case.

Two-level solution algorithm

Finally, the *PSCS* solution of optimization problem *OP* without control and state constraints will be the following.

Step 1.

At level 1 (lower level) solve M difference Riccati matrices equations (18), or, for time-invariant subsystems, algebraic Riccati equations. (Calculating $P_i[k]\ \forall\ i$).

Step 2.

Set the iteration index $l = 1$. At level 2 (higher level) start with an initial guess of the trajectories $\overline{x}_1^l[k]$, $\overline{u}_1^l[k]$ $\alpha^l[k]$, $\beta^l\ [k]$ for the first subsystem.

Step 3.

At level 1 using the trajectories supplied by the level 2 $\overline{x}_1^l[k]$, $\overline{u}_1^l[k]$, $\alpha^l[k]$, $\beta^l[k]$, determine $x_1^l[k]$ and $u_1[k]^l$ using (16)–(19).

Send results $x_1^l[k]$ and $u_1^l[k]$ send to level 2.

Step 4. At level 2, determine coordinated parameters for the second subsystem $\overline{x}_2^l[k]$, $\overline{u}_2^l[k]$, $\alpha^l[k]$, $\beta^l[k]$ using (10-13), (14) and information from level 1.

Step 5. At level 1 compute (16-19) for second subsystem using $P_2[k]$, $\overline{x}_2^l[k]$, $\overline{u}_2^l[k]$, $\alpha^l[k]$, $\beta^l[k]$. Send results $x_2^l[k]$, $u_2^l[k]$, send to level 2.

Steps 4 and 5 repeat for the all other subsystems and finish by computing $x_M^l[k]$, $u_M^l[k]$ and sending values to level 2.

Step 6. Set $l = l + 1$. At the level 2 coordinated parameters $\overline{x}_1^{l+1}[k]$, $\overline{u}_1^{l+1}[k]$, $\alpha^l[k]$ and $\beta^l[k]$ update for first subsystem .

Step 7.

Calculate

$$e^{l+1} = \parallel F^{l+1} - F^l \parallel,$$

where $\parallel . \parallel$ - indicates Euclid norm, $(F^l)' = \Big[(\overline{x}^l)'[k]\ (\alpha^l)'[k]\ (\overline{u}^l)'[k]\ (\beta^l)'[k]\Big]$. If $e^l \leq \epsilon$ (ϵ – is very small), stop, else - go to step 3, to new calculate circle.

5 Problem with inequality constraints

In this section *PSCS* is extended to developed hierarchical algorithms for solving a discrete optimal control problem (1) – (4) with state and control constraints.

The main idea of this procedure is to add to the above two-level *PSCS* a separate special intermediate computational level by converting constraints to equality-type ones. Writing the necessary conditions of optimality for the EOP leads to following relations.

$$\begin{aligned}
\overline{x}[k] &= x[k]; \\
\alpha[k] &= -Q_{2xd}(x[k]-\overline{x}[k]) + \frac{\partial\psi(\overline{x},\overline{u},k)}{\partial\overline{x}} + \left[\frac{\partial\varphi(\overline{x},\overline{u},k)}{\partial x}\right]\lambda[k+1]; \\
\overline{u}[k] &= u[k]; \\
\beta[k] &= -Q_{2ud}(u[k]-\overline{u}[k]) + \frac{\partial\psi(\overline{x},\overline{u},k)}{\partial\overline{u}} + \left[\frac{\partial\varphi(\overline{x},\overline{u},k)}{\partial u}\right]\lambda[k+1];
\end{aligned} \tag{20}$$

From the above necessary condition of optimality we can obtain iterative algorithms of the third, higher level:

$$\begin{bmatrix} \alpha[k] \\ \beta[k] \\ \overline{x}[k] \\ \overline{u}[k] \end{bmatrix}^{l+1} = \begin{bmatrix} \hat{\alpha}[k] \\ \hat{\beta}[k] \\ x[k] \\ u[k] \end{bmatrix}^{l}. \tag{21}$$

