Every Stabilizing Dead-Time Controller has an Observer-Predictor-Based Structure

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Abstract

This paper considers the stabilization problem for systems with a single delay h in the feedback loop. The state-space parametrizations of all stabilizing regulators are derived. These parametrizations have simple structures and clear interpretations. In particular, it is shown that every stabilizing controller consists of a delayed state observer, an h time units ahead predictor and a stabilizing state feedback. Some applications of the proposed parametrization are discussed.

1 Introduction and problem formulation

Dead-time systems are the systems with a time delay between control inputs and measured outputs (Palmor, 1996). Such time delays appear frequently in industrial processes, economical and biological systems. Moreover, dead-time systems may serve as an alternative of high-order or infinite-dimensional models for describing complicated physical phenomena (Zwart and Bontsema, 1997).

In this paper we are concerned with the (internal) stability of dead-time systems in a general linear fractional transformation (LFT) framework. To describe the problem, consider first a general LFT in Fig. 1(a), where $\tilde{P}$ and $\tilde{K}$ are LTI generalized plant and controller, respectively. Recall (Green and Limebeer, 1995) that the LFT in Fig. 1(a) is said to be *internally stable* if the nine transfer matrices mapping $\tilde{w}$, $\tilde{v}_1$, and $\tilde{v}_2$ to $\tilde{z}$, $\tilde{y}$, and $\tilde{u}$ are stable, that is they belong to H^∞. In the case when $\tilde{P}(s)$ is rational (i.e., finite dimensional) the stability problem for the setup in Fig. 1(a) is well studied, see, e.g., (Green and Limebeer, 1995; Zhou *et al.*, 1995). Namely, the necessary and sufficient conditions for the stabilizability exist and the set of all $\tilde{K}$ which internally stabilize the system is parametrized (the so-called *Youla parametrization*). The results are particularly elegant in the state-space setting, where the Youla parametrization has a nice interpretation in terms of the observer-based structure.

A general dead-time system in the LFT setup is shown in Fig. 1(b), where P is a rational part of the generalized plant with the transfer matrix

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right], \tag{1}$$

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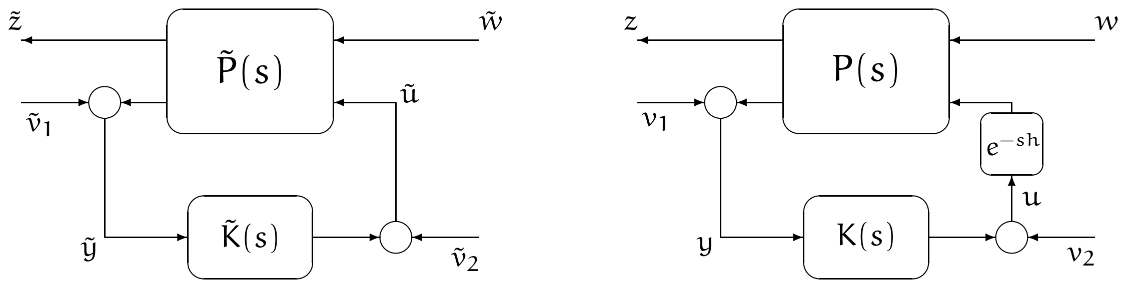

(a) General setup

(b) Dead-time setup

Figure 1: LFT stability setup

the controller K is supposed to be *proper*, and e^{-sh} is the delay element. The following stability problem is dealt with in the paper:

SP: Given the dead-time setup in Fig. 1(b) with the rational part of the generalized plant (1), find conditions when the closed-loop system is internally stable and characterize all proper controllers stabilizing the system.

Note, that the system in Fig. 1(b) belongs to the class of infinite-dimensional systems. The stability of infinite-dimensional systems can in principle be analyzed using the coprime factorization approach (Vidyasagar, 1985), much in parallel to the analysis of the finite-dimensional systems. The factorization approach, however, appears to be too general for the system in Fig. 1(b). Consequently, the parametrization of all stabilizing controllers resulting from the factorization approach destroys simple and physically meaningful structure of the problem. It is therefore of interest to treat SP using more problem-oriented approaches which exploit the structure of the system to a maximum extent.