To find $z[k]$, $\rho[k]$, $\gamma[k]$, $\mu[k]$ we will use, as in (M.S.Mahmoud and A.S.Fawsy, 1984), the following recurrent gradient-type routine:

$$\begin{bmatrix} z[k] \\ \rho[k] \\ \gamma[k] \\ \mu[k] \end{bmatrix}^{s+1} = \begin{bmatrix} z[k] \\ \rho[k] \\ \gamma[k] \\ \mu[k] \end{bmatrix}^{s} +$$

$$\begin{bmatrix} \Delta_1\rho[k]z[k]; \\ \Delta_2(u^T[k]u[k]-(u_m+u_M)^Tu[k] \\ +u_m^Tu_M+z^T[k]z[k]); \\ \Delta_3\mu[k]\gamma[k]; \\ \Delta_4(x^T[k]x[k]-(x_m+x_M)^Tx[k] \\ +x_m^Tx_M+\gamma^T[k]\gamma[k]); \end{bmatrix}^{s} \tag{22}$$

where Δ_1, Δ_2, Δ_3, Δ_4 - are step lengths along the direction of the gradients, s- iteration index.

The right part in (22) was obtained from the following necessary conditions:

$$\begin{aligned}
\partial H/\partial z &= 2\rho[k]z[k], \\
\partial H/\partial\rho &= u^T[k]u[k]-(u_m+u_M)^Tu[k] \\
&\quad +u_m^Tu_M+z^T[k]z[k], \\
\partial H/\partial\gamma &= 2\mu[k]\gamma[k], \\
\partial H/\partial\mu &= x^T[k]x[k]-(x_m+x_M)^Tx[k] \\
&\quad +x_m^Tx_M+\gamma^T[k]\gamma[k].
\end{aligned} \tag{23}$$

Finally, the equations for calculating $x[k+1]$ and $u[k]$ will be:

$$x[k+1] = A_dx[k] + B_du[k] + \varphi(\overline{x},\overline{u},k), \quad x[0] = x_0; \tag{24}$$

$$\begin{aligned}
u[k] &= -D^{-1}B_d^TPA_dx - D^{-1}\big[-Q_{2ud}\overline{u} \\
&\quad +B_d^TP\varphi(\overline{x},\overline{u},k) + B_d^Tf[k+1] \\
&\quad +\beta-\rho(u_m+u_M)\big],
\end{aligned} \tag{25}$$

$$\begin{aligned} P[k] &= Q_{1x} + Q_{2x} + A_d^T P[k+1] A_d + I2\mu[k] \\ &\quad - A_d^T P[k+1] B_d D^{-1} B_d^T P[k+1] A_d, \\ P[N] &= Q_{1d} + Q_{2d}, \end{aligned} \tag{26}$$

$$\begin{aligned} f[k] &= -Q_{2x}\overline{x}[k] + \alpha[k] + A_d^T f[k+1] \\ &\quad -\mu[k](x_m + x_M) \\ &\quad -A_d^T P[k+1] B_d D^{-1} \Big[-Q_{2ud}\overline{u} \\ &\quad + B_d^T P[k+1]\varphi(\overline{x},\overline{u},k) + B_d^T f[k+1] \\ &\quad + \beta[k] - \rho[k](u_m + u_M) \Big] \\ &\quad + A_d^T P[k+1]\varphi(\overline{x},\overline{u},k), \\ f[N] &= -Q_{2d}\overline{x}[N]. \end{aligned} \tag{27}$$

Here

$$D = R_d + Q_{2ud} + 2\rho[k]I + B_d^T P B_d.$$

A structural feature of the derived equations (26) allows us to obtain positive benefits connected with the calculations facility by special adaptation of weighted matrices Q_{2xd} and Q_{2ud} in equivalent optimization problem criterion.

If it assumed that

$$Q_{2xd}[k] = Q'_{2xd}[k] - 2I\mu[k], \tag{28}$$

$$Q_{2ud}[k] = Q'_{2ud}[k] - 2I\rho[k]. \tag{29}$$

Then we can obtain the Riccati equations which are independent of the iteration parameters $\mu[k]$ and $\rho[k]$:

$$\begin{aligned} P[k] &= Q_{1x} + Q'_{2xd}[k] + A_d^T P[k+1] A_d \\ &\quad - A_d^T P[k+1] B_d D_0^{-1} B_d^T P[k+1] A_d, \\ P[N] &= Q_{1d} + Q_{2d}, \end{aligned} \tag{30}$$

where

$$D_0 = R_d + Q'_{2ud}[k] + B_d^T P B_d.$$

Also as in the previous section the special choice of the matrices $Q'_{2xd}[k]$ and $Q'_{2ud}[k]$ influences the speed of convergence of the computing procedure and, also it can be shown that for finding solutions on the lower level it is necessary to solve a set of algebraic but not difference Riccati matrices equations.