2 Parametrization

To solve SP we need a preliminary result, which establishes that the stability problem for the dead-time setup in Fig. 1(b) can be reduced to an equivalent problem for the setup in Fig. 1(a) with *rational* (i.e., delay-free) $\tilde{P}$. To this end, define the following auxiliary system:

$$\tilde{P}(s) = \begin{bmatrix} \tilde{P}_{11}(s) & P_{12}(s) \\ P_{21}(s) & \tilde{P}_{22}(s) \end{bmatrix},$$

where $\tilde{P}_{11}(s)$ and $\tilde{P}_{22}(s)$ are some rational proper LTI systems and the non-diagonal blocks are the same as for P. The lemma below, the first part of which is just an extension of (Curtain and Zhou, 1996, Lemma 2.2) and (Zwart and Bontsema, 1997, Lemma 3.11), plays a key role in the discussion to follow.

Lemma 1. *Let $\tilde{P}_{11}$ and $\tilde{P}_{22}$ be such that the transfer matrices $\Xi_{11}(s) \doteq P_{11}(s) - e^{-sh}\tilde{P}_{11}(s)$ and $\Xi_{22}(s) \doteq \tilde{P}_{22}(s) - e^{-sh}P_{22}(s)$ are stable. Then*

$$\tilde{K}(s) \doteq \big(I - K(s)\Xi_{22}(s)\big)^{-1}K(s) \tag{2}$$

is proper iff so is $K(s)$ and K internally stabilizes the system in Fig. 1(b) iff $\tilde{K}$ internally stabilizes the system in Fig. 1(a). Furthermore, for any K the following equality holds:

$$\mathcal{F}_\ell\big(P(s), e^{-sh}K(s)\big) = \Xi_{11}(s) + e^{-sh}\mathcal{F}_\ell\big(\tilde{P}(s), \tilde{K}(s)\big). \tag{3}$$

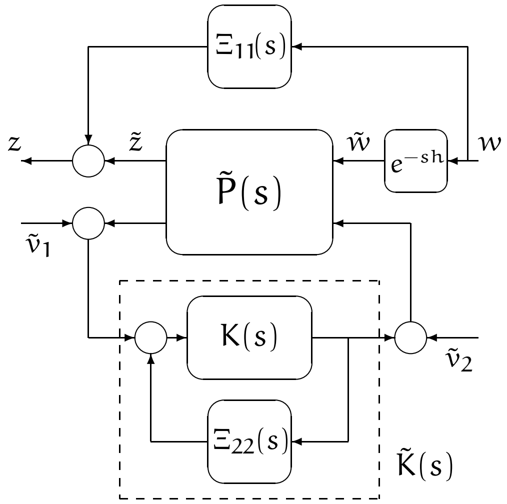

Figure 2: Equivalent setup

Proof. Consider the system in Fig. 2. Using standard manipulations with block-diagrams one can show that this system is equivalent to that in Fig. 1(b), where $\tilde{v}_1 = v_1 + \Xi_{22} v_2$ and $\tilde{v}_2 = v_2$. Thus, the connection between the input and output signals in Fig. 1(a) and 1(b) is as follows:

$$\begin{bmatrix} \tilde{w} \\ \tilde{v}_1 \\ \tilde{v}_2 \end{bmatrix} = \begin{bmatrix} e^{-sh} & 0 & 0 \\ 0 & I & \Xi_{22} \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} w \\ v_1 \\ v_2 \end{bmatrix}$$

and

$$\begin{bmatrix} \tilde{z} \\ \tilde{u} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & -\Xi_{22} \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} z \\ u \\ y \end{bmatrix} + \begin{bmatrix} -\Xi_{11} & 0 & 0 \\ 0 & 0 & \Xi_{22} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ v_1 \\ v_2 \end{bmatrix},$$

from which the equivalence between the stability of these systems follows immediately. Since $P_{22}(s)$ is strictly proper so is $\Xi_{22}(s)$ and then the inverse in (2) is well defined for any proper $K(s)$ and is proper, but not strictly proper. The properness of $\tilde{K}(s)$ in (2) is then equivalent to the properness of $K(s)$. Finally, equality (3) follows directly from Fig. 1. $\square$

Remark 2.1. Note, that if $\mathcal{P}_{22}$ is stable, then in Lemma 1 one can simply chose $\tilde{\mathcal{P}}_{22} = \mathcal{P}_{22}$. In this case, the modified controller $\tilde{\mathcal{K}}$ is just the famous Smith predictor (Smith, 1957) (see also (Palmor, 1996) where numerous generalizations are discussed) for $\mathcal{P}_{22}$. Actually, the rationale behind the proof of Lemma 1 is the same as in the Smith predictor case: to draw the delay out of the feedback loop. Thus, in general case $\tilde{\mathcal{K}}$ can be thought of as a generalized Smith predictor.

As seen from Lemma 1, the stabilizability of the dead-time setup in Fig. 1(b) can be reduced to the stabilizability of the standard finite-dimensional system in Fig. 1(a). The latter problem, in turn, can be solved using standard approaches. Thus, the SP can be reduced to a simpler problem for which the solution is known. The central step in this direction is to find causal $\tilde{P}_{11}$ and $\tilde{P}_{22}$ such that Ξ_{11} and Ξ_{22} are stable.

Consider the following candidates for $\tilde{P}_{11}$ and $\tilde{P}_{22}$:

$$\tilde{P}_{11}(s) = \left[\begin{array}{c|c} A & e^{Ah}B_1 \\ \hline C_1 & 0 \end{array}\right], \tag{4a}$$

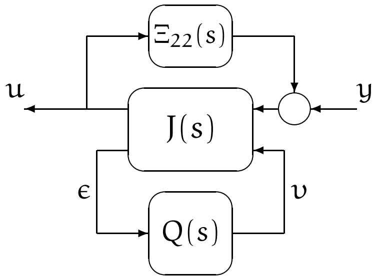

Figure 3: Youla parametrization for the dead-time setup

(its impulse response is a "truncation" of the impulse response of $e^{sh}P_{11}$ to $\mathbb{R}^+$) and

$$\tilde{P}_{22}(s) = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 e^{-Ah} & 0 \end{array}\right], \tag{4b}$$

(its impulse response is a continuation of the impulse response of $e^{-sh}P_{22}$ to the whole semi-axis $\mathbb{R}^+$). It can be verified that in this case the transfer matrices

$$\Xi_{11}(s) = D_{11} + C_1(sI - A)^{-1}(I - e^{-(sI-A)h})B_1 \tag{5a}$$

and

$$\Xi_{22}(s) = C_2 e^{-Ah}(I - e^{-(sI-A)h})(sI - A)^{-1}B_2 \tag{5b}$$

are entire functions of s (actually, they have finite impulse responses with the support in $[0, h]$) and thus both belong to H^∞. Hence, the transfer matrices in (4) satisfy the assumption of Lemma 1. The auxiliary generalized plant has then the following transfer matrix:

$$\tilde{P}(s) = \left[\begin{array}{c|cc} A & e^{Ah}B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 e^{-Ah} & D_{21} & 0 \end{array}\right]. \tag{6}$$

Now, the finite dimensional (6) is stabilizable by an output feedback controller iff the pair (A, B_2) is stabilizable and the pair $(C_2 e^{-Ah}, A)$ is detectable. It is clear that the latter condition is equivalent to the detectability of the pair (C_2, A). To solve SP one therefore needs to parametrize the set of all stabilizing controllers for plant (6) and then feed the control signal back through the gain Ξ_{22}. This leads to the following theorem:

Theorem 1. *There exists a proper* $K(s)$ *achieving internal stability of the setup in Fig. 1(b) iff* (A, B_2) *is stabilizable and* (C_2, A) *is detectable. Further, let* F *and* L *be such that* $A + B_2F$ *and* $A + LC_2$ *are Hurwitz. Then all controllers solving* SP *can be parametrized as the transfer matrix from* y *to* u *in Fig. 3, where*

$$J(s) = \left[\begin{array}{c|cc} A + B_2F + e^{Ah}LC_2e^{-Ah} & -e^{Ah}L & B_2 \\ \hline F & 0 & I \\ -C_2e^{-Ah} & I & 0 \end{array}\right]$$

and

$$\Xi_{22}(s) = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 e^{-Ah} & 0 \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & 0 \end{array}\right]$$

and with any $Q(s) \in H^\infty$. Furthermore, the set of all closed-loop transfer matrices from w to z achievable by an internally stabilizing controller is

$$\mathcal{F}_\ell\big(P(s), e^{-sh}K(s)\big) = \Xi_{11}(s) + e^{-sh}\big(T_{11}(s) + T_{12}(s)Q(s)T_{21}(s)\big), \tag{7}$$

where

$$\begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & 0 \end{bmatrix} = \left[\begin{array}{cc|cc} A + B_2F & -B_2F & e^{Ah}B_1 & B_2 \\ 0 & A + e^{Ah}LC_2e^{-Ah} & e^{Ah}(B_1 + LD_{21}) & 0 \\ \hline C_1 + D_{12}F & -D_{12}F & 0 & D_{12} \\ 0 & C_2e^{-Ah} & D_{21} & 0 \end{array}\right]$$

and

$$\Xi_{11}(s) = \left[\begin{array}{c|c} A & B_1 \\ \hline C_1 & D_{11} \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & e^{Ah}B_1 \\ \hline C_1 & 0 \end{array}\right].$$

Proof. The result follows by application Theorems 12.8 and 12.16 in (Zhou *et al.*, 1995) to (6). ☐

Some remarks are in order:

Remark 2.2. The generator of all stabilizing controllers J and the generators of all closed-loop maps T_{ij} have the same structure and are of the same dimensions as the corresponding transfer matrices in the delay-free case. Moreover, since $\Xi_{11}(s)$ is an entire function and $A+e^{Ah}LC_2e^{-Ah}$ is similar to $A + LC_2$ the closed-loop poles in the dead-time setup do *not* depend on the delay h.

Remark 2.3. As in the Smith predictor case, the internal feedback through Ξ_{22} is added to predict the output of P_{22}. Unlike the Smith predictor, which has infinite impulse response and thus is sensitive to the instability in the plant, Ξ_{22} is an FIR system. The latter guarantees the stability of the internal feedback (see the discussion in (Palmor, 1996, §10.10)). In fact, Ξ_{22} is the minimum variance h-unit predictor for P_{22}.

The presence of the predictor in the feedback loop is intriguing. Indeed, since the feedback loop contains the delay one would expect that a stabilizing controller attempts somehow to "predict" the process behavior h time units ahead. Yet the role of the predictor Ξ_{22} in Theorem 1 is not evident. The reasoning above suggests that there might be a different form of the parametrization in Theorem 1, which shows clearly the prediction nature of the controller.

The goal of the rest of this section is to show that any stabilizing controller for the dead-time process in Fig. 1(b) does attempt to predict its state vector. To this end, let us move the delay e^{-sh} to the measurement y (this can always be done since K is LTI), that is assume that only $y(t-h)$ is available for the controller at time t. Let x_J be the state vector of J and define the following function:

$$\eta(t) \doteq e^{-Ah}\left(x_J(t) - \int_{t-h}^{t} e^{A(t-\tau)}B_2u(\tau)d\tau\right).$$