A convergence of the given three-level procedure can be shown as in (B.M.Mirkin, 1986).

As a result, the solution (OP) is :

$$u_i[k] = -D_i^{-1} B_i^T P_i x_i[k] + u_{ic}[k]. \tag{31}$$

It is easily seen that the structure of the optimal control of systems with inequality constraints is identical in structure to that obtained in the case without constraints.

It consists of an optimal state feedback for each subsystem of an additional open-loop compensation function $u_{ic}[k]$ that coordinates the interaction of the subsystems within the overall system. The open-loop component is the time function which is determined after finishing the iterative three-level solution procedure.

Three-level solution algorithm

A summary of the algorithmic procedure is:

Step 1.

For each i- th subsystem level 1 solve the family of M dynamic Riccati equations (30), or, for time-invariant subsystems, algebraic Riccati equations.

Step 2.

Set the iteration index $l = 1$. Start with an initial guess for the trajectories $\overline{x}_1^l[k]$, $\overline{u}_1^l[k]$, $\alpha_1^l[k]$, $\beta_1^l[k]$ at level 3 for first subsystem.

Step 3.

Set the iteration index $s = 1$. At level 2 guess initially the trajectories $z[k]$, $\rho[k]$, $\gamma[k]$, $\mu[k]$.

Step 4.

Define matrices $Q_{2xd}[k]$ and $Q_{2ud}[k]$ from (28) and (29) using the trajectories supplied by the higher levels. Then compute $x[k]$ and $u[k]$ from (24) – (27) and send to level 2.

Step 5.

The iteration number $s = s + 1$. At level 2 compute the new trajectories $z^{s+1}[k]$, $\rho^{s+1}[k]$, $\gamma^{s+1}[k]$, $\mu^{s+1}[k]$ using (22). If $\| h^{s+1} - h^s \| \leq \epsilon$, go to step 6, otherwise go to step 4 with new h^{s+1}, $(h = \left[z^T \rho^T \gamma^T \mu^T\right]^T)$.

Step 6.

At level 3 update the vector of coordinated parameters $\overline{x}_i^l[k]$, $\overline{u}_i^l[k]$, $\alpha_i^l[k]$, $\beta_i^l[k]$ for next i-th subsystem from (21).

Steps 3–6 repeated $\forall i \in [1, M]$ and finished by computing $x_M^l[k]$ and $u_M^l[k]$.

Step 7.

The iteration index $l = l + 1$. If $\| f^{l+1} - f^l \| \leq \epsilon'$, stop, else go to step 3 with new prediction vector f_1^{l+1} $(f_1^l = \left[\overline{x}_1^l[k]\ \alpha_1^l[k]\ \overline{u}_1^l[k]\ \beta_1^l[k]\right]')$.

6 Conclusion

In the present work hierarchical algorithms with parallel-sequential coordination scheme are proposed for optimal control of interconnected discrete dynamic large-scale systems with control and state constraints.

Synthesis of algorithms based on goal function adaptation in a specially formulated intermediate equivalent optimization problem in three (or two) levels.

The iterative solving parallel-sequential coordination scheme takes in to account that coordinated parameters used in computation are different for each subsystem. An important aspect of this procedure is a smaller number of iterations.

This scheme applied to the synthesis of algorithms for the optimal control of large-scale non-linear dynamic systems with and without control and state inequality constraints.

A special choice of weighting matrices in the equivalent optimization problem enables us to obtain a Riccati equation of the standard kind, or, in other words, to calculate feedback coefficients, independent of the iteration parameters of the intermediate level. As a result the control law has two components: feedback and a coordinating element.

Proposed algorithms have shown computational benefits.

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