Denote also $y_f \doteq \Xi_{22}u$. Then using the fact that $y_f(t) = C_2e^{-Ah}\int_{t-h}^{t} e^{A(t-\tau)}B_2u(\tau)d\tau$ one gets:

$$\begin{aligned}\dot\eta(t) &= e^{-Ah}\big(A + B_2F + e^{Ah}LC_2e^{-Ah}\big)x_J(t) - L\big(y(t-h) + y_f(t)\big) + e^{-Ah}B_2v(t) \\ &\qquad - e^{-Ah}A\int_{t-h}^{t} e^{A(t-\tau)}B_2u(\tau)d\tau - e^{-Ah}B_2u(t) + B_2u(t-h) \\ &= A\eta(t) + B_2u(t-h) - L\big(y(t-h) + y_f(t) - C_2e^{-Ah}x_J(t)\big) \\ &= A\eta(t) + B_2u(t-h) - L\big(y(t-h) - C_2\eta(t)\big).\end{aligned}$$

The equation above is actually the observer for the *delayed* plant state $x(t-h)$. Proceeding further, one gets:

$$\begin{aligned}\epsilon(t) &= -C_2 e^{-Ah} x_J(t) + y(t-h) + y_f(t) \\ &= -C_2 \eta(t) + y(t-h),\end{aligned}$$

that is ϵ is the innovation for the delayed observation. Finally, the state vector of J, which is

$$x_J(t) = e^{Ah}\eta(t) + \int_{t-h}^{t} e^{A(t-\tau)} B_2 u(\tau) d\tau,$$

is clearly just the h-*units predictor* for η. We thus have the following result:

Theorem 2. *All controllers of the form $e^{-sh}K(s)$ internally stabilizing the dead-time system in Fig. 1(b) have the following observer-predictor-based (O&P) form:*

$$\begin{aligned}\dot{x}_o(t) &= Ax_o(t) + B_2 u(t-h) - L\big(y(t-h) - C_2 x_o(t)\big) && \textit{(observer)} \\ x_p(t) &= e^{Ah} x_o(t) + \int_{t-h}^{t} e^{A(t-\tau)} B_2 u(\tau) d\tau && \textit{(predictor)} \\ u(t) &= Fx_p(t) + \upsilon(t),\end{aligned}$$

where $\upsilon = Q\epsilon$, where $\epsilon(t) = y(t-h) - C_2 x_o(t)$ is the innovation and $Q(s) \in H^\infty$ but otherwise is arbitrary.

Theorem 2 shows that every stabilizing delayed controller is a combination of a stable observer of the delayed plant state, an h-ahead predictor of the observer state, and a stabilizing state (prediction) feedback plus a weighted (by a stable weight) observation error. This structure is similar to the standard observer-based structure of the set of all stabilizing controllers for systems without dead-time. The difference is caused by the fact that controller has delayed information about the process and thus only the delayed process state can be observed. Hence, in order to implement a state feedback law, the controller attempts to "compensate" the lack of information by the use of a prediction.

Note, that when $Q = 0$ the controller in Theorem 2 is actually the controller proposed by Furukawa and Shimemura (1983). Such a structure in (Furukawa and Shimemura, 1983) was postulated and then the closed-loop stability was proved. It therefore seemed that the O&P-based structure is just one possible choice among others and the question of whether this choice is justifiable or not was still open (cf. (Palmor, 1996, §10.9.3)). Theorem 2 shows clearly that the O&P-based controller structure is inherent in the context of the dead-time control, much like the observer-based one is in the delay-free case.

3 Applications

In this section two application of the results presented in the previous section are outlined. In §3.1 the H^2 optimization problem for the dead-time systems is treated, and in §3.2 the robust stability against additive or multiplicative uncertainties in the rational part of the dead-time system is studied.

3.1 H^2 optimization

The solution to the H^2 (LQG) problem for the dead-time systems has been known for at least 20 years (Kleinman, 1969). Thus, our goal is this subsection is not to present new results, but rather to show that the parametrization of the closed-loop transfer matrices in Theorem 1 enables one to obtain the solution using remarkably simple arguments. Moreover, we shall explicitly characterize the deterioration (with respect to the delay-free case) of the H^2 performance due to the delay in the loop.

First, to guarantee the boundedness of the H^2 norm assume that $D_{11} = 0$. Consider the closed-loop transfer matrix (7). This parametrization has an important property that the two terms in its right-hand side are *orthogonal* in H^2. Indeed, the first term has the impulse response with support in the interval $[0, h]$, whereas the second—in $[h, \infty)$. Consequently, for every stabilizing controller one can write:

$$\begin{aligned}\|\mathcal{F}_\ell(P, e^{-sh}K)\|^2_{H^2} &= \|\Xi_{11}\|^2_{H^2} + \|e^{-sh}\mathcal{F}_\ell(\tilde{P}, \tilde{K})\|^2_{H^2} \\ &= \|\Xi_{11}\|^2_{H^2} + \|\mathcal{F}_\ell(\tilde{P}, \tilde{K})\|^2_{H^2},\end{aligned}$$

where $\tilde{P}$ and $\tilde{K}$ are defined by (4) and (2), respectively. Since Ξ_{11} does not depend on the controller, the H^2 optimization for the dead-time system is reduced to that for the finite-dimensional plant $\tilde{P}$. The latter, in turn, can be solved using standard methods (Green and Limebeer, 1995; Zhou *et al.*, 1995).

The reasoning above yields in principle a complete algorithm of calculating both the optimal controller and the optimal cost. The latter will consist of three components: the norm of Ξ_{11} and components related to the state feedback and the Kalman filtering. Taking into account (6), one can see that all these three components depend on the delay h. This conforms well with the results of Kleinman (1969). On the other hand, the O&P-based structure of the controller in Theorem 2 has an interesting property: there only the predictor depends on h, whereas the parameters of the observer and the state feedback do not. This suggests that the optimal H^2 cost can also be decomposed into two parts, one of which is a function of h and another one depends only on the rational part P of the plant. The goal of the rest of this subsection is to show that this is indeed true.

To clarify the derivation assume without loss of generality (Zhou *et al.*, 1995) that

$$D_{12}'\begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \quad \text{and} \quad D_{21}\begin{bmatrix} B_1' & D_{21}' \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}.$$

Then, the solution of the H^2 problem for the plant $\tilde{P}$ requires the following two algebraic Riccati equations (ARE's):

$$\begin{aligned}&X_hA + A'X_h + C_1'C_1 - F_h'F_h = 0, && \text{where } F_h \doteq -B_2'X_h, \\ &AY_h + Y_hA' + e^{Ah}B_1B_1'e^{A'h} - L_hL_h' = 0, && \text{where } L_h \doteq -Y_he^{-A'h}C_2'.\end{aligned}$$

The subscript in X_h and Y_h emphasizes the dependence of $\tilde{P}$ on h (thus, the delay-free case corresponds to X_0 and Y_0, respectively). Note, that $X_h = X_0$ and $Y_h = e^{Ah}Y_0e^{A'h}$. The optimal H^2 performance for the generalized plant $\tilde{P}$ is then (Green and Limebeer, 1995, §5.4.2):

$$\begin{aligned}\mathcal{J}_h^2 &= \operatorname{tr}\big(e^{Ah}B_1B_1'e^{A'h}X_h + F_h'F_hY_h\big) \\ &= \operatorname{tr}\big(B_1B_1'X_0 + F_0'F_0Y_0\big) + \operatorname{tr}\big(B_1B_1'(e^{A'h}X_0e^{Ah} - X_0) + F_0'F_0(e^{Ah}Y_0e^{A'h} - Y_0)\big).\end{aligned}$$

The first term in the last expression is just the optimal H^2 performance in the delay-free case, $\mathcal{J}_0^2$. To handle the second term the following well known result (see, e.g., (Rugh, 1996, Ex. 7.12)) is required:

Claim 1. *Let* M *be the solution of the Lyapunov equation* $MA_\alpha + A'_\alpha M = Q_\alpha$. *Then*

$$e^{A'_\alpha h} M e^{A_\alpha h} - M = \int_0^h e^{A'_\alpha t} Q_\alpha e^{A_\alpha t} dt.$$

It is readily seen that X_0 satisfies the Lyapunov equation in Claim 1 subject to $A_\alpha = A$ and $Q_\alpha = F'_0 F_0 - C'_1 C_1$, while Y_0—subject to $A_\alpha = A'$ and $Q_\alpha = L_0 L'_0 - B_1 B'_1$. Then,

$$\mathcal{J}_h^2 = \mathcal{J}_0^2 + \operatorname{tr}\Big(F_0 \int_0^h e^{At} L_0 L'_0 e^{A't} dt\, F'_0\Big) - \operatorname{tr}\Big(B'_1 \int_0^h e^{A't} C'_1 C_1 e^{At} dt\, B_1\Big).$$

Since the last term in the right-hand side above is exactly $\|\Xi_{11}\|_{H^2}^2$, we get the following lemma:

Lemma 2. *The optimal* H^2 *performance achievable in the dead-time system in Fig. 1(b) is*

$$\mathcal{J}_{\mathrm{opt}}^2 = \mathcal{J}_0^2 + \int_0^h \operatorname{tr}\big(F_0 e^{At} L_0 L'_0 e^{A't} F'_0\big) dt,$$

where $\mathcal{J}_0^2$, F_0, *and* L_0 *are the optimal cost, the optimal state-feedback gain, and the Kalman filter gain, respectively, in the delay-free* ($h = 0$) *case.*

The quantity $\int_0^h \operatorname{tr}\big(F_0 e^{At} L_0 L'_0 e^{A't} F'_0\big) dt$ can thus be thought of as the price of delay in the H^2 control.

3.2 Robust stability

Robustness is one of the most important characteristics of any control system. Special care to the robustness properties should be taken for prediction-based control schemes, since the prediction is inherently an open-loop process. The Smith predictor, for instance, might become *practically unstable* if designed without taking into account modeling uncertainties (Palmor, 1996).

On the other hand, Zwart and Bontsema (1997, §3.3) pointed out that if the nominal plant P_0 is stable, then the Smith predictor $\frac{K}{1-KP_0(1-e^{-sh})}$ guarantees the same robustness margin (against additive uncertainties in P_0) as its generator K in the delay-free case. Below we shall show that this property holds in the case of multiplicative uncertainties as well.

The result is actually a straightforward consequence of parametrization (7). To see this, consider the following three perturbed systems, corresponding to the cases of additive, input multiplicative and output multiplicative uncertainties, respectively:

$$\begin{aligned} P_{\Delta,a}(s) &= \big(P_0(s) + W_1(s)\Delta(s)W_2(s)\big)\, e^{-sh}, \\ P_{\Delta,im}(s) &= P_0(s)\big(I + W_1(s)\Delta(s)W_2(s)\big)\, e^{-sh}, \end{aligned}$$

and

$$P_{\Delta,om}(s) = \big(I + W_1(s)\Delta(s)W_2(s)\big) P_0(s)\, e^{-sh},$$

where the nominal plant P_0 is assumed to be *stable*, the LTI uncertainty Δ is assumed to belong to the open unit ball in H^∞ but otherwise is arbitrary and W_1 and W_2 are stable weighting transfer functions reflecting *a-priory* information about the uncertainty. It is well known (Zhou *et al.*, 1995) that a perturbed system is stabilized by a controller K for all $\Delta \in BH^\infty$ iff the H^∞

norm of the closed-loop system from w to z in Fig. 1(b) (for a specially constructed rational part of the generalized plant P) is smaller than 1. For the three types of systems $P_{\Delta,\bullet}$ described above the generalized plant is to be chosen as follows:

$$P_a(s) = \begin{bmatrix} 0 & W_1(s) \\ W_2(s) & P_0(s) \end{bmatrix},$$
$$P_{im}(s) = \begin{bmatrix} 0 & W_1(s) \\ P_0(s)W_2(s) & P_0(s) \end{bmatrix},$$

and

$$P_{om}(s) = \begin{bmatrix} 0 & W_1(s)P_0(s) \\ W_2(s) & P_0(s) \end{bmatrix}.$$

The important common property of these transfer matrices is that their "P_{11}" blocks are zero. Consequently, Ξ_{11} in Theorem 1 is zero. Furthermore, since the systems above are stable, one can trivially chose $F = 0$ and $L = 0$ in Theorem 1. One can easily verify that in this case $T_{11} = 0$, $T_{12} = P_{12}$, and $T_{21} = P_{21}$ and hence the closed loop transfer matrix from w to z becomes

$$\mathcal{F}_\ell\big(P(s), e^{-sh}K(s)\big) = e^{-sh}\, P_{12}QP_{21} = e^{-sh}\,\mathcal{F}_\ell\big(P(s), \tilde{K}(s)\big),$$

where $\tilde{K}$ is defined by (2). Since e^{-sh} is inner, we finally get:

$$\|\mathcal{F}_\ell\big(P(s), e^{-sh}K(s)\big)\|_{H^\infty} = \|\mathcal{F}_\ell\big(P(s), \tilde{K}(s)\big)\|_{H^\infty}.$$

The equality above says actually that any H^∞ norm of the transfer matrix from w to z achievable for the delay-free system can also be achieved for the dead-time system by a predictor-based controller and this fact does *not* depend on the delay h. In the context of the robust stabilization this implies that if the nominal plant is stable, then any robustness margin against both additive and multiplicative perturbations in the rational part of the plant achievable for the system without the delay can also be achieved in the presence of a delay.

Note, that above we considered the case of *unstructured* uncertainties only. The result, however, is also true for any *structured* LTI Δ's that fit into the "complex μ" framework. This follows from the fact that the inner e^{-sh} is scalar and thus does *not* affect the lower bound of μ, which, in turn, is equal to μ itself (Zhou *et al.*, 1995, Theorem 11.4).

Acknowledgment

The first author would like to thank Dr. Héctor Rotstein for sharing a part of his expertise in μ.

References

Curtain, R. F. and Y. Zhou (1996). "A weighted mixed-sensitivity H^∞-control design for irrational transfer matrices," *IEEE Trans. Automat. Control*, **41**, no. 9, pp. 1312–1321.

Furukawa, T. and E. Shimemura (1983). "Predictive control for systems with time delay," *Int. J. Control*, **37**, no. 2, pp. 399–412.

Green, M. and D. J. N. Limebeer (1995). *Linear Robust Control*, Prentice-Hall, Englewood Cliffs, NJ.

Kleinman, D. L. (1969). "Optimal control of linear systems with time-delay and observation noise," *IEEE Trans. Automat. Control*, **14**, no. 5, pp. 524–527.

Palmor, Z. J. (1996). "Time-delay compensation — Smith predictor and its modifications," in *The Control Handbook* (S. Levine, ed.), CRC Press, pp. 224–237.

Rugh, W. J. (1996). *Linear System Theory*, Prentice-Hall, Upper Saddle River, NJ, 2nd edn.

Smith, O. J. M. (1957). "Closer control of loops with dead time," *Chem. Eng. Progress*, **53**, no. 5, pp. 217–219.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*, The MIT Press, Cambridge, MA.

Zhou, K., J. C. Doyle, and K. Glover (1995). *Robust and Optimal Control*, Prentice-Hall, Englewood Cliffs, NJ.

Zwart, H. and J. Bontsema (1997). "An application driven guide throught infinite-dimensional systems theory," in *Plenary Lectures and Mini-Courses, ECC'97* (G. Bastin and M. Gevers, eds.), pp. 289–328